Chapter 5
The Resources in *Te Reo Māori* for Students to Think Mathematically

Te reo Māori, like all languages, contains features that can be used to support thinking in mathematics. Some features exist traditionally within the language, such as logical connectives, and others have become newly available with the development of the mathematics register. The challenge continues to be one of identifying these features so that they can be used in such a way that the integrity of the language is maintained and that the benefit to students when doing mathematics is realised. For second-language learners of *te reo Māori*, such as most teachers and students in *kura kaupapa Māori*, the influence of English often makes it difficult for them to appreciate the features in Māori which could contribute to mathematical thinking. Once the features have been identified, there are further challenges in being able to understand why some terms are difficult to learn. The ultimate aim is to support students to think mathematically through explaining and justifying what they know.

Thinking mathematically is about using mathematical understandings to create mathematical solutions to problems. Using a symbolic interaction perspective, Erna Yackel (2001) observed that

> [s]tudents and the teacher give mathematical explanations to clarify aspects of their mathematical thinking that they think might not be readily apparent to others. They give mathematical justifications in response to challenges to apparent violations of normative mathematical activity. (p. 14)

Language, including diagrams and symbolic equations, is more than just the vehicle for the thinking. The linguistic features of a language support or constrain the way that ideas are discussed. Halliday (1978) summarised how languages both reflect and shape different worldviews of people from different cultures:

> Languages have different patterns of meaning – different 'semantic structures', in the terminology of linguistics. These are significant for the ways their speakers interact with one another; not in the sense that they determine the ways in which the members of the community *perceive* the world around them, but in the sense that they determine what the members of the community *attend to*. (p. 198)

However, recent work suggests that even when a language has limited counting words, speakers can still complete enumeration activities (Butterworth & Reeve, 2008). This reinforces the fact that language can constrain but not predetermine

T. Meaney et al., *Collaborating to Meet Language Challenges in Indigenous Mathematics Classrooms*, Mathematics Education Library 52,
DOI 10.1007/978-94-007-1994-1_5,

what can be seen and acted upon. Thinking mathematically involves being able to perceive a situation and recognise how mathematics could be utilised to resolve an issue within that situation. Burton and Morgan (2000) stated that “[t]he language used in mathematical practices, both in and out of school, shapes the ways of being a mathematician and the conceptions of the nature of mathematical knowledge and learning that are possible within those practices” (p. 445).

English-medium mathematics education research has suggested for some time that language has a considerable impact on the teaching and learning of mathematics (Cocking & Mestre, 1988; Ellerton & Clements, 1990; Durkin & Shire, 1991). Although the focus of much early research was on the specific vocabulary terms (Love & Tahta, 1991), this was replaced by an interest in the features of English that are significant in explanations and justifications in mathematics, which support the solving of problems. For example, Bills (2002) highlighted certain word-level features as being useful indicators of students’ mathematical thinking. Personal pronouns (“you” and “I”), present tense, and logical connectives such as “because”, “so”, and “if” were more likely to be found in appropriate answers to mental arithmetic questions (Bills, 2002).

Using slightly different resources to those in English, the mathematics register in *te reo Māori* supports mathematical thinking in a different manner. In the next section, extracts from transcripts of lessons and staff meetings illustrate how the mathematics register in *te reo Māori* is used for thinking mathematically. Although we concentrate on spoken language in this chapter, mathematics is often done in conjunction with some form of written text, and we have included these when relevant. Chapter 6 focuses more on how writing in *te reo Māori* contributes to students’ thinking mathematically.

Identifying relevant features in the mathematics register that support mathematical thinking needs to be done in conjunction with fulfilling the aims of *kura kaupapa Māori*. Thus, the use of the mathematics register should help students achieve academically, but also support the revival of *te reo Māori* by using it to fulfil a range of different functions. Nevertheless, there is still room for improvement in how the students use the mathematics register, and this is an ongoing challenge.

Resources in *Te Reo Māori*

Given that Māori-immersion education was set up to reverse the decline in Māori language (Spolsky, 2003), it has been recognised that there is a need to ensure that “the authenticity of the language is maintained” (Christensen, 2003, p. 12). Māori mathematical discourse has several distinct characteristics that are similar to those found in the English discourse. It is conceptually dense and jargon-filled (Halliday, 1978; Pimm, 1987; Dale & Cueras, 1987). There are also linguistic characteristics specific to *te reo Māori* which can be used to discuss mathematics and, when continuously used, can enhance the learning of *te reo Māori* (Barton et al., 1998). For example, a very important construction in Māori, and one which is used more frequently than its English equivalent, is the passive tense (Harlow, 2001). A feature of mathematics is that there is an inherent requirement to perform certain actions – to add, to multiply, to increase, to find out, to solve, and so on. In *te reo Māori*, when an action is required, in most cases a passive tense is used. Māori passives have a variety of suffixes, and there are some restrictions on their use. Therefore, learning mathematics in the medium of Māori supports the learning of this very important linguistic construction. Similarly, Māori verbal numerical markers do not have English language counterparts and differ according to function of the grammatical expression (Trinick, 1999). For example, the verbal particle *ka* is used when counting, *e* when quantifying, and *kia* when expressing a need for a certain number of things. In *te reo Māori*, numbers are preceded by a range of particles depending on the function and context (Barton et al., 1998).

