Chapter 88
Models for Multiple Attribute Decision Making with Intuitionistic Trapezoidal Information

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Abstract The aim of this chapter is to investigate the multiple attribute decision making problems to deal with supplier selection with intuitionistic trapezoidal fuzzy information, in which the information about attribute weights is completely known, and the attribute values take the form of intuitionistic trapezoidal fuzzy number. We developed a multiple attribute decision making method by closeness to ideal alternative in intuitionistic trapezoidal fuzzy setting. Then, we calculate the distances between the ideal alternative and all alternatives to determine the ranking of all alternatives. Finally, an example about supplier selection is shown to highlight the procedure of the proposed algorithm.

Keywords Multiple attribute decision making · Supplier selection · Intuitionistic trapezoidal fuzzy number · Weight information · Distances measure

88.1 Introduction

Atanassov [1, 2] introduced the concept of intuitionistic fuzzy set (IFS) characterized by a membership function and a non-membership function, which is a generalization of the concept of fuzzy set [3] whose basic component is only a membership function. The intuitionistic fuzzy set has received more and more attention since its appearance [4]. Later, Atanassov and Gargov [5] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a

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generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers. Xu [6] developed some aggregation operators with interval-valued intuitionistic fuzzy information. Xu [7] investigated the interval-valued intuitionistic fuzzy MADM with the information about attribute weights is incompletely known or completely unknown, a method based on the ideal solution was proposed. Wang [8] investigated the interval-valued intuitionistic fuzzy MADM with incompletely known weight information. A nonlinear programming model is developed. Then using particle swarm optimization algorithms to solve the nonlinear programming models, the optimal weights are gained. And ranking is performed through the comparison of the distances between the alternatives and idea/anti-idea alternative. Shu et al. [9–12] gave the definition and operational laws of intuitionistic triangular fuzzy number and proposed an algorithm of the intuitionistic fuzzy fault-tree analysis.

The aim of this chapter is to investigate the multiple attribute decision making problems to deal with supplier selection with intuitionistic trapezoidal fuzzy information, in which the information about attribute weights is completely known, and the attribute values take the form of intuitionistic trapezoidal fuzzy number. We developed a multiple attribute decision making method by closeness to ideal alternative in intuitionistic trapezoidal fuzzy setting. Then, we calculate the distances between the ideal alternative and all alternatives to determine the ranking of all alternatives.The remainder of this chapter is set out as follows. In the next section, we introduce some basic concepts related to intuitionistic trapezoidal fuzzy numbers. In Sect. 88.3 we introduce intuitionistic trapezoidal fuzzy multiple attribute decision making problems with completely known weight information.Then, we calculate the distances between the ideal alternative and all alternatives to determine the ranking of all alternatives. In Sect. 88.4 we conclude the chapter and give some remarks.

88.2 Preliminaries

In the following, we shall introduce some basic concepts related to intuitionistic trapezoidal fuzzy numbers.

Definition 1 Let $\tilde{a}$ is an intuitionistic trapezoidal fuzzy number, its membership function is [10–12]:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a}{b-a}\mu_{\tilde{a}}, & a \le x < b; \\ \mu_{\tilde{a}}, & b \le x \le c; \\ \frac{d-x}{d-c}\mu_{\tilde{a}}, & c < x \le d; \\ 0, & others. \end{cases} \tag{88.1}$$

Its non-membership function is:

$$v_{\tilde{a}}(x)=\begin{cases}\frac{b-x+v_{\tilde{a}}(x-a)}{b-a}, & a_1 \le x < b;\\ v_{\tilde{a}}, & b \le x \le c;\\ \frac{x-c+v_{\tilde{a}}(d_1-x)}{d_1-c}, & c < x \le d_1;\\ 0, & \text{others}.\end{cases} \tag{88.2}$$

where $0 \le \mu_{\tilde{a}} \le 1; 0 \le v_{\tilde{a}} \le 1$ and $\mu_{\tilde{a}} + v_{\tilde{a}} \le 1; a, b, c, d \in R$. Then $\tilde{a} = \langle([a, b, c, d]; \mu_{\tilde{a}}), ([a_1, b, c, d_1]; v_{\tilde{a}})\rangle$ is called an intuitionistic trapezoidal fuzzy number.

