Chapter 16
On Stability, Passivity and Reciprocity Preservation of ESVDMOR

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Abstract The reduction of parasitic linear subcircuits is one of many issues in model order reduction (MOR) for VLSI design. This issue is well explored, but recently the incorporation of subcircuits from different modelling sources into the circuit model has led to new structural aspects: so far, the number of elements in the subcircuits was significantly larger than the number of connections to the whole circuit, the so called pins or terminals. This assumption is no longer valid in all cases such that the simulation of these circuits or rather the reduction of the model requires new methods. In Feldmann and Liu (ICCAD '04: Proceedings of the 2004 IEEE/ACM International Conference on Computer-Aided Design, 88–92, 2004), Liu et al. (ICCAD '05: Proceedings of the 2005 IEEE/ACM International Conference on Computer-Aided Design, 821–826, 2004; Integr. VLSI J. **41**(2): 210–218, 2008) the extended singular value decomposition based model order reduction (ESVDMOR) algorithm is introduced as a way to handle this kind of circuits with a massive number of terminals. Unfortunately, the ESVDMOR approach has some drawbacks because it uses the SVD for matrix factorizations. In Benner (Mathematics in Industry 14, 2009), Schneider (Matrix decomposition based approaches for model order reduction of linear systems with a large number of terminals, 2008) the truncated SVD (TSVD) as an alternative to the SVD within the ESVDMOR is introduced. In this paper we show that ESVDMOR as well as

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the modified approach is stability, passivity, and reciprocity preserving under reasonable assumptions.

Keywords Model reduction · Model order reduction · Many terminals · Large number of inputs/outputs · Passivity · Stability · Reciprocity · Power grids

16.1 Introduction

MOR has become an important tool in the preprocessing of circuit simulation over the last decades. The original model which results from the mathematical modeling with the help of, e.g., modified nodal analysis [11] has to be simplified due to its complexity. One special part of this simplification for VLSI design is the MOR of parasitic linear interconnect circuits [7]. These circuits appear in form of substructures in the design of integrated circuits (ICs). They contain linear elements which do not necessarily have large influence on the result of the simulation.

There are applications in which the structure of these parasitic linear subcircuits has been changing recently in the following sense. So far, the number of elements in these interconnect circuits was significantly larger than the number of connections to the whole circuit, the so-called pins or terminals. This assumption is not true anymore in many cases. Power grid networks which supply elements of large circuits with energy are of this special form [22, 24]. Often these power grids are realized as an extra layer of elements in between the layers of transistors. The fact that they need connections to the elements for power supply explains the high number of pins. The same problem appears in connection with synchronization. In clock distribution networks, the clock signal is distributed from a common point to all the elements that need it for synchronization [14]. For simulating these special subcircuits, new methods are needed. In some applications, a lot of terminal signals behave similarly so that it is possible to compress the input-/output -matrices of the transfer function in such a way that the I/O behavior can be realized through a few so-called virtual inputs/outputs. As a consequence we deal with these virtual terminals, the number of which is much less than the original number of terminals. This allows the use of well known MOR methods like balanced truncation or Krylov subspace methods to reduce the number of inner nodes. The basic work was done a few years ago with the introduction of the (E)SVDMOR approach [6, 15–17, 23]. In this paper we review the existing ESVDMOR approach [6, 17] and discuss stability and passivity of the computed reduced-order model. As most linear subcircuits with a massive number of pins represent RC(L) circuits and contain no active devices, they are modelled as passive and thus stable systems. Therefore, it is generally important to compute reduced-order models that share these properties with the original model.

In the following section, we review the fundamentals of the ESVDMOR approach. We introduce the moments of a transfer function of the circuit describing system and show how to use the information in these moments in order to reduce the number of terminals. In this way we achieve a very compact model. The preservation of stability and passivity in the reduced-order models is the topic of Sect. 16.3. We introduce basic definitions and prove that the ESVDMOR approach is stability, passivity, and reciprocity preserving under certain conditions. A numerical example shows the passivity preservation. In Sect. 16.4, we sum up the results and outline future research activities.