Concerns have been raised about the possible implications for *te reo Māori* as a consequence of its use for discussing mathematics (Barton et al., 1998). For example, it would be a great pity if the grammatical structures used in English to discuss mathematics were superimposed onto *te reo Māori*, so that the language became a Māorified version of English. Whilst *te reo Māori* is traditionally characterised by the liberal use of passive verbs, some writers argue that many contemporary speakers and learners of *te reo Māori* have an inability to make use of these passives (Barton et al., 1998; Harlow, 2001). English is much more likely to use the active tense in situations, where native speakers of *te reo Māori* would apply passives. Christensen (2003) found that the Māori-immersion teachers who had learnt mathematics in English resisted discussing mathematics in the passive voice.

> Difficulties were experienced because in Māori the words do not follow the sequence of the written symbols, as they do in English. English was also seen to be more concise than Māori. For this reason, many teachers and students simply follow the linguistic structure of English, using Māori words. For example, an addition problem is written in symbols as $3 + 2 = 5$. In English it is most common to say this as it is written, symbol for word, *three plus two equals five.* In Māori it is linguistically correct to begin with the verb *tāpirihia te toru me te rua, ka rima.* However, many teachers and students have adopted the English structure, saying *toru tāpiri rua ka rima.* While it may be pragmatic to accept this borrowed linguistic structure as an example of language change resulting from contact between English and Māori, it is unclear whether such a borrowed structure used specifically for pāngarau [mathematics] could transfer across to general language use. (Christensen, 2003, p. 37)

Therefore, the problem may not necessarily be the inability to use the passive voice, but rather the inability to choose when it is appropriate to apply it. In English, mathematics is often discussed without reference to an active participant as the action has been included in a nominalisation, and the verb identifies the type of relationship involved (Meaney, 2005a, 2005b). The natural use of the passive voice in *te reo Māori* may well support the same conceptualisation more easily, so it may be valuable for students to learn how to use it in *te reo Māori* from an early age.

Rather than have to make Māori sound like English in order to discuss mathematics, we argue that the authentic resources within contemporary *te reo Māori* can provide students with resources to think mathematically. In the following sections, we outline some of these resources.

Linguistic Markers

One beneficial resource is the linguistic markers within *te reo Māori* that forewarn listeners about the type of information that is to follow. These markers assist listeners' thinking, because they add meta-level information about the importance of what they are receiving. Although English has some ways of forewarning listeners about the type of information that is to follow, there does not seem to be the diversity that is available in *te reo Māori*. One of these markers, *kē,* tells the listener that what is to follow is unexpected. Another, *arā*, is used to emphasise that an elaboration is following.

Y6Teacher: Ānei tētahi o ngā ahutoru, arā, te koeko tapawhā, mahara? (2005 lesson)	Y6T: Here is one of the 3D shapes, [namely] the square pyramid, remember?

This utterance began the first of this teacher's filmed lessons at Te Koutu in 2005, and referred to material covered in the previous lesson. The teacher highlighted one term *ahutoru* (three-dimensional shape) as the word that needed to be recognised and understood by the students. *Arā* then emphasised that an elaboration was coming. Although the term was used in a previous lesson, the teacher assumed that many of the students still needed to have it highlighted.

In the next extract, which also comes from the same teacher's 2005 set of lessons, the teacher's language suggests that she expected some students to struggle to follow the logic in the argument being presented. She used words and commands to ensure that they paid attention to the important sections.

Y6Teacher: Tekau ngā tapa, tekau ngā mata me ngā akitu, tekau mā rua ngā tapa, tāpirihia kia rua, ā, ka tekau mā rua kē tērā. Heoi anō, i mutu i te karaehe, i kā mai kē tētahi; "Whaea, kei te hē tētahi o ngā mahi, me kā, ngā kaute, kua hē tētahi o ngā wāhanga." Ko [Ākonga 1] tērā, he aha tāu i kite ai?	Y6T: 10 sides, 10 faces and vertices, 12 sides add another 2, that's 12. However, at the end of the class, someone said "Whaea, some of the working out is incorrect, according to calculations one side is incorrect". That's [Student1], what did you discover?
Student1: E waru ngā tapa?	Student1: Eight sides.
Y6T: E hia?	Y6T: How many?
Student1: E waru ngā tapa.	Student1: Eight sides
Y6T: Me whai kē mehemea kei te tika ia. Tahi, rua, toru, whā, rima, ono, whitu, waru, nā reira, kāore ko te tekau. Nā reira, kei te tika te maha o ngā mata me ngā akitu?	Y6T: Follow along to see if he's got it right. One, two, three, four, five, six, seven, eight. Therefore it's not ten. Therefore, is the number of sides and vertices, correct?
Students: Āe!	Students: Yes!
Y6T: Āta whakaaro koa!	Y6T: Please think carefully

Students: Āe!	Students: Yes!
Y6T: Āe, i te mea he aha tētahi atu huarahi i kite kē?	Y6T: Yes, because what other way did you discover to solve the problem?
Students: Tāpirihia te rima ki te rima?	Students: Add five to five?
Y6T: Nā reira, kei te kōrero, i rongo koe, koutou? I a ia e kā ana? Kōrero mai anō koa, tama.	Y6T: Therefore, you're talking, did you, all of you, hear what he was saying? Tell us again, please.
Students: Tāpirihia te rima ki te rima?	Students: Add five to five?
Y6T: Tāpirihia te maha o ngā mata ki te maha o ngā akitu, kua puta kē ko te tekau, nēhā? Te maha o ngā tapa, me kā, waru ināianei.	Y6T: Add the number of faces to the vertices and it will be 10. The number of the sides, we need to say, is eight.

