Chapter 13
Model Reduction Methods for Linear Network Models of Distributed Systems with Sources

Stefan Ludwig and Wolfgang Mathis

Abstract Distributed systems with distributed sources are modeled as large electrical networks with linear RLC-elements, independent sources and pins for the connection with other network models. Often these networks are too large to be simulated efficiently. Model reduction can be used to reduce these networks while approximating the behavior at the pins for the connection with other nonlinear and linear network models. In the standard model reduction the independent sources are extracted and connected by ports with the RLC-part of the network which is to be reduced. This extraction only enables a weak reduction for networks with a large number of independent sources, as the number of ports is very high. In this article an efficient reduction of networks with a large number of sources is proposed by taking the waveforms of the sources into account. The method is based on the reduction of the dimension of the function space of the waveforms with the help of function approximation. The basis functions of the function approximation are used in the network, which results a lower number of independent sources. Extracting this lower number of independent sources and reducing the network enables a higher model reduction with state of the art techniques. With the proposed method a smaller size and higher accuracy of the reduced model can be achieved. The validity and efficiency of the proposed method is shown by reducing example models.

Keywords Reduced order modeling · Linear RLC networks with sources · Generalized state space systems · Terminals · Impedance · Modified model analysis

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P. Benner et al. (eds.), *Model Reduction for Circuit Simulation*,
Lecture Notes in Electrical Engineering, 74, DOI: 10.1007/978-94-007-0089-5_13,

13.1 Introduction

In the modeling of distributed systems very large linear RLC-networks are created. For these very large networks a smaller network with approximated behavior at some nodes of interest, typically the pins for the connection with other network models, is required to enable fast simulations. With the help of model order reduction (MOR) and network synthesis it is possible to find such a reduced network [3, 7, 10, 24]. Replacing the original network with the reduced network enables investigations with a lower effort for simulations.

For distributed systems with distributed sources the modeled networks also contain, next to passive RLC-elements a high number of independent sources. These models of distributed systems with distributed sources typically occur in the IC design and verification as for example in the modeling of power grids, substrate couplings and conducted emission behavior [14, 19, 23, 33]. In the standard approach the independent sources are extracted from the reducible RLC-part of the network and connected by ports. With this extraction only the passive RLC-part is reduced and passivity can be preserved during a MOR [2, 24, 28, 32]. For networks with a large number of independent sources this extraction produces a large number of ports of the network to be reduced. A large number of ports is a strong limitation for the MOR [20, 30]. In this article we will propose an additional step prior to the MOR to overcome this limitation. In this step the dimension of the function space of the waveforms of the independent sources is lowered. Realizing this smaller function space as network elements and replacing the original sources by controlled sources results in a lower number of independent sources. Extracting this lower number of independent sources leads to a lower number of ports of the reducible network. In addition, network properties like passivity and reciprocity are preserved in the reducible part. The behavior at the pins for the connection with other nonlinear elements like drivers and loads is preserved in this first port reduction step but the behavior of the distributed sources is approximated. The resulting reducible network with a lower number of ports can now more efficiently reduced with state of the art MOR techniques by approximating the behavior at the pins.

This article is structured as follows. In Sect. 13.2 the basics of model reduction are shown. In Sect. 13.3 a description of a large number of independent sources as additional step before an efficient model reduction is proposed. The efficiency of the complete reduction process is shown by reducing two example networks in Sect. 13.4. The article is concluded in Sect. 13.5.

13.2 Background for Model Reduction of Linear Networks

This article will focus on distributed systems modeled as linear positive real valued RLC-networks with independent current sources. The impedance description of the network is considered. However, all presented methods are also valid for the admittance or hybrid case and for networks with independent voltage sources.

For MOR the electrical circuit is described by differential algebraic equations. With the help of modified nodal analysis, circuit equations in the frequency domain of order N in the form of

$$(s\mathbf{C}+\mathbf{G})\mathbf{x}=\mathbf{B}\mathbf{u}, \quad \mathbf{y}=\mathbf{L}^{T}\mathbf{X} \tag{13.1}$$

are generated. The real positive definite matrices $\mathbf{C}, \mathbf{G} \in \mathbb{R}^{N\times N}$ contain the stamps for the capacitors, resistors and inductors. $\mathbf{u}, \mathbf{y}$ are the Laplace transformed p input currents and output voltages respectively, which are connected with the system vector $\mathbf{x}$ by the system matrices $\mathbf{B} \in \mathbb{R}^{N\times p}$ and $\mathbf{L} \in \mathbb{R}^{p\times N}$. The resulting impedance transfer function is given with

$$\mathbf{Z}(s)=\mathbf{L}^{\mathbf{T}}(s\mathbf{C}+\mathbf{G})^{-1}\mathbf{B}. \tag{13.2}$$

With the help of MOR a reduced system in the form of Eq. 13.1 is built by approximating the transfer function behavior. MOR algorithms based on the projection are commonly used for the reduction of large models. The advantage of the projection methods is the simple implementation and the low computational effort for the construction of the reduced model. For the reduction a projection matrix $\mathbf{T} \in \mathbb{R}^{N\times n}$ has to be generated. By changing the system vector $\widetilde{\mathbf{x}}=\mathbf{T}^{T}\mathbf{x}$ the reduced system matrices of order n

$$\widetilde{\mathbf{C}}=\mathbf{T}^{T}\mathbf{C}\mathbf{T}; \quad \widetilde{\mathbf{G}}=\mathbf{T}^{T}\mathbf{G}\mathbf{T}; \quad \widetilde{\mathbf{B}}=\mathbf{T}^{T}\mathbf{B}; \quad \widetilde{\mathbf{L}}=\mathbf{T}^{T}\mathbf{L} \tag{13.3}$$

are calculated [24]. If the projection matrix is real, this method preserves the passivity of suitably described systems [24]. For the generation of the projection matrix different methods are used.

