Didactic Incidents: A Way to Improve the Professional Development of Mathematics Teachers

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Abstract In this chapter the professional development of teachers is observed through the joint work of researchers and teachers. In the particular context of the European project EdUmatics, which focuses on mathematics education in a computer environment, the collaboration between researchers and teachers has helped both to build innovative situations and also to better understand the difficulties involved in the introduction of technology in classrooms. The theoretical framework of the theory of didactic situations, didactic incidents and documentational genesis allows the construction of analyses in order to better understand the students' and teacher's joint action and so to enhance teachers' professional development. We highlight both the consistency of the framework and the contributions of our findings to the professional development of teachers.

Keywords Didactics incidents • Documentational genesis • Milieu • Theory of didactic situations

Introduction

The EdUmatics project[1] was a place of multiple collaborations: collaboration between researchers; collaboration between researchers and teachers; and collaboration between teams of different European countries. At the beginning of the project these collaborations could not be taken for granted and their achievement has depended on a set of local and global conditions. One of the most important challenges was to

[1] 50324-UK-2009-COMENIUS-CMP; European Development for the Use of Mathematics Technology in Classrooms, http://www.edumatics.eu.

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take into account the professional development of teachers involved in the project. As full partners of the project, the schools play an important role in the development of the EdUmatics resources, and the teachers not only experimented in their classrooms with new and original lessons, but also participated to the global construction of an in-service on-line course for others. The particular context of technology added a complexity even if most of the teachers involved in the project were, from the beginning, highly experimental teachers.

In this chapter, I would like to emphasise the relationship between professional development and analysis of classroom situations. To this end, I will present the frameworks of didactic incidents and perturbations, which describe and help understand the dynamics of the relationship between teaching and learning in a perspective of documentary genesis of teacher and students.

Theoretical Framework to Approach the Complexity

The starting point of the EdUmatics project resided in the premise that '*recent studies in Mathematics Education show that, despite many national and institutional actions within the EU aiming to integrate ICT into mathematics classrooms, such integration in secondary schools remains weak.*' Research has shown that beyond some contextual problems (computer availability, technical difficulties…), the professional activity of teachers who integrate technology in their lessons is complex, both in terms of internal reasons (linked to the mathematical and technological knowledge, to the conceptions of mathematics as well as of teaching mathematics) and external ones (institutional, social or material constraints) (Rodd and Monaghan 2002; Lagrange and Degleodu 2009). In order to understand and describe this complexity and to facilitate the dissemination of professional skills leading to integration of technology into mathematics classes, the two theoretical approaches of the Theory of Didactic Situations (TDS) and of documentational genesis appeared to be appropriate. The first, through the concepts of milieu and of didactic incidents, makes it possible to take both the point of view of teachers and students in a given situation, from the design of the situation to its implementation in the class. The second considers the technology, not only as an artefact tending to become an instrument, but also more widely as a resource tending to become a document.

The Concept of Milieu

The Theory of Didactic Situations (Brousseau 1986, 2004) provides powerful tools to describe the dynamics of the interactions between teacher and students in the classroom. This theory develops a model of teaching and learning of mathematics through the description of a 'game' where teachers and students win when students learn, that is to say, when students modify their knowledge. The game must lead to new knowledge that replaces or completes a previous knowledge, and the game must encompass all the possibilities of the teaching situation. Obviously, speaking of game involves speaking of players, of playground and of rules; the players are both the teachers and the students, with different roles. The rules are modelled by the *didactic contract*, that is to say the part of the relationship between students and teachers concerning the knowledge and the responsibility of each of them in the construction of this knowledge. In a Piagetian perspective, knowledge is built in a process of adaptation and equilibration in response to environmental constraints. The environment, the playground in which (and against which) the players play is called the *milieu*; the *milieu* is thought of, designed, organised and observed by the teacher and when students play the game, the *milieu* responds to the students' actions. The didactic situation can initially be defined as the description of the interactions between players in a particular playground. The model gives the situation a central role and, obviously, the *milieu* is an important part of the success of the game. But the previous definition of situation is not sufficient to describe the activity of the actors and their positions within the milieu. Different paradoxes become apparent: a student can develop knowledge in some situations without knowing that this knowledge is socially shared; the role of the teacher is then to recognise and to institutionalise this knowledge (in a phase of institutionalisation) and this important and often neglected part of the situation will be revisited later in the chapter. A second paradox of teaching situations is that all didactic systems possess the project of their extinction, the built knowledge having to be used outside the interactions with the teacher in the particular institution of the school. To this end, *a-didactic* situations are, in a sense, a model in a didactic situation of the real interactions between subject and environment. In such a situation, students are face to face with the milieu and act on it in a situation of action, formulating the knowledge in a situation of formulation and building relationships between mathematical objects in a phase of validation. Defined for the first time as "*the antagonist system of the previously taught system*" (Brousseau 1986, p. 340), the milieu appears to be more complex when the positions of teachers and students are included within the model. "*But a milieu without didactic intentions is clearly insufficient to infer all of the student cultural knowledge that you want it to achieve*" (Brousseau 1986, p. 297).

A didactic situation is, by definition, not static and the dynamic has to be represented relative to the position of the players in the playground. This shows the necessity of structuring the milieu relative to these positions. The concept of milieu and its structuring is well adapted to understanding the situation from its design to its implementation in classrooms (Margolinas 2004) and makes it possible to analyse 'ordinary' classrooms. We speak of ordinary classrooms to distinguish didactic engineering where the construction of the situation is devolved to the researcher in contrast to those where the construction of the situation is devolved to the teacher. At each level, the milieu includes not only material objects but also *naturalised knowledge*, conceptions, beliefs, artefacts, numerical tools and so on. The naturalised knowledge is defined as the knowledge which is familiar enough to be used naturally, for example elementary arithmetic for students starting to learn algebra, or Euclidean geometry in the context of learning hyperbolic geometry.

