Introduction

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Teacher education is an important issue for society and it is framed within cultural, social, political and historical contexts. In recent years international research in mathematics education has offered a range of theoretical perspectives that provide different and interrelated frames and viewpoints (Ball and Bass 2003; Clark and Hollingworth 2002; Davis and Simmt 2006; Jaworski 1998; Wood 2008). The role of digital technologies within this discourse has an increasing relevance as the society and government place demands on teachers to integrate technology into their classroom practices so that students can experience its potential as a powerful learning tool (Drijvers et al. 2010; Lagrange et al. 2003; Trouche 2004).

Many of the chapters in this book open by stating how, despite over 20 years of research and curriculum development concerning the use of technology in mathematics classrooms, there has been relatively little impact on students' experiences of learning mathematics in the transformative way that was initially anticipated. The direct response to this has been an increase in research that focuses on the role of the teacher within technology-mediated lessons, in addition to the need for governments and schools to justify their expenditure on educational digital technologies. Many researchers in the past have discussed the role that teachers might or could play in the technology-rich classroom – exploring, for

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example, how the computer might blur the distinction between teacher and students (Papert 1980) or how the computer might even *become* the teacher (Pimm 1983) – but it is only recently that more systematic study of the unique demands and opportunities of the teacher teaching with digital technology had been undertaken.

Research on the use of technology in the mathematics classroom has traditionally focused very strongly on the affordances of particular software environments (in addition to hardware configurations) as well as on the ways such software affects or even changes the nature of mathematical objects and relations (see Sinclair and Jackiw 2005). When the research lens is trained on the classroom teacher, however, the emphasis can shift away from the technology, the associated tasks for students, and even the mathematical concepts at play, as it is hard to maintain a deep focus on multiple aspects. This will be evident in the chapters that follow. However, many of the authors have provided more information about the tasks that featured in their research as additional materials that can be accessed on the Springer webpages associated with the book.

In compiling an edited volume such as this, which features research from Australia, Canada, England, France, Italy, Netherlands, Mexico, New Zealand and the United States, it is inevitable that there is a broad range of terminology adopted. For example, the word technology is used to mean software, programs, applets, applications, courseware, display technology and hardware. Similarly, this research domain has its own set of vocabulary and constructs. To assist the wider understanding of this domain within an international context, we offer a Glossary chapter, which can be found on the Springer website (http://extras.springer.com*), compiled by the editors and authors through a collaborative process of exchange of ideas.

Many of the chapters within the book make reference to examples of particular teachers using particular tasks that have incorporated technology in some way. In each case it will be important to consider the particular context for the associated research as, in many cases, the researchers are focusing on teachers' existing or developing practice, rather than exemplifying forms of 'best practice'. As such, each of the classroom tasks described within each chapter must be interpreted within its specific context and it is for the reader to actively question the appropriateness of the task or the technology under scrutiny. Alongside this, the relevance of the cultural, social, political and historical context relating to each particular classroom cannot be overlooked.

A Journey Through the Text

The book has been divided into three main parts, which are preceded by an opening chapter that plays the dual role of inviting the reader into the context of doing and teaching mathematics with digital technologies and of alluding to some of the opportunities and tensions that such work entails. In his chapter 'Interactions between teacher, student, software and mathematics: Getting a purchase on learning with technology', John Mason shifts your attention away from the classroom-based

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contexts contained within the remaining chapters of the book and invites you to engage with three different *e-screens* that are accessible from the Springer website (http://extras.springer.com*). Mason's chapter will challenge you to consider how to make sense of your mathematical experiences from the perspectives of several contrasting structural frameworks. He offers these in the spirit of his own work with pre-service and in-service teachers, which is centrally about helping them learn to attend to the mathematical ideas at play in the technological setting.

Following this opening chapter, the structure of the book unfolds as follows: Part I consists of six chapters that draw on a range of research perspectives (including grounded theory, enactivism and Valsiner's zone theory) and methodologies (including questionnaires, interviews, video analyses and longitudinal observations) that provide an overview of current practices in teachers' use of digital technologies in the classroom and explore possibilities for developing more effective practices. Part II gathers six chapters that share many common constructs (such as *instrumental orchestration*, *instrumental distance* and *double instrumental genesis*) and research settings that have emerged from the French research community, but have also been taken up by other colleagues. The two papers in Part III provide more meta-level considerations of research in the domain by contrasting different approaches and proposing connecting or uniting elements.

