Chapter 2
Novel Methods for Extending Optical Code Set for Coherent OCDMA

Chen Peng

Abstract Compared with that used in incoherent system, the optical code (OC) for coherent OCDMA system is analyzed in definition, characters, correlation performances and construction method. Aperiodic correlation property of OC is newly defined, and made the criterion for the judgment of OC' performance in a coherent OCDMA system. A novel differential algorithm is firstly proposed for extending OC set with optimal aperiodic auto-correlation property. With such algorithm ten iterative shows that global aperiodic auto-correlation property and capacity are respectively twice and 3713 bigger than original 127-chip Gold sequence. In order to obtain targeted user number's codes in a finite code set with optimal aperiodic cross-correlation property of OC, a novel cross searching algorithm is firstly given. As an example, 49 codes with good aperiodic cross-correlation property (>22) and aperiodic auto-correlation property (>40) are selected from 129 Gold codes.

Keywords Optical code division multiple access · Aperiodic correlation · Differential algorithm · Optical code

2.1 Introduction

Optical code division multiple access (OCDMA) is considered as one of the most promising technologies for the next generation broad-band access network, OCDMA schemes are classified as either incoherent or coherent OCDMA based

C. Peng (✉)
PLA Commanding Communications Academy, 45 No. Jiefang Park Road, Wuhan, China
e-mail: bigroc.chen@163.com

on their operation principle. The incoherent techniques, which work on an optical-power- intensity basis, process OCs in a unipolar (0, 1) manner, and coherent techniques, working on a field-amplitude basis, process OCs in a bipolar (+1, −1) manner all optically [1–3]. Incoherent OCDMA which Optical Orthogonal Code (OOC) is widely used in has been studied since 1990s [4, 5], and there are many constrainers and bounds on size of these codes in the literature [2]. Recently, coherent OCDMA using ultra-short optical pulse has been receiving increasing attention due to the advances in reliable and compact en/decoder devices and detection schemes [1, 6–8], however the OC set appropriate for coherent OCDMA has not yet been created. Gold codes as an effective alternative are widely used for coherent OCDMA at present [3, 9, 10], but Gold which is designed for the optimization of periodic correlation property cannot realize the optimization of aperiodic correlation property for coherent OCDMA [10], so an appropriate method to select OCs for coherent OCDMA is very necessary.

2.2 Definition

Codes for coherent OCDMA based on temporal phase en/decoding is bipolar, code (1, 0) is respectively relevant to (+1, −1). Sequential pulses within a bit interval are copies of a unitary optical pulse, and therefore they are coherent. However, pulses from different bit interval are incoherent since they come from different origin, so aperiodic correlation is appropriate for measuring the codes' property of coherent OCDMA. If code is denoted by (+1, −1), aperiodic correlation is defined as follows.

Aperiodic auto-correlation:

$$R_{xx}(m) = \sum_{i=1}^{N-|m|} x_i x_{i+|m|} \quad m = 0, \pm 1, \pm 2, \cdots, \pm(N-1) \tag{2.1}$$

Aperiodic cross-correlation:

$$R_{xy}(m) = \sum_{i=1}^{N-m} x_i y_{i+m} \quad m = 0, 1, 2, \ldots, N-1$$
$$R_{xy}(m) = \sum_{i=1}^{N-|m|} y_i x_{i+|m|} \quad m = 0, -1, -2, \ldots, -(N-1) \tag{2.2}$$

Bipolar periodic correlation is given and followed as a contrast. Periodic auto-correlation:

$$R_{xx}(m) = \sum_{i=1}^{N} x_i x_{i+|m|} \quad m = 0, \pm 1, \pm 2, \cdots, \pm(N-1) \tag{2.3}$$

Periodic cross-correlation:

$$R_{xy}(m) = \sum_{i=1}^{N} x_i y_{i+m} \quad m = 0, 1, 2, \cdots, N-1$$

$$R_{xy}(m) = \sum_{i=1}^{N} y_i x_{i+|m|} \quad m = -1, -2, \cdots, -(N-1)$$ (2.4)

$$x_i, y_i = 1 \, or \, -1 \quad x_i = x_{i+N} \quad y_i = y_{i+N}$$

Aperiodic and periodic correlations are clearly distinguished from their definitions.

