Chapter 8
Secondary Teachers' Beliefs About Teaching Applications – Design and Selected Results of a Qualitative Case Study

Frank Förster

Abstract As a result of the standard-based curricula, in several countries secondary teachers' beliefs about applications or modelling have developed in the scope of mathematics education. In contrast, German secondary teachers rarely integrate applications or modelling into their instructional practice. This research report is focused on teachers' beliefs that hinder or promote integrating applications or modelling into their teaching practice. The objective of this approach was to reconstruct the teachers' belief systems concerning applications. The undefined term "beliefs" is specified by the psychological construct of "subjective theories." In this chapter, results with reference to the subjective theories of teachers with respect to modelling will be presented. Furthermore, some recommendations concerning teacher training will be sketched.

1 The Call for Applications in Mathematics Curriculum is Quite Old!

> *Everything flows*. The way of teaching mathematics undergoes steady changes. To be more precise, there are two aspects in particular that have been altered over the years at a slow but steady pace. Firstly, one strives to simplify the teaching subjects. … Secondly, the approach to teaching mathematics increasingly seeks to adapt the needs of everyday life. This effort is reflected especially in the selection and status of the so-called real-world problems. (Heinrich Kempinsky 1928, p. 9. Translation by author).

Today, applications are in vogue: Even if there were always oscillations between utilitarian periods and puristic periods in teaching mathematics over the last 100 years (Kaiser 1995; Niss 2000) and, after a "weaker period" in the 1990s, today applications are brought back into focus by the TIMSS and PISA discussion – as one can see by newer German schoolbooks or in recent German curricula and didactical journals.

F. Förster (✉)
Technische Universität Braunschweig, Braunschweig, Germany
e-mail: f.foerster@tu-braunschweig.de

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But applications still miss out in the mainstream classroom at least in German secondary schools: For years we asked freshmen at the Technical University of Braunschweig: “What applications of mathematics do you remember from school?” The most common answer was: “None.” Often applications were mentioned as *pure mathematical applications* like curve sketching as an “application of differential calculus.” There are still only a few research studies to support this thesis, but a lot of direct or indirect evidence by many studies (e.g., BIQUA 2007; Hiebert et al. 2003; Hugener 2008; Kaiser 1999; Neubrand 2004; Stigler and Hiebert 1999).

The burning issue is: Will there be changes made by the new German standards-based curricula (i.e., Bildungsstandards, KMK 2004; the Kerncurricula e.g., for Niedersachen, Niedersächsisches Kultusministerium 2006)? Not necessarily, we think, and particularly not automatically, as the following analysis will show.

2 A First Root Cause Analysis: Focusing on Teachers

> How teachers make sense of their professional world, the knowledge and beliefs they bring with them to the task, and how teachers’ understanding of teaching, learning, children, and the subject matter informs their everyday practice are important questions that need an investigation of the cognitive and affective aspects of teachers’ professional lives (Calderhead 1996, p. 709).

Is it possible to identify issues that can explain the gap between the educational demands for applications and modelling and the instructional practice in the classroom? We use the following didactic triangle (see Fig. 8.1) as a simple model for teachers’ actions (Tietze et al. 1997, pp. 74 f.). The model incorporates the mathematical subject, students, and teacher and, as indicated by the arrows, the interactions between teacher and students, and within the group of students and the involvement of the teacher and the student with the subject. From the qualitative point of view, the dialogue between the persons involved and the subject matter is a dialogue that can change both person and subject matter. Last but not least, we take into account the conditions that frame school teaching.

The key persons in changing or reforming mathematics education and to apply new curricula are teachers (e.g., Fernandes 1995; Wilson and Cooney 2002).

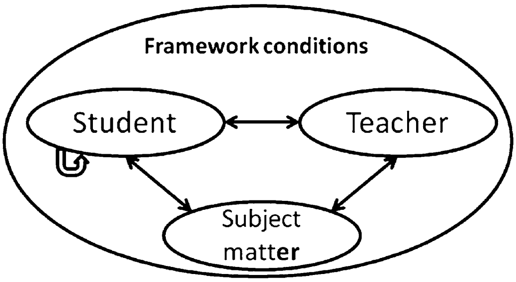

Fig. 8.1 Didactic triangle

Focusing on teachers as the main actors for planning and performing teaching in school is just *one* possible perspective. One could also focus on the *framing* by investigating the effect of curriculum, assessment, or even social expectations (of parents, society, or even the business world) on mathematical teaching. One could focus on *subject matter* as the context of schoolbooks or learning material for teaching applications that also have a significant influence. Alternatively, one could focus on the *students*, including their mathematical competencies, their attitudes to applications and modelling, and the students' expectations of their mathematical teaching as the research of Maaß (2004) has shown, a factor, that should not be underestimated.

