Chapter 23
Statistical Models in Plant Diagnosis and Calculating Recommended Nitrogen Rates

Paulo Cezar Rezende Fontes, Marcelo Cleón de Castro Silva, and Glauco Vieira Miranda

Abstract The potato, *Solanum tuberosum* L., is an important crop in Brazil and in the world. In addition to other factors, the potato plant needs an adequate nutrient supply. Best N management in potato plants is aimed not only to improve tuber yield and quality but also to increase N fertilizer use efficiency and to reduce environmental risk. It is common to use mathematical models in establishing relationships between N rate and crop yield and plant N content. Those relationships are essential to best N management in potato plant at diagnosis and recommendation phases. In establishing a plant index it is necessary to adjust the data using some mathematical model. Therefore, either in the assessment of plant index or in the rate recommendation it is necessary to select a model. In the text will be discussed the relationship between potato yield and nitrogen rates obtained by different mathematical models and how the model chose affects plant nitrogen indices under Brazilian conditions.

23.1 Introduction

The potato, *Solanum tuberosum* L., is an important crop in Brazil and throughout the world. In addition to other factors, the potato plant needs an adequate nutrient supply. Nitrogen (N) is one of the nutrients of greatest impact on crop productivity. The N effect on potato tuber yield has been well documented worldwide (Meyer and Marcum 1998; Bélanger et al. 2000; Rodrigues et al. 2005; Silva et al. 2007). N has a marked effect on the vegetative and reproductive plant compartments. It is essential for the fast cycle and high growth rate of the potato plant. A higher N

P.C.R. Fontes (✉) • M.C. de Castro Silva • G.V. Miranda
Departamento de Fitotecnia, Universidade Federal de Viçosa,
s/n. 36570-000 Estado de Minas Gerais, Brazil
e-mail: pacerefo@ufv.br

Z. He et al. (eds.), *Sustainable Potato Production: Global Case Studies*,
DOI 10.1007/978-94-007-4104-1_23, © Springer Science+Business Media B.V. 2012

availability has positive effects on stem and leaf growth and intercepted radiation, which generally leads to high tuber yield and N accumulation (Nunes et al. 2006).

Low N supply will not only result in lower yield but will also reduce tuber size due to reduced leaf area and early defoliation. On the other hand, excess of N leads to dry matter yield in other parts of the plant than the tubers (Goffart et al. 2008). When superfluous, N promotes excessive stolon and leaf growth, delays both leaf maturation and tuber differentiation, and reduces the length of the tuber bulking period, tuber solid contents and yield. Additionally, the excess of N remains available in the soil to be leached.

Best N management in potato plants is aimed not only to improve tuber yield and quality but also to increase N fertilizer use efficiency and to reduce environmental risk. It is common to use mathematical models in establishing relationships between N rate and crop yield and plant N content. Those relationships are essential to best N management in potato plant at diagnosis and recommendation phases. All nitrogen recommendation approaches, *ex post*, *ex-ante* and "when necessary" imply uncertainty about which model should be chosen and usually local and temporal variations are neglected.

23.2 Nitrogen Rate Recommendation

In an optimistic view, the N rate recommended by the extension service (*ex ante*) is based on the yield potential, soil organic matter and N use efficiency (NUE). Usually the *ex ante* recommendation is based on the yield potential and the NUE (50%). In a broad sense, under Southeastern Brazilian conditions it is necessary to apply 1.0 kg of N fertilizer for the expected yield of 190 ± 40 kg potato (Fontes 1997).

Alternatively, the recommendation may be based on a mathematical function previously obtained in experiments where the effects of N fertilizer rates on crop yield were evaluated. The relationship between fertilizer application and crop yield is generally represented by a mathematical function which seeks to estimate the optimal rate or the maximum economic rate of N (Zimmermann and Conagin 1986; Seefeldt et al. 1995). As the mathematical function is *ex-post* chosen, this is an *ex-post* recommendation.

