Teachers' Instrumental Geneses When Integrating Spreadsheet Software

Mariam Haspekian

Abstract The spreadsheet is not *a priori* a didactical tool for mathematics education. It may progressively become such an instrument through the process of professional geneses on the part of teachers. This chapter describes the beginning of such a genesis, and presents some results concerning teachers' professional development with ICT by examining the outcomes of two different sets of data. Theoretical notions, such as instrumental distance and double instrumental genesis supported the analysis of data leading to a comparison of a teacher integrating spreadsheets, for the first time in her practices, with the practices of teachers who are more expert with spreadsheets. The similarities found in the ways they use the tool leads to some hypotheses on the importance of these common elements as key issues in teachers' ICT practices.

Keywords Mathematics teaching and learning • Teaching practices • ICT integration • Professional learning of mathematics teachers • Technology-mediated classroom practices • Spreadsheet • Professional/personal instrument • Double instrumental geneses (professional/personal) • Instrumental distance • Novice/expert teacher

Introduction

Around the 1980s, the idea that ICT could serve school learning, in particular mathematical learning, began to develop. Nowadays the use of ICT in classrooms is prescribed in the curricula of many countries and it includes detailed recommendations for teachers (Eurydice 2004, p. 24). However, many reports comment upon the

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poor integration of ICT in mathematics teaching. After an enthusiastic period in which the benefits of the use of ICT for learning mathematics have been claimed, researchers now describe a phenomenon of disappointment. It is a fact that the potential for ICT use in mathematics is rather poorly exploited and that ultimately, technology integration is very limited. For example, using data from PISA 2003, Eurydice (2005) reported that fewer than half of the students (from a more than 90 000 students survey) were familiar with activities with spreadsheets such as plotting a graph. One reason for this, which has been suggested by many studies, is the 'teacher barrier' (see for instance Ruthven 2007 or Balanskat et al. 2006). Hence it seems crucial to advance our knowledge of teachers' 'usual practices' alongside their technology-mediated ones: How do ICT practices develop and evolve in time? What do we know about the instrumental geneses with ICT and about teachers' resistances? My own doctoral research (Haspekian 2005a) led me to look for reasons beyond those that are often cited: lack of time, lack of training, lack of material, conservatism etc. Without denying these factors, my research claimed that there are deeper reasons for teachers' resistance, related to the *impact* that technology has on the mathematics to be taught, and the difficulty, for teachers in managing this impact. Therefore, it remains important to advance our understanding of this impact and the ways that teachers account for it.

With this purpose in mind, this chapter aims to provide an insight into teachers' practices with technology by comparing the results of different studies concerning a common technology, the spreadsheet (Haspekian 2005a, 2006, 2011). The first study formed different elements of my doctoral study. These were: an observation of a teacher, called Ann,[1] who was integrating spreadsheet for the first time in her practices; and an inquiry interviewing and comparing pre-service teachers with teachers who were considered 'experts' with spreadsheets.[2] The second and third studies resulted from a different research project observing ICT sessions in ordinary classrooms, during which I happened to return to Ann's classroom. Thus, I had the opportunity to observe her practice a year later. Consequently, these three studies provided an opportunity to make an interesting comparison concerning teachers' practices with spreadsheets at different stages of integration within mathematics teaching:

- Pre-service teachers that were novice in teaching and in using spreadsheets in mathematics teaching,
- Teachers who are expert with spreadsheets and teaching mathematics using spreadsheet;
- A teacher who is neither a novice, nor an expert with ICT in general.

This comparison involved two theoretical frameworks. The instrumental approach (Artigue 2002; Guin et al. 2004), which was developed around the concept

[1] The name taken in the initial French research is 'Dan'; in this chapter, it is translated to 'Ann' as the teacher is a woman.

[2] This term is explained in section 3.

of instrumental genesis, supported an analysis of the impact of the spreadsheet on mathematics. This led me to determine both the didactical potential of spreadsheets and the difficulties that might occur as the spreadsheet changed the mathematics to be taught. The second framework was the didactic and ergonomic approach (Robert and Rogalski 2002), which helped to describe teachers' activity. In the second section of this chapter, this is used alongside the instrumental approach to understand Ann's evolution over two years. The third section probes Ann's practices more deeply by comparing her evolution with the practices of the 'expert' teachers. This will highlight some results about the development of ICT use in teachers' practices concerning the way that their practices evolve and the difficulties they encounter when integrating spreadsheet technology.

ICT and Mathematics Education: The Case of the Spreadsheet

An increasing number of technologies can be found in today's mathematical school landscape, from pocket calculators adapted for the elementary school through to universities' virtual learning environments that include interactive exercises and complete courses for various domains of mathematics. In France, the spreadsheet is officially prescribed for use in junior high and high schools, especially for the teaching and learning of algebra. However, this tool was neither created for, nor has it been adapted to, mathematics learning. The origins of the spreadsheet are, quite remote from the educational world, in accountancy (see Bruillard and Blondel 2007 for a historical and economical approach of the creation of the spreadsheet). Yet, to know how to calculate with a spreadsheet, in particular by using a formula, is a competency required in the curricula of an increasing number of countries worldwide (Pelgrum and Anderson 2001). Prior to the existence of spreadsheets, the use of computer tools required competencies in programming and thus, the learning of a programming language. The spreadsheet provided, for the first time, a way to avoid the need to program, leading Baker and Sugden (2003, p. 18) to say, "Nowhere is its application becoming more marked than in the field of education". However, in spite of some isolated experiments to adapt them for education, the spreadsheet remains a tool for the business world, with an increasingly sophisticated set of functionalities that have been designed in response to business rather than educational demands.

The poor integration of spreadsheets within mathematics teaching contrasts with other educational software such as dynamic geometry software.[3] This seems to offer a contradiction in that, even if some researchers question the relevance of

[3] There is no research at world scale comparing integration of geometry software and spreadsheets, but all local studies that can be found indicate a better penetration of geometry software than spreadsheets (see the examples cited in Haspekian 2005a).

spreadsheets in mathematics education, the majority of the research highlights the potential benefits of the spreadsheet for students. A brief synthesis on this theme turns the attention to the teaching and learning of algebra. The next section examines these tendencies in the light of the instrumental approach in order to analyse further the characteristics and complex relations of the spreadsheet with mathematics.

Potential Uses of the Spreadsheet for Mathematics Learning: An Overview of Research Literature

I begin by asking "What mathematical topics can be engaged through the use of spreadsheets at school?" The field that comes to mind most naturally is that of statistics. However, a closer examination of the operations of the spreadsheet reveals the algebraic nature of such activity. Without going into technical details,[4] one can note that from a historical point of view, the relation with algebraic concepts had been long identified. According to Bruillard and Blondel (2007):

> le premier tableur connu serait le 'calcolatore tabulare meccanico automatico' ou calculateur tabulaire mécanique automatique de Giovanni Rossi (1870), qui a permis une avancée décisive dans la relation entre l'algèbre matricielle et les matrices comptables. (Cilloni and Marinoni 2006; Cilloni 2007)
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> The first known spreadsheet would be the 'calcolatore tabulare meccanico automatico' or automatic mechanical tabular calculator from Giovanni Rossi (1870), who permitted a key advance in the relationship between matrix algebra and financial matrices. (Cilloni and Marinoni 2006; Cilloni 2007)

The ability to link cells by formulas is an effective feature of the spreadsheet that many research studies have affirmed to offer potential to support the learning of algebra (algebraic objects, modes of treatment, problem solving) by analysing the new opportunities that spreadsheets offer alongside the operational constraints of their use. The new possibilities concern:

- The interactivity, allowing feedback richer than paper and pencil (for example, the numeric feedback of a formula helps students to conjecture or detect errors);
- The capacity for calculation (automatic recopying of formulas, and instantaneous display of results);
- The articulation of multiple registers of representation (natural language, formulas, numbers and graphics).

The benefits of spreadsheets, which can derive from the constraints of use, relate to both the symbolic language and the methods for solving mathematical problems with them. The symbolic requirement is due to the tool itself as opposed to didactic contract that is usually entered into when students begin to encounter algebra involving the unmotivated use of letters that competes with non-algebraic

[4] The reader can find a brief explanation of the basic functionalities, in a didactic approach, in Haspekian 2005a, pp.18–23.

strategies.[5] Spreadsheets also compel students to plan their work, organise their worksheet and, in doing so, anticipate the possible feedback from the technology.

For most researchers (Ainley et al. 2003; Arzarello et al. 2001; Capponi 2000; Dettori et al. 2001; Rojano and Sutherland 1997), these potential benefits place the spreadsheet between arithmetic and algebra. This intermediate position is seen to be ideal for the learning of algebra. For instance, Rojano and Sutherland (1997) conclude that the spreadsheet supports a smooth transition for pupils' initial numeric methods towards algebraic ones. In a previous study I showed that by comparing arithmetic, algebraic and spreadsheet solution methods for the same problem,[6] the spreadsheet adds some algebraic characteristics to an arithmetic procedure (Haspekian 2005b). For others, spreadsheets could help to overcome the semantic/syntactic difficulties of algebra. In Arzarello et al. (2001), the complexity of algebra is interpreted as a difficulty for pupils to enter the 'game of interpretation' between the algorithmic and symbolic functions of algebra. The various registers of representation of the spreadsheet are then seen as a tool helping the pupils to enter this 'game' through the construction and interpretation of formulae.

These potential benefits of spreadsheets contrast with the previous discussion of their weak integration. In the reality of the classroom, after having been introduced to them within the study of algebra, students use them rarely during their time at secondary school. The results of the DidaTab project (Bruillard et al. 2008) showed that the high school students from regions where the spreadsheet is most used do not have higher competences than average, except for the competencies of selecting and formatting cells. More generally, the research concludes that all of the 288 students involved in the study:

> seem to manage the 'surface' components, such as formatting the cells and the tables, but the mastery of the essential functioning of the spreadsheet, the writing of formulas, and the knowledge of its constituent elements (operators, operands, references, functions…) is not demonstrated by the large majority of students.

