Chapter 9
Sensitivity of Neurons Exposed to AC Induction Electric Field

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Abstract Neuronal coding is one of the characteristics that exhibit the response of the neuron to the external stimulus. Based on a simplified Hodgkin–Huxley model, this paper firstly establishes a new neuronal model under the effect of alternating current (AC) induction electric field, and investigates the mechanism of neuronal encoding and the sensitivity of firing rate to noises. According to the model established, this paper obtains the bifurcation point of the model with the membrane voltage–time curves Then by the data analysis of the mean firing rate-electric field frequency or the mean firing rate-amplitude curves, it obtains the relationship between the neuronal firing rate and noises as well as the electric field intensity, proving that the fluctuations of electric field frequency or amplitude can affect the sensitivity of neurons.

Keywords Sensitivity · Mean firing rate · AC induction electric field · OU noise

9.1 Introduction

Neurons are the minimal unit of structure and function in the nervous system, determining the specific characteristics of brains, such as memory and cognition. When subjected to certain stimulus, neurons will generate spiking. Different action potential sequence represents the firing pattern induced by different stimulation, namely neural coding, which is one of the most fundamental problems in the

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nervous system [1]. The information coding of neurons changes when stimulation parameters change, such as input and noises, which is called sensitivity.

A large number of theoretical studies and experimental results show that the external electrical field can change the firing characteristics of neurons, thereby change the function and properties of the nervous system [2]. On the one hand, the electromagnetic radiation of the external electric field will change the normal properties of neurons, leading the production of some neuropathies [3]. On the other hand, the electrical stimulus has become one of the main approaches of physical therapy for neuropathies, such as transcranial magnetic stimulation, TMS for short [4], and electro-axupuncture therapy. Since neurons are in an environment with a variety of noises, the noises may change the initial firing time of neurons, thereby affect neuronal coding and transmission of information [5], here it is called sensitivity. Lundstrom et al. interpreted neuronal sensitivity to mean or variance as conferring on the neuron integrator-like or differentiator-like properties [6]. Jing Yang et al. investigated the "critical sensitivity" phenomena of neural firing pacemaker via physiological experiments [7]. Although some literature refers to sensitivity of neurons, but all do not explore the neuronal sensitivity under AC induction electric field.

This chapter establishes a new model of neurons under external electric field on the basis of HH model, introduces the mean firing rate to describe the output of a neuron, and analyze neuronal sensitivity under noisy AC electric field from the view of comparing F–A, F–f_e curves, then reveal the regulation of AC induction electric field to neuronal sensitivity.

9.2 Model Simplification

Hodgkin–Huxley Model is a four-dimensional model with the features of multi-variable, nonlinearity and strong-coupling. In order to investigate the sensitivity, this paper makes some simplification of the HH model: eliminate the time-dependence of m considering the relatively small time constant of m, adopt the m's stable value m_∞ rather than m, and make h linearly related to n [8], thereby reduce the order of the model. Then superimpose a induced voltage V_e on the membrane of the neuron which represents the effect of electromagnetic radiation, while noises are superimposed on stimulus current I in the term of I_{noise}, then obtains the following two-dimensional model:

$$\begin{aligned} C\frac{dV}{dt} &= -G_{Na}m_\infty^3 h(V+V_e-E_{Na}) - G_k n^4(V+V_e-E_k) \\ &\quad - G_{leak}(V+V_e-E_{leak}) + (I+I_{noise}) + I_e \\ \tau\frac{dn}{dt} &= n_\infty - n \end{aligned} \tag{9.1}$$

Where $C = 1\text{nF/cm}^2$, $G_{Na} = 50\text{mS/cm}^2$, $G_k = 36\text{mS/cm}^2$, $G_{leak} = 5\text{mS/cm}^2$, $E_{Na} = 50$ mV, $E_k = -77$ mV, $E_{leak} = -54$ mV, $k_m = 7$, $V_m = -40$ mV, $V_n = -45$ mV, $\tau = 5$ ms. There are five other relational expressions as below:

$$
\begin{aligned}
m_\infty &= 1/(1 + \exp((V_m - V)/k_m)) \\
n_\infty &= 1/(1 + \exp((V_n - V)/k_n)) \\
h &= 0.89 - 1.1n \\
V_e &= A\sin(\omega t) \\
I_e &= C\frac{dV_e}{dt} = CA\omega\cos(\omega t)
\end{aligned}
$$

The model has two variables: a fast variable V is the membrane voltage of the model, while a slow variable n is a combined variable denoting the probability of the Na^+ channel inactivation and K + channel activation. τ is the time constant of recovery variable n. V_e is external AC induction electric field. I_e is the current flowing across the membrane capacitance caused by induction electric field. I_{noise} represents the external OU noise whose intensity is determined by variable D.