16.2 The Extended SVDMOR Approach

We consider a linear time-invariant continuous-time descriptor system, which can be represented as

$$\begin{aligned} C\dot{x}(t) &= -Gx(t) + Bu(t), \quad x(0) = x_0, \\ y(t) &= Lx(t), \end{aligned} \tag{16.1}$$

with $C, G \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m_{\mathrm{in}}}$, $L \in \mathbb{R}^{m_{\mathrm{out}}\times n}$, $x(t) \in \mathbb{R}^n$ containing internal state variables, $u(t) \in \mathbb{R}^{m_{\mathrm{in}}}$ the vector of input variables, e.g., the terminal currents, $y(t) \in \mathbb{R}^{m_{\mathrm{out}}}$ being the output vector, e.g., the terminal voltages, $x_0 \in \mathbb{R}^n$ the initial value and n the number of state variables, called the *order* of the system. The original linear system (16.1) to be reduced has the following transfer function in frequency domain:

$$H(s) = L(sC + G)^{-1}B. \tag{16.2}$$

The number of inputs m_{in} and the number of outputs m_{out} are not necessarily equal. Further on we can define the ith block moment of (16.2) as

$$\mathbf{m_i} = L(-G^{-1}C)^i G^{-1}B, \quad i = 0, 1, \ldots \tag{16.3}$$

in terms of $\mathbf{m_i}$ as an $m_{\mathrm{out}} \times m_{\mathrm{in}}$ matrix

$$\mathbf{m_i} = \begin{bmatrix} m^i_{1,1} & m^i_{1,2} & \cdots & m^i_{1,m_{\mathrm{in}}} \\ m^i_{2,1} & m^i_{2,2} & \cdots & m^i_{2,m_{\mathrm{in}}} \\ \vdots & \vdots & \ddots & \vdots \\ m^i_{m_{\mathrm{out}},1} & m^i_{m_{\mathrm{out}},2} & \cdots & m^i_{m_{\mathrm{out}},m_{\mathrm{in}}} \end{bmatrix}. \tag{16.4}$$

Note that the moments in (16.3) are equal to the coefficients of the Taylor series expansion of (16.2) about $s = 0$. The expansion about $s = s_0 \neq 0$ leads to *frequency shifted moments* defined as

$$\mathbf{m_i}(s_0) = L(-(s_0C + G)^{-1}C)^i (s_0C + G)^{-1}B, \quad i = 0, 1, \ldots \tag{16.5}$$

In the ESVDMOR method [6, 17], the information of a combination of these moments to create a decomposition of (16.2) is used in the following way. For being able to allow terminal reduction for inputs and outputs separately, w.l.o.g. we use r different block moments forming two moment matrices, the input response matrix M_I and the output response matrix M_O, as follows:

$$M_I = \begin{bmatrix} \mathbf{m_0} \\ \mathbf{m_1} \\ \vdots \\ \mathbf{m_{r-1}} \end{bmatrix}, \quad M_O = \begin{bmatrix} \mathbf{m_0}^T \\ \mathbf{m_1}^T \\ \vdots \\ \mathbf{m_{r-1}}^T \end{bmatrix}. \tag{16.6}$$

Note that it is also possible to use different numbers of block moments to create M_I and M_O and that column k of M_I represents the coefficients (moments) of the series expansion of (16.2) at all outputs corresponding to input k. Similarly, each column k of M_O represents the coefficients of output k corresponding to all inputs. We assume the number of rows in each matrix to be larger than the number of columns. If not, r has to be increased.

Applying the SVD to these matrices, we can obtain a low rank approximation

$$M_I = U_I \Sigma_I V_I^T \approx U_{I_{r_i}} \Sigma_{I_{r_i}} V_{I_{r_i}}^T, \quad M_O = U_O \Sigma_O V_O^T \approx U_{O_{r_o}} \Sigma_{O_{r_o}} V_{O_{r_o}}^T, \tag{16.7}$$

where

- $U_I = \left[U_{I_{r_i}}, U_{I_{(rm_{\text{out}}-r_i)}} \right]$ is an $r \cdot m_{\text{out}} \times r \cdot m_{\text{out}}$ orthogonal matrix,
- $V_I = \left[V_{I_{r_i}}, V_{I_{(m_{\text{in}}-r_i)}} \right]$ is an $m_{\text{in}} \times m_{\text{in}}$ orthogonal matrix,
- $U_O = \left[U_{O_{r_o}}, U_{O_{(rm_{\text{in}}-r_o)}} \right]$ is an $r \cdot m_{\text{in}} \times r \cdot m_{\text{in}}$ orthogonal matrix,
- $V_O = \left[V_{O_{r_o}}, V_{O_{(m_{\text{out}}-r_o)}} \right]$ is an $m_{\text{out}} \times m_{\text{out}}$ orthogonal matrix,
- Σ_I and Σ_O are $r \cdot m_{\text{out}} \times m_{\text{in}}$ and $r \cdot m_{\text{in}} \times m_{\text{out}}$ diagonal matrices,

