Chapter 4
What Do Mathematics Students Say About Mathematics Internationally?

Introduction

In this chapter we continue our investigation of students' views about mathematics. Here we move to the second phase of our project, involving a much larger and more diverse group, over 1000 undergraduate students from five universities in five different countries – indeed, five different continents. The previous chapters have investigated the views of a small number of undergraduate students studying mathematics as a major. Information on their ideas about mathematics, learning mathematics and using mathematics was collected from a small number of semi-structured interviews and analysed using a phenomenographic approach. By extending our research to an international undergraduate group, we are able to check whether the results that we obtained with our 22 mathematics major students in an Australian university can be supported with a much broader sample group.

In order to achieve this generalisation, we have to change some features of our earlier study. The most obvious one is that it would not be practicable to carry out so many interviews, so instead we have gathered students' views using written, open-ended questions. We are still interested to see what new ideas may come up from the broader group in the study, so we give them the opportunity to write down their thoughts, yet we realise that for practical reasons we can ask them only a few questions about their views of mathematics, and their responses may only be fairly short written statements, making it more difficult to identify their conception of mathematics. However, we are not starting from scratch – we already have a categorisation for conceptions of mathematics from our small group of undergraduates, and our aim is to investigate whether those categories can be used to classify the ideas from this much larger group.

Another feature of this larger sample is that they are not all mathematics majors: the group includes students studying mathematics as a 'service' subject in areas such as engineering, commerce and computer science, as well as some who are studying mathematics as part of a teaching qualification. For such students, the use of mathematics

L.N. Wood et al., *Becoming a Mathematician: An international perspective*,
Mathematics Education Library 56, DOI 10.1007/978-94-007-2984-1_4,

is one component of their professional life, rather than its centre. This might result in a narrower range of conceptions of mathematics amongst groups of students for whom mathematics was a component rather than the focus of their professional preparation – though this is not our general experience in phenomenographic studies. We have shown in Chap. 2 that engineering and sports science/tourism students showed the same range of conceptions of mathematics as did mathematics major students. In previous research, we have shown that this situation holds also for statistics, with majors and service students showing the same range of conceptions of the discipline, from narrowest to broadest views (Petocz and Reid 2005). Further, in a quite different field, we have found that primary and secondary school children show the same range of conceptions of the notion of 'environment' (Loughland and et al. 2002), and that the views of adults can be adequately classified using the same sample space (Petocz and et al. 2003).

Our results show that the original sample space can be applied successfully to the larger group, though we have made two modifications. Firstly, we identified a conception of mathematics ('number') that was narrower than the 'components' conception. The two conceptions share a fragmented view of mathematics, though the narrower 'number' conception lacks the notion that 'components' are part of a coherent mathematical investigation. When we analysed the results from the interviews with mathematics major students, we did not distinguish this narrowest conception as the sample did not contain any student who viewed mathematics exclusively at this level. Secondly, we decided to separate the intermediate 'modelling' conception into applied and pure aspects, referred to here as 'modelling' and 'abstract' respectively. This was foreshadowed in our earlier analysis, and in the team's early discussion of the interview transcripts, when we referred to models as being representative of specific situations or universal principles.

In the next sections of this chapter we report on the categorisations of students' conceptions of mathematics and the relationships between these categories, supported by quotes from their written responses to the open-ended questions. We use standard quantitative techniques to investigate the relationships between the conceptions of mathematics and other variables such as university, year and area of study. We summarise students' views about their future use of mathematics, obtained from the responses to two of the survey questions. Finally, we discuss the implications of these results for teaching and learning university mathematics. By the end of the chapter, we are in a position to assess the contribution of adding an international perspective to our overall aim of investigating the process of becoming a mathematician.

The Methodology of Our Study

In this second phase of our study of students' ideas about mathematics, we broadened our focus from a small group of undergraduate mathematics majors at an Australian university to a much larger sample of students studying mathematics as a major or as a service component in courses such as engineering, commerce and computer science, or as part of a mathematics education degree. With our international team, we developed an open-ended questionnaire that was completed by 1182 students from five universities on five continents – the University of Technology, Sydney (UTS) in Australia; the University of Pretoria, South Africa; the University of Ulster, Northern Ireland; Concordia University, Montreal, Canada and the Universiti Brunei Darussalam, Brunei. All these countries have historical links to the British Empire, and hence their educational systems derive in some part from the British system. Nevertheless, their present situations are quite different, and include for the majority a non-English national language.

Students in relevant classes were informed of the aims of the research, were invited to participate and, if they agreed to do so, completed the survey in class. While the survey questions were presented in English, the language of instruction in the class, students were told that they could respond in their preferred language, and some students from the South African and Canadian samples replied in Afrikaans and French respectively. Although participation was voluntary, the majority of students in each class supported the research program by completing the survey. The survey consisted of general demographic questions, asking students their age, sex and language background, and their degree and year of studies; these were followed by three open-ended questions. The first and most important question was: *What is mathematics?* The responses to this question form the focus of the first part of the present chapter. The two additional questions were: *What part do you think mathematics will play in your future studies? … in your future career?* A summary of the responses to these questions is presented later in this chapter.

To analyse the data from the first question, we used an approach based on a knowledge of the previously-developed phenomenographical categories of conceptions of mathematics (as we have described them in Chap. 2). These categories were used as a starting point, with the possibility of modifying or augmenting them if necessary. Student responses from the different institutions were randomly rearranged, making it impossible to determine to which institution, sex or study field the student belonged. The responses were labelled so that they could later be 'un-randomised' and linked to the demographic information for quantitative analysis. Four members of our international research team (Leigh Wood, Ansie Harding, Johann Engelbrecht and Geoff Smith) studied each response separately and suggested categories describing the variation in students' conceptions of mathematics. These categories substantiated and in some cases extended those found in the previous interviews undertaken with the Australian undergraduates, and they were refined and checked by repeated reading and discussion. The final categories were confirmed by identification of appropriate illustrative quotes from the questionnaires. Following this, each analyst coded the responses into the categories identified and compared results to check the consistency of the coding. Where there was a difference of opinion, we were usually able to reach a consensus after further discussion. On those occasions (fewer than 4% of cases) where consensus could not be reached, the response was omitted from the analysis.