Lundstrom et al. [9] classify the neurons into three types: Type A, B+ and B− on the basis of how their F–I curves response to fluctuation input. This chapter focuses on the firing sensitivity of Type A neuron, choosing GNa and τ as neural firing pacemaker, letting GNa = 50 mS/cm^2, $\tau = 5$ ms in Eq. 9.1. The noise used here is equivalent to an Ornstein–Uhlenbeck process which is called OU noise. The mean of OU noise is zero, the noise intensity D is variable. The Eq. 9.1 is solved via a fourth-order Runge–Kutta solver with a 0.02 ms step.

9.3 Analysis of Neuronal Sensitivity

9.3.1 Firing Patterns

Firstly, we obtain the Voltage–time curves (V–t curves) of the model under the direct current (DC) I without noises and induction electric field (Fig. 9.1). It shows that the firing pattern of the model neuron changes with the increasing stimulus: the neuron exhibits single-spiking when $I < 38.2$ μA/cm^2 and begins periodic firing when $I = 38.2$ μA/cm^2, then the firing rate increases with increasing current exhibiting continuously periodic firing. It is indicated that $I = 38.2$ μA/cm^2 is the bifurcation point of the neuron, where the firing pattern transforms from single-spiking to periodic firing.

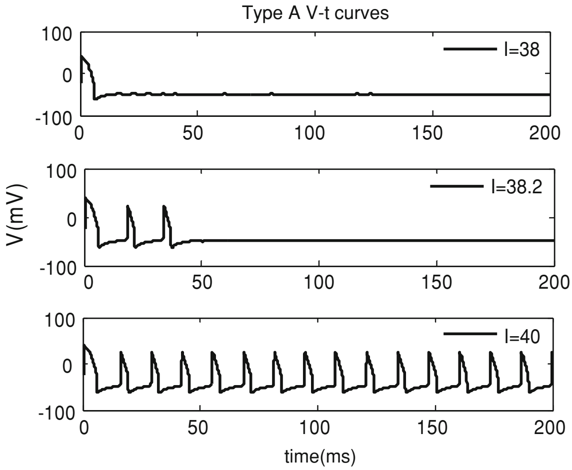

Fig. 9.1 Comparing the V–t curves of Type A neuron when I = 38, 38.2, 40 μA/cm^2

9.3.2 Sensitivity Under DC Electric Field

Analyze the sensitivity of Type A neuron under the input of noisy DC via the firing rate of different noises [9]. The noise intensity is taken as $D = 0$, 10, 20, 30 μA/cm^2 respectively, then obtain a series of firing rate by gradually change the DC stimulus I under each value of D in Fig. 9.2. Figure 9.2 shows: Type A neuron has different firing rate under different intensity of noises when $I \leq 40$ μA/cm^2,which indicates that noises have some effect on the firing pattern of Type A neuron and increase the firing rate, Type A neuron has a little sensitivity to noises; when $I \geq 40$ μA/cm^2 Type A neuron has almost the same firing rate under different intensity of noises, indicating that noises have no effect on the firing rate and Type A neuron has no sensitivity to noises which implies that Type A neuron has self-adaptation.

9.3.3 Sensitivity Under AC Induction Electric Field

The current is taken as the bifurcation point of Type A, *namely* $I = 38.2$ μA/cm^2, while AC Voltage $V_e = A\sin(2\pi ft)$.

1. **Fixed the amplitude *A* and change the frquency f_e**

Fixed A, then obtain the firing rate–voltage frequency curves F–f_e under noise intensity $D = 0$, 10, 20, 30 μA/cm^2 respectively (Fig. 9.3): (a) Firing rate and voltage frequency have a nonlinear relationship. By comparing the F–fe curve of no noise in Fig. 9.3 (blue solid line) can it show that noises can increase the firing rate of Type A neuron obviously, and exhibit the sensitivity with certain laws. The sensitivity of Type A neuron to noises is in a fluctuating state transform between