The most common method is that the projection matrix is generated by using a single-point Krylov subspace [24]. It can be shown, that the moments of the reduced system match the moments of the original system to some order [24]. As an extension the union of multiple Krylov subspaces at different expansion points is used to generate the projection matrix [7], which is in the following named MP-Krylov. With this type of projection matrices the order of the reduced model is proportional to the number of ports. For models with a large number of ports, this represents a strong limitation for the possible order reduction [30].

Another approach for the generation of the projection matrix is proposed in [27] with Poor Man's Truncated Balanced Realization (PMTBR) using congruence transformation (Section V.E in [27]). An approximation to the controllability Gramian is generated and an approximation to the Hankel singular values is calculated. As shown in [28, 29] the approximated Hankel singular values are used for order and error control. With this projection matrix the order does not directly depend on the number of ports. However, there is an indirect dependence on the number of ports and the possible order reduction [27].

For investigations with electrical simulators the order reduced system is synthesized with network-synthesis algorithms as described for example in [20, 25, 35].

As these methods are only applicable to networks with a relatively low number of ports some advanced MOR methods dealing with networks with a large number of ports are developed. The underlying principles can be divided into two methods: using properties of the network structure and using input information. The methods presented in [9, 18] belong to the first class and use the correlation of the ports which is efficient if the models are regularly structured. The same class includes methods of dividing the network into substructures with a lower number of ports, as for example presented in [5]. For the class of methods where input information is used, for example the input correlated TBR method where the dependencies of the input excitations are taken into account in the model reduction process is presented in [31]. The reduced order analysis methods presented in [16, 17, 34] rely on determined inputs at all ports of the network, which is more likely for a simulation method as all inputs have to be determined and no connection with other nonlinear network elements is possible. The method presented in [21] uses the information of parameterized loads connected to the reducible network for a more efficient reduction with parametric MOR methods. In [20] a method based on the determined behavior of independent sources for reducing the number of ports is presented, which allows for the connection with other network models at some specified nodes but is limited to groups of internal sources with equal or proportional waveforms.

13.3 Description of Distributed Sources

In this article a generalization of the method of [20] is presented which is not restricted to groups of independent sources with proportional waveforms and includes waveforms that can be decomposed into or approximated by a small set of basis functions. Another advantage of this method is that unlike in [9, 18] no limitations for the structure of the linear network are necessary, Unlike in [16, 17, 34] not all inputs of the model have to be determined during the reduction which allows for the connection with other network models in simulations. The disadvantage is that only the number of ports where the input is determined is reduced and thereby the method is designated for networks with a large number of independent sources and a low number of pins for the connection with other network models. The method is an additional prior step to enhance the MOR.

In the standard description the independent sources are extracted from the RLC-network and connected through ports with the remaining network (Fig. 13.1). In this way the reducible part of the network contains only positive real valued RLC-elements and thereby the network is passive and reciprocal. Passivity can be preserved during MOR using adequate reduction algorithms [2, 24, 28, 32] as well as reciprocity [10]. For networks with a large number of independent sources, the disadvantage of this extraction is the large number of ports which limits the order reduction [8, 20, 30].

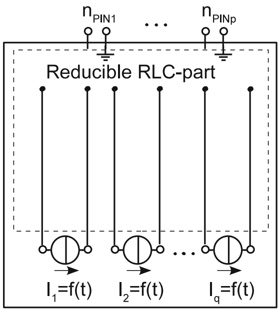


Fig. 13.1 Reducible network with extracted independent sources and a large number of ports

In this article a description of the sources in the network, which results in a lower number of independent sources while approximating the behavior at the pins is presented. The description of the sources is a prior step to the MOR and meets the following requirements: Firstly, preserve reciprocity and passivity of the reducible network in order to enhance network synthesis and guarantee stable time-domain simulations of the reduced model. Secondly, preserve properties of matrices $\mathbf{C}, \mathbf{G}$ like (semi-) definiteness for passivity-preserving reduction [10, 24] and (J-) symmetry and typical block structure to enhance network synthesis after the reduction [10, 25, 35]. The general structure of the network is shown in Fig. 13.2 for an RLC-network with q independent current sources.

Firstly we will have a look at the waveforms of the independent sources. A reduction of the dimension of the function space based on decomposition and approximation of the waveform functions is used. The q waveforms of the independent sources of the network are described by superposition with the weights $w_{i,l}$ of b basis functions

$$I_l \approx \sum_{i=1}^{b} w_{i,l} I_i(t) \quad 1 \le l \le q. \tag{13.4}$$

For this function approximation several methods are known in the fields of approximation theory, black box modeling and artificial neural networks. In the following some of the most often used methods are presented.

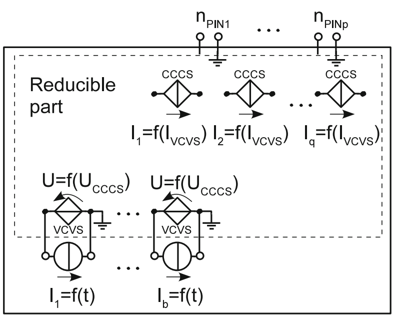


Fig. 13.2 Passive and reciprocal reducible network with replaced sources and a small number of ports

A special and most simple case is the grouping of equal or proportional waveforms as presented in [20], where one representative waveform of every group is used as basis function. The number of groups specifies the number of necessary basis functions.