Part I

The first two chapters in this part both provide a snapshot of the ways in which large numbers of teachers are currently using digital technologies. In her chapter 'Exploring the quantitative and qualitative gap between expectation and implementation: A survey of English mathematics teachers' uses of ICT', Nicola Bretscher surveys 188 secondary teachers' technology use in England with the aim of exploring the gap between the reality and the potential of ICT use in the mathematics classroom. Rather than taking ICT broadly as the unit of analysis, Bretscher's survey made important distinctions between software and hardware use, as well as between teacher-centred use of technology (whole classroom settings with data projectors and interactive whiteboards) and student-centred use (on laptops or in computers labs). Bretscher follows Remillard's (2005) socio-cultural approach in her study of teachers' technology use; this approach focuses more on how technology gets used as a resource in teaching, amongst other resources, which have institutional, contextual and historical dimensions, and not just cognitive ones. The results of her survey show a predominant use of IWBs in teacher-centred classroom environments, with relatively little use of mathematical software (such as graphing software and dynamic geometry software). Bretscher discusses the three factors that lead to the statistically significant differences between using IWBs in a whole-class context and giving students direct access to ICT in a computer suite: teachers' confidence in using ICT; teachers' perception of the difficulty of classroom management; and the amount of curriculum material covered in ICT lessons. These factors should be

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very important for professional development, and are worth bearing in mind in the following chapters in which more qualitative approaches are used to study the conditions of digital technology integration.

In the chapter entitled 'Teaching with digital technology: Obstacles and opportunities', Michael Thomas and Joann Palmer consider the types of obstacles – which they define as "anything that prevents an affordance-producing entity being in a classroom situation" (p. 72) – to research secondary teachers' implementation of digital technologies. In order to study the indicators of teacher progress in implementation of technology use, the authors introduce the construct of Pedagogical Technology Knowledge (PTK), which they define as including not only proficiency in using technology but also understanding of the techniques required to build didactical situations incorporating it. Unlike TPACK and instrumental genesis, PTK attends also to the personal orientations of teachers and, thus, to the role that beliefs and attitudes play within technology integration. With a focus on this aspect of teacher orientation, Thomas and Palmer report on two studies that sought to examine the importance of teacher confidence in the growth of PTK. One study involving 22 secondary teachers shows that, for these teachers, there is a correlation between strong confidence in one's ability to teach with graphing calculators and a more positive attitude toward technology and its use in the learning of mathematics. A follow-up study involving 42 female secondary teachers confirmed the strong correlation between confidence in using technology in the mathematics classroom and PTK. These findings have important implications for professional development, which the authors outline in their final section.

The rate at which technology evolves is such that there is an abundance of practice-based theories, which are offered to the community as a set of ideas that might prompt further research and discourse. The contribution by Allan Bellman, Wellesley Foshay and Danny Gremillon in the chapter 'A developmental model for adaptive and differentiated instruction using classroom networking technology', is offered in this context; they suggest a progression in teachers' development concerning their uses of a particular technology that can make students' learning outcomes more visible in the classroom. Drawing on grounded theory, and using extensive longitudinal observations, this chapter offers a model that could be used to consider how teachers might become masters of a given technology that has been designed to treat assessment differently.

In the chapter 'Integrating technology in the primary school mathematics classroom: The role of the teacher', María Trigueros, María-Dolores Lozan and Ivonne Sandoval examine the role of the teacher when integrating technology in the primary mathematics classroom. Guided by an enactivist perspective, this chapter examines the three different uses of technology that can occur in the classrooms – *replacement*, *amplification* and *transformation* – and relate them to five aspects of the role of the teacher in terms of communication of mathematics, interaction with students, validation of mathematical knowledge, the source of mathematical problems, and the actions and autonomy of students. The authors provide three case studies, each of which tends toward one of the three uses of technology (though all move between at least two different uses). They also emphasise the way in which

the characteristics of the technology used has a strong influence on the role of the teacher. Thus, their chapter shows also how any approach to technology integration for teachers must be attentive to the kinds of digital technologies being used.