A didactic situation is thus defined as the interactions between players (teacher and students), and the playground, including knowledge and other artefacts, and it responds according to the position of players. The construction of knowledge moves

Table 1 The structuring of the milieux (From Margolinas 2004)

Level	Student	Teacher	Situation	Milieux
M + 3: Design	–	T + 3: Noospherian	S + 3: Noospherian situation	Upper-didactic levels
M + 2: Project	–	T + 2: Developer	S + 2: design situation	
M + 1: Didactic	St + 1: Reflexive	T + 1: Projector	S + 1: Project situation	
M0: Learning	St0: Student	T0: Teacher	S0: didactic situation	
M − 1: Reference	St − 1: Learner	T − 1: Observer	S − 1: Learning situation	Lower didactic levels
M − 2: Objective	St − 2: Acting	–	S − 2: Reference situation	
M − 3:	St − 3: Objective	–	S − 3: Objective situation	

through a dynamic process that takes into account both students and teacher in different positions, from the teacher in the situation of designing activities to the situation of the student confronted with the *material milieu*. At almost every level, teacher and students have a role to play. Table 1 summarises the structuring of the milieu. It is a nested structure, the level n situation being the milieu of the level $n+1$ situation. Thus, for example, the didactic situation (S0) is the description of the interactions between the teacher in the position of teaching, the students and the milieu. The milieu is, in that case, the learning situation where the teacher in the position of observer (T − 1) interacts with a student (St − 1) in the position of learner, discovering new knowledge through the interaction with the reference situation.

It is possible to read this table from the bottom, taking the point of view of students who face a material milieu made up of objects (files, geometrical tools, calculators…), knowledge or conceptions, and which is devoid of any didactic intention. Typically, when students come into the classroom and discover the theme of the lesson, from a sheet of paper with the wording of a problem or exercises, or a file uploaded on a computer, all this is part of the material milieu. Before any interaction, this milieu has no didactic intention. The interactions with the teacher, the feedback of this material milieu make sense when the students are confronted with knowledge and are able to access a reference situation, which is the situation of experiments with material objects (computer, calculator, ruler, compass etc.) and mathematical objects (circle, equality, equation, operation, which are constitutive of the mathematical situation or problem). In the learning situation, students relate the result of the experiments with knowledge, the milieu of this situation being constituted of the relationships of the mathematical experiments, their results and the student's knowledge. The didactic situation, S0, is the situation in which the teacher's teaching intentions encounter the student's learning will. It is the place of institutionalisation where the operational knowledge becomes a social and shared knowledge in a particular institution.

Symmetrically, the situation S + 3 is called the '*noospherian*' situation. The word 'noosphere' (from the Greek νόοζ: intellect or intelligence and σφαίρα: field, social circle), originating from the theory of didactic transposition (Chevallard 1985) designates a level of institutional organisation where knowledge to be taught is defined separately from academic knowledge in a social construction. The S + 3 situation, as

Ca: […] What are you doing?

JC: I don't know, I try... you must find something... (He is calculating with letters.)

Ca: ab minus a minus b over ab; a square, 2 is missing...

[…]

JC: b minus a equals ab, well... No, b plus a equals ab, so minus b minus a equals minus ab

S: That doesn't get us anywhere!

JC: Hence, um, then... (he continues the calculation and writes a=b/(b-1) and b=a/(a-1)...)

S: What are you doing?

JC: I don't know.

S: It's impossible to find something!

Fig. 1 Three students exploring the problem of Egyptian fractions

well as the S – 3 situation is not finalised, that is to say it is not directly linked to a particular situation, but more generally refers to the teachers' conceptions both of mathematics (epistemological conceptions, mathematical knowledge) and of teaching (learning hypothesis: constructivism, situated learning, transfer of learning).

The S + 2 situation or design situation is the situation in which the teacher designs an activity for generic students building on work already done in the classroom. It is the situation where the teacher makes choices (didactic variables, elements of the material milieu) using his/her set of resources (see below). The S + 1 situation takes into account the actual students' interpretations of the didactic intentions alongside the mathematical knowledge of concern. In this situation, the student is conceptualised as an actor engaged in his/her own learning and this position is directly linked to the didactic contract built between the didactic intentions of the teacher and the student's desire to learn as illustrated in the abstract of Fig. 1. This description of the *milieux* provides an opportunity to conduct two kinds of analysis: one, starting from the point of view of the teacher, called the descendant analysis; and the second starting from the point of view of the student, called the ascendant analysis.

This table has to be considered in a dynamical way, each actor moving from one position to another in and outside school. The three situations S – 1, S – 2 and S – 3 constitute what Margolinas called the *lower didactic levels* which differ from the *a-didactic* situations in the context of ordinary classrooms. The lower didactic levels of a situation may lead students to meet new knowledge but sometimes, lead students to operate with almost consolidated knowledge without encountering the new knowledge. In that case, the situation brings into play only two levels of the situation, the levels –3 and –2 in which the confrontation with the material and objective milieu involves only naturalised knowledge and a stationary or static process. Such situations are called *nil-didactic situations* and can be illustrated by the following episode in which students try to solve the following problem: Is it possible to find two different natural integers a and b such that $1/a + 1/b = 1$?

The three students Ca, JC and S try to calculate algebraically without success because their algebraic knowledge is not sufficient, for example it should be possible to extend the reasoning of JC:

$a = b/(b - 1)$ but b and $b - 1$ are relatively prime because of the Bezout's relation: $b + (-1)(b - 1) = 1$ hence $b - 1$ divides b if and only if $b = 2$ and $a = 2$ which cannot be kept because a and b are distinct.

The material milieu of students does not allow them to carry on calculating and the objective milieu lets them calculate without any chance of reaching an algebraic solution. They continue to calculate with no result but they are not in contradiction with the didactic contract because this kind of calculation can be considered as legitimate in the classroom. This particular phenomenon is called *didactic bifurcation* and results from a gap between the teacher's intention and what comes to the students' minds to do. When the teacher gives students a problem, he/she plans on his/her teaching intentions, that is to say his/her will to modify the system of knowledge of students. He/she builds a didactic situation by designing the milieu of the situation. In their position as objective students, students may ignore or be ignorant of the teacher's intentions but may, however, guess them as reflexive students and in turn project their own objective situation. There is bifurcation when, confronted with this material milieu, students invest a different reference situation from that specified in the teacher's intentions as illustrated in the previous analysis.