For sources with waveforms described by continuous piecewise-linear (PWL) functions an exact decomposition into (finite) ramp functions is possible, e.g. described in [1]. The number of basis functions is now given by the number of points which are used for the waveform description. If the number of points describing the waveform is lower than the number of sources a reduction of the function space is achieved. If the PWL functions are periodic, the basis functions can also chosen as periodic ramp functions and thereby the number of basis functions is given with the number of data points in one period of the waveform. For discontinuous PWL functions the representations of [6] can be used. If the waveforms are piecewise constant functions, shifted bar functions can be used as basis functions.

If waveforms are specified in a limited range of time, for example if the waveforms are generated from measurements or modeling in a finite time range, several classes of basis functions used in the field of approximation theory and artificial neural network modeling can be used. For every waveform the approximating basis functions can be obtained as algebraic polynomials to any degree of accuracy as stated in the Weierstrass approximation theorem. An example is the well known Taylor series expansion. Radial basis functions which are used in the black box modeling are also suitable as basis functions, as they are universal approximators [11, 26]. Another class of basis functions are the sigmoid functions, which are almost always capable of universal approximation [13]. For periodic or time- or band-limited waveforms the approximation with a fourier series expansion can be used. The basis functions are then trigonometric functions. The series converges almost everywhere to the waveform functions which is stated by the Carleson–Hunt theorem and thereby a finite number of basis functions is sufficient.

The discrete wavelet transform, which is widely used in signal- and data processing can also be used for the reduction of the dimension of the function space. With this transformation one dimensional wavelets are used as basis function for the approximation as for example presented in [22].

For this approximative methods the number of necessary basis functions depends on the desired accuracy of the approximation of the waveforms. For the number of basis functions a tradeoff between the number of basis functions and the desired accuracy has to be found. For a higher accuracy a higher number of basis functions is necessary and the reduction of the dimension of the function space is lower.

Any other method of decomposition into or approximation by a finite number of basis functions is feasible, as long as the number of basis functions b is smaller than the number of independent sources q in the network. As for most approximation methods the number of basis functions depends on the desired accuracy and the number of basis functions has to be lower than the number of independent sources waveforms this method is most suitable for networks with a large number of independent sources.

The inclusion of the reduced dimension function space in the network model in the form of Fig. 13.2 is divided into the following four steps. Independently on the decomposition or approximation method for every basis function an independent current source with the basis waveform is added in the first step in the network model, resulting in b additional independent sources. In the second step all q original independent current sources are replaced by current controlled current sources (CCCS), controlled by the b additional independent sources. The gains of the CCCS are given by the weights $w_{i,l}$ in Eq. 13.4. In the third step, voltage controlled voltage sources (VCVS), connected with the additional independent sources, are used for the preservation of passivity and reciprocity if the gains are chosen according to the gains of the CCCS, as is shown later on in this section. In the last step, only the b additional independent current sources are extracted and connected through ports with the reducible part of the network. In this way a reducible network with a reduced number of ports is generated. The resulting reducible network also contains controlled sources in addition to the RLC-elements and is called network with replaced sources throughout the article.

In the following the description of the reducible network with replaced sources and a lowered number of ports is given, whereas the system matrices $\mathbf{C}, \mathbf{G}$ have the typical properties of pure RLC-networks like (J-)symmetry, (semi-)definiteness and block structure. An RLC-network with N nodes, p pins and q independent current sources, which waveforms are the superposition of b basis waveforms, is considered. For the generation of the system matrices of the network with replaced sources the incidence matrices $\mathbf{K}_C, \mathbf{K}_R, \mathbf{K}_L, \mathbf{K}_{\text{CCCS}}, \mathbf{K}_{\text{pin}}$ for capacitors, resistors, inductors, CCCSs and pins are used. The matrices $\widehat{\mathbf{P}}_{\text{CCCS}} \in \mathbb{R}^{q\times b}, \widehat{\mathbf{P}}_{\text{VCVS}} \in \mathbb{R}^{b\times q}$ contain the parameters of the gains of the controlled sources. The branch constitutive relations are given with

$$\begin{aligned} \mathbf{i}_C = s\widehat{\mathbf{C}}\mathbf{u}_C, \quad \mathbf{i}_R = \widehat{\mathbf{G}}\mathbf{u}_R, \quad \mathbf{u}_L = s\widehat{\mathbf{L}}\mathbf{i}_L \\ \mathbf{i}_{\text{CCCS}} = \widehat{\mathbf{P}}_{\text{CCCS}}\mathbf{i}_{\text{VCVS}}, \quad \mathbf{u}_{\text{VCVS}} = \widehat{\mathbf{P}}_{\text{VCVS}}\mathbf{u}_{\text{CCCS}}. \end{aligned} \tag{13.5}$$