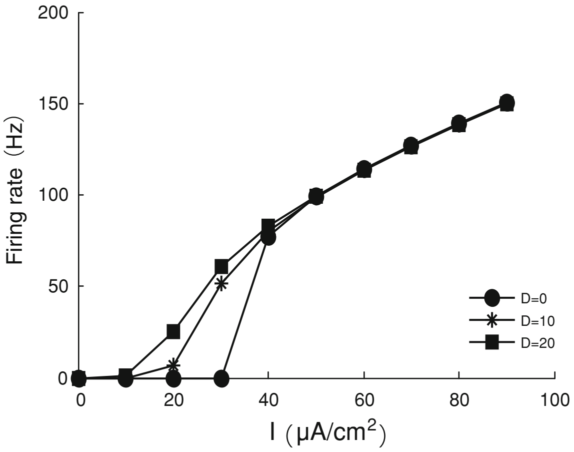

Fig. 9.2 F–I curves of Type A neuron under the input of different noisy DC. The three curves (*star, square, circle curves*) correspond to noises of $D = 0$, 10, 20, 30 μA/cm^2 respectively

strong (at $f_e = 0$, 10, 30, 50 Hz) and weak (at $f_e = 20$, 40, 60 Hz). (b) Neurons show little sensitivity to noises, and have slight sensitivity only in the case of $f_e \in [0, 30]$ Hz. When $f_e \in [0, 30]$ Hz, firing rate has an inverse proportion to voltage frequency, while has a direct proportion when $f_e \in [30, 60]$ Hz. Comparing with the F–fe curve of no noise indicates that the range of inverse proportion is decreased and the range of direct proportion is increased. Noises make the firing rate of Type A neuron obviously decrease (c–f).

Type A neuron has almost the same sensitivity curves of different noises which exhibit no strong sensitivity.

In conclusion, when $A \leq 20$ mV, adding noises can obviously change the firing rate of neurons, indicating that noisy AC electric field has effect on the sensitivity of the neuron in this case, increases the sensitivity of Type A neuron that has no sensitivity under the input of high noisy DC and make Type A neuron exhibits various sensitivity. While $A \geq 20$ mV, Type A neuron has slight sensitivity in the case of fe $\in [0, 20]$ Hz or fe $\in [0, 10]$ Hz and exhibits no sensitivity that has no obvious changes of the firing rate in the case of fe ≥ 20 Hz because of the Type A's self-adaptaion.

Fixed the the frequency f_e and change the amplitude A

Fixed the frequency f_e, then obtain the firing rate-voltage amplitude curves F–A with noise intensity $D = 0$, 10, 20, 30 μA/cm^2 respectively (Fig. 9.4). (a) The neurons exhibit weaker sensitivity in the case of $A \in [0, 10]$ mV while exhibit no in the other range. Firing rate has an inverse proportion to voltage amplitude and firing rate in the range of $A \in [17, 25]$ mV is decreased compared with the one that has no effect of noises, increased in the other range. (b) Similar to (a), the neuron exhibits weaker sensitivity in the case of $A \in [0, 10]$ mV while exhibits no in the other range. Firing rate in the range of $A \in [16.5, 30]$ mV is decreased compared with the one that has no effect of noises, increased in the other range. In the case of $A \geq 30$ mV, the effect of the AC electric field is so strong and the neuron has the