Capponi (2000) adopts a more moderate position about the potentiality for spreadsheets. His view is that the intermediate position of the spreadsheet between arithmetic and algebra may allow the pupil remain entirely on the arithmetic side without ever noticing the algebraic aspects.[7] Capponi quotes, for example, the display or editing of a formula which centres the user on the numeric aspects (computation results, designation of numbers) to the detriment of the underlying algebraic aspects (formulas, and cell references that play the role of variables).

So the question becomes, how can we support pupils to build algebraic knowledge with this tool? All of the above-mentioned researchers underline the

[5] One can see in Coulange (1998) at which point the algebraic methods rest on rules of didactic contract and remain fragile for pupils ages 15–16 who, facing atypical problems, provide correct answers in rupture with the algebraic rules of the didactic contract.

[6] Analyse/synthesis, trial/refinement and equations.

[7] because the algebraic character of the formulas is restricted to their utility in carrying out and automating calculations, the focus is not on providing an operational language to analyse and handle relations (Capponi and Balacheff 1989).

importance of the didactical design of the situations but say little about these situations, such as how to create them, and on which variables to focus the teaching. In many spreadsheet resources that have been published on professional websites one can identify the mathematical variables used, while the 'instrumental' variables (the tool features) remain mostly implicit. Yet, if these elements are not examined, they may generate misunderstandings, resulting in the pupils using spreadsheets in ways 'other than' what is expected. The organisation of the teaching (didactical and mathematical), the way the tool is introduced, its links to mathematics, the techniques taught, their links with the mathematical techniques already learned (or to be learned) in paper and pencil environment, the role of the teacher and her didactic managements are all elements that must be created by the teacher. For instance, how and when does the teacher introduce into the lesson the important technical specificities of spreadsheets, such as the functionality of dragging? How does the teacher structure the teaching so that the ideal didactic potential of the spreadsheet becomes actual? Again, the question of linking the tool features with mathematical concepts arises, revealing that the work will be different from work in the paper and pencil environment. What exactly are these differences and what impact could they have? These questions echo those that were central to research leading to the instrumental approach (Artigue 2002; Lagrange 1999; Drijvers 2000; Guin et al. 2004). This particular theory showed the importance of instrumentation and its relation to conceptualisation within CAS environments, another type of tool, like spreadsheets, that was not initially created for teaching. These issues lead directly to the theoretical construct that is *instrumentation*, which allows us to understand more clearly the problems of technological integration, by showing the need to take account of the process of *instrumental geneses*.

The Instrumental Approach: Some Theoretical Elements

ICT use in mathematics education is a domain within the more general area of technology use in human activity, which has been studied within the field of cognitive ergonomics. A psychological and socio-cultural theory of instrumentation, developed in this field, provides a frame for tackling the issue of learning in complex technological environments (Vérillon and Rabardel 1995; Rabardel 1993, 2002). The instrumental approach in didactics took some elements of this frame, including two of its key ideas: the artefact/instrument distinction, and the fact that using a tool is not a one-way process; rather, there is dialectic between the subject acting on/her personal instrument and the instrument acting on the subject's thinking.[8] Within the

[8] Because of this dialectic "it is not possible to clearly distinguish between these two processes" (Trouche 2004).

activity of a subject, an artefact[9] becomes an instrument through a long individual process of instrumental genesis, which combines two interrelated processes: 'intrumentalisation' (the various functionalities of the artefact are progressively discovered, and may be transformed in personal ways) and 'instrumentation' (the progressive construction of cognitive schemes of instrumented actions).

The two processes also indicate that the instrumental geneses are not *neutral* for the subject: instruments have impact on *conceptualisation*. For example, using a graphic calculator to represent a function may play on pupils' conceptualisations of the notion of limit. This idea of non-neutral 'mediation' provides a way to report on the strong overlaps that exist, and have always existed, between mathematics and the instruments of the mathematical work. This idea has been used in several research studies on symbolic calculators in mathematics education (Artigue 2002; Lagrange 1999; Drijvers 2000; Guin et al. 2004, Trouche 2004).

In what follows I articulate in more detail the two notions that were used.

Instrumental distance (Haspekian 2005b), which will be used to analyse relations between spreadsheet and mathematics.

Instrumental genesis which will give more precisely a phenomenon of *double instrumental genesis* within the context of analysing teaching practices. Indeed, for students, the spreadsheet may become a mathematical instrument through an instrumental genesis. However, as a spreadsheet is not by definition a didactical tool to serve mathematics education, it also has to progressively become such an instrument during a professional genesis on the part of teachers (Haspekian 2006). These are two different instruments, which both exist for the teacher.

Instrumental Distance

In French curricula, dynamic geometry software is prescribed with as much emphasis as spreadsheets. However, the former find a better integration in mathematics classrooms than the second does. The notion of *distance* to the referential environment seems to play an important role in the explanation of this phenomenon (Haspekian 2005a). It intends to take into account, beyond the 'computer transposition' (Balacheff 1994), the set of changes (cultural, epistemological or institutional) introduced by the use of a specific tool in mathematics 'praxis'. For a given tool, if the distance to the 'current school habits' is too great, this acts as a constraint on its integration (Haspekian 2005b). On the other hand, the didactical potential of technology relies on the distance it introduces regards to paper-pencil mathematics as, for instance, by providing new representations, new problems, increasing calculation possibilities, etc. This is the case for the dynamic figures in geometry software, with respect to the static figures in paper-pencil geometry.

[9] We limit ourselves to the case of the material artefacts, but the ergonomic approach is extended to 'psychological' artefacts: symbols, signs, cards, etc.

The didactic potential of these dynamic objects and their benefits for students' learning have been evidenced by many research studies, (see for example Laborde 2001). For the concept of 'figure', a central object in geometry, the dynamic geometry does not only broaden the conception of such objects but it offers a representation that corresponds more closely to the abstract concept of 'figure' than its paper-pencil equivalent. The dynamic dimension helps to realise the famous distinction of *spacial drawing/geometrical figure* (Laborde 2001; Parzysz 1988; Laborde and Capponi 1994). One can also consider the interesting possibility of creating new types of geometrical problems for students by varying the different tools available in the toolbars of this software. Geometric construction problems can be completely different as a result of the suppression of traditional geometric tools or through the addition of new tools by the creation of macro-constructions.

Four types of elements have been brought out that can generate such instrumental distance (Haspekian 2005a). Some of these elements relate directly to the *computer transposition*, such as the representations and the associated symbolism. Some others are of different nature: *institutional*, or *didactical* (vocabulary, field of problems whose solution they allow, etc.), and *epistemological* (i.e. what gives a tool an epistemological legitimacy). For example, the vocabulary in spreadsheets is far from the mathematical one; teachers must even create it for themselves.[10] There is no official reference to help the mathematics teacher to relate this vocabulary (and the objects within spreadsheets) to their mathematical equivalents. Many questions arise for teachers, such as:

- What is a cell?
- Is it a variable?
- What is a column (or a row)?
- Is it a set of several variables, or another representation of a *unique* variable?
- What is a relative address? Is there an algebraic equivalent?
- What is 'filling/dragging down' (a gesture embodying the concept of formula?)
- Is the numeric feedback: a number? a result of a formula? the permanent appearance of the cell containing a formula whereas the formula itself would be its temporary appearance? etc.

In fact, beyond the computer transposition that modifies the mathematical objects, the modification, from an institutional point of view, actually concerns the whole ecology of these objects as the tasks, techniques, and theories can all be modified. The idea of '*distance*' reflects this gap between the praxeologies[11] associated to two different environments (considering paper-pencil as a peculiar environment of mathematical work). As for the epistemological aspect, distance relates to the teachers' personal component (their representations of mathematics, of teaching,

[10] This raises difficulties for teachers, see the experiment described in Haspekian 2005b.

[11] Mathematical objects are not isolated, in educational institutions they live through mathematical and didactical organisations that are praxeologies: a quadruplet composed of tasks, techniques, technologies (discourse about the techniques: explanations, justifications…) and theories. See (Chevallard 2007).

Fig. 1 A2 is the cell argument, B2 calculates the square of the value in A2

	A	B
1		
2	5	= A2^2

of the role this tool plays in the development of mathematics etc.). This idea is developed later in the chapter.

In what follows, I apply this instrumental approach to the spreadsheet for the teaching and learning of algebra in order to study the impact of the spreadsheet on algebra (the objects, techniques and symbolisations) through the notion of *distance* between paper-pencil algebra and algebra with spreadsheets. The relationship between spreadsheets and mathematics is not simple as mathematical knowledge is needed to achieve spreadsheet mastery.

Mathematics Within Spreadsheet Objects

Some computer characteristics within spreadsheets do not strictly correspond to mathematical knowledge transposed to a computer environment, or even to a computer transposition of school knowledge, however they are linked with mathematics. The basic principle of the spreadsheet, which consists of connecting cells by formulas, gives an example of these objects, linking spreadsheets to the domain of algebra. This particular relationship with mathematics is precisely the reason why many studies in didactics from different countries give spreadsheets a positive role in the learning of elementary algebra, identifying them as tools of an arithmetic-algebraic nature (Ainley (1999); Arzarello et al. (2001); Capponi (2000); Dettori et al. (1995) or Rojano and Sutherland (1997)). But, in spite of the apparent simplicity of use of speadsheets, it is not so evident for teachers to take advantage of their characteristics. In (Haspekian 2005a) I showed that the tool generates some complexity as it transforms the objects of learning and the solution strategies by creating new modalities of actions, new objects, and by modifying the usual objects, such as: variable, unknown, formula; and equation.

For example, in the paper and pencil environment, variables in formulae are written by means of symbols (generally a letter for the school levels concerned here). This 'letter variable' relates to a set of possible values (here numerical) and it exists in reference to this set. In a spreadsheet, let us take for example the formula for square numbers. Figure 1 shows a cell argument A2 and a cell B2 where the formula was edited, referring to this cell argument.