Documentational Genesis and Incidents

Resources taken in a general meaning "not limited to curriculum material, but including everything likely to intervene in teachers' documentation work: discussions between teachers, orally or on line; students' worksheets, etc." (Gueudet and Trouche 2009, p. 200) are part of the milieu either for teachers in the upper levels and for students in the lower levels. The documentational genesis is an extension of instrumental genesis (Rabardel 1995; Rabardel and Pastré 2005), which has been adapted to mathematics education (Artigue et al. 1998; Artigue 2007; Drijvers and Trouche 2008). In this model an artefact (a tool, a thing…) becomes an instrument as the result of a long process in which the artefact modifies the activity of the actor (instrumentalisation) while the actor shapes the artefact for his/her use (instrumentation).

Considering the available resources as artefacts, documentational genesis models a process where instrumentalisation conceptualises the appropriation by the subject of the resource and the instrumentation describes the influence of the resources on the subject's activity. At a given time, resources become a document when combined with schemes of utilisation. However, the process is ongoing and the document becomes a resource for the ongoing process. Combining *documentational genesis* and the concept of *milieu* provide an opportunity to follow two dynamical processes, making it possible to better understand the game of knowledge construction. Particularly, new types of calculators are artefacts tending to become instruments but also resources tending to become documents because the internal properties are more than mere properties of calculation or representation. For example, the possibility to organise and share files within the machine and with other calculators or computers adds to the calculator documentational properties. In our experiments, students worked with TI-Nspire CAS, which is a novel handheld device for several reasons:

- The handheld exists as an extension of the software available on computer;
- Files can be organised into a directory tree;
- Different representational environments (graphical, geometrical, CAS, spreadsheet) can be easily connected.

When considering a dynamic process, it is natural to focus on moments of rupture or of clashes, moments where the dynamics changes, where a new direction is followed. Such an event can be seen as an event that the actors did not foresee. Clark-Wilson (2010) has introduced the concept of ‘hiccup’. “The hiccup is defined as a perturbation experienced by the teachers during lessons that is stimulated by their use of the technology and which illuminates discontinuities in their knowledge” (p. 217). In the perspective of professional development, the hiccups conceptualise the moment where a teacher becomes aware of a phenomenon. This notion appears as a methodological tool to emphasise an epistemological rupture in the development of professional skills of mathematics teachers in connection with an IT environment based on multi-representation. Either the teacher does not have an available answer and simply postpones the treatment of the hiccup or seeks to provoke a dialogue in order to overcome the difficulty, or alternatively has a *well-rehearsed* repertoire of responses that are used to deal with the problem. This repertoire is built over time and can be considered as a part of the teacher’s set of resources. Sabra (2011) distinguishes between individual and community incidents and explores the relationship between the individual and community documentations of mathematics teachers. He defines and *individual documentary incident* as an event, which can be seized by the teacher, leading to a reorganisation of his/her system of resources, and *collective documentary incident* as an event bringing in a community documentation system as a resource that leads to the reorganisation of the community documentation. While building on this work, I diverge from it by considering the incident from the point of view of the teacher and the student or, more precisely, from the point of view of the interactions within the couple (teacher, student) in relationship with the didactic milieu.

Another difference builds on the fact that a didactic incident can be ‘invisible’ for both teachers and students and the *didactic perturbations* that follow can be a source of misunderstanding between them. The concept of *didactic incident* (Aldon 2011) has been defined as an event of the didactic system that modifies the dynamics of the situation. I have distinguish different types of didactic incidents:

- An *outside incident* corresponds to an event not directly linked to the situation but often important in the classroom, for example the presence of an observer in the classroom, the mobile phone of a student that is ringing. This type of incident can amplify a previously caused perturbation;

- A *syntactic incident* linked to the conversion between semiotic registers of representations; in a technological environment, the incidents mainly come from feedback from the machine, or from the conversion of a register into the specific language of the software;
- A *friction incident* corresponds to the confrontation of two situations of different levels (cf. Table 1); such an incident may be caused by a change in the position of the teacher who moves from a position T-*n* to a position T-*n* + *1* or T-*n* − *1* or when, in the interactions, students' positions are different;
- A *contract incident* occurs when an event breaks or modifies significantly the didactic contract; this modification disrupts the trajectory of the dynamics and is strongly correlated with the appearance of didactic bifurcations where students invest a nil-didactic situation;
- A *mathematical incident* when a mathematical question is asked without answers.

Following the incidents, and in a perspective of joint action (Sensevy 2007), actors (students and/or teacher) may have different answers, modifying the milieu, or reorganising the development of the lesson or changing responsibilities within the situation. In the relationship between student and teacher, the kind of answer (or the absence of an answer) can deeply change the dynamics of the class and lead to a didactic bifurcation.

Analysis of a Situation

The Context and the Methodology

Methodology can be defined as the shape that is given to research to try to answer a question in a given framework. Choices have to be made and are interrelated with context and research questions. In the research presented in this Chapter, I wanted to: observe an 'ordinary' classroom in the sense that the responsibility for the teaching lies with the teacher; and focus on the uses of the technology in the class without being distracted by mathematics teaching difficulties.

I also wanted a *micro-view*, allowing me to capture events as they happened and a *macro-view*, allowing me to track changes over time. It is the reason the methods were chosen in order to address this challenge, that is to say, to catch incidents that are unpredictable and to follow their dynamics during the school year. Three different classes from two schools have been observed over a period of 3 years. Classes have been chosen from 16–18 year old students on a scientific course (last two grades of high school in France). In each school, one teacher was observed. The teachers were both experienced. In the first school, the teacher did not have much expertise of technology integration but in the second school, the teacher was an expert at teaching with technology. The timeframe for the data collection is summarised in Table 2. Two kinds of data were

Table 2 Data collection's timetable

First year (T: teacher, St: student, Obs: Observation in the classroom)							
	Dec.	Jan.	Feb.	March	April	May	June
Observations	–	Obs 1		Obs2	Obs3	Obs 4	–
Handheld contents	X	x	x	x	X	x	x
Interviews	–	T	–	T	T	T	St
Questionnaires (St)	Q1	–	–	–	–	–	Q2
Second year (T: teacher, St: student Obs: Observation in the classroom)							
	Dec.	Jan.	Feb.	March	April	May	June
Observations	–	–	–	–	Obs1	Obs2 Obs 3	–
Handheld contents	–	–	X	X	X	X	X
Interviews	–	–	–	T	T	T	St
Third year (T: teacher, St: student Obs: Observation in the classroom)							
	Oct.	Nov.	Dec		April		
Observations	Obs1	Obs 2	Obs 3	–	–		
Handhelds contents	X	X	X	–	X		
Interviews	T	T	T	–	St		

collected, the first by direct classroom observations during the year and the second by asking the teacher and students to provide additional data that included:

- Teachers were asked to fill in a small journal and agreed to answer interviews before and after observations in class;
- Students agreed to send me the content of their handhelds at regular intervals and were interviewed at the end of the year.

