# Chapter 1
# The Need for Novel Model Order Reduction Techniques in the Electronics Industry

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**Abstract** In this paper, we discuss the present and future needs of the electronics industry with regard to model order reduction. The industry has always been one of the main motivating fields for the development of MOR techniques, and continues to play this role. We discuss the search for provably passive methods, as well as passivity enforcement methods that are currently being developed. Structure preservation is another important research topic, for which new concepts are being developed. This also holds for the calculation of delays in long interconnect lines, a topic that leads to an entirely new type of methods. Topics that are still in their infancy are model order reduction for parameterized and nonlinear problems. We will discuss what the needs of the industry are in all of these fields, show specific applications and what has been achieved so far. The paper is meant as a guideline for future research, not as a detailed survey of existing methods.

## 1.1 Introduction

Both in the area of dynamical systems and control, and in numerical analysis, a wealth of model order reduction (MOR) techniques have been developed over the years. Balanced truncation, Krylov subspace methods, proper orthogonal decomposition and other SVD-based methods are just a few classes of methods that have been described. The electronics industry has been one of the main providers of

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motivating examples, and indeed model order reduction has found widespread use in this industry.

The starting point for MOR in the electronics industry is usually attributed to a method termed asymptotic waveform evaluation (AWE) [33]. The underlying idea of this method is simple, approximating the moments of the transfer function of the system. The idea being that moments will decay, so that calculating a sufficient but finite number of moments will eventually lead to an accurate approximation of the transfer function. Soon after its publication, it was realized that the method suffers from numerical problems that are similar to those encountered when applying the power method to calculate eigenvalues and eigenvectors of a system. The columns of the coefficient matrix generated to calculate the solution will tend more and more towards the eigenvector corresponding to the largest eigenvalue, which means that we will soon find this matrix to be very ill-conditioned. This is exactly what is observed when applying AWE: after 8–10 iterations, the process stagnates, and no additional useful information about moments is being generated. This stagnation is graphically illustrated in Fig. 1.1.

In order to overcome these numerical difficulties associated with AWE, Feldmann and Freund [13] proposed the use of Krylov subspace type methods. Within the area of numerical linear algebra, such methods had been suggested in the 1950s, and shown to be capable of avoiding the ill-conditioning of the coefficient matrices encountered in the power method. In fact, Lanczos already used a similar technique to determine eigenvalues and eigenvectors of symmetric matrices. Again, the idea is fairly simple: rather than generating a sequence of vectors that quickly become (almost) linearly dependent, after each iteration an orthogonalisation step is performed. The effect is that the same subspace is produced, but now with an orthonormal basis. Stated differently, information already contained in previous iterates is projected out of the newly generated vectors, and only the new information is taken into account. Not surprisingly, this also provided the solution to the problems arising in AWE. The method suggested by Feldmann and Freund was termed

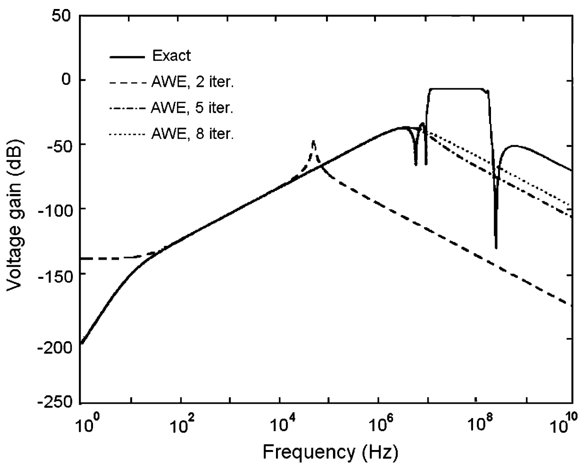



**Fig. 1.1** Stagnation of AWE method [13]

Padé-via-Lanczos (PVL) [13], and shown to match the moments of the transfer function. It demonstrated that Krylov subspace methods, developed mainly in the area of numerical linear algebra, can also be used to perform model order reduction.

With the advent of PVL, the way was paved for new developments in MOR. In the past decade, the electronics industry has continued to formulate challenging research topics in the field of model order reduction. Although PVL eliminated the problem of ill-conditioning of the coefficient matrix associated with the calculation of the moments of the transfer function, it turned out to lead to non-passive approximations in some cases. Indeed, it is fair to expect that reduced order models for passive systems should also be passive. Thus, the invention of PVL was soon followed by the search for provably passive Krylov subspace methods. In 1998, the first of such methods was published (PRIMA [32], based on the Arnoldi process), soon followed by the Laguerre-type methods [27, 30]. PVL itself was also remedied, although the SyPVL and SymPVL methods [15] do not seem to have found widespread use; this also holds for Laguerre-type methods. Other work that should be mentioned here is that by Antoulas and Sorensen on passivity preservation based on methods using spectral zeros, in which forward and adjoint systems are treated together [3].

Since the beginning of the new century, developments in MOR for problems in the electronics industry have been stepped up. Engineers formulated more and more constraints and requirements, thereby formulating many interesting topics for researchers in the field of model order reduction. An important question was, for example, how to assess the accuracy of methods. Or, equivalently, how can we guarantee that the reduced order model generated is a sufficiently accurate approximation of the original transfer function. For methods based on Krylov subspaces, this question has not been answered so far, so that the iterative process is usually stopped when the difference between subsequent iterations is below a certain threshold. Methods used in the area of dynamical systems and control, most prominently those based on Hankel singular values, are equipped with good and reliable error estimates. The counterside is that these methods involve the solution of Lyapunov equations, which is limited to fairly small systems. Hence, research is now concentrating on extending the applicability of these methods to larger systems, often making use of the sparsity [7].

