Sanskrit Prosody, Piṅgala Sūtras and Binary Arithmetic

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Abstract

In India, the science of prosody, had its beginnings in the Vedic period and has been held in high esteem, being regarded as a *vedāṅga* or a limb of the *Veda.* The earliest work on prosody was by Piṅgala which is generic and on which all the subsequent works are based. It is an amazing fact that this early work already deals with matters relating to problems of combinatorics. The main aim of this paper is to give a brief description of this work of Piṅgala and discuss in detail the mathematics arising out of it.

§1. Introduction

Classical Sanskrit composition is of two kinds: *gadya* (prose) and *padya* (poetry). Sanskrit prosody, the study of the metrical scanning of Sanskrit poetry, called *chandas śāstra*, has its beginnings already in the vedic times. In his classic work, *Vedic Metre in its Historical development*, first published in 1905, E. V. Arnold ([3]) begins with high praise for the *Ṛgveda* by remarking that *The Rigveda is not a book, but a library and a literature.* At the end of the first chapter, he adds: *To whatever conclusions we may be further led in detail, it must be plain that as works of mechanical art, the metres of Rigveda stand high above those of modern Europe in variety of motive and in flexibility of form. They seem indeed to bear the same relation to them as the rich harmonies of classical music to the simple melodies of the peasant. And in proportion as modern students come to appreciate the skill displayed by the Vedic poets, they will be glad to abandon the easy but untenable theory that the variety of form employed by them is due to chance, or*

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the purely personal bias of individuals; and to recognize instead that we find all the signs of a genuine historical development, that is of united efforts in which a whole society of men have taken part, creating an inheritance which has passed through generations from father to son, and holding up an ideal which has led in turn to seek rather to enrich his successors than to grasp at his own immediate enjoyment. If this was so, then the vedic bards are also to be counted amongst 'great men and.....such as sought out musical tunes and set forth verses in writing'.

To quote another great British Indologist on the Indian contributions to metres, here is what H. D. Colebrooke has to say in his essay entitled *On Sanscrit and Prakrit poetry*, published in "Asiatic Researches" in 1808:xxx (and reprinted in *Essays on History, Literature and Religion of Ancient India* ([9])): *The prosody of Sanskrit is found to be richer than that of any other known language, in the variations of the metre, regulated, either by quantity, or by the number of syllables both with or without rhyme and subject to laws imposing in some instances rigid restrictions, in others, allowing ample latitude.*

A few words about the arrangement of the article: Since a general reader, (for instance a mathematician) to whom this article is addressed, may not be familiar with principles of prosody in general and Sanskrit prosody in particular, this article begins with a rather discursive account of some aspects of vedic prosody and also introduces some basic facts on the rules of Sanskrit prosody in the first few preliminary sections. The main theme of the paper which is the discussion of the classical work of Piṅgala on prosody is taken up, beginning §5. The crucial section is §8 , from the point of view of a mathematician, which deals with the combinatorial aspects of Piṅgala's work.

§2. The beginnings of prosody

As we said earlier, the beginnings of Sanskrit prosody go back to the vedic times. The *Brāhmaṇas* speak eloquently of the origins of metres, colouring them with mysticism. The study of prosody has been held, right from the early times, with the greatest of esteem; At the end of section 7 of *prapāṭhaka 1* of the *Nidāna Sūtra* ([16]), we have for instance the following stanza:

chandasāṃ vicayaṃ jānan yaḥ śarīrādvimucyate *chandasāmeti sālokyamānantyāyāśnute śriyam*

छन्दसां विचयं जानन् यः शरीराद्विमुच्यते।
छन्दसामेति सालोक्यमानन्त्यायाश्नुते श्रियम्॥

Translated roughly into English the above stanza reads:

One who a has deep knowledge of Chandas, shares, after liberation from his body, the same abode of the Chandas, acquiring eternity, glory and beatitude

Prosody has been described as the feet of the *Vedas*; it is thus one of the limbs of the *Veda* - a *vedāṅga.* In *Pāṇinīya śikṣā*, we have the following verse:

Chandaḥ pādantu vedasya hastau kalpo'tha paṭhyate
Jyotiṣāmayanaṃ cakṣurniruktaṃ śrotramucyate

छन्दः पादन्तु वेदस्य हस्तौ कल्पोऽथ पठ्यते ।
ज्योतिषामयनं चक्षुर्निरुक्तं श्रोत्रमुच्यते ॥

Chandas are the feet of the Vedas, Kalpa the hands, Astronomy the eyes and Nirukta the ears.

