Chapter 34 Characterising and Developing Teachers' Knowledge for Teaching Statistics with Technology

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Abstract Developing the pedagogical expertise needed to effectively engage students in learning statistics with technology requires teachers to have a depth of knowledge about statistics, technological tools for exploring statistical ideas, and of pedagogical issues related to teaching and learning statistics with technology. In this chapter, a framework for a specialised knowledge that is called technological pedagogical statistical knowledge (TPSK) is presented and examples of how aspects of this type of knowledge may assist a teacher are provided. Implications for training teachers are described.

1 Introduction

Many international organisations and curricula promote the use of technology in teaching and learning statistics. The GAISE project (Franklin et al., 2005) and the 2008 Joint ICMI/IASE Study Conference call for teachers to have a deeper understanding of statistics and an ability to use technology tools. Although technologies are becoming more prevalent in classrooms, teachers' abilities to use these tools effectively in lessons depends on many factors, including their: (a) statistical knowledge, (b) understanding of how to use technology to explore statistical ideas, and (c) understanding of pedagogical issues related to teaching statistics. These factors impact teachers' decisions, and will ultimately affect whether the use of technology will enhance or hinder students' learning of statistics. This chapter provides a framework that integrates these three factors, a discussion of issues to consider in developing knowledge for teachers of statistics, and examples of teacher education efforts that appear promising.

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1.1 Teachers' Pedagogical Content Knowledge

Teacher education and research has been greatly influenced by Shulman's (1986) pedagogical content knowledge (PCK) as an integration of teachers' content understandings with knowledge needed in teaching. For example, Simon (1995) described important components of a mathematics teaching cycle that include a teacher's knowledge of content, activities and representations, students' learning of particular content, and a teacher's hypotheses about students' knowledge.

Recently, Hill, Ball, and Schilling (2008) have extended PCK to describe mathematical knowledge needed for teaching, which includes constructs such as common content knowledge (content considered to be commonly used by many) and specialised content knowledge (content knowledge needed in the practice of teaching). Groth (2007) used these two constructs to hypothesise what statistical knowledge might be needed for teaching. In particular, he drew upon differences between mathematics and statistics as fields of study and ways of thinking (delMas, 2004; Rossman, Chance, & Medina, 2006; see also Gattuso & Ottaviani, in this book) and gave examples of specialised knowledge needed that was mathematical and non-mathematical. For example, identifying difficulties students may have in constructing algorithms for generating random data is mathematical in nature, but deciding if data collection should include random sampling or random assignment is a non-mathematical task (see Godino et al., and Callingham & Watson, in this book for other analyses of PCK components).

1.2 Teachers' Technological Pedagogical Content Knowledge

The teaching and learning of mathematics and statistics has been greatly influenced by technology (e.g., Ben-Zvi, 2000; Chance, Ben-Zvi, Garfield, & Medina, 2007; Heid & Blume, 2008; Pratt, Connor, & Hunt, this book). Others have also pondered how technology influences teaching and learning and have described technological pedagogical content knowledge (TPCK, see Fig. 34.1) as a type of knowledge needed to effectively use technology to teach specific content (Koehler & Mishra, 2005; Niess, 2005, 2006; American Association of Colleges for Teacher Education Committee on Innovation and Technology, 2008; Mishra & Koehler, 2008).

Niess (2005, 2006) describes four aspects that comprise teachers' TPCK that include a focus on understanding: (a) how to teach a subject with technology, (b) instructional strategies and representations, (c) students' thinking with technology, and (d) curriculum materials that integrate technology. Niess et al. (2009) have since recast these four aspects as being specific to mathematics and have proposed standards and indicators for mathematics teachers' TPCK. In what follows, the notion of TPCK specifically for teachers of statistics is described.



Fig. 34.1 Components of technological pedagogical content knowledge

2 A TPCK Framework for Teachers of Statistics

Rather than describing each of the seven distinct types of knowledge emphasised in the TPCK framework (Fig. 34.1), the framework described here focuses on three important types of knowledge for specifying TPCK for the teaching of statistics. Every aspect of this framework is focused on knowledge needed in the practice of teaching, and thus some pedagogical component is blended in each aspect of the framework, albeit not always an explicit focus. The development of teachers' technological pedagogical statistical knowledge (TPSK) is conceptualised as three layered circles with a foundation focused on teachers' statistical knowledge (Fig. 34.2).