Different models, *ex-post* chosen, provide differing values for the estimated optimum fertilizer rate (Nelson et al. 1985; Fontes and Ronchi 2002; Berzsenyi and Dang 2006) and the model chosen largely determines the maximum economic N rate or economically optimal fertilization rate (ENR) affecting the crop profitability and may cause adverse impact on the environment. To illustrate the impact of model choice on estimating N rate, Olness et al. (1998) compared the relative accuracy of three models on 48 corn data sets. In about one-third of the cases all models performed about equally well. The ENR depended on several factors (soil texture, tillage, hybrid, climatic zone) including the model. This renders broad generalizations of ENR quite misleading.

Another approach is applying an intentionally small N rate before planting and to decide, in real time, the supplemental N needs of the crop (Fontes and Araujo 2006). This decision should be based on a plant nitrogen index at the appropriate time. Schepers et al. (1992) called "fertilization when necessary" the decision of topdressing fertilizer based on plant N status. To answer the question how much N should be applied it is necessary to construct an algorithm, a finite sequence of operations including a plant N index.

In theoretical terms, the N rate should be dictated by demand-capacity-efficiency factors combination based on the complex soil-plant-environment. The factors combination may lead to an algorithm. In the algorithm construction process at least three types of information are utilized: (1) plant N demand; (2) soil capacity to provide N; and (3) fertilizer N use efficiency. Ideally, the N rate to be applied as fertilizer should be estimated using the assessment of the integrated system in both providing (soil, water and incorporated organic residues) and demanding N (yield potential of the cultivar in a given production system) modulated by the processes efficiency (Fontes and Araujo 2007). By combining strategies it is possible to reduce the potential for nitrate leaching in the potato crop, but the progress of research has not allowed a quick and ready solution to prevent the potential for nitrate leaching to groundwater in certain regions of the world (Shrestha et al. 2010).

There are several models available in the literature, with different qualitative and quantitative approaches, especially for the estimation of the demand-capacity terms. Normally, variable effects are known qualitatively but the interactions among them are unknown, making the model empirical. The deterministic mathematical models, with one or more variables, are hardly compatible with a set of data showing high variability and low correlations. Still, the mechanistic and deterministic models are useful in knowledge and information systematization and organization. But there is always the challenge of how to parameterize the various processes occurring in biological systems.

Some models attempt to quantify the various factors that affect the demand-capacity processes in an attempt to infer the efficiency of the combination. Normally, those factors are reflected in a specific N index evaluated at the appropriate time in the plant. This index is simultaneously utilized in the N diagnosis and prescription processes, integrants of an algorithm.

In practical terms, the N plant index should enable answering the following questions: (a) is it necessary to fertilize (especially valid question when the N rate applied was the recommended and not an intentionally small rate at planting) and (b) how much N should be applied? In precision agriculture, these questions should be answered in real time. In most Brazilian conditions, up to 25 days after potato plant emergence (time of side dress application*)* is necessary to determine answers to the two above questions.

In establishing the plant N index it is necessary to adjust the data using some mathematical model. Therefore, either in the assessment of plant index or in the rate recommendation it is necessary to select a model.

23.3 Selection of Mathematical Model: The N Rate

There are several classifications for mathematical models, for example, static, dynamic, linear, deterministic, stochastic, empirical, mechanistic, and others. In part, the reliability of the information obtained with the aid of a model depends on the fitness of the model to the experimental data. The term model is also adopted for the representation of a system with flow diagrams where several mathematical submodels and algorithm may be involved in implementing the several factors that explain the system.

In the present paper there is interest in explaining the potato yield as a function of the N rates. This relationship can be described by a mathematical model using the quantitative variable N rate as independent variable and yield as dependent variable. Selecting the most appropriate model to describe the relationship between crop yield and fertilizer rate is not an obvious decision (Bock and Sikora 1990; Angus et al. 1993). There seems no possibility of standardizing a specific model to describe the plant response to N rates. The main reason is that the type of curve needed is inherently dependent on the variation in soil N availability reflecting the added rates.

Generally, in research reports there is little description of how the model was chosen. There is no statistical basis for selecting one functional form over another across all sites and years (Rajsic and Weersink 2008). The choice of a statistical model should be based on some criteria, such as a biological explanation of the phenomenon; the significance of the regression mean square; F-statistic significance or lack of fit; high coefficient of determination (R^2) and the significance of the regression parameters. Besides these criteria, it would be recommended to consider the maximization of productivity and profit. These issues will be addressed in the following example.