Another constraint that was formulated fairly recently is the preservation of the structure of systems. Again, this is a natural requirement. This has led to the development of classes of structure-preserving methods, of which SPRIM [17] and structure-preserving PMTBR [14] are examples. Although these methods have provided new ideas to perform MOR for electronic systems, new developments are still needed in this area. One of the problems is that the methods suggested so far destroy the structure of the incidence matrix that relates the different types of unknowns (such as currents and voltages), leading to a fully dense system of couplings. This also holds for the method described by Vandendorpe and Van Dooren [45] for interconnected systems, although, at first glance, this method appears to preserve the sparse interconnections. Also interesting in this context is the paper [16].

Despite all efforts, many unsolved problems remain, and the electronics industry is anxiously waiting for solutions. In this paper, the demands of the electronics industry with respect to model order reduction will be detailed. In Sect. 1.2, we briefly sketch where these needs arise, and place the problem in its context. We start by discussing, in Sect. 1.3, the problems of passivity and realization, the latter meaning that the mathematically reduced order model should be cast into a physically realizable model mimicking properties of the original model. In Sect. 1.4 it will be shown that structure preservation is still not in a state that is acceptable to the business, and that different concepts and methods are needed.

Another important problem is that of the reduction of networks with many inputs and outputs, the topic of Sect. 1.5 Krylov methods cannot handle such problems, and methods that make use of singular value decompositions are not applicable in most cases. We will also discuss methods for delay equations, a fairly recent development in MOR for long interconnect lines. Section 1.6 gives some details on the requirements associated with this problem.

Last, but certainly not least, the state of nonlinear and parameterized model order reduction techniques is far from mature enough to help the industry cope with their problems in behavioral and response surface modeling. Section 1.7 discusses some of the issues, and the state-oft-the-art in this important research area. Future developments will probably be concentrated in this area. Section 1.8 is the conclusion of this paper, with a summary of the current and future needs of the electronics industry.

It should be noted that the discussion in this paper will not be exhaustive; several other problems in the electronics industry lead to requirements for model order reduction techniques, and often these are the subject of research. Furthermore, the main emphasis will be on Krylov subspace methods, whence some readers may feel that solutions can also be provided by methods originating from the area of dynamical systems and control theory. Besides not being exhaustive, the discussion will also not be elementary, in the sense that some background knowledge by the reader is required, both in the electronics industry and in the area of model order reduction. For the latter, this background information can conveniently be obtained from the surveys found in the books [4, 8, 37, 43]. For the former, a wealth of books is available, as well as many websites.

## 1.2 Mathematical Problems in the Electronics Industry

The electronics industry is dealing with the design of computer chips. As is well known, the development over the years can be fairly well described by Moore's Law, as displayed in Fig. 1.2. It has various possible interpretations, but the bottom line is that every 18 months a new generation of chips is borne that is twice as fast as the previous generation. The dimensions are dramatically reduced from generation to generation, and the operating frequencies rise exponentially.


**Fig. 1.2** Moore's law

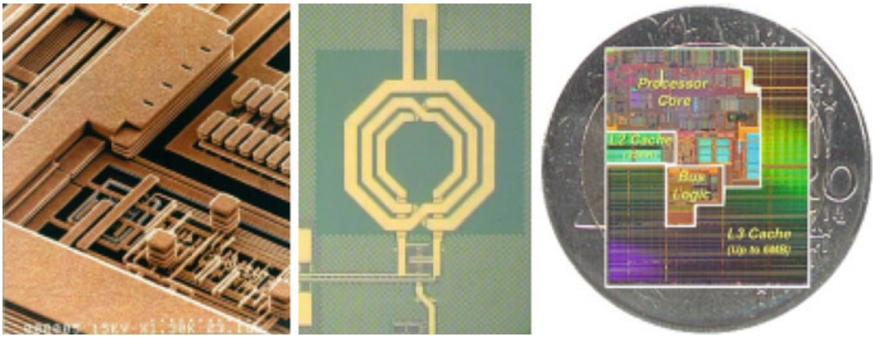



**Fig. 1.3** Illustration of the scale problem in the electronics industry

Dealing with computer chips means having to deal with complexity. Simulation of the behavior of semiconductor devices, electronic circuits and interconnect structures is a huge mathematical problem, in which we need to deal with scales that vary up to $10^{11}$. This scale problem is illustrated in Fig 1.3. In the left picture, we see a close-up of the wiring in the interconnect structure, with details of 100 nanometers. The middle picture shows a 50 GHz integrated inductor, for which the scale is in the order of 500 μm. The picture on the right shows a close-up of the Intel Itanium chip (source: McKinley, 2002) for which approximately 10 km of interconnect are needed to connect all components.

Fortunately, we see a development in numerical methods that is similar to Moore's law. Since the advent of iterative methods for solving linear systems, occurring at the core of all simulation programmes, the size of systems that can be solved has increased exponentially. Most notable developments are preconditioned conjugate gradient methods and the multigrid method. Figure 1.4 shows the development of these methods in a period of 30 years (see also [49]).