For the second and third questions a content analysis (Weber 1990) seemed more appropriate than a phenomenographic analysis, particularly as we had no previously-developed outcome space on which to base analysis. We returned to the original interview study, which contained some relevant comments about future use of mathematics, and identified various themes that were then used in the content analysis of the survey responses. The unit of analysis was the sentence (or occasionally, group of sentences) that each student wrote in the survey concerning their future use of mathematics. Again, the same four team members independently classified the students' responses according to the themes identified and compared results. All differences were resolved by discussion by the team.

To some extent, the form of the survey inclined students to write less rather than more, and hence their statements were in some cases too brief to accurately determine their conception of mathematics, or their views about its future use. However, there were many students who wrote more lengthy responses to one or more of the questions – over 100 words in some cases, although the average was around 20 words per question. Although we used the students' responses to determine their categorisation, it is an unavoidable limitation of our methodology that such short responses could have resulted in wrong allocations. During interviews, such problems can be resolved by further questioning, so that even if a student starts with a short 'definition' of mathematics as *"a subject involving numbers"*, or a brief statement that mathematics will play *"a major part"* in their future professional work (quotes from the survey responses), their underlying ideas can be explored in greater depth.

Results from the Survey – Conceptions of Mathematics

In this section we describe the categories that we found, illustrate them with quotes from students and summarise the relationships between them. Some of the responses to this question consisted of short statements without details, or longer but evasive statements, and so we were unable to categorise them into conceptions. Some examples are: *"a language"*, *"a science"* and *"such a question could require a very complex answer which I feel I am not both educated and wise enough to answer"*. Other responses gave information on students' attitudes to rather than conceptions of mathematics, for example, *"hard stuff"*, *"a waste of time"* and *"my first, my last, my everything"*. The majority of responses, however, could be attributed to one of the conceptions of mathematics identified in the previous interview study, or a conception derived from those. We identified five qualitatively different categories (the numbering has been chosen to highlight the consistency with the previous results from the interviews).

(0) Mathematics Is About Numbers

In this conception, students consider mathematics to be connected with numbers and calculations. Mathematics is manipulation with numbers with no essential advance beyond elementary arithmetic. People mention numbers, calculations, sums and basic operations, usually fairly briefly, as the following examples show.

- Mathematics is where we do calculations and deal with numbers.
- The study of numbers in different formats.
- It is a study of the science of numbers and manipulation of numbers.
- Mathematics is the understanding of numbers and calculations.
- Mathematics is a subject that is set up and designed to help your knowledge of arithmetic. It is a very important subject for everybody who is situated at a school or college as it is a compulsory unit which deals with numbers and multiplication which is needed in future life.

The last quote is more lengthy, and talks about the importance of mathematics, though it retains a focus on number (or arithmetic).

(1) Mathematics Is About Components

Here, students view mathematics as a toolbox to be dipped into when necessary to solve a problem, or a disparate collection of isolated techniques unrelated to real-world applications. Students may list some of the components, or mention formulas, equations and laws.

- Mathematics is a collection of tools that you can use to solve problems.
- Maths is a selection of theorems and laws, which help solve equations and problems.
- Mathematics is a study involving analysis, statistics, algebra and other calculations.
- Mathematics is the use of numbers and letters to solve equations and problems so that objectives can be achieved.
- Mathematics is the use of numbers to solve problems and engineering tasks through the use of processes and formulas.
- It is the field where we learn to add, subtract, multiple and divide correctly. We also learn how to put data onto graphs and, well I don't know, I think it also has got to do something with logic, and probability. Since people or rather mathematicians said maths is the universal language, I was interested in know[ing] more about it.

This last and most extensive quote also illustrates the hierarchical nature of the conceptions, containing obvious reference to the idea of mathematics as number.

(2a) Mathematics Is About Modelling

In this conception, students link mathematics to the world using the notion of modelling. They make strong connections between mathematics and the physical world, which can be described, perhaps imperfectly, by mathematics. Mathematics is seen as a human endeavour invented to describe the world. Here are some illustrations of the conception:

- The attempt to explain the physical laws, patterns of the physical world by algebraic and numerical means.
- Mathematics, and especially actuarial mathematics, is the model set up to analyse and predict real world events.
- Mathematics is a way to solve problems presented by physics, chemistry, finance and many other fields. It is a way to model the world, so we can understand it better.
- Mathematics is the study of modelling real life issues. Because we are extremely uncertain about the future, the models we derive in mathematics [are] actually a relief for future uncertainties.
- A modelling tool. We use it to predict and determine things such as how a beam will react under stress, e.g. it would be less costly to predict and model the above on paper rather than building first.
- It is the study that provides you with the knowledge to explain things in a form of an equation, for example, the number of accidents in one place can be represented by a graph that is defined by a mathematical equation, and of course simple arithmetic.

Again, the last quote gives a hint of the hierarchy, referring to *"simple arithmetic"*.