The relationship between potato yield and N rates obtained by different mathematical models will be described. This relationship was obtained from research conducted in a Red-Yellow Podzolic Cambic soil where five N rates (0, 50, 100, 200 and 300 kg ha^{-1}), as ammonium sulfate, furrow applied, were evaluated (Silva et al. 2007). Potato 'Monalisa' was cultivated under irrigated conditions and 114-day-growth cycle. After natural canopy drying, the tubers were harvested, remaining in the field around an hour and then weighed. Tuber yield data were submitted to analysis of variance procedures and to linear and nonlinear regression analysis and curve fitting using the SAS and SAEG programs. Six mathematical models were selected to relate yield and N rate: linear plateau, quadratic plateau, Mitscherlich, sigmoidal, square root and quadratic. The first four are nonlinear models and the two last are linear. For each considered model four variables were estimated: (a) the maximum N rate; (b) the maximum physical yield of tubers; (c) money spent on N fertilizer; (d) money left over after selling the potato and paying the N fertilizer. The economic optimum N fertilization rate (ENR) was also estimated for quadratic model at unfavorable and favorable potato price conditions in Brazil (Fig. 23.1).

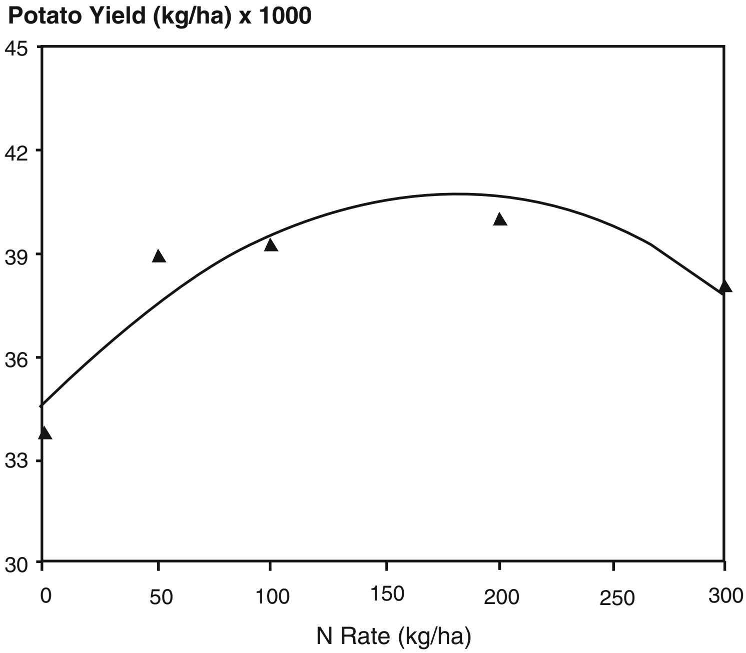

Fig. 23.1 Relationship between potato yield and nitrogen rate as described by the quadratic model

The mathematical expressions of the models are below.

1. Linear plateau, defined by Eqs. 23.1 and 23.2:

$$Y = a + bX, \text{ if } X < C \tag{23.1}$$

$$Y = P, \quad \text{if } X \geq C \tag{23.2}$$

Where Y is the tuber yield (kg ha^{-1}); a and b are intercept and linear coefficient, respectively; X is the N rate (kg ha^{-1}); the constant C is the intersection point of the linear model with the plateau; P is the potato yield when it reaches the plateau.

2. Quadratic plateau, defined by Eqs. 23.3 and 23.4:

$$Y = a + bX + cX^2 \text{ if } X < C \tag{23.3}$$

$$Y = P, \quad \text{if } X \geq C \tag{23.4}$$

Where c is the quadratic coefficient and the others terms were defined in Model 1.

