Chapter 15
Complementing Mathematical Thinking and Statistical Thinking in School Mathematics

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Abstract The introduction of statistics into school curriculum within the mathematics subject poses multifaceted problems to mathematics teachers. This chapter first discusses the relevance of developing mathematical and statistical literacy in schools, and secondly reflects on some current recommendations to teach statistics in the school mathematics and challenges faced in the training of teachers. Then the chapter underlines differences between mathematical and statistical thinking and suggests that, taking account of their specificities, it is possible to generate teaching strategies that allow the harmonious development of both mathematical and statistical thinking in school. Some implications for teacher training are finally included.

1 Introduction

In the last few decades of the twentieth century unprecedented innovation in society and, in particular, globalisation catalysed by modern telecommunications did justify a new perceived complexity of reality, enhancing “the central importance of mathematics and its applications in today’s world with regard to science, technology, communications, economics and numerous other fields” (United Nations Educational, Scientific and Cultural Organisation [UNESCO], 1997). The basic aim of mathematics in this changing society had already been formulated, in the United States of America, within the National Council of Teachers of Mathematics

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(NCTM) Standards (1989), where the concept of "mathematical power" was presented as the notion of empowering the students to be able to apply their mathematical knowledge, concepts, and ability in problem solving, communication and reasoning.

In 2001, "use mathematics to solve problems and communicate" (Stein, 2001, p. 17) was listed as one of the 16 Equipped for the Future (EFF) standards needed by adults to effectively carry out their different roles in society. The use of knowledge in concrete situations – where the active participation of individuals is required – stresses the concept of "competence". For mathematics the curriculum shift from content topics to competences has many consequences from the pedagogical point of view both for teachers and for students, and implies a new teaching/learning style relying particularly on problem-posing and problem-solving teaching methods. In particular, giving evidence of workplace needs, Steen (2003) regarded mathematical thinking as "an essential component of virtually every competency. Reasoning, making decisions, solving problems, managing resources, interpreting information, understanding systems, applying technology – all these and more build on quantitative and mathematical acumen" (p. 56). When the information to be dealt with is quantitative in nature and keen quantitative discernment is required, statistics and statistical thinking play an important role in the mathematical curriculum.

This chapter aims to emphasise the necessity of complementing statistical thinking and mathematical thinking in school and generating didactic strategies allowing statistics and mathematics to evolve together, in a harmonious way.

2 Why Statistics is Taught in School Mathematics

To be part of a modern society in a competent and critical way requires citizens to know and to interpret collective/social phenomena in a broad sense, and understand the variability, dispersion, and heterogeneity which cause uncertainty in interpreting, in making decisions, and in facing risks. To pose and solve problems in everyday life may require data collection and the ability to analyse the data in order to get information to be interpreted and used in suitable ways.

However, in reality citizens will seldom have the opportunity to control all stages of the statistical process of inquiry, particularly when they have at their disposal only data collected, organised and interpreted by others to address others' aims. In this case, statistical competences and thinking become more and more important as they encourage caution before using those data in a superficial way. That is why modern citizens require both basic knowledge of statistics and statistical concepts, and also statistical thinking.

The role of data, statistics and probability in school curriculum has been recognised in the Organisation for Economic Co-operation and Development

(OECD) study titled Programme for International Student Assessment (PISA). Mathematical literacy is one of the three domains assessed by PISA in order to measure how well young adults, at the age of 15, are prepared to meet the challenges of today's knowledge society. According to the project:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (OECD, 2006, p. 72)

For PISA assessment purposes the mathematical content that a person might utilise in solving a problem has been organised by four overarching ideas: *space and shape*, *change and relationships*, *quantity* and *uncertainty* (OECD, 2006). The three first ideas form the heart of any mathematics curriculum, but it is not the same for the fourth. The recognition from OECD that dealing with uncertainty is essential in everyday life is obviously of primary importance in promoting the teaching of statistics and elements of probability theory in school mathematics.

3 Teaching Statistics in the Mathematics Classroom

Statistics is appearing more and more in school curricula; in some countries, statistics has recently even entered the curricula of elementary schools. The situation in various countries is described in the first chapter of this book. Statistics in schools is linked to mathematics so mathematics teachers are responsible for its implementation.