The following analysis integrates the interviews (students and teacher), the content of the handheld device and classroom observations with a focus on one particular lesson that was conducted during the European project EdUmatics. In this project a university-school pairing in one country worked closely with a university-school pairing from a second country. The pair ENS de Lyon-Lycée Parc Chabrière was coupled with the pair University of Torino-Liceo Scientifico Copernico. From a perspective of international experimentation, a classroom activity was designed by the Italian team and adapted to the French context. In the text that follows, Jean, the (male) French teacher started from the original idea to build his own didactic situation, taking into account the French curriculum and his 16–17 years old students (who were all following a scientific pathway). Before and after the lesson Jean took part in an interview and the lessons were videotaped. The analysis has been constructed from these interviews and on the transcripts of the lessons.

The mathematical situation was developed around the notion of sequences, aiming to lead students to find a mathematical description of sequences of natural numbers from the following prompt, which was presented in the written scenario shown in Table 3.

Table 3 Task given to the students

Alberta (A), Bruno (B), Carla (C), Dario (D), Elena (E) and Federico (F) (pseudonyms) are exploring the set of natural numbers and each one identifies a sequence. Here are the sequences identified
A: 1, 2, 3, 4, 5, …, …
B: 3, 6, 9, 12, 15, …, …
C: 5, 8, 11, 14, 17, …, …
D: 1, 3, 6, 10, 15, …, …
E: 3, 9, 27, 81, 243, …, …
F: 2, 3, 5, 7, 11, …, …
Individually
What is, in your opinion, the sixth number that each of the six friends will insert next and how do you find it?
Do your previous answers change if we ask you to write the tenth number in each sequence? And the fortieth? Why?
By groups
Explain your answers to your friends. Discuss the different solutions and give a common answer for the group. If you can't, express your disagreement
In your opinion, will someone among A, B, C, D, E, F, eventually find the number 1275 in his/her sequence? If yes, after how many steps?
Describe the method you use
Can you answer the same questions with the number 2187?

The students were invited to discuss in groups the solution and answer some supplementary questions leading to a more formal definition of the sequences. The complete scenarios in Italian and in French are available in Appendices 1 and 2.

The Analysis

In this section, we develop the analysis of the French situation, starting from descendant and ascendant analysis (an *a priori* analysis) and continuing with the analysis of the incidents (and *a posteriori* analysis).

Descendant Analysis

In the situation level S + 3 the teacher considers that each new concept requires a process of discovery on the part of students and a preliminary research problem will highlight the students' knowledge and their difficulties. This research problem and the class situation aim at supporting a future lesson by providing a point of references within the students' memories throughout the sequence of lessons. In the interviews, Jean said: "Later in the class; I just have to refer to the problem and for students it makes sense".

In addition, T + 3 considers that technology has to be integrated into the everyday functioning of the classroom. All students in his classroom possess a TI-Nspire™ handheld and students have the opportunity to use them in almost every lesson as a natural and familiar tool. As Jean explains: "I use calculators very often, not necessarily for a long time, but just to verify something or to illustrate a property, or… Students are used to working with it".

In his role as T+2: Developer the teacher is in a particular position because of his work in the EdUmatics project since he agreed to adapt the activity proposed by the Italian team. T + 2 organised the situation as a research problem based on the initial wording and adapted to his students' knowledge according to the French curriculum. In the French curriculum students had not had any formal lessons on sequences previously, but this kind of problem (What is the next term?) is often used in magazines for young people. The teacher therefore organised the wording as a challenge, taking into account this cultural familiarity, but with precise questions in order to lead students to a formal definition of the concept of sequence. He also sought different mathematical possibilities to answer the questions, noticing in particular that there is no unique answer. For example, the first sequence 1, 2, 3, 4, 5 can be continued with the number 7 considering the sequence of prime power p^k (p prime and $k \geq 0$) and the second sequence (3, 6, 9, 12, 15) can be continued with 20 considering the sequence defined by $a(1) = 3$ and $a(n) = a(n-1)$ + greatest prime factor of $a(n-1)$ and so on.

The teacher in the position of T + 1, within the project, chose to conduct this lesson in a particular room where computers were available along with space for group work. Computers as well as calculators were available but with no direct instruction about how to use them. It is interesting to note that the material milieu that the teacher wanted for students included different kinds of tools but also the freedom to consider these tools as useful or not for the resolution of the problem and for the documentation. Also, the teacher in position T + 1 wrote worksheets for students in order to allow and to encourage them to write their answers individually and in groups; these sheets are part of the material milieu as well as the common knowledge about sequences described above.

Ascendant Analysis

In the material milieu of students there are digital artefacts, namely the two-page description of the problem and sheets on which they would produce their report. Knowledge of students in the position St − 3 on the subject of sequences is non-existent in the school context, but as already said, present in a cultural and emotional context. The students' ability regarding the technology is sufficiently high to consider the artefact as an instrument permitting calculation in a familiar context. It is also a part of the set of resources that students may use if needed.