With Kirchhoff's Current Law $\mathbf{K}\mathbf{i} = 0$ we get

$$\begin{pmatrix} \mathbf{K}_C & \mathbf{K}_R & \mathbf{K}_L & -\mathbf{K}_{\text{CCCS}} & -\mathbf{K}_{\text{pin}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{i}_C \\ \mathbf{i}_R \\ \mathbf{i}_L \\ \mathbf{i}_{\text{CCCS}} \\ \mathbf{i}_{\text{pin}} \\ \mathbf{i}_{\text{VCVS}} \\ \mathbf{i}_b \end{pmatrix} = \mathbf{0} \tag{13.6}$$

with $\mathbf{I}$ as the identity matrix. Notably the network is divided into two parts which are not interchanging charges. The first part contains the RLC-elements, the CCCSs and the pins and the second part the VCVS and the additional independent sources. The voltages are defined as

$$\begin{pmatrix} \mathbf{u}_C \\ \mathbf{u}_R \\ \mathbf{u}_L \\ \mathbf{u}_{\mathrm{CCCS}} \\ \mathbf{u}_{\mathrm{pin}} \\ \mathbf{u}_{\mathrm{VCVS}} \end{pmatrix} = \begin{pmatrix} \mathbf{K}_C^T & \mathbf{0} \\ \mathbf{K}_R^T & \mathbf{0} \\ \mathbf{K}_L^T & \mathbf{0} \\ \mathbf{K}_{\mathrm{CCCS}}^T & \mathbf{0} \\ \mathbf{K}_{\mathrm{pin}}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{u}^{\phi} \\ \mathbf{u}_{\mathrm{VCVS}}^{\phi} \end{pmatrix} \tag{13.7}$$

where $\mathbf{u}^{\phi}$ contains the potentials of the first part of the network and $\mathbf{u}_{\mathrm{VCVS}}^{\phi}$ contains the potentials of the second part of the network. We define a system of circuit equations with

$$\overbrace{\begin{pmatrix} s\mathbf{K}_C\widehat{\mathbf{C}}\mathbf{K}_C^T + \mathbf{K}_R\widehat{\mathbf{G}}\mathbf{K}_R^T & \mathbf{K}_L \\ -\mathbf{K}_L^T & s\widehat{\mathbf{L}} \end{pmatrix}}^{(s\mathbf{C}+\mathbf{G})} \overbrace{\begin{pmatrix} \mathbf{u}^{\phi} \\ \mathbf{i}_L \end{pmatrix}}^{\mathbf{x}} = \overbrace{\begin{pmatrix} \mathbf{K}_{\mathrm{CCCS}}\widehat{\mathbf{P}}_{\mathrm{CCCS}} & \mathbf{K}_{\mathrm{pin}} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}}^{\mathbf{B}} \overbrace{\begin{pmatrix} \mathbf{i}_b \\ \mathbf{i}_{\mathrm{pin}} \end{pmatrix}}^{\mathbf{u}} \tag{13.8}$$

$$\underbrace{\begin{pmatrix} \mathbf{u}_{\mathrm{VCVS}} \\ \mathbf{u}_{\mathrm{pin}} \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} \widehat{\mathbf{P}}_{\mathrm{VCVS}}\mathbf{K}_{\mathrm{CCCS}}^T & \mathbf{0} \\ \mathbf{K}_{\mathrm{pin}}^T & \mathbf{0} \end{pmatrix}}_{\mathbf{L}^T} \underbrace{\begin{pmatrix} \mathbf{u}^{\phi} \\ \mathbf{i}_L \end{pmatrix}}_{\mathbf{x}} \tag{13.9}$$

Since the currents $\mathbf{i}_b$ and $\mathbf{i}_{\mathrm{pin}}$ are the inputs $\mathbf{u}$ of the system, the voltages $\mathbf{u}^{\phi}$ and inductor currents $\mathbf{i}_L$ form the system vector $\mathbf{x}$ and the voltages across the additional independent sources $\mathbf{u}_{\mathrm{VCVS}}$ and at the pins $\mathbf{u}_{\mathrm{pin}}$ are the outputs $\mathbf{y}$, the system has the form of equations (13.1) with $\mathbf{C}, \mathbf{G}$ having the typical MNA properties of pure RLC-networks without controlled sources. The controlled sources only appear in the matrices $\mathbf{B}$ and $\mathbf{L}$. The gains $\widehat{\mathbf{P}}_{\mathrm{VCVS}}$ of the VCVSs are now chosen with

$$\widehat{\mathbf{P}}_{\mathrm{VCVS}} = \widehat{\mathbf{P}}_{\mathrm{CCCS}}^T \tag{13.10}$$

and thereby the condition $\mathbf{B} = \mathbf{L}$ holds. With this description it is shown, that the controlled sources which normally influence the matrix $\mathbf{G}$ can be described in a way, where only $\mathbf{B}, \mathbf{L}$ are changed and properties of $\mathbf{C}, \mathbf{G}$ like definiteness, symmetry and structure of pure RLC-networks are conserved, enhancing the reduction.

This system of the network with replaced sources now has, in addition to the ports for the pins, only b ports for the independent sources which is less than the number of ports q if all sources were extracted. In the following, it is shown that this reducible network is passive and reciprocal.

Theorem 1 *The network with replaced sources is passive.*

Proof For the proof of passivity, the conditions for passivity have to be considered. The necessary and sufficient condition for the passivity of the impedance transfer function $\mathbf{Z}(s)$ of the model is positive-realness [15]:

1. $\mathbf{Z}(s^*) = \mathbf{Z}^*(s)$ where * is the conjugate complex operator
2. $\mathbf{Z}(s)$ is positive, that means $\mathbf{a}^H[\mathbf{Z}(s) + \mathbf{Z}^H(s)]\mathbf{a} \geq 0$, with H as Hermitian operator is satisfied for all complex s with $Re(s) \geq 0$ and for all finite complex vectors $\mathbf{a}$

Since the matrices stay real while replacing the independent current sources, the first condition is still fulfilled.