Curriculum developers suggest a data-oriented approach to teaching statistics (Moore, 1997; Burrill & Camden, 2005) where students should formulate research questions; design investigations; collect data using observations, surveys, and experiments; describe and compare data sets; and propose and justify conclusions and predictions based on data.

The GAISE project, for example, has developed useful guidelines for statistics education (Franklin et al., 2005). However, as discussed in the Joint ICMI/IASE Study Conference (Batanero, Burrill, Reading & Rossman, 2008), these recommendations are seldom followed and doing statistics too often becomes synonymous with doing computations and following protocols. Consequently, students finishing high school understand very little statistics and are usually unable to utilise it in a critical way.

The problem is that the teachers generally have no preparation for teaching statistics, little knowledge about statistics and almost never any training in statistics education. They need a framework for understanding statistics, so that they can understand where their students are coming from and where they are going (Ottaviani, Peck, Pfannkuch, & Rossman, 2005). Although there has been a lot of progress in the implementation of statistics in the school curriculum, statistics education for future teachers is almost non-existent.

The situation is serious for elementary teachers who have little or no experience in this field, and often demonstrate little interest in mathematics although they have to teach it. The situation is not much better for secondary teachers. Their mathematical knowledge is more important but in some ways, particularly if mathematics is seen in a formalistic view, this may even hinder their grasp of statistics. Most trainee secondary teachers will follow a course in statistics but very few teacher training programmes include the didactic of statistics. In fact, mathematic educators often casually admit their lack of qualification in the subject.

In addition to gaps in teachers' statistical knowledge, negative attitude and beliefs towards statistics complicate the situation. "Negative attitudes are linked to perceived difficulty, lack of knowledge and overly formal learning experience" (Estrada & Batanero, 2008, p. 5). Meletiou (2003) argued that beliefs about the nature of mathematics affect instructional approaches and curricula in statistics, and act as a barrier to the kind of instruction that would provide students with the skills necessary to recognise and intelligently deal with uncertainty and variability. Although the teaching of mathematics has undergone many changes and proposes a constructivist approach, long-held beliefs and attitudes of teachers are difficult to change. Statistical concepts linked to context should be approached as social constructs, following the way suggested by the data-oriented approach. In reality, concepts are too often presented to students without any links to the real-world context or at the most within artificial examples and using a traditional and procedural approach that in many cases meet students' and parents' expectations.

Obviously, knowing the theory of statistics is not enough to teach it. Teachers must have the opportunity to develop their own statistical thinking. The education of pre-service and in-service teachers has to be taken seriously. According to Batanero (2008), initial and continuing teacher training courses for mathematics teachers need to be redesigned completely. Future teachers must experience the same activities proposed for students and experience the same difficulties, but obviously teacher knowledge needs to be broader and deeper than that of the students they are teaching. In fact, many teachers have no experience with data analysis and do not understand the role of variability and the idea of distribution, which are key concepts for the development of statistical thinking.

Today, teacher training is mostly under the auspices of mathematics educators. However, statisticians involved with statistics education, and statistics educators must cooperate and be involved in developing resources for teachers including high-quality teaching materials that, promoting the issue of teaching statistics, could help motivate students to learn mathematics. To achieve these goals it is fundamentally necessary to describe characteristics of statistics and to identify differences and similarities between mathematical thinking and statistical thinking.

4 Statistics and Statistical Thinking

If statistics is different from mathematics, what is statistics? As is the case for any science, to define statistics is difficult. In recent years it has been recognised that:

> Statistics has developed from two disciplines: The mathematical study of probabilities and chance events and the scientific attempt to draw conclusions from data in the face of inevitable error and imprecision. Modern statistics does not simply apply mathematical results to determine the properties of particular statistical methods; it includes a concern for discerning, describing, and confirming patterns and relationships in data. (Thisted & Velleman, 1992, p. 41)

In fact, “statistics makes a heavy and essential use of mathematics, yet has its own territory to explore and its own core concepts to guide the exploration” (Cobb & Moore, 1997, p. 814). It “is a subject whose goal is to solve real-world problems” (Moore & Cobb, 2000, p. 617). Statistics may be considered both as a discipline in itself and as a technique: “A special technique suitable for the quantitative investigation of mass or collective phenomena, those phenomena (…) whose measurement requires a collection of observations” (Gini, 1966, p. 17).

