Chapter 9
Secondary Teachers' Beliefs on Modelling in Geometry and Stochastics

Boris Girnat and Andreas Eichler

Abstract This chapter presents two combined qualitative studies on secondary teachers' beliefs on modelling in geometry and stochastics. The teachers' views on modelling, which are described in detail, differ considerably in both parts of mathematics from a pragmatic approach to modelling. In case of elementary geometry, a conflict with a traditional view on geometry is detected and elucidated. In case of stochastics, the need for data and real situations are revealed as controversial. The chapter ends with the invitation to analyse the parts of factual school mathematics including teachers' beliefs more specifically, that is, to compare applied-oriented aims with other didactical requests, and to design tasks which are supposed to be a response to the teachers' hesitations on modelling analysed before.

1 Teachers' Beliefs and Individual Curricula

"That what teachers believe is a significant determiner of what gets taught, how it gets taught, and what gets learned in the classroom" (Wilson and Cooney 2002). Based on this rationale, teachers' beliefs have become a vivid research focus of mathematics education (Philipp 2007). In this chapter, we will present the core results of two combined studies concerning secondary teachers' beliefs on applications in geometry and stochastics, respectively. The studies rest upon small samples (less than 18) and follow a qualitative methodology based on in-depth interpretations of semi-structured interviews. All the teachers consulted are employed at German higher-level secondary schools (Gymnasien).

The studies share the same theoretical framework and research question, namely, the reconstruction of teachers' individual curricula on teaching geometry and stochastics. Individual curricula are supposed to possess similar constituents and the same purpose as written curricula to guide the instructional practice to specific goals of

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education. Intended curricula are the "blue prints" of individual curricula, that is, the teachers' instructional intentions whether they can be implemented in classroom practice exactly or not (Eichler 2007). Hence, our questions were directed to teaching goals, teaching methods, and the students' learning. The studies suggest that applied-oriented goals are seen as subordinate ones among others and that there are significant differences in elementary geometry, analytical geometry, and stochastics. These differences, but also some unsuspected similarities between analytical geometry and stochastics, lead to the decision to present these studies combined and to deliberate on the special position of elementary geometry.

2 Theoretical Background, Data, and Evaluation

The design of the interviews and the interpretation of the data are based on the *research programme of subjective theories* (Groeben et al. 1988), which is intended to reconstruct the background theories that professionals use to manage their job-related behaviour. According to this background theory, the interviews are designed as semi-structured ones. They start with open questions on the professionals' intentions and knowledge and lead to confronting questions, derived from literature, subsequently. All the interviews were held and transcribed by the authors. The participants were not chosen by specific criteria, but volunteered for the interviews in response to an impersonal invitation. In our cases, we began with open questions on the teachers' goal of education and confronted them with divergent opinions (cf. 3). To summarise, the interviews involve questions about (1) goals of the mathematics curriculum, (2) goals of the geometry and stochastics curriculum, (3) content of the geometry or stochastics curriculum, and (4) students' learning and teaching methods. The evaluation process is guided by a so-called *dialogue-hermeneutic methodology* (Groeben and Scheele 2001), which contains two steps: Firstly, the interpreter explicates the central subjective notions by "defining" paraphrases and links between them similar to a concept map and reconstructs the argumentative structure of each interview by a hierarchical diagram, containing top-level goals of education on the highest level and derivative goals, contents, and methods on lower levels (Eichler 2007, cf. Fig. 3 for an example). These diagrams are intended to express the implicit means-ends relations the teachers take for granted when structuring their classroom practice. Secondly, the paraphrases, concept maps, and diagrams are discussed with the teachers; and the teachers have the chance to approve, to dismiss or to change the researcher's suggestion. This is the dialogical part of the methodology, which is intended to enforce the reliability.