The ratio of aperiodic auto-correlation peak and the maximum of wing (P/W) and the ratio of aperiodic auto-correlation peak and the maximum of aperiodic cross-correlation (P/C) are the measurement of aperiodic auto- and cross-correlation properties respectively [3, 11].

$$P/W = \left(\frac{R_{xx}(0)}{\text{Max}(|R_{xx}(m)|)}\right)^2 \quad m = \pm1, \pm2, \cdots, \pm(N-1)$$ (2.5)

$$P/C = \left(\frac{R_{xx}(0)}{\text{Max}(|R_{xy}(m)|)}\right)^2 \quad m = 0, \pm1, \pm2, \cdots, \pm(N-1)$$ (2.6)

Analyzing the features of aperiodic auto- and cross-correlation, some basic rules are listed as follows:

Proposition 1:
Code (x1, x2, ..., xN) and (xN, xN−1, ..., x1) have the same aperiodic auto-correlation property.
Proposition 2:
Code (x1, x2, ..., xN) and (−x1, −x2, ..., −xN) have the same aperiodic auto-correlation property.

From Proposition 1 and Proposition 2, asymmetric code has the same aperiodic auto-correlation property as three other codes.

Proposition 3:
Code (x1, x2, ..., xN) and (xN, x1, ..., xN−1) have the different aperiodic auto-correlation property. That is the obvious difference between aperiodic and periodic correlation property.

2.3 Differential Algorithm

Codes based on the optimization of aperiodic correlation have not been created and it is impossible to judge aperiodic correlation property of all codes, so it is very urgent to discover an appropriate method for realizing the optimal aperiodic

Fig. 2.1 Flow chart of differential algorithm
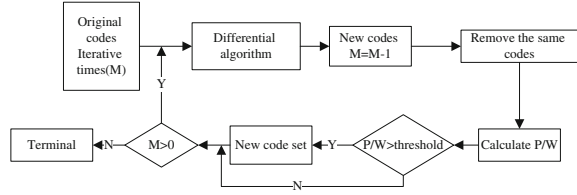
correlation property of OCs. Differential algorithm is proposed to extend code set having good aperiodic auto-correlation property, its fundament is to obtain new codes from original codes using differential algorithm, and then decide whether to save these new codes through judging *P/W* one by one, so the number of new code set can be gradually grown under the control of iterative times (as Fig. 2.1).

The differential formula is defined as follows:

$$b_k = a_k \oplus b_{k-1} \, b_k = 1 - a_k \oplus b_{k-1}$$
$$b_0 = 1 \text{ or } 0 \, k = 1, 2, \cdots\cdots, N \tag{2.7}$$

Where $\oplus$ denotes modulo 2 addition, and a_k, b_k represent for the kth element of original code and new code respectively, so four new codes are generated from one code through the proposed differential algorithm.

Differential algorithm can promise good aperiodic auto-correlation property of new codes through setting appropriate threshold. As an example, 18 Gold codes with 127-chip which are randomly selected from a set of Gold codes are used as original codes, and the targeted threshold of aperiodic auto-correlation property is 50. Figure 2.2 shows that the obtained code number grows with the iterative times. After 10 times, we get 67162 codes whose aperiodic auto-correlation property are all beyond 50. Obviously, capacity is 3713 times bigger than original codes and can be farther grown by adding iterative times. To some extend the number of obtained codes is divergent for this threshold which is receivable for coherent OCDMA system, the result shows that differential algorithm is appropriate to extend code set. Figure 2.3 shows the comparison of P/W between a set of Gold codes which is used for selecting original codes and obtained codes by differential algorithm. Bold line and thin line give the P/W frequency of new codes and the set of Gold codes whose capacity is 127×129 because of proposition 3. Figure 2.4 also shows that P/W of that set of Gold codes and new codes is concentrated in 40 and 80 respectively, so global P/W of OC set is distinctly improved by differential algorithm.

Each user is assigned a unique signature OC which can distinguish itself from other users in the OCDMA system [12], every user has a unique couple of encoder and decoder designed by the assigned code, encoders are used for encoding the optical pulse from every user, and then decoders which are matched for different encoders are used for decoding the encoding signal, so original pulse signal from different users are recovered respectively. In the circumstance of ignoring noise, interrupts among users' decoding signal are dominant for recovering signal pulse,

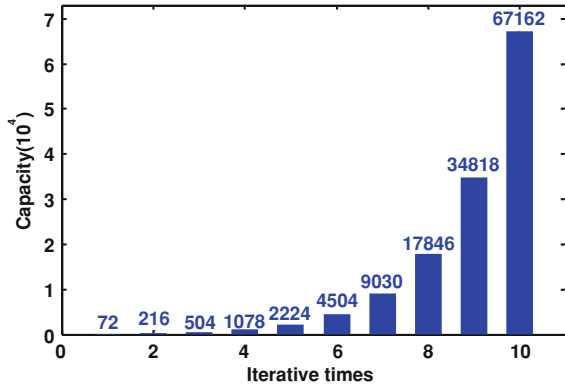**Fig. 2.2** Capacity increased
with iterative times

