Teaching Mathematics with Technology at the Kindergarten Level: Resources and Orchestrations

Ghislaine Gueudet, Laetitia Bueno-Ravel, and Caroline Poisard

Abstract In this chapter we study the use of software in mathematics by French kindergarten teachers who are working with 5 and 6-year-old children. We retain the theoretical perspective of the documentational approach, considering that teachers interact with a variety of resources, including technology. These interactions lead to the development by the teachers of documents, associating resources and professional knowledge. We focus here on the way teachers organise the available resources, for a given mathematical objective through the orchestrations they choose. By focusing on three teachers in particular, we identify different types of orchestrations, evidencing teacher agency and a specific attention to individual children's differences. Teacher knowledge of different kinds (pedagogical knowledge, knowledge about curriculum material, knowledge about the teaching of numbers at kindergarten) influences the choice of orchestration.

Keywords Abacus • Documentational approach • Genesis • Kindergarten • Orchestration • Resources

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G. Gueudet (✉) • L. Bueno-Ravel
CREAD, University of Brest, IUFM Bretagne, site de Rennes,
153 rue Saint Malo, 35000 RENNES, France
e-mail: Ghislaine.Gueudet@bretagne.iufm.fr

C. Poisard
CREAD, University of Brest, IUFM Bretagne, site de Quimper,
8 rue Rosmadec BP 301, 29191 QUIMPER cedex, France

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Teacher Resources and Orchestrations

Instrumental Orchestrations, Orchestration Types

The concept of instrumental orchestration (introduced in Trouche 2004) develops the instrumental approach for the study of teaching and learning of mathematics to include a focus on technology integration. The instrumental approach (Verillon and Rabardel 1995) distinguishes between a given artefact (here, a mathematical software) and an instrument developed by the subject (here, a student) using this artefact. Along with one's use of the artefact, in different contexts for a similar aim, one develops knowledge about the artefact itself, about its use, and also other kinds of knowledge (in particular, here, mathematical knowledge). The instrument is composed of the artefact and the knowledge developed. The process of development of this instrument is called an instrumental genesis. Instrumental orchestration describes how a teacher guides the instrumental geneses of the children, using a given piece of software. It comprises two aspects: a didactical configuration and an exploitation mode. A didactical configuration is an arrangement of artefacts in the environment, while an exploitation mode refers to the way the teacher decides to exploit this didactical configuration. The configuration and the exploitation mode are not only material organisations, but they also encompass precise didactical objectives, in terms of the mathematical knowledge at stake in the situation.

Drijvers (2012) has refined and clarified the concept of orchestration. He introduces in particular a third element of orchestration, the didactical performance, in order to distinguish between what has been planned and what actually happens in class. The didactical performance "involves the ad hoc decisions taken while teaching on how to actually perform in the chosen didactic configuration and exploitation mode: what question to pose now, how to do justice to (or to set aside) any particular student input, how to deal with an unexpected aspect of the mathematical task or the technological tool, or other emerging goals" (Drijvers 2012, p. 266). In addition, having followed several teachers, Drijvers characterises seven different orchestration types: Technical-demo (demonstration of tools techniques by the teacher); Explain-the-screen (whole-class explanation of what happens on the screen); Link-screen-board (explanation by the teacher of the link between the screen and mathematics written on the board); Discuss-the-screen (whole-class discussion about what happens on the computer screen); Spot-and-show (showing interesting student's work); Sherpa-at-work (a student carries out actions requested by the teacher); and Work-and-walk-by (the children work individually on computers; the teacher observes their work and intervenes if necessary).

In our study, we adopt this definition of orchestration, and the idea of characterising orchestration types. We assume from the beginning that orchestration types observed at kindergarten will be different from the ones observed in previous studies, which all take place at the secondary school level. Some differences may arise due to the fact that the orchestrations are at the kindergarten level where, in France, there is much less whole-class teaching and less mathematics written on the board.

Orchestration Choices, Resources and Geneses

Orchestrations can be considered as the choices made by teachers about the use of technology in their classrooms. Which factors are likely to influence these choices? Ruthven (2012) proposes the following five dimensions related to classroom practices of a teacher integrating technology: the working environment; the resource system; the activity format; the time economy; and the curriculum script (described as a set of goals and actions). These dimensions can also enlighten the orchestration choices. The working environment, that is the material conditions of teacher work, is certainly important. In France, only a few kindergartens have a computer lab; in most cases, there are one or two computers in each classroom, and not always a digital projector. It certainly contributes to the rarity of technology use. But, as Ruthven demonstrates, this material aspect does not explain everything.

In our work, we focus mostly on the resource system and on the curriculum script, as well as, more generally, teachers' professional knowledge. We use the perspective introduced by the documentational approach (Gueudet and Trouche 2009). In their professional activity teachers interact with a great variety of resources, including curriculum material, children's work and software. Clark-Wilson (2010), when considering teachers' use of technology, demonstrates that teacher knowledge shapes their use of technology and that, simultaneously, the use of technology contributes to teacher learning. Instrumental geneses (Verillon and Rabardel 1995) also occur for teachers engaged in their professional activity, and using technology. Similar processes happen when teachers interact with textbooks (Remillard 2012). The development of the documentational approach is based on accounting for this multiple resource use and learning. This approach considers that, for a given professional aim, the teacher interacts with sets of resources such as textbooks, official texts, websites and software. If a teacher has already taught this topic, he/she certainly also uses previous notes and children' worksheets in preparing future lessons. When he/she creates the lesson in class, children' productions and reactions also constitute resources. All this belongs to what we call the *teacher's documentation work*. Teacher knowledge intervenes in this work. On the one hand, the knowledge influences the use of resources (this part of the process is called *instrumentalisation,* referring to the instrumental approach); on the other hand, the use of resources leads to evolution of the knowledge *(instrumentation*). We call this process *a documentational genesis*. Within such geneses, for different teaching objectives, in different classes, the teachers constitute a resource system, which is an organised set of resources, transformed in the course of their use in class (Gueudet and Trouche 2012). We consider that orchestration choices are influenced by teacher professional knowledge and by their resource system. We especially focus on the knowledge linked with the mathematical content (but we do not refer to precise categories, like those proposed by Ball et al. (2008)).

Within this perspective, the research questions addressed in this paper are:

- Which orchestrations do kindergarten teachers choose when using technology in their teaching of mathematics? Is it possible to identify 'orchestration types'? If so, do these resonate with those that Drijvers (2012) identified at the secondary school level?

– Which of the following factors influence these choices of orchestrations: the resources and their features (the software in particular) within an instrumentation process? or the teacher knowledge within an instrumentalisation process (and if so, which kind of knowledge)?

In order to answer these questions, we draw on data gathered during a design and research project that we now present.

The 'Mathematical Package' Project

Our work takes place in the frame of a contract with the French education ministry and the French Institute for Education (IFÉ), 'the mathematical package' project, which aimed to design mathematical tools and situations for the teaching of mathematics at the kindergarten level and in grades 1 and 2. In France, kindergartens reside within primary schools and most children attend from the age of three. They comprise three classes: young section, middle section and older section. Our work for kindergarten concerns only the older section (children aged 5–6).