Here again the variable is written with symbols (those of the spreadsheet language) and exists, as with the paper and pencil environment, in reference to a set of possible values. But this referent set (abstract or materialised by a particular value, e.g. 5 in Fig. 1) appears here through an intermediary, the cell argument A2, which is simultaneously:

- An abstract, general reference: it represents the variable (indeed, the formula does refer to it, making it play the role of variable);

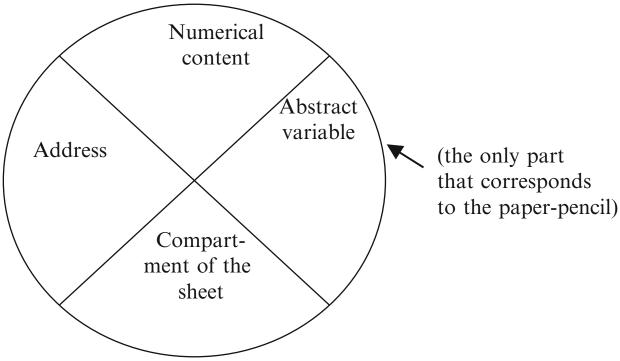

Fig. 2 The 'cell variable'

Table 1 The distance between different 'algebraic worlds'

'Values' of algebra	In paper-pencil environment	In the spreadsheet environment
Objects	Unknowns, equations	Variables, formulae
Pragmatic potential	Tool for resolution of problems (sometimes involving proof)	Tool of generalisation
Process of resolution	*Algorithmic* process, application of algebraic rules	Arithmetical process of trial and improvement
Nature of solutions	Exact solutions	Exact or approximate solutions

- A particular concrete reference: here, it is a number (in case nothing is edited, some spreadsheets attribute the value 0);
- A geographic reference (it is a spatial address on the sheet);
- A material reference (as a compartment of the grid, it can be seen as a box).

So, whereas in a paper and pencil environment, we would place a set of values, here we have an overlapping cell argument, bringing with it, besides the abstract/general representation, three other representations that do not have an equivalent representation in paper and pencil (Fig. 2). Other examples of spreadsheets' impact on algebra are given in Haspekian 2005a.

From an institutional point of view, these changes have different impacts depending on the range of ways that algebra is introduced. As one of the previous ICMI studies has showed (Stacey et al. 2004), different aspects of algebra can be focused on: a tool of generalisation; a tool of modelling; or a tool to solve arithmetical, geometrical or everyday life problems through the, so called, Cartesian *analytical method*. Depending on the focus, different mathematics are brought to the fore: variables, formulae and functions on one hand; unknowns, equations and inequalities on the other hand. The traditional French school culture adopts the analytic approach. The resolution of various problems through the solving of equations is emblematic of pupils' introduction to algebra. Table 1 provides a brief insight into the distance between the algebraic culture in the French secondary education and the algebraic world that is characteristic of spreadsheets.

Beyond the vocabulary, it is the whole set of the 'valued algebraic' objects that is modified in the spreadsheet environment. Within the paper and pencil algebra of junior high schools in France, the move is from algebra as a tool of resolution where equations and unknowns are valorised, towards the algebra of variables and formulae in their functional aspect, where algebra is more seen as tool of generalisation.

Overall, the mathematical culture sustained by spreadsheets is an 'experimental' one of approximations, conjectures, graphical and numerical resolutions, implementing everyday life/concrete problems, statistics, etc. Thus, this vision does not fit with the one usually attached to traditional mathematics in the secondary school of the French education system.

What Are the Consequences of Such Changes for the Teaching?

The idea of *distance* allows one of the conditions of viability of an instrument in teaching to be translated by considering the whole set of modifications that it introduces, not only at the level of computer transposition, but also through the cultural, epistemological and institutional aspects (Haspekian 2005b).

In the case of the spreadsheet for algebra, this distance seems to play a role in the teachers' resistances to its use because they have to grant to it a personal legitimacy, as the institutional legitimacy (the programs) or the social legitimacy (stemming from it as a modern tool that is used widely in industry) are not sufficient. Hence, the mediative, cognitive and personal components of the teachers (their history, perceptions of teaching, of algebra, etc.) come into play here. This also partly explains why not all instruments are treated alike in mathematics teaching and learning! Do teachers consider this distance 'legitimate' with regard to their epistemology of mathematics on the one hand, and to the didactic potential they foresee on the other hand? The interviews carried out with novice teachers (Haspekian 2005a) show that this is not self-evident. Furthermore, if a certain distance is necessary for the tool be seen to be interesting, this distance involves a mathematical and didactic reorganisation and thus an additional workload for the teacher. As we saw above, not only are there new praxeologies to create (that the programs and the resources, however many, are not enough to release) but additional tasks arise for teachers as they consider the management of pupils' instrumental geneses in a new environment. Last, but not least, this management should lead pupils to mathematical concepts (variable, formula, etc.) that remain relevant to the traditional paper-pencil environment.

Finally, the integration (or not) of a new tool requires equilibrium between the various elements. Do the teacher's own convictions about the expected benefits and/or the official directions to use the tool counterbalance the additional workload he/she can foresee in that task of integration? Moreover, a phenomenon of *double genesis* can come into play and add further complexities for teachers who are not very familiar with the tool, which is described later in the chapter. For the spreadsheet, one can assume that the praxeologies are far from the mathematical and didactic organisations currently practiced within early algebra in France.

This idea of *instrumental distance* prompts a number of questions concerning spreadsheet integration within mathematics education such as: do the many resources available to teachers consider it? and How do teachers who have integrated spreadsheets take advantage of this distance in their practices?

The next section reports on a case study involving an experienced teacher during the first two years of her integration of spreadsheets into her teaching, showing that the evolution during the second year moves precisely in the direction of reducing this distance.

Understanding Practices with ICT: A Case Study on Integrating Spreadsheets

Taking into account the idea of distance, I turn to the question of the teaching practices, with some additional tools to support the associated analysis.

In a study concerning teachers' initial training involving the integration of CAS calculators, Trouche (2004, p. 307) had already noticed the importance of two factors relative to the teachers themselves: their degree of mastery of the tool and the range of their positivity or negativity of the representation/conception of its integration.[12] In the same way, the numerous works analysing practices inspired by the *double approach* (Robert and Rogalski 2002) underline that teachers' activity is not only related to the mathematical content to be taught or the learning experiences of the students but also to a number of teacher-related factors such as individuals exercising a job which has its own constraints and freedoms. When considering ICT integration, it is relevant to take this personal component into account.

Additional Theoretical Elements to Analyse teachers' Practices

The didactic and ergonomic approach (Robert and Rogalski 2002) is an interesting theoretical support for the analysis of teachers' practices as it frames teacher's activity through different components, one of which is this important *personal component*. By turning the spotlight onto this personal component and because we want to take into account teachers' apprehension of the instrumental issues, I distinguish a professional instrument from a personal one (Haspekian 2006) and consider their corresponding instrumental geneses, professional and personal.

Didactic and Ergonomic Approach

The didactic and ergonomic approach analyses practices by means of five components: *cognitive, mediative, institutional, social* and *personal* (Robert and Rogalski 2002). The *cognitive* and *mediative* components relate to the choices made by the

[12] The words 'representation' and 'conception' are not problematised in this chapter and used in their common senses.

teacher in the spatial, temporal and mathematical organisation of the lessons. These choices are made according to the teacher's *personal component*. The personal component relates to the teacher as a singular subject with his/her own history, practices, vision of mathematics, way of conceiving mathematics learning, teaching, etc. Yet, the personal factor is not the only one to consider. Teachers are not completely free in their choices as they are more or less constrained by *institutional* and *social* dimensions. The institutional and social dimensions relate to the curricula, lesson duration, school social habits, mathematics teachers' habit, etc.

In the case of ICT practices, instrumental aspects seem to interfere with each of these components (Haspekian 2005a). In particular, the personal component plays a crucial role in determining whether ICT in mathematics teaching is supported. For example, teachers integrate ruler and compass without any problem as they are accepted as part of the mathematical culture. This might be because historically, the ruler and compass played an essential and epistemological role in the development of mathematics. (Chevallard 1992) This role and the number of mathematical problems generated by these traditional tools serve to legitimise their place in mathematics education. Is it the same for spreadsheets? How is their introduction in mathematics teaching justified? Do teachers feel this tool relevant to their mathematics and the ways they learned, learn, do and teach mathematics?

The consideration of these questions led to the use of the instrumental approach to analyse more locally some of the phenomena observed with ICT practices, in particular the teachers' professional instrumental genesis with the spreadsheet.

Professional Instrumental Genesis

This case study shows that, at the early stages, the way that teachers orchestrate and support pupils' instrumental geneses evolves year by year. Starting from the premise that the spreadsheet as an instrument for the teacher, which allows her to achieve some teaching goals, the process of instrumental genesis is considered *from the teacher's perspective* (Haspekian 2006). The same artefact, the spreadsheet, becomes an instrument for pupils' mathematical activity and an (other) instrument for teacher's didactical activity. Thus, when applying the instrumental approach to the spreadsheet as a *teaching* instrument created by the teacher through a professional genesis, two processes are highlighted:

- A process of instrumentalisation as teachers instrumentalised the tool in order to serve didactic objectives. It is transformed from its initial functions and its didactical potential is progressively created (or discovered and adapted in the case of an educational tool).
- A process of instrumentation in which the teacher, as a subject, is required to incorporate within her (already stable) teaching schemes some new schemes that integrate the use of the tool. Progressively, the teacher will specify the use of the tool for a particular class of situations (like, for example, "take advantage of spreadsheet for algebra learning") and organise her activity in a way progressively stable for this class of situation (Ann's case already shows some regularities from year 1 to year 2).

The instrument that is created as a result of this process of professional genesis (for instance the 'spreadsheet as a tool to teach algebra') is different from the instrument built through a *personal* genesis (the spreadsheet as a tool of personal work of calculation, plotting, data treatment, etc.). From the same artefact, two instrumental geneses (that may have interferences/interactions on each other) lead to two different instruments. The spreadsheet in these two situations is not at all *the same instrument*. The second one is close to the instrument we want pupils to build. The teacher's professional genesis with the tool is much more complicated as it includes the pupils' instrumental geneses. Here again, the phenomena are imbricate and interfering.