When considering the mathematical problems in more detail, we should distinguish various levels, as indicated by the scale issue discussed before. At the top

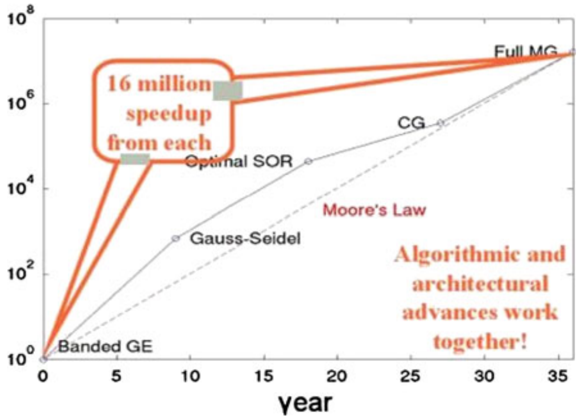



**Fig. 1.4** "Moore's law" for numerical methods in solving linear systems

level, we need to deal with electronic circuit simulation which encompasses large (millions) discrete networks containing resistors, capacitors, inductors, diodes, transistors, and other components. The discretisation of this system is fairly simple, as it obeys Kirchhoff's laws. The models used for active devices, such as diodes and transistors, are extremely nonlinear, meaning that non-standard methods are needed to solve the associated nonlinear systems. The linear systems generated in this process often have a hierarchical structure and are sparse, but may also be indefinite. Transient simulations are also non-trivial, due to the sudden changes occurring due to externally applied signals. AC analysis for high frequencies requires the solution of large complex linear systems. Pole-zero analysis and harmonic balance are also worth mentioning, but there are some comments to be made here. Normal AC analysis has a matrix $\omega C + G$ in which $\omega = i \cdot 2\pi f$. Clearly, here the size of the system equals the size of C and G (i.e. defined by the design). In Harmonic Balance more harmonics are involved in one coupled system. So here the number of harmonics influences the size of the system. In nonlinear problems high frequency effects may result in the need to increase the number of harmonics. In Periodic AC (PAC), i.e. studying small perturbations after linearizing around a PSS solution, two frequencies have to be taken into account: that of the PSS solution and the one of the perturbation of the right hand side. Usually, a high frequency PSS solution needs many time points for a proper time-domain representation and to be able to study the effects of the perturbations to sum and difference frequencies $f_{\text{osc}} \pm f_{\text{pert}}$. The reader is referred to [9, 26], where MOR is studied for systems arising in PAC analysis.

Not surprisingly, the development of robust and efficient methods for electronic circuit simulation has taken a considerable number of years (since the 1970s). Even nowadays it is certainly not a trivial task, with new developments still being published, and new simulation software containing the latest advances in numerical methods being brought to the market. The additional benefits of using MOR is also a key topic in recent years.

At a lower level, we encounter the simulation of individual semiconductor devices. This is again a complicated mathematical problem. Often, one uses the model consisting of three coupled partial differential equations, together constituting the drift-diffusion model. The system is singularly perturbed, meaning that special discretization techniques are needed to obtain accurate solutions on relatively coarse meshes. The resulting discrete system of equations is again extremely nonlinear, requiring special numerical methods for their solution. Damped Newton methods were developed, as well as nonlinear block Gauss-Seidel methods, the latter being known in the field as Gummel's method. Nonlinear variable transformations have also been shown to provide successful solution techniques. In the 1980s and 1990s, many researchers have been working on this problem, both from academia and within large electronics companies like IBM, AT&T, and Philips. Although the set of methods for the drift-diffusion equations is by now fairly well accepted and stationary [39], this is certainly not the case for the extended models such as the energy transport model or the hydrodynamic set of equations [2].

Although electronic circuit simulation and semiconductor device simulation are now at a fairly mature level, new problems have occurred in the mean time. At high frequencies, electromagnetic effects in the interconnect structure (see Fig. 1.5) may start to influence the behavior of circuits and devices. This leads to delay of signals, malfunctioning devices, and undesired effects in the substrate. The Maxwell equations need to be solved to address these issues, either by using a full wave solution using methods like FDTD, FIT, BEM, and FVM, or approximate methods like PEEC [35]. The resulting extremely large systems must be reduced to acceptable size to be coupled with the underlying circuit. These are complex linear systems, which are difficult to solve at high frequencies. The eigenvalues depend on the frequency, and iterative methods have difficulties to solve such systems.

An important problem for the discussion in subsequent sections is that, even if we would be able to solve such large systems by iterative or direct methods, circuit

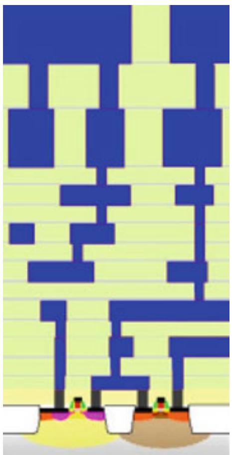


**Fig. 1.5** Interconnect structure above the silicon containing the circuit and devices

simulation programmes will not be able to cope with these large systems. The problem is that a coupled simulation needs to be carried out, and the commonly used way to obtain this coupling is to translate the discretized electromagnetic system into a linear circuit, and solve this in conjunction with the original non-linear electronic circuit using a circuit simulator. As this is not a feasible approach, one needs to first reduce the electromagnetic system to an acceptable size. This is one of the reasons why model order reduction has received much attention within the electronics industry. Reducing the electromagnetic system is indeed possible, as we are only interested in the dominant electromagnetic effects that influence the underlying circuit. Therefore, not all detail contained in the full discretized system is needed, and the use of model order reduction is fully justified.