The study presented here draws on the work of a group of teachers and researchers where the teachers created lessons in their classes that were observed and videotaped by researchers and subsequently discussed during working group meetings. The teachers wrote descriptions of their lessons, with the aim of sharing them with other colleagues. As researchers, we participated in the group. We presented several pieces of software to teachers, including those considered in this paper, but we did not intervene in the teachers' choices or on their lessons with this software. Our intervention in the design concerned more the format of the lesson descriptions in particular, the relevant categories. At the same time we studied the material produced by the group with the aim of analysing orchestrations and documentational geneses. This process was supported by the completion of a questionnaire by the teachers that asked about their use of resources and, in particular, the technological ones.

The data we gathered for each teacher were:

- Notes from the group meetings (always taken by a researcher);
- Videos of the lessons, with accompanying field notes of the observing researcher;
- Resources used by the teacher in her preparation, produced by the teacher for the children, and produced by the children during the lesson;
- Lesson plans elaborated by the teacher for colleagues (some of these descriptions are written individually and others by several teachers working together);
- Questionnaires completed individually by the teachers;
- Children's work.

The data was analysed with two specific aims. Firstly we wanted to describe the orchestrations developed by the teachers. The observations, videos and lesson descriptions provided us with information about the teachers' configurations, exploitation modes and their didactical performances. An orchestration can correspond to a short amount of time spent in class (some of the orchestrations described by Drijvers (2012) lasted only 10 min). In our work we generally considered longer time periods because we were interested in the descriptions produced by the teachers. Naturally these descriptions cannot exist within a timescale of a few minutes.

Secondly, we wanted to understand the factors shaping these orchestrations, especially those linked with teacher professional knowledge and with teacher resource systems. Hence we analysed the teachers' questionnaire responses alongside their lesson descriptions and the other resources. The lesson descriptions were essential data because the teachers themselves wrote them, so they did not contain the researchers' interpretations. They also provided evidence of what the teacher thought important to emphasise to their colleagues. We consider that such descriptions are linked with a teacher's knowledge, and in particular the knowledge developed during the use of the software. Naturally, descriptions proposed by teachers carry specific biases as they correspond more to the view of the teacher on his/her teaching than to the actual practice. However, a comparison of the descriptions alongside the videos shed light on the teachers' orchestration choices.

We worked with seven teachers during the whole academic year 2011–2012. In this chapter we select the cases of three of these seven teachers; all of whom have 12-15 years experience at kindergarten level. None of them had used software in their mathematics teaching before the beginning of the research. One teacher used the abacus, both material and virtual and the other teachers both used a specially designed software program.

Instrumental Orchestrations: Two Case Studies at the Kindergarten Level

Learning Numbers with the Virtual Abacus

In this section we focus on Deborah, a kindergarten teacher and a member of the research group. The classroom work with the abacus (which we will refer to as the abacus-lessons) lasted 12 sessions and involved number sense. After presenting an outline of these lessons, we focus on specific aspects of it: the introduction of the virtual abacus and the interaction between the teacher and the children.

The Chinese abacus, both virtual and material, was the central resource in the lessons we followed. The virtual version used by Deborah was developed by Sésamath – IREM of Lille and is available online.[1] On the virtual abacus, the children can move one or several beads by clicking on a bead with the mouse. They have some feedback from the software as they can verify their work by using the icon 'see number', which is written in numeral form. One important feature of the software is that there is no possibility for the teacher to save student work. The teachers have three opportunities to find out what the children have done: they can observe them manipulating the abacus, they can ask a child to show a manipulation on the board using the digital projector or interactive white board (IWB, see below) or they can offer a paper and pencil task as the only way to keep a record of the work done.

[1] http://cii.sesamath.net/lille/exos_boulier/boulier.swf

Fig. 1 Deborah's class organisation before the IWB: teacher using a digital projector (*left*) and the children manipulating the physical abacus (*right*)

Fig. 2 Deborah's classroom organisation with the IWB and laptops (*left*) and the children manipulating the virtual abacus (*right*)

The Abacus-Lessons in Deborah's Class

During the sequence of lessons, the general classroom organisation was the same: the 24 children sat in three groups of eight and Deborah spent 20 min with each group, thereby repeating the session three times. The school had a computer room with an IWB but Deborah thought this solution would be too complicated for young children. The [http://cii.sesamath.net/lille/exos_boulier/boulier.swf] abacus-lessons took place in Deborah's regular classroom. However, the classroom equipment changed in April 2012, about half way through the lessons. From November to March, the classroom was equipped with a digital projector (session 1–7, Fig. 1), and from April to June with an IWB and eight laptops (session 8–12, Fig. 2).

This change in the material environment resulted in changes in Deborah's exploitation mode, which we identify as a process of instrumentation. During sessions 1–7, the children learned to show and read numbers on the Chinese abacus, with one physical abacus per student. They manipulated the physical abacus to show a number and the digital projector allowed the teacher to show a correction with the virtual abacus. Deborah also asked the children to read numbers shown on the virtual abacus. So, for the first seven sessions, the children did not manipulate the virtual abacus. For the last five sessions (8–12), the resources available were the virtual abacus, the IWB and also student worksheets (Appendix 2). The sessions were organised in two phases. The first phase was an introduction during which Deborah asked the children to show numbers on the virtual abacus displayed on the IWB, and a discussion about the different possibilities for showing numbers was organised. This phase is what we call an 'investigation approach' (Poisard and Gueudet 2010), when the children are asked to show their work and to argue about the validity of suggestions. During the second phase, the children were asked to show and read numbers on a hand-out given by the teacher. Each student has a computer and can use the virtual abacus as a help to answer questions (Appendix 2). During the last two sessions, Deborah introduced a paper 'abacus-book' in the class representing numbers on the abacus and the equivalent numbers in numeral. When a number was shown on the virtual abacus, the page concerning the number was put on the board as a record of different ways to show numbers when needed.

Introduction of the Software

We observed two important moments during the initial introduction of the software in the classroom. The first moment occurred during session 1, when the children worked with the physical abaci. They did not directly manipulate the software but they were asked to read numbers projected using the software. Some children had difficulties connecting the horizontal physical abacus and the vertical virtual one. Moreover, the gestures needed are different between the two abaci. For example, to show seven on a physical abacus, only one gesture is needed (beads are carried on the central bar, pinching them between the forefinger and the thumb) while two gestures are needed on the virtual abacus (two moves with the mouse, above and below the central bar). The children overcame these difficulties quickly.

During sessions 1–7, Deborah planned a didactic configuration and an associated exploitation mode, where the children used the physical abacus. After organising a discussion and argumentation session between the children, Deborah used the virtual abacus as a means of collective explanation and correction (because it permits a projection, visible for all, and also because of the 'display the number' facility). There was in this case, in session 1, a short Technical-demo orchestration, where the teacher presented the features of the virtual abacus. Then most of the abacus use during sessions 1–7 corresponded to an 'Explain-the-screen' orchestration (Drijvers 2012), since explanations given by Deborah exceeded the technical aspects and also comprised mathematical knowledge. The following extract corresponds to such an Explain-the-screen orchestration in Deborah's class (the French version of the extracts is provided in Appendix 3; this is our translation). This discourse took place at the beginning of session 3 and it was stimulated by the children discussed how to show 5 on the physical abacus. Deborah was sitting with the children and she moved to the screen to show the children' suggestions.