In the reference situation, it is possible to think that St − 2, playing with sequences, is going to construct criteria for a valid response and confront them with the objective situation at level S − 3. The material milieu in itself is unable to validate the answer (and the validation is impossible because of the different possible answers). However, the wording asking for an answer for large numbers or questioning the presence in the sequence of large numbers (1275, 2187, …) is an element of the material milieu that generates feedback, not on the values but on the process of calculation. In particular, the use of software (spreadsheet, computer algebra system, numerical calculating) brings immediate feedback inasmuch as the correct syntax can be implemented in the machine. There is a necessary translation in S − 2 from the semiotic register of representation of natural language into the semiotic register of representation of the software, if software is used, or of the algebra, if specifications are algebraically performed.

In the learning situation, the interactions ‘against’ the reference milieu permit both individual validation (it is possible to find a method of calculation to obtain the nth term of the sequence) and collective validation (it is possible to clarify and to explain this process). In the S − 0 situation, the intentions of the teacher may meet the learning acquisition of students accomplished during the a-didactic phase. Formalising and institutionalising results can then restart the problem in a new situation that includes, within the material milieu the institutionalised knowledge that has to be assimilated in order to promote its naturalisation.

Analysis of Key Incidents in the Classroom

The two previous analyses are *a priori*, but now I turn to analyses made after observations in the classroom. This approach reveals a potential gap between the analysis and the contingency and makes it possible to analyse the cause of the bifurcations and the role of the teacher in maintaining the dynamics of the situation. The complete analysis is not reported in this chapter but concentrates on the different kinds of incidents, in an attempt to illustrate the different types and the perturbation that follows.

The teacher’s introduction was short, less than 3 min. During this time, Jean gave out the first worksheet (Appendix 1) and students worked individually for 5 min. Before asking students to work in groups, Jean placed the calculator and the software in the material milieu of students, saying, “The software is installed on computers, OK, you can use either computers or your handheld”.

The research observation within this lesson concerns a group of four students, two boys (B1 and B2) and two girls (G1 and G2) working as shown on Fig. 2

The observation in the group of students shows that the devolution of the problem is properly executed, even if the goal is not yet clear (Fig. 3):

The word ‘people’ designates here the future readers of the report, including the teacher, of course, but also other students and this refers to the established didactic contract in Jean’s classroom. It is interesting to see in this short extract different positions in the structure. B2 seems to be in the S + 1 situation, thinking about the situation given by the teacher (‘we must explain’, ‘It helps explain!’) whereas G2 and G1 focus on the objective situation (‘the gap between numbers’) which is characteristic of an incident of friction.

Fig. 2 The group working together

B2: People will say, yes but how many, we must explain!

G1: No matter!

G2: Yes, precisely, we explain, just here, the gap between numbers.

[…]

G2: But, the difference between two numbers, we found it, at the beginning of the sequence.

B2: Yes, but people don't know,... It helps to explain! […]

Fig. 3 The devolution and the negotiation of the didactic contract

B1: You do the first gap, it's equal to three minus two, and after, little by little, you add.

G2: The next one is easy because we just have to multiply by three.

Fig. 4 An incident of contract

B2: I don't find anything, at least we can say one, one, two, two, four four...

G2: I conclude like that, but... perhaps is it one, two two, four, six six. Perhaps there is only one four as there is only one one.

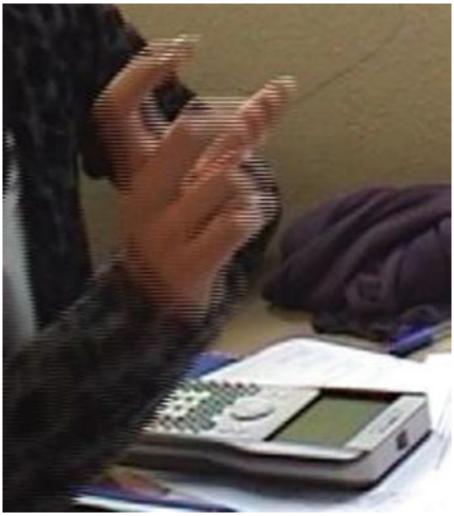
Fig. 5 A discussion as a consequence of the incident

The first incident of contract occurs very quickly when B1 gives an answer for the sequence D (Fig. 4):

In this episode, B1 is seeking a closed formula whereas G2 is seeking a recursive definition. They cannot understand each other and this might be due to a discrepancy between their individual understandings of the aim of the problem. But the perturbation offers possibilities of discussions and lasts a long time until they realise that the two definitions are possible (Fig. 5):

The consequence of these incidents is a discussion in the group about the problem itself, which helps the students to make the goal of the problem clearer and contributes to the devolution of the situation. The proposed milieu is sufficiently adapted to support changes in the position of students but also strong enough to interact with students and to facilitate discussions. In this case, incidents encountered in the lower didactic levels were the driving force behind the dynamic making it possible for the students to engage thoughtfully on the problem.

Fig. 6 The mime with fingers to indicate the recursion



G2: It is not billion, million, perhaps? […] What comes after billion?

B2: There's a trillion?

Fig. 7 A digression as a nil-didactic situation

Another interesting and important set of incidents comes in the phase of action when students try to answer the question: "is 2187 present in the different sequences?" B1 and B2 try to use the calculator whereas G1 and G2 work with paper and pencil. The difficulty for B1 and B2 is to translate the recursive definition of sequences given in the register of natural language into an algebraic register and finally into the register of the calculator's syntax. Figure 1 shows the gesture that goes with the trial of translation (Arzarello & Robutti 2010) (Fig. 6).

B1 takes his calculator: I'm sure, there are sequences in it…
B2: Yes sure!
B1: But where?

The calculator is part of the material milieu and the syntactic incident leans towards a nil-didactic situation: B1 and B2 use their calculator to evaluate 3^{40} and digress by talking about the huge number they obtain and reading the number aloud (Fig. 7):

The consequences of the incident diverts the students from the aim of the problem and the difficulties of translation between registers of representation lead students back to the objective situation.

In the lower didactic levels, didactic incidents play two different roles depending on whether the milieu reacts. In the first example, the feedback of the milieu constitutes a guide and the incidents present an amplification of the dynamic whereas in the second example, the technical incidents bring the students back to the objective situation. The calculator's syntax is not sufficiently naturalised to become the place of experiments and remains an obstacle to reaching the learning situation.