(2b) Mathematics Is About the Abstract

The emphasis in this conception is on mathematics as a logical system or structure, the 'pure' models that are a counterpart to the 'applied' models in the previous conception. Applications and modelling techniques may be recognised but are regarded as secondary to the abstract structure of the mathematics. Mathematics is the 'other', perhaps even a kind of game of the mind, somehow pure and abstract. The following quotes illustrate the conception:

- Conceptual thought and logical development of ideas.
- An intellectual pursuit. Conceptual understanding and the application of techniques.
- Maths is a way to describe a perfect world. A way to put something on paper that is impossible in the real world. Thus, maths is something that only exists in theory. A very happy place.
- It is the way in which an abstract problem gets turned into a logical numerical analysis and from which a logical conclusion can be made. It's the logical way of solving an abstract problem through various methods.

- Mathematics is not so much a subject as a way of thinking. It is based on logical thought used to solve complex problems. It is neither just creative nor just methodical but rather a combination. It has little to do with numbers and much to do with principles.
- The abstract yet the most fundamental science in the world. The basic tools for analysing other subjects. Though other subjects will study the real thing (theorems, phenomena, particles …), mathematics focuses on providing logical and numerical ways for your research in others.

(3) Mathematics Is About Life

In this broadest conception, students view mathematics as an integral part of life and a way of thinking. They believe that reality can be represented in mathematical terms in a more complete way than the modelling conception. Mathematics mediates their way of thinking about reality. They may make a strong personal connection between mathematics and their own lives. This is what some students have written illustrating this conception:

- A way of life – [I] don't have a definition handy at present though.
- Mathematics is the language of nature. It is the way in which nature is ruled by God.
- Mathematics is a way to approach life in an analytical manner as to support and formalise natural processes. In a sense it is a way to understand how life works.
- Mathematics is life. Without it life would be a misery. It helps us understand many concepts in life. it even helps in science subjects, especially physics.
- Mathematics is an essential part of everyone's life. It is used either directly or indirectly (was done by someone else) in every facet of our lives. Most technology uses mathematics either in developing the strategy or in implementing a task. Maths is what makes the world function.
- Mathematics is the key factor involved in almost all lines of work, buildings, banking, accountancy, etc. Maths helps society to build in units and provides society with a backbone to all careers. It is a combination of numbers and equations which helps find calculations and answers to lengths, statistics, etc.

The first couple of quotes are very short, maybe even glib, but they give a clear flavour of the 'life' conception. The others develop the idea of the universal aspect of mathematics and its connection with professional and personal life. Interestingly, the last quote includes reference to the 'numbers' and 'components' conceptions. And here is a final interesting quote; it also includes reference to the narrowest conceptions, as well as illustrating aspects of mathematics as modelling. Yet it contains both a range of modelling situations (reminiscent of Eddie's quote from Chap. 2

Table 4.1 Distribution of conceptions of mathematics

Conception	N	%
0. Number	109	9
1. Components	515	44
2a. Modelling	235	20
2b. Abstract	165	14
3. Life	71	6
(Missing/uncoded)	87	7
Total	1,182	100

illustrating the 'life' conception) and also a reference to the role of mathematics in *"everyday life"*:

- It is series of calculations and formulas that aims to achieve business needs and everyday needs. It is an art because there are many ways to solve a problem. It is a language because you have to learn a lot of jargon. Research, we use mathematics to determine the optimal solution to companies' problems. In additional to that, mathematics has also played a vital role in our everyday life. Housewives use mathematics to do household accounting, budgeting. Students use it to pass their examinations.

These conceptions have a clear hierarchical relationship, inherited from the outcome space found in the earlier interview study, with two modifications. The narrowest conception is 'number', which was not identified as a separate conception in the interview transcripts (though our previous studies of students' views of statistics, which could be viewed as a component of mathematics, did include a 'numbers' conception – Reid and Petocz 2002). This is followed by the 'components' conception, then the 'modelling' and 'abstract', and finally the broadest 'life' conception. We regard the 'modelling' and 'abstract' conceptions to be at the same hierarchical level: one describes modelling applied to the real world, while the other refers to abstract (mathematical) structures and ideas. Neither seems to include the other logically, and the empirical evidence – both from these surveys and the previous interviews – leads us to place them at the same level, as two aspects of the same idea.

The distribution of these conceptions is shown in Table 4.1. Due to the larger numbers of students involved in the open-ended survey, we are able for the first time to make meaningful quantitative comment about the distribution of conceptions. The 'components' conception is the most common in this group, followed by 'modelling', and the 'life' conception is the least common.

Results from the Survey – Future Use of Mathematics

The framework for our content analysis of students' views about their future use of mathematics is grounded in comments that students made in our original interview study. Although they were not specifically asked about the role of mathematics in their future, many of the interviews did include some responses that seemed to relate to their careers. Surprisingly, some students said that they had no idea what role mathematics would play in their future careers, even though they were majoring in an area of mathematics. Our content analysis of these comments could be grouped into four categories: unsure; procedural skills ('knowing how'); conceptual skills ('knowing that'); and professional skills ('knowing for'). We have found these skills categories useful in assessing the development of generic skills in the context of undergraduate business education (Wood et al. 2011). The categories allow us to describe the range of ideas that students had, and form a basis for checking the validity of the later (and much briefer) questionnaire responses.

Unsure: Some students had little idea of what they might be doing as a mathematician. It was sometimes even difficult for them to see themselves as a mathematician or know what they could offer an employer with their mathematical skills.

> *Candy*: I'm not exactly sure myself, so I can't really imagine what it will be like to work as a mathematician, or be recognised as a mathematician until I graduate, and a lot of people wouldn't even realise, they will be probably thinking 'what can I do, what can a mathematician, like, offer me?', in a sense, if you know what I am saying. It's not like, oh, accountant, lawyer, like that's just straight away 'oh, I need one of those', but like with a mathematician, 'what can I do with a mathematician, what do I need one for?', you know. So I'm not exactly sure, because right now that's what I think as well.