3. Mitscherlich, defined by Eq. 23.5:

$$Y = A / \left(1 - e^{-c(X+b)}\right) \tag{23.5}$$

Table 23.1 Name and mathematical expression of the adjusted models to the relationship between potato yield (kg ha^{-1}) and nitrogen rate (kg ha^{-1})

Model name	Mathematical model expression	R^2
1- Linear plateau	$Y = -83.5986 + 0.0057X$	0.09
2- Quadratic plateau	$Y = 33813 + 178.1X - 1.4937X^2$	0.92
3- Mitscherlich	$Y = 39090/(1 - e^{-1(X+2.0025)})$	0.92
4- Sigmoidal	$Y = 37850 + \frac{200}{1 + e^{-[(X-Xo)/1]}}$	0.92
5- Square root	$Y = 33790 - 44.9876X + 1039.7792X^{1/2}$	0.97
6- Quadratic	$Y = 34608 + 68.8565X - 0.1938X^2$	0.87

Source: Adapted from Silva et al. (2007)

Where A is the maximum expected yield in response to N; c and b are constants and correspond to the N fertilizer efficiency coefficient and the estimated N availability in the soil, respectively; X and Y were defined above.

4. Sigmoidal, defined by Eq. 23.6

$$Y = Yo + \frac{a}{1 + e^{-[(X-Xo)/b]}} \tag{23.6}$$

Where Yo is the yield obtained with the initial rate (g/plant); a and b are non-linear regression model parameters; Xo is the initial N rate or 0 kg ha^{-1}; X and Y were defined above.

5. Square root, defined by Eq. 23.7:

$$Y = a + bX + cX^{1/2} \tag{23.7}$$

Where Y, b, c and X have been defined previously.

6. Quadratic, defined by Eq. 23.8:

$$Y = a + bX + cX^2 \tag{23.8}$$

Where Y, b, c and X have been defined previously.

The models fitted to experimental data (Table 23.1) were evaluated by the following criteria: the significance of the regression mean square (QMRr); significance of the F-statistic or lack of fit (FA); high coefficient of determination (R^2); significance of the regression parameters using the t-test at 1, 5 and 10% and F at 1 and 5% probability (T′ and T″); fidelity to the observed data (FTR). The results are shown (Table 23.2).

In linear plateau, quadratic plateau, Mitscherlich and sigmoidal models the biological explanation of the phenomenon is dependent on the actual rate-yield curve, mainly at the highest N rate. So they were classified as unfaithful to the observed data – FTR – (Table 23.2). Smaller QMRr indicates model better fit the data. The T values showed differences between the N rates evaluated. The coefficient of determination (R^2) is the measure of correlation between N rate and tuber yield. Several models (linear plateau, quadratic plateau, quadratic, square root and

Table 23.2 Regression mean square (QMRr), lack of fit (FA), coefficient of determination (R^2), significance of the regression parameters (T′ and T″) and fidelity to the observed data in the six models

	Criteria					
Models	QMRr	FA	R^2	T′	T″	FTR
1- Linear plateau	17676	NC	0.09	NC	NC	No
2- Quadratic plateau	952641	NC	0.92	NC	NC	No
3- Mitscherlich	639648	NC	0.92	NC	NC	No
4- Sigmoidal	11679632	NC	0.92	NC	NC	No
5- Square root	246196	ns	0.97	6.6**	8.5*	Yes
6- Quadratic	1583441	ns	0.87	3.5***	3.1***	Yes

NC not considered, *ns* not significant by F test
*, **, and *** significant by t test at 1, 5 and 10% probability, respectively
Source: Adapted from Silva et al. (2007)

Table 23.3 Estimated maximum nitrogen rate (DMN), maximum tuber yield (PMFT) and the cost of the nitrogen fertilizer (GAN) in the six models

Models	DMN ($kg\ ha^{-1}$)	PMFT ($kg\ ha^{-1}$)	GAN ($U\$\ ha^{-1}$)
1-Linear plateau	50.00	33,493	97
2-Quadratic plateau	59.64	39,125	116
3-Mitscherlich	65.70	39,125	128
4-Sigmoidal	65.70	39,125	128
5-Square root	133.53	39,797	259
6-Quadratic	177.57	40,720	345
7- Without fertilizing with N	0	33,813	0,00

Source: Silva et al. (2007)

Mitscherlich) were also evaluated by Cerrato and Blackmer (1990) to describe the corn yield response to N rates. The authors obtained R^2 values ranging from 79 to 84 but it was not a reliable criterion for the model selection and the economically optimal N rate identification.