The process of statistical investigation begins with some study questions providing a basis for the design used to produce data, it goes on with the collection of the data, their exploration and description, and eventually formal inductive inference is required if conclusions are needed about the population or process from which the data were drawn. The interpretation of the results coming from the data is the crucial point where statistics comes in touch again with the questions that started all the process. Only at this point does it become evident whether both the statistical methods used and the statistical reasoning followed were effective in solving the problems giving rise to the study. The investigative cycle: from problem, to data (collected, analysed and reported), to problem forms the core of statistical thinking. In this vision of statistics, there are concepts – such as centre and variability – and measures of concepts – such as arithmetic mean, median, mode and standard deviation, interquartile range, range – and not just numbers and formulae.

To debate the differences between statistics and mathematics is important for statistics educators who need “to carefully define the unique characteristics of statistics and in particular the distinction between statistical literacy, reasoning and thinking” (Garfield & Ben-Zvi, 2007, p. 380). Each of these three capabilities can be differentiated according to the level of statistical tools and concepts people understand and the connections people are able to make among them. The focus in this chapter is on statistical thinking.

To simplify the comparison with mathematical thinking, this chapter uses the definition of statistical thinking proposed by Scheaffer (2003): “Data analysis and statistical thinking … develop knowledge, beliefs, dispositions, habits of mind, communication capabilities and problem solving skills that people need to engage effectively in quantitative situations arising in life and work” (pp. 146–147), particularly in those situations involving processes and their variation.

An active group of educators, psychologists, and statisticians have studied and examined the change in the importance given by the statistics instruction when evolving from the statistical techniques, formulas, computations and procedures towards conceptual understanding of statistics (Garfield & Ben-Zvi, 2004), and have also made connections between the research results and practical suggestions for teachers (Garfield & Ben-Zvi, 2008).

5 Differences Between Mathematical Thinking and Statistical Thinking in School

In some ways mathematical thinking and statistical thinking may appear contrary, but when we underline their differences, we will see that they may support each other. Where mathematics exploits deductive reasoning, statistics uses more inductive reasoning. While mathematics promotes abstraction, statistics insists on interpretation in context. Variation and measurement are dealt with differently in the two disciplines. In summary, reasoning in mathematics and statistics is different. A more comprehensive picture of the situation can be found in Rossman, Chance, and Medina (2006) and Scheaffer (2006).

Although, more and more mathematics educators encourage a constructivist approach for mathematics in the classroom, teaching too often is dominated by presenting deterministic procedures even if most curricula propose a broader view of mathematics. "One question has one answer". Traditional teaching is all too often focused on developing procedures to solve closed problems. Even in the so-called open-ended problems, the solutions are often predetermined. This misleads students who look for "what the teacher wants". Mathematics is about logical and deductive reasoning, modelling, optimising, and proving results that come logically from axioms and definitions. Although not all mathematics teaching in schools follows this line, it is too often procedural, allocating more space to calculation than to understanding. However, more and more mathematics educators and researchers are rejecting the traditional approach and proposing that learning mathematics should develop the ability to create mathematical models of real phenomena, pose hypotheses and verify them using mathematical tools (Sierpinska & Kilpatrick, 1998). In statistics, the same question with the same data may lead to different ways of analysing and different solutions that are equally defendable. This requires inductive reasoning, working with randomness, dealing with counterintuitive results, drawing uncertain conclusions, and interpreting results.

Mathematics and statistics are different in the ways that they use numbers. Mathematics mostly deals with numbers, their operations, generalisations and "abstractions", while for statistics numbers are "data linked to a context", which is essential to statistical reasoning as well as to mathematical modelling. When doing statistics, one must know the nature of data, and where and how they are produced, to be able to go on with the analysis and to draw some conclusions. Mathematics, on the contrary, may rely on context for motivation in the classroom, or as a source of research problems, but its goal is abstracting, finding patterns and generalising. The context has to be put to one side to grasp the model or the structure. To synthesise: "In data analysis the emphasis is on answering real questions rather than trying to fit those questions into established theories" (Scheaffer, 2003, p. 145).

