Chapter 86
Optimal Detection of Distributed Target with Fluctuating Scatterers

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Abstract In the non-Gaussian clutter modeled as a spherically invariant random vector, the optimal detection of a distributed target is addressed with fluctuating scatterers in high resolution radar scenarios, by exploiting the generalized likelihood ratio test design procedure and the binary integrator. The formula relating the false alarm probability to the detection threshold is given, which implies the constant false alarm rate property with respect to both the clutter covariance matrix structure and the clutter power level. Moreover, the optimal detection parameter is also obtained for a distributed target with fluctuating scatterers. Finally, the performance assessment conducted by Monte Carlo simulation confirms the effectiveness of the proposed detectors.

Keywords Optimal detection · Distributed target · Fluctuating scatterer · Binary integrator

86.1 Introduction

The point-like target detection against Gaussian clutter for the traditional low-resolution radar has been addressed partly in [1]. However, a high-resolution radar can resolve a target into a number of scatterers, which is referred to as a distributed target [2, 3]. Increasing the radar range resolution can reduce the amount of energy per cell backscattered by distributed clutter and can enhance the radar detection

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X. He et al. (eds.), *Computer, Informatics, Cybernetics and Applications*, Lecture Notes in Electrical Engineering 107, DOI: 10.1007/978-94-007-1839-5_86,

performance largely by appropriate detection strategies [4, 5]. Whereas, at the higher range resolution, the radar system receives target-like spikes that result in non-Gaussian observations, which can be suitably modeled by a spherically invariant random vector (SIRV) [6–8].

In this work, the optimal detection of a distributed target is addressed with fluctuating scatterers, by exploiting the generalized likelihood ratio test (GLRT) design procedure [9] and the binary integrator (BI).

86.2 Problem Formulation

It is assumed that data are collected from N sensors and the problem of detecting the presence of a target across K range cells $z_t s$, $t = 1, \ldots, K$, is dealt with. It is supposed that the possible target is completely contained within those data [10]. Herein, the clutter-dominant environment is considered, and the internal noise is ignored. Hence the detection problem can be formulated as the following binary hypotheses test

$$\begin{aligned} H_0 &: z_t = c_t, & t = 1, \ldots, K \\ H_1 &: z_t = \alpha_t p + c_t, & t = 1, \ldots, K \end{aligned} \tag{86.1}$$

where $\mathbf{p}$ denotes the normalized steering vector, such that $\mathbf{p}^{\mathrm{H}}\mathbf{p} = 1(\cdot)^{\mathrm{H}}$implies conjugate transpose), and the $\alpha_t s$, $t = 1, \ldots, K$ are unknown parameters accounting for both the target and the channel effects. Note that, for the uniform linear array, $p = \left(1\,, e^{j\phi},\, e^{j2\phi}, \ldots, e^{j(N-1)\phi}\right)^T \Big/ \sqrt{N}$, where ϕ denotes a constant phase shifting and $(\cdot)^{\mathrm{T}}$ represents transpose.

The clutter returns are modeled as a SIRV for representing non-Gaussian clutter [6]. Thus the N-dimension clutter vector $\mathbf{c}_t$ at range t can be given by

$$c_t = \sqrt{\tau_t} \cdot \eta_t, \;\; t = 1, \ldots, K + R \tag{86.2}$$

where $\eta_t = (\eta_t(1), \eta_t(2), \ldots, \eta_t(N))^T, \;\; \eta_t(n)s, \;\; n = 1, \ldots, N$ are zero-mean complex circular Gaussian random variables (RVs) with variance equal to one, and the texture component τ_t is a semipositive real RV with probability distribution f_τ, which is called mixing distribution. Moreover, $\boldsymbol{\eta}_t$ and τ_t are assumed to be independent from range cell to range cell. Here an $N \times N$ clutter covariance matrix structure $\boldsymbol{\Sigma}$ associated with $\eta_t, s, \;\; t = 1, \ldots, K + R$ is defined as

$$\Sigma = E\left\{\eta_t \eta_t^H\right\} \tag{86.3}$$

where $\boldsymbol{\Sigma}$ is the positive definite and Hermitian matrix.

It is assumed that the underlying mixing distribution f_τ is unknown. Thereby each component of clutter vector $\mathbf{c}_t$ is modeled as conditionally Gaussian with the unknown variance τ_t. It is also assumed that α_t is unknown but $\mathbf{p}$ is known.