St1: Thirteen!

St2: Fourteen!

Teacher (joking): Thirteen, fourteen, well, good prices!

St3: Fifteen!

Teacher: Another answer? "What do you say St"?

St4: We can't know...

Teacher: We can't know? Well, what does it mean, we can't know? (Hubbub) Wait, wait, one after the other!

St1: We don't have enough information.

Teacher: Why do you have enough information for the others?

St1: They were linear.

Teacher: You say, they were...?

St1: Linear.

Teacher: Linear?

St3: Yes, you know, at the beginning there's two, then...

St2: It's always the same thing...

St5: Constant.

Teacher: It is always the same thing. It is constant, … yes?

St6: For the others, there was a logical sequence

Teacher: And now, why are you sure it is not a logical sequence?

St: We are not sure. We have not enough information.

Fig. 8 The debate about the prime numbers' sequence

The second part of the observation concerns a common phase where Jean wants to institutionalise, firstly the two possible definitions of a sequence (recursive or using a closed formula) and, secondly, the possibility of having several different and correct answers for a problem. Despite all his efforts, Jean does not succeed in the second aim with the first five sequences. From the point of view of students, there is only a unique possibility:

A: 1, 2, 3, 4, 5, **6**
B: 3, 6, 9, 12, 15, **18**
C: 5, 8, 11, 14, 17, **20**
D: 1, 3, 6, 10, 15, **21**
E: 3, 9, 27, 81, 243, **729**

On the other hand, the discussion is strong when the sixth sequence comes up for discussion; students have never formally studied the prime numbers even if they know the definition. The result is that different responses emerge around the classroom (Fig. 8).

The debate is about the place of different didactic incidents, which are all visible and allow Jean to institutionalise the second point even if he does not take into account the vagueness of vocabulary. Linearity is seen as regularity or a logical sequence as a result of a known formula. The incident of contract occurs because of the distance of the students from the mathematical thinking; in a 'typical' mathematics class each problem has a unique answer and thinking about the possibility of having different answers goes against the students' conceptions of mathematics. This conception is unsettled by the prime number sequence which is not sufficiently familiar to students to remain in the material milieu and the experiments on numbers lead to negotiation about the incident of contract to a new didactic contract.

Even if all the details of the analysis of incidents cannot be reproduced here, it is possible to draw a conclusion that is illustrated by the previous extracts. An important issue that is raised by this observation is the confirmation of the constructive dimension of didactic incidents, which in several cases have revived the students' work. Mathematical incidents, provided that they become visible for students and teacher, appear as prompts that link knowledge and experiments on mathematical objects and they facilitate the transition from the objective situation to the learning situation. Incidents of contract allow a renegotiation of the contract in the classroom and promote a step back in relation to the didactic situation. In contrast, syntactic incidents have not been able to be overcome and instead have played out, in this observation, as a brake on the dynamics of research. This conclusion points to the need to better understand the place of technology in the set of resources of both the teacher and the students.

Technology in the Set of Resources

In previous research, I concluded that:

> […] the documentational geneses become distinct and separated processes for teacher and students. These processes are confronted with each other only in a collective domain and concern mainly the property of the creation. The communication and cognitive properties (memorization and organization of ideas) seem to remain private but are important parts of the documentational genesis. (Aldon 2010, p. 746)

Incidents created by the gaps between the private, collective and public use of calculators had been highlighted by looking at the content of calculators and the activity of students working on a task. It is quite clear that the calculator belongs to the set of resources of the teacher and in this sense, it is part of the milieu of design. At the same time, it belongs to the material milieu of the objective situation. More precisely, the calculator can become a document useful in the situation of reference and the situation of learning if, and only if, it belongs, for the teacher, to the milieu of the project and to the didactic milieu. In other words, in the perspective of the integration of the calculator in the mathematics lesson, it remains compulsory to negotiate the didactic contract, including the different properties of calculators, not

E6: Well, for the functions, with my old calculator, I type the function, Graph and I have the curve, whereas, with this one, I don't know, you must define it...

E5: There are many steps...

E6: Yes, there is a lot of things to do, just for one result, whereas with my calculator, you type your calculation, you have your result, that's all!

E5: It's faster...

I: And do you remember the moment you said: I don't want this calculator!

E5: Very quickly, yes, we must use menu, then this place, then click everywhere, we had a long course to do a calculation that can be done very quickly with our calculator.

E6: Yes, it was a lesson at the beginning of the year, about functions, we spent two hours with the calculator, it really bugged me. It put me off this calculator.

Fig. 9 Interview of two students who do not use the TI-Nspire technology

T: Then you open the catalogue and type the first letter of the command, well for the moment, R and you just have to go down, OK, you see Randint, it's here. Well. (he is doing on the computer whilst speaking)	E1: We have to type a blank. E2: Do you think that? E1: It's six. E2: Yes, randint one six minus randint one six? E1: And, how do you type the absolute value? E1: It doesn't work.
T: Well. I have simulated the throw of a dice. The question now is: how are you going to simulate the throw of two dice and how will you obtain the value of the difference of the greater minus the smaller?	E2: (watching to the screen of E1's calculator) Missing? E1: and now it gives six, Ahhh! E2: Ahhhh! E1: It doesn't work! E2: Too many arguments! E1: I can't do that!

Fig. 10 Crossed dialogues of teacher and students

only as a tool becoming an instrument in specific situations, but also as a resource becoming a document available in the set of resources of students and the teacher.

In the following example, the global consequence of an incident is illustrated. E5 and E6 are two students who do not want to use the TI-Nspire and prefer their old calculator, in fact a TI-82 (E5) and a Casio Graph 35 (E6). See (Fig. 9):

It is interesting to set this dialogue against the observation which took place at the beginning of the year where the teacher is speaking to the whole class whilst students work with their calculator (Fig. 10):

The discrepancy between the talk of the teacher and the students' difficulties is clear. The syntactic incident is caused by students' incomprehension of the machine's feedback. At first, instead of typing randint(1,6), E1 typed randint 1 6. The feedback of the machine was *Missing*), but the bracket was not read by the students. E1 tried to type brackets but finally obtained fresh feedback, which he could not interpret. This kind of incident may lead to a rejection of the technology, as E5 and E6 said.