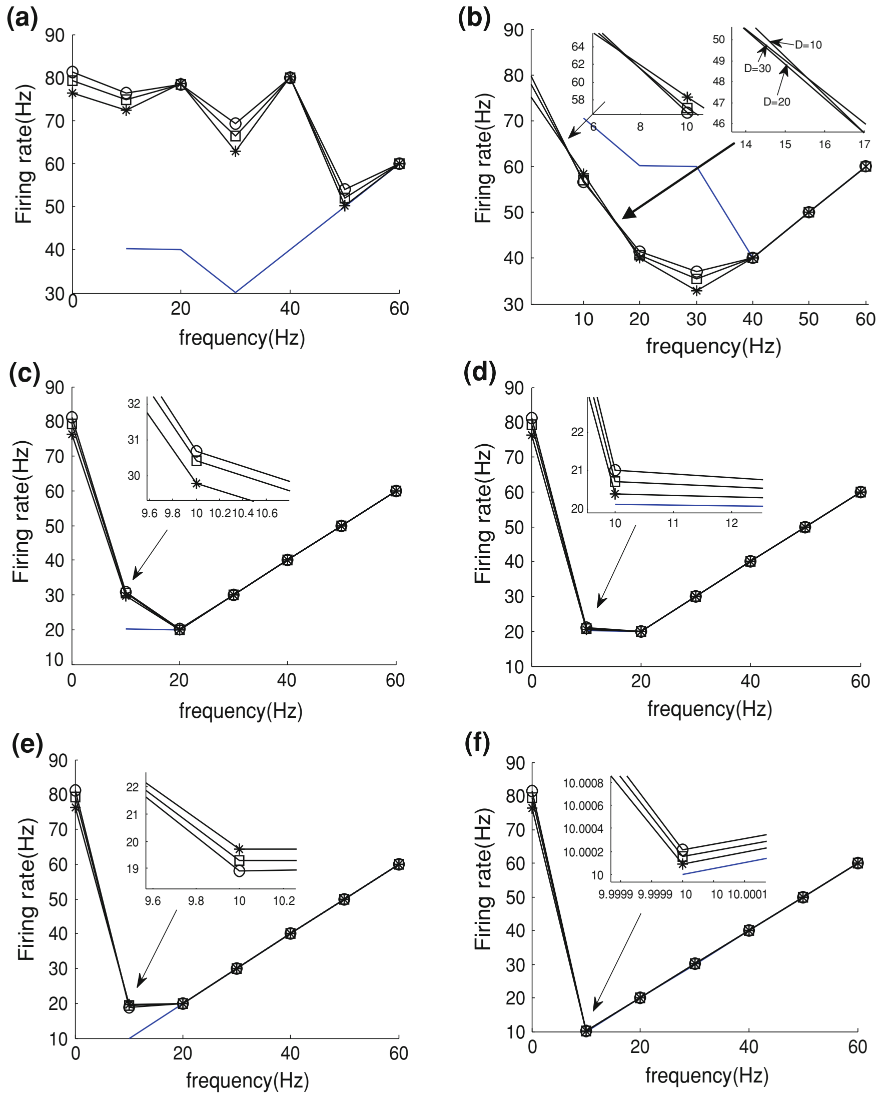

Fig. 9.3 F–fe curves of Type A neuron under different noises of each fixed amplitude A . **a** A = 10 mV. **b** A = 20 mV. **c** A = 30 mV. **d** A = 40 mV. **e** A = 50 mV. **f** A = 60 mV. The three curves (*star*, *square*, *circle curves*) correspond to noises of D = 0, 10, 20, 30 μA/cm^2 respectively

adaptation that the firing rate of the neuron equals to the frequency of external electric field and does not change any more. (c) when $A \in [0, 30]$ mV, the neuron exhibits relatively strong sensitivity to noises and firing rate increases with the decrease of the voltage frequency. When $A \geq [0, 30]$ mV, the neuron exhibits no