For now, the focus will be on *teachers* as the main actors in the educational process. Two, at first glance, very simple questions arise: *Do not teachers want to teach applications and modelling in the classroom?* Which would correspond with the teachers' motives? Or: *Cannot they teach applications and modelling?* Taking into account the application competencies of the teachers and/or the objective or subjectively felt barriers that hinder them from teaching applications.

3 Why a Qualitative Case Study?: Methodology and Methods

Questionnaire-based quantitative studies of teachers' cognitions and attitudes to applications and modelling show that the great majority have a quite positive attitude to applications and want to increase the number of applications but see a lot of barriers connected with applications in the classroom and even self-distrust of their own application competencies. Barriers mentioned are, for example, "too few materials for teaching applications," "applications are hard stuff and therefore only relevant for high-performers," "applications are difficult to assess" and most of all: "There is not enough teaching time for applications" (e.g., Grigutsch et al. 1998; Humenberger 1997; Tietze 1990, 1992; Zimmermann 2002).

From this quantitative research, a lot of questions remain open, especially: *If teachers want to teach applications and modelling in the classroom, why do they not create the framing conditions to reach their goals?* And that's where the qualitative study starts with the following research questions: (1) What are the teachers' reasons to integrate or to ignore modelling in their teaching practice? (2) Is it possible to identify issues that can explain the gap between the educational demands of modelling and the instructional practice?

3.1 Theoretical Constructs

The research is based on the following constructs: Teaching and planning of teaching are actions (and not behaviour) of teachers (*Theory of action*, Hofer 1986). Sources and reasons for actions are not observable, but have to be reconstructed by interpretation (*Interpretative paradigm*, Wilson 1973). We have an *epistemological*

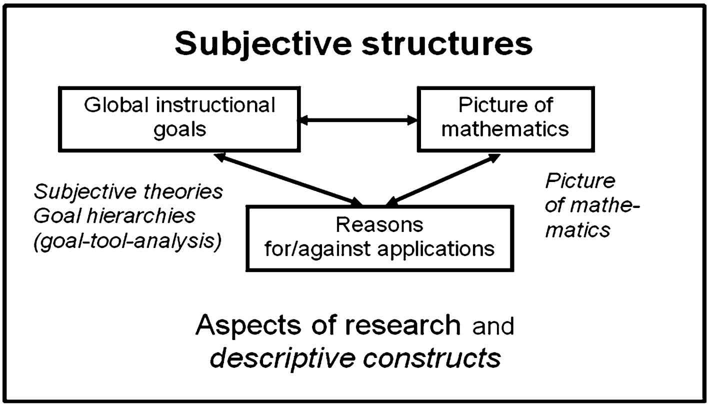

Fig. 8.2 Subjective structures

conception of man (psychology of the *reflexive subject*, Groeben and Scheele 1977, 2000), which means especially that researcher and researched are structurally equal, so it is possible to communicate about the reasons and intentions of the teachers – and to validate these reconstructions.

Building on results of research on beliefs (Leder et al. 2002; Thompson 1984, 1992), the *aspects of research* (see Fig. 8.2) are the following: We look for "global instructional goals," the "picture of mathematics," and the "reasons for or against applications," as well as the connections between them. To describe these aspects, we use the following *descriptive constructs*, which are explained below: "Subjective theories" as a background theory for beliefs, "goal hierarchies" and again the "picture of mathematics" as a descriptive tool. All together, we call it the *subjective structure* of a teacher.

Subjective theories resemble scientific theories in structure as well as in function (explanation, prediction, technology) but are, in comparison to scientific theories, less coherent and consistent, are usually implicit and not explicit, and have an important function of orientation for the teachers (Groeben and Scheele 1977, 2000). Furthermore, most importantly, they are *subjective* and not *objective* theories.

To structure the subjective theories we use *goal hierarchies*, which we gain by *goal-tool-argumentation* ("Ziel-Mittel-Argumentation" according to König 1975; q.v. Scheele and Groeben 1988). Added are specific assumptions about the structure and the function of subjective theories (Groeben et al. 1988) which we use but will not mention any further in this chapter.