Thus, a teacher's statistical knowledge needed to engage in statistical thinking is the largest of our "sets." This illustrates that a teacher's statistical knowledge and thinking abilities are paramount for their knowledge of anything related to pedagogy or the use of technology in teaching statistics. The inner-most layer represents elements of TPSK and is a subset of the sets in the outer two circles, meaning TPSK is founded on and developed with teachers' knowledge in the outer two sets of technological statistical knowledge (TSK) and statistical knowledge (SK). In addition, developing TSK and SK is essential to, but not sufficient for, teachers having the specialised TPSK.

2.1 Statistical Knowledge and Thinking as Foundational

For many teachers, engaging in statistical thinking is a different process than typically used in teaching and learning mathematics (delMas, 2004; Pfannkuch & Ben-Zvi, this book; Rossman et al., 2006). Thus, it is important to engage teachers as active



Fig. 34.2 Framework for teachers' technological pedagogical statistical knowledge

learners and doers of statistical practices and explicitly develop an understanding of and disposition towards statistical thinking as "an understanding of why and how statistical investigations are conducted and the 'big ideas' that underlie statistical investigations" (Ben-Zvi & Garfield, 2004, p. 7). The remaining five aspects are adapted from Wild and Pfannkuch (1999) and build off Pfannkuch's (2008) description of how to engage teachers in the "game of statistics" (p. 1).

To engage in statistical thinking, teachers should recognise the need for properly collected data to examine situations and make decisions, rather than relying on personal experiences or anecdotal evidence. Teachers should then be able to engage in transnumeration (Wild & Pfannkuch, 1999) as a process of transforming a representation between a real system and a statistical system with the intention of engendering understanding (Pfannkuch & Wild, 2004). Thus, teachers should be able to collect measures, represent them meaningfully with graphs and computed statistical measures, and translate their interpretations back to the context.

Statistics is founded on the fact that variations exist in phenomena and that one must use non-deterministic models and explanations to describe such phenomena with attention to variation (e.g., Moore, 1997; Wild & Pfannkuch, 1999; Ben-Zvi & Garfield, 2004; Shaughnessy, 2006). As statistical thinkers, teachers should notice variation in contexts and use strategies to reduce or eliminate sources of variation in data collection, where possible, and use models that take into account other sources of variation when making predictions or explanations (Pfannkuch, 2008).

With a focus on statistical models, teachers should be able to focus more on aggregate-based reasoning, rather than data as individuals (Konold & Higgins, 2003). Considering data in the aggregate can allow one to characterise group propensities that can include attention to centres, spread, outliers, clusters, intervals, or residuals. In accord with the notion of active graphing used by Ainley, Nardi, and Pratt (2000), teachers should not consider a statistical model such as a graph or a measure of

centre as merely indicating a result of a statistical analysis, but rather as a means to reason with to tell a bigger story of the phenomenon under study.

It is fundamental to integrate a teacher's knowledge of a context in a statistical investigation. The context of data, and the reason for undertaking a statistical investigation, should always influence a teacher's thinking, choice of strategies and methods, representations, statistical summaries used, and interpretations made.

2.2 Technological Statistical Knowledge

Tools such as graphing calculators, spreadsheets, and statistical packages such as *SPSS*, *SAS*, or *Minitab*, have become commonplace in many tertiary and some secondary contexts. Newer educational technologies such as *Fathom*, *TinkerPlots*, and *Probability Explorer* are available and allow for dynamic control over data – meaning that as data changes, representations of that data dynamically update. For example, in *Probability Explorer* and *Fathom*, as data is randomly generated, graphs can be simultaneously "building" so that variability in a distribution can be analysed as sample size increases. Further, several tools allow users to drag data points within a graph and notice the effect on tabular representations and measures.

The availability of technologies for today's work of doing and teaching statistics calls for attention to what specialised knowledge teachers need about technology that is particular to statistics (see Pratt, Connor, & Hunt, in this book). Building from the work of Pea (1987) and Ben-Zvi (2000) provides a useful lens on ways to amplify or reorganise one's work with technology. According to Pea, technology tools are typically used in two different ways: to amplify our abilities to solve problems or reorganise the way we think about problems and their solutions. The notion of amplifier and reorganiser is used as a lens to consider the five aspects of TSK adapted from Chance et al. (2007).

The idea of an *amplifier* is that the tool expedites a process that could be completed without its use. For example, technology tools can be used to automate many activities such as, quickly organising data, generating lists of pseudorandom numbers, computing measures, and generating graphs. By automating the tasks of computing or generating graphs, technology affords an opportunity to focus on conceptual understanding and more time to engage in exploratory data analysis (Konold & Higgins, 2003).