The importance of the knowledge of *chandas* for understanding the *Vedas* is emphasised in *Bṛhaddevata* (a text dealing with the gods of the *Ṛgveda*, supposedly written by that ancient venerable vedic seer Śaunaka, verse 136, VIII, ([5]) as follows:

aviditvā ṛṣiṃ chando daivataṃ yogameva ca
yo'dhyāpayejjapedvāpi pāpīyāñjāyate tu saḥ

अविदित्वा ऋषिं छन्दो दैवतं योगमेव च ।
योऽध्यापयेज्जपेद्वापि पापीयाञ्जायते तु सः ।

One who teaches or recites the Veda without having proper knowledge of the applications, the seers, metres and Gods, becomes indeed a sinner.

As one of the earliest references to prosody, we have a verse (I.1.5) in the *Muṇḍakopaniṣad* ([10]), which lists *chandas* as essential for attaining the "lower knowledge", the higher one being that of the *Brahman.*

tatrāparā ṛgvedo yajurvedo sāmavedo atharvavedaḥ
śikṣā kalpo vyākaraṇaṃ niruktaṃ chando jyotiṣamiti

तत्रापरा ऋग्वेदो यजुर्वेदो सामवेदो अथर्ववेदः ।
शिक्षा कल्पो व्याकरणं निरुक्तं छन्दो ज्योतिषमिति ॥

The six *vedāṅgas* which were considered essential for the understanding of the *Vedas* are as mentioned above: *śikṣā*, phonetics; *kalpa*, the knowledge of the sacrificial rites; *vyākaraṇa*, grammar; *nirukta*, etymology; *chandas*, prosody; and *jyotiṣa*, astronomy.

These *vedāṅgas*, whose beginnings can be traced already to the *Brāhmaṇas* and the *Āraṇyakas*, did not originally refer to independent branches of knowledge but were only indicated as fields of study, essential for the understanding of the *Vedas*. As time went by, it was realised that there was a real need to develop them as auxiliary subjects associated with the study of the *Vedas*. Hence, independent texts were written in the (mnemonic) *sūtra* style to expound these subjects. ([28]) . It is perhaps worthwhile to mention, by the way, that the *sūtra* style of writing is something unique to Indian literature. A very succinct definition of a *sūtra* is found ([14]) in the *Viṣṇudharmottara purāṇa* and runs thus:

alpākṣaram asandigdhaṃ sāravat viśvatomukham
astobham anavadyaṃ ca sūtraṃ sūtravido viduḥ

अल्पाक्षरं असन्दिग्धं सारवत् विश्वतोमुखं ।
अस्तोभम् अनवद्यं च सूत्रं सूत्रविदो विदुः ॥

A sūtra should have the least number of syllables, should contain no doubtful words, no redundancy of words, should have unrestricted validity, should contain no meaningless words and should be faultless!

The study of prosody which began in the vedic period evolved to apply to classical Sanskrit and to the Prakrit poetry as well and had its continued impact on the poetry of the later ages too.

To quote an example, the *ślokas* of the epics like the *Rāmāyaṇa* and the *Mahābhārata* are derived by and large from the vedic metre *anuṣṭup*. Indeed the vedic metre *anuṣṭup* came to be monopolised by the poets of the classical age. On the other hand, the vedic metres *triṣṭup* and *jagatī* led to metres used by poets and bards at the courts of various kings.

In this connection, it is amusing to note, parenthetically, that the great poet Kālidāsa (who himself is probably the author of a work entitled *Śrutabodha* [21] on classical metres - though this work is attributed by some to Vararuci) employs a vedic metre in a very appropriate context in his great play *Abhijñāna Śākuntalam*.