In the present example, with each model but Mitscherlich and Sigmoidal the maximum N rate (DMN), the maximum tuber yield (PMFT), and the cost of the N fertilizer (GAN) were estimated. With asymptotic models it is not possible to calculate the maximum, so it was utilized at 90% of the estimated maximum for Mitscherlich and Sigmoidal models. The results are shown in Table 23.3. Models with distant R^2 values can estimate close values for the estimated DMN which ranged from 50 to 178 kg ha^{-1} depending on the model (Table 23.3). With the quadratic model, the estimated maximum N rate was 178 kg ha^{-1} leading to a maximum tuber yield of 40.7 Mg ha^{-1}.

With the quadratic model, the maximum economic N rate or economically optimal fertilization rate (ENR) was calculated, which was defined as the rate of N application where U$1 of additional N fertilizer returned U$1 of potatoes, and it describes the minimum rate of N application required to maximize economic return (Colwel 1994). ENR was the point where the last increment of N returns a yield

Table 23.4 The lowest and the highest relative price of nitrogen to potato price in two nitrogen fertilizer, from January to October 2010 in Brazil

	Relative price of N/potato price	
Nitrogen fertilizer	Lowest	Highest
Ammonium sulfate	2.22	6.90
Urea	1.76	5.63

Table 23.5 Value receipt from the potato sale (RCVB) and money left over after selling potatoes and paying the nitrogen fertilizer (SAPN) under favorable and unfavorable scenario of potato price in the six models

	Unfavorable scenario		Favourable scenario	
	RCVB	SAPN	RCVB	SAPN
Models	(U$ ha^{-1})			
1- Linear plateau	11,821	11,765	23,642	23,586
2- Quadratc plateau	13,809	13,693	27,618	27,502
3- Mitscherlich	13,809	13,681	27,618	27,502
4- Sigmoidal	13,809	13,681	27,618	27,502
5- Square root	14,046	13,786	28,092	27,832
6- Quadratic	14,372	14,027	28,743	28,399
7- No fertilizer N	11,934	11,934	23,868	23,868

Source: Adapted from Silva et al. (2007)

large enough to pay for the additional N. ENR was calculated by setting the first derivative of the N response curve equal to the ratio between the cost of fertilizer and the price of potatoes. The resulting equation was solved for the ENR. Price ratio was the ratio of N fertilizer price to potato tuber price (U$/kg ÷ U$/kg), in two potato price scenarios, favorable and unfavorable. For the calculations, N price was U$ 1.94/kg. Potato prices were $ 0.35/kg (unfavorable scenario) and U $ 0.71/kg (favorable scenario). The estimated ENR value was 163 or 171 kg ha^{-1} in unfavorable or favorable potato price scenarios, respectively. For reference, the lowest and the highest relative price of N to potato price in two fertilizer sources in Brazilian conditions are shown (Table 23.4).

The value received from the potato sale (RCVB) was calculated by multiplying PMFT by the potato price. Also evaluated was the amount of money left over after selling potatoes and paying the N fertilizer (SAPN), which was obtained from DMN and the corresponding tuber yield. Fertilizer application costs were considered equal at all N rates and any yield variation does not imply extra costs. Results are shown in Table 23.5. The SAPN would be highest with the quadratic model (Table 23.5). Moreover, a higher amount of N estimated by the quadratic model in relation to the square root could be an insurance against possible losses of N. This probably did not occur due to several conditions, among them the N source, ammonium sulfate, applied in the furrow, a loamy soil and the drought period, only 255 mm of rainfall during the growing period supplemented by irrigation.

Three statistical models (quadratic, square root and exponential-Mitscherlich) were compared to describe the potato yield response to N rates at planting, in Canada

(Bélanger et al. 2000). High values of R^2 for the three models were found and the highest optimal N rate was estimated by the quadratic model, followed by the square root and the exponential model. That is, the estimated N rate for the potato crop depended on the mathematical model as also mentioned by Neeteson and Wadman (1987) in Netherlands.