In the chapter 'Technology integration in secondary school mathematics: The development of teachers' professional identities', Merrilyn Goos uses the construct of *teachers' pedagogical identities* to prompt our thinking about the process through which two Australian secondary mathematics teachers (who are new to the profession) develop the afore mentioned identities as they begin to use technology in their classrooms. She defines technology in a broad sense to include display technologies in the classroom and various mathematical programs and applications. Goos' theory emanates from a socio-cultural view of both teachers' and students' learning and it adapts Valsiner's zone theory, as well as her earlier work, to provide a framework that takes account of the way that the technology alters the teachers' role and the factors that influence how the teachers adopt the technology. Goos' expansion of Valsiner's zones lead her to define a teacher's *zone of free movement* and *zone of free action* as a complex system that overlaps with the teacher's *zone of proximal development*, which can be used to explain how beginning teachers are able to develop innovative practices that involve technology.

We move to a university mathematics department setting in the chapter 'Teaching roles in a technology intensive core undergraduate mathematics course', as Chantal Buteau and Eric Muller describe the on-going development of a computer programming course for undergraduate mathematicians in the Canadian setting, with an emphasis on the roles of the course tutors and the department in which they operate. The mathematics department's goal was to achieve "an education of mathematics majors and prospective teachers of mathematics that would empower them to develop, implement, and use their own interactive mathematical objects". The authors use the context of the *Mathematics Integrated with Computers and Applications* courses, which have been in development since 2001, to raise issues around the associated course design and implementation.

Part II

Most of the chapters in this part are inspired in one way or another by Trouche's (2004) notion of *instrumental orchestration*, which is a construct that is based on the ergonomic framework of the instrumental approach. It articulates teachers' work before and during their activity with students, which is described in terms of schemes of action. In the chapter 'Digital technology and mid-adopting teachers' professional development: a case study', Paul Drijvers, Sietske Tacoma, Amy Besamusca, Cora van den Heuvel, Michiel Doorman and Peter Boon combine instrumental orchestration with elements of TPACK (a framework that is described in more detail also in the chapter 'Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice') to describe the practices that teachers may develop when they

use technology in their classrooms, and how these practices change over time when working in a community of teachers. These practices are described in terms of the use of seven different orchestrations. The authors stress the importance of supporting mid-adopting teachers in their professional development concerning technology, rather than focusing on the introduction of new technologies to non-expert teachers. Many of the orchestrations they describe are used in subsequent chapters of this part, and further developed to better suit different contexts.

In the chapter 'Teaching mathematics with technology at the kindergarten level: Resources and orchestrations', Ghislaine Gueudet, Laetitia Bueno-Ravel and Caroline Poisard seek to adapt Drijvers' et al. orchestrations to the context of the kindergarten mathematics classroom. Indeed, one of their aims is to study the kinds of orchestrations that might be specific to this level of schooling, in contrast to those described in the context of secondary school mathematics teaching. The authors provide case studies of three kindergarten teachers using two different applications, one a digital abacus and the other a game focusing on using number as position. They find new orchestrations in these kindergarten classrooms that are related to the differences in the resource system in the kindergarten classroom as well as in the importance of verbalisation for these children who do not yet read or write. Their chapter shows the need for theory development that is attentive to context – in this case the grade level of the students.

The chapter entitled 'How do teachers integrate technology in their practices? A focus on their instrumental geneses' moves to the context of the secondary mathematics classroom, where Mariam Haspekian describes a new study that expands upon her doctoral work in which she introduced two constructs, *instrumental distance* and *double instrumental genesis*, focusing on the use of spreadsheets within mathematics education. These constructs, originating from French research based on the activity theoretic approaches first proposed by Vygotsky, offer insight into the processes through which teachers appropriate the spreadsheet for use as a mathematical and pedagogical resource. Her case study of an experienced secondary mathematics teacher illuminates aspects of the teachers' practices.

There is resonance between the work of Haspekian and that of Alison Clark-Wilson, whose research also concerns the professional learning of experienced secondary mathematics teachers. Clark-Wilson's longitudinal study of two teachers focuses on aspects of what and how the teachers learnt as they began to use a complex multi-representational technology within an English school setting. In her chapter 'A methodological approach to researching the development of teachers' knowledge in a multi-representational setting', Clark-Wilson illuminates the methodology through which her construct of the *hiccup* became evident, highlighting the challenges that studies into teachers' knowledge development present to researchers. Her approaches, which are framed within the instrumental perspective, offer insight into a researchers' thinking in designing systematic and objective research protocols.