Model order reduction is not only used for the reduction of the electromagnetic system generated by the interconnect structure, but also by the substrate region below the individual semiconductor devices. This area of a chip also needs to be co-simulated with the electronic circuit, in order to account for signal propagations taking place within the substrate region. Often, this leads to fairly simple resistor or RC networks, the problem being that these systems are again very large. Hence, model order reduction is needed to reduce these substrate systems to acceptable size, so that they can also be co-simulated with the electronic circuit.

Having demonstrated the need for model order reduction, in the remainder of this paper we will address several issues that are encountered when performing this task. Once again, we stress that the discussion is not exhaustive. There are many papers describing the research on suitable MOR methods used in the electronics industry, and the reader is referred to these for more information.

## 1.3 Passivity and Realizability

As described in the introduction, the Padé-via-Lanczos method [13] was the starting point for many new developments in numerical methods for model order reduction. As we also discussed, it was soon realized that the method can generate non-passive reduced order models despite the fact that the original system was passive. The search for provably passive methods was started, and methods like PRIMA [32] and Laguerre-type methods [27, 30] are now routinely used. Thus, the problem appears to have been solved satisfactorily, and no further research is needed.

However, the latter is not entirely the case. The first problem encountered in practice is the possibility that the discretisation of the original problem does not guarantee a passive system. This may be caused in electromagnetic simulations, for example, by a mesh that is too coarse. Another example is when using the boundary element method for the simulation of printed circuit boards, when the fourfold integrals have not been calculated to sufficient accuracy (this is especially crucial for the diagonal elements, which contain singular integrals). In these cases, there is little hope that the reduced system will be passive.

A related problem is that the reduced order model is solely based upon experimentally obtained results, for example measurements of S-parameters. Again, due to inaccuracy in the data, the state space system constructed from these may have poles in the wrong halve of the complex plane, and be non-passive. Again, this leaves not much hope for passivity of the reduced order system that should capture the dominant behavior of the experimentally generated data (see also [28]).

It should be noted that a tendency is observed to increasingly perform MOR on data obtained fully or partially from experiments. Antoulas and his group [31] have done work on Loewner matrices and shifted Loewner matrices, that can conveniently be used to generate reduced order models based on available data. These methods split the data into two disjoint sets, and then perform a computational procedure using these so-called Loewner matrices. The data may come from measurements, and can also be combined with simulation results. In fact, one could also perform a huge number of simulations, and use these for the set of data. These Loewner approaches lead to linear state space models that interpolate exactly [5, 28].

From the foregoing it is clear that passivity is certainly not a closed chapter in the book of MOR! However, the emphasis has shifted from provably passive methods to techniques for passivity enforcement. This topic is receiving quite some attention in recent years. It is also an important topic, as it is essential that rational models be passive in order to avoid unstable time domain simulations such as shown in Fig. 1.6. Thus, several researchers are concentrating on passivity enforcement methods, some from the numerical analysis point of view, others in the area of dynamical systems and control. For example, in [19], building on earlier work (2004), a general class of a posteriori passivity enforcement algorithms is considered based on iteratively perturbing the Hamiltonian matrix. The authors present a frequency-weighting scheme leading to the definition of a modified Gramian that, when employed during passivity enforcement, effectively leads to control of the relative error.

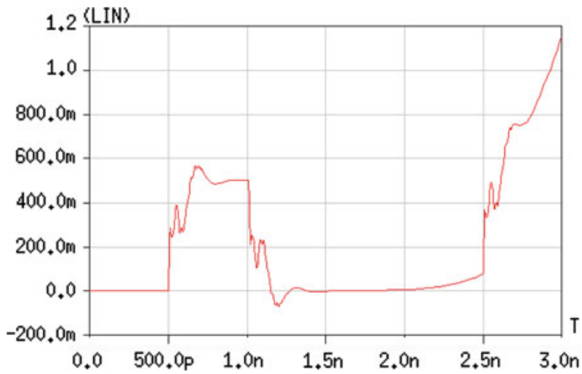



**Fig. 1.6** Unstable time domain simulation for non-passive reduced order model of a low pass filter (see also [44])

A different technique is described in [23], where a fast approach for passivity enforcement of pole-residue models is obtained by perturbing the eigenvalues of the residue matrices, as opposed to the existing approach of perturbing matrix elements. This leads to large savings in computation time with only a small increase of the modeling error. This fast residue perturbation (FRP) approach is merged with the modal perturbation technique, leading to fast modal perturbation (FMP). Usage of FMP over FRP achieves to retain the relative accuracy of the admittance matrix eigenvalues. A complete approach is obtained by combining the passivity enforcement step with passivity assessment via the Hamiltonian matrix eigenvalues and a robust iteration scheme, giving a guaranteed passive model.

Also worth mentioning is the work in [36]. A nice and recent survey of passivity enforcement methods is provided in [20], and also at the recent SPI conference [25] interesting new developments were presented. There are, however, some remaining issues that are important from the electronics industry point of view. Most methods developed so far trade accuracy for passivity. In other words, the reduced order model can be made passive only at the expense of lower accuracy. This is not surprising, as the model was constructed in such a way that its transfer function gives a sufficiently accurate approximation of the transfer function of the original system. By changing the location of only a few poles, namely those causing the non-passivity, the accuracy may be seriously affected. A 5% change in the S-parameters (to obtain values not exceeding 1) will often lead to at least 5% loss of accuracy. This is not acceptable from an engineering point of view. The only solution for this problem appears to be an optimization approach that specifies the constraints, and changes the location and residues of all poles, until both passivity and sufficient accuracy are obtained. Techniques suggested so far avoid this optimization problem by performing iterations on a limited number of poles, but we feel it is necessary to invest in the more expensive approach. If this is achieved, then the chapter of passivity can be closed, and the engineering community within the electronics industry will also be much happier.