3 Applications and Model Building

We now describe the topics used for confronting questions on the applied-oriented aspects during the interviews. We assume that the main questions of teaching applied-oriented mathematics are as follows: What is the *relationship*

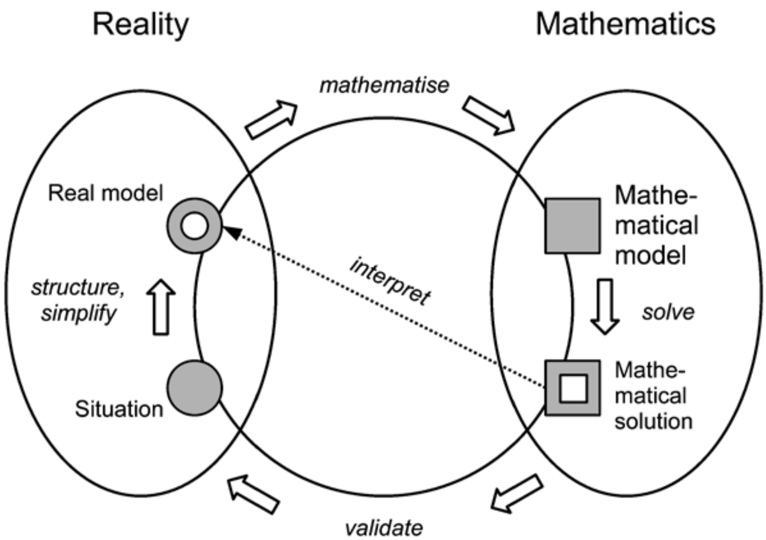

Fig. 9.1 Modelling cycle used in our studies

between *general* mathematical concepts or theories and *specific empirical knowledge* on singular situations (Kaiser 1995)? Is mathematics seen from a *static* or *dynamic* point of view (Hersh 1986)? In how far do the teachers' ideas match the concept of modelling, which is often seen as "one of the main components of the theory for teaching and learning mathematical modelling" (Kaiser et al. 2006, p. 82)?

The concept of modelling is typically explained by one of the common *modelling cycles* (Kaiser 1995). To leave the teachers room for personal perspectives, we used a simple version of these cycles (Fig. 9.1). In addition to *conceptual* topics, we were also interested in *normative* aspects. These opinions were analysed against the background of Kaiser-Meßmer's classification (Kaiser-Meßmer 1986): The extremities are seen in the *pragmatic* and the *scientific-humanistic* approach. Whereas the latter emphasises mathematical concepts, theories, and taxonomies, using real-world situations as subordinate tools to develop mathematical concepts based on manifold realistic associations, the pragmatic view stresses empirical knowledge and a reflection on the relationship between mathematics and reality on a meta-level: (1) Utilitarian aims: The real-world situation and the gain of empirical knowledge are taken seriously. (2) Methodological aims: It is a goal to achieve general competencies and meta-knowledge about applying mathematics. (3) Scientific aims: Applied mathematics is to be perceived as model building, which includes reflections on modelling and an introduction of its basic concepts in classroom practice.

We were interested in localising the teachers' standpoints in this area of tension and in finding reasons *why* a teacher prefers one or the other position by posing questions derived from the topics above during the interview. The questions are not quoted here literally, since they vary from interview to interview in some minor details according to the open structure of the method, which provides questions merely as adapted responses to the teachers' former statements. In terms of intended curricula, this task consists of in reconstructing the location of applying mathematics within the teachers' intended curricula and in revealing connections and conflicts with other goals.

4 Geometry

The study on geometry consists of nine interviews. We refer to the corresponding teachers by the letters A to I. The findings on elementary geometry, taught from grade 7 to grade 10, differs from the ones on analytical geometry, taught from grade 11 to grade 13. Hence, we present them separately.

4.1 Elementary Geometry

Seven of nine teachers express a seemingly paradoxical opinion: They regard geometry as an applied part of mathematics par excellence, but not as very suitable for model building, though being open-minded about modelling in other parts of mathematics.

Mr. A: I think, the better applications can be found in algebra or stochastics, per cent calculations, linear optimisation. It is important to get a deeper insight into reality by modelling. In geometry, there are such things as dividing a pizza by a compass. I saw a trainee teacher do so. That's ridiculous.

Mr. B: Geometry as a tool to get access to the real world is not in the first place, and it is rightly not in the first place. An application is useful to introduce a new subject, to legitimise it, and to test the competencies of this field by realistic tasks in the end. But in between, a lot has to be done without any reference to the real world, detached from these accessory parts which are not important to the mathematical model.

Mr. F: Applications are motivating, but it is important to me that my pupils also switch to an abstract level, practise pure geometry. In order to do so, concrete figures, measuring and so on are rather obstacles than aids.

Mr. C: If someone asserted in [the] case of the Pythagorean Theorem "Proved by measuring, the theorem holds", then something of value would disappear, something which is genuinely mathematical. … If geometry just consisted of measuring, calculations, drawing, constructing, and land surveying, then I would regard it as poor.