By contrast, in the chapter 'Teachers and technologies: Shared constraints, common responses', Maha Abboud-Blanchard adopts a meta-level approach in her cross-analysis of the outcomes of three different French studies that each researched aspects of the process of teachers' integration of technology within mathematics classrooms.

Two of these studies involved empirical research in secondary classrooms concerning the use of a computer algebra system, dynamic geometry program and web-based resource, whilst the third concerned a meta-analysis of over 600 individual publications on this theme. Abboud-Blanchard, who uses the construct of the *double approach* (Robert and Rogalski 2002), shows how it is possible to identify the characteristics of ordinary teachers' uses of technology and contrast the resulting outcomes in terms of the different components of their practice.

The chapter 'Didactic incidents: A way to improve the professional development of mathematics teachers', by Gilles Aldon, connects to the other chapters in this part through some shared theoretical perspectives. It is based on the instrumental approach, emanating from the viewpoints of *documentational genesis*, and the theory of didactic situations (*milieu*). Aldon introduces the ideas of *didactic incidents* and *perturbations*, theoretical constructs that help to explain the dynamic relation between teaching and learning from the perspective of documentary genesis that is contextualised within a milieu. These ideas support the analysis of teaching and learning as it underpins the construction of knowledge in a dynamic way and as processes that evolve over time, with the possibility of enhancing teachers' professionalism. The data and examples are taken from the European Union Comenius funded project EdUmatics, and provide evidence of this dynamism. The transformation of a resource into a document, or of an artefact into an instrument, when schemes of utilisation are activated, are not stigmatised once and for all, but they are seen as an interrelated set of on-going processes.

Part III

In the chapter 'Meta-didactical transposition: A theoretical model for teacher education programme', Ferdinando Arzarello, Ornella Robutti, Cristina Sabena, Annalisa Cusi, Rossella Garuti, Nicolina Malara and Francesca Martingone highlight the need to take the complexity of teacher education into account with respect to the institutions in which teaching operate, alongside the relationships that teachers must have with these institutions. This chapter also considers the evolution of the professional role of a teacher both as an individual within classes, and as member of a community of teachers. To address this need, the authors use Chevallard's (1985, 1992, 1999) *Anthropological Theory of Didactics* (ATD), which is mainly centred on the transposition of mathematics created by the teacher with the students. It is applied to teacher education, that is, to teachers as learners in a community, in which they improve their professionalism using technologies and resources, discussing among themselves and other communities, particularly that of researchers. The result of this study is the presentation of a model entitled the *Meta-Didactical Transposition,* which describes the evolution of teacher education over time, by analysing the different variables involved: *components* that change from external to internal; *brokers* who support teachers interacting with them; and *dialectic interactions* between the community of teachers and researchers. This chapter is

particularly useful to help researchers interpret the process of teacher education, not only in a particular national context, but in various situations, because this process is usually characterised by praxeologies that are adopted and evolve over time, as well as increases in teachers' awareness when they come into contact with the community of education researchers. Moreover, the model usefully describes the role that institutions may have in this process, especially as, in every country, teacher education programme often start from specific directives given by a central institution that is linked to schools or to the academy.

The chapter entitled 'Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice', by Kenneth Ruthven, compares and contrasts some of the main frameworks that are currently being used in research on teacher expertise in the context of digital technology integration (and within this book). These frameworks are all relatively new and each attends to a somewhat different aspect of the teaching activity. In this chapter, Ruthven analyses each of these frameworks and shows, through examples, how they function differently – both in terms of their epistemological assumptions and their intended unit of analysis – as research tools. All three approaches that Ruthven discusses emerge out of research focused at the secondary level but involving a range of different digital technologies.