For convenience, let $\tilde{a} = ([a, b, c, d]; \mu_{\tilde{a}}, v_{\tilde{a}})$.

Definition 2 Let $\tilde{a}_1 = \left([a_1, b_1, c_1, d_1]; \mu_{\tilde{a}_1}, v_{\tilde{a}_1}\right)$ and $\tilde{a}_2 = \left([a_2, b_2, c_2, d_2]; \mu_{\tilde{a}_2}, v_{\tilde{a}_2}\right)$ be two intuitionistic trapezoidal fuzzy number, and $\lambda \ge 0$, then [24–26]

(1) $\tilde{a}_1 + \tilde{a}_2 = \left([a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; \mu_{\tilde{a}_1} + \mu_{\tilde{a}_2} - \mu_{\tilde{a}_1} \cdot \mu_{\tilde{a}_2}, v_{\tilde{a}_1} \cdot v_{\tilde{a}_2}\right)$;
(2) $\tilde{a}_1 \cdot \tilde{a}_2 = \left([a_1 \cdot a_2, b_1 \cdot b_2, c_1 \cdot c_2, d_1 \cdot d_2]; \mu_{\tilde{a}_1} \cdot \mu_{\tilde{a}_2}, v_{\tilde{a}_1} + v_{\tilde{a}_2} - v_{\tilde{a}_1} \cdot v_{\tilde{a}_2}\right)$;
(3) $\lambda\tilde{a}_1 = \left([\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1]; 1 - \left(1 - \mu_{\tilde{a}_1}\right)^{\lambda}, v_{\tilde{a}_1}^{\lambda}\right)$;
(4) $\tilde{a}_1^{\lambda} = \left(\left[a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}\right]; \mu_{\tilde{a}_1}^{\lambda}, 1 - (1 - v_{\tilde{a}_1})^{\lambda}\right)$

Definition 3 Intuitionistic trapezoidal fuzzy positive ideal solution and intuitionistic trapezoidal fuzzy negative ideal solution are defined as follows:

$$\tilde{r}^{+} = ([a^{+}, b^{+}, c^{+}, d^{+}]; \mu^{+}, v^{+}) = ([1, 1, 1, 1]; 1, 0) \tag{88.3}$$

Definition 4 Let $\tilde{a}_1 = \left([a_1, b_1, c_1, d_1]; \mu_{\tilde{a}_1}, v_{\tilde{a}_1}\right)$ and $\tilde{a}_2 = \left([a_2, b_2, c_2, d_2]; \mu_{\tilde{a}_2}, v_{\tilde{a}_2}\right)$ be two intuitionistic trapezoidal fuzzy number, then the normalized Hamming distance between $\tilde{a}_1$ and $\tilde{a}_2$ is defined as follows [26]:

$$\begin{aligned} d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{8}\Big(&\left|\left(1 + \mu_{\tilde{a}_1} - v_{\tilde{a}_1}\right)a_1 - \left(1 + \mu_{\tilde{a}_2} - v_{\tilde{a}_2}\right)a_2\right| + \Big|\left(1 + \mu_{\tilde{a}_1} - v_{\tilde{a}_1}\right)b_1 \\ &- \left(1 + \mu_{\tilde{a}_2} - v_{\tilde{a}_2}\right)b_2\Big| + \left|\left(1 + \mu_{\tilde{a}_1} - v_{\tilde{a}_1}\right)c_1 - \left(1 + \mu_{\tilde{a}_2} - v_{\tilde{a}_2}\right)c_2\right| \\ &+ \left|\left(1 + \mu_{\tilde{a}_1} - v_{\tilde{a}_1}\right)d_1 - \left(1 + \mu_{\tilde{a}_2} - v_{\tilde{a}_2}\right)d_2\right|\Big). \end{aligned} \tag{88.4}$$