A topic that is closely related to passivity is that of realizability. A well known theorem tells us that passive systems are automatically realizable. In his thesis, Brune (1931) shows that any passive transfer function can be realized by some passive network. He also provides rules for the construction of such networks, a drawback being that he is forced to employ transformers with perfect coupling. From an engineering point of view, this is undesirable, and it is more attractive to use so-called Foster synthesis, dating back to Foster (1924).

Although this may, therefore, appear to be a problem solved long ago (see also [21] for a survey dating back to 1957), again this is not the case at closer inspection. Foster realization (or similar techniques) often leads to an extremely large number of resistive, capacitive and/or inductive elements. Thus, the size of the reduced order model is dramatically increased after realization. This is mainly due to the fact that the matrices in the reduced order model often have lost sparsity, so that all unknowns in the reduced order model are mutually coupled. Indeed, this dense structure is one of the stumbling blocks for efficient circuit synthesis of reduced order models. Other obstacles are that there may be negative values of resistors, capacitors and inductors, and, last but absolutely not least, that the methods often work only for single input single output (SISO) systems. For multiple input multiple output (MIMO) systems, the approaches often fail, or have not even been described for this case.

Recently, a method that makes use of "un-stamping" was published by Yang [50]. The method is termed RLCSYN, and heavily relies on block structure preserving MOR methods (see next section). This may indeed be the only way to avoid the mutual coupling between all elements in the reduced order system. So far, however, experiences with the method are lacking, and time will tell whether it is the solution to the aforementioned stumbling blocks.

A point of discussion in the electronics industry is, of course, whether or not one needs to realize/synthesize the reduced order models. The reasoning is simple: suppose we have an electronic circuit, and are able to reduce it using mathematical techniques. If we perform the intermediate step of realizing the reduced order model, we obtain a smaller circuit that can be treated by a circuit simulator. But the circuit simulator will immediately start "discretizing" the circuit, translating the equations back to the mathematical reduced order model. Hence, it appears unnecessary to make the effort of realizing and/or synthesizing a reduced order model. However, there is a need within the electronics industry. A synthesized reduced order model can provide more insight to engineers and designers than the reduce1d order model in mathematical form. In fact, as we shall see in Sect. 1.7, the compact models constructed by device physicists for transistors and diodes are translations of exactly these insights. Thus, there is a need in the industry for such realization and synthesis techniques. It would be even better if this could be done for parameterized reduced order models.....

## 1.4 Structure Preservation

When the discussions about passivity were drastically reduced after the publication of PRIMA and the Laguerre-type methods, a new topic emerged in the MOR community. In fact, this was initiated by the needs of the electronics industry, where it is a given that electronic circuits are formulated in terms of voltages and currents. Hence, the coefficient matrices in the state space systems can be regarded as consisting of blocks that correspond to these two types of unknowns. Unfortunately, conventional MOR methods do not make use of this knowledge. In fact, Krylov subspace methods like PVL immediately destroy this underlying structure, by mixing information that is of "current type" with that of "voltage type". This implies that the unknowns in the reduced order model do not have a physical interpretation.

Clearly, the foregoing is rather undesirable, and there is a clear need in the electronics industry to maintain the underlying structure of the problem within the context of MOR. This has led to new methods that are now termed structure-preserving MOR methods. The first to be published was that of Roland Freund, termed SPRIM [17]. The idea is simple, yet very clever: first, a Krylov subspace is generated using conventional MOR methods. However, rather than using the vectors generated as a basis for the Krylov subspace, Freund suggests to split each vector into two vectors, by splitting it into a vector containing only voltage unknowns, and a vector containing only current unknowns. In the vector containing the current unknowns, the voltage entries are all set to zero, and similarly for the vector containing the voltage unknowns. In this way, a basis is formed that consists of twice the number of basis vectors as the basis for the space constructed originally. The big advantage is, however, that it can now be shown that there will be no mixing of voltages and currents, meaning that it should be possible to assign a physical meaning to each of the vectors.

Unfortunately, despite the nice properties of SPRIM (such as matching double the number of moments, see [17]), the method was not felt by the industry to be the ultimate answer to the question of structure preservation. In fact, it was soon noticed that the incidence matrix, i.e. the matrix that relates voltages and currents, is destroyed completely. Originally, it is a very sparse matrix with at most two elements per row (1 and $-1$, relating branches and nodes), but using SPRIM it immediately turns into a dense matrix. This means that all currents and voltages in the reduced system are coupled, which is to be avoided in view of subsequent realization and synthesis (see Sect. 1.3).

Only recently, a remedy for the aforementioned problem has been presented. The method in [6] uses a clever transformation of variables before the reduction process starts, and then uses SPRIM. The transformation used is that which translates branch currents, arising in the frequently used MNA formulation of electronic circuits, into loop currents. Such transformations are also very helpful in methods for the solution of indefinite linear systems, as has been shown in [38]. In fact, long time ago, circuit simulation was usually done using the so-called "mesh method", which is precisely using loop currents rather than branch currents. With the advent of fast computers, the preference became MNA and using branch currents, but from a mathematical point of view, this is certainly not the preferred choice. In any case, Bai et al. [6] make use of the loop currents, the big advantage being that the incidence matrix becomes an identity matrix. That is, the (1,2)-block in the matrix will be identity, the corresponding (2,1) block is not affected. Hence, we immediately see a drawback of the method, namely it destroys the original symmetry.