The second condition of positive-realness can be written as

$$\mathbf{a}^H[\mathbf{Z}(s) + \mathbf{Z}^H(s)]\mathbf{a} = \mathbf{a}^H\mathbf{L}^T[(s\mathbf{C} + \mathbf{G})^{-1} + (s\mathbf{C} + \mathbf{G})^{-H}]\mathbf{Ba}. \tag{13.11}$$

With $\mathbf{B} = \mathbf{L}$ a finite complex vector $\mathbf{b} = \mathbf{B}a = \mathbf{L}a$ is defined and the transfer function with replaced independent current sources is positive

$$\mathbf{b}^H[(s\mathbf{C} + \mathbf{G})^{-1} + (s\mathbf{C} + \mathbf{G})^{-H}]\mathbf{b} \geq 0. \tag{13.12}$$

Since the controlled sources are described in a way that does not influence the system matrices they stay positive definite and real while replacing the independent sources. This means that the passivity of the network is conserved, as long as $\mathbf{L} = \mathbf{B}$ holds, which is guaranteed by Eq. 13.10. □

Theorem 2 *The network with replaced sources is reciprocal.*

Proof For the proof of reciprocity, the condition for reciprocity has to be considered. The condition for a model to be reciprocal is that the impedance transfer function is symmetrical [4, 15]:

$$\mathbf{Z}(s) = \mathbf{Z}^T(s). \tag{13.13}$$

This condition can be written as

$$\mathbf{L}^T(s\mathbf{C} + \mathbf{G})^{-1}\mathbf{B} = \mathbf{B}^T(s\mathbf{C} + \mathbf{G})^{-T}\mathbf{L}, \tag{13.14}$$

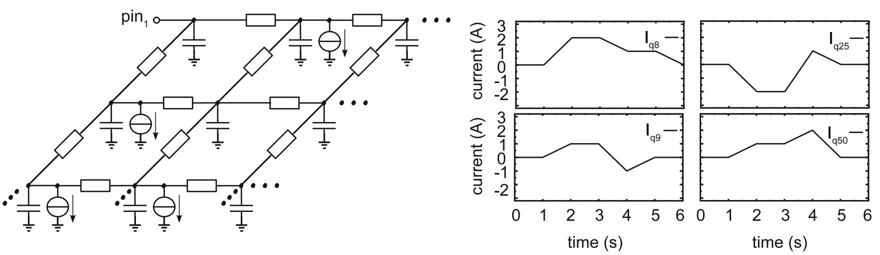
which is fulfilled since system matrices $\mathbf{C}$ and $\mathbf{G}$ can be written in symmetric form by multiplying $\mathbf{C}, \mathbf{G}$ with an appropriate diagonal matrix $\mathbf{J}$ and with Eq. 13.10 the condition $\mathbf{B} = \mathbf{L}$ still holds. □

13.4 Examples

13.4.1 RCI-Grid

As an example, an RCI-grid network as shown in Fig. 13.3a which is widely used in power grid simulations [14, 23] is utilized. All reductions and simulations of this example are performed on an actual personal computer platform.[1] The example

[1] CPU: Core2Duo 2.66 GHz, RAM: 2GB



(a) Section of the network structure **(b)** Some of the 55 current sources waveforms

Fig. 13.3 Example 50 × 50 RCI-grid network. **a** Section of the network structure. **b** Some of the 55 current sources waveforms

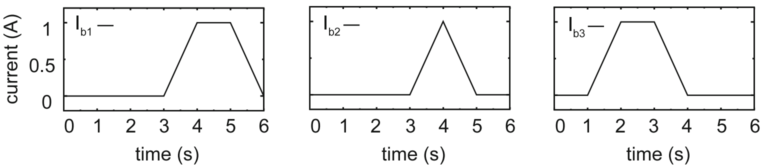


Fig. 13.4 The three basis waveforms of the RCI-grid example using decomposition

grid with 50 × 50 nodes contains 4900 resistors and 2500 capacitors and two pins at two opposite corners for the connection of nonlinear driver models.

The values of the elements are randomly chosen in the ranges $1 \pm 0.5\,\Omega$ and $0.1 \pm 0.05\,\mathrm{F}$ for the resistors and capacitors, respectively. $q = 55$ independent current sources with PWL-waveforms are connected between arbitrary chosen nodes in the network and the ground node. Some PWL-waveforms used in the example are shown in Fig. 13.3b. All PWL-waveforms are decomposed into the three basis-waveforms shown in Fig. 13.4.

With the standard method of extracting all independent sources the system has 57 ports. Only three independent sources are necessary if the waveforms of the independent sources are decomposed into the three basis-waveforms shown in Fig. 13.4 and the network with replaced sources of Sect. 13.3 is used. This results in five ports of the reducible part of the network. By this network alteration the behavior at the two pins of the model is not changed as the decomposition is exact. The transfer function of the system of the network with replaced sources can be calculated almost four times faster before the reduction with MOR due to the lower number of ports (Table 13.1). The network with extracted sources as well as the network with replaced sources are reduced with the PMTBR algorithm as described in Sect. 13.2. The 25 frequency points used in the reduction process are equally distributed in the frequency range $10^{-9} \leq s_i \leq 10^{9}$. The reduced systems are generated within less than one minute, which is not critical as the model reduction process is done offline before the simulation. In Fig. 13.5 the approximated Hankel singular values of both systems are shown. The singular values of the system with replaced sources decay faster. This means, that for a given