whereas

- $\Sigma_{I_{r_i}}$ and $\Sigma_{O_{r_o}}$ are $r_i \times r_i$ and $r_o \times r_o$ diagonal matrices,
- $V_{I_{r_i}}$ and $V_{O_{r_o}}$ are $m_{\text{in}} \times r_i$ and $m_{\text{out}} \times r_o$ isometric matrices that contain the dominant column subspaces of M_I and M_O,
- $U_{I_{r_i}}$ and $U_{O_{r_o}}$ are $rm_{\text{out}} \times r_i$ and $rm_{\text{in}} \times r_o$ isometric matrices that are not used any further. $r_i \leq m_{\text{in}}$ and $r_o \leq m_{\text{out}}$ are the numbers of significant singular values as well as the numbers of the reduced virtual input and output terminals. Due to the fact that the important information about the dependencies of the I/O-ports is hidden in the matrices $V_{I_{r_i}}^T$ and $V_{O_{r_o}}^T$, approximations of B and L using the factors in (16.7) lead to

$$B \approx B_r V_{I_{r_i}}^T \quad \text{and} \quad L \approx V_{O_{r_o}} L_r. \tag{16.8}$$

Here, $B_r \in \mathbb{R}^{n \times r_i}$ and $L_r \in \mathbb{R}^{r_o \times n}$ are computed by applying the Moore-Penrose pseudoinverse [18] (denoted by $(\cdot)^+$) of $V_{I_{r_i}}^T$ and of $V_{O_{r_o}}$ to B and L, respectively. In detail, that means

$$B_r = BV_{I_{r_i}}(V_{I_{r_i}}^T V_{I_{r_i}})^{-1} = BV_{I_{r_i}}^{T+} = BV_{I_{r_i}} \tag{16.9}$$

and

$$L_r = (V_{O_{ro}}^T V_{O_{ro}})^{-1} V_{O_{ro}}^T L = V_{O_{ro}}^+ L = V_{O_{ro}}^T L, \tag{16.10}$$

since $V_{I_{r_i}}$ and $V_{O_{r_o}}$ are isometric. As a consequence we get a new internal transfer function $H_r(s)$ by using the approximation

$$H(s) \approx \widehat{H}(s) = V_{O_{ro}} \underbrace{L_r(G + sC)^{-1}B_r}_{:=H_r(s)} V_{I_{r_i}}^T. \tag{16.11}$$

This terminal reduced transfer function $H_r(s)$ can be further reduced to

$$\tilde{H}_r(s) = \tilde{L}_r(\tilde{G} + s\tilde{C})^{-1}\tilde{B}_r \approx H_r(s) = L_r(G + sC)^{-1}B_r \tag{16.12}$$

by some established MOR method, e.g., balanced truncation or a Krylov subspace method, see [2, 4, 20] for introductions to linear model reduction techniques. We end up with a very compact terminal reduced and reduced-order model $\tilde{H}_r(s)$, i.e.

$$H(s) \approx \widehat{H}(s) \approx \widehat{H}_r(s) = V_{O_{ro}} \tilde{H}_r(s) V_{I_{r_i}}^T. \tag{16.13}$$

Note that the well-known SVDMOR approach [6] can be considered as a special case of ESVDMOR, using only one moment and one SVD, e.g. $r = 1$, and using $\mathbf{m}_0$ as moment

The ESVDMOR approach as described above is not appropriate for really large-scale circuit systems as it employs the full SVD of the moment matrices. Observing that the computation of the reduced-order model only needs the leading block-columns $U_{I_{r_i}}, V_{I_{r_i}}, U_{O_{ro}}, V_{O_{ro}}$ of the orthogonal matrices computed within the SVD, the expensive SVD can be replaced by a truncated SVD which can be computed cheaply employing sparsity of the involved matrices using Krylov subspace or Jacobi–Davidson methods [12, 13]. Efficient algorithms based on the first approach are suggested in [5, 21] while the Jacobi–Davidson version is under current investigation.

16.3 Stability, Passivity, and Reciprocity

In this section we establish some facts on the preservation of stability, passivity, and reciprocity in ESVDMOR reduced-order models.