This was part of a discussion of Euler's rule (Vertices + Faces – Edges = 2) applied to a pyramid and how some of the previous day's work had been incorrect. The *kē* highlighted for the listeners that they should notice and be surprised by what followed. It acted as a scaffolding device for students' listening so that they could understand the differences between what had been said on the two days. This was further emphasised by the teacher with the command *Āta whakaaro koa* (Please think carefully) which occurred a few turns later. Once the student had responded to the initial question, the teacher re-emphasised the need to listen. She then had the student repeat what he had said. These examples suggest that the teacher was confident that the students would understand what was being discussed but, because of its complexity, she needed to remind them to be careful so that they would not miss valuable information.

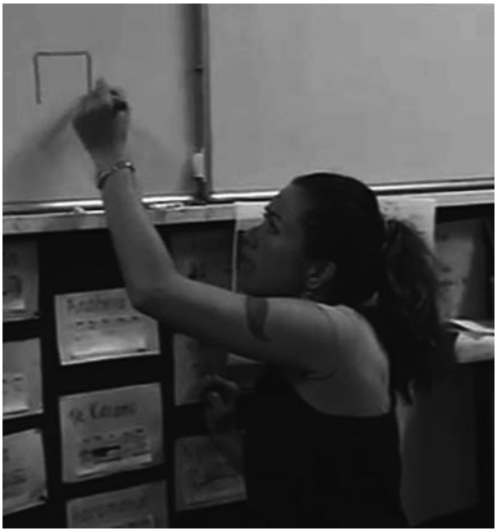
Linguistic markers, such as *arā* and *kē*, which forewarn listeners that important information is to follow, can help students to focus on what the speaker feels they should be paying attention to. These markers were used by the teachers and did not appear to the same degree in students' oral explanations and justifications. If students could learn to make use of these resources, they would not only be showing a rich command of *te reo Māori* but also a more in-depth understanding of the mathematics that they were describing.

Transparency Within Terms

As is discussed in Chapter 2, the development of terms for the mathematics register in *te reo Māori* was done in such a way that the meaning of the terms should be transparent to the learner. In this section we look at how the teachers at Te Koutu made students aware of this transparency.

In the following extract, the teacher explicitly made the students aware of how the label for a "square" in *te reo Māori* provided them with the clues about its definition. The word for square in Māori is *tapawhā rite*, which literally means "four equal sides". She emphasised the features of a square through words, symbols, and diagrams.

I kā koe i mua he tapāwha rite. He aha te tikanga o tērā? He pai ngā ingoa Māori no te mea ka whakamārama i te āhua i roto i te ingoa, nē? He tapawhā rite. He aha te tikanga o te rite? [draws shape on board] He ōrite te aha? He rite ngā taha. Mehemea ka whakamahia au taku rūri. . . he rite ia taha? Nā reira he tapawhā. . .. He tapawhā. . . he tapawhā rite, na te mea he ōrite ngā taha.	You said earlier it was a square. What does that mean? Māori names are good because the shape is explained in the name, isn't it? A square. What is the meaning of "same"? [draws shape on board] What is the same? The sides are the same. If I use my ruler are the sides the same? Therefore, it's a quadrilateral . . . a quadrilateral. . . It's a square, because all the sides are the same.



If students are not familiar with, or do not use the cognates of mathematical terms in their conversational language, they are unlikely to benefit from these everyday meanings when the words are introduced into a mathematical setting. For instance, if students do not have *horahanga* in their conversational language, which means a "spreading out [of food]", then they are unlikely to see a connection with its mathematical meaning of "area".

The transparency of the mathematical meaning of the terms has the potential to support students in thinking mathematically. Yet, this is unlikely to happen without instruction. The teachers at Te Koutu felt that the students had to learn the conversational meaning for such terms as *mua* ("before" or "in front of") and *muri* ("after" or "behind") before these terms could be used for talking about "the number before" or the "number after". Christensen (2003) noted that the teachers in the Poutama Tau – a professional development project on numeracy – also struggled with these terms. In the diagnostic interview that teachers gave students as part of Poutama Tau, they had to ask:

> Kia tatau whakamuri mai i te 23. (Count backward from 23.)
> Kia tatau whakamua koe, atu i te 10. (Count forwards from 10.)
> Some teachers recognised that the use of these words in Māori is different from their equivalent use in English and that this may be one of the reasons for confusion, especially for teachers and students whose stronger language is English. (p. 37)

Although the transparency of some terms such as *tapawhā rite* (square) supports students' understanding and thinking in mathematics, not all terms that were chosen for transparency turned out to be as transparent. As described in Chapter 9, not all of the teachers at Te Koutu were aware of how the terms had been constructed. Consequently, they were still grappling with how best to support students to gain the vocabulary to achieve academically, and to use *te reo Māori* fluently in a range of contexts. This challenge will be ongoing.