Mrs. G: Besides proof abilities, problem solving is in fact the most important thing I want to convey in my lessons on geometry.

Table 9.1 Differences between modelling and proving or problem solving

	Model building	Proving/problem-solving task
Object of interest	Singular situation	General theorem or configuration
Access to objects	By measurement/experience	By construction descriptions
Building a real model	By simplification	Not allowed
Mathematical treatment	Inventing a mathematical model	Using known operators
Validation	Empirically	By deductive arguments

Summarising these quotations, our teachers do not see mathematics education from a *comprehensive applied-oriented approach*, but as *split* into the common disciplines of school mathematics. Insofar, applied-oriented goals are not top-level aims, but have to find their places within the *local curricula of the particular disciplines*. The range of goals is occupied by several categories, mainly abilities in proving, defining, problem solving, and constructing. Applying geometry is only a further goal among others; and deduction and problem solving are seen as the main objectives of geometry than getting "access to the real world". Insofar, certain unease about teaching geometry applied-oriented arises from the various goals of education to handle in conjunction. Additionally some conflicts go deeper and are bounded to a *classical Euclidean view* on geometry (Girnat 2009a): Even though geometry is applied, the justification of every assertion has to be done purely deductively on known axioms and theorems, whereas referring to experience is regarded as a sign of a deficient understanding. Hence, some essential parts of a modelling cycle are in contrary to the settings of a proving or problem-solving task (cf. Holland 2007, pp. 170–195) (Table 9.1).

It may be comprehensible to avoid geometrical applications "in between" to prevent students getting confused by different standards and challenges of modelling, proving, and problem solving. This challenge is suspected to be unique to geometry, since it seems to be the only part of school mathematics which allows regarding its objects "naturally" from two different perspectives (Girnat 2009a): from a theoretical Euclidean point of view and from a more empirical perspective of modelling. Since teachers have to fulfil both of them, the academic debate is requested to state an answer on how to deal with these disparities.

As a further finding, it is interesting to see what most of our teachers perceive as "good" geometrical applications. Typically, the examples possess a two-step structure: In the first step, geometry is used to calculate some boundary conditions, for example, some lengths, areas, or volumes. Afterwards, these values are committed to a second step, which normally includes a non-geometrical question, for example, some price, weight, or velocity calculations. Especially Mr. A mentioned that the interesting insights primarily arise in the second step. Even in case of optimisations (when a geometrical value is adjusted afterwards), the issues and structure of model building typically arise only in the second step, whereas in the first one, the geometrical background is taken for granted. Here, a static

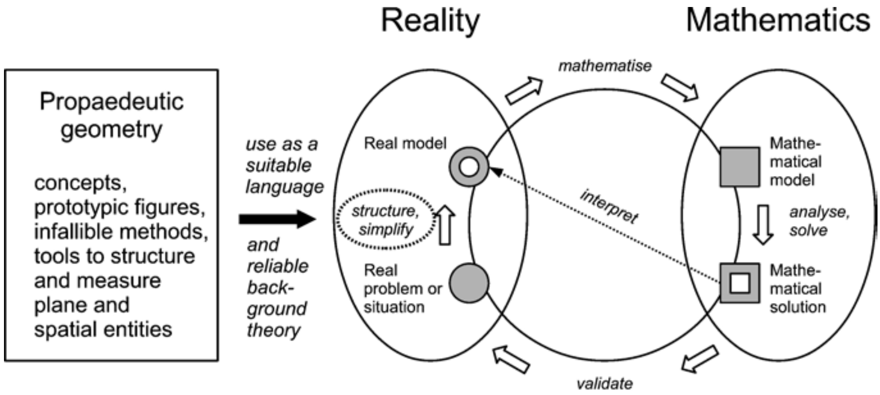

Fig. 9.2 Geometry as propaedeutic to model building

view of mathematics is predominant, forming geometry as being "propaedeutic" to model building. Insofar, this view of modelling can be illustrated as follows (Fig. 9.2).