This notion of *double instrumental genesis* together with the *didactic and ergonomic approach* is used in the next section to analyse the observation of a teacher who is integrating the use of a spreadsheet in mathematics. The case of the spreadsheet provides a good amplification of the phenomena that play in the development of ICT practices for at least two reasons. Firstly, the spreadsheet is a professional tool without any *a priori* didactical functionality. In this case, the instrumental distance is not negligible and plays a considerable role in the difficulties surrounding the integration of spreadsheets. Secondly, the teacher has to turn this non-educational tool into a didactical instrument through a process of professional genesis, a process made more complex by this instrumental distance.

A Case Study: Ann's Practices and Evolution in ICT Integration

The next section reports the data and subsequent analyses of a study that observed how a very experienced teacher integrated spreadsheets within her practices for the first time and the evolution of this integration during the subsequent year.

The Data

Ann is not a trainee; she has taught mathematics for more than 10 years but is not an expert in the use of technology within mathematics teaching and learning. She has already some experience of dynamic geometry software and now she is beginning to integrate spreadsheets in her classroom. In this first year, Ann's choices were motivated by her participation in a 1-year research project that focused on spreadsheet use for learning *algebra* (Haspekian 2005a). The data that was collected included: observations of all of her spreadsheet lessons (6 sessions); teacher interviews before and after each session; and the students' spreadsheet files. At the end of the research, an interview collected Ann's thoughts and feelings about this experience.

After the completion of the research, Ann continued to use spreadsheets in the following year. During this second year, I observed and recorded her first spreadsheet session and the subsequent session in a paper and pencil environment. I collected the problems as they were given to the students and the associated homework, and I carried out some interviews concerning her intentions for this second year.

Table 2 Ann's approach to the introduction of spreadsheet in her teaching

Use of spreadsheet	Year 1 of the introduction of the spreadsheet	Year 2 of the introduction of the spreadsheet
Variations		
Class level	7th Grade (12 year old)	8th Grade (13 year old)
Old/new content	New	Old
Mathematical domain	Algebra	Statistics
Spreadsheet location	Limited to computer classroom	Computer/ordinary classroom
Synthesis	No	Yes
Interactions teacher-students	Mostly individual	Individual and collective
Use of the video and collective presentation	Piloted by teacher, limited role	Teacher and student. Important role
Students configuration	Work by pairs	Work by pairs + collective work: one student at the board
Regularities		
Maths objectives, teacher aims	Algebra	
Additional material	Worksheet for pupils and pre-organised spreadsheet file	
Institutionalisation	In an ulterior lesson, in ordinary classroom	

The resulting analyses showed an evolution of her practice. This evolution converges towards the characteristics of experts' practices described in the next section.

During the second year, Ann introduced the spreadsheet not within algebra but within statistics (headcounts, frequencies and cumulative frequencies), after having seen these notions in paper and pencil environment. In this context, some of the observed elements were surprising as the lesson revealed very little statistical content and mostly centred on the tool use and functionalities, revealing unexpected mathematics such as notions of variable, formula and the distinction between numeric and algebraic functions. Of course, this reflects the influence of the first year of her experience, centred on algebra, but this does not explain the complete evolution (variations and regularities) summarised in Table 2 of Ann's choices for introducing spreadsheets.

In both years, Ann met the institutional demand to integrate spreadsheets within her mathematics teaching but the way that she did this was different in each year. Table 2 shows an evolution of two components. The mediative and cognitive components have evolved with respect to the chosen mathematical domain, the way that the spreadsheet was introduced and the level of the class that was chosen. This prompts the questions: Why did she evolve, and how can we state more specifically her professional genesis with the tool?

Ann's Professional Genesis with the Spreadsheet as a Didactical Tool

In both years, Ann's activity with the spreadsheet is oriented by the goal of using it to teach algebraic concepts such as variables and formulae, for example, by using the copy function, or by profiting from the numerical feedback to infer the equivalence of two formulae.

This brings into play some usage schemes[13] concerning the material and organisational aspects that are being developed from one session to another towards a more stable set of practices that concern: integrating the tool within a larger set of instruments (with the data projector); using the data projector at the beginning of the lesson to make collective explanations; requiring the pupils to communicate and work in pairs; giving an instruction sheet and a pre-built file to save time and regularly clicking on a cell to check whether pupils have edited a formula or numerical operation, or the numerical result.

In Ann's case, this professional genesis was not independent from her personal genesis with spreadsheet as the observations show how these interfered (i.e. they interacted in a relational sense) with each other.[14] These interferences were made more complex by the fact that she wanted her pupils to manipulate the spreadsheet for themselves (one could imagine a spreadsheet usage only under a teacher's control) and learn mathematics as a result of this activity. As already stated, as the pupils' instrumental geneses forms part of the teacher's professional genesis with the tool this leads to another interference.

Observation of some of Ann's activity in these first two lessons in her second year result from these interferences and an example of this now follows.

The Interferences Between the Teachers' Double Instrumental Genesis and the Pupils' Instrumental Geneses

As already mentioned, Ann chose to introduce the spreadsheet to a different class within the domain of statistics. Figure 3 is an abstract of the task she developed for her pupils that shows the corresponding spreadsheet file with the pre-edited formula built by Ann:

It is interesting to notice that Ann modified this file three times. In its first version, the formula calculating the frequency (in B7) was **= B6*100/50**. This formula, if copied along row 7 calculates the correct frequencies for the corresponding data of row 6. But it is not adequate regarding the question b.[15]

The day before the lesson, Ann realised the mistake and changed the formula to **= B6/F6*100**. She confided that she did not yet feel very comfortable with spreadsheets. Her own instrumental genesis with spreadsheets as a mathematical instrument probably plays a role here as we also see that the key point of the problem comes from the spreadsheet as a *didactic-oriented* instrument. From the point of view of the spreadsheet as a *calculation-oriented* instrument, the formula was adequate. The didactical aim (showing the mathematical dependency between

[13] Rabardel (2002) distinguishes the *usage schemes* (related to the *material* dimension of the tool) from the *schemes of instrumented action* (related to the global achievement of the task, with goals and intentions).

[14] It may not be the case for all teachers: unlike Ann's case, the first instrument can be already constituted in a more advanced way, long before trying to make it a didactical instrument.

[15] The formula refers to the value 50 for the total. If one changes the value of any headcount, then the total will change and the formula becomes wrong.

Step 2: use of formulae and the fill command

distance (km)	0<d ≤5	5<d ≤10	10<d ≤15	15<d ≤20	total
headcounts	16	14	12	8	50
frequencies (%)	32				100

1) a) What is the total number of items? _______________

Where is this number located? What is the formula to calculate it? _____________

b) If one changes the headcounts for 0<d ≤5, does the frequency change? _________

	A	B	C	D	E	F
1						
2						
3	**Etape 2 :**		**calcul de fréquences**			
4						
5	distance (km)	0<d ≤5	5<d ≤10	10<d ≤15	15<d ≤20	total
6	effectif	16	14	12	8	=SOMME(B6:E6)
7	fréquence (%)	=B6/F6*100				100

Fig. 3 Ann's final version of the formulae

numbers and frequencies) led Ann to ask the question b which resulted in an incorrect formula. She did not realise this when she first constructed her formula. At that moment, the personal instrument stands at the front of the scene, and obscures the professional instrument and its associated didactical aims (the question b.).

Interference between the personal and the professional instrument can be seen again within the continuation of the story. The new formula, =B6/F6*100, is now adequate for question b, but still not convenient if we consider the next question (Fig. 4) for inverse reasons! Ann wants pupils to copy the formula in order to fill row 7 and meet this filling functionality with the automatic increasing of cell references (B6 becomes C6…). This time, this is part of her goals for students' instrumental geneses.

The formula above, if copied along row 7, is no longer valid, as the cell referring to the total, F6, will change into G6, H6… along the row. A solution to this problem is to fix the cell F6 in the recopy by using the $ symbol. But Ann did not want this functionality to appear in the first spreadsheet session as it was above the level of instrumentation she wanted for her pupils at that moment. When she built her new formula for question b, the $ symbol was not in her mind and she did not include it, forgetting that it would create false results at question 3. The day before the session, we had a phone call to finalise our meeting during which she realised the new issue and included the $ symbol as a last-minute decision.

Thus, this time the formula was 'wrong' with regards to an instrumental goal, that is the use of the $ symbol was above Ann's instrumental objectives and she did not have it in her mind. It is neither easy nor trivial to adapt to meet all of the constraints,

3) Complete the table using the formula in B7:
Recopy the formula on the right. (see instructions below for the "cell recopy")
What is the formula contained in C7? D7? E7?

Fig. 4 The next stages of the task

particularly as she had already changed her very first version of formula for a mathematical aim and now she had to change it again for an instrumental aim. This time, the professional-oriented instrument overrode the personal one, by taking into account pupils' geneses and the level of instrumentation that she wanted them to reach.

These successive formulae disrupted the session and finally Ann put the $ sign into the formula but expected to avoid speaking about it with the pupils. Unfortunately, it arose of course during the session! Being compelled by pupils' questions to explain, she only said that it is not important to write it with a paper and pencil environment. Then, when a pupil came to the board to write the spreadsheet formula, he forgot the $, the 'division by zero Error' appeared after filling and Ann said "*now you're happy?*" but did not explain the message nor the division by zero.[16] In that sense, the perturbation due to the '$' sign appears as one of Clark-Wilson's lesson hiccups (Clark-Wilson 2010b) defined as:

> These were the perturbations experienced by the teachers during the lesson, triggered by the use of the technology that seemed to illuminate discontinuities in their knowledge and offer opportunities for the teachers' epistemological development within the domain of the study (Clark-Wilson 2010b, p. 138).

Interpretation of the Complex and Divided Geneses on the Part of the Teacher

The example above shows how the double genesis on the teacher side may interfere with pupils' geneses. The spreadsheet's constraints interacted with the teacher's goals and didactical expectations (she wanted to introduce a basic level of spreadsheet functionalities but did not want to go any further). This is evidence that she has not yet turned her personal instrument into a mathematics-teaching instrument. This process is made more complex by the different geneses at stake. As we saw in the example, it is constrained by:

- The teachers' aims for the mathematical learning, i.e. concerning statistics and algebra.
- The pupils' instrumentation that is, how to support pupils' mathematical work through their interactions with the spreadsheet i.e. the mathematical headcount-frequency dependence through the change of the frequency cell after changing the value of the headcount cell.