Deborah: You suggested to me… Laurie, you suggested to me to move the five beads on the red rod, ok. Why? [*The unit rod is red on the virtual abacus and others are green. Deborah moves 5 as five 1-unit counters on units*].

Laurie: Because, the ones above, they are useless.

Deborah: Because you think the above beads, they are useless. Do you agree with Laurie's choice?

Some children:	No!
Deborah:	No?
A student:	Because the above beads, they mark 5.
Deborah:	We are going to check, What did I ask to show?
A student:	Five!
Deborah:	Very well. [*Deborah activates the icon "see number" and 5 appears on screen*]
Some children:	Five.
Deborah:	So, is Laurie's choice right?
Few children:	Yes!
Deborah:	Now, there is another possibility for five. Some children moved one 5-unit counter that means one above bead. [*Deborah shows on screen number 5 as one 5-unit counter, the icon "see number" is inactivated.*] So, how many ways can we show five? Maëlle?
Maëlle:	Two.
Deborah:	Yes, two ways. Either we activate the five lower beads, or I activate one of the higher beads. [*Deborah shows on screen the two possibilities.*]

In this extract, Deborah's didactical performance corresponds to what she had planned before the lesson. Her aim was to elicit the two ways to show five: with five 1-unit beads or with one 5-unit bead. The children participated in the discussion and Deborah used their suggestions; but their responsibility remains limited, which means that this orchestration is more teacher-centred. We observed an evolution towards the children having more responsibility during the second half of the abacus lessons.

Using the Software on Laptops and on the IWB

In session 8, the children started to manipulate the software using the laptops and the IWB and they discovered specific features of the virtual abacus software. We identify in session 8 a Technical-demo orchestration (Drijvers 2012), which involved the demonstration of tool techniques by the teacher (not for the whole class, but for the group of eight children involved). This was very quick as children were able to manipulate the virtual abacus with no technical obstacles. The only technical point concerned the use of the IWB pencil, which they overcame with ease.

During sessions 8–12, Deborah circulated amongst children and watched their individual work, on the laptops or on paper, providing help if needed. This corresponded to Drijver's 'Work-and-walk-by' orchestration, when the teacher follows the individual work of the children. We can also identify a 'Discuss-the-screen' orchestration as children were asked to come to the IWB to display and argue in support of their suggestions as to how to show eight, as in the following extract:

Deborah:	I would like you to show… Eight! We think about… How do we do eight? [*Some children want to immediately give answer*] Eight is? -
Some children:	Five and three!
Deborah:	Show me with your hands. Five and three! Kevin. [*Kevin goes to the board and he activates one 5-unit counters and three 1-unit counters (the third bead, one gesture).*] Five and three. Yes, you activated five and three. [*Deborah goes closer to the board to show the activated beads.*] Do you agree with his choice? Is there another solution? Another way to show number eight? Number eight?
Some children:	Yes.

Deborah: Yes Anaïs. [*Anaïs goes to the board and takes the pencil*]. Go on, you did not push hard enough, I think. [*Anaïs activates three 1-unit counters, within three gestures: one, two, three, and one 5-unit counter.*] So, it is because you, you activated the beads one after the others. It is very well. Maëlle. [*Maëlle goes to the board and activates three 1-unit counters in the tens and one 5-unit counter in the units, it marks 35, the icon "see number" is activated.*] No. [*Maëlle tries with three 1-unit counters in the tens, and five 1-unit counters in the units, it marks 35 as well.*] You activated indeed eight beads, but did you show number eight?

Maëlle: No. [*Looking at number 35 written in numeral on board*]

Deborah: Did you understand your mistake?

Maëlle: Yes, three and five, it makes 30 and 5!

Deborah: Three and five, it makes 35. [*Showing the two different rods*]. And above all, they are not located on the same rod.

In this extract, Deborah's didactical performance partly corresponds to what she planned, which was to find different ways to display 8 on the abacus. Nevertheless, the children participated more than in the previous extract as they display the numbers for themselves on the board. We also observe that she reacts in the moment to a mistake (anticipated, on a general level, because it is a classical difficulty with the abacus) arising from a confusion between the number of beads and the value they represent. We will focus below on the interactions between Deborah and the children in terms of how children's reactions constitute resources for Deborah.

Interactions Between Children and the Teacher

Clark-Wilson (2010) demonstrates how hiccups, in mathematics lessons using technology, lead to evolutions in the teacher practice and thus form part of a teacher's professional development. She proposes a classification of such hiccups, several categories of which correspond to unplanned teacher-children' interactions. We do not use the concept of hiccup here. Nevertheless, we obtain similar results, which we interpret as documentational geneses, with the children's productions and reactions constituting central resources for the teacher. We presented above the example of a local adaptation to a student's answer. Over a longer timescale, Deborah also changed her plans for progression within the abacus-lessons as a result of her observations of the children's work.

Deborah first centred the tasks proposed to the children on 'show a number on the abacus'. Her observations of the children over several sessions led her to also propose work on the task 'read a number shown on the abacus'. From sessions 8–12, children were asked to achieve two tasks on paper (Appendix 2): to read numbers (from abaci images on printed hand-outs) and write them in numerals; and to draw beads on empty abaci, corresponding to a number written as a numeral. Deborah first thought to ask children to complete both paper-based tasks at each session. But it appeared to be too difficult for the children when the numbers were above 5, so Deborah chose to alternate the paper tasks in the following sessions (session 9–12). The computer was then used as a possible means of support to the children alongside this paper and pencil task. Hence, Deborah modified her plans as a result of the knowledge she gained from her interactions with children.

During sessions 8 and 9, she observed that some children were still encountering difficulties. For this reason, she elaborated an 'abacus book', which was first used within session 11. This book (on paper) presented all of the possibilities to show numbers up to 10 on the abacus. This kind of book, generally used as a record of class work, is commonly used at this level in French classrooms. This can be considered as part of the process of instrumentalisation. Deborah has professional knowledge about the possibility to use such books in order to keep a collective record of class work, which she applies to the abacus, as a result of the difficulties encountered by some children. After the introduction of this book, it appeared that most children were able to read numbers proposed by the teacher for paper work (without using the virtual abacus). A few children manipulated the virtual abacus to verify their results. More precisely, in session 11 all of the children were able to recognise a 15 shown in the three different ways on the abacus. Deborah considered that the abacus lessons were useful for the children in learning about numbers. She planned to use the abacus next year and even to dedicate more than 12 sessions to it. In her opinion, it should be an everyday tool for the children.

We now consider a second case study, which involved a different software program, used over a much shorter time period.