Procedural skills: Some students had the idea that mathematics could be used as a toolbox of procedures from which to select as needed in their further career. The tools may be simple or complex, but they remain tools, and only certain isolated skills or techniques are regarded as relevant.

> *Sujinta*: It's like a toolbox, you are getting a lot more tools and in, like if you are a carpenter or something, like you did, before you had a maths degree, you just had a screwdriver, where you come back and now you have a screwdriver, a Phillips head, you've got pliers to do different things, you've got saws to shorten things, so like simplifying things, you've got a lot more tools there to play with, so it makes you a lot, well, when you get into the workplace, you are much more of an attractive sort of employee to have, yeah.

Conceptual skills: Some students focused on the idea that studying mathematics develops conceptual skills such as the logical thinking associated with the mathematical approach – solving mathematical problems and the notion of mathematical proofs.

> *Hsu-Ming*: [What are you aiming to achieve through learning mathematics?] Nothing specific, I guess it's more of a way of thinking, a thought process, rather than anything specific, or I can't come out saying, I've not come into this particular course wanting to learn one specific thing or many specific things. So I guess I'd have to say, and its rather vague, a generalised thought process.

> *Brad*: I think it's one of the fundamental things, because mathematics is all based on proof. We, somebody, notices something happens in this particular case and then they sit down and establish whether it will happen in every single case, and all those proofs are based on logic.

Professional skills: Some students pointed to the generic benefit of studying mathematics, rather than to any specific role for the mathematics. These included generic problem solving (as opposed to the skill of solving specific mathematical problems), analytical and communication skills.

> *Monique*: The problem solving, the analytical skills, the, the decision making and some, be able to use like, be able to use for example the statistical packages that, and we learn that at uni, we are learning it right now, so I think it will be useful when I find work.

> *Dave*: I guess statistical consulting is something that interests me and it seems for that the skills you need are relatively narrow. I guess first and foremost you need the communication skills to, you know, figure out what is going on and relate it to your, whoever's employing you, your customer.

The responses in the survey addressed the role of mathematics in students' future studies and in their future careers. These were obviously seen by students as related, and many students gave the same or similar responses to both questions. Using the framework of unsure, specific procedural or conceptual skills and generic professional skills, we now present some quotes from the survey responses illustrating students' comments on their future use of mathematics. The framework is extended by a final category where students wrote of the essential role of mathematics, though without any detail.

Unsure: Some students felt uncertain, or even said that they had no idea at all, about their future use of mathematics.

- I don't know what career I am going to pursue so I do not know.
- Do I look like a fortune teller?
- Dunno, get back to me in about 10 years.
- To be honest I'm not entirely sure. Originally I thought it would have minimal, since I thought I would concentrate on the IT part of my studies. But I'm now open to any job opportunities math may open for me.

Procedural skills: This is the view of mathematics as a toolbox. Students believed that they would select from a range of mathematical procedures as they needed them, and that only certain isolated skills or techniques were relevant.

- Mathematics will provide a way to find values and prices of various financial instruments.
- I will use it to measure materials and work out dimensions of materials.
- A large part as calculations have to be carried out in order to carry out safety checks.
- It will be important for calculating stresses, loads and forces on structures.
- For calculating money in business if I have one. Also when I need to calculate currency exchange rate in overseas. Basically for the calculation of the daily use products.

Conceptual skills: These statements are based on the idea that mathematics develops logical thinking and mathematical problem-solving skills, which are useful for deepening understanding in other disciplines or professions.

- I will be required to assess the financial positions of funds and also be able to analytically make decisions based on a firm foundation.

- Quick thinking, ease with tackling difficult life problems, ability to think on an intense level.
- It will help us to think more logically and therefore able to solve problems more efficiently.
- Logic, since I'm taking computer science.
- The methods of solving problems, the many ways to derive to a solution, the discovering of new methods of solving them will help me enhance my mind to work faster as I go along to further my study in maths.

Professional skills: Here students see the importance of mathematics as providing a range of generic skills for their future profession, whatever that may be. They refer to communication and presentation skills, high-level numeracy skills, and generic problem-solving skills. Also included here are comments about the generic usefulness of mathematics in studies or the professional workplace, though without any specific details, as illustrated by the last quote.

- It will allow me to make better judgements based on a sound knowledge of various types and forms of data. My presentation skills may also be improved through the use of graphs, charts, etc.
- Maths helps me to become a good teacher and a better person, not left behind by much more educated people.
- It is important that I have a sound understanding of maths as it will allow me to evaluate and communicate ideas and problem solving throughout my future career.
- Huge! But I fear that my limited knowledge of IT programming and hard math as well as the increasing capabilities of PCs will render my maths skills too rudimentary and expensive to be of any real use. I hope to take a sort of reasoning or method of problem solving away from this rather than a bag of tricks and clever manipulations.

Necessary: Another group of responses talked about the necessary role of mathematics in a student's future studies or professional life, though without giving any further detail. This category seems to be an artefact of the survey methodology, asking students to respond in a few words or sentences to the questions. Follow-up questions (in an interview) would have been able to decide why such students thought that mathematics had an essential role.

- A very large part as I plan on being a statistician.
- As an actuary, I think it is obvious that it will be essential.
- It will play a major role in my professional career and it is essential in most, if not all, professional jobs. Therefore it is essential to have.

Uncoded: Finally, some of the students wrote responses that did not fit into the framework. Some of these just stated 'none', 'a small part' or 'a large part' with no further elaboration. This includes a small group of students who believed that they will not use much mathematics as it will be superseded by technology, and this may be a result of their experience with computer programs such as Maple and Mathematica.