A quadratic model was used to describe the yield response of potato cultivars to N fertilizer rates (0 to 300 kg ha^{-1}) under Brazilian conditions (Fontes et al. 2010). For the maximum marketable tuber yields, the optimum fertilization rates were 168, 212, 175, and 193 kg ha^{-1} of N for Ágata, Asterix, Atlantic, and Monalisa, respectively. For these cultivars at the optimum N fertilization rate the predicted marketable yields were 33.1, 32.3, 33.3, and 25.9 Mg ha^{-1}, respectively. The economic optimum N fertilization rates ranged from 147 to 201 kg ha^{-1} depending upon cultivar and relative prices of N and potato tubers. Depending on the cultivar, under favorable price conditions (low N price and high tuber price), the economic optimum N fertilization rates to be applied by potato growers were 92–95% of the estimated N fertilization rate for obtaining the maximum potato yield. Under unfavorable conditions (high N price and low potato tuber price) the economic optimum N fertilization rates to be applied should be decreased to 86–92% of the rates for maximum yield. Usually, with crops of high value, as potato, the fertilizer price has less impact on the most economic rate than for crops with lower value.

In another potato crop study under Brazilian conditions, among the regression models tested (linear, quadratic and square-root), the quadratic model was more appropriate to describe the relationship between N rates (0 to 400 kg ha^{-1}), as urea, and yield of potato cultivars (Coelho et al. 2010). Nitrogen rates at 297 and 250 kg ha^{-1} provided the highest commercial potato yield of Agata (45.1 Mg ha^{-1}) and Asterix (46.5 Mg ha^{-1}).

The quadratic model is not always the best choice to represent the relationship between N rate and yields, as was found with corn plants (Cerrato and Blackmer 1990; Bullock and Bullock 1994). They found that the quadratic plateau was the most appropriate model.

Depending on many factors such as pre-crops, tuber yield, rainfall, soil type, cultural practices, season year, spacing, source, and cultivar involved in the experiments, the N fertilizer use efficiency in our conditions has been 190 ± 40. That is, for each kg of N fertilizer added the yield has been 190 ± 40 kg of potato. So, to produce 30 Mg ha^{-1} of potato it will be necessary to use from 143 up to 200 kg ha^{-1} of N. In Minas Gerais State, the recommendation has been 190 kg ha^{-1} of N for the 30 Mg ha^{-1} target yield (Fontes 1999).

23.4 Model Selection: Plant N Status

The potato plant N status (ENP) can be monitored by several direct and indirect methods. The main ones are the analysis of N content in the leaf dry matter, the petiole sap nitrate content, and leaf chlorophyll content with several studies trying to use leaf spectral reflectance indices determined by a spectroradiometer or a digital camera (Wu et al. 2007; Goffart et al. 2008; Zebarth et al. 2009; Cohen et al. 2010; Busato et al. 2010; Fontes 2011).

Almost all tests to assess the ENP employ a reference or critical value to assist in making the decision to side dress N in the potato crop. Several factors affect the critical value among them the mathematical procedures employed to calculate its value (Fontes 2001). This was also shown by Fontes and Ronchi (2002) in a study that aimed to establish critical values for several plant N indices. Plant indices assessed included chlorophyll meter (SPAD) readings, petiole sap nitrate content (PSNC), and N contents in the leaf dry matter (ORNL) under different soil and nutrient solution conditions, and determined by three different statistical procedures.

In the procedure designated as ‘one’, linear, quadratic, square root, potential, exponential, hyperbolic, logarithmic and cubic root models were fitted to statistically significant data using N level as the independent variable. The best fitting model with biological explanation of the phenomenon was used to estimate the maximum shoot dry weight (SDW) obtained by equating the first derivatives of the best fitting model to zero, solving for X, substituting the X values into the model and solving for Y. To estimate SPAD, PSNC, and ORNL critical values (CV) in both experiments, N rate associated with maximum shoot dry weight (CV100) was introduced into the best fit model previously determined, which correlates SPAD, PSNC, and ORNL to N rate. The model also was used to determine the SPAD, PSNC, and ORNL critical values associated with 99.9, 99, 95, and 90% of the maximum SDW.