A brief example is given. From an interview there is the statement: "Applications motivate the students." This statement is now transferred into a descriptive sentence: "If one wants to motivate students one can teach applications." From this description, two prescriptive sentences can be derived: "One should motivate students" and "One should teach applications" as shown in schema in Fig. 8.3.

So, we have "motivation" as a *goal* and the "teaching of applications" as a *tool* to achieve "motivation" in this hierarchy. By adding further information from the

	Goal Level	Goal/Tool-Level
Prescriptive sentence	One should *motivate students.*	
Descriptive sentence	If one wants to *motivate* students …	… one can *teach applications.*
Prescriptive sentence		One should *teach applications.*

Fig. 8.3 An example of goal-tool-argumentation

interview, we can develop a chain of arguments and a more or less complex *goal-tool-structure* – either in search of higher goals (*goal-perspective*) or in search of suitable tools (*tool-perspective*).

To sum up with the example, the teacher might have also said: "Applications convey a representative picture of mathematics". There are at least two ways to interpret this and you have to reconstruct from the data, which interpretation is adequate. (1) To teach a representative picture of mathematics and to motivate the students are goals at the same level and for both goals the teaching of applications is a suitable *tool*. (2) To teach a representative picture of mathematics is the main goal. The tool for this goal (i.e., applications) is now becoming a *goal for the next level*. The first interpretation of the sentence "Applications motivate the students" is altered to "If you teach applications you motivate students" which means application leads to motivation as a consequence. The important difference between these interpretations is that in the first case we can see applications as a *tool* and in the second one as a *goal* in the hierarchy.

Note: Due to space restriction, we leave out an explication of the construct "picture of mathematics" in this chapter – roughly speaking, it is a mixture of *beliefs about mathematics* and *content knowledge* of applications (see Förster 2008).

3.2 Study Design

Understanding action as an inner process depending on situations determines an inquiry in the form of case studies (Stake 2000). The definition of the cases is according to theoretical sampling (Glaser and Strauss 1967). The main study involved the questioning of eight *in-service teachers* grade 7–13 of secondary schools (A-level) in Northern Germany with several years of teaching experience (at least 2, up to 20 years). Data were mainly collected by *(in-depth) interviews* up to 4 h with open and semi-structured parts. The interviews were prepared by evaluating a *standardized questionnaire* (mainly statistical data such as age of the teacher, type of school, and also schoolbooks and teaching material used).

Interpretation was based on an adoption of *qualitative content analysis* (Mayring 1995) and methods of *qualitative teaching research* (e.g., Corbin and Strauss 2008; Jungwirth and Krummheuer 2008) and went through the following steps: transcription of the interview, sequential interpretation of the transcription (to gain the subjective theories), summing up global analysis (to gain patterns in the goal hierarchy), intrapersonal description of the teachers (to gain the subjective structure of the teacher), and interpersonal analysis (to gain different types of subjective structures).

4 Discussion and Some Selected Results

The following discussion focuses on one main result of the study. The process of interpretation of the interviews is not outlined. While primarily one case will be discussed, some results of other cases are used to complete the case description.

Figure 8.4 shows an "overview map" derived from the goal-tool-structure of Teacher A representing the global instructional goals, as part of the subjective structure. Omitted are subsidiary goals, the connections between the two excerpts, as well as most of the substructure of the connections within the excerpts.

Teacher A's main instructional goals are *significance for the future* and *fulfillment of curriculum plan*. Significance for the future means *university and vocational preparation* and *school attainment*. Firstly, students have to learn *logical thinking*. This goes with the transfer hypothesis that logical thinking in mathematics leads to logical thinking towards everyday-life problems. That of course is objectively not true – but that does not matter to Teacher A as long as his subjective structure is coherent! (An explanation of why in the subjective structure of Teacher A's logical thinking can be achieved by *accuracy in working* would take too long to explain here, but it "makes sense" within the subjective structure (see Förster 2008).

Looking at the *School attainment*: For Teacher A "no one is left behind" in his teaching – to ensure students pass the examinations means *repetition and exercising* and *training of techniques* as a *stockpiling of knowledge*. This altogether needs a lot of *teaching time* – which of course is not an explicit goal, but a *necessary tool* in the goal-tool hierarchy – and the needed time is missing for other goals.