[16] Increment of references after filling makes the formula refer to empty cells. By default, empty cells are treated in formulas as if they contain the value 0, this option that can be changed.

- The pupils' instrumentalisation, that is the choice of functionalities to be used, the desired schemes of use, i.e. relative references and the automated increments for cell references using the copy function, but not yet the absolute references, the $ sign and its specificity in the filling of formulae.

The simultaneous management of these constraints is not easy as the spreadsheet is not *a priori* a didactical instrument. According to Rabardel's theory (Rabardel 2002) Ann's case study on making the spreadsheet a didactic instrument shows that such an instrument is, as any instrumental genesis, only developed progressively in a long-term and complex process. Here, the teacher's and the students' personal instrumental geneses are elements that are adding complexity to this professionally oriented genesis.

How to Understand Ann's Evolutions?

The way that Ann evolved from the first year to the second is related to this professional instrumental genesis.

In the previous section, using both the notions of distance and double instrumental genesis, I have described the beginning of such a genesis and analysed locally the associated complexity through the case of Ann's use of spreadsheet. In particular, the way that teachers orchestrate and support pupils' instrumental geneses evolves year after year.

Ann's goal is to use the spreadsheet to teach algebraic concepts and she develops some instrumented schemes of action for this that concern the material aspects, the organisation of the sessions and the orchestration of pupils' instrumental geneses. Ann's practice with speadsheets includes, for instance, the following elements that emerged during the first year (not necessarily since the beginning) and seemed to stabilise in the second year:

- Using a data projector at the beginning of the session to make collective explanations;
- Requiring pupils to communicate and work in pairs;
- Giving pupils a sheet of instructions and a pre-built computer file to save time;
- Regularly 'click' on individual cells to check whether pupils have edited a formula or numerical operation, or even directly the numerical result.

Some other elements of her orchestrations were modified during the second year:

- The use of the spreadsheet with a higher level of class, i.e. with Grade 8 instead of Grade 7;
- Fewer 'new' concepts were introduced at one time, i.e. the introduction of the spreadsheet and the introduction of new mathematical notions;
- She changed the mathematical domain, i.e. it was introduced within with statistics, which seemed to Ann to be more appropriate than algebra;
- A deeper articulation was made between social and individual schemes, something that Trouche (2005) has stressed the importance of within the process of instrumental geneses. In the interview, Ann said she had not organised enough moments of 'mutualisation' (whole class discussions) and she explicitly wished to take care of this point in the second year.

The next section observes these evolutions more closely, and shows that they all appear to converge in the direction of *reducing* the instrumental distance.

Changing the Class Level: Higher Level of Class

This modification comes with the change of the mathematical domain. In the French curriculum the spreadsheet is explicitly mentioned for the teaching and learning of statistics for Grade 8 pupils. In the Grade 7 curriculum the spreadsheet appears in a more general and vague way and teachers are required to reflect more deeply to define its potential for the learning of mathematical notions. These notions appear more distant from spreadsheet mathematics than within the Grade 8 curriculum, where the spreadsheet is more clearly specified with respect to precise mathematical notions. Thus, by choosing this level Ann was able to reduce the distance and match the official prescriptions more easily. In addition, during year 1 Ann did not find the Grade 7 pupils' instrumentalisation process easy. The pupils had difficulty in filling cells, selecting a single cell and editing a formula. Older pupils seemed to be more skilful and problems that were linked to instrumentalisation should interfere less with the mathematical work. With Grade 7, the manipulations of the tool seemed more difficult and the tool appeared less transparent.

The 'Old/New' Knowledge Game with Respect to the Mathematical and Instrumental Content

During year 1, Ann introduced a new instrument at the same time as she introduced some new mathematical content (algebraic notions). The relationship between the old knowledge and the new knowledge is different in year 2, which tends to reduce the instrumental distance by lessening the amount of newness. For example, all of the mathematical notions at stake in the spreadsheet session (headcounts, frequency, cumulative frequency) had already be seen previously by the pupils in the paper and pencil environment. This experience (new environment with 'already-seen' concepts) will then serve Ann as a base to introduce algebraic notions (new concepts in an 'already-seen' instrument).

Domain Changing

There are at least three reasons why the mathematical domain chosen by Ann in year 2 also reduces the distance with respect to algebra. The domain of statistics is usually seen to conform more closely to the representations within a spreadsheet than the domain of algebra. Furthermore, institutional pressure is less important in statistics than algebra, which is a more classic and traditional domain that is strongly linked to paper and pencil mathematics. On the contrary, nowadays statistics is more aligned to the use of technology. Finally, within the language of the spreadsheet, one can find terms that are more commonly used within statistics whereas the distance to the traditional vocabulary of algebra is wider (and more important) (Haspekian 2005b).

Moments of Mutualisation and Articulation with Paper and Pencil Mathematics

In her second year, Ann introduced some moments of mutualisation during her spreadsheet sessions. In the interview, she affirmed her will to increase the similarity between these sessions and the traditional ones. She felt that it was necessary to increase the links to the paper and pencil mathematics. For example, she started the sequence with a paper and pencil session, then revisited the same notions in a spreadsheet session, and then returned to the work done with spreadsheet within a subsequent paper and pencil session.

Thus, at a range of different levels, Ann's modifications tended to minimise the spreadsheet's instrumental distance. All of these actions contributed to reduce the distance with paper-pencil and to mix in a greater proximity the mathematics within these two environments.

Another notable development is that Ann's evolution gains some characteristics of experts' practices, as evidenced in the research. This is explored in the next section.

Bringing Together the Results from Different Research

In this section, I am bringing together Ann's case study with the results of a second research study. This latter research studied the practices of what we have called *expert* teachers, that is, non-novice teachers who have been integrating ICT and spreadsheet for a long time and who are also 'ICT trainers' and 'spreadsheet trainers' within the context of mathematics teacher training. By comparing the practices of these expert teachers alongside the practices of pre-service teachers, I have highlighted some overarching characteristics of practices with ICT.

An interesting outcome of this cross analysis is that Ann's evolution with the spreadsheet converges towards the characteristics of experts' practices. The next section presents this in more detail by first giving some results and regularities found in the data collected with *expert* and novice teachers.

Some Characteristics of Experts' Practices with ICT

Are there regularities of practice amongst teachers who have successfully integrated the spreadsheet? In making a comparison with novice teachers, what are the characteristics of the expert teachers' practices that seem to contribute fundamentally to their success? How do they manage the pupils' instrumental geneses? And how do they take into account the instrumental distance generated by the spreadsheet? In order to answer these questions, I looked for regularities at the following levels: in teachers' conceptions; in the evolution of their practices; and in the changes that resulted from this evolution. The notions of *coherence* and *stability* as defined by Robert & Rogalski can enlighten these questions:

> the coherence of the system of the practices of a teacher (…) would prevent the introduction of inconsistent elements with this system (Robert and Rogalski 2002, p. 521).

Within an alternative theoretical framework, the considerations of Lagrange are in the same direction. Lagrange (2000) underlines that the introduction of a tool into mathematics lessons generates an upheaval of the *praxeologies*, which may hinder its integration into the practices. How did expert teachers deal with these obstacles?

As said in the introduction, I carried out questionnaires and interviews with trainees and expert teachers. The questionnaire for trainees contained 41 questions divided in three parts (see Appendix). The first was general information about the teacher (age, training, etc.), the second concerned their general opinions about the use of technology and the third concerned their use of spreadsheet in mathematics classroom and their opinions about this. There were 23 questionnaires returned by the trainees and four additional group discussions (in groups of 3 or 4) were held in which we allowed the trainees to discuss their answers to parts 2 and 3 of the questionnaire in order to gain a better understanding of their opinions. The questionnaire given to the expert teachers was an identical one and six individual interviews lasting 2–3 h were conducted about their effective practices with ICT and spreadsheets. We also collected all of their teaching materials, which evidenced their progression in use of the spreadsheet, examples of tasks, frequency of use, etc.

The research study compares the trainees with the experts (Haspekian 2005a) and outlines some common findings about the novices, such as their obvious difficulties in perceiving the tool's potential and to conceive mathematical and classroom organisations, which as yet they had not seen or experienced. It also suggested some convergence of practice amongst the experts that can be connected to their successful integration of spreadsheets.

The first result concerns the nature of the tasks chosen for a spreadsheet use. Parts 2 and 3 of the questionnaire included a set of different spreadsheet tasks that included very basic use of the spreadsheet as a calculator to a more interesting use that took greater advantage of the spreadsheet's potential. These latter tasks were based on research situations mentioned in Capponi 2000, Arzarello et al. 2001, and Rojano and Sutherland 1997, and they had been analysed by their authors as being positive for mathematics learning. In the questionnaire we presented different ways of using spreadsheets and asked the teachers to choose which of these situations they found interesting for mathematics teaching and learning. The results of this study concurred with those from other research (Laborde 2001; Monaghan 2004), that is novice teachers who are non-expert in the use of the spreadsheet have difficulty in realising the potential of the tool and in identifying interesting situations for its use. The choices and underlying rationales of the beginner teachers were *systematically opposed* to those of the expert teachers, which corresponded to the interesting situations. Thus, the teachers' first approach to the use of spreadsheets did not take advantage of the tool's potential. As Artigue recalls, the observed (and quite understandable) tendency amongst novice users is to use technological tools not for their epistemic value (as a support to understand mathematical objects) but only for their pragmatic value (to produce results quickly and easily) within tasks that are very similar to those given in traditional paper and pencil tasks (Artigue 2002).