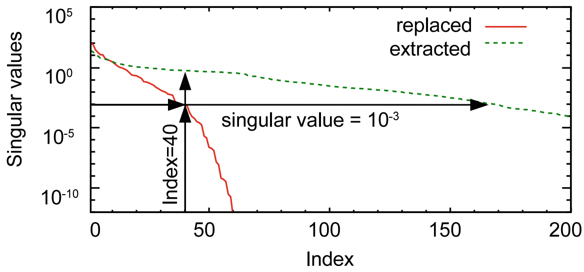


Fig. 13.5 Hankel singular values approximated with PMTBR for the 50 × 50 RCI-grid

Table 13.1 Comparison of reduction results of the 50 × 50 RCI-grid

Original order, $N = 2{,}500$	Ports, p	Reduced order, n	Reduction (%)	Reduction time (s)	MATLAB speed-up	Accuracy
Extracted sources	57	2,500			1×	Exact at the pins
Extracted sources, reduced with PMTBR	57	165	93.4	30	9.4×	Very good
Extracted sources, reduced with PMTBR	57	40	98.4	30	42×	Bad
Replaced sources	5	2,500			3.8×	Exact at the pins
Replaced sources, reduced with PMTBR	5	40	98.4	2	52×	Very good

maximum error the network with replaced sources can be more efficiently reduced than the network with extracted sources. Likewise for a given reduced order the reduced model of the network with replaced sources is more exact than the reduced model of the network with extracted sources. Firstly both systems are reduced to a maximal singular value of 10^{-3}, resulting in an order of $n = 40$ for the system with replaced sources and $n = 165$ for the system with extracted sources. The MATLAB-speed-up of both reduced systems for the calculation of the transfer function in several frequency points is shown in Table 13.1. Both reduced systems with the maximal singular value of 10^{-3} show a good agreement with the original system, as shown for the transfer function Z_{11} in Fig. 13.6. However, since the reduced system of the network with replaced sources is smaller, it can be simulated faster than the reduced system of the network with extracted sources (52× in comparison to 9.4×, Table 13.1). Secondly, both systems are reduced to the same order of $n = 40$. The speed-up of the system of the network with extracted sources is almost as high as for the system of the network with replaced sources (42× and 52×, Table 13.1). Nevertheless, the reduced system of the network with replaced sources is more accurate (Fig. 13.6) than the reduced system of the network with extracted sources.

With this example it is shown that the proposed replacing of independent sources as an additional step before the reduction can lead to smaller and therefore faster models than the standard method of extracting the sources. It was also

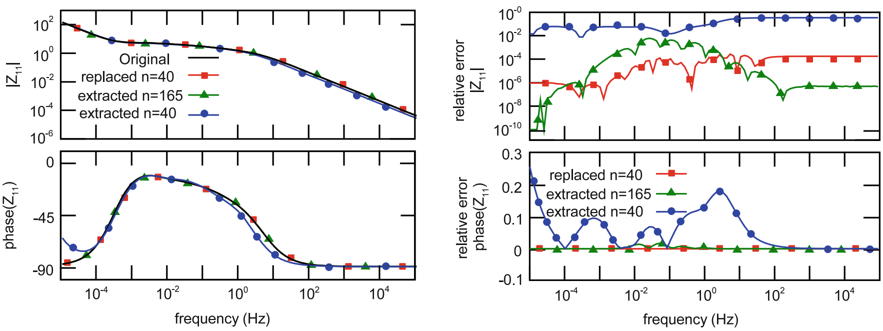
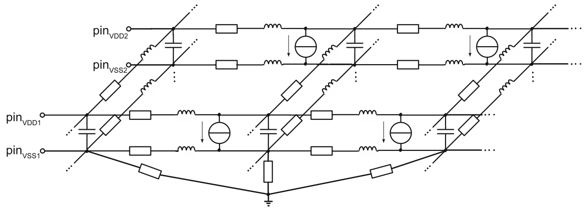


Fig. 13.6 Magnitude and phase of transfer function Z_{11} of 50 $\times$ 50 RCI-grid

Fig. 13.7 Section of the IC conducted emission network



shown that for the same size of the reduced system our proposed method can lead to more accurate reduced models.

13.4.2 IC Conducted Emission Model

As a second example for a network with a large number of independent sources an industrial model describing the IC conducted emission (ICEM) behavior of the Infineon 32 Bit microcontroller TC1796 is utilized [33]. All reductions and simulations of this example are performed on an actual server platform.[2] The network is generated with the Infineon Technologies AG Germany EXPO-Tool [12]. In Fig. 13.7 a small section of the network is shown. The model contains more than 35,000 passive RLC-elements and 328 independent current sources. For the connection with the printed circuit board model the network contains 61 pins. The circuit equations of the complete model have an order of 25,062.