When Śakuntalā is about to leave the hermitage of the sage Kaṇva to go to meet her husband Duṣyanta, Kaṇva offers a benediction, which is set in the following beautiful stanza in the vedic metre *triṣṭup*, with 11 syllables in each of its four *pādas*.

amī vedīm paritaḥ klṛptadhiṣṇyāḥ
samidvantaḥ prānta saṃstīrṇa darbhāḥ
apaghnanto duritaṃ havyagandhaiḥ
vaitānāstvāṃ vahnayaḥ pāvayantu

Translated in to English, the stanza reads:

May these sacrificial fires, fixed in their places around the altar, nourished by holy wood, with the darbhā grass strewn around their boundaries, removing sin by the fragrance of the oblations, purify thee!

अमीवेदिं परितः क्लृप्तधिष्ण्याः
समिद्वन्तः प्रान्त सम्स्तीर्ण दर्भाः ।
अपघ्नन्तो दुरितं हव्यगन्धैः
वैतानास्वां वह्नयः पावयन्तु ॥

§3. Units of prosody, the syllables

The etymology of the Sanskrit word for prosody *chandas* traces it to various roots, for instance, it can de derived from the root *chad*, which means "to cover"; incidentally, this is not the only possible etymological derivation; there are several other possibilities too! Whatever be the etymology of *chandas* and the consequent derived meanings, it denotes the science of syllables in verses.

A syllable (*akṣara* in Sanskrit) is a vowel with or without one or more consonants. A syllable is called a *laghu* (short), (denoted by l), if it consists of a short vowel followed by at most one consonant. A syllable which is not a *laghu* is called a *guru* (long), (denoted by g). But there is a proviso by which even a short syllable will be treated as long while scanning, when it is followed by a conjunct consonant, an *anusvāra* (a nasal) or a *visarga* (an aspirant). Unlike in classical Sanskrit prosody, where the nature of the syllables is also an important aspect of prosody, vedic metre is governed solely by the number of syllables in a verse, called the *length* of the metre. (a verse is called a *pāda* in Sanskrit), which forms the basic unit of Sanskrit poetry. Verses combine to form a *ṛk*, or a stanza, which is a unit of a vedic hymn.

A stanza consists, generally, of not less than three and not more than fifteen verses. A stanza may consist of metrically identical (*sama*) or metrically different (*viṣama*) verses. Two or three stanzas combine to form a strophe.

The following is an example of a *ṛk* in *gāyatrī* metre (a stanza with three verses each of which has 8 syllables):

agnim īle | puróhitam |
yajñásya de | vam ṛtvijam |
hótāram ra | tnadhā́tamam |

which has the following arrangement of 8 syllables in each of its verses:

$$\not{g}\quad l\quad g\quad g\quad l\quad g\quad l\quad \not{l}$$
$$\not{g}\quad g\quad l\quad g\quad l\quad g\quad l\quad \not{l}$$
$$\not{g}\quad g\quad g\quad g\quad l\quad g\quad l\quad \not{l}$$

(According to one of the rules of prosody, the first and the last syllables of a verse are ignored for scanning purposes.)