This observation is interesting for two reasons: It could give some advice to manage the disparities between modelling and a Euclidean view on geometry: In joining both steps, a static and dynamic view and a pragmatic and scientific-humanistic approach can be combined in the same task. The reason why we call this use of geometry propaedeutic and why this function cannot be integrated into the modelling cycle under "mathematise" is as follows: Geometrical concepts and theorems are already used to structure and to simplify the real situation, that is, to build the *real* model. Hence, they are prior to any kind of mathematisation in the sense of the modelling cycle. This observation seems to be unique to geometry, since geometrical terms are part of the vocabulary we naturally use to describe the objects surrounding us and, therefore, geometry has a different, and quasi-unavoidable reference to reality, more than other parts of mathematics (Girnat 2009b). Thus, it is questionable if it makes sense to distinguish between a real model and a mathematical model or even to use the word "model" at all as far as geometry alone is concerned.

4.2 Analytical Geometry

Goals of education are manifold in elementary geometry, and, hence, our teachers' opinions cannot to be described by a single model. On the contrary, in the case of analytical geometry (AG), it is possible to present a *single* curriculum which eight of nine teachers possess. The basic structure can be described by a hierarchical diagram of educational goals (Fig. 9.3).

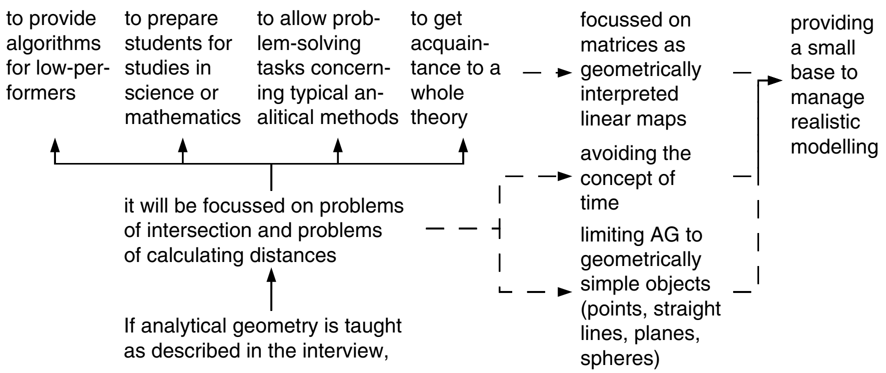

Fig. 9.3 Main aspects of the predominant intended curriculum in analytical geometry

The uniformity may be enhanced by the fact that the school leaving certificate is more standardised than the examinations in the lower secondary school. Nevertheless, the limitation to issues of intersection and distances and the focus on problem solving is openly approved:

Mr. B: Analytical geometry, that is just the metric Euclidean geometry: relative positions, calculating angles, distances, intersections.

Mr. C: Calculating distances without knowing the methods completely and, finally, inventing a formula to calculate distances, these are the things the focus has to be on.

In Fig. 9.3, the disadvantageous consequences of such a curriculum on applied mathematics are marked by dotted lines, forming the "trilemma" of application in analytical geometry:

1. If AG is taught as described in Fig. 9.3, it will be difficult to find realistic applications.
2. If AG is enriched by the concept of time and some basic physical theory instead, most of our teachers will regard AG as unfair to pupils who have not chosen physics and to teachers who do not teach science as a second subject.
3. If AG is enriched by parts of linear algebra which are not interpreted geometrically, but as tools of social and biological science instead, most of our teachers will regard AG as mathematically too simple or will fear an inappropriate restriction of the "real core" of AG or will accuse these applications as being not realistic, since "not everything in the world is linear" (Mr. A).

In contrast to elementary geometry, the main obstacles for applications do not bear on two opposing approaches to geometrical objects, but on the focus on problem solving and on preparing pupils for academic studies.

5 Stochastics

The settings of the second study are equal to the first one, but rest upon 17 interviews. The teachers are denoted by a to q. In contrast to geometry, the teachers' intended curricula are more diversified, and all the possible combinations of Kaiser's classification are instantiated, leading to the following prototypes (Eichler 2007): Shown in Table 9.2.

Both types of the humanistic-scientific approach provide statements which are familiar from the geometrical part of this chapter: Traditionalists approve insights into formalism and mathematical theories; structuralists tend to stress problem solving. The only difference is the fact that these opinions are not supported by a traditional educational theory, as it exists in the case of a Euclidean view on geometry and provides some kind of legitimisation to these points of view. Much more interesting are the more pragmatic types, which are in principle open minded to an applied-oriented approach, including "real" model building. But two of Mrs. f's tasks and Mr. d's comments are indicators for a different interpretation:

Mrs. f: Task 1: "In a German city, 30% of the population are infected with HIV, …"; task 2: "The probability of a hamburger having two slices of tomatoes is 10%. In case you buy three hamburgers, …"

Mr. d: And that's what I am trying to illustrate here as well, that you get models of approach this way, but of course become better afterwards … that there are quite often problems you can solve with maths, … that students are enabled to categorise mathematical models better.