The method proposed by Bai et al. [6] is certainly promising, and in a further publication it has been shown to be capable of addressing the realization problem discussed in the previous section. However, it is too early to state that all problems have been solved. There is still the clear need in the electronics industry to address the structure preservation issue, and it is the feeling of the author that other methods need to be explored. For example, when considering an electronic circuit, it usually contains more branches than nodes. This means that we have more unknowns of the type "current" than of the type "voltage". Structure preserving methods suggested so far generate the same number of basis vectors of both types. This seems unnatural. Adding a new basis vector for the voltages, should probably

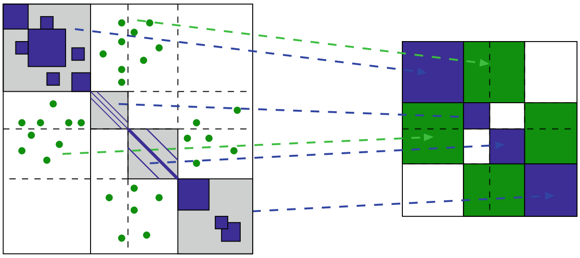


**Fig. 1.7** Structure preserving MOR for coupled systems

lead to a number of basis vectors added for the currents. So that, in the end, having a full basis for the voltages at the same time leads to a full basis for the currents. In the presently used methods, this is not the case. Most probably, the answer needs to be found in graph theory, but currently it is not known how to do this.

In the foregoing, we have concentrated on electronic systems containing currents and voltages, reflecting a structure consisting of two types of unknowns. Of course, the structure may be much more refined. From a theoretical point of view, we could have interconnected systems as considered by Vandendorpe and Van Dooren (see [37], Chap. 14). More general (but very much related to that of Vandendorpe and Van Dooren) is the approach considered in [14]. The graphical interpretation shown in Fig. 1.7 clearly shows the idea: keep the structure of the original systems, and do not destroy it during the MOR phase. The picture also shows the drawback of the method developed so far, namely that the coupling blocks become dense after reduction. Here, a combination with the ideas of Bai et al. [6] could provide a solution. However, as before, we feel that other methods, possibly from graph theory, need to be investigated in order to arrive at a fully acceptable solution.

Summarizing, the problem of structure preservation remains unsolved, despite various useful attempts. The feeling within the electronics industry is that novel concepts and ideas are needed.

## 1.5 Reduction of MIMO Networks

Although not mentioned explicitly, so far the discussion has been limited to systems that are of single input single output type. Of course, several of the methods presented can also be applied to the MIMO case, but often this will reveal some hidden problems. One very clear disadvantage of Krylov subspace methods applied to MIMO systems is that the number of basis vectors added to the subspace is equal to the number of inputs and outputs of the system. In the SISO case, one vector is added at the time, but in the MIMO case, many vectors are added in each iteration. Thus, the dimension of Krylov subspaces quickly becomes large, in many cases even prohibitively large. In [27], this problem has been analyzed, and a block-Arnoldi algorithm was proposed that reduces the blocks of vectors that is added per iteration. Experiments conducted with this method show the advantage, but it is too marginal to be very effective in practical situations. Hence, new concepts need to be developed.

One new concept that has recently been investigated, and published in another chapter in this book, is the use of a mixture of graph theoretical and numerical methods to perform some kind of "preconditioning" of the original system. In fact, it is similar to the successive node elimination (SNE) technique [41]. Instead of applying Krylov subspace methods to the system, related to iterative solution methods for linear systems, the method employs a more direct approach of eliminating unknowns, corresponding to direct solution techniques for linear systems. We do not wish to repeat the information that is in the chapter elsewhere in this volume, and refer the reader to it for more details. What can be said is, however, that there is a very clear and relatively urgent need by the electronics industry to be able to drastically reduce such networks with many so-called "ports". As has been demonstrated in Sect. 1.2, both for the interconnect system and for the substrate, huge resistive (and sometimes RC networks) are generated that need to be co-simulated with the underlying electronic circuit. This is only feasible in present-day circuit simulators if these huge networks are reduced very substantially.

## 1.6 MOR for Delay Equations

A fairly recent development is concerned with delay extraction-based passive macromodeling, associated with multiconductor transmission line type interconnects. High-speed effects influencing a signal propagating on an interconnect could be multifold, such as delay, rise time degradation, attenuation, crosstalk, skin effect, overshoots, undershoots, ringing, and reflection. Concerning propagation delay, a signal traversing from one end of a transmission line to the other end takes a finite amount of time, resulting in the signal experiencing a certain amount of delay. In addition, the signal may encounter rise time degradation, where the rise time at the receiver end is larger than the rise time at the source end. Rise-time degradation further adds to the overall delay experienced by the signal.

Concerning signal reflection and the associated ringing, this can also severely distort signal propagation at higher frequencies. The prime cause of this is the discontinuity in characteristic impedance of the transmitting line, due to the impedance variation on a line taking place over a certain length. This can occur due to the change in the medium along the length of the signal trace, which may have to traverse several layers.

It turns out that conventional model order reduction techniques do not work well for this case. characterized by multiport (Y, Z, S, or the transfer function) tabulated parameters. In [10] an algorithm is described that approximates the logarithm of the transfer function using a low-order rational function. Subsequently, the DEPACT (delay extraction-based passive compact transmission line macromodel) algorithm is applied to obtain a passive and causal macromodel for SPICE simulation. This method leads to compact, low-order macromodels that are more reliable for time-domain simulations.