In the procedure designated ‘two’, the initial steps were the same as in ‘one’, but the best fitting model was chosen among only linear, quadratic and cubic models. In the procedure designated ‘three’, all models listed in procedure one were fitted to SPAD, PSNC and ORNL as independent variables (X) and the SDW as the dependent variable (Y).

In each experiment, the best fitting model within the range of observed X values was used to estimate SPAD, PSNC and ORNL critical values at CV100, CV99.9, CV99, CV95, and CV90. There were considerable disagreement among the statistical procedures, substrates and yield levels selected to estimate critical plant N indices, indicating a need to emphasize them when setting critical values. As expected, all critical N indices in tomato plants grown in soil and nutrient solution were higher when 100% maximum shoot dry weight was selected compared to lower percentage of the maximum shoot dry weight.

Selecting higher maximum values for the critical value imply higher N rate. Using a lower optimum N rate prevents over-fertilization but highest yields can not be assured. As the price of N fertilizer is relatively low in relation to potato, a high percentage of the maximum yield should be chosen. But using enough N fertilization to reach 100% of the maximum yield is usually not economically and ecologically optimal. The impact of uncertainty on the optimum N fertilization rate and agronomic, ecological and economic factors was discussed by Henke et al. (2007).

23.5 Conclusion

The complex relationships between N rate and crop yield and plant N content can be explaining by a model. Models are simplifications to facilitate understanding, organizing, reasoning and eventually allow the prediction of certain complex relationships. Those relationships are essential to best N management in potato plant at diagnosis and recommendation phases. Therefore professionals are involved in the selection of more appropriate models either in the assessment of plant index or in the rate recommendation.

Acknowledgments To CNPq and FAPEMIG by awarding research productivity grants and for financial resources. To UFV for the physical and professional infrastructures.

References

Angus JF, Bowden JW, Keating BA (1993) Modeling nutrient responses in the field. Plant Soil 155(156):57–66

Bélanger G, Walsh JR, Richards JE, Milburn PH, Ziadi N (2000) Comparison of three statistical models describing potato yield response to nitrogen fertilizer. Agron J 92(5): 902–908

Berzsenyi Z, Dang QL (2006) Use of various functions to analyze the fertilizer responses of maize (Zea mays L.) hybrids in long-term experiments. Acta Agron Hung 54(1):1–14

Bock BR, Sikora FJ (1990) Modified-quadratic/plateau model for describing plant responses to fertilizer. Soil Sci Soc Am J 54(6):1784–1789

Bullock DG, Bullock DS (1994) Quadratic and quadratic-plus-plateau models for predicting optimal nitrogen rate of corn: a comparison. Agron J 86(1):191–195

Busato C, Fontes PCR, Braun H, Cecon P (2010) Seasonal variation and threshold values for chlorophyll meter readings on leaves of potato cultivars. J Plant Nutr 33(14):2148–2156

Cerrato ME, Blackmer AM (1990) Comparison of models for describing corn yield response to nitrogen fertilizer. Agron J 82(1):138–143

Coelho FS, Fontes PCR, Puiatti M, Neves JCL, Silva MCC (2010) Dose de nitrogênio associada à máxima produtividade econômica de tubérculos de cultivares de batata e índices do estado de nitrogênio da planta. Rev Bras Ciênc Solo 34(4):1175–1183

Cohen Y et al (2010) Leaf nitrogen estimation in potato based on spectral data and on simulated bands of the VENμS satellite. Precis Agric 11(5):520–537

Colwel JD (1994) Estimating fertilizer requirements: a quantitative approach. CAB International, Wallingford

Fontes PCR (1997) Preparo do solo, nutrição mineral e adubação da batata. Universidade Federal de Viçosa, Viçosa

Fontes PCRB (1999) In: Ribeiro AC et al (eds) Recomendações para o uso de corretivos e fertilizantes em Minas Gerais. 5a Aproximação. CFSEMG, Viçosa