Table 4.2 Distribution of views of future role of mathematics

Role of mathematics	Studies		Career	
Unsure	83	7%	55	5%
Procedural skills	327	28%	347	29%
Conceptual skills	82	7%	69	6%
Professional skills	264	22%	182	15%
Necessary	120	10%	243	21%
(Missing/uncoded)	306	26%	286	24%
Total	1,182		1,182	

- Not very big. Computer programs will do the work and applications for you.
- I don't think it will be as much as how important it is now in my studies because computers nowadays are capable of doing most things to do with engineering.

The group of students who participated in the survey had a range of views about the role that mathematics would play in their future studies and career, and these have been classified using categories that were consistent with statements made by the smaller group of students in the interview study. Again, the results can be presented quantitatively (Table 4.2).

There are some similarities in students' views of the role of mathematics in their future studies and career; indeed, many students gave the same or similar response to both questions. It is interesting that a small but not negligible group stated that they didn't know what role mathematics would play in their future. The most common response, of more than one-quarter of students, described a procedural role of mathematics, which seems consistent with a fragmented view of the discipline, though a substantial proportion of students focused on professional skills or described the necessary role of mathematics in their future.

Factors Associated with Broader Conceptions of Mathematics

With information collected from more than 1000 students, the survey gave us the opportunity to carry out quantitative as well as qualitative analyses. This section presents quantitative analyses of the responses to the first question, *What is mathematics?*, from which students' conceptions of mathematics were identified. The demographic variables collected in the survey (personal information on age, sex and language background, and academic information on degree, year of study and university) were categorised as shown in Table 4.3. We were interested to examine whether any of these variables were correlated with broader as opposed to narrower conceptions of mathematics. In particular, we wanted to check whether later-year students were more likely to show broader conceptions than were students at earlier stages in their studies. In this context, we defined the 'modelling', 'abstract' and 'life' conceptions

Table 4.3 Variables used in the quantitative analyses

Variable	Levels
Conception	0 = number, components; 1 = modelling, abstract, life
Age	0 = up to 25 years; 1 = mature aged (over 25)
Sex	0 = male; 1 = female
Language	0 = English; 1 = non-English speaking background (nesb)
Degree	1 = mathematics, 2 = other 'service' (computing, commerce, etc.); 3 = mathematics education; 4 = engineering
Year	1 = first year; 2 = second year; 3 = third year; 4 = fourth year
University	1 = Ulster; 2 = Pretoria; 3 = UTS; 4 = Brunei; 5 = Concordia

as the broader ones, and 'number' and 'components' as the narrower ones. The analysis was carried out using two complementary statistical techniques – logistic regression and classification trees. However, we do not believe that each individual student's conception of mathematics could be unambiguously determined from a relatively short written response to a question. This results in lack of precision of measurement and contributes to the residual variability in the models.

In logistic regression, we model the odds of a student showing a broader rather than a narrower conception of mathematics in terms of the other factors. Technically, odds is defined as the ratio of the probability that an event occurs (in this case, a broader conception) to the probability that it doesn't occur (in this case, a narrower conception). While probabilities are by definition constrained to be between zero and one, the odds scale uses any non-negative number to measure chances. The potential explanatory factors are checked in the model for statistical significance – that is, for a relationship stronger than one that could be attributed to chance alone. The methods used are described in standard books such as Agresti (1996), and the fact that this was an observational study rather than an experiment implies that those factors found significant should be interpreted as relationships rather than causes. The analysis was carried out using the statistical package SPSS version 18 (IBM SPSS 2011), and the results of the model that examines the joint effects of all the factors on the response is shown in Table 4.4. Standard statistical tests indicated that the overall fit of the model was adequate and a significant improvement on the null model (the one without any demographic explanatory variables), and it was able to correctly classify over two-thirds of the students (68%) in terms of their conceptions of mathematics on the basis of their demographic information.

The results (see Table 4.4) show that university and year were the most significant factors (each with $p < 0.001$), followed by more marginal effects of language background and sex. The odds ratios show the multiplicative effect of each factor on the odds of a broader rather than a narrower conception of mathematics. Because odds are measured on an open scale (from zero upwards), multiplying odds by any positive number will result in another value of odds (this would not work with probabilities on a zero to one scale). Compared to Ulster, Brunei is not significantly different, but students at Pretoria are almost 12 times as likely to describe a broader

Table 4.4 Results from binary logistic regression analysis

Factor	Interpretation	Odds ratio(s)	95% Confidence interval	p-value
Age	Mature aged compared to young	1.43	(0.85, 2.40)	0.18
Sex	Females compared to males	0.74	(0.55, 1.00)	0.048
Language	nesb compared to English	0.65	(0.47, 0.91)	0.011
Degree	Compared to mathematics …			0.27 (overall)
	Other 'service'	0.68	(0.44, 1.06)	
	Mathematics education	1.35	(0.49, 3.75)	
	Engineering	0.74	(0.48, 1.13)	
Year	Compared to first year …			<0.001 (overall)
	Second year	1.49	(0.91, 2.43)	
	Third year	2.33	(1.49, 3.64)	
	Fourth year	2.50	(1.32, 4.72)	
University	Compared to Ulster …			<0.001 (overall)
	Pretoria	11.71	(6.83, 20.08)	
	UTS	4.67	(2.40, 9.09)	
	Brunei	1.09	(0.39, 3.05)	
	Concordia	3.23	(1.93, 5.41)	

conception of mathematics, and those at UTS and Concordia about five times and three times as likely, respectively. Compared to first-year students, those in second year are 50% more likely (that is, 1.5 times as likely) to describe a broader conception of mathematics (though this is not significantly more so), and those in third and fourth year are 2.3 and 2.5 times as likely, respectively. This seems to support the idea that students are more and more likely to describe broader conceptions of mathematics in later years of study, with the largest increase between second and third year.