Similar to this illustration, even the pragmatic teachers of our study differ from some essential properties of the applied-oriented approach mentioned above: At first, some of the teachers, like Mrs. f, take empirical knowledge on a specific situation not very seriously and replace real data by partly ludicrous dummy data, starting the modelling cycle at a simulated, not realistic "real model" and taking the "recognition of the need for data" as an essential topic of the current debate on stochastics ad absurdum (Wild and Pfannkuch 1999).

Table 9.2 Prototypes of teachers' intended curricula

	Static	Dynamic
Humanistic-scientific	*Traditionalists*: establishing a theoretical base, including algorithmic skills and insights into the abstract structure of mathematics, not involving applications.	*Structuralists*: encouraging students' understanding of the abstract system of mathematics in a process of abstraction, starting from applications.
Pragmatic	*Application preparers*: making students grasp the interplay between theory and applications (first theory, then applications).	*Every-day-life-preparers*: developing statistical methods in a process, making students cope with real stochastic problems and criticise them.

Furthermore, as seen in Mr d's quotation, the methodological aims are partly turned into their opposite: There is no process which consists of analysing a situation and *inventing* a fitting model. Instead, there are some *pre-established* models, and the students' task is to recognise properties of the situation in order to *choose* an "adequate" model and, then, to work the chosen one over for deepening the *mathematical* understanding of this model. Why this model may be empirically adequate is not discussed. For these reasons, "absurd" data are sufficient as "model indicators"; but exactly the aspect of *building* a model and evaluating its *empirical* relevance is suspended, which leads to omitting its most important meta-scientific feature: Building a model is typically not determined into one direction; but just the insight that there are many possibilities to treat a situation mathematically and that there is no "unique solution" is avoided by several teachers. As a result, the intended or unconscious scientific aim of such an approach is not perceiving applied mathematics as model building, but as choosing fitting operators (in disguise of known models) in the sense of problem solving. Only a few teachers mentioned some data-related aspects, like Mr. d, connected to real situations in a process where mathematics is seen as a tool to describe the world. But even in these cases in which developing mathematical methods is not the primary goal in itself, but as a tool to enable students to cope with real problems, the process of building a model is not detectable. Overall, this is an interesting consequence, also perceivable in geometry: Scientific aims of applying mathematics are typically not pursued on an *abstract level* (like the process of model building as a *general* approach to applied mathematics), but on more concrete ones which are *bounded to specific disciplines*: In stochastics, that means the selection of the fitting model (like the correct urn problem or the adequate average); in geometry, there are problems of measurement, choosing an adequate formula, or dividing an object into known figures.

6 Conclusions

Our studies underline the importance on empirical investigations of teachers' beliefs: Although our teachers try to match the same written curriculum, the outcome differs considerably. The focus on intended curricula has served as a useful tool to reveal the reasons why the written curriculum is interpreted differently. These findings are not only a preliminary work to design representative studies on larger samples, but highlight some crucial topics worth discussion: In case of elementary geometry, model building is in conflict with aspects of traditional approaches to geometry and with educational goals of proving and problem-solving tasks. These oppositional requests have to be clarified in the academic debate and to be balanced for a realisable combination in practice (cf. 2.1). In case of stochastics and analytical geometry, the main question is: Do we have convincing rationales for emphasising modelling in a strict sense instead of only using mathematical applications to motivate and illustrate mathematical content? Our studies namely suggest that teachers mostly plan their lessons in view of the mathematical content and think in separated mathematical subdisciplines, leading to a preference for content specific, purely mathematical problem solving or even schematic tasks (cf. 2.2 and 3). This invokes two challenges: Firstly, it seems advisable to consider the various parts of school mathematics more differentiated and to integrate and balance didactical requests which are not focussed on applications. Secondly, it poses the question if there are really convincing examples which are both realistic applications and fruitful occasions to establish a broad theoretical background of mathematics for every discipline (i.e., algebra, geometry, stochastics, and analysis), for every grade, and for proving and problem-solving tasks or if the teachers' hesitation indicates a lack of mathematically rich applied-oriented tasks.

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