According to the previous assumptions, the probability density function (PDF) of $\mathbf{z}_t$s, $t = 1, \ldots, K$ under each hypothesis is given by [11]

$$f(\mathbf{z}_t|\Sigma, \tau_\mathbf{t}, \mathrm{H}_\mathbf{0}) = \frac{1}{\pi^N \tau_t^N \det(\Sigma)} \times \exp\left[-\frac{1}{\tau_t}\mathbf{z}_t^H \Sigma^{-\mathbf{1}} \mathbf{z}_\mathbf{t}\right] \tag{86.4}$$

$$f(z_t|\Sigma, \alpha_\mathbf{t}, \tau_\mathbf{t}, \mathrm{H}_1) = \frac{1}{\pi^N \tau_t^N \det(\Sigma)} \times \exp\left[-\frac{1}{\tau_t}(\mathbf{z}_\mathbf{t} - \alpha_\mathbf{t}\mathbf{p})^H \Sigma^{-\mathbf{1}}(\mathbf{z}_\mathbf{t} - \alpha_\mathbf{t}\mathbf{p})\right] \tag{86.5}$$

where $\det(\cdot)$ denotes determinant.

According to the Neyman-Pearson criterion, the optimal solution to the hypotheses testing problem (86.1) is the likelihood ratio test, but for the case at hand, it cannot be implemented due to total ignorance of the parameters $\{\alpha_t|t = 1, \ldots, K\}$ and $\{\tau_t|t = 1, \ldots, K + R\}$. We resort, instead, to GLRT-based decision schemes [9].

86.3 Binary Integrator

In this section, with the known matrix $\mathbf{\Sigma}$, the BI is introduced.

To simplify the analysis, only one scatterer is supposed to occupy one resolution cell. In most cases of target scattering, the scatterers may occupy only a fraction of the K range cells. Furthermore, the echo amplitudes of range cells occupied by target scatterers are significantly greater than that of range cells with clutter only.

The traditional detection strategies of a point-like target only utilizes target energy in a single range cell, and may fail for distributed targets. In order to make the best of target energy in all K resolution cells of the range extent of target, we can accumulate target scatterers by BI, after single target scatterer detection in each range cell.

With known $\mathbf{\Sigma}$, the derivation of target scatterer detection in single range cell is begun by writing the GLRT as follows [3]

$$\frac{\max\limits_{\tau_t} \max\limits_{\alpha_t} f(\mathbf{z}_\mathbf{t}|\Sigma, \alpha_\mathbf{t}, \tau_\mathbf{t}, \mathrm{H}_\mathbf{1})}{\max\limits_{\tau_t} f(\mathbf{z}_\mathbf{t}|\Sigma, \tau_\mathbf{t}, \mathrm{H}_\mathbf{0})} \tag{86.6}$$

By replacing the unknown parameters with their maximum likelihood estimates under each hypothesis, the GLRT statistic for target scatterer detection in single range cell can be denoted as

$$\lambda_1(\mathbf{z}_\mathbf{t}) = -N \ln\left[1 - \frac{\left|\mathbf{p}^\mathbf{H}\Sigma^{-\mathbf{1}}\mathbf{z}_\mathbf{t}\right|^2}{\left(\mathbf{z}_\mathbf{t}^\mathbf{H}\Sigma^{-\mathbf{1}}\mathbf{z}_\mathbf{t}\right)\left(\mathbf{p}^\mathbf{H}\Sigma^{-\mathbf{1}}\mathbf{p}\right)}\right] \tag{86.7}$$

Therefore, the first detection threshold T_1 of BI for the given first false alarm probability P_{fa1} can be given by [5]

$$P_{fa1} = \exp[-(N-1)T_1/N] \tag{86.8}$$

Set

$$d_t = \begin{cases} 1, & \text{if } \lambda_1(\mathbf{z}_t) > T_1 \\ 0, & \text{otherwise} \end{cases}, t = 1, \ldots, K \tag{86.9}$$

Herein, the BI (or M/K detector) is designed to detect all scatterers for a distributed target. The detection decision is based on at least M threshold crossings, out of K observations [12], where K is the integrated cell number and M $(1 \leq M \leq K)$ is the threshold of BI. The first threshold level of a single range cell must be determined, for the given M and K, to produce the desired integrated false alarm probability P_{fa2}. Since the choice of M affects this result, each M requires a different first threshold. It is necessary to determine an optimal or nearly optimal value for the parameter M.