Logical Connectives

Western mathematics utilises many types of relationships at different levels. At one level is the nature and origin of mathematical objects and their relationship with language. For example, numbers are related to other numbers by such relations as "greater than" (*nui ake*), "less than" (*iti iho*), and "equal to" (*ōrite ki*). Additionally, a "relation" in Western mathematics can be defined as a set of ordered pairs {(1,3), (2,6), (3,9)...}. In this relation, the ordered pair is connected by a mathematical relationship of multiplying by three.

The syntax of the language describes these mathematical relationships (Carrasquillo & Rodriguez, 1996). One syntactical device is the logical connectives; these are words or expressions between clauses or sentences that are used to join or connect ideas that have a logical relationship (Dawe, 1983). The types of relationships indicated by these expressions include time and space, enumeration and exemplification, amplification and contrast, inference and summation, cause and effect, etc. Within each relationship category, the logical connectives, which join the ideas or clauses, are used differently, with different grammar and punctuation.

Logical connectives determine what can be inferred from these relationships, and mathematical reasoning relies heavily on their use. Research has shown that when students read mathematics text and/or engage in mathematical conversations in English, they must be able to recognise logical connectors, and the situations in which they appear (Spanos, Rhodes, Dale, & Crandall, 1988). Solomon and O'Neill (1998) argued that "[m]athematics cannot be narrative for it is structured around logical and not temporal relations" (p. 217). Generally in narratives, cohesion is achieved by placing a series of events in a timeline. In mathematics, cohesion is achieved by logically joining separate ideas together. For example, in problem solving "[a] convincing argument makes a clear connection, using reasoning, between what is known about a problem and the suggested solution" (Meaney, 2007, p. 683). Logical relationships are timeless and, although time markers are common in recounts, they are inappropriate in discussing mathematics. For English speakers, Esty (1992) stressed the importance of "five key logical connectives: 'and', 'or', 'not', 'if . . . then' and 'if and only if' ", which provided mathematics students with an understanding of when equations were true and, therefore, provided the limits of their generalisations.

For Māori, relations are also important and vary to suit different contexts. The word "relation" can be translated as either *whanaunga* or *pānga*. However, both these words are context specific. *Whanaunga* is a generic term applied to kin of both sexes related by marriage, adoption, and descent. This word implies some human kinship relation. *Whānau* terms are considered inappropriate to use when describing "relationships between mathematical objects" (Trinick, 1999); it is more appropriate to use terms like *pānga* (a connection) or *tūhono* (join), for non-kinship/human relations.

Te reo Māori has an abundance of logical connectives that illustrate the range of possible relations. Table 5.1 is a sample of *te reo Māori* connectives.

Table 5.1 Logical connectives in *te reo Māori*

Relationship category	Logical connectives	English translations
Time	*kia*	when, until – used for future time
	rā anō	right to, as far as, since long ago
	i	while, during
	ina	for, since, inasmuch as, when, if, and when.
	muri	after, afterwards, the time after, the sequel – often modified by *mai, iho*, or *atu*.
	ka . . . ana	when, whenever
	tonu & rawa	"as soon as" and "by the time"
Mathematics example		
I a koe e whakaroa ana nga taha ka aha?		While you were making the sides longer, what happened?
Ina tango te rima ka . . .?		If [you] subtract the five, then . . . [what happens]?
Tāpiri tonu te whitu ka tau tōrunga.		As soon as [you] add the seven [it] becomes positive.
Causal (Reason and Purpose, Cause and Effect)	*kia. . . .ai*	so that
	e. . . .ai	in order, whereupon . . . that
	na te mea	because
	nō reira	therefore, thereby, that's why, so, consequently, for that reason, hence, thus, accordingly.
Mathematics example		
Nō reira kei hea pea tona tuaka hangarite?		Therefore, where perhaps is the line of symmetry?
Kia tuhia te rārangi e hono ai ēnei kotinga e rua.		Draw the line in order to join these two bisectors.
Tāpirihia kia rua kia nui atu ai te roa.		Add two so that the length is longer.
Adversative (unexpected result, contrast, opposition)	*ahakoa tonu*	even though, even so
	ahakoa	although, notwithstanding, despite, even though,
	kē	whatever, no matter, in spite of, nevertheless indicate difference or unexpectedness.

Table 5.1 (continued)

Relationship category	Logical connectives	English translations
Mathematics example		
Kua roa kē tēnei i tēnā.		This has become longer that that.
Ahakoa he roa atu he nui atu te horahanga o tēnei.		Although it's longer, the area of this is greater.
Condition	*mehemea*	If
	ki te	Condition about the future
Mathematics example		
Ki te tāpiri i te rua ka waru.		If [you] add two [you] get eight.