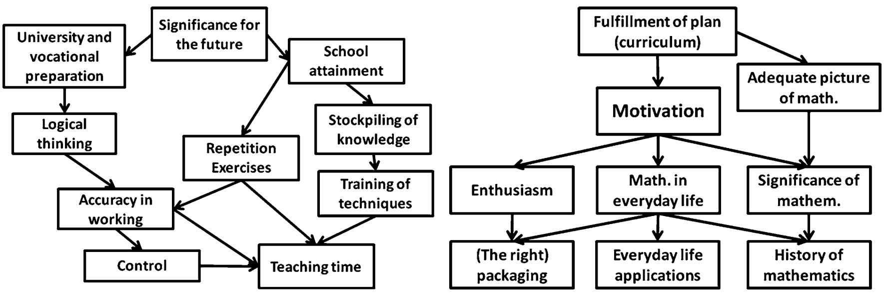

Fig. 8.4 Global goals of Teacher A (excerpts)

Where are the applications? We look at the right side of Fig. 8.4: *Fulfillment of plan* goes hand in hand with the goal (!) to *motivate the students* for the content they have to learn at any rate. One way to motivate the students is to teach simple *everyday-life applications* – simple enough to motivate. However, Teacher A can also show the *significance of mathematics* to be motivated for example by episodes of *mathematical history*, which can be told to the students "in passing." Last but not least, the *enthusiasm* of the teacher can motivate by his choosing the *right packaging* for mathematical content. We will come back to these aspects, but first have a look at the type of applications Teacher A uses in his teaching: *Everyday life applications* have to be simple to understand (nonmathematical background) and simple to teach (teaching time). As a consequence, there can be no complex modelling. Examples are height determinations of trees, simple financial mathematics (especially interest calculations), volume of a conically shaped wine glass, volume of a cigar, the Pythagorean knotted rope, pieces of cake (fractions) and some applications in physics. These examples correspond very well with Teacher A's *subjective definition of application* derived from the interviews: "Everything (!), that students know from their everyday life (or can at least imagine) and that can be associated with mathematics, is (!) an interesting application."

This brings us to the *right packaging*: Teacher A mentions a task from a schoolbook: In a picture, an expander is hanging from the ceiling of the room. By Hooke's law with a proportional function the elongation of the expander, respectively, the weight-force, can be computed by elongation = const. × weight of person. After a short explanation of the expander and after introducing "Silke and Dirk," this is a quite normal, fairly boring word problem. So what's the point to Teacher A that's worth mentioning this task in the interview? Teacher A transfers the situation in the picture into his classroom. He brings an expander with him. He lets the student try out the expander and finally he screws the expander to the ceiling of the classroom and comes to the same questions – but with the real expander. Surely not a real-world problem, but his students are interested by the packaging. As mentioned before, time for applications is limited by the main goals – so for this teacher applications are merely a *tool* for motivation.

To explain (or at least illustrate) the origin of the three different types of "teacher's subjective substructures according to applications," we are closing with a brief look at the position of applications in the hierarchy of goals of two other teachers and summarize these three types in the following Fig. 8.5.

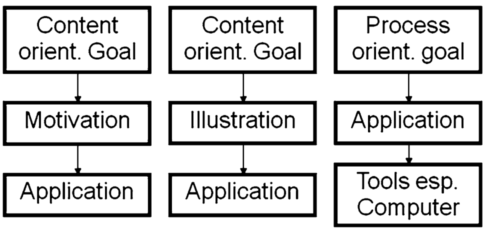

Fig. 8.5 Position of applications in the hierarchy of goals

Teacher B sees *applications* as a central goal of teaching corresponding with the goal (and tool) of *teaching applied mathematics*. The goals have higher aims as *empowered citizen*, *problem solving* (inside and outside mathematics), *a positive attitude towards mathematics*, and *learning to ask questions*. This teacher also has high *professional pretension* and he gets the needed teaching time by using the handheld *computer* as a tool in the hands of the students. *Motivation* is not a goal, but a consequence of the other goals.

In contrast, Teacher C is a *structuralist* sensu (Eichler 2007), happy with context-free mathematics. Therefore, you have to search for applications, which are merely a *tool* for *illustrating mathematical content* and quite isolated from the rest of the goal hierarchy.

The first column of Fig. 8.5 leads to attaching word problems to a traditional form of mathematical teaching without modelling. The second propagates context-free problems – and sporadic use of applications – corresponding with very high, nearly unrealisable expectations in terms of a realistic context for the applications. The third approach allows more complex applications, sometimes corresponding with high expectations of the mathematics involved in the applications. For type one and three, there will be no search for tools to overcome the barriers that hinder these teachers to have applications in their teaching.