Automation in technology also facilitates a person's capability to visualise abstract concepts and serve as a *reorganiser*, such as taking advantage of dynamic dragging capabilities to illustrate the effect of an outlier on a measure of centre in a univariate distribution. Through dynamic features of dragging, linking of multiple representations, and overlaying measures on graphs, technology can be used in ways that extends what we may be able to do without technology to help reorganise and change a student's or teacher's statistical conceptions. For example, overlaying statistical measures such as means and regression line on a graphical representation can help change the way teachers and students conceptualise these measures in relation to a bivariate distribution, particularly since the statistical measures update as data is changed

by dragging points in the graph. This visualisation is not possible without technology and can provide a way of reorganising one's conceptions of bivariate distributions.

Technology can be used to view and design simulations that can enhance the study of random processes and statistical concepts such as sampling distributions (Chance et al., 2007). The flexibility of many simulation tools allow for: (a) algorithms and models to be used to input the properties of a theoretical distribution that would control the pseudorandom number generation, (b) controlling parameters such as sample size, and (c) displaying graphs generated in real time. A teacher who uses technology in their own statistical investigations will have first-hand knowledge of the power of using simulations as a pedagogical tool.

Technology also facilitates the use of large messy data sets gathered and accessible through the Internet (see Hall, in this book). Longstanding projects such as Census at School and newer projects such as Experiments at School (Connor, Davies, & Holmes, 2006) demonstrate an advantage of the Internet to gather and access data of interest to students. Knowing how to gather real data from the Internet and how to transform it into usable data in a particular piece of software or downloading it into a graphing calculator is a useful skill. In addition, teachers should develop an ethical disposition concerning the use of public data, citing sources, and being wary of data that has already been transformed by others with particular agendas. At the same time, teachers also need to consider characteristics of data sets that can be used to bring different statistical ideas to the fore. For example, data sets with a skewed tendency are typically good for investigating the usefulness of different measures of centre.

Chance et al. (2007) support the use of technologies such as discussion forums, Wikis, interactive whiteboards, and self-assessment software to promote collaboration and student involvement. While these tools can help in course management and engage students in learning and assessment, we also support a focus on collaborative tools for data collection, analysis, and visualisation. Examples of collaborative tools to collect or simulate data include networked graphing calculators (e.g., TI-Navigator[™] systems), Experiments at School (www.experimentsatschool.ntu.ac.uk) GoogleDocs spreadsheets (docs.google.com), and networked computing and simulations tools such as HubNet, which uses *Netlogo* (www.ccl.northwestern.edu/netlogo/). Such technologies can be used to have individuals contribute data from local simulations to be aggregated as a class. Collaborative tools can promote a community approach to generating and analysing data that can foster both small-group investigations at a local machine and whole-group discussions to consider the phenomena in an aggregate.

2.3 Technological Pedagogical Statistical Knowledge

The ultimate goal in the preparation of teachers of statistics is to develop a specialised subset of knowledge for teachers representing TPSK (Lee & Hollebrands, 2008a). This knowledge encompasses TSK and SK (see Fig. 34.2). While pedagogical issues and implications may be implicit in aspects of teachers' SK or their TSK, pedagogy comes to the fore when considering the particular subset of TPSK.

TPSK can allow teachers to consider how students think and reason about statistics with and without technology. This implies that they have the specialised content knowledge that Groth (2007) hypothesised as particular to statistics teachers, and that they are familiar with common ways that students may approach statistical tasks. For example, teachers should know that students often initially consider data representing a characteristic of an individual (e.g., Sally's height), and have difficulty viewing data as an aggregate where they consider the entire distribution (Konold & Higgins, 2003). They should also know how technology can promote different reasoning that may facilitate a transition to aggregate-based thinking. For example, when examining distributions graphically, students can characterise the data with such ideas as "bins" (intervals) (Rubin & Hammerman, 2006) and a "modal clump", that is, a small range of data that contains many data points within a data distribution (Konold & Higgins, 2003), rather than initially focusing on computing statistical measures.

Teachers can also use TPSK to consider how technology can facilitate and support students' statistical thinking, and in essence become designers of a conceptual space for students to learn powerful ideas (Pratt, 2008). For example, teachers would know how to use *a dynamic statistical tool* to highlight a region of data in a distribution, compute the number and proportion of data within the region, and use this process to support students' natural tendencies to describe a distribution's centre using a "modal clump", to complement a formal measure of centre. This aspect of TPSK necessarily encompasses components of teachers' TSK (e.g., automation of graphs, data exploration, visualising concepts) and SK (e.g., transnumeration, consideration of variation).