Logical connectors in *te reo Māori*, as in any other language, are acquired and mastered by children as part of their language development in and out of school. An examination of Te Koutu teacher and student talk shows that the basic and more frequent connectors are acquired early in this development, such as *anō* (again) or *engari* (but). Other connectors are mastered much later, if at all, and only after students have being exposed to a variety of language learning situations. For example, Uenuku, who teaches the older students, frequently uses the particle *ai* in his mathematics talk. *Ai* is a particle of great use, particularly in the older generation of speakers of *te reo Māori*. It mainly represents the English "who", "which", and "what", and has reference to the time, place, manner, cause, means, intention, and so on of an action (Harlow, 2001). This connector is almost absent in the talk of teachers of younger students. It is unlikely that these students will learn how to use this particle appropriately without modelling from their teachers.

Linguistic Complexity

Even when the features in *te reo Māori*, which would be useful in thinking mathematically, have been identified, there can be difficulties in learning them because of their complexity. An extended debate in English-medium mathematics education has focused on what features of the mathematics register are difficult for students to learn. Was it the difficulty of the mathematical concept, which made the language hard to acquire, or was it the mathematical language itself which contributed to the problems in understanding the mathematical concept? In this debate, there is an awareness of how the contexts in which the mathematical language was acquired contributed to the ease with which it was learnt.

In an early study, Knight and Hargis (1977) posited that since mathematics is a study of relationships, comparative structures are an essential and recurring component of mathematical language. Nevertheless, they also argued that comparative structures are difficult for many students to master. In contrast, Walkerdine (1988) suggested that rather than some terms being more conceptually difficult for children to master, it was the context in which terms were learnt that contributed to children's difficulties. For example, small children often exhibited much more difficulty with the concept of "less" than with the concept of "more". This had led to suggestions that it was cognitively more difficult to master. However, when children's lives were examined, there were few instances when children asked for less, but many instances of children asking for more. Consequently there is a need to consider how language is used in children's lives, both in and out of school, to better understand what aspects of the mathematics register are the most difficult to master.

In Māori-medium mathematics education, there has been less research into the linguistic features that may be difficult for students to learn. Hereafter we outline some suggestions about the features, which may cause problems for Māori-medium students. We include our reasons as well as some examples of these features from recorded lessons and meetings.

1. Some of the features in *te reo Māori*, particularly the particles, are not always semantically transparent and can have a variety of meanings. This can prove a challenge to teachers and students alike. For example, the word *ki* has many major functions, and many different grammatical constructions. In modern *te reo Māori* mathematics register, the word *ki* has taken on heightened significance, more so than in common daily usage. This is because of its role in introducing an instrumental phase, that is, the thing by means of which some action is carried out. For example, *Whakareatia te 5 ki te 4* means that the 4 acts on (replicates/multiples) the 5 because of the position of the word *ki*. However, *ki* as a preposition also means: "motion towards a place", or "on to", or "in the event of", or "according to".

Kua haere ***ki*** *Rotorua.*	They have gone *to* Rotorua.
Kua paea te waka ***ki*** *te ākau.*	The boat is stranded *upon* the beach.
Ki *te puia he uka, he aha ngā putanga e taea ana?*	*If* a coin is tossed, what results are possible?
Ki *a Uenuku, he nui atu tēnei.*	*According* to Uenuku, this is bigger.

Another example of connectors that can prove challenging is the set of particles common in teachers' mathematics talk: *nei, nā*, and *rā*. Their basic meaning is as a locative particle to indicate position near the speaker (*nei)*, position near the person being spoken to (*nā)*, and position distant from both, (*rā*).

E hia ngā mata nei?	How many [geometric] faces are there?
He nui atu te koki rā i tēnei?	Is this angle [over there] bigger than this angle [by us]?

As well as these spatial relationships, *nei* and *rā* can be used to imply "nearness to" and "distance from" the present time.

Nō mua atu rā	some time ago, some time before
I te rā nei	today

These particles are also often used with pronouns and personal nouns to strengthen and emphasise the reference to "me", "us", or "you".

Ki ahau nei	in my view/opinion

Additionally, *nā* has several different functions. Without a macron to indicate a lengthened vowel sound, *na* is used at the beginning of a narrative, to call attention or to introduce some new element or emphatic statement, to which special attention needs to be drawn. This is a device teachers use frequently to signal to students that they are going to introduce a new or additional idea.

Na, ko te rīrapa, anā, he momo ara, nē?

Now, the maze, well, it's a sort of pathway, isn't it?

Na, ka haere atu koe ki tatahi, nēhā? Ka kite i ngā anga ma e toru nēhā.

Now, you go to the beach, yes? You will see three white shells, yes.

Very often, in teacher mathematical talk, a question is asked inviting the students to agree with, and/or to support a particular statement. In questions, which serve this purpose, it is very common to use a device called a tag at the end of the sentence (Harlow, 2001). In the two earlier examples, the tag used is *nē* or *nēhā*. The root word is *nē* and can be followed by *rā* or *hā*, depending on the dialect of the speaker.