Variability and variation are found in mathematics and in statistics but with a different sense. In the mathematics classroom students study the dependence of one variable on the other, and the form of the link between the variables. In statistics variability, that is the propensity of the observations for one data set to change, is a fundamental idea supporting the concept of distribution. Looking at averages without taking account of variability (spread) is useless and will not lead to the understanding of the distribution, thus missing the whole pattern.

Furthermore mathematics and statistics have a different approach to measurement. In mathematics, measurement goes with spatial configurations, and their transformation, and abstraction. For example, at secondary school in a geometry problem there is no need for rulers to show that two sides of a triangle have equal length; the equality of the length can be deduced from hypotheses, definitions, and theorems. Although a figure may help understanding or finding the proof, its measures do not need to be accurate and can be assessed approximately. Because statistics is mostly about understanding, measuring and describing the real world, taking valid measurements is crucial. In any investigation, the study question has to be well formulated and the data have to be accurate.

6 Advantages of Doing Statistics and Mathematics Together in School

Despite the differences between statistical thinking and mathematical thinking, there are certain advantages in studying statistics in the mathematics classroom. First, statistics can stimulate motivation and develop problem-solving abilities such as posing questions, analysing, representing and communicating quantitative information. If well chosen and close to students' interests, context, which is essential to statistics, often has a positive effect on students' motivation and involvement also in mathematics. According to Kranendonk (2006), students playing with real data that makes sense connect with these data, and they get curious and often go beyond what they were asked to do. Finding a new interest can modify a negative attitude towards mathematics. Using context also agrees with new curricula in mathematics that advocate problem solving imbedded in real-world situations.

Second, much of statistics involves posing questions and finding ways to answer them. A problem leading to the collection of data and analysis, even if elementary, will enrich a child's mathematical thinking. At the beginning, there is no need for complex mathematics, but only for the ability to classify and group. The ability to formulate a question and to be critical about it can be practised even by kindergarten children (Schwartz, 2006) and will transfer to the study of mathematics; is it not said that, in problem solving, when the question is posed the problem is almost solved?

Third, statistical analysis is not a linear process. After collecting and grouping data, analysis comes next. Comparing groups, looking at the characteristics of the distribution, identifying clusters, outliers, examining the differences in the medians, means, modes and measures of spread may suggest a "rerouting" in the analysis of data. To go back and forth to find a solution or a proof can also be very helpful in mathematics, but in the classroom it is unusual and not often shown. Instead, the result is traditionally exposed in a straightforward manner without revealing the trials and errors preceding the optimal solution shown on the board, as it would come from some magical inspiration leaving the students helpless.

Finally, the construction of representations is essential in the study of data. Not only does the representation have to be adequate and complete, but it helps to visualise statistical distributions and give evidence to relationships among variables. Different representations may also lead to a different grasp of the distribution. This is useful also in mathematics where a graphical representation is a necessary prerequisite to modelling, even if "the standard mathematical models ignore data production" (Cobb & Moore, 1997, p. 807).

The competence to communicate mathematical results is nowadays part of the mathematics curricula recommendations. In statistics, interpreting and communicating the results to answer the original question follows statistical analysis. It requires convincing with "numerical" arguments placed in their context and is completed with a discussion of the various possibilities investigated, thus assuming (more or less consciously) variability and probability. Again, the development of this competency may benefit both disciplines.

The points underpinned above are about conceptual understanding and thinking both of statistics and of mathematics. Besides, it should not be forgotten that when going through various statistical procedures, a lot of mathematics is applied. From elementary arithmetic (especially proportional reasoning) to advanced functions (e.g., the least squares method), many examples of mathematical concepts and tools are employed while doing statistics and mathematical learning can only profit from this use (Gattuso, 2006, 2008). Teachers must be aware of the benefits of making statistics part of their mathematics teaching but at the same time be familiar with the specificity of each discipline.