Ruthven begins with the Technology, Pedagogical Content Knowledge approach (TPACK), which complements, with the addition of the 'technology component', the widely-used PCK framing of the knowledge that mathematics teachers need to teach. Using examples from research, Ruthven shows how this framework is used to "signal the need to consider technological, pedagogical and epistemological aspects of the knowledge underpinning subject teaching and their interaction in general terms" (p. 380) as part of the other components of knowledge, but that it provides only a "rather coarse-grained tool for conceptualising and analysing teacher knowledge" (p. 380). He then considers the Instrumental Orchestration approach, which also draws on an existing framework (instrumental genesis) focusing on student use of digital technologies (rather than on teacher expertise, as in PCK). This approach has been successful in providing a fine-grained analysis of the organisation of classroom activity around the use of a tool. Researchers using this approach have developed a typology of orchestrations that have enabled the identification of more general classroom patterns as well as comparison across different teachers and/or classrooms. As Ruthven writes, the typology "makes visible an important dimension of the professional knowledge that teachers participating in trialling had employed or developed in order to incorporate use of these digital technologies into their practice" (p. 384). While the Instrumental Orchestration approach used to study the integration of new technologies depends on the development of certain knowledge-for-teaching, Ruthven's third framework, the Structuring Feature of Classroom Practice approach, has a different purpose, which is to support the identification and analysis of teaching-with-technology expertise. This approach has evolved from prior research attending not only to teacher mathematics knowledge but also to classroom organisation and integration. It offers five structuring features of classroom practice that shape the ways in which teachers integrate new

technologies. This approach provides a more differentiated characterisation of some of the key features of the instrumental orchestration approach.

As you read through the chapters and encounter these different approaches (or, indeed, other ones), you might consider some of the particular orientations toward the phenomenon of technology integration in mathematics teaching that Ruthven has outlined, asking yourself perhaps why a particular approach was chosen and what it offers as well as obscures in terms of understanding the phenomena at hand. For beginning researchers, this chapter offers an (often missing) critical comparison between competing frameworks that should make it easier to select the one more appropriate for your research questions and goals.

In the 'Conclusion', the editors make some concluding comments about the similarities and differences between the theoretical constructs, contexts and implications of the book's chapters, for example the ways in which the various constructs might be used to help to shape future research (and its associated methodologies) concerning the appropriation of mathematical digital technological tools in a range of educational settings. In addition, the editors look holistically at the implications of the various constructs on the design, content and implementation of professional development for teachers. Finally, the editors suggest how the individual chapters or combinations of chapters might be used within teaching sessions aimed at the intended readership of this book, that is to say researchers, Masters' or Doctoral students and pre-service and in-service teachers.

References

Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In B. Davis & E. Simmt (Eds.), *Proceedings of the 2002 annual meeting of the Canadian Mathematics Education Study Group Edmonton* (pp. 3–14). Alberta: CMESG/GDEDM.

Chevallard, Y. (1985). *Transposition Didactique du Savoir Savant au Savoir Enseigné*. Grenoble: La Pensée Sauvage Éditions.

Chevallard, Y. (1992). Concepts fondamentaux de la didactique: perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques, 12*(1), 73–112.

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques, 19*(2), 221–266.

Clark, D., & Hollingworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18*, 947–967.

Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics, 61*(3), 293–319.

Drijvers, P., Kieran, C., & Mariotti, M. A. (2010). Integrating technology into mathematics education: Theoretical perspectives. Mathematics education and technology-rethinking the terrain. *New ICMI Study Series, 3*(Part 2), 89–132.

Jaworski, B. (1998). Mathematics teacher research: Process, practice and the development of teaching. *Journal of Mathematics Teacher Education, 1*, 3–31.

Lagrange, J.-B., Artigue, M., Laborde, C., & Trouche, L. (2003). Technology and mathematics education: A multidimensional study of the evolution of research and innovation. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Second international handbook of mathematics education* (pp. 239–271). Dordrecht: Kluwer Academic.

Papert, S. (1980). *Mindstorms: Children, computers and powerful ideas*. London: Harvester Press.
Pimm, D. (1983). Self images for a computer. *Mathematics Teaching, 105*, 40–43.
Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211–246.
Robert, A., & Rogalski, J. (2002). Le système complexe et coherent des pratiques des enseignants de mathématiques: une double approche. *Revue canadienne de l'enseignement des sciences, des mathématiques et des technologies, 2*(4), 505–528.
Sinclair, N., & Jackiw, N. (2005). Understanding and projecting ICT trends in mathematics education. In S. Johnston-Wilder & D. Pimm (Eds.), *Teaching secondary mathematics effectively with technology* (pp. 235–252). Maidenhead: Open University Press.
Trouche, L. (2004). Managing complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*, 281–307.
Wood, T. (Ed.). (2008). *The international handbook of mathematics teacher education* (Vol. 1–4). West Lafayette, USA: Purdue University/Sense.