§4. Some works on prosody other than Piṅgala's

As we shall notice presently, Piṅgala wrote a definitive work (in *sūtra* style) on prosody, probably around the middle of the third century B.C. As is the case with such definitive works, (for example the *Aṣṭādhyāyī* of Pāṇini), Piṅgala's work systematises and improves upon the work of many earlier authors on the subject. The names of Yāska and the otherwise unknown prosodists like Saitava, Rāta, Māṇḍavya, Tāṇḍī, Krauṣṭiki and Kāśyapa are mentioned as some of those who preceded him. Like Pāṇini once again, who dealt with classical Sanskrit grammar rather than vedic grammar, Piṅgala, though he begins his work with vedic metres, deals for a substantial part with classical metres. It should be remarked that works like the *Chando viciti* (called *Tatva subodhinī*), which is a part of the *Nidāna Sūtra* (which is a *śrauta sūtra* of the *Sāmaveda*, and is supposed, according to some, to have been written by the great Patañjali, who wrote the *Mahābhāṣya* - the "great commentary" - on Pāṇini's *Aṣṭādhyāyī*), *Ṛkprātiśākhya* (written by the venerable Śaunaka), *Sāṅkhyāyana Brāhmaṇa*, associated to the *Ṛgveda* and *Ṛksarvānukramaṇī*, also deal with various aspects of vedic metres. The *Agni purāṇa, Nārada purāṇa, Garuḍa purāṇa*, and the *Viṣṇudharmottara purāṇa*, *Nāṭya Śāstra* by Bharata, and Varāhamihira's *Bṛhat saṃhitā* are some of the fairly old texts which have separate sections dealing with Sanskrit prosody. Subsequent to the classic work of Piṅgala, apart from commentaries on it, like that of Halāyudha (called *Mṛta sañjīvanī* ([11]), that of Yādava Prakāśa ([19]), there have been many authors like Kedāra Bhaṭṭa ([25]), Svayambhū (847 A.D.), Kṣemendra (1100 A.D.) and Gaṅgādāsa (1500 A.D.)([6]) and others ([26]), who have written texts on prosody. As we mentioned earlier, even Kālidāsa is said to be the author of the text *Śrutabodha*, dealing with classical Sanskrit prosody. All of these are heavily influenced by the monumental work of Piṅgala. There have also been many Jain authors who have written on prosody, like the author of Jānāśrayī (6th to the 7th century A.D)([12]), Jayakīrti (1000 A.D.) Jayadeva (1000 A.D.) and that polymath from Gujarat, Hemacandra (1088–1172 A.D.)([7]).

§5. Piṅgala, the author of Chandas Sūtra

As is the case with many of the ancient personages in India, very little is known about Piṅgala himself except that he was highly venerated and referred to as *Piṅgalācārya* or *Piṅgala Nāga.* (*Nāga* in Sanskrit means a serpent and serpents are supposed to be endowed with great wisdom). Some think that he was identical with Patañjali the author of the *Mahābhāṣya.* Ṣadguruśiṣya in his commentary (1187 A.D.) on *Ṛganukramaṇī* refers to Piṅgala as *pāṇinīyānuja* which can be interpreted to mean that Piṅgala was a younger contemporary of Pāṇini or even that he was the younger brother of Pāṇini. Though, conjecturally, it is thought that Piṅgala lived in the middle of the third century B.C.,the precise period of Piṅgala is hard to determine. Most probably, Piṅgala was a younger contemporary of Pāṇini and belonged to the third century B.C. With reference to his place of birth, we are equally ignorant, though it is surmised that he might have been born somewhere on the west coast of India. That he lived near a coast is perhaps obliquely corroborated by the statement in the *Pañcatantra* (2,36) (cf. [18], p. 255) about the manner in which Piṅgala met his death. Stressing the theme that even the meritorious ones can not take it for granted that they are safe from assault, it is mentioned there

chandojñānanidhiṃ jaghāna makaro velātaṭe piṅgalam

Translated into English, it reads *Piṅgala, the repository of the knowledge of metres was killed by a crocodile on the sea shore.* The full verse in fact says that Pāṇini was killed by a lion, Jaimini by an elephant and Piṅgala by a crocodile. Albrecht Weber in his book, "Uber die Metrik der Inder" ([27]), guesses that this enumeration is perhaps in the order of time and therefore Piṅgala probably was later in time than Pāṇini and Jaimini.

§6. Piṅgala's Chandas Sūtra

However uncertain one is about Piṅgala as a man and his life history, his work on *chandas* (in eight chapters, containing 315 *sūtras*) is very much extant and has been commented upon, as we said earlier, by several distinguished authors including Halāyudha (11th century), Yādava Prakāśa (11th century), the latter being the well known teacher of Ramānuja. As we also mentioned, there are several later texts on Sanskrit prosody based on Piṅgala's work, one of the most important one being by Kedāra