16.3.1 Stability

In order to discuss stability of descriptor systems we need the following definition and lemma which can be found, e.g., in [4].

Definition 1 *The descriptor system* (16.1) *is called asymptotically stable if* $\lim_{t\to\infty} x(t) = 0$ *for all solutions* $x(t)$ *of* $C\dot{x}(t) = -Gx(t)$.

Lemma 1 *Consider a descriptor system* (16.1) *with a regular matrix pencil* $\lambda C + G$. *The following statements are equivalent*:

1. *System* (16.1) *is asymptotically stable.*
2. *All finite eigenvalues of the pencil* $\lambda C + G$ *lie in the open left half-plane.*

Using the results from Lemma 1 we are able to formulate the following theorem:

Theorem 1 *Consider an asymptotically stable system*(16.1) *with its transfer function* (16.2). *The ESVDMOR reduced-order system corresponding to* (16.13) *is asymptotically stable iff the inner reduction* (16.12) *is stability preserving.*

Proof It is obvious that none of the approximations (16.8), (16.11) and (16.13) change the eigenvalues of $\lambda C + G$. With Lemma 1 and the assumption that (16.12) is stability preserving it directly follows that the ESVDMOR approach is stability preserving. □

A possible stability preserving model reduction method that can be applied along the lines of Theorem 1 is balanced truncation for regular descriptor systems, see [3, 4].

16.3.2 Passivity

Showing that the ESVDMOR approach is passivity preserving turns out to be more difficult. First we note that a system is *passive* iff its transfer function is positive real [1]. The following definition of positive realness can be found, e.g., in [8].

Definition 2 The transfer function (16.2) is positive real iff the following three assumptions hold:

1. $H(s)$ has no poles in $\mathbb{C}_+ = \{s \in |\mathrm{Re}s > 0\}$, i.e. the system is stable if additionally there are no multiple poles on $i\mathbb{R}$,
2. $H(\bar{s}) = \overline{H(s)}$ for all $s \in \mathbb{C}$,
3. $\mathrm{Re}(x^H H(s)x) \geq 0$ for all $s \in \mathbb{C}_+$ and $x \in \mathbb{C}^m$.

For passive systems we have to assume that the number of inputs is equal to the number of outputs: $m_{\mathrm{in}} = m_{\mathrm{out}} = m$. Due to the fact that we often deal just with parasitic linear RLC circuits we furthermore assume $L = B^T$ such that

$$H(s) = B^T(sC + G)^{-1}B. \tag{16.14}$$

As a result of Modified Nodal Analysis (MNA) modeling the system has a well defined block structure [8] and we get

$$\begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}\dot{x} + \begin{bmatrix} G_1 & G_2 \\ -G_2^T & 0 \end{bmatrix} \quad x = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}u, \qquad (16.15)$$
$$y = [\,B_1 \quad 0\,]x,$$

where G_1, C_1, C_2 are symmetric, $G_1, C_1 \geq 0$ (i.e., both matrices are positive semi-definite), and $C_2 > 0$, i.e., C_2 is positive definite. Moreover, as before, the matrix pencil $\lambda C + G$ is assumed to be regular. It is easy to see that under these assumptions, $H(s)$ is positive real and thus the system is passive [10].

Theorem 2 *Consider a passive system of the form* (16.15). *The ESVDMOR reduced system* (16.13) *is passive iff the inner reduction* (16.12) *is passivity preserving.*

Proof If we can show that $\widehat{H}_r(s)$ in (16.13) is positive real, we have shown that the reduced system is passive, see Definition 2. The moments of (16.15) are

$$\mathbf{m_i}(s_0) = B^T(-(s_0C+G)^{-1}C)^i(s_0C+G)^{-1}B, \qquad (16.16)$$

with $0 \leq s_0 \in \mathbb{R}$ and $\det(s_0C+G) \neq 0$.

Following a technique which can be found, e.g., in [9] we define $J = \begin{bmatrix} I & 0 \\ 0 & -I. \end{bmatrix}$. The properties of G_1, C_1, and C_2 as well as the fact that $J = J^T$ and $J^2 = I$ lead to the following rules:

R1: $J = J^{-1}$,
R2: $JC = CJ$, hence $JCJ = C$,
R3: $(s_0C+G)^T = s_0C + JGJ = s_0JCJ + JGJ$, hence $(s_0C+G)^{-T} = (s_0JCJ + JGJ)^{-1} = J^{-1}(s_0C+G)^{-1}J^{-1} = J(s_0C+G)^{-1}J$,
R4: $B = JB$, and
R5: for every matrix X and Y, $(-X^{-1}Y)^i = X^{-1}(Y(-X)^{-1})^iX$ holds.