In the analysis of the expert teachers' practices and the subsequent comparison of these findings with the novice practices, a set of common characteristics appears (for more detail on this see Haspekian, (2005a)). This prompts the question as to whether there are fundamental elements contributing to teachers' success in the integration of spreadsheets. The first element is the importance of taking into account not a single tool but a system of instruments. This confirms the importance of the *instrumental distance* as these characteristics are a way to minimise the distance imposed by the spreadsheet. Another common characteristic was the fact that, using this system of instruments, these teachers play an *old/new game* concerning the mathematical content with equal attention to the various technological tools that they integrate. This means that they alternate new/old instruments with new/old content and do not try to introduce, for example, a new instrument with new concepts. This game also helps to articulate the work involving the technology with the paper and pencil work.

These two characteristics provide an economic way to both manage the class in ICT sessions, and to manage the pupils' instrumental geneses. For example, concerning the mathematical content, one teacher said that it offered "a way of making revisions by bringing something more". Another said that he had "the same notions presented in two different environments". A third *expert* teacher who was interviewed said that she systematically works on the same notion using by hand methods after an ICT session, and combines paper-calculator-spreadsheet and so on: "I make links non-stop, again and again…"

For all of these expert teachers, the integration of the spreadsheet is based upon this orchestration of a whole system of instruments. As they perceive the spreadsheet as more complex, they introduce it to their pupils after other software. This allows:

- **Time saving** on the management of the class in ICT sessions (introduce the classroom, organise the didactic contract, etc.);
- **Time saving** with respect to the instrumental geneses with the spreadsheet as some aspects have been addressed through other technological tools (physical manipulation of the materials, the computer room, virtual manipulation of files, etc.).

Within the common characteristics, we also found an increased attention paid to the questions of *mutualisation* and *socialisation*, which was accomplished in two ways. Firstly, the expert teachers all organised their sessions with the pupils working in pairs and secondly, the teachers have developed the habit to use the data projector in order to mutualise or bring together the scattered knowledge of the pupils leading to more homogenous mathematical and instrumental knowledge.

Table 3 resumes the common characteristics that appear to contribute fundamentally to the expert teachers' successful integration of the spreadsheet:

→ the taking into account of a system of instruments, including the articulation with the paper and pencil environment;
→ a game of *old/new,* which is played at both the level of the mathematical content and at the level of the instrument;
→ a certain art/skill to know how to mix these two games,
→ the use of *mutualisation* and *socialisation* (students work in pairs, use of the data projector).

Table 3 Some common elements found in experts' practices

What is noticeable is that some connections can be seen then between these characteristics and Ann's evolution of practice as a result of the changes she introduced in the second year.

Reducing Instrumental Distance: Towards Experts' Practices

In the analysis of the expert teachers, there were some common characteristics in their successful integration of ICT, in particular concerning spreadsheets. In this section, I will show that Ann's evolution, as analysed previously, *tends towards* some of these characteristics and gives an indication of the importance of these characteristics.

First, as seen in both cases, we find the tendency to minimise the instrumental distance. Actually, some of Ann's evolutions can be explained in terms of a *reduction* of the distance, either by making this distance more explicit or by increasing the times when she alternated the work in both the spreadsheet and paper and pencil environments, which enriched both of them. This mixing of different environments and, in particular, the articulation within the paper and pencil environment, appeared precisely as a common characteristic of the teachers who have integrated the spreadsheet successfully. Thus, it is interesting to notice that Ann's professional genesis follows the same path (even though she did not achieve a level of expert practice with respect to all characteristics). For instance, the moments of mutualisation and articulation with paper and pencil mathematics by Ann are more successful in the second year, whereas she did not pay much attention to this in the first year.

The *old/new* game mentioned above is another characteristic found in the expert teachers' practices. They manage ICT integration by adjusting and adapting the degree of novelty to incorporate a degree of complexity of the tool. When introducing a complex artefact such as the spreadsheet, they choose familiar content, which has already been introduced within the paper and pencil environment. Once the students have more familiarity with the spreadsheet with more familiar mathematical content, they use it subsequently to develop new mathematical knowledge.

Again, it can be noted that Ann's evolution is moving in that direction. In the first year, she introduced both the spreadsheet and a *new* mathematical domain (algebra), whereas in the second year she changed her approach to introduce spreadsheets by choosing an *old* mathematical domain, statistics. The pupils, having already seen statistics in a paper and pencil environment, then meet the new instrument, a spreadsheet, in the context of old content. Ann's long term intention, as stated in her interview, is to use the spreadsheet within the context of algebra, but now she intended to do this after the pupils have seen spreadsheets in another area of mathematics (an *old* one) to avoid introducing both new artefact and new contents.

Of course, when I observed Ann at the beginning of the second year, she had not achieved all of the common characteristics of the expert teachers as listed in Table 3, but this is not surprising. She was at a stage within her professional genesis with the spreadsheet where she was integrating it for the second time in her career. It is predictable that her practices are not completely stabilised and that these will continue to evolve. For instance, for the expert teachers, the game 'old/new' concerns not only the mathematical content and not only one tool, but a complex system of instruments that incorporate paper and pencil articulations. Expert teachers do not expect pupils to first meet computers through the use of spreadsheets but with other software, such as dynamic geometry software, which presents a smaller instrumental distance than the spreadsheet. In that way, pupils meet the computer classroom, the basic instructions about the use of the computers, the files, the opening and closing sessions, the articulation within the paper and pencil environment, the work in pairs, and so on, with a software that seems easier to integrate than the spreadsheet. Once they are used to these basic manipulations and orchestrations on a more familiar *old* instrument, they are ready to meet a new, more difficult one, such as the spreadsheet.

Discussion and Perspectives

In the section, I will come back to the general purpose of this work, which was to gain a better understand of teachers' practices with technology and the process of their instrumental geneses. To this aim, the previous sections have introduced some important elements and lead me to draw conclusions on their instrumental professional geneses with ICT, which I will discuss here.

I have analysed Ann's evolutions in terms of a *reduction* of the instrumental distance, either by making this distance more explicit, or by multiplying the opportunities to alternate work in the two environments, enriching both of them. This distance is more or less important, depending upon the tool. The integration of spreadsheets in the teaching and learning mathematics constitutes a significant creative task for teachers as the tool is not given with any didactical functionality. It requires a professional instrumental genesis on the teacher's side that differs from the teacher's personal genesis with the tool (even if they interfere) and different again from that of the pupils. Here again, one can hypothesise that a professional instrumental genesis with dynamic geometry software is easier.

These combined considerations helped the analysis of Ann's genesis and the conclusion that Ann tended to acquire in her evolution some of the characteristics found as commonalities among the expert teachers as follows:

- Articulation with paper-pencil mathematics;
- Moments of mutualisation and socialisation;

- The game old/new, concerning the mathematical content (not yet on the instruments for Ann).

These are all included in the experts' characteristics Table 3. The inverse is not true because Ann did not demonstrate all the characteristics of the experts. For example, in her evolution, this exploitation of different instruments to facilitate the introduction of spreadsheets does not appear yet, but it seems reasonable to think that one does not gain all of the characteristics of the expert teachers after only 1 year. This instrumental professional genesis is a long process, as is any instrumental genesis. This raises questions for the professional training of teachers such as: How to take into account the importance of working within a system of instruments instead of the isolated tools? How to take into account the *socialisation* dimension? Is it possible through these improvements to shorten the time needed for the instrumental professional genesis?

I conclude on the fact that these results are at the stage of hypotheses, as key issues in ICT integration. To extend this result, a larger scale study is needed with more than six expert teachers, and with some observations of their actual practices in the classrooms. The fact that Ann's evolution tends towards some of their common characteristics is a simple indication that these elements may constitute good *candidates* of ICT practices, but this hypothesis does requires further research.

Other questions remain for research. For example, concerning ICT integration and evolutions of teachers' practices, a criterion which we have seen as important in this chapter is the notion of instrumental *distance*. If it does reveal itself as a source of difficulty for teachers, then it is crucial to advance in the comprehension of ICT impact on mathematics and the way teachers take into account instrumental distance, drawing some important characteristics from experts' practices. However, it is also necessary to determine which elements may counterbalance this distance and may support the process of tool integration, such as institutional injunctions, or the tool's epistemic value and its didactical design. As technology evolves, the instrumental distance can thus be important for educational tool designers. As for the epistemological legitimacy, it also relates to teachers' representations and beliefs about ICT and mathematics. This dimension has been investigated in other research, see for instance Norton et al. (2000), who conclude that teachers' resistance is related to their beliefs about mathematics teaching and learning. If knowledge and beliefs about teaching mathematics with ICT are actual barriers, can this dimension be considered in teachers' training and how?

Finally, the issue of 'isolated' potential of technology for mathematics education does not solve the problem of their integration in teaching practices (for example in teaching algebra in the case of the spreadsheet), due to this instrumental distance. Several questions remain and a better understanding of the characteristics of experts' practices and of course the way to develop these, may be important also in a training perspective. This remains an open field for further research.

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Appendice-Extract of the Questionnaire Trainees and Experts

When several answers are possible, please number them according to you preferences: 1 for the 1rst, 2 etc.

PART I:

14. A **priori**, a) Which maths content do you think is more appropriate for using spreadsheet in the mathematics class? *(number if several answers)* [ ]Algebra [ ]Arithmetics [ ]Statistics [ ]Calculus [ ]Probabilities [ ]Others :

b) For which topic *(1choice or more)*? Simulations [ ] Fonctions [ ] Introducing Algebra [ ] Series [ ] Implementing algorithms [ ] Problems of approximations [ ] Resolution of algebraic problems [ ] Others [ ] :

19. How do you envisage the use of ICT in your current and future teaching ? *(number if several answers)*

[ ] rather ponctually, as a outil to free oneself from tedious calculations so that pupils focus their work on the concepts

[ ] rather for individual help and remediation for pupils who have difficulties in mathematics

[ ] rather well integrated in my year progression, as a new tool to create learning situations

20. A priori, do some tools appear to you easier to integrate than others? (*number from the most the the less)*:

[ ] dynamic geometry software (as Cabri or Géoplan) [ ]spreadsheet (as Excel) [ ]internet [ ]calculators

21. Vos professeurs avaient d'autres programmes et manières d'enseigner. Auriez-vous aimé procéder comme eux ?

22. Your teachers had other programs and ways of teaching. Would you have liked to proceed like them? [] yes, I would have liked much [] that is equal for me [] no, especially not. Explain:

PART II: "Fictitious Teachers": A, B, C, D, E and F are 6 fictitious maths teachers. The first five ones have never really integrated ICT in their teaching, hjere are their arguments:

<u>A:</u> *This year, I was firmly decided to use ICT with my pupils, but the key of the computer classroom seemed so difficult to obtain that I finally gave up*

<u>B:</u> *I have already thought about having a computer session for my pupils. But, I felt afraid not to know well how to manage technocally (especially that some pupils manage surely better than me) so I gave up*

<u>C:</u> *I wish I could use ICT to teach mathematics, but this requires too much time of preparation!!*

<u>D:</u> *I do not have anything against using ICT with my pupils, but I've never been trained to integrate it in my teaching. If I were suitably prepared for that, I would do it readily "*

<u>E:</u> *Honesly, I feel as abnormal incorporating ICT in our programs because our role is to teach mathematics. It is obvious that if I had the choice,, I would teach mathematics without using any ICT*

23. *Have you already felt or thought like one of these teachers? (number if several answers)* **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ]
Was that concerning a particular tool ?