More recent developments can be found in [25], where several papers discuss the issue of MOR for delay equations. The most commonly used approach nowadays appears to be an adapted version of vector fitting. Rather than using only rational functions, it is suggested to have an expansion which is a mixture of exponential and rational terms. The exponential terms then account for the delay, and can account also for reflections. The “coefficients” of these exponential terms are then rational functions as used in the vector fitting approach.

Although simulations carried out so far suggest that the foregoing approach is the way to go, there is also the clear disadvantage that the methods are only suited for delay equations. Stated differently: if one includes the exponential terms, one can account for effects in long interconnect lines, but this expansion is not very suitable for short lines. If one does not include the exponential terms, the normal vector fitting approach is used, and one will find that a prohibitively large number of terms is needed in the expansion when used for long interconnect lines. Hence, the essential question is, whether a combined approach can be found that addresses both long and short interconnect lines. This is a clear need in present-day simulations of interconnect structures, also for extraction programmes [24].

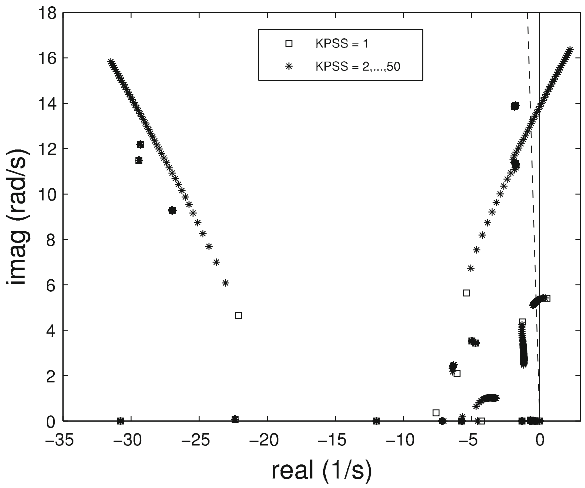
## 1.7 Parameterized and Nonlinear MOR

As will have become clear from the foregoing sections, the state-of-the-art concerning MOR for linear problems is relatively well developed, although there are still many open questions and remaining problems. For parameterized and nonlinear problems, model order reduction techniques are still in their infancy.

When looking at parameterized MOR, the electronics industry has clear needs. Modern analog designs are often built using the concept of programmable cells (PCells, PCircuits) and the ROD/Skill programming language. All of these introduce parameters, either in the definition of the circuit sub blocks (Pcells), or the relation between sub blocks (ROD, relative object design). Often, these sub blocks are optimized with respect to a certain objective function on the so-called schematic level, but optimization needs to be performed again on the higher layout level, in order to account for parasitic effects that might take place due to interactions between the different components. Clearly, these problems cannot be addressed by an MOR technique that only uses numbers. We need to be able to perform MOR also when unknown parameters are in the system.

So far, several researchers have addressed this problem, but the techniques suggested are often not very general, or not capable of solving the aforementioned complex problems. Obvious extensions using assumptions on the linearity are often suggested, whereas what is needed are new concepts. Rather than using Taylor expansions in the parameters, leading to a multitude of cross terms, it is the

**Fig. 1.8** Dependence of poles on parameters




feeling of the author that we need a careful analysis of the poles of the transfer function in terms of the parameters. A development in this direction that is felt to be very promising is the sensitive pole algorithm (SPA), that shows the dependence of the location of poles on parameters. In Fig. 1.8 an example is shown (see [34]).

It is exactly this dependence that is important. For the Maxwell equations, for example, part of the eigenvalue spectrum appears to be independent of the parameter "frequency", whereas another part is heavily dependent on the frequency. Hence, it should be possible to develop MOR methods that make use of this knowledge, and only consider the poles that are most sensitive to changes of parameters. This would be the author's advice to researchers working in the area of parameterized MOR. The electronics industry would be extremely delighted when better methods, not as involved as the methods suggested so far, would "see the light". A potentially very interesting development is that using symbolic approximations. The reader is referred to [40, 48] for more information.

For nonlinear problems, the situation is similar to that of parameterized MOR. Some methods have been suggested, but often these are based on linearizations of the underlying problem. This can never be effective for general nonlinear problems, and is far from the needs of the electronics industry. In the area of Krylov subspace type methods, the trajectory piecewise linear method (TPWL) (Fig. 1.9) appears to be most popular; see also the chapter by Striebel in this volume. That chapter contains a wealth of information on the state-of-the-art of model order reduction for nonlinear problems [42]. In the area of dynamical systems and control, methods developed by Fujimoto and Scherpen [18] and Verriest [46] are promising, although so far the methods are limited to small nonlinear systems.

The question is, however, whether one can expect to be able to develop MOR methods for general nonlinear systems. In the author's opinion, larger nonlinear systems can only be tackled by using specific information about the underlying

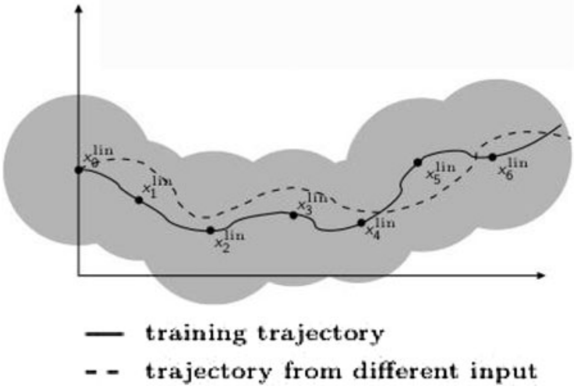



**Fig. 1.9** Schematic illustration of the TPWL method

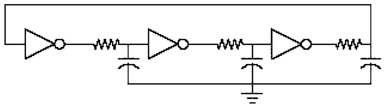


**Fig. 1.10** Simple oscillator

nonlinear system. A nice example is provided by the simulation of voltage controlled oscillators. These are devices that are used frequently in electronic designs. Initiated by Roychowdhury [12], tremendous achievements have been obtain in the reduction of such systems. In Fig. 1.10 a simple oscillator schematic is shown, whereas in Fig. 1.11 the behaviour is shown in case it is in the so-called unlocked state.