Clearly the syntactic incidents are inherent in the use of technology in the classroom. Taking into account the perturbations, consequences of incidents are essential to limit their long-term effects from the point of view of:

- teachers' professional development by increasing the *response repertoire* (Clark-Wilson 2010);
- students by increasing the registers of representation of studied mathematical objects.

The classroom management and the orchestration of a mathematical situation in a digital environment (Trouche 2004) accentuate the importance of the teachers' responsibilities with respect to the instruments and show the necessity of including the analyses of such situations' in the process of teacher development.

Teacher Development

In this section, I would like to emphasise the links between the analysis, the observation, the feedback and the professional development of teachers. Starting from observations in the classroom and interviews with Jean, a French teacher involved in the EdUmatics project, I will show how and why the collaborations introduced at the beginning of the chapter contribute to the professional development of teachers as well as to strengthening theoretical approaches.

Collaboration Between Researchers and Teachers

One of the important aspects of the EdUmatics project was to enable teachers and researchers to work together on the implementation of lessons using technology. Even though these work habits are already widely implemented in France in the network of IREM (Institut de Recherche sur l'Enseignement des Mathématiques/ Research Institute on Mathematics Education), the particular experience of the EdUmatics project provided valuable information for the professional development of teachers. The confrontation of teachers' professional skills with analyses based on theoretical frameworks helped both to increase the skills and refine the theoretical tools.

An *a priori* analysis sufficiently complete to embrace the mathematical aspects, the didactic characteristics and the pedagogical modalities give the design of a lesson a new dimension, as Jean says: "To predict, to analyze and to find solutions to all the difficulties, pedagogical as well as technical is something demanding and interesting". The contrast between this *a priori* analysis and the reality of the classroom shows that the theoretical tools are consistent and the possibility to see '*live*' the occurrences of predicted events modifies significantly the teaching approach. During the interviews with Jean, before and after the class observations, the two analyses (*a priori* and *a posteriori*) were shared and discussed with him; as shown in Table 2, the data collection tended to catch the evolution either for students or the teacher over the long term. Jean commented on the benefits of this *a priori* analysis "When I see in the classroom some attitude that the *a priori* analysis had predicted and for which a solution was already ready, it's reassuring and very satisfying in my professional practice […] Several times, later, in my classroom, I surprise myself in remembering this moments and I modified my attitude to take into account the observations".

The work done in the project and the collaboration between researchers and teachers developed an awareness of professional gestures. The analysis using the concept of incidents illuminates different processes occurring in the classrooms and, more particularly, the place and the role of technology in the development of both teaching and learning. In addition, the observations highlight the role of didactic incidents in the students' construction of knowledge, particularly in the lower didactic levels. But in order to become shared knowledge in the classroom and, more generally, to become a potential naturalised knowledge, the knowledge that students encounter must be recognised as legitimate. The institutionalisation of knowledge in the course of acquisition is essential and this institutionalisation is typically the responsibility of the teacher who needs to recognise, to interpret, to organise and to transform the *knowledge in action* from what the teacher at level T − 1 observed in the learning situation into what students must know and learn. Players win not only because they reach the end of the game but also because they know how and why they win. Students have to transform their *knowledge in action* into shared knowledge and teachers have to understand the key elements of the situation allowing this knowledge construction, or perhaps the key elements that prevent them reaching their initial didactic intentions.

Working in a technological environment adds to this institutionalisation knowledge, being directly linked with the technology in use. One of the main difficulties is surely to recognise the different knowledge that students act upon during the phase of action in lower level situations. The *a priori* analysis and the feedback of what happens in real classrooms shine light on the actual activity of students and the knowledge that has to be institutionalised. In the last interview, Jean said: "In fact, when you are in my classroom I see things that I didn't see usually. Sometimes, I'm not happy with my lesson, but you say that a student or a group of students work on this or that; I know then that I've not wasted my time."

Giving teachers this opportunity, at least once, is surely a fundamental aim of teacher development, but in an 'ordinary' classroom this awareness is a key element of the modification of schemes. Giving tools which make it possible to observe and analyse what happens in the class can augment the *response repertoire* of teachers. The framework of *didactic incident* may increase the awareness of teachers interpretations of students' work when they are in a position of observer (T − 1) and facilitate the institutionalisation of knowledge directly linked with the actual activity of students. The design of our part of the EdUmatics course takes into account the common analysis. Future research should concern the construction of tools facilitating the incident analysis by teachers themselves.

The Documentational Genesis

A second aspect that occurs from the observations in Jean's school concerns the documentary role of the digital artefacts. The different properties of digital documents are described by Pédauque (2006):

> The two cognitive functions, mnemonics and organization of ideas, seem to be the fundamental basis for the documentary production. […] The function of creativity comprising enrichment due to the domain of interest related to the document surpasses that kind of organization just mentioned. […] The third and last constituting function of the documentary production is the transmission function. (pp. 5–6).

The technological context shows the four dimensions present in this handheld device seen as a resource, and the phenomenon of documentational genesis builds on these properties in different meditational contexts. The cognitive properties of storage and organisational ideas are built in parallel and remain in a private domain, both for teachers and for students. On the other hand, the properties of creation and communication are built in the collective domain. The method(ology) allows the researcher to follow the joint documentational genesis of teacher and students by entering into private domains, particularly regarding the contents of handheld devices. The handheld with its computational and representational properties, along with its properties of storage and communication, prefigures digital resources that may be available in coming years. The documentational genesis of such an artefact may not be understood without taking into account the domains of mediation, whether private, collective or public. The handheld appears then to be at the crossroads between the teacher's teaching intentions and the students' learning intentions, that is to say, at the core of the didactic game. Different trajectories are sources of tension and generate didactic incidents that deeply affect interactions in the classroom, interactions between teacher and students, and also interactions between teacher, students and artefact. The integration of digital resources in the mathematics classroom cannot be achieved by considering only one property but, on the contrary, by thinking globally about the integration of all properties in the learning game. In the upper didactic levels, incidents call into question the teacher's personal epistemology and contribute to professional development.