24. According to you, which one could concretely happen to you? **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ] *(number if several answers)*

25. Que pensez-vous plus précisément de chacune des déclarations ? *A – B –C – D et E*

26. According to their declaration, **which** of these 5 teachers appears to you as beeing: *(number if several answers)*:

a) the least fictitious: **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ] **b)** the most representative: **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ]

27. Which of them do you feel the most resembling? **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ] *(number if several answers)*

28. Which of them do you feel the less resembling? **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ] *(number if several answers))*

29. And you? What is your position concerning the use of ICT in mathematics teaching?

30. In general, how do you see the introduction of computer tools into mathematics teaching? (use 5 or more adjectives to describe your feeling) (if needed, put at the back any other comment)

31. Lastly, teacher F says: "*I do see well the value of teaching mathematics with ICT, I would like, for example, use the spreadsheet with my 8th grade pupils, who have already an experience of it"*

A colleague proposes 2 activities to him, each of them having an instruction sheet to guide pupils and a spreadsheet file prepared by the teacher:

<u>1:</u> "Formulas" : ***A ready-made worksheet contains a formula in B3, the pupils must initially identify that the value in B3 depends on the values in A1 and C1, then identify the formula and use it to answer the question 2.***

Instructions:

1. Replace 8 and 9 by other integers and observe what happens.
2. What numbers must the cells A1 and C1 contain for having 50 in cell B3? And 100? and 300? Can we get all the integers? Explain.

Spreadsheet file ready made

	A	B	C
1	8		9
2			
3		43	

(the formula in B3 is here "=2*A1+3*C1")

2 : **"Theorem of Pythagore" :** ***Students draw on paper 5 right-angled triangles, then use a spreadsheet file ready-made by the teacher to calculate automatically the squares of the three sides and the sums of the numbers***

Instructions:

1. Draw 5 right-angled triangles of different measures and fill the columns B, C D *the spreadsheet calcules automatically the results in columns E, F, G and H*
2. What do you notice ?

Spreadsheet **(formulas in E, F, G et H are ready made):**

	A	B	C	D	E	F	G	H
1		Mesures des 2 côtés de l'angles droit		Mesure de l'hypoténuse	Carrés des côtés de l'angle droit		Somme de ces carrés	Carré de l'hypoténuse
2								
3	Triangle 1							
4	Triangle 2							
5	Triangle 3							
6	Triangle 4							
7	Triangle 5							
8								

Which one would you advise him according to the different hem ay have: mathematical interest, classroom management, easyness to integrate in a progression (*explain your choice for each criterion*)

32. What about you ? If you had to, which one of these 2 activities would you be ready to implement in the classroom? rather 1 [] rather 2 [] Why?

33. Which one would you rather have as a student? rather 1 [] rather 2 [] Why.

PART III: "A supposition…"

Suppose that tomorrow you want or need (for whatever reason) to make your Grade 8 pupils use the spreadsheet to write algebraic formulas (they have experience in spreadsheets but little knowledge in algebra). Your objective is that the pupils find, from the given spreadsheet-file (a sequence of consecutive integers), the formula "2n+1" as the general expression of an odd number. A colleague proposes to you the two following statements:

	A	B
1		
2	0	
3	1	
4	2	
5	3	
6	4	
	…	

Statement A: Enter in the cell B2 "=2*A2+1", pull down the filling handle. What do you notice? Can you explain it?

Statement B: From the numbers of the column A, find a general formula that makes in column B a sequence of odd numbers.

34. According to you, which are the advantages and disadvantages of these 2 statements?

35. With the same file and the same objective, which of the 2 statements would you use unchanged (without any modification)? [ ]rather **A** [ ] rather **B** [ ]**None:** : I would have modifications to be made, here is my statement:

Topics: writing of formulas.

Objective: Find, through the use of the spreadsheet, the formula "2n+1" as a general expression of an odd number

File given :

	A	B
1		
2	0	
3	1	
4	2	
5	3	
6	4	
	…	

Personal statement :

36. Let's try to build up a sequence:

a) How do you introduce the selected activity? With what set of instructions?

b) A priori which are the possible strategies of the students, and with which functionalities of the spreadsheet?

c) What are the foreseeable difficulties and which help can you bring so that the objective is achieved?

d) In view of b) and c) (foreseeable strategies, difficulties and assistances) describe the way you would management the chosen activity (your role, that of the students, the different phases of the sequence…)

37. What would you write in the copybook about this activity?

38. Which precise continuation would you give to this activity?

39. On the topic "introduce students to algebraic work", if your objective was to make students comfortable **in the writing of algebraic expressions to solve problems by equations and handle variables in formulas**, which progression would you build around this activity (notions, concepts that you plan before, after)?

Progression		
Before: - …… - ……	**Odd numbers activity**	**After:** - …… - ……

40. Let us go back to current reality: today, would you use the activity you have chosen? yes [ ] non [ ]

41. Why?

Annexe A- Le questionnaire initial intégral

Consigne : quand plusieurs réponses sont permises, les classer par ordre de préférence:1 pourla 1ère, 2etc...

PREMIERE PARTIE : Vous connaître : Madame [ ] Monsieur [ ] Année de naissance :……….

1. Dans quel type d'établissement enseignez-vous ? ZEP [ ] Difficile [ ] Sensible [ ] Normal [ ] Bon [ ]

2. Avec quelle(s) classe(s) ? Pour combien d'heures de cours/ soutien/ aide individualisée etc ?

Classe : ……… pour ……….h……de cours et …….h………de……………………….

Classe : ……… pour ……….h…… de cours et …….h………de……………………….

3. Vous êtes : Certifié [ ] Agrégé [ ] Dernier diplôme obtenu :………………………………………

4. Avez-vous exercé une autre profession avant d'être enseignant ? oui [ ] non [ ]

Si oui, laquelle ?…………………………………………………………………

5. Avez-vous déjà enseigné auparavant ? oui[ ] non[ ] Si oui, combien d'années ?………Dans quelles classes ? ………………......

6. Vous intéressez-vous à : L'histoire des mathématiques [ ] L'informatique [ ] La didactique [ ]

Autre(s)[ ] :……………………………………………………………………………………………………….

7. a)Connaissez-vous la littérature enseignante ? oui [ ]laquelle :……………………… non [

b)Consultez-vous régulièrement des sites enseignants ? oui[ ]lesquels :……………………………… non [ ]

8. Lisez-vous des revues en lien avec les mathématiques ? Jamais [ ] Parfois [ ] Régulièrement [ ]

9. Au cours de votre scolarité au collège ou au lycée, avez-vous eu l'occasion ?

- d'utiliser en classe de mathématiques un logiciel d'enseignement ? oui [ ]Préciser :………………………. non [ ]
- de consulter des sites web concernant les mathématiques ? oui [ ] non [ ]

10. Au cours de vos études (hors IUFM) avez-vous eu des formations en informatique ou suivi personnellement des cours d'informatique (à l'université ou avec des organismes privés) ? oui [ ] non [ ]

Préciser :……

11. Equipement : possédez-vous un ordinateur personnel ? oui [ ] non [ ] une adresse électronique ? oui [ ] non [ ]

12. Connaissances informatiques : Savez-vous	oui	un peu	non
déplacer, copier, supprimer un fichier ?			
créer un document texte (ex : avec Word) ?			
créer un tableau dans un document texte ?			
mettre en forme (styles…) un document texte ?			
créer automatiquement une table des matières ?			
utiliser un éditeur d'équation pour écrire des formules mathématiques dans un texte ?			
composer des pages html ?			

13. Tableur : Avant l'IUFM saviez-vous	Avant			Et maintenant ?		
	oui	un peu	non	oui	un peu	non
- saisir et utiliser des formules dans un tableur ?						
- créer un graphique à partir de données saisies dans un tableur ?						

14. A **priori**, a) à quel domaine des mathématiques l'usage du tableur vous semble-t-il le plus approprié ? *(1 choix ou plus, dans ce cas numéroter)* [ ]Algèbre [ ]Arithmétique [ ]Statistiques

[ ]Analyse [ ]Probabilités [ ]Autres :…………………………………………

b) pour quelle partie *(1choix ou plus)*? Simulations[ ] Fonctions[ ] Introduction de l'algèbre[ ]

Mise en oeuvre d'algorithmes[ ] Suites[ ] Problèmes d'approximations[ ]

Résolution de problèmes d'algèbre[ ] Autres[ ] :……………………………………………

15. Enseignement : Connaissez-vous	oui, très précisément	un peu, vaguement	non, pas du tout
16. les logiciels installés dans votre établissement ?			
17. le matériel informatique (salles, postes…) dont il est équipé ?			