There is also a marginal indication that non-English speaking background (nesb) students are less likely to describe broader conceptions of mathematics, possibly due to students having trouble reading or expressing themselves in English (note that there was no problem translating those responses that were written in Afrikaans or French, since our team included members who are fluent in those languages). There is also a marginal indication that females are somewhat less likely than males to describe broader conceptions of mathematics. An interesting observation is that age and degree are not significant explanatory variables, although the odds ratios seem to be in the 'expected' directions, higher for mature-aged students and those studying mathematics education, lower for those studying mathematics as a service subject.

This model could be refined by the deletion of non-significant factors (degree and age) and the addition of possibly significant interactions between factors (none was found). When we investigated such refinement, the overall results did not change. Only one observation might be useful: removing age and degree, and combining the (small number of) fourth-year students with those from third year showed that students in second-year were 1.7 times as likely, and those in third/fourth-year were 2.8 times as likely as first-year students to describe broader conceptions of mathematics (and in each case the figure is statistically significant).

An alternative approach is afforded by techniques of data mining, particularly methods using classification trees. In this approach, the set of data is progressively sub-divided based on the values of the explanatory factors into groups that are more homogeneous in terms of conceptions of mathematics. The relative importance of each factor is assessed in terms of how much it contributes to splits into more homogeneous subgroups. This data mining approach is independent of distributional assumptions about the data, deals with missing values using 'surrogate' variables, uses automatic 'cross validation' of models, and is able to investigate local rather than global structure to identify important interactions. CART (Classification and Regression Trees) models were introduced by Breiman et al. (1984), and this analysis was carried out using the package based on their work, CART version 6 (Salford Systems 2011).

The input to the data mining investigation was the same set of response and explanatory variables (shown in Table 4.3) as for the logistic regression. The results of the analysis are presented in the form of a classification tree (Fig. 4.1) that shows which factor and which values were the most important separators at each step of the process. The first split was on the basis of university – Ulster, Brunei and Concordia (with overall 33% broader conceptions) were separated from Pretoria and UTS (with overall 67%), and this latter group was left untouched by further splits. In the first group of universities, first-year students (with overall 28% broader conceptions) were then separated from higher-year students (with overall 44%). This latter group was split on the basis of degree, with mathematics students (overall 51% broader conceptions) separated from the others (overall 42%). In this group, the younger age group was selected out, and then further broken down in terms of degree (mathematics education versus the service courses including engineering) and by language background.

With this classification tree, CART was able to classify 67% of the students in terms of their conception of mathematics, an overall result almost identical to that from the logistic regression. Yet the different methodology showed some important differences of focus. It identified university as the most important factor, followed by year of study, in exactly the same way as the logistic regression analysis, but it indicated only a minor role for the other variables, including language background that was identified as marginally significant in the logistic regression. The similarity in these results increases our confidence in the analysis. In terms of the overall results, Pretoria and UTS were identified as universities with a high proportion of students showing broader conceptions of mathematics, and the *local* analysis focused then on the three other universities, Ulster, Brunei and Concordia, identifying locally-important variables that correlated with broader conceptions. However, when the data from Pretoria and UTS were analysed separately, in order to examine the possible effects of factors on that branch, the most important factor was age, followed by year of study and language background (younger students in first and second years, particularly