If every independent source is extracted and connected through a port with the reducible part of the network, the circuits equations contain 389 ports including the 61 pins. With respect to their prescribed waveforms the current sources of the network are grouped into eight groups and therefore eight additional independent sources are necessary. Replacing the 328 independent sources with CCCSs and

[2] CPU: 8$\times$ DualCore, Intel Harwich Platform, RAM: 64GB

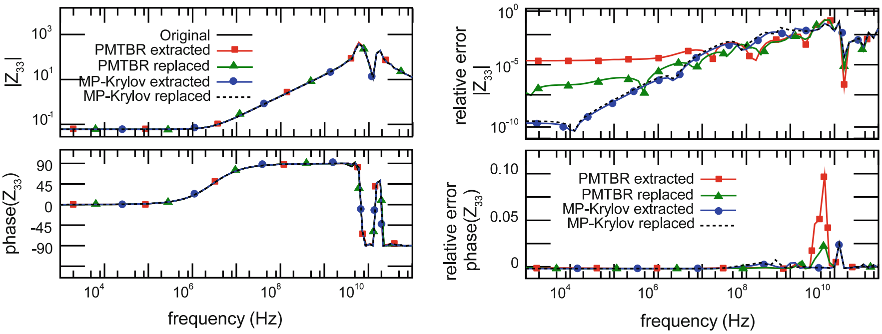


Fig. 13.8 Magnitude and phase of transfer function Z_{33} of ICEM

extracting the eight additional independent sources, as described in Sect. 13.3, results in a reducible network with only 69 ports, including the 61 pins. The network with extracted as well as the network with replaced sources are reduced with the MOR algorithms described in Sect. 13.2. The results for the order reduction are presented in Table 13.2. All models show a close alignment in the frequency range of interest as shown for the arbitrary chosen transfer function Z_{33} in Fig. 13.8.

For the MP-Krylov reduction two expansion points, $s_1 = 2\pi 10^3, s_2 = 2\pi 10^9$ are used where in every point one moment is to be matched. For preservation of reciprocity the projection matrices are split according to [10]. As the reduced models are generated once and used often the necessary time of below one minute for the reduction process is not critical. As there is a direct dependency of the order of the reduced system on the number of ports with MP-Krylov, there is a significant difference in the reduced order for both systems. Since the system of the network with extracted sources has more than five times as many ports as the system of the network with replaced sources, the reduced model is over five times the size of the reduced model of the network with replaced sources. This leads to much faster simulations of the reduced model of our proposed method for the calculation of the transfer function in several frequency points with MATLAB (132× in comparison to 2.8×, Table 13.2).

Fig. 13.9 Hankel singular values approximated with PMTBR for the ICEM

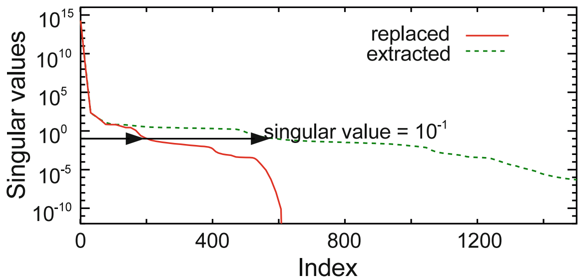


Table 13.2 Comparison of the reduction results of the ICEM

Original order, $N = 25{,}062$	Ports, p	Reduced order, n	Reduction (%)	Reduction time	MATLAB speed-up	Accuracy
Extracted sources, reduced with MP-Krylov	389	1,556	93.8	30 s	2.8×	Very good
Replaced sources, reduced with MP-Krylov	69	276	98.9	10 s	132×	Very good
Extracted sources, reduced with PMTBR	389	590	97.6	2 h	33×	Good
Replaced sources, reduced with PMTBR	69	201	99.2	5 m	188×	Good

For PMTBR 20 frequency points are equally distributed in the range $10^1 \leq s_i \leq 10^{10}$. For the reduction prior to the simulation of the system of the network with extracted sources around five minutes and for the system of the network with replaced sources around two hours are necessary. With reduction using PMTBR the reduced order does not directly depend on the number of ports and the reduced order is chosen according to the Hankel singular values. The approximation to the Hankel singular values calculated during PMTBR is shown in Fig. 13.9. It can be seen, that the singular values for the network with replaced sources decay faster than for the network with extracted sources. The largest singular value is set to 10^{-1}, which results in a reduced order of 201 and 590 for the replaced and extracted sources network, respectively. With this MOR algorithm the reduced model of the network with replaced sources is three times smaller than the reduced model of the network with extracted sources and the achieved speed-up in calculating the transfer function behavior with MATLAB is 188× in comparison to 33× (Table 13.2).

With this IC emission model example is has been shown that the proposed network with replaced sources can lead to much smaller and therefore faster reduced models than the standard method of extracting all sources.

13.5 Conclusion

In this article an efficient description of independent sources in electrical networks that can achieve a higher model reduction is presented. The dimension of the function space of the waveforms of the sources is reduced with the help of function decomposition and approximation. A network with a reduced number of independent sources, whereas the behavior at the pins of the network is preserved is generated. This network can be more efficiently reduced with practically all model reduction techniques. The validity and superior efficiency of the proposed method compared to standard methods is shown by reducing two example networks.