18. Votre tuteur utilise-t-il l'outil informatique ? très régulièrement [ ] assez souvent [ ] jamais [ ]

19. Savez-vous s'il y a des logiciels de mathématiques accessibles aux élèves au CDI ? je sais [ ] je ne sais pas [ ]

20. Comment envisagez-vous l'usage de l'ordinateur dans votre enseignement actuel et futur ? *(numéroter si plusieurs réponses)*
[ ] plutôt ponctuellement, comme outil pour se dégager des calculs fastidieux et concentrer le travail sur les concepts
[ ] plutôt pour l'aide individualisée et la remédiation avec les élèves en difficultés
[ ] plutôt bien intégré à ma progression annuelle, comme nouvel environnement pour créer des situations d'apprentissage
21. A priori, certains outils vous paraissent-ils plus faciles à intégrer que d'autres ?*classer du plus facile(1) au moins facile(4) :*
[ ]logiciel de géométrie dynamique (tels Cabri ou Géoplan) [ ]tableur (tel Excel) [ ]internet [ ]calculatrice

22. Vos professeurs avaient d'autres programmes et manières d'enseigner. Auriez-vous aimé procéder comme eux ?
[ ]oui, j'aurais beaucoup aimé [ ]ça m'est égal [ ]non, surtout pas. Expliquer :...
....

DEUXIEME PARTIE : « Des professeurs fictifs »

A, B, C, D, E et F sont 6 professeurs de mathématiques fictifs du second degré. Les 5 premiers n'ont jamais réellement intégré l'informatique dans leur enseignement, voici leurs arguments :

<u>A:</u> *« Cette année, j'étais fermement décidé à utiliser l'informatique avec mes élèves, mais la clé de la salle semble si difficile à obtenir que j'ai finalement abandonné »*

<u>B:</u> *« J'ai déjà envisagé de préparer une séance informatique pour mes élèves. Mais, j'ai eu peur de ne pas bien savoir me débrouiller techniquement (surtout que certains élèves se débrouillent sûrement mieux que moi) alors j'ai renoncé »*

<u>C:</u> *« Je voudrais bien utiliser l'informatique pour enseigner les mathématiques à mes élèves, mais ça demande trop de temps de préparation !! »*

<u>D:</u> *« Je n'ai rien contre utiliser l'informatique avec mes élèves, mais on ne m'a jamais formé à l'intégrer dans mon enseignement. Si j'étais convenablement formé, je le ferais volontiers »*

<u>E:</u> *« Franchement, je ressens comme anormale la présence de l'informatique dans nos programmes car notre rôle est d'enseigner les mathématiques. Il est évident que si on me donnait le choix, j'enseignerais à mes élèves les mathématiques sans faire intervenir l'informatique »*

23. Avez-vous déjà ressenti ce qu'exprime l'un d'eux ? *(numéroter si plusieurs réponses)* **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ]
Envisagiez-vous un outil informatique en particulier ?...

24. Selon vous, cela pourrait-il concrètement vous arriver ?**A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ] *(numéroter si plusieurs réponses)*
25. Que pensez-vous plus précisément de chacune des déclarations ? *(compléter au dos si besoin)*
A :...
....
B :...
....
C :...
....
D :...
....
E :...
....
26. D'après leur déclaration, **lequel** des 5 enseignants vous paraît être *(numéroter si plusieurs réponses)* :
a) le moins fictif : **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ] **b)** le plus représentatif : **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ]
27. Duquel vous sentez-vous le plus proche ? **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ] *(numéroter si plusieurs réponses)*
28. Duquel vous sentez-vous le moins proche ? **A**[ ] **B**[ ] **C**[ ] **D**[ ] **E**[ ] *(numéroter si plusieurs réponses)*
29. Et vous ? Quelle est votre position concernant l'utilisation d'un ordinateur dans les cours de mathématiques ?
....
....
30. En général, comment ressentez-vous l'introduction de l'outil informatique dans les programmes de mathématiques ? (utiliser 5 adjectifs, ou plus, pour décrire votre sentiment) (si besoin, mettre au dos tout autre commentaire)
....
31. Enfin, **<u>le professeur F dit :</u>** *« Je vois bien à quoi l'informatique peut servir, je voudrais, par exemple, utiliser le tableur avec mes 4è qui en ont déjà une expérience»*

Un collègue lui propose 2 activités que voici : (*Il s'agit à chaque fois d'une séance en salle informatique où les élèves ont une fiche élève guidant leur travail et un fichier-tableur préparé par le professeur)*

1 : "Formules" : ***Une feuille de calculs toute prête contient une formule en B3, les élèves doivent dans un premier temps repérer que la valeur en B3 dépend des valeurs en A1 et C1 puis identifier la formule et l'exploiter pour répondre au 2.***

Fiche Elève :

1. Remplacer 8 et 9 par d'autres nombres entiers et observer ce qui se passe.
2. Que placer dans les cellules A1 et C1 pour obtenir 50 dans la cellule B3? Et 100 ? et 300 ?
   Peut-on obtenir tous les nombres entiers ? Expliquer

Tableur **(feuille prête, formule déjà créée):**

	A	B	C
1	8		9
2			
3		43	

*(la formule entrée en B3 est ici"=2*A1+3*C1')*

2 : "Théorème de Pythagore" : ***Les élèves tracent d'abord sur papier 5 triangles rectangles puis calculent les carrés des 3 côtés grâce à une feuille de calculs déjà prête qui calcule des carrés et des sommes de nombres.***

Fiche Elève :

1. Tracer 5 triangles rectangles de mesures différentes et compléter les colonnes B, C et D *le tableur calcule automatiquement les résultats en E, F, G et H*
2. Que remarque-t-on ?

Tableur **(feuille prête, formules dans E, F, G et H déjà créées):**

	A	B	C	D	E	F	G	H
1		Mesures des 2 côtés de l'angles droit		Mesure de l'hypoténuse	Carrés des côtés de l'angle droit		Somme de ces carrés	Carré de l'hypoténuse
2								
3	Triangle 1							
4	Triangle 2							
5	Triangle 3							
6	Triangle 4							
7	Triangle 5							
8								

Laquelle lui conseilleriez-vous en fonction des différents critères qu'il peut avoir : intérêt mathématique, gestion de classe, facilité à l'intégrer dans une progression *(expliquer votre choix pour chaque critère)* :

……

……

……

……

……

……

……

32. Et vous, s'il le fallait, laquelle seriez-vous prêt à mettre en œuvre en classe ? plutôt la 1[ ] plutôt la 2[ ]

Pourquoi ?………………………………………………………………………………………………………

……

……

……

……

……

……

……

33. Laquelle auriez-vous préféré avoir en tant qu'élève ? plutôt la 1[ ] plutôt la 2[ ] aucune [ ]

Pourquoi ?………………………………………………………………………………………………………

……

TROISIEME PARTIE : « Une supposition… »

Supposons que, demain, vous vouliez ou deviez (pour une raison quelconque) utiliser le tableur pour faire travailler vos élèves de 4[e] (ayant déjà une expérience du tableur mais peu de connaissances en algèbre) sur l'écriture de formules algébriques. Votre objectif est que les élèves trouvent, à partir du fichier-tableur ci-contre (une suite d'entiers consécutifs), la formule **« 2n+1 »** comme expression générale d'un nombre impair. Un collègue vous propose les 2 énoncés suivants :

	A	B
1		
2	0	
3	1	
4	2	
5	3	
6	4	
	…	

Enoncé A : Dans la cellule B2, tape : « =2*A2+1 », tire la poignée de recopie vers le bas. Que remarques-tu ? Peux-tu l'expliquer ?

Enoncé B : A partir des nombres de la colonne A, trouver une formule générale qui donne, dans la colonne B, des nombres impairs.

34. Quels sont, pour vous, les avantages et inconvénients de ces 2 énoncés ?

	Avantages	Inconvénients
Enoncé **A**		
Enoncé **B**		

35. En gardant le même fichier et le même objectif, lequel des 2 énoncés donneriez-vous à vos élèves tel quel (sans aucune modification) ? [ ]Plutôt **A** [ ]Plutôt **B** [ ]**Aucun** : j'aurais des modifications à apporter, voici mon énoncé :

Thème : écriture de formules.

Objectif : Trouver, grâce au tableur, la formule « 2n+1 » comme expression générale d'un nb impair

Fichier donné :

	A	B
1		
2	0	
3	1	
4	2	
5	3	
6	4	
	…	

Enoncé personnel :

36. Essayons de construire un scénario…:

a) Comment introduisez-vous l'activité choisie ? Avec quelle (s) consigne(s) ou contrat de travail ?……………………..
……….
……….

b) A priori quelles sont les stratégies possibles des élèves, et avec quelles fonctionnalités du tableur ?
……….
……….
……….
……….

c) Quelles sont les difficultés prévisibles et quelle aide pouvez-vous apporter pour que l'objectif soit atteint ?
……….
……….
……….
……….

d) Au vu du b) et c) (stratégies, difficultés et aides prévisibles) décrire la gestion que vous feriez de l'activité choisie à travers les grands moments a priori du scénario : votre rôle, celui des élèves, les différentes phases de la séance suivant les changements de prise de parole (professeur/élève)
……….
……….
……….
……….

37. Que feriez-vous écrire dans le cahier de cours concernant cette activité ?……………………………………………
……….
……….
……….
……….

38. Quelle suite précise donneriez-vous à cette activité ?………………………………………………………………
……….
……….
……….
……….

39. Sur le thème : « faire entrer les élèves dans un travail algébrique », quelle progression construiriez-vous autour de cette activité (notions, concepts que vous planifiez avant, après) si votre objectif final est que **les élèves soient à l'aise dans l'écriture d'expressions algébriques pour résoudre des problèmes par équations ou manipuler des variables dans des formules ?** *(compléter au dos un tableau suivant ce modèle)*

Progression		
Avant : - …… - ……	**L'activité des nombres impairs**	**Après :** - …… - ……

40. Retournons à la réalité actuelle : aujourd'hui, utiliseriez-vous l'activité que vous avez choisie ? oui [ ] non [ ]

41. Pourquoi ?………………………………………………………………………………………………………
……….
……….
……….
……….

Merci de votre collaboration

Si besoin, seriez-vous d'accord pour que l'on vous contacte pour un entretien ?(téléphone ou mail)…………………………

Noter ici et au dos toute remarque supplémentaire que vous avez envie d'exprimer :

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