The theory developed for oscillators is based on insight in the behaviour of such devices. The circuit describing these oscillators often consist of about 100 components, but their behaviour can be reduced to a single equation describing the phase noise. In fact, reduction to a single equation is possible as one knows that only the eigenvalue 1 plays a role in the final behaviour, all other modes will damp out. This example illustrates that it can be very effective to use detailed knowledge in order to obtain an adequate and effective reduction of the nonlinear system.

The electronics industry needs more of these specific ideas for the reduction of nonlinear components. In fact, physicists and device engineers have performed model order reduction already since the 1980s, but their activities are termed "compact modeling" rather than reduced order modeling. Initially, compact models were made in the form of so-called Gummel–Poon models (see Fig. 1.12).

More generally, the idea of compact modeling is to capture the full nonlinear behaviour of a semiconductor device into a model consisting of resistors, capacitors, inductors and simple nonlinear components like diodes, containing many parameters such as channel width, channel length, and doping profile. Compact modeling has become a discipline in itself, a good and solid reference being [29]. In recent years, efforts have mainly concentrated on compact models for the

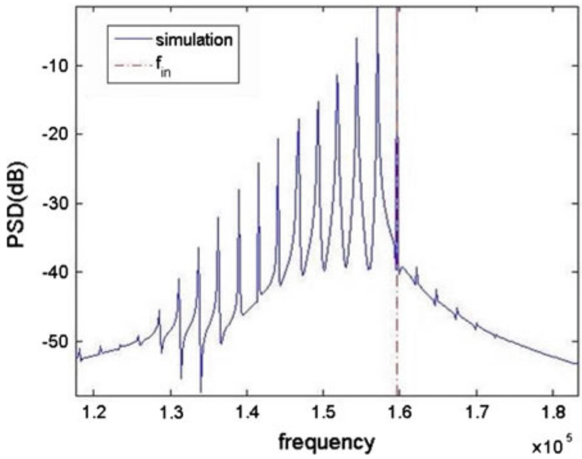



**Fig. 1.11** Frequency domain behaviour of unlocked oscillator

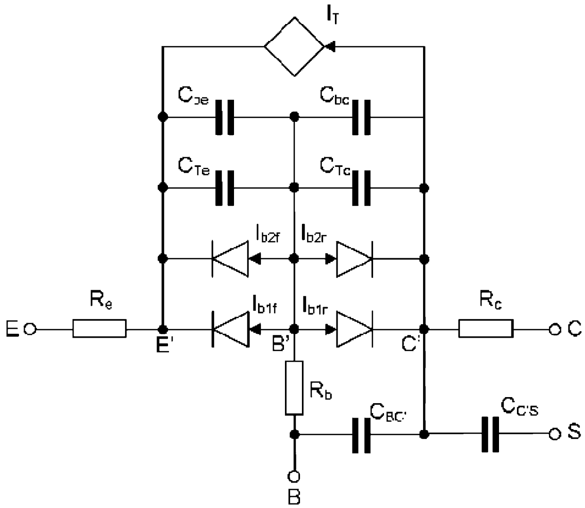



**Fig. 1.12** Gummel–Poon model used in compact modeling of semiconductor devices

various generations of MOS transistors, building blocks of most electronic circuits nowadays. The Penn State Philips (PSP) model is the most recent model, now being accepted as the world standard [1].

It would be great if the MOR community could provide ideas for the automatic generation of such compact models. Thus far, such models have been constructed manually, using much insight and a huge number of experimental results being generated over years. In view of the latter, it is to be expected that successfully generated reduced order models must also make use of huge numbers (millions) of measured or simulated (via semiconductor device simulation) results.

A final remark is that the work presented in [11, 46, 47] is also of very much interest in the context of nonlinear MOR, and might provide the breakthroughs the industry is waiting for.

## 1.8 Summary: Present and Future Needs of the Electronics Industry

In this chapter, we have touched upon many subjects within the field of model order reduction as viewed from the electronics industry. The conclusion is that the area of linear problems is fairly well developed, but that many problems still remain, with new research topics being generated regularly. In the areas of parameterized and nonlinear model order reduction, many problems remain, and these areas still need many new ideas in order to get the status of being mature.

Specific topics that we identified in this paper, and are worth stressing again:

- Good and reliable error estimates for Krylov subspace based MOR techniques
- Passivity enforcement in such a way that accuracy is not traded for passivity
- More natural structure preserving methods that use different spaces for currents and voltages
- Efficient reduction methods for systems with multiple inputs and outputs
- An MOR approach that can address both long and short interconnect lines, i.e. is suitable for situations with and without delay
- Parameterized model order reduction based on the sensitive pole algorithm

The ultimate challenge, however, is to be able to automatically generate the nonlinear device models that can currently only be constructed by device physicists!

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