The d_ts, $t = 1, \ldots, K$ are inputted into the M/K detector. Moreover, the hypothesis that a distributed target is present is tested as follows

$$\lambda_2 = \sum_{t=1}^{K} d_t \underset{H_0}{\overset{H_1}{\gtrless}} T_2 \tag{86.10}$$

where the second detection threshold of BI is given by

$$T_2 = M, \;\; 1 \leq M \leq K \tag{86.11}$$

Accordingly, the second false alarm probability P_{fa2} is simply expressed as

$$P_{fa2} = \sum_{k=M}^{K} P_{fa1}^{k} \left(1 - P_{fa1}\right)^{K-k} K!/(k!(K-k)!) \tag{86.12}$$

where P_{fa1} is the first false alarm probability in (86.8). For the given overall false alarm probability $P_{fa} = P_{fa2}$, M and K, the first false alarm probability P_{fa1} can be determined from (86.12) either iteratively or approximately [12]. Finally, the first threshold T_1 can be computed from (86.8) for the given P_{fa1}. It is shown that both of two detection thresholds T_1 and T_2 are independent of $\mathbf{\Sigma}$ and τ_t, s, $t = 1, \ldots, K$ It implies that, with known $\mathbf{\Sigma}$, the BI is constant false alarm rate (CFAR) with respect to both the clutter covariance matrix structure and the clutter power level.

86.4 Optimal Detection Threshold

In this section, the optimal threshold M (M_{opt}) of BI is calculated for distributed target detection.

The matrix $\mathbf{\Sigma}$ is assumed to be Toeplitz. The clutter samples were generated assuming an exponential correlation structure, i.e., the matrix $\mathbf{\Sigma}$ has elements [8]

Table 86.1 Values of P_{fa1} with different M for P_{fa}=10^{-4} and K=15

M	1	2	3	4	5
P_{fa1}	6.6670×10^{-6}	9.8006×10^{-4}	6.1472×10^{-3}	1.70846×10^{-2}	3.38243×10^{-2}
M	6	7	8	9	10
P_{fa1}	5.60672×10^{-2}	8.3561×10^{-2}	1.16188×10^{-1}	1.53997×10^{-1}	1.97231×10^{-1}
M	11	12	13	14	15
P_{fa1}	2.46399×10^{-1}	3.02418×10^{-1}	3.66965×10^{-1}	4.4346×10^{-1}	5.4117×10^{-1}

$$[\Sigma]_{i,j} = \gamma^{|i-j|},\ 1 \le i, j \le N \tag{86.13}$$

where γ is the one-lag correlation coefficient.

The distribution f_τ is modeled as a Gamma distribution with the following PDF

$$f_\tau(x) = (L/b)^L x^{L-1} e^{-(L/b)x} \Big/ \Gamma(L),\ x \ge 0 \tag{86.14}$$

where $\Gamma(\cdot)$ is the gamma function, b indicates the mean of the distribution, and L controls the deviation from Gaussian statistics.

It is assumed that each of the K range cells has a clutter component and each of the h_0 target range cells has a signal component. The quantity σ_s^2/σ_c^2 indicates the average signal-to-clutter ratio (SCR) per range cell taken over K range cells, where σ_s^2 and σ_c^2 indicate the average signal and clutter power per range cell respectively. The returns from target scatterers are modeled as independent and identically distributed (IID) zero-mean complex circular Gaussian RVs with the variance σ_s^2. It means that the target amplitude fluctuates with Rayleigh law over range cells. Moreover, the input SCR of distributed target detectors is defined as [10]

$$SCR = \sigma_s^2 \mathbf{p}^{\mathrm{H}} \mathbf{\Sigma}^{-1} \mathbf{p} / \sigma_c^2 \tag{86.15}$$

For the given P_{fa}, M and K, the first detection threshold of BI can be computed from (86.8). Moreover, P_{d}s for all detectors are estimated based on Monte Carlo simulation. For analytical convenience, the P_{fa1} with different M is given for $P_{\mathrm{fa}} = 10^{-4}$ and $K = 15$ in Table 86.1.

For only one scatterer is supposed to occupy one resolution cell, we just consider the values of M for $M \le h_0$. For the space consideration, herein, we only present some representative M_{opt} for $h_0 = 2, 5, 8$. In addition, M_{opt} for other values of $h_0 \le K$ has the similar rules and can be calculated from the resultant equation of M_{opt} with respect to h_0.

In Fig. 86.1, the plots of P_{d} versus SCR_{in} of BI ($M = 1, 2$) are given for $N = 2$, $L = 1$, $\gamma = 0$, $K = 15$ and $h_0 = 2$. It is observed that, the BI with $M = 2$ outperforms the BI with $M = 1$. We determine that $M_{\mathrm{opt}} = 2$ for $h_0 = 2$. With the other preferences same as Figs. 86.1 and 86.2 refers to the detection performance of BI ($M = 1, 2, 3, 4, 5$) for $h_0 = 5$. It highlights that the BI with $M = 1$ performs worst, and the performance gets better as M increases. Moreover,