5 Conclusion

> Quite clearly, there is a fundamental need to understand everything that underlies the way in which mathematics teachers approach their subject before suggestions and recommendations concerning good classroom practice can be made. (Eichler 2007, p. 208).

There is an unexpected *high consistency* between the picture of mathematics, the global goals concerning teaching and the selection, and reasons for or against applications. It was not expected to be so clearly defined. This is also important because one aspect of the research on teachers' beliefs is the conclusion that they have a high impact on students' beliefs (Chapman 2001). And, the students of today are the teachers of tomorrow. *Motivation* is *the* dominant argument for applications. The assumption, mathematics is per se formative toward active, creative, and flexible individuals, makes applications as an independent and clear goal abundant. And, therefore *potential applicability* of mathematics is sufficient for these teachers – in correspondence with their picture of mathematics. *Realistic modelling* and further educational demands do not play any (important) role in the classroom and there is often a mixture of *applications* and *applied mathematics*.

Interesting is the role of the *second teaching subject* of the German teachers. We expected higher competence in applications especially with physics and other natural science teachers, but we also found limiting factors as the immense time exposure for the second subject (especially physics) when taught as an experimental subject and different approaches that teachers have to applications in their different subjects – "applications in physics: of course" but "applications in mathematics: no need for them."

Tertiary Education has effects on teaching of applications: For instance, application examples often come from the teachers' own university education. Also the *German traineeship for teachers* has effects, because frequently the trainees are encouraged by the instructors to teach applications. However, if the closer contact with applications does not start until this traineeship, this "retrofitting" of competencies in applications is considered by the teachers as amateurish (dilettantish) and after their traineeship they will not teach applications any further. So a positive attitude to applications and knowledge about applications and modelling should be set up in school or in university study at the latest. Starting late will be too late!

References

BIQUA (2007). Die Bildungsqualität von Schule. Abschlussbericht. http://www.ipn.uni-kiel.de/projekte/biqua/index.html. Accessed 10 September 2010.

Calderhead, J. (1996). Teachers: Beliefs and knowledge. In D. C. Berliner (Ed.), *Handbook of education* (pp. 709–725). New York: Macmillan.

Chapman, O. (2001). Understanding high school mathematics teacher growth. In M. Heuvel-Panhuizen (Ed.), *Proceeding of the 25th PME Conference* (Vol. 2, pp. 233–240). Utrecht: PME.

Corbin, J., & Strauss, A. L. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks: Sage.

Eichler, A. (2007). Individual curricula: Teachers' beliefs concerning stochastics instruction. *International Electronic Journal of Mathematics Education, 2*(3), 208–225.

Fernandes, D. (1995). Analysing four preservice teachers' knowledge and thoughts through their biographical histories. In L. Meira & D. Carraher (Eds.), *Proceedings of the 19th PME Conference* (Vol. 2, pp. 162–169). Recife: PME.

Förster, F. (2008). Subjektive Strukturen von Mathematiklehrerinnen und -lehrern zu Anwendungen und Realitätsbezügen im Mathematikunterricht. In H. Jungwirth & G. Krummheuer (Eds.), *Der Blick nach innen: Aspekte der alltäglichen Lebenswelt Mathematikunterricht – Band 1/2*. Münster: Waxmann.

Glaser, B., & Strauss, A. (1967). *The discovery of grounded theory*. Chicago: Aldine.

Grigutsch, S., Raatz, U., & Törner, G. (1998). Einstellungen gegenüber Mathematik bei Mathematiklehrern. *Journal für Mathematik-Didaktik, 19*(1), 3–45.

Groeben, N., & Scheele, B. (1977). *Argumente für eine Psychologie des reflexiven Subjekts*. Darmstadt: Steinkopff.

Groeben, N., & Scheele, B. (2000). Dialogue-hermeneutic method and the "research program subjective theories". *Forum Qualitative Social Research 1*(2). http://www.qualitative-research.net/index.php/fqs/article/view/1079/2354. Accessed 10 September 2010.

Groeben, N., Wahl, D., Schlee, J., & Scheele, B. (1988). *Forschungsprogramm Subjektive Theorien – Eine Einführung in die Psychologie des reflexiven Subjekts*. Tübingen: Francke.

Hiebert, J., et al. (2003). *Teaching mathematics in seven countries*. Washington: NCES.

Hofer, M. (1986). *Sozialpsychologie erzieherischen Handelns*. Göttingen: Hogrefe.

Hugener, I. (2008). *Inszenierungsmuster im Unterricht und Lernqualität*. Münster: Waxmann.