Fig. 2.3 Comparison
between a set of gold and the
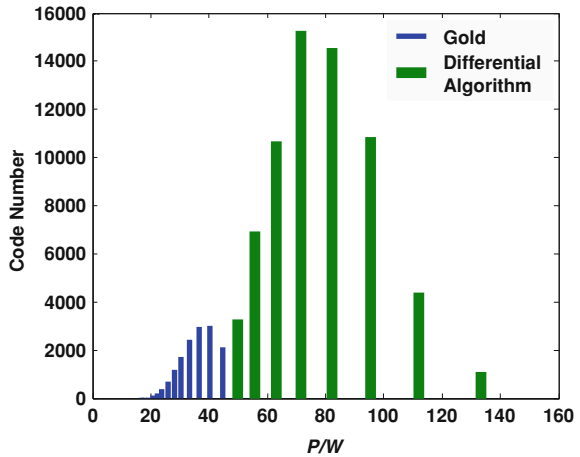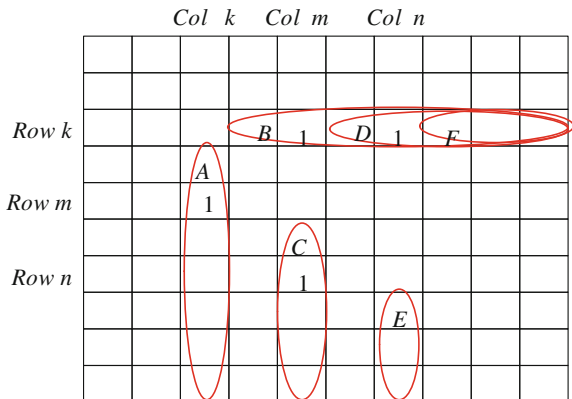result of differential
algorithm

Fig. 2.4 Diagram of cross
searching algorithm

and P/C of different user codes are the measurement of interrupts in coherent OCDMA system, so it is critical for coherent OCDMA to search codes having big P/C of each other.

2.4 Cross Searching Algorithm

In essence, the process of searching codes for OCDMA is to drag a sub-set which has enough big *P/C* of each other from code set having big *P/W*. In this paper, cross searching algorithm is given to do this work. Now we have *M* codes which *P/W* satisfy the demand, the flow of choosing *N* codes whose *P/C* > *V* from *M* codes as follows:

(1) Built a M×M zero-matrix, the value of (i, j) is prepared for the P/C of code i and code j, k = 1;

(2) Calculate the values of P/C between code k and code k + 1, k + 2, …, M, through judging if P/C > V, respectively put 1 or 0 to the relevant position of matrix (k, k + 1), (k, k + 2), …, (k, M) (region B), region A is the same as B, calculating K(1) which denotes the number of nonzero in k-th row and saving nonzero column number (m, n, …). If K(1) > N − 1 continue next step, else k = k + 1, and return to (2);

(3) Calculate the values of P/C between code m and code m+1, m +2, …, M, through judging if P/C > V, respectively put 1 or 0 to the relevant position of matrix (m + 1, m), (m + 2, m), …, (M, m) (region C), calculating K(2) which denotes the number of the same element whose value is 1 between (k, m + 1), (k, m + 2),…, (k, M) (region D) and (m + 1, m), (m + 2, m), …, (M, m) (region C) and saving the same element (n, l,…). If K(2) > N−2, so two codes are chosen, continue next step, else k = k + 1, and return to (2);

(4) Calculate the values of P/C between code n and code n + 1, n + 2, …, M, through judging if P/C > V, respectively put 1 or 0 to the relevant position of matrix (n + 1, n), (n + 2, n), …, (M, n), calculating K(3) which denotes the number of the same element whose value is 1 between (k, n + 1),(k, n + 2),···(k, M) (region F) and (n + 1, n), (n + 2, n), …, (M, n) (region E) and saving the same element (l, …). If K(3) > N − 3, so three codes are chosen, continue next step, else k = k + 1, and return to (2);

(5) Next step is the same as 3 or 4, and terminate until N codes are chosen.

Figure 2.4 shows the process of searching, ellipses show the region of codes needed to be filled with appropriate value 1 or 0 in every step.

The algorithm reduces effectively the unnecessary calculating time by judging the possibility of finding enough optical codes in next steps when every new code is added to the selected code set, and promises that P/C of the code set is beyond threshold by judging the P/C between every new code and the chosen codes.