7 Implications for Teacher Training

An important key to the development of statistics teaching is teacher training. Well-prepared teachers will willingly include statistics in their teaching. With adequate training, teachers will be more confident and they will be able to encourage students to speculate and explore phenomena, create their own data representations, make and test their own conjectures, use appropriate technological tools, and spend time on discussion and reflection instead of limiting the students to the practice of procedural skills and execution of calculations.

Teachers surely need to acquire statistical knowledge and develop their statistical thinking, but they also need training in the didactic of statistics to be able to follow their students' learning and reasoning and be able to spontaneously take advantage of classroom situations to promote student learning. The didactic of statistics will introduce teachers to misconceptions, difficulties, and common errors involved in learning statistics and will propose ways to handle them, thus allowing teachers to develop the self-assurance needed to teach adequately.

Also, it is important to pay attention to the teachers' concerns about leaving out some mathematical content by assuring and showing them that, while doing statistics, they are really doing a lot of mathematics. It is also necessary to match mathematical concepts to their applications in statistics so that one supports the development of the other (Dunkels, 1990). Statistics can contribute to the learning of mathematics by introducing mathematical concepts in realistic and motivating contexts. Measurement of phenomena (such as bullying, free time, fertility, poverty), proportional reasoning and percentages, graphical displays, averages, data modelling, and inductive reasoning are all points of contact and tension between statistics and mathematics in school (Biehler, 2008). Research is necessary to understand how to transform a possible uneasy junction of such different disciplines into a fruitful one. This may require statistics educators to work side by side with mathematics educators, respecting each other and showing how concepts and knowledge of the two disciplines may evolve together in the classroom in a harmonious way (Ottaviani, 2008).

During their training in statistics teachers should also be exposed to the use of technological tools. Technology, in fact, can assist students in "doing" and "seeing" statistics and in reflecting on data. Different kinds of statistical tools exist. Some are useful to visualise data and to analyse it in a simple way, some are more suitable for developing an understanding of data and data exploration, and others are more useful for understanding concepts connected to probability distributions. Besides this, the Internet offers a large set of downloadable data to support exploratory data analysis and to assist in understanding variability (Garfield & Ben-Zvi, 2004). By navigating the Internet it is possible to find resources for teachers to use in classrooms or improve their knowledge of statistics and resources for those training the teachers. In particular, the International Statistical Literacy Project (ISLP), under the umbrella of the International Association for Statistical Education provides an online repository of national and international activities to disseminate statistical thinking (www.stat.auckland.ac.nz/~iase/islp/). Through this, teaching statistics in school mathematics has the added bonus of students acquiring a greater familiarity with technological instruments used in everyday life.

Teacher training should also include discussion on assessment methods. Mathematics teachers are used to utilising multiple choice, "right or wrong" answer or short answer questions, thus focusing on accuracy of computation, correct application of formulas or correctness of graphs and charts. These kinds of questions are not useful when statistical thinking is involved. To get information about students' statistical reasoning processes requires the teachers to identify assessment methods that can reveal student understanding of basic statistical concepts such as variability, visual representation of data, centre and spread (Gal & Garfield, 1997). The importance of assessment is evident when we notice that teachers are more and more motivated to do a better job with statistics as long as assessing the achievement of statistical curriculum is required.

The support of statistics educators and practising statisticians for mathematics teachers is essential to help them cope with their new role as statistics teachers.

8 Conclusion

The "marriage" of statistics and mathematics in schools is difficult particularly due to the school teachers' general lack of statistical knowledge that makes it hard for them to develop their own statistical thinking. In fact, this chapter shows that mathematics and statistics are different, at least, in the way that reasoning takes place, in the way they use numbers, in the way that variability and variation are taken into account, and in their approach to measurement. However, there are good reasons for mathematics school teachers to teach statistics in their classes, such as while students are doing statistics they are really doing a lot of mathematics, and students with a negative attitude towards mathematics can find a new interest. For statistics to be taught in an adequate way in school mathematics will take a long time. A lot of research activity needs to be carried out by statistics educators in collaboration with mathematics education in order to offer mathematics teachers appropriate instructional resources and strategies. There is no doubt that to have statistical thinking diffused in society is fundamental so that both pre-service and in-service mathematics teachers have to receive high-quality training in statistics.

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