Learning Numbers with the 'Passenger Train'

This section considers another software program, Passenger Train [http://python.bretagne.iufm.fr/docenligne/marene/Train_des_Lapins_Online_2012-10-05.html], which was chosen by two other teachers, Chloe and Mia, who were also members of the project group. The related classroom observations began in January 2012. Mia had a 'double level' class, with eight children aged 3 (young section) and twenty children of 5–6 years (older section). Chloe had a class of older children (5–6 years). The Passenger Train program was designed as a game and it corresponds nevertheless to a precise mathematical learning situation. We claim that the children will develop mathematical meanings within this playful context, which is set within a perspective that is relevant for young children as evidenced by previous research (Van Oers 2010).

Main features of the Passenger Train program

This freeware program was designed to focus on the specific function of numbers as indicators of a position on a number line. The children's task was to seat one to three passengers (rabbits) in the same passenger car of an empty train and to match those in a reference train. The freeware program enables two modes of use: 'discovering' mode (the reference train remains visible, Fig. 3) and 'learning' mode

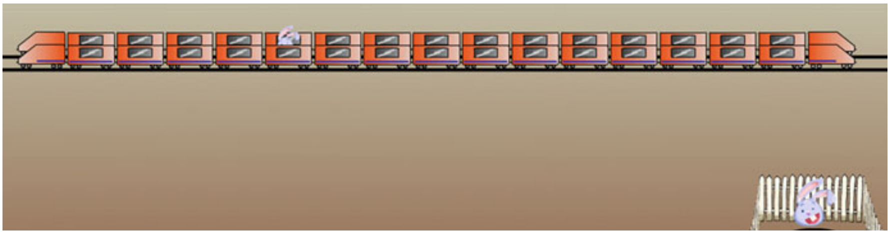


Fig. 3 The ‘passenger train’, ‘discovering’ mode

Fig. 4 Student ‘score’ sheet

(the reference train disappears when the empty train arrives). The discovering mode corresponds to the appropriation of the task, whereas the learning mode is designed for the learning of the mathematical knowledge.

In a previous research study (Bueno-Ravel and Gueudet 2009), we have shown that software features that provide teachers with *instrumented teaching techniques* for managing the students’ heterogeneity promoted its integration into teachers’ practices. Two main technical aspects supported this integration: the possibility to personalise through the choice of different settings for different children; and having access to the outcomes of the children’s work on the computer. When working with the passenger train program the settings of the software can be customised by choosing: the number of passenger cars (from 10 to 30); the position of the rabbit in the train (near a locomotive, in the middle of the train, random); and the number of rabbits to place (from 1 to 3). Students’ choices are not stored, so teachers do not yet have access to the outcomes of the children’s activity unless they are observing them. Nevertheless, to progress from one attempt to another, the children have to fill in a score sheet (Fig. 4). The results of the last ten attempts appear in this score sheet, which provides some information for the teacher.

The Passenger Train Lessons

In Chloe and Mia’s classrooms, the working environment and activity format (Ruthven 2012) were very similar. In both cases, the computer equipment was limited; one computer in Chloe's classroom, one to three computers in Mia’s one and neither classroom had a digital projector. The activity format followed a typical approach in French kindergarten classrooms, involving whole-class discussion and group work. The groups, comprising of 4–6 children, practise the same activities in succession. There are two adults in the class, the teacher and an assistant so it is usual for two of the groups to have adult supervision, whilst the two remaining groups work by themselves. Chloe's and Mia’s Passenger Train lessons are described in detail within Appendix 4. These lessons lasted five or six sessions. Each session was repeated four times during a week (in France, children go to school 4 days a week so teachers usually organise a group rotation each day for a same session, and a session lasts 1 week).

Introduction of the Software

Chloe’s Class

Chloe introduced the mathematical situation, and the associated tools (paper materials and the software) simultaneously. She began by manipulating the software while the children watched. Then each student, in turn, tried to use the software, with the teacher nearby to help in case of difficulties. Within the software, each rabbit must be moved precisely using the mouse, which can be difficult for some children. When Chloe was not with a group of children in which some children had difficulty when moving a rabbit, she ‘left’ part of the responsibility of the technical knowledge to the children who had mastered the movement. They were in charge of helping others if needed.

Mia’s Class

Mia introduced the mathematical situation with the paper material alone and she chose to start using the software in session 2. Even though she had organised group work for this session, she decided to introduce the software in a whole classroom setting. She sat near the computer and manipulated the mouse while all the children watched the computer screen, sitting on two rows of benches and on tables, in front of the screen, with the lights off (Fig. 5). Mia decided to introduce the software in a whole classroom setting because she thought that if she introduced it in a work group, the children working autonomously on other tasks would be more interested by the software than by the work they had to do.

For the introduction of the software (S1 for Chloe and S2 for Mia), the didactical configurations used were different, but we identify nevertheless the same two types

Fig. 5 Mia introduced the software in whole class (S2)

of orchestration *Technical-demo* and *Explain-the-screen* (Drijvers 2012): Our analysis points out that, contrary to the conclusions of Drijvers et al. (2010), the didactical configuration of the Technical-demo orchestration does not necessarily include "facilities for projecting the computer screen and a classroom arrangement that allows the children to follow the demonstration" (p. 219). What is clearly at stake, for this orchestration, is to design a didactical configuration allowing each student to follow the demonstration. Mia and Chloe both overcame the lack of a digital projector through their choices of classroom arrangement. In both cases, the main didactical objective that influenced the didactical configuration was that in order for the children to work independently of the teacher with the software, the teachers have to ensure that the children are 'technically' autonomous and that they understand how to realise the tasks on their own. So, as exploitation modes, teachers pay particular attention to the explanation of the tasks, the features of the interface and the related actions. The emphasis on the task explanation is necessary as the children are not able to read yet and must not see the software merely as a game.

We have shown that Mia and Chloe created different didactical configurations even though their working environment was similar. These choices were dependent on each teacher's professional knowledge about student behaviours and the pedagogical organisation of the classroom. Nevertheless, we notice that they both had anticipated the importance of guiding children's geneses by providing good explanations, even though the use of the software did not seem too complex.

Orchestrating Student Heterogeneity: Fostering Autonomy

We have identified two new orchestrations types in Chloe and Mia's classes: *Autonomous-use* and *Supported-use*. These orchestrations appeared more noticeably during sessions 3 and 4 in Chloe's class and during sessions 2 and 3 in Mia' class

(Appendix 4). Chloe and Mia designed these orchestrations in order to solve professional problems, which were how to manage the children's heterogeneity and, more precisely, how to provide support to the children most in need? In both cases, they responded by making themselves available for the children most in need. This implies that their main didactical objective in relation to these two new orchestrations was to design an orchestration that allows the teacher to be near the children most in need.

Autonomous-Use

As the children who do not have any difficulties are able to work autonomously with the software, the teacher is 'discharged' and can devote herself to the children who were most in need. The didactical configuration comprises at least one computer, a prior identification of the children who do not have difficulty through a diagnostic evaluation (instrumented or not) and individual or paired-work with the software. As an exploitation mode, the teacher mainly needs to anticipate and organise rotation (if needed) of the children working autonomously with the software. Of course, teachers do not leave the older children in complete autonomy. They occasionally check their work and intervene, when necessary, in relation to the mathematical or technical content. When teachers check the work of these children, the orchestration is similar to Drijver's (2012) 'Work-and-walk-by' orchestration type.