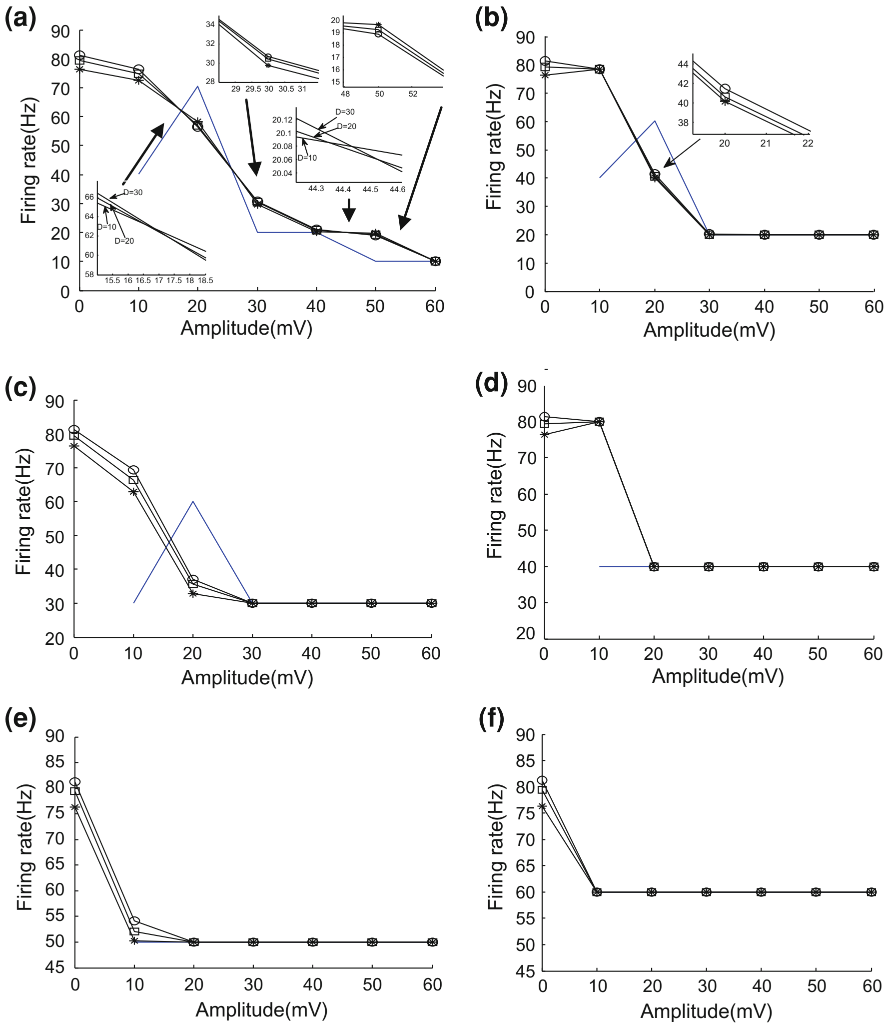

Fig. 9.4 F–A curves of Type A neuron under different noises of each fixed frequency. **a** fe = 10 Hz. **b** fe = 20 Hz. **c** fe = 30 Hz. **d** fe = 40 Hz. **e** fe = 50 Hz. **f** fe = 60 Hz. The three curves (*star*, *square*, *circle curves*) correspond to noises of D = 0, 10, 20, 30 $\mu A/cm^2$ respectively

sensitivity and the firing rate equals to the frequency of external electric field. Firing rate in the case of $A \in [15.5, 30]$ mV is decreased compared with the one of no effects of noises, while increased when $A \in [0, 15.5]$ mV. The result in the case of $A \geq 30$ mV is similar to (b) and (d). In the case of $A \in [0, 10]$ mV, the firing rate increses as the noise intensity increses, and the more stronger the noise is, the more the firing rate increases. And the neuron has slight sensitivity in this region.

When $A \geq 10$ mV,the neuron has adaptation that the noise has no effect on the firing rate. As the amplitude increases, the firing rate decreases until $A = 20$ mV, it equals to the external voltage frequency. (e) The neurons exhibit a little strong sensitivity in the case of $A \in [0, 20]$ mV and the firing rate has an inverse proportion to the voltage amplitude. When $A \in [0, 20]$ mV, noises make the firing rate increase compared with the one of no noisy effects. While $A \geq 20$ mV, the neuron exhibits no sensitivity and has adaptation that the firing rate equals to the voltage frequency. (f) The neurons exhibit a little weaker sensitivity in the case of $A \in [0, 10]$ mV and the firing rate in this range increases because of noises. As the amplitude A increases to 10 mV, the neuron exhibit adaptation that the firing rate remain the frequency of the external electric field.

In conclusion, noises make Type A neuron exhibits relatively strong sensitivity in the range of lower voltage amplitude, that is $A \in [0, 10]$ mV with $f_e = 10$, 20, 40 or 60 Hz, $A \in [0, 20]$ mV with $f_e = 50$ Hz, $A \in [0, 30]$ mV with $f_e = 30$ Hz. And in these range the firing rate increases compared with the one that has no influence of noises—the stronger the noise intensity is, the more the firing rate increases. In the case that AC electric field has large amplitude or frequency (refer to the analysis above), Type A neuron has self-adaptation that exhibit no sensitivity.

9.4 Conclusions

This paper firstly establishes a new neuronal model under the effect of AC induction electric field on the basis of a simplified HH model, then investigates the coding mechanism of Type A neuron to the stimulus and the sensitivity of firing rate to OU noises when the neuron is in an AC induction electric field. Obtain the conclusions as follows: Under the noisy AC electric field, Type A neuron has certain sensitivity in the case of $A \in [0, 10]$ mV or $A \in [10, 20]$ mV with $f_e \in [0, 50]$ Hz or $A \in [20, 30]$ mV with $f_e \in [0, 30]$ Hz. While Type A neuron exhibits no sensitivity in the case of $A \in [10, 20]$ mV with $f_e \geq 50$ Hz or $A \in [20, 30]$ mV with $f_e \geq 30$ Hz or $A \geq 30$ mV with any frequency. Because when the intensity of the electric field is low, noises superimposed on stimulus current have larger effect on the neuron that make the firing rate change, thereby make the neuron exhibit certain sensitivity, and when the intensity of the electric field is too high, the effect of the noise is much weaker than the one of the electric field, so the effect of noise is ignored and the neuron is equivalent to only have the effect of electric field stimulus, namely the neuron has self-adaptation. Therefore, in this case, no matter how the intensity of the noise changes, the firing rate has no change maintaining the frequency of external electric field. Namely, the neuron has no sensitivity.

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