References

1. Aliprantis, C., Harris, D., Tourkey, R.: Continous piecewise linear functions. Macroecon. Dyn. **10**, 77–99 (2005)
2. Antoulas, A.: A new result on passivity preserving model reduction. Syst. Control Lett. **54**(4), 361–374 (2005)
3. Antoulas, A.C.: Approximation of Large-Scale Dynamical Systems. Society for Industrial and Applied Mathematics, New York (2005)
4. Belevitch, V.: Classical Network Theory. Holden-Day, San Francisco (1968)
5. Benner, P., Feng, L., Rudnyi, E.: Using the superposition property for model reduction of linear systems with a large number of inputs. In: International Symposium on Mathematical Theory of Networks and Systems (2008)
6. Chua, L., Kang, S.: Section-wise piecewise-linear functions: canonical representation, properties, and applications. Proc. IEEE **65**, 915–929 (1977)
7. Elfadel, I.M., Ling, D.D.: A block rational Arnoldi algorithm for multipoint passive model-order reduction of multiport RLC networks. In: International Conference Computer-Aided Design (1997)
8. Feldmann, P.: Model order reduction techniques for linear systems with large number of terminals. In: Design, Automation and Test in Europe (2004)
9. Feldmann, P., Liu, F.: Sparse and efficient reduced order modeling of linear subcircuits with large number of terminals. In: International Conference on Computer-Aided Design of Integrated Circuits and Systems (2004)
10. Freund, R.W.: SPRIM: structure-preserving reduced order interconnect macromodelling. In: IEEE/ACM International Conference on Computer Aided Design (2004)
11. Hartman, E., Keeler, J., Kowalski, J.: Layered neural networks with gaussian hidden units as universal approximations. Neural Comput. **2**, 210–215 (1990)
12. Hesidenz, D., Steinecke, T.: Chip-package emi modeling and simulation tool 'EXPO'. In: International Workshop on Electromagnetic Compatibility of Integrated Circuits, Munich, Germany (2005)
13. Hornik, K., Stinchcombe, M., White, H.: Multilayer feedforward networks are universal approximators. Neural Netw. **2**, 359–366 (1989)
14. Kozhaya, J., Nassif, S., Najm, F.: A multigrid-like technique for power grid analysis. IEEE Trans. Comput. Aided Design Integr. Circuits Syst. **21**, 1148–1160 (2002)
15. Kuh, E.S., Rohrer, R.A.: Linear Active Networks. Holden-Day, San Francisco (1967)
16. Lee, Y.M., Cao, Y., Chen, T.H., Wang, J., Chen, C.C.-P.: Hiprime: hierarchical and passivity preserved interconnect macromodeling engine for rlkc power delivery. In: International Conference on Computer-Aided Design of Integrated Circuits and Systems (2005)
17. Li, D., Tan, S.D., McGaughy, B.: ETBR: extended truncated balanced realization method for on-chip power grid network analysis. In: Design, Automation and Test in Europe (2008)
18. Liu, P., Tan, S.D., McGaughy, B., Wu, L., He, L.: Termmerg: an efficient terminal-reduction method for interconnect circuits. IEEE Trans. Comput. Aided Design Integr. Circuits Syst. **26**, 1382–1392 (2007)
19. Ludwig, S., Mathis, W.: Reduction of network models of parasitic coupling effects in mixed-signal VLSI circuits. In: International Symposium on Theoretical Electrical Engineering (2009)
20. Ludwig, S., Radić-Weissenfeld, L., Mathis, W., John, W.: Efficient model reduction of passive electrical networks with a large number of independent sources. In: IEEE International Symposium on Circuits and Systems (2008)
21. Ma, M., Khazaka, R.: Model order reduction with parametric port formulation. IEEE Trans. Adv. Packaging **30**, 763–775 (2007)
22. Mallat, S.: A Wavelet Tour of Signal Processing. Academic Press, New York (1999)
23. Nassif, S., Jozhaya, J.: Fast power grid simulation. In: Design Automation Conference (2000)

24. Odabasioglu, A., Celik, M., Pileggi, L.T.: PRIMA: passive reduced-order interconnect macromodeling algorithm. IEEE Trans. Comput. Aided Design Integr. Circuits Syst. **17**, 645–654 (1998)
25. Palenius, T., Roos, J.: Comparison of reduced-order interconnect macromodels for time-domain simulation. IEEE Trans. Microw. Theory Tech. **52**, 2240–2250 (2004)
26. Park, J., Sandberg, I.: Universal approximation using radial-basis-function networks. Neural Comput. **3**, 246–257 (1991)
27. Phillips, J.R., Silveira, L.M.: Poor mans's TBR: a simple model reduction scheme. IEEE Trans. Comput. Aided Design Integr. Circuits Syst. **24**, 43–55 (2005)
28. Phillips, J.R., Daniel, L., Silveira, L.M.: Guaranteed passive balancing transformations for model order reduction. IEEE Trans. Comput. Aided Design Integr. Circuits Syst. **22**(8), 1027–1041 (2003)
29. Phillips, J.R., Zhu, Z., Silveira, L.M.: Model Order Reduction, chap. PMTBR: A Famliy of Approximate Principal-Components-Like Reduction Algorithms. Springer (2008)
30. Silva, J.M.S., Villena, J.F., Flores, P., Silveira, L.M.: Outstanding Issues in Model Order Reduction. Math. Ind. **11**, 139–152 (2007)
31. Silveira, L., Phillips, J.: Exploiting input information in a model reduction algorithm for massively coupled parasitic networks. In: Design Automation Conference (2004)
32. Sorensen, D.: Passivity preserving model reduction via interpolation of spectral zeros. Syst. Control Lett. **54**(4), 347–360 (2005)
33. Steinecke, T., Goekcen, M., Hesidenz, D., Gstoettner, A.: High-accuracy emission simulation models for VLSI chips including package and printed circuit board. In: IEEE International Symposium on Electromagnetic Compatibility, Honululu, Hawaii, USA (2007)
34. Wang, J., Nguyen, T.: Extended krylov subspace method for reduced orderanalysis of linear circuits with multiple sources. In: Design Automation Conference (2000)
35. Yang, F., Zeng, X., Su, Y., Zhou, D.: RLCSYN: RLC equivalent circuit synthesis for structure-preserved reduced-order model of interconnect. In: IEEE International Symposium on Circuits and Systems (2007)