Supported-Use

For the children most in need the teacher stays nearby and provides help to the children as they work on the software. The didactical configuration comprises at least one computer, a prior identification of the children who do have difficulties through a diagnostic evaluation (instrumented or not) and teacher's presence (Fig. 6). In an exploitation mode, the teacher has to anticipate the main difficulties children can encounter and the remediation needed. These difficulties can concern the manipulation of the software, the mathematical content being addressed or the prerequisite mathematical content. In this orchestration, the teacher usually intervenes individually with children. However, we do not include this orchestration in a *Work-and-walk-by* type. Indeed, the didactical objectives of these two orchestrations differ significantly. For the *Supported-use* orchestration, the didactical objective is an explicit choice to support children most in need. For the *Work-and-walk-by* orchestration, Drijvers (2012) shows that the fact that "many teachers seem to prefer individual interactions to whole-class teaching" (p. 271) explains the high frequency of this orchestration. He does not take into account the possibility that handling the children's heterogeneity might explain such an orchestration choice.

The professional problem we have identified (how to deal with children's heterogeneity and precisely how to support children most in need) is very general. The pedagogical organisation of the class divided into four groups working in

Fig. 6 Mia is helping a student who has been identified as 'most in need', while a student on her left is working autonomously

parallel on different tasks can be interpreted as part of the teachers' answers to this problem. So *Autonomous-use* and *Supported-use* orchestrations are not specific of the use of software. However, we have identified two features of the Passenger Train program that facilitates such orchestrations:

- Passenger Train (paper or software) is a self-validating situation, supporting autonomous use;
- The Passenger Train provides the validation for the children's answers (Bueno-Ravel and Gueudet 2009) and allows many attempts, which also supports the autonomous work of children.

Autonomous-use orchestration leads Chloe and Mia to think about the design of resources that will enable them to keep a trace of the children's autonomous work as the Passenger Train software does not store their results. Within the teachers' joint description of their lessons that was written for colleagues, they proposed three resources that might overcome this problem:

- A worksheet on which children note each attempt, and whether they succeed or not;
- A road map on which children note each attempt, if they succeed or not, and indicate the parameters of the software (number of rabbits, number of passenger cars);
- An observation grid for teachers that highlights the main procedures and mistakes children might encounter.

We interpret this process as a genesis, encompassing intertwined instrumentation and instrumentalisation processes. Chloe and Mia know how important it is to have access to the children's work, particularly for children who cannot yet write.

Influence of a Digital Projector or IWB: Planned Orchestrations

In our analysis of the abacus-lessons, we have mentioned that the installation of an IWB in Deborah's classroom led her to modify her orchestration choices. (Chloe's and Mia's schools were not equipped with IWBs or digital projectors). We assume that had Chloe and Mia had access to projection facilities they would have modified their orchestrations. Within their lesson descriptions they suggested 'Sherpa-at-work' orchestrations for colleagues who had projection facilities. Indeed at interview, when asked how they would adapt the Passenger Train lessons, they said that having a digital projector or an IWB facilitated the introduction of the software, "At least one and a half hours and three work groups are saved during the software presentation phase if you have a digital projector […] The large screen caught the attention of the children […]" (our translation). However, most of their writings are centred on joint work concerning procedures or synthesis phases, instrumented by the Passenger Train software. "(digital projector and IWB) are interesting for synthesis and institutionalisation phases; the different strategies can be illustrated by the children, and, the attractiveness of the Passenger Train is a real context to facilitate talk […]" (our translation). They developed three variations of *Sherpa-at-work* orchestration: (1) one Sherpa-student, 'guided' by the teacher, (2) one Sherpa-teacher, following on the computer actions that a student shows on the wide screen and (3) a Sherpa-pair, one student following on the computer actions that the other student shows on the wide screen (or the student near the wide screen follows the actions of the student on the computer).

The didactical objective of this orchestration is to foster verbal and non-verbal interactions (e.g. showing how to count the passenger cars with a finger) in the whole classroom setting in order to institutionalise expert procedures. Chloe and Mia have been led to develop such exploitation modes to make sure that children will learn something using this software and not only remember their successes or failures at the game: 'the teacher must be vigilant about the joint verbalisation (between the computer screen and the wide screen) and the reproduction of the counting mode on the wide screen […]. Without such precautions, only the final result (the place of the rabbit and the validation) will be visible to the eyes of the whole class' (our translation).

Drijvers et al. (2010) have pointed out that, in a working context offering projection facilities, even though teacher guidelines describing *Sherpa-at-work* orchestration are given to teachers before their lessons, this type of orchestration is ignored 'to a certain extent'. They report that teachers' orchestration choices are consistent with 'their regular habits and their view on mathematics teaching'. In order to follow the lesson descriptions elaborated by Mia and Chloe it is necessary to be aware of research specific to the kindergarten school context. We will return to this perspective in our conclusion. In the next section, we discuss central issues emerging from both case studies.

Discussion

In this section we draw on the case studies in order to enlighten general issues about teachers' use of software in mathematics at the kindergarten level and, in particular, with 5–6-year-old children. Naturally, the scope of our study remains limited as we observed only a few teachers who are experienced and involved in a design and research group. Nevertheless, some of the facts that we observed are not restricted to their specific case. We first present issues which are directly linked with specific aspects of this level of schooling and then others concerning orchestration more generally.

Articulation of Resources

Kindergarten teachers in France usually use textbooks for their teaching of mathematics, which include a teacher's guide and specific worksheets for the children. These textbooks suggest exercises and some of them also include mathematical situations, developed in the course of collaborations between teachers and researchers. Moreover, diverse physical materials such as games, cards, tokens, cubes and figurines are also included. In previous studies we noticed that for the secondary school teachers' resource system, the articulation between a textbook and a software program is an important factor for technology integration (Gueudet and Trouche 2012). We observe the same here at the kindergarten level. For Chloe, the 'passenger train' is associated with a game on paper that she had already used before and for Deborah, two exercises from textbooks are associated with the work on the abacus (sessions S3 to S6). Moreover, many other kinds of material are included and the virtual abacus is naturally articulated with the physical abacus, whilst the Passenger Train software is used alongside the corresponding situation on paper. In fact the computer becomes one of many artefacts living in the classroom, which probably contributes to the richness and complexity of the didactical configurations, and thus of the orchestrations.

Types of Orchestration at Kindergarten Level

From the outset, we expected the orchestrations at this very early level to be significantly different from the orchestrations observed by Drijvers (2012) at the secondary school level. Some differences are linked with the available material. On the one hand, there are often only a few computers available in the classrooms; on the other hand, as mentioned above there is a wide variety of material used by the teachers at this level. So the computer is one resource, amongst many others. Nevertheless, the projection facilities seem to lead to a central role played by an image on the computer screen, projected for the whole class, or at least for a half-class. Differences are also linked with the importance of verbalisation in these class levels; it seems to foster orchestrations encompassing this verbalisation such as *Discuss-the-screen*.