Bhaṭṭa (12-13th Century). We note also that in the *Agni purāṇa* ([1]), chapters 327-334 give a summary of the *Chandas Śāstra* as expounded by Piṅgala, beginning with a description of prosody thus:

chando vakṣye mūlajaistaiḥ piṅgaloktaṃ yathākramam

छन्दो वक्ष्ये मूलजैस्तैः पिङ्गलोक्तं यथाक्रमं ।

In Varāhamihira's *Bṛhatsaṃhitā* ([23]) in section 104, which deals with *grahagocarādhyāya* (movements of planets), verse 58, emphasising the rule of prosody (already found in the first chapter of Piṅgala's *Chandas Sūtra*), reads:

prakṛtyāpi laghuryaśca vṛttabāhye vyavasthitaḥ
sa yāti gurutāṃ loke yadā syuḥ susthitā grahāḥ

प्रकृत्यापि लघुर्यश्च वृत्तबाह्ये व्यवस्थितः ।
स याति गुरुतां लोके यदा स्युः सुस्थिता ग्रहाः ॥

Very much like the final syllable in a verse which is deemed long by the rules of prosody even if it is short, a person though of mean birth, and reprehensible in character, becomes respectable in this world, if the planets are favourable.

§7. A brief discussion of Piṅgala's Chandas Sūtra

As we said earlier, Piṅgala's *Chandas Sūtra* contains 315 *sūtras* distributed over eight chapters. Among these, the *sūtras* of the first three chapters and the first seven *sūtras* of the fourth are devoted to vedic metres. As mentioned before, the two basic building blocks of Sanskrit prosody are the *guru* (g) and the *laghu* (l). These correspond to the Greek syllables: *thesis* and *arsis*. From these, the following groups of disyllables can be built:

g	g	–	which in Greek is the disyllable	*spondee*
l	g	–	"	*iambic*
g	l	–	"	*trochaeus*
l	l	–	"	*pyrrhic*

Obviously, the number of trisyllables is eight and are as written below:

Trisyllable	Greek name	Sanskrit name
g g g	*molossus*	*magaṇa*
l g g	*bacchius*	*yagaṇa*
g l g	*amphimacer*	*ragaṇa*
l l g	*anapaestus*	*sagaṇa*
g g l	*antibacchius*	*tagaṇa*
l g l	*amphibrachys*	*jagaṇa*
g l l	*dactylus*	*bhagaṇa*
l l l	*tribrachys*	*nagaṇa*

In the first chapter of his work, Piṅgala gives the mnemonics *ma, ya, ra, sa, ta, ja, bha, na* to the set of trisyllables written above. Any trisyllable is called a *gaṇa*, so that the trisyllables are denoted, respectively by *magaṇa, yagaṇa, ragaṇa, sagaṇa, tagaṇa, jagaṇa, bhagaṇa, nagaṇa.* These are referred to in the Piṅgala's *Chandas Sūtra* as *aṣṭau vasava iti.* Piṅgala remarks that these gaṇas along with the *guru* and *laghu* form the basis of all prosody.

Many works on prosody, like *Vṛtta Ratnākara* of Kedāra Bhaṭṭa (1150 A.D.), the commentary of Yādava Prakāśa (circa 1050 A.D.) and many other commentators of Piṅgala's work have the following couplet which expresses poetically the pre-eminence of the above ten units of prosody:

myarastajabhnagairlāntaiḥ ebhirdaśabhirakṣaraiḥ
samastaṃ vāṅmayaṃ vyāptaṃ trailokyamiva viṣṇunā

म्यरस्तजभ्नगैर्लान्तैः एभिर्दशभिरक्षरैः ।
समस्तं वाङ्मयं व्याप्तं त्रैलोक्यमिव विष्णुना ॥

The world of speech is enveloped by the ten units ma, ya, ra, sa, ta, ja, bha, na, g and l, like Lord Viṣṇu permeating the three worlds.

(The same statement is also made by Ṣadguruśiṣya in his commentary of *Ṛksarvānukramaṇī.*) We quote another *śloka* given by Kedāra Bhaṭṭa in his *Vṛtta Ratnākara*, which gives a mnemonic for the eight *gaṇas*:

ādimadhyāvasāneṣu bhajasā yānti gauravam *yaratā lāghavaṃ yānti manau tu guru lāghavam*

आदिमध्यावसानेषु भजसा यान्ति गौरवं ।
यरता लाघवं यान्ति मनौ तु गुरुलाघवं ॥

A rough translation in to English of the above *śloka* reads:

The guru moves into the first, middle and the last position in bha, ja and sa. The laghu moves into the first, middle and the last positions in ya, ra and ta. ma and na represent all gurus and all laghus.