As an example, 49 OCs P/W > 40 and P/C > 22 are obtained by cross searching algorithm from 129 Gold codes. Figure 2.5 shows the aperiodic

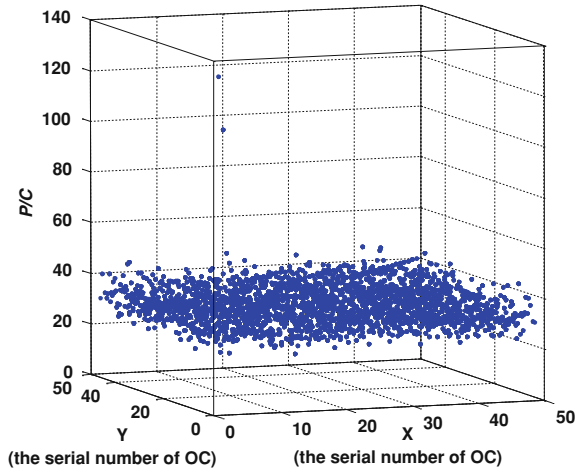**Fig. 2.5** Aperiodic correlation properties of 49 OC

Fig. 2.6 OC subset used widely in OCDMA system

P/C >	17.9					20.5		
P/W >	17.9	30	35	38	42	35	38	42
Number of codes	129	110	82	48	29	33	22	17

correlation, and the value of diagonal denotes P/W of codes, and the value of (x, y) denotes the P/C between code x and code y. We can take a comparison between above result and OC set (as Fig. 2.6) which is widely used in OCDMA system [11], there are only 17 OC that satisfy the P/W > 42 and P/C > 20.5. Our result has the obvious advantage comparing to that, so the differential and cross searching algorithm we proposed are very useful.

2.5 Conclusion

Coherent OCDMA system based on Super Structured Fiber Bragg Gratings (SSFBG) and Planar Lightwave Circuits (PLC) has been the most promising technologies because of the advantages of all optical processing, soft capacity on demand, protocol transparency. An important factor to realize OCDMA system is the OCs' optimization of aperiodic correlation property. The paper obtains three fundamental rules from the definition of aperiodic correlation, and proposes a novel differential algorithm for extending the code set and improving the global aperiodic auto-correlation property, and cross searching algorithm is firstly given for the optimization of aperiodic cross-correlation property. The result shows that

the OC set obtained by our methods is obviously better than the used OC set in capacity and aperiodic correlation property. This improvement means that we establish good base for next work to realize multi-user coherent OCDMA system.

References

1. Taro H, Xu W, Naoya W (2005) Ten-user truly asynchronous gigabit OCDMA transmission experiment with a 511-chip SSFBG en decoder. J Light Technol 24:95
2. Reza O, Kumar P (2006) Spreading sequences for asynchronous spectrally phase encoded optical CDMA. ISIT, Seattle
3. Xu W, Matsushima K, Kitayama K (2005) High-performance optical code generation and recognition by use of a 511-chip 640-Gchip/s phase-shifted super structured fiber Bragg grating. Opt Lett 30:355
4. Zhang J (1998) Application of strict optical orthogonal codes to asynchronous all-optical CDMA networks, ICCT'98
5. Zhang J (1999) Flexible optical fiber CDMA networks using strict optical orthogonal codes for multimedia broadcasting and distribution applications. Trans Broadcasting 45:106
6. Xu W, Kitayama K (2004) Analysis of beat noise in coherent and incoherent time-spreading OCDMA. J Light Technol 22:2226
7. Grunnet-Jepsen A, Johnson AE (1999) Demonstration of all-fiber sparse lightwave CDMA based on temporal phase encoding. Photonics Technol Lett 11:1283
8. Kitayama K, Xu W, Naoya W (2006) OCDMA over WDM PON-solution path to Gigabit-symmetric FTTH. J Light Technol 22:1654
9. Teh PC, Petropoulos P, Ibsen M, Richardson DJ (2001) A comparative study of the performance of seven- and 63-chip optical code-division multiple-access encoders and decoders based on super structured fiber Bragg gratings. J Light Technol 9:1352
10. Koji M, Xu W, Kutsuzawa S et al (2004) Experimental demonstration of performance improvement of 127-Chip SSFBG En/Decoder using apodization technique. Photonics Technol Lett 16:2192
11. Xu W, Koji M, Akihiko N et al (2004) High reflectivity superstructured FBG for coherent optical code generation and recognition. Optical Express 12:5457
12. Ma W, Chao Z, Hongtu P, Lin J (2002) Performance analysis on phase encoded OCDMA communication system. J Light Tech 20:798