Conclusion

The exploitation in teacher education of the frameworks of the Theory of Didactical Situations (TDS), didactic incidents and documentational genesis, should make it possible to build a detailed analysis of situations in ordinary classrooms in a technological environment. The observation of interactions within the classrooms through didactic incidents and the understanding of joint documentational geneses of students and teachers are two parts of the same methodological tool aiming at better understanding the didactic game.

The descendant and ascendant analyses assist the *a priori* analysis to take into account the role and the place of both teacher and students in the didactic game, and the incident analysis refines the *a posteriori* analysis. Inter-connecting the two analyses constitutes a tool for teachers in the preparation of lessons and in the understanding of what happens in the classroom. The typology of didactic incidents can be extended and refined to allow easy and more operational identification for new teachers in a perspective of understanding the dynamics of the classroom. It can also become a tool for regulating those dynamics within the classroom. Finally, connecting local incidents to global phenomena resulting from differences in the documentational geneses of teachers and students makes it possible to better understand the place of digital artefacts in the classroom.

New hypotheses that result from this research are about documentational geneses and the possible conflicts between the point of view of students, teachers and society as a whole. Further research might involve clarifying the role and the learning potential of digital artefacts in a digital age and reorganising the importance of teacher development in their usage.

Appendix 1

N.B. In questa attività, sia nei lavori individuali, sia in quelli di gruppo, potrai utilizzare, se lo desideri, gli strumenti informatici che ritieni più opportuni. Nei lavori di gruppo, nel caso in cui opinioni discordanti dovessero rimanere tali anche dopo un confronto, riportatele sul foglio di lavoro.

Situazione

Alberta (A), Bruno (B), Carla (C), Dario (D), Elena (E) e Federico (F) stanno esplorando la successione dei numeri naturali, studiando le proprietà dei numeri che la costituiscono. Le modalità di esplorazione, pero, sembrano molto diverse fra loro, anche se tutte sono caratterizzate da una forte sistematicità. Ecco i numeri che i sei amici prendono in considerazione:

A: 1, 2, 3, 4, 5, …, …
B: 3, 6, 9, 12, 15, …, …

C: 5, 8, 11, 14, 17, …, …
D: 1, 3, 6, 10, 15, …, …
E: 3, 9, 27, 81, 243, …, …
F: 2, 3, 5, 7, 11, …, …

Proposta di lavoro

Attività 1 (individuale)

Qual è, secondo te, il sesto numero che ciascuno dei sei amici prenderà in considerazione? In caso di risposta affermativa scrivilo e cerca di spiegare come/cosa hai fatto. In caso di risposta negativa, spiega perché non riesci a individuarlo.

Le tue precedenti risposte cambierebbero se ti venisse chiesto di individuare il decimo numero? E il quarantesimo? Spiega perché.

Attività 2 (di gruppo: 3 studenti)

Parlando uno alla volta, spiegate ai vostri compagni di gruppo come avete risposto alle domande dell'attività 1. Discutete sulle eventuali differenze. Riuscite a produrre una risposta condivisa di gruppo? In caso di risposta affermativa, riportatela sul vostro foglio; in caso di risposta negativa, riportate i punti di dissenso rimasti dopo la discussione.

Attività 3 (di gruppo)

C'è qualcuno, fra A, B, C, D, E, F che, secondo voi, prima o poi, troverà, nella sua successione, il numero 1275? In caso di risposta affermativa, dopo quanti passi?

Giustificate la risposta e precisate le strategie utilizzate per rispondere. Come cambierebbero le vostre risposte se le domande fatte sul numero 1275 fossero fatte sul numero 2187?

È possibile trovare un numero naturale diverso da 0 tale che nessuno, fra A, B, C, D, E ed F, prenderà mai in considerazione? Giustificate la vostra risposta.

Esiste almeno un numero naturale che non potrà mai essere raggiunto da B, né da C, né da D, né da E, né da F? In caso di risposta positiva, trovatelo e spiegate come avete fatto. In caso di risposta negativa, spiegate perché, secondo voi, tale numero non esiste.

Appendix 2

À suivre…

Partie 1

En travaillant sur l'ensemble des nombres naturels, Alberta (A), Bruno (B), Carla (C), Dario (D), Elena (E) et Federico (F) ont chacun créé une suite de nombres. Ils ont tous suivi un processus de construction différent mais systématique.

Voilà les cinq premiers nombres que chacun des six amis a écrit:

- A : 1, 2, 3, 4, 5, …, …
- B : 3, 6, 9, 12, 15, …, …
- C: 5, 8, 11, 14, 17, …, …
- D: 1, 3, 6, 10, 15, …, …
- E: 3, 9, 27, 81, 243, …, …
- F: 2, 3, 5, 7, 11, …, …

1. Êtes-vous capable d'écrire le sixième nombre qui selon vous a été créé par chacun des six amis ?
   Si oui, expliquez comment vous avez fait.
   Si non, expliquez les raisons qui vous empêchent de répondre.
2. Vos réponses précédentes changeraient-elles si on vous demandait d'écrire le dixième nombre ? Et le quarantième ? Pourquoi ?
3. Y a t-il quelqu'un parmi A, B, C, D, E, F qui selon vous, tôt ou tard, trouvera dans sa suite le nombre 1275 ? Si oui, lequel (ou lesquels) et après combien d'étapes ? Justifiez votre réponse et décrivez la méthode qui vous a permis de répondre.
   Pouvez-vous alors répondre aux mêmes questions avec le nombre 2187 ?

À suivre…

Partie 2

4. Les méthodes que vous avez utilisées précédemment vous permettent-elles de calculer le 70ème, le 200ème, le 1000ème nombre de chaque suite ?
   Si oui, calculez ces nombres, si non essayez de modifier vos méthodes pour les obtenir.
5. Essayez, en utilisant la calculatrice, de donner une représentation graphique de ces suites.

6. Les méthodes que vous avez utilisées précédemment vous permettent-elles de demander à votre calculatrice de calculer ces nombres ? Si oui, écrivez le calcul demandé.
   Sinon, dire pourquoi ces méthodes utilisées ne le permettent pas.

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