The blackboard remains an important resource in the classroom, but for children aged 5–6 who cannot read, it is mostly filled with images, only some of these being connected to mathematics, which can explain the absence of the *Link-screen-board* orchestration. The material is not the only cause for the differences we observed. As mentioned in the previous section, there is much less whole classroom teaching and much more group work at this level. For the introduction of a new software program, the *Technical-demo*, *Explain-the-screen* and *Discuss-the-screen* orchestration types introduced by Drijvers (2012) are still present, if we assume that we can adapt them to groups of children (while they were introduced with a whole classroom presentation). But we also observed new orchestration types linked with the usual group work organisation, where children work on different tasks: the *Autonomous-use* and the *Supported-use*. We do not claim that these types of orchestrations are absent at the secondary school level. Nevertheless, they are probably much more frequent at the kindergarten level because of the usual *activity format* (Ruthven 2012), which includes group work on a regular basis and also because of the importance of managing heterogeneity at this school level.

The Software, Shaping the Orchestration Choices?

One of the questions raised by Drijvers in his work is the influence of the software's features on the orchestration choices. Our data clearly demonstrate such an influence, in several directions. Concerning the presence of the teacher with children working on the software, we noticed that it clearly depends on the feedback offered, or not, by this software. Naturally, if the software provides feedback, the teacher's presence is needed less. With the virtual abacus, the number inscribed is displayed, and children can compare it to the number they want to reach. Nevertheless, they will never receive a message such as 'wrong number displayed', so the teacher may still need to intervene. Another important aspect of the orchestration that depends on the software features is the presence of recording worksheets to complement the work on the computer. If the software provides access to the children's answers, these written notes are needed less, especially at this very early level. As the children cannot write, there is no objective linked with the writing of a mathematical procedure, but only the need for the teacher to have access to potential mistakes (if the rabbit has been misplaced, in which carriage was it? If a number has not been correctly inscribed, was there confusion between the rods etc.). We assume that a change in the software, permitting the recording of children's productions, would certainly change the orchestrations, requiring less written records.

Orchestration and Teacher Knowledge

Teacher Knowledge Shaping Orchestration Choices

The issues discussed above provide evidence that orchestration choices depend on the available material, the number of computers and the features of a software program. Orchestration choices also depend on the teacher's resource system and activity format. We argue that these are also linked with teacher knowledge and beliefs and give some important examples below. In these examples, we infer teacher knowledge by comparing the notes taken during the group's meetings, the lesson plans designed by the teacher, for herself and for colleagues, and the classroom videos.

Deborah has professional knowledge about number sense and its difficulty for children. In her orchestrations, she plans exploitation modes where she emphasises the different values of the beads depending on the particular rods, using the virtual abacus projected on a screen. This orchestration choice is clearly a consequence of teacher knowledge about number sense, about children's difficulties with it, and about the abacus.

For Chloe and Mia, their knowledge about the children's need to learn to use numbers as indicators of both quantity and position, determined their choice of the Passenger Train software, especially as only a few situations exist that enable work on the second aspect. Chloe and Mia placed great importance on the management of heterogeneity within the classroom. Thus they used the software as a resource to support this management. They developed a document, with a 'resource' part including the software, the associated children's worksheets, and all the material elements of the didactic configuration. This document also included professional knowledge about the management of heterogeneity as they considered that the skilled children could work by themselves whilst the teacher has to stay with the others, a choice that can also be considered as pedagogical knowledge.

Teacher Learning

During the lessons we observed unplanned elements within the teachers' didactical performances. We also observed evolutions in their creation of the orchestrations. The teacher-children's interactions were a major source for these evolutions alongside the major resource provided by the children's outputs. We consider this as evidence of teacher learning, involving different kinds of knowledge. The teachers certainly learned about children's reasoning. For example, in the case of the abacus, Deborah noticed that there were three possible explanations for how to display five on the abacus:

- Counting reasoning: Five one-unit counters are activated and displayed by five gestures by counting: one, two, three, four, five.

- Grouping reasoning: Five one-unit counters are activated and displayed in one gesture: the fifth bead is activated.
- Calculating reasoning: One five-unit counter is activated.

Deborah planned to revise her lesson plan for the abacus-lessons to include an additional objective to hold a discussion with the children to encourage them to reason in these three ways and make the connection with the calculations. In the case of the Passenger Train Chloe discovered that the children used two correct procedures, which were to use the number of the passenger car hosting the rabbit or to count the empty passenger cars on the left. In fact the use of number as memory of position is always linked with its use as memory of quantity. She also plans to discuss with the children these two possibilities during her next Passenger Train lessons.

In Deborah's class, the evolutions also concerned the responsibility given to the children, with a tendency towards giving them more responsibility. This evolution, from teacher-centred to children-centred orchestrations, can be a consequence of the participation of the teachers in the group (Drijvers observes that the teachers involved in his experiment proposed more student-centred orchestrations than in their usual practice). We consider anyway that it provides evidence for teacher learning and about the possibility to leave some responsibilities to the children when working in mathematics with technology.

Our focus here was not on teacher knowledge evolution and documentational geneses. Additional research would be needed to study these geneses and the links with the orchestration evolutions. Geneses are indeed long-term processes and a follow-up of teachers during several school years is necessary for their analysis. This will be the subject of another study; the same holds for the appropriation, by other teachers, of the resources designed during this project.

Conclusion: Orchestrations at the Kindergarten Level

In her review of research papers about the use of technology in the teaching of mathematics, Joubert (2013) identifies 'orchestrating learning' as the central theme, present in 74 % of the papers she considers. She also observes that less than 5 % of the papers concern primary school or kindergarten. Levy and Mioduser (2010) demonstrate that, in the context of kindergarten children learning mathematics with digital artefacts, the learning environment is of vital importance. Thus answering the question proposed here (§ 1.2) (which orchestrations are chosen by kindergarten teachers when using technology in their teaching of mathematics and which factors shape these choices of orchestrations?) is of central importance within mathematics education research.

Most of the orchestration types introduced by Drijvers from the secondary school context can also be observed at the kindergarten level, with adaptations resulting from the available material or from a usual activity format. In our work, the *Link-screen-board* orchestration does not intervene, but this might be a specific feature of the kindergarten context. We identified new types of orchestrations: *Autonomous-use* and *Supported-use*. These two types of orchestrations are linked with one of the teacher's objectives when orchestrating the teaching of mathematics, which is to take account of individual children's differences. The study of kindergarten, or primary school classrooms has shown that the teachers seem to focus their orchestrations more towards these differences. This issue resonates with a recommendation formulated by Hoyles et al. (2004), in their comments about the use of orchestration, to study the integration of digital technologies: "the individual difference is not something to be minimized or avoided, it is an inevitable part of orchestration itself" (p. 320). Our work certainly supports this claim.

The teachers also proposed variations on the *Sherpa-at-work* orchestration. We hypothesise that, at least in the French teaching context, the kindergarten (or primary school) level permits the development of more orchestration types than at the secondary school level. Kindergarten teachers follow the children for the whole day (a school day lasts 6 h), and are free to organise this time. They also teach several subjects which, in turn, certainly leads to different activity formats. This can foster the development of a specific teacher agency concerning orchestration; this hypothesis naturally needs to be investigated further.