2. Many logical connectors in *te reo Māori* have comparable, yet different variants in low- and high-frequency use, such as "if" which can be *mehemea* in high-frequency use, and *ki te* in low-frequency use. There are a number of different words, which translate to the English word "if", and some care is needed in selecting the form to use on any particular occasion, since they are not all equivalent in meaning (Harlow, 2001).
3. Some logical connectors are polysemic in structure and are made up of two or three words together such as *ki te* previously described. Others commonly used in teacher talk include *heoi anō* (however, so much for that) or *nā reira* (therefore, that's why, so, consequently, for that reason). The individual words have their own meaning, but the new multi-lexical form has a new function.
4. As noted earlier, the syntax of mathematics is seen commonly as the language that describes relationships. Often, mathematics discussion involves understanding a number of related ideas in one sentence. A challenge for students is to master the correct word order to illustrate the desired relationship between the main idea (contained in the main clause) and the modifying or supporting idea(s) (subordinate clause). A simple sentence consists of a single clause, for example, *tāpiritia te 5 ki te 2* (add 5 to 2). However, mathematics discussion involves much more than simple sentences and often requires the joining of a number of ideas to create complex sentences. For example, *I te tuatahi, me tāpiri te 5 ki te 2 kia kimi ai te otinga* (First, add 5 to 2 to find the answer). "To find the answer" is

a subordinate phrase. "First" as the logical connective joins the ideas in "add 5 to 2" in a logical manner "to find the answer" in order for the sentence to make sense. Syntactically, some of the relationships between the main clause and the subordinate clause are linguistically more difficult to master than others. These include the following:

Clauses of purpose
Kia tāpiri ngā tau matitahi i te tuatahi kia ngāwari ai te kimi i te whakaotinga.
Add the single digits first **so that** it is easier to work out the answer.
Particular relative clauses
These are clauses whose function is to qualify the noun.
Haere ki te whare e tū nei te hui.
Go to the house [where the meeting is taking place].

Learning How to Give Spoken Explanations

In the next chapter we look extensively at students' written explanations and justifications. Students usually begin to explain and justify their reasoning through speaking before they put their thoughts into writing. As a research area, speaking about mathematics came to the fore in the late 1980s with the publication of David Pimm's 1987 book *Speaking Mathematically*.

Since then, research, with English as the language of instruction, has tended to focus on dialogical structures in mathematics classrooms, and their contribution to students' mathematical understanding (see Nathan & Knuth, 2003; Bill, Leer, Reams, & Resnick, 1992; Moskal & Magone, 2000; White, 2003; Tanner & Jones, 2000).

The role of the teacher in supporting students to talk about the mathematics they were engaged with has been a focus in this research. Early on this research identified the typical teacher–student exchange known as the IRF (initiation – response – feedback) exchange (Mehan, 1979). The teacher asks a question and sometimes leaves a sentence incomplete. The students are expected to provide a response, and then the teacher gives either explicit feedback, through affirmation or negation of the response, or indirect feedback by asking a new question. Nathan and Knuth (2003) discussed the difficulties that teachers had in reconciling the need to accept all students' responses (social scaffolding) and the need to ensure that mathematical ideas were central in these responses (analytical scaffolding). In order for teachers to persuade children to take risks and put forward their ideas, teachers sometimes accept all of the students' responses. However, Khisty and Chevl (2002) showed that unless the talk within the classroom focused on developing mathematical understandings, then students were unlikely to gain anything from the talking.

There have been a number of critiques of the IRF exchange, which suggest that it is unlikely to lead to improved mathematical understandings. Wood (1998) criticised the use of leading questions where the student simply provided a one-word answer to questions because they did not push students to think mathematically. She stated, "[A]lthough the teacher may intend that the child uses strategies and learns about the relationship between numbers, the students need only to respond to the surface linguistic patterns to derive the correct answers" (p. 172). She suggested an alternative pattern, that she labelled "focusing", would be more effective in promoting learning: "A high level of interaction between the teacher and students creates opportunities for children to reflect on their own thinking and on the reasoning of others" (Wood, 1998, p. 172).

For students to give explanations and justifications, they need to understand how they are constituted and to see them as essential components of doing and learning mathematics. "The understanding *that* students are expected to explain their solution is a social norm, whereas the understanding of *what counts* as an acceptable mathematical explanation is a sociomathematical norm" (Yackel, 2001, p. 14). Students need to learn how to phrase explanations and justifications, but teachers also need to expect that children will provide these as part of each mathematics lesson. Gibbons (1998) in studying students' acquisition of the English register for a science topic found that "as the discourse progresses . . . , individual utterances become longer and more explicit, and this occurs as the students begin to formulate explanations for what they see" (p. 109). Gibbons suggested that teacher requests for explanations were what triggered students to move from the "doing" to the "thinking" in their learning.

At Te Koutu, teachers recognised that children needed to explain their understandings as a normal part of a mathematics lesson. This awareness was linked to the teachers in the primary section of the school being involved in a New Zealand–wide professional development program on numeracy, Poutama Tau. In this program, teachers learnt about the need to have children explain their strategies when solving arithmetic problems.

Y4 Teacher: It was all the little words too that they got mussed up on like *atu* (away), *mai* (towards), *i* and *ki*. Yeah, and I noticed with Student1, he is a good mathematician but his language lacks and when it came to the actual explaining of how he did it he couldn't really explain but he can do it in his head but he can't explain because his language is quite poor, actually, I was talking to somebody about. I think it is *te reo* in the home too isn't that strong.

Tamsin: Because I was also talking to Y7 Teacher today and your kids now, you have been forcing them to explain themselves for a while.

Y7 Teacher: Yep

Tamsin: Could you talk a little bit about the consequences of that just to. . .