A straightforward calculation employing these rules shows that

$$\begin{aligned}
\mathbf{m_i}^T(s_0) &= (B^T(-(s_0C+G)^{-1}C)^i(s_0C+G)^{-1}B)^T \\
&= B^T(s_0C+G)^{-T}\{(-(s_0C+G)^{-1}C)^i\}^TB \\
&\overset{(R5)}{=} B^T(s_0C+G)^{-T}\{(s_0C+G)^{-1}(C(-(s_0C+G)^{-1}))^i(s_0C+G)\}^TB \\
&= B^T(s_0C+G)^{-T}(s_0C+G)^T\{(C(-(s_0C+G)^{-1}))^i\}^T(s_0C+G)^{-T}B \\
&= B^T\{(C(-(s_0C+G)^{-1}))^i\}^T(s_0C+G)^{-T}B \\
&\overset{(C=C^T)}{=} B^T((-(s_0C+G))^{-T}C)^i(s_0C+G)^{-T}B \\
&\overset{(R3)}{=} B^T(-J(s_0C+G)^{-1}JC)^i(J(s_0C+G)^{-1}J)B \\
&\overset{(R5)}{=} B^TJ(s_0C+G)^{-1}(JC(-J(s_0C+G)^{-1}))^i(s_0C+G)J(J(s_0C+G)^{-1}J)B
\end{aligned}$$

$$\overset{(R1,R2)}{=} B^T J(s_0C+G)^{-1}(C(-(s_0C+G)^{-1}))^i(s_0C+G)(s_0C+G)^{-1}JB$$
$$= B^T J(s_0C+G)^{-1}(C(-(s_0C+G)^{-1}))^i JB$$
$$\overset{(R4)}{=} B^T(s_0C+G)^{-1}(C(-(s_0C+G)^{-1}))^i B$$
$$\overset{(R5)}{=} B^T(-(s_0C+G)^{-1}C)^i(s_0C+G)^{-1}B$$
$$= \mathbf{m_i}(s_0).$$

It follows from (16.6) that $M_I = M_O$, such that

$$V^T_{I_{r_i}} = V^T_{O_{ro}} = V^T_r \tag{16.17}$$

with the help of (16.7). It is easy to see that

$$\widehat{H}(s) = V_r B^T_r (G+sC)^{-1} B_r V^T_r, \tag{16.18}$$

with B_r analogously to (16.9). Hence, if the model reduction method used in (16.12) leads to a positive real transfer function of the reduced-order model, passivity is preserved. □

Remark 1 As a simple numerical example we show a RC circuit called *rc549*, which is also investigated in [21] and the Section by Antoulas/Lefteriu in this book. It is provided by the Infinoen Technologies AG, Munich, Germany. The order of the corresponding descriptor system is $n = 141$, the number of terminals is $m = 70$. Due to simplicity we use $H_{DC} = m_0(s=0)$ as basis for the matrices in (16.6) and we abstain from the reduction step in (16.12). Following Definition 2, we show that the Hermitian part of the transfer function on the imaginary axis is positive semi-definite, i.e., $H_H = H(j\omega) + H(j\omega)^* \geq 0$. Figure 16.1 shows the

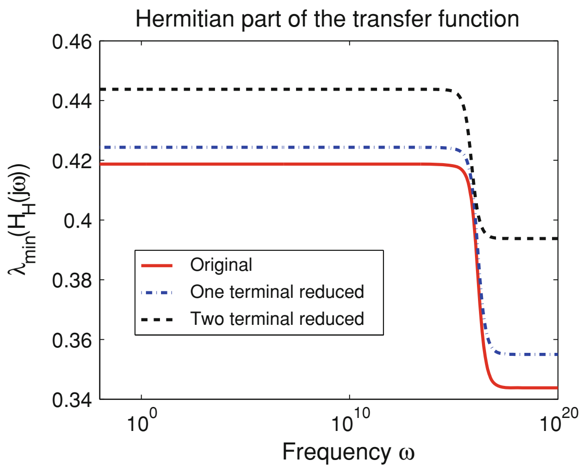

Fig. 16.1 Smallest eigenvalues of the hermitian part of the transfer function of *rc549* depending on the frequency ω