Teachers are continually planning lessons for students and evaluating and choosing curricula materials for use in their classroom. In these contexts, teachers are again working as designers (Pratt, 2008) and using TPSK in their daily work. They should be able to draw upon elements of their TSK and SK that facilitates an appropriate use of technology that can positively affect students' learning of statistics. For example, consider a teacher in the midst of a lesson on least squares regression where she ascertains that her students are having difficulty understanding the concept of a residual. She uses the dynamic dragging capabilities in *Fathom* to provide a demonstration of the residual plot and how it relates to the position of a moveable line overlaid on a scatter plot (Fig. 34.3). The teacher uses vertical translations of the moveable line to illustrate how the residual plot responds if she places the moveable line entirely above all data points. She anticipates this may help students understand why the corresponding residuals would have a negative numeric value.

3 Developing Teachers' TPSK

There is an increasing trend for teacher preparation programmes to include a focus on the use of technology for teaching mathematics (e.g., Powers & Blubaugh, 2005) and research that suggests that mathematics teachers may struggle in learning how to use technology in their teaching (Zbeik & Hollebrands, 2008; Niess et al., 2009). Both Niess et al., and Zbiek and Hollebrands propose a model of how mathematics teachers may develop as technology-using teachers, with early stages including



Fig. 34.3 Visualising residuals in Fathom

playing with the technology and using it for their personal use for doing mathematics. Latter stages include integration of tools and a focus on teachers' assessment and evaluation of how technology may be impacting students' learning.

The responsibility for developing teachers' TPSK does not only lie with teacher educators. Mathematicians and statisticians who teach statistical content courses are very influential, because this is where teachers can develop their SK and TSK as learners of statistics. Thus, courses and textbooks that include the use of educational software such as *Fathom*, rather than only statistical packages such as *MiniTab*, are needed (e.g., Rossman, Chance, & Lock, 2001). Materials for use in pedagogy-focused courses can also be helpful.

To help develop teachers' TPSK, materials should engage teachers in statistical thinking as doers of statistics with a variety of technology tools and then provide opportunities for teachers' to reflect on their own learning and to consider pedagogical issues. The work of Graham (2006), Ben-Zvi (2008), Lee and Hollebrands (2008b), Pratt (2008), and Lee, Hollebrands, and Wilson (2010) have similar aims and focus on the use of technology and pedagogical considerations in teaching and learning statistics.

For example, Table 34.1 provides two tasks that could be used to develop teachers' TPSK. Both tasks are posed to teachers after they have explored a probability context by designing and using simulations in their own learning.

Because many teachers struggle with focusing on students' thinking, the use of artefacts of practice, such as video cases and students' work, can be helpful to engage teachers in discussions about pedagogical decisions that may affect students' understanding (e.g., Groth, 2008; Wilson, 2008). The use of these artefacts can help teachers make sense of students' work and develop an understanding of how students reason about statistical ideas (Hollebrands, Wilson, & Lee, 2007; Groth, 2008; Wilson, Lee, & Hollebrands, 2011).

Table 34.1 Tasks to develop TPSK

Example 1. (Graham, 2006, p. 183)

- a. How could you use the simulation possibilities of a graphics calculator to help learners test the statement "a six is hard to get?" Hint: You need to generate a number of dice rolls and see whether the outcome "6" is under-represented.
- b. Some people might claim that the calculator provides a more satisfactory microworld than a spreadsheet for investigations like these. See what you think after trying this task.

Example 2. (Lee, Hollebrands, & Wilson, 2010, p. 114)

- Suppose a university gives away token spirit gifts to all incoming freshmen. They get to randomly choose one of three cards. Each card displays a different gift: keychain, window decal for a car, t-shirt with college logo.
- a. Explain how you would help students use the graphing calculator to simulate this context. Explicitly describe what the commands represent and how the students should interpret the results.
- b. Suppose that after conducting the above simulation for 30 freshmen choosing a card, a student gets a result that a t-shirt was chosen 20% of the time. The student is surprised and claims that this proportion is too low from the expected 33.3%. What might this student be misunderstanding? What are some questions you could pose and further simulations you could suggest that might help this student?

4 Conclusion

Preparing teachers of statistics to use technology appropriately in their classrooms is a difficult and important task. While statistical knowledge is foundational in the TPSK framework, it is also important to engage teachers in opportunities to develop technological statistical knowledge and TPSK. This implies that the community of those involved in educating teachers of statistics (e.g., mathematics educators, statistics educators, mathematicians, statisticians) should also have opportunities to develop their own TPSK and join together in efforts to create our next generation of teachers of statistics.

Acknowledgements The preparation of this manuscript was supported by the National Science Foundation (DUE 04-42319 and DUE 08-17253; Online: ptmt.fi.ncsu.edu).

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