Concerning factors shaping the orchestrations, we recorded influences from the material environment, the available resources (in particular from the software features), the usual activity formats, and from teacher knowledge, with all of these being tightly intertwined. Different kinds of teacher knowledge come into consideration: pedagogical knowledge about the management of heterogeneity; knowledge about curriculum material (mathematical exercises available in textbooks, for example); and knowledge about children's possible mistakes and difficulties. Knowledge about the teaching and learning of numbers at this school level (importance of number sense, distinction between number as memory of quantity and as memory of position) was especially important for the teachers' choice of a given software program. The kindergarten teachers involved in our study were not specialists in mathematics. In spite of this, the mathematical content and its didactical aspects were central in their choices. In this chapter we did not focus on professional development. Nevertheless, we point out that our observations are coherent with those of Erfjord et al. (2012), who comment that when involved in a research group concerning the orchestration of mathematical activities, kindergarten teachers adopt an inquiry stance. It seems to contribute, in particular, to the development of their awareness of the mathematical ideas involved. However, the identification of the specific interventions of different kinds of teacher professional knowledge, and how they articulate with instrumental knowledge about the possible use of a given software program, requires an additional study.

Much research about the use of technology for the teaching of mathematics to young children, in particular at the kindergarten level, is still needed. We consider that the perspective of the documentational approach is fruitful for these studies. Especially at the kindergarten level and in primary school, technology is only one teaching resource amongst many others, and investigating teachers' work and professional development requires taking into account their interactions with these resources.

Acknowledgements We warmly thank for their contribution all the members of the 'mathematical package' project. We especially thank Typhaine Le Méhauté and Michel Guillemeau for their support in the design of the Passenger Train software.

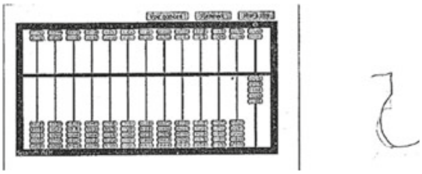
Appendix 1: The Abacus-Course in Deborah's Class (5–6 Years Old)

		Resources used by the teacher and children
	Session title	Exploitation mode
S1	Discovering the abacus: setting numbers up to 6	Resources: Virtual abacus and digital projector (teacher) Physical abacus (children) Teacher textbooks for preparation Exploitation: One physical abacus per student Children have to set numbers on the physical abacus and correction is made on the virtual abacus by teacher. Children do not use the virtual abacus before S8 Children have to read numbers set on the virtual abacus by teacher
S2	Discovering the abacus: setting and reading numbers up to 6	
S3	Setting and reading numbers (up to 12). Adding (1 + 2, 3 + 1, etc.) with the game greli-grelo (from teacher textbook): teacher has beads in hands and children are asked to say the total number of beads. Verification is made on physical abacus by children	
S4	Setting and reading numbers (up to 25). Adding with the game greli-grelo	
S5	Setting and reading numbers. Adding with the treasure game (from teacher textbook): children win golden coins with a dice and they are asked to say the total number of coins. Verification is made on the physical abacus by children	
S6	Setting and reading numbers. Adding with the treasure game	
S7	Setting and reading numbers. Adding with the treasure game	
S8	Setting and reading numbers (0–5)	Resources: Virtual abacus and IWB for both teacher and children Pencil and paper work prepared for children Sessions 11 and 12: the 'abacus book' Exploitation: One computer per student with the virtual abacus (to help children to fill the paper work) Sessions' introduction on the virtual abacus by teacher and children (set numbers in different ways with the IWB) Individual work for children (pencil-paper and virtual abacus on computer)
S9	Reading numbers (5–10)	
S10	Setting numbers (5–10)	
S11	Reading numbers (10–15)	
S12	Setting numbers (10–15)	

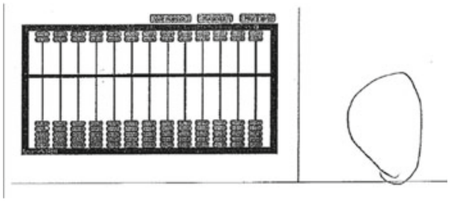
Appendix 2: Examples of children' Work on the Chinese Abacus, Deborah's Class (5–6 Years Old)

Session 8

1. Read a number: the examples of 5 and 0

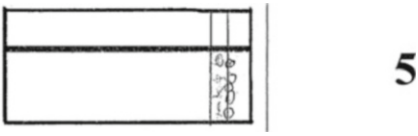


This student recognises number 5 set on the abacus but wrote it in 'a mirror form' which is usual at this level

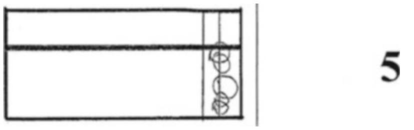


Children learn to recognise a particular number that is 0

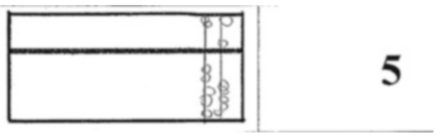
2. Set a number: the example of 5



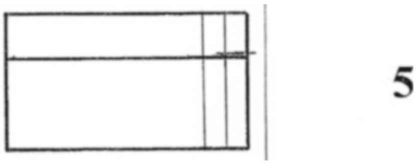
This student drew the beads between two rods and after on the unit rods, activating five one-unit-counter



This student first wrote the numeral 5 on the unit rod and the teacher asked to draw the beads



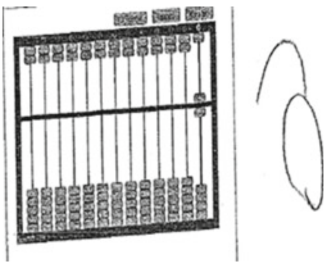
This student drew the activated beads and the non activated as well. Most children drew only activated beads spontaneously



This student drew one five-unit-counter on the units rods to set 5

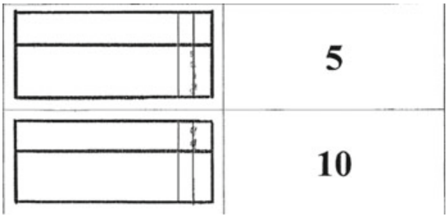
Session 9

1. Read a number: the example of 6

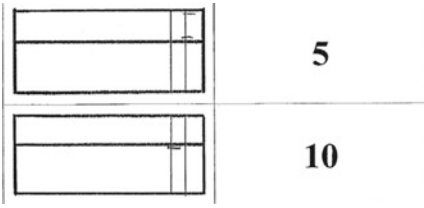


This student recognised number 6 set on the abacus but wrote it in the wrong way (see session 8)

2. Set a number: the examples of 5 and 10 (for the same student)

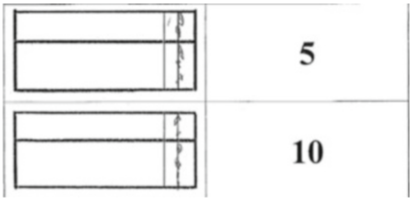


Here the number 5 is set as 5 one-unit counters and 10 as 2 five-unit counters. Only activated beads are drawn



Here the number 5 is set as 1 five-unit counter and a non activated bead is drawn. The number 10 is set in the economical inscription i-e 1 one-unit counter in the tens. To represent beads, this student draws a short line

Here beads are circles (not filled) and the two numbers are set in the economical way



To set number 5, 5 beads are activated: there is a misunderstanding between quantity of activated beads and the quantity represented by the 5-unit counters. To set 10, it seems that the seven beads per rod are activated

Appendix 3: Deborah's Class Transcriptions in French

Session3: Beginning of the session. A discussion about how to set 5 on the physical abacus raised between children. Deborah was sitting with the children and goes to the screen to show children' propositions.