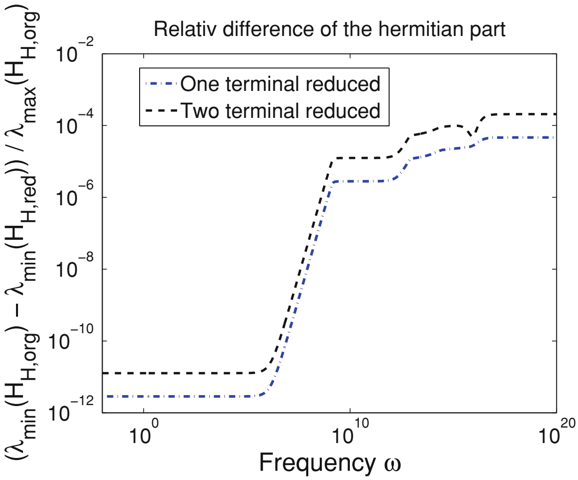

Fig. 16.2 Relative difference correlated to the largest eigenvalue of the original transfer function

smallest eigenvalue of H_H of the original transfer function and of the terminal reduced transfer functions $H_r(s)$. Although the frequency range is much larger than important in applications, this smallest eigenvalue grows depending on how much terminals are reduced. The relative difference of these smallest eigenvalues of the reduced systems to those of the original system is shown in Fig. 16.2. We recognize that this difference depends on the spectrum of the eigenvalues of $H(s)$ but does not play a too important role in the aspect of passivity preserving. Note, that we add numerically zero eigenvalues in (16.13) which do not destroy the positive semi-definiteness of the transfer function.

16.3.3 Reciprocity

Another important property of MOR methods is reciprocity preserving, which is a requirement for synthesis of the reduced order model as circuit. We assume the same setting as in Sect. 16.3.2. An appropriate definition can be found, e.g., in [19].

Definition 3 A transfer function (16.14) is *reciprocal* if there exists $m_1, m_2 \in \mathbb{N}$ with $m_1 + m_2 = m$, such that for $\Sigma_e = \text{diag}(I_{m_1}, -I_{m_2})$ and all $s \in \mathbb{C}$ where $H(s)$ has no pole holds

$$H(s)\Sigma_e = \Sigma_e H^T(s).$$

The matrix Σ_e is called *external signature* of the system. A descriptor system is called reciprocal if its transfer function is reciprocal.

As a consequence, a transfer function of a reciprocal system has the form

$$H(s) = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ -H_{12}^T(s) & H_{22}(s) \end{bmatrix}, \tag{16.19}$$

where $H_{11}(s) = H_{11}^T(s) \in \mathbb{R}^{m_1,m_1}$ and $H_{22}(s) = H_{22}^T(s) \in \mathbb{R}^{m_2,m_2}$, which means that the transfer function is some kind of symmetric.

Theorem 3 *Consider a reciprocal system of the form* (16.15). *The ESVDMOR reduced system* (16.13) *is reciprocal iff the inner reduction* (16.12) *is reciprocity preserving.*

Proof Due to the reciprocity of the original system, the corresponding transfer function (16.14) has the structure given in (16.19). Equation 16.18 shows that none of the steps in ESVDMOR destroy this symmetric structure if (16.12) preserves reciprocity. □

16.4 Remarks and Outlook

We have reviewed the ideas of ESVDMOR and that this approach is stability, passivity, and reciprocity preserving under reasonable assumptions described in Sect. 16.3.2. As a direction of future research, it would be desirable to have a proof of passivity preservation for more general settings, e.g., a more general structure than in (16.15).

In [5] it is pointed out that an explicit computation of the moments in (16.3) would be numerically unstable and too expensive. In order to avoid this problem, a numerical solution by using the truncated SVD based on the implicitly restarted Arnoldi method is presented there. As an alternative, we also investigate the Jacobi–Davidson approach (JDSVD) [12] for computing the TSVD. In this context, the computation of the residual and the solution of the correction equation within JDSVD are based on iterative methods. This offers a large potential for increased efficiency in ESVDMOR due to the usual structures of circuit equations that can be exploited here.

An automatic error control which regulates the number of significant singular values as well as the best choice of the decomposition method and the number of used moments are other interesting topics for future research.

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