Y7 Teacher: Yeah, forcing them to explain everything that they do, it doesn't matter what it is, whether it is number, we are looking at algebra and statistics. So it doesn't matter what they are doing, they have to explain every answer that they ever get, the same sort of scenario. For some, it is quite easy to explain it in words. For some, the language is just not good enough for them to do that.

So in that sense, although they are writing stuff, they are also explaining it. Now those ones that are pretty good at explaining themselves in writing, it is quite easy for them to explain it talking as well, verbally, I mean. Whereas those ones that are a bit slower, they have to read what they have written and in same cases that is an opportune time to fix up what they have actually said.

Y1 Teacher: We do that in *Poutama Tau* anyway. We always ask them how they got that answer, "*pēhea koe e mōhio ai*" [how do you know?] things like that. And it's also getting them to, because they will only give you the straight answer like the basic answer but you get them to repeat it. Like you say "*kei hea te tūru*?" [where is the chair?] and they will say "*kei kō*" [over there] and you go "*whakamārama mai*" [explain it to me] or "*kei te taha o te tēpu*" [at the side of the table], you know. You are just getting them to use it even with the little ones as they are going up, so they do get to the higher levels. "*Kei te mārama, kei te pai pea tō reo*" [Do you understand, is your language okay]? Yeah, to *whakahāngai* [relate] to them too. When I do lessons with them I also think how I am going to *whakahāngai ngā kōrero ki a rātau* [relate what is being discussed to them], to relate to them, how it is going to relate to them? Like shapes, naming all the shapes in the classroom, . . . using the language because my ones can't write either. They are not, some of them are starting to write, but it is getting them to talk about it.

Tony: These are great you know, some of the language teaching stuff in this is pretty good. (Meeting Sept. 2008)

The teachers could see the potential in having the students explain their thinking. However, it is clear that students' lack of exposure to *te reo Māori* outside of the classroom was a challenge that teachers had to address. This is discussed further in Chapter 10. An example of a classroom exchange can be seen in the following extract from a lesson that was recorded in 2006.

Y6Teacher: Kotahi rau, rima tekau mā ono, anā, āe, ngāwari tērā! Nō reira he mea ngāwari tērā Student1?	Y6T: One hundred, fifty six, that's easy, therefore that's easy.
Student1: Āe!	Student1: Yes!
Y6T: He aha ai?	Y6T: Why?
Student1: Nō te mea ka mōhio ko te tekau whakarau tekau mā toru ko te 'tahi rau, toru tekau, anā ka mōhio ko te rua whakarau tekau mā toru ko te rua tekau mā ono, anā, me tāpiri noa iho i te 'tahi rau toru tekau ki te rua tekau mā ono.	Student1: Because you know that ten times thirteen is one hundred and thirty, you know that two times thirteen is twenty six. You only need to add one hundred and thirty to twenty six.
Y6T: Āe, ka pai, [I] kite au i tērā!	Y6T: Yes that's good, I see it now!

This child was able to give an explanation about how to do the calculation. Gradually, the teachers were beginning to expect students to give these explanations, and the students were beginning to know that they had to give them. When the teachers started requiring students to provide this information, students often gave answers to questions about what had they done as “I just knew it” or “I just guessed”. Nevertheless, even in 2008, the teachers were still the most dominant speakers in most mathematics lessons. They did find that it was easier to have students give verbal explanations and justifications around regular writing activities, and this is discussed in more detail in the next chapter.

Kanikani Pāngarau – Dancing Mathematics

As well as using writing to support students giving explanations and justifications, the teachers in the junior end of the school involved students in thinking mathematically through an activity known as Kanikani Pāngarau (mathematical dancing). This activity was taken from the New Zealand television programme *Toro Pikopiko* and was initiated by the teacher Horomona Horo.

Students learnt a series of movements for each of the numbers from zero to ten. They also learnt symbols for the four operations (+, –, ×, and ÷) and for the equals sign. Children were then given problems by the teacher through movement and asked to provide an answer by also using movements themselves. Figure 5.1 shows Horomona illustrating a problem with children watching and then writing down their answers.

Addison and Te Whare (n.d.), the originators of Kanikani Pāngarau, explained that it was based on the principles of *kapa haka* where specific words were represented by specific actions. *Kapa haka* is a traditional team dance that is often performed competitively (Murray, 2000). In the *haka*, actions emphasise the sung or chanted words (Matthews, 2004). However, as Matthews noted, “[I]t was the body that was the instrument and vessel of delivery” (p. 9). In Kanikani Pāngarau, the actions must carry all the meaning. The audience is expected to respond in kind, making the expression of meaning paramount. The children at Te Koutu loved their involvement in Kanikani Pāngarau, and although it was only concerned with basic facts, it did seem to resonate with the children’s cultural background.

Kinaesthetic involvement is believed to support students’ understanding and was labelled by Howard Gardener as an intelligence-kinaesthetic intelligence (Touval & Westreich, 2003). Sellarés and Toussaint (2003), in considering why some algorithms in computational geometry failed, found that these were based on kinaesthetic heuristics rather than logico-mathematical ones. However, the kinaesthetic-based algorithms were much faster, even when they were incorrect, thus suggesting that they are more computationally efficient. Sellarés and Toussaint suggested that there is a need for algorithm designers to bridge the gap between the two types of heuristics in order for efficiency to be combined with accuracy. Although Kanikani Pāngarau deals with much simpler mathematics, there is potential for it to be a support for students to think mathematically if it is further developed.