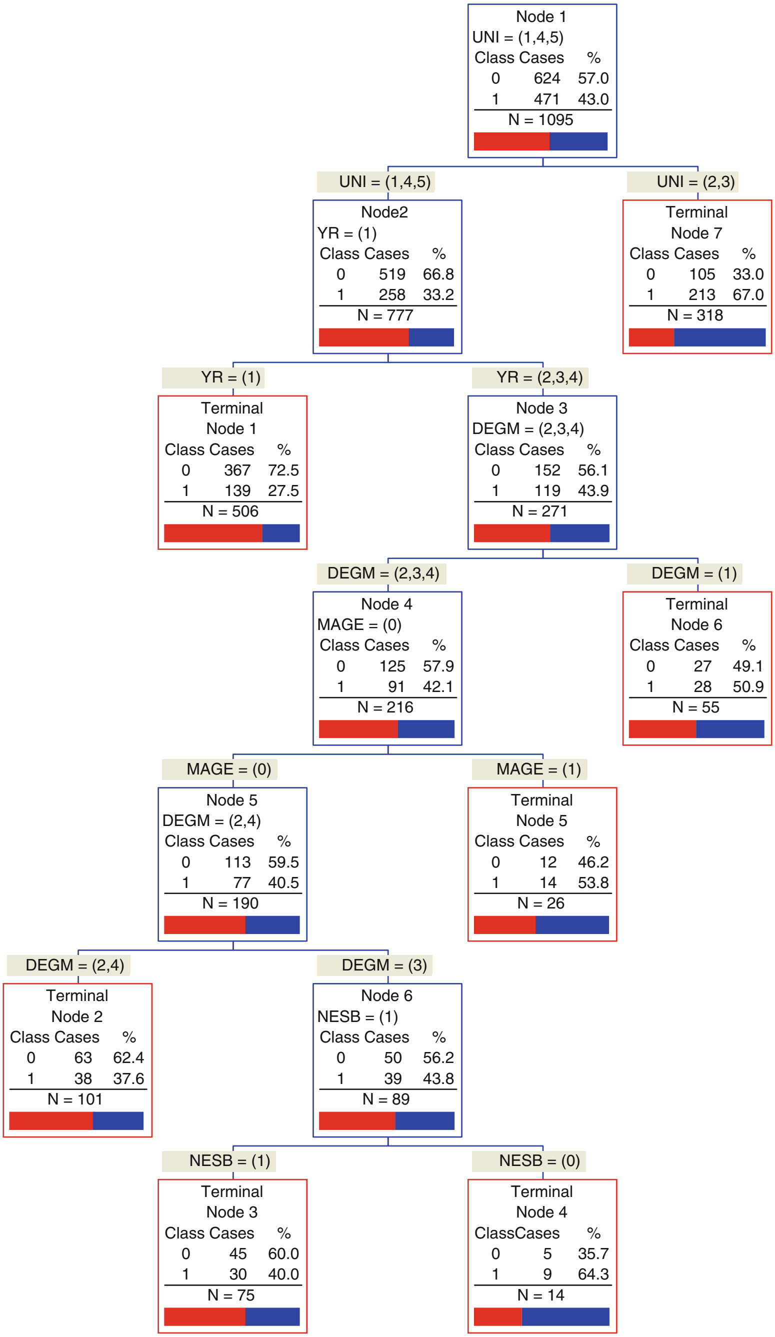

Fig. 4.1 Results from the CART modelling

those with nesb background, were less likely to show the broader conceptions of mathematics). This may explain the higher importance placed by the logistic regression model, which dealt with all the data globally, on language background.

Discussion

The analysis of responses to the open-ended survey has added some important information to our overall investigation of students' views of mathematics and their ideas about the role that mathematics will play in their future. The responses to the first question, *What is mathematics?*, have confirmed and extended our outcome space for conceptions of mathematics with a much larger and more diverse sample of students, including many for whom mathematics is only a component of their professional preparation. Although students who major in mathematics are often seen as the more important group, mathematically speaking, those students who study mathematics as a professional component form the much larger group. One change in the outcome space is that the intermediate conception of mathematics as modelling has been separated into two conceptions, 'modelling' and 'abstract', that comprise the practical models of specific aspects of reality and theoretical models of the logical and abstract nature of mathematics. This division corresponds to some extent with the classical notion of applied and pure mathematics, and was discussed – though not formalised – during the analysis of the interview transcripts. We regard these two conceptions as being at the same level in the hierarchy. The other change is the identification of the 'mathematics is about numbers' as a narrower conception than 'components', and the most limited conception in the hierarchy. We did not identify this 'numbers' conception in the analysis of the interviews, possibly because the participants were (a small number of) later-year students majoring in mathematics, and possibly because in such interviews a brief statement of the 'number' conception would be amplified in further discussion.

We also classified students' views about the role of mathematics in their further studies and future professional life. In this case, there did not seem to be any hierarchical structure in the phenomenographic sense, so we utilised a content analysis that suggested a framework of procedural, conceptual and professional skills, as well as groups of students who were unsure about the role of mathematics, and those who were convinced about its utility but gave no details about how they would use mathematics. We were able to identify some appropriate quotes from the interviews to illustrate students' use of the various skills, and then use the framework to classify the majority of the much briefer survey responses.

As well as confirming and extending the outcome space for conceptions of mathematics, and identifying a categorisation for utility of mathematics, the survey results allow us to investigate the distribution of views about mathematics and its use. Over 90% of responses could be classified as to students' conception of mathematics. The 'components' view was the most common, and together with the 'number' view, just over half of the students (53%) showed fragmented conceptions of mathematics. Only a small group of students (6%) seemed to hold the broadest 'life' conception. In terms of the future use of mathematics, we were able to categorise only around three-quarters of responses, and over half of the responses gave evidence for one of the groups of skills. Of these, the reference to procedural skills was most common, mentioned by more than half the students who wrote about one of the skills, with smaller numbers pointing to professional or conceptual skills. A small group of students (5–7%) wrote that they were unsure about their future use of mathematics, while a larger group was convinced that it would be essential, though they did not give any reason for this view. Thus, we have evidence that a significant proportion of students are unsure of the uses to which they will put the mathematics that they are studying, or at least, are unable or unwilling to articulate their views on this topic.

While we increased our knowledge of students' views, the survey methodology has some limitations. Some students wrote over 100 words in response to individual questions, but the average response was much lower (around 20 words) and there were students who wrote only one or two words. This, and the impossibility of follow-up questions, means that there is a degree of imprecision in our categorisations, even though our procedures ensured reliable coding of responses. Nevertheless, since we were able to classify the majority of responses, we carried out further quantitative investigation of students' conceptions of mathematics, identifying the factors that were associated with the broader conceptions ('modelling', 'abstract' and 'life') rather than the narrower conceptions ('number' and 'components').

The results from modelling using logistic regression and classification trees point out that there seem to be substantial differences between universities represented in the study. Around two thirds of the students at Pretoria and UTS indicated broader conceptions of mathematics, while at the other universities (Ulster, Brunei and Concordia) only around one third of students indicated the broader conceptions. This could be due to national differences, since each university was located in a different country. Such differences include societal views about the nature, usefulness and status of mathematics and its applications: a recent European study (Dahlgren et al. 2007) found such national differences in disciplines including psychology and political science. They may also be reflected in different entry requirements for the various mathematics programs. Different views about appropriate roles for males and females in mathematical professions may also be pertinent. For instance, the largest group at the Universiti Brunei Darussalam was studying mathematics education, and about four-fifths of them were women. By contrast, the largest group at the University of Ulster was studying Engineering, and more than four-fifths of them were men. It is sometimes suggested that students' views about mathematics derive from their lecturers' ideas. While lecturers' conceptions may have an effect on their students' views, the situation is much more complex. Students in later years would have had several different lecturers, and even those in first year would have been exposed to different teachers during their schooling.