88.3 Models for Multiple Attribute Decision Making with Intuitionistic Trapezoidal Fuzzy Information

The following assumptions or notations are used to represent the MADM problems with completely known weight information in intuitionistic trapezoidal fuzzy setting. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \ldots, G_n\}$ be the set of attributes. Suppose that $\tilde{R} = \left(\tilde{r}_{ij}\right)_{m \times n} =$

$\left(\left[a_{ij}, b_{ij}, c_{ij}, d_{ij}\right]; \mu_{ij}, v_{ij}\right)_{m\times n}$ is the intuitionistic trapezoidal fuzzy decision matrix, $\mu_{ij}^{(k)} \in [0,1]$, $v_{ij}^{(k)} \in [0,1]$, $\mu_{ij}^{(k)} + v_{ij}^{(k)} \le 1$, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$, $k = 1, 2,\ldots,t$. The information about attribute weights is incompletely known. Let $w = (w_1, w_2,\ldots,w_n)$ be the weight vector of attributes, where $w_j \ge 0$, $j = 1,2,\ldots,n$, $\sum_{j=1}^{n} w_j = 1$.

In the following, we develop a practical method for solving the MADM problems, in which the information about attribute weights is completely known, and the attribute values take the form of intuitionistic trapezoidal fuzzy variables. The method involves the following steps:

Step 1. Suppose that $\tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n} = \left(\left[a_{ij}, b_{ij}, c_{ij}, d_{ij}\right]; \mu_{ij}, v_{ij}\right)_{m\times n}$ is the intuitionistic trapezoidal fuzzy decision matrix, $\mu_{ij}^{(k)} \in [0,1]$, $v_{ij}^{(k)} \in [0,1]$, $\mu_{ij}^{(k)} + v_{ij}^{(k)} \le 1$, $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$, $k = 1,2,\ldots,t$. The information about attribute weights is incompletely known. Let $w = (w_1, w_2,\ldots,w_n)$ be the weight vector of attributes, where $w_j \ge 0$, $j = 1,2,\ldots,n$, $\sum_{j=1}^{n} w_j = 1$.

Step 2. Let $\tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n} = \left(\left[a_{ij}, b_{ij}, c_{ij}, d_{ij}\right]; \mu_{ij}, v_{ij}\right)_{m\times n}$ be the intuitionistic trapezoidal fuzzy decision matrix, the ideal alternative can be defined as follows:

$$A^{+} = \left(\tilde{r}_1^{+}, \tilde{r}_2^{+},\ldots,\tilde{r}_n^{+}\right),$$

where $\tilde{r}_j^{+} = ([a^{+}, b^{+}, c^{+}, d^{+}]; \mu^{+}, v^{+}) = ([1,1,1,1]; 1, 0)$.

Step 3. Utilize the weight vector $w = \left(w_1, w_2,\ldots,w_n\right)$ and utilize the (88.4) to derive the distances$d(A_i,\ A^{+})(i = 1,2,\ldots,m)$, by which we can get the ranking of all alternatives $A_i(i = 1,2,\ldots,m)$.

$$d(A_i, A^{+}) = \sum_{j=1}^{n} w_j d\left(\tilde{r}_j^{+}, \tilde{r}_{ij}\right) \tag{88.5}$$

Step 4. Rank all the alternatives $A_i(i = 1,2,\ldots,m)$ and select the best one(s) in accordance with $d(A_i, A^{+})(i = 1,2,\ldots,m)$. The smaller $d(A_i, A^{+})$, the better the alternatives A_i.

Step 5. End.

88.4 Conclusion

In this chapter, we have investigated the multiple attribute decision making problems to deal with supplier selection with intuitionistic trapezoidal fuzzy information, in which the information about attribute weights is completely known, and the attribute values take the form of intuitionistic trapezoidal fuzzy number. We developed a multiple attribute decision making method by closeness to ideal alternative in intuitionistic trapezoidal fuzzy setting. Then, we calculate the distances between the ideal alternative and all alternatives to determine the ranking of all alternatives. At last, a practical example about supplier selection is provided to illustrate the proposed method.

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