Fontes PCR (2001) Diagnóstico do estado nutricional das plantas. Editora UFV, Viçosa

Fontes PCR (2011) Nutrição mineral de plantas: avaliação e diagnose. Arka Editora, Viçosa

Fontes PCR, Araujo C (2006) Use of a chlorophyll meter and plant visual aspect for nitrogen management in tomato fertigation. J Appl Hortic 8(1):8–11

Fontes PCR, Araujo C (2007) Adubação nitrogenada de hortaliças: princípios e práticas com o tomateiro. Editora UFV, Viçosa

Fontes PCR, Ronchi CP (2002) Critical values of nitrogen indices in tomato plants grown in soil and nutrient solution determined by different statistical procedures. Pesqui Agropec Bras 37(10):1421–1429

Fontes PCR, Braun H, Busato C (2010) Economic optimum nitrogen fertilization rates and nitrogen fertilization rate effects on tuber characteristics of potato cultivars. Potato Res 53(2):1–13

Goffart JP, Olivier M, Frankinet M (2008) Potato crop nitrogen status assessment to improve N fertilization management and efficiency: past-present-future. Potato Res 51(3–4):355–383

Henke L, Breustedt G, Sieling K, Kage H (2007) Impact of uncertainty on the optimum nitrogen fertilization rate and agronomic, ecological and economic factors in an oilseed rape based crop rotation. J Agric Sci 145(5):455–468

Meyer RD, Marcum DB (1998) Potato yield, petiole nitrogen, and soil nitrogen response to water and nitrogen. Agron J 90(3):420–429

Neeteson JJ, Wadman WP (1987) Assessment of economically optimum application rates of fertilizer N on the basis of response curves. Fertil Res 12:37–52

Nelson LA, Voss RD, Pesek J (1985) Agronomic and statistical evaluation of fertilizer response. In: Engelstad OP (ed) Fertilizer technology and use, 3rd edn. S.S.S.A, Madison

Nunes JCS, Fontes PCR, Araújo EF (2006) Potato plant growth and macronutrients uptake as affected by soil tillage and irrigation systems. Pesqui Agropec Bras 41(12):1787–1792

Olness A, Evans SD, Alderfer R (1998) Calculation of optimal fertilizer rates: a comparison of three response models. J Agron Crop Sci 180(4):215–222

Rajsic P, Weersink A (2008) Do farmers waste fertilizer? A comparison of ex post optimal nitrogen rates and ex ante recommendations by model, site and year. Agric Syst 97(1–2):56–67

Rodrigues MA, Coutinho J, Martins F, Arrobas M (2005) Quantitative side dress nitrogen recommendations for potatoes based upon crop nutritional indices. Eur J Agron 23(1):79–88

Schepers JS, Francis DD, Vigil M, Below FE (1992) Comparison of corn leaf nitrogen concentration and chlorophyll meter readings. Commun Soil Sci Plant Anal 23(17/20):2173–2187

Seefeldt SS, Jensen JE, Fuerst EP (1995) Log-logistic analysis of herbicide dose–response relationships. Weed Technol 9(2):218–227

Shrestha RK, Cooperband LR, Macguidwin AE (2010) Strategies to reduce nitrate leaching into groundwater in potato grown in sandy soils: case study from North Central USA. Am J Potato Res 87(3):229–244

Silva MCC, Fontes PCR, Miranda GV (2007) Modelos estatísticos para descrever a produtividade de batata em função da adubação nitrogenada. Hortic Bras 25(3):360–364

Wu J, Wang D, Rosen CJ, Bauer ME (2007) Comparison of petiole nitrate concentrations, SPAD chlorophyll readings, and QuickBird satellite imagery in detecting nitrogen status of potato canopies. Field Crop Res 101(1):96–103

Zebarth BJ, Drury CF, Tremblay N, Cambouris AN (2009) Opportunities for improved fertilizer nitrogen management in production of arable crops in eastern Canada: a review. Can J Plant Sci 89(2):113–132

Zimmermann FJP, Conagin A (1986) Ajuste de modelos polinomiais do 2º grau em pesquisas com fertilizantes. Pesqui Agrop Bras 21(9):971–978