Humenberger, H. (1997). Anwendungsorientierung im Mathematikunterricht – erste Resultate eines Forschungsprojekts. *Journal für Mathematik-Didaktik, 18*(1), 3–50.

Jungwirth, H., & Krummheuer, G. (2006/2008). *Der Blick nach innen: Aspekte der alltäglichen Lebenswelt Mathematikunterricht – Band 1/2*. Münster: Waxmann.

Kaiser, G. (1995). Realitätsbezüge im Mathematikunterricht – Ein Überblick über die aktuelle und historische Diskussion. In G. Graumann et al. (Eds.), *ISTRON* (Vol. 2, pp. 66–84). Bad Salzdetfurth: Franzbecker.

Kaiser, G. (1999). *Unterrichtswirklichkeit in England und Deutschland*. Weinheim: Beltz, Deutscher Studien Verlag.
Kempinsky, H. (1920, 1928). *Sachaufgaben für das erste Schuljahr*. Leipzig: Dürr'sche Buchhandlung.
KMK. (2004). *Bildungsstandards im Fach Mathematik für den Mittleren Schulabschluss*. München: Wolters Kluwer.
König, E. (1975). *Theorie der Erziehungswissenschaft Band 2 – Normen und ihre Rechtfertigung*. München: Fink.
Leder, G. C., Pehkonen, E., & Törner, G. (Eds.). (2002). *Beliefs: A hidden variable in mathematics education?* Dodrecht: Kluwer.
Maaß, K. (2004). *Mathematisches Modellieren im Unterricht – Ergebnisse einer empirischen Studie*. Hildesheim: Verlag Franzbecker.
Mayring, P. (1995). *Qualitative Inhaltsanalyse: Grundlagen und Techniken*. Weinheim: Deutscher Studien Verlag.
Neubrand, M. (Ed.). (2004). *Mathematische Kompetenzen von Schülerinnen und Schülern in Deutschland. Vertiefende Analysen im Rahmen von PISA 2000*. Wiesbaden: Verlag für Sozialwissenschaften.
Niedersächsisches Kultusministerium (Ed.). (2006). Kerncurriculum für das Gymnasium Schuljahrgänge 5–10 Mathematik Niedersachen. Hannover: Unidruck. http://db2.nibis.de/1db/cuvo/datei/kc_gym_mathe_nib.pdf. Accessed 10 September 2010.
Niss, M. (2000). Applications of mathematics "2000". Moments of mathematics education in the twentieth century pp. 271–284. http://www.mathunion.org/fileadmin/ICMI/files/ Digital_Library/Other_ICMI_Conferences_Proceedings/Proc_EM_ICMI_Symp.pdf. Accessed 7 August 2009.
Scheele, B., & Groeben, N. (1988). *Leitfaden zur Ziel-Mittel-Analyse (ZMA)*. Tübingen: Francke.
Stake, R. E. (2000). Case studies. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 435–508). Thousand Oaks: Sage.
Stigler, J. W., & Hiebert, J. (1999). *The teaching gap*. New York: The Free Press.
Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*(2), 105–127.
Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.
Tietze, U.-P. (1990). Der Mathematiklehrer an der gymnasialen Oberstufe – Zur Erfassung berufsbezogener Kognitionen. *Journal für Mathematik-Didaktik, 11*(3), 177–243.
Tietze, U.-P. (1992). *Materialien zu: Berufsbezogene Kognitionen, Einstellungen und Subjektive Theorien von Mathematiklehrern an der gymnasialen Oberstufe, Band 1–3*. Göttingen: Universität Göttingen.
Tietze, U.-P., Klika, M., Wolpers, H., & Förster, F. (1997). *Mathematikunterricht in der Sekundarstufe II*. Braunschweig/Wiesbaden: Vieweg.
Wilson, T. P. (1973). Theorien der Interaktion und Modelle soziologischer Erklärung. In Arbeitsgruppe Bielefelder Soziologen (Ed.), *Alltagswissen, Interaktion und gesellschaftliche Wirklichkeit, Bd. 1* (pp. 54–79). Reinbek: Rowohlt.
Wilson, M. C., & Cooney, T. (2002). Mathematics teacher change and development. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education*? (pp. 127–147). Dordrecht: Kluwer.
Zimmermann, B. (2002). Vorstellungen über Mathematik und Mathematikunterricht von Lehrerinnen und Lehrern verschiedener Schularten. *MU, 48*(4/5), 7–25.