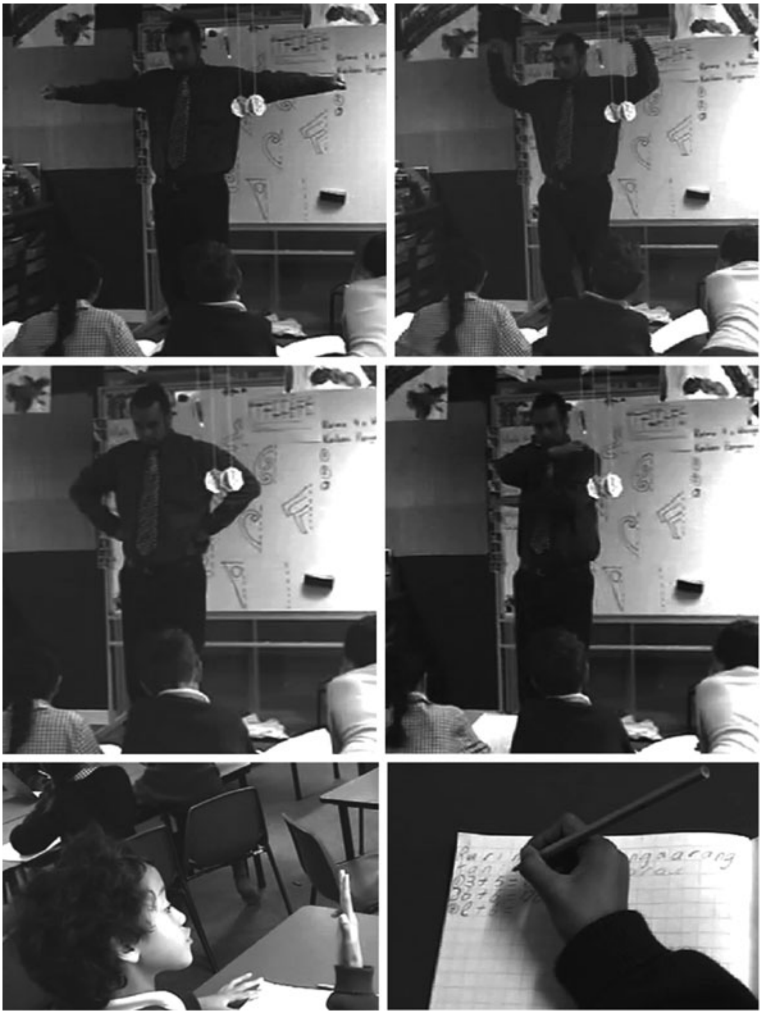


Fig. 5.1 *Kanikani Pāngarau*

Kinaesthetic activities related to gestures that accompany speaking have begun to be researched (see Roth, 2001; Radford, 2003a, 2003b). This research into gestures has concentrated on how extra meaning is added to oral descriptions of mathematical ideas and how it supports students to understand what they are learning. Using culturally appropriate movements to provide extra layers of meaning may well contribute to students being able to think mathematically. However, much more work needs to be done to identify the actions that teachers and students are already using to support mathematical thinking in classrooms, not just those used in *Kanikani Pāngarau*. By working with teachers and students, it would be possible to determine the most effective way that these actions could be used to support students.

Such a challenge is only just being recognised as having potential for improving students' understanding.

Meeting Challenges Around Thinking Mathematically

Thinking mathematically in *te reo Māori* has not yet been fully explored. Different aspects of this are at different stages in the "overcoming challenges cycle" described in Chapter 1.

The mathematics register in *te reo Māori* has features that have the potential to be a useful tool in supporting students to think mathematically. Linguistic markers and logical connectives that are in *te reo Māori* can be useful for linking ideas logically. There may well be other supportive features, which are yet to be identified, but the issue of using such features is a challenge that has been recognised.

As well, there remains a challenge to have students realise the potential from using these features when thinking mathematically. This involves determining potential difficulties in learning aspects of the mathematics register and then looking at how these can be overcome. We are still at an early stage in meeting this challenge. As Christensen (2003) described, there is some resistance to using traditional features of *te reo Māori* with some teachers using English grammatical structures with Māori words. Yet as teachers begin to insist on students explaining and justifying their mathematical understandings, the need for the features within *te reo Māori* may become more self-evident.

The interactions between teachers and students indicate that these traditional features to some extent are being utilised already. However, until the education system as a whole recognises this utility, many will continue to see English as the more appropriate language for thinking about mathematics. Thus, this challenge is far from being met.

Other aspects of Māori language and culture are only now being identified at Te Koutu as having potential, but are yet to be explored fully. They offer openings, which, if followed up, can provide unexpected solutions in meeting the challenge of both supporting students to think mathematically and ensuring the integrity of the language. There is a need for more research to support a better understanding of their efficacy. For example, the use of actions that are part of cultural activities such as *kapa haka* may also have a greater use beyond Kanikani Pāngarau. Unless further work is done to explore this area, it is unlikely that its potential will be realised. This is something that needs further consideration in the coming years.