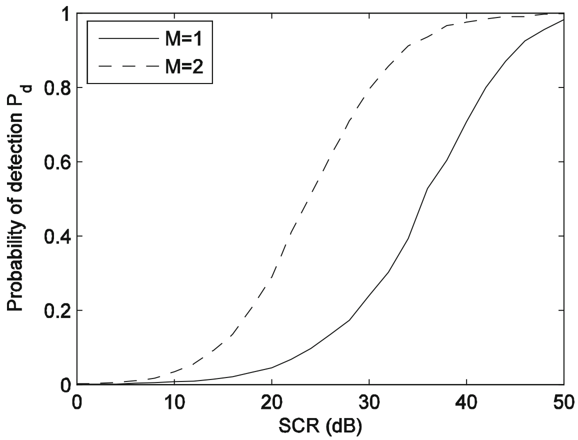

Fig. 86.1 P_d versus *SCR* of BI for N=2, L=1, γ=0, K=15, M=1,2, h_0=2, P_{fa}=10^{-4}

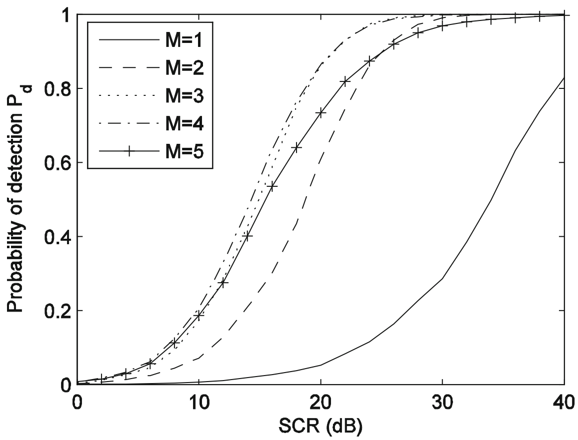

Fig. 86.2 P_d versus *SCR* of BI for N=2, L=1, γ=0, K=15, M=1,2,3,4,5, h_0=5, P_{fa}=10^{-4}

the BI with $M = 4$ performs best, but the performance gets worse as M increases for $M \geq 4$. Hence, we determine that $M_{opt} = 4$ for $h_0 = 5$.

Furthermore, with the other preferences same as Figs. 86.1, 86.3 refers to the detection performance of BI ($M = 1, 2, 3, 4, 5, 6, 7, 8$) for $h_0 = 8$. It is indicated that, for $M \leq 5$, the performance gets better as M increases, however, for $M \geq 5$, the performance gets worse as M increases. Thereby we determine that $M_{opt} = 5$ for $h_0 = 8$. In like manner, we also calculate the optimal M for other values of $h_0 \leq K$, and the values of M_{opt} with different h_0 are given for $K = 15$ in Table 86.2. It is observed that M_{opt} is a monotonically increasing function of h_0.

According to Table 86.2, it is concluded that, for $1 < h_0 \leq K$, M_{opt} satisfies

$$M_{opt} = round(h_0/2 + 1) \tag{86.16}$$

where $round(\cdot)$ denotes rounding the parameter to the nearest integer.

Fig. 86.3 P_d versus *SCR* of BI for $N = 2$, $L = 1$, $\gamma = 0$, $K = 15$, $M = 1,\ldots,8$, $h_0 = 8$, $P_{fa} = 10^{-4}$

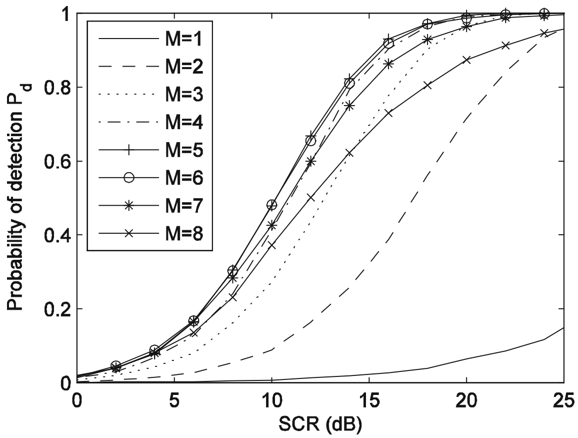

Table 86.2 Values of M_{opt} with different h_0 for $K = 15$

h_0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
M_{opt}	1	2	3	3	4	4	5	5	6	6	7	7	8	8	9

86.5 Conclusions

The optimal detection of a distributed target is addressed with fluctuating scatterers, by exploiting the GLRT design procedure and the BI. The formula relating the false alarm probability to the detection threshold of BI implies the CFAR property with respect to both the clutter covariance matrix structure and the clutter power level. Moreover, the optimal parameter of BI is also obtained by performance assessment.

Acknowledgments This work was supported by National Natural Science Foundation of China (61032001) and Scientific Research Foundation of Naval Aeronautical and Astronautical University for Young Scholars (HYQN201013).

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