Deborah:	Vous m'avez proposé… Laurie, tu m'as proposé d'activer les cinq boules de la tige rouge, d'accord… Pourquoi ? *[La tige des unités est rouge sur le boulier virtuel, les autres sont vertes, Deborah active 5 comme cinq unaires dans les unités].*
Laurie :	Parce que celles du haut, elles servent à rien.
Deborah:	Parce que tu penses que celles du haut, elles ne servent à rien. Est-ce que vous êtes d'accord avec le choix de Laurie?
Quelques élèves:	Non!

Deborah: Non?
Une élève : Parce que les boules du haut, elles valent 5.
Deborah : On va vérifier, j'avais demandé de faire combien ?
Une élève : Cinq !
Deborah : Très bien. *[Deborah active l'icône « voir nombre » et le chiffre 5 apparaît à l'écran]*
Plusieurs élèves : Cinq.
Deborah: Alors, est-ce que le choix de Laurie est juste?
Quelques élèves : Oui !
Deborah : Maintenant, il y a une autre possibilité par cinq. Certains élèves ont activé une quinaire, c'est-à-dire une boule du haut. *[Deborah montre à l'écran le chiffre 5 comme une quinaire, l'icône « voir nombre » est désactivé].* Alors, combien on a de possibilités pour inscrire cinq ? Maëlle ?
Maëlle : Deux.
Deborah : Oui, deux façons. Soit on active les cinq boules du bas, soit j'active une boule du haut. *[Deborah montre à l'écran les deux possibilités]*

Session 9. Children come to the IWB to set 8. « Discuss-the-screen » orchestration raises.

Deborah: Je voudrais que vous activiez… Huit ! On réfléchit, comment est-ce qu'on fait huit ? *[Certains élèves veulent donner immédiatement la réponse].* Huit c'est ?
Quelques élèves : Cinq et trois ! Montrez-moi avec vos mains. Cinq et trois ! Kevin. *[Kevin va au tableau, il active une quinaire et trois unaires (3ème boule, un geste)]*
Deborah : Cinq et trois. Tu as bien activé cinq et trois *[Deborah s'approche du tableau pour montrer les boules activées]*. Vous êtres d'accord avec son choix ? Est-ce qu'il y aurait une autre solution ? Une autre façon d'inscrire le chiffre trois ? Le chiffre huit ?
Quelques élèves : Oui
Deborah : Oui, Anaïs. [Anaïs va au tableau et prend la crayon]. Vas-y, tu n'as pas appuyé assez fort, je pense.
[Anaïs active trois unaires, en trois gestes : un deux, trois, puis une quinaire]
Deborah : Alors, c'est parce que toi, tu as activé les boules les une à la suite des autres. C'est très bien. Maëlle.
[Maëlle va au tableau et active trois unaires dans les dizaines et une quinaire dans les unités, ce qui fait 35, l'icône « voir nombre » est activé.]
Deborah : No
[Maëlle essaie avec trois unaires dans les dizaines et cinq unaires dans les unités, ce qui fait également 35.]
Deborah : Ah, tu as activé en effet huit boules, mais est-ce que tu as inscrit le nombre huit ?
Maëlle : Non. *[Regardant le nombre 35 écrit en chiffres au tableau]*
Deborah : Est-ce que tu as compris ton erreur ?
Maëlle : Ah oui, trois et cinq, ça fait 30 et 5 !
Deborah : Trois et cinq, ça fait 35. [Montrant les deux différentes tiges.*]* Et surtout elles ne sont pas situées sur la même tige.

Appendix 4: The 'Passenger Train' in Chloe and Mia's Classes (5–6 Years Old)

The 'passenger train' in **Chloe's class** (5–6 years old)

	Configuration/Exploitation mode	Resources used by the teacher
S1	**Presentation** of 'passenger train' on paper (by the teacher, for a group of six children)	Trains and rabbits on paper
	Presentation of the 'passenger train' on the computer, by the teacher, then successively by each student, working in a pair (the four other children are spectators)	Computer with the software
S2	**Learning phase** (30 min): The children are grouped by six and organised in pairs inside the group	Trains and rabbits on paper
	Synthesis: whole class, when all the groups have done the 'passenger train' situation	
S3	**Learning phase** (30 min): The children are grouped by six and organised in pairs inside the group	Trains and rabbits on paper
	Training on the software: individual work for advanced children	One computer with the software
	Synthesis: whole class, when all the groups have done the 'passenger train' situation	
S4	**Training on the software** (10 min): pair work for most of the children, in autonomy, rotation each 10 min. Morning	One computer with the software
	Scaffolding (30 min): group work for children who have difficulties, with the teacher. Afternoon	Trains and rabbits on paper + Computer with the software
	Synthesis: whole class, when all the groups have done the 'passenger train' situation	
S5	**Training on the software** (10 min): pair work (homogeneous), in autonomy, rotation each 10 min. Teacher chooses the difficulty level (numbers of passenger cars, numbers of rabbits) according to the level of the 'pair'	One computer with the software
S6	**Training on the software** (10 min): pair work (homogeneous), in autonomy, rotation each 10 min. Teacher chooses the difficulty level (numbers of passenger cars, numbers of rabbits) according to the level of the 'pair'	One computer with the software

The 'passenger train' in **Mia's class** (5–6 years old)

	Configuration/Exploitation mode	Resources used by the teacher
S1	**Diagnostic assessment:** children in groups of five or six and work individually	Trains and rabbits on paper One computer with the software (Diagnostic assessment was made on computer for one group of five children)

(continued)

(continued)

	Configuration/Exploitation mode	Resources used by the teacher
S2	**Presentation** of the 'passenger train' on the computer, by the teacher, for the whole class	One computer with the software
	Learning phase (30 min): children in groups of five or six. Children with no difficulties work in pairs taking turns on the computer (in autonomy). The others (one or two children) work individually, teacher is nearby	Three computers with the software
	Synthesis: whole class, when all the groups have done the 'passenger train' situation	
S3	**Training on the software** (30 min): pair work for most of the children, in autonomy, rotation each 10 min	Three computers with the software
	Scaffolding (30 min): group work for children who have difficulties, with the teacher	
S4	**Training on the software**: pair work (homogeneous), in autonomy, rotation. Teacher chooses the difficulty level (numbers of passenger cars, numbers of rabbits) according to the level of the 'pair'	Three computers with the software Training sheet created by Mia
	Training on the paper: individual work (homogeneous)	
S5	**Training on the software** (10 min): pair work (homogeneous), in autonomy, rotation each 10 min. Teacher chooses the difficulty level (numbers of passenger cars, numbers of rabbits) according to the level of the 'pair'	One computer with the software

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