Maybe the most important result from the quantitative modelling is the strong indication that there are higher proportions of students with broader conceptions of mathematics at later years of study (although as this was not a longitudinal study, we cannot conclude that *individual* students are broadening their conceptions). Students in second year are 50% more likely than those in first year to show broader conceptions of mathematics, and those in third and fourth year are almost three times as likely to do so. This is surely what we would like to see; that during the course of their university studies our students are broadening their view of the discipline of mathematics. However, we should note that at each university, in each year level and in each degree, students indicated the full range of conceptions of mathematics, from ‘number’ to ‘life’. The hints of sex, language background and degree differences, and the national and cultural differences between universities, suggest areas for further research, since our results are not extensive enough to allow more definite conclusions.

Implications for Teaching and Learning

The results from the open-ended survey reinforce the teaching and learning strategies suggested in the two previous chapters. Because of the hierarchical nature of the conceptions, we would prefer our students to hold broader conceptions of mathematics, at least by the time they finish an undergraduate degree. This may be achieved by designing curricula and using pedagogical approaches that will allow students to develop deeper understandings. This should include the strategy of introducing our students to the full range of conceptions about mathematics and giving them opportunities to discuss the nature and implications of these conceptions. While this is important for all students, it is particularly important for students of mathematics education, as they will be educating the next generation of students.

The quantitative results send a strong message that our students in every year group and degree program hold widely differing ideas about the nature of mathematics and its use in their studies and future careers. While our results show that there is a pleasing increase in the proportion of students who have broader conceptions of mathematics in later years of study, the full range of variation is present in classes at all levels; it is simply not true that all students enter university with the narrowest ideas about mathematics and leave with the broadest ideas. By the time they enter university, students have already developed conceptions of mathematics, which may be reinforced or modified during tertiary study. With appropriate teaching methods and learning materials, we can encourage students towards the broader conceptions – and the hierarchical nature of the conceptions implies that they will then be aware of the full range and be able to make use of them as appropriate.

As an example, if we focus solely on the techniques of mathematics and regularly ask rote and procedural questions, we will encourage a ‘component’ (or even narrower) view of mathematics. Such an approach may even lead to the sorts of views about mathematics being a ‘waste of time’ that some of the students in our study seemed to hold. On the other hand, if we focus on the broader role of mathematics in real or realistic contexts, we will encourage students towards the broader conceptions. Such an approach will benefit from making explicit connections between students' courses and the world of professional work. As mathematics lecturers, we can design learning tasks that model the way that mathematicians work in industry and academia in order to give students an idea of the way mathematics is used in their future professions. For example, arranging a student conference on some mathematical topic, with students carrying out the planning, writing, reviews and presentations, can introduce academic aspects of the discipline. A film clip presenting some aspect of professional work as an engineer or a statistician with follow-up discussion and questions (for example, Wood et al. 2000; Petocz et al. 1996) can model professional roles and highlight the diverse range of skills (much more than mere technical competence with the mathematics) needed in a typical work situation.

The significant differences between universities in terms of the proportion of students holding the broader conceptions of mathematics sound a positive note in this endeavour. These may be the result of national differences in the way that students are prepared for studying mathematics at university, or they may be due to the practices and pedagogical approaches at the different universities. While these factors may be hard to change, the fact that there are such substantial differences implies the possibility of changing pedagogical and institutional approaches to emphasise the broadest views of and roles for mathematics.

Summary and Looking Forward

In this chapter we have described the results of the second stage of our investigations: a survey of students' views of mathematics and its future use in their studies and profession, undertaken with a sample of over 1000 students from five different universities in five different countries. Although we were unable to get detailed information from a sample of this size, we have balanced that by obtaining a much larger amount of information that is still in the form of responses to open-ended questions, supported by demographic details for each student. This has allowed us to confirm and extend our previous qualitative results concerning conceptions of mathematics, and then investigate these results using quantitative methods – logistic regression and classification trees.

The quantitative analysis has revealed two important aspects. Firstly, there seem to be substantial differences between universities in terms of the proportion of students holding the broadest conceptions of mathematics. Allowing for differences in year, sex, language, degree and age – which the logistic model does – three of the universities have one third, and the other two universities have two thirds of their students indicating the broader conceptions, and this result is quite unlikely to have occurred by chance. Such differences seem likely to result from some feature of the national context of each university, or the specific approach taken by each university to their mathematics education. In either case, it affords the possibility of making positive changes. Secondly, the analysis shows that, allowing for differences in the other variables, students in later years are more likely to hold broader conceptions of mathematics: about 50% more likely in second year, and almost three times as likely in third or fourth year, than in first year. This is certainly a good sign, and something that we would hope to result from our mathematics courses. We need to remember, however, that only a longitudinal study could establish that students were broadening their views as they progressed through their degrees, and also that almost half of final-year students are currently finishing their courses with the narrower views of mathematics as 'number' or 'components', some of them majors in mathematics education or mathematics itself.

In the following chapter, we continue our studies of undergraduate students and report on the construction of a survey instrument, based on the statements that we have obtained from students in interviews and open-ended surveys, that can be used as a tool to track the development of the broader ideas that are so important in their future professional lives.

Note: Some of the material in this chapter was previously published in Petocz, P., Reid, A., Wood, L. N., Smith, G. H., Mather, G., Harding, A., Engelbrecht, J., Houston, K., Hillel, J., & Perrett, G. (2007). Undergraduate students' conceptions of mathematics: An international study. *International Journal of Science and Mathematics Education*, *5*, 439–459 and in Wood, L. N., Mather, G., Petocz, P., Reid, A., Engelbrecht, J., Harding, A., Houston, K., Smith, G. H., & Perrett, G. (2011). University students' views of the role of mathematics in their future. *International Journal of Science and Mathematics Education*, *9*(1). Online at http://springerlink.com/content/a255j102v01653xh/

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