There are many features of Sanskrit prosody which distinguish it from the Greek. Greek prosody had its origin in music and dance, whereas in India, prosody began with the vedic chants. Also, whereas in Greek prosody, scanning is achieved though the analysis of the position and nature of disyllables, in Sanskrit, it is through the analysis of trisyllables and the two single syllables g and l.

We include at this point a few facts on vedic as well as classical prosody. In general, metrical music deals with three factors: the sound value of a syllable, syllabic quantity and the time taken for the utterence of a syllable. In vedic metres, the music depends only on the modulation of the voice in the pronunciation of the syllables; the essential features of the syllables, namely whether they are short or long do not matter. On the other hand, the music of classical metres depends on the essential features of the syllables, their variations and their order of succession. Hence, in classical prosody, a single letter could not be the unit of a metrical line as in vedic metres. A mere mention of the number of syllables which are all independent units sufficed to give an idea of the metrical line in the vedic metre and there was no need to give the essential features of the letters nor was it necessary to say how they were related to each other. But both these points required to be stated in the case of classical metres. Hence a method had to be found for scanning classical metres. Units of two syllables and their fourfold combinations are a choice and such a choice was indeed tried out by a Jain prosodist, as has been pointed out by H. D. Velankar in his book *Jayadāman*([13]). But these were found to be inadequate to express the basic constituents of the music of a metre, especially in the case of longer verses. So a new unit had to be found by the classical prosodists, which was neither too long nor too short. In ancient India, the number 3 was the least number which denoted multiplicity; the number 2 did not really signify plurality and indeed enjoyed too special a status. This is perhaps one of the reasons why, as H. D. Velankar suggests in his *Jayadāman*([13]), groups of three syllables were chosen by the ancient prosodists of India for scanning classical metres.

The second chapter of Piṅgala's *Chandas Sūtra* introduces and discusses various aspects of the seven basic vedic metres: *gāyatrī, uṣṇik, anuṣṭup, bṛhatī, paṅkti, triṣṭup* and *jagatī*. *Gāyatrī* consists generally of three *pāda*s of eight syllables each and hence has 24 syllables in all, and from then on, the number of syllables in these metres increases by 4 at a time, so that *uṣṇik* has 28 syllables, *anuṣṭup* has 32, *bṛhatī* has 36, *paṅkti* has 40, *triṣṭup* has 44 and *jagatī* has 48. Eight different varieties of these metres, are also discussed. Thus the seven basic vedic metres are divided into eight forms each, and totally there are 56 different kinds of metres.

In the third chapter, the notion of a *pāda* (foot) in Sanskrit prosody (which is very different from the notion of a 'foot' in Greek prosody) is discussed. Rules regarding filling of a *pāda* are also discussed. For example, in the *gāyatrī* when the number of syllables falls short of the required number of eight syllables, as in the following:

tatsaviturvareṇyam

तत्सवितुर्वरेण्यं

where there are only seven syllables, one should scan it as:

tatsaviturvareṇiyam

तत्सवितुर्वरेणियं

changing *y* to *iy*.

In this chapter, nine forms of the *gāyatrī* metre in terms of the number of *pāda*s are described. To give an example, one could have a *gāyatrī* stanza containing four *pāda*s with six syllables each, which is called *catuṣpāda gāyatrī*. Halāyudha, in his commentary gives such an example from the *Atharvaveda, Kāṇḍa 6, Sūkta 1.1* ([4]).

It is interesting to note that a non vedic *catuṣpāda gāyatrī* stanza (attributing it to the *Pāñcālās*) is also quoted in the *Nidāna Sūtra (prapāṭhaka 1, Chando viciti)*([16]), whose meaning is unclear.

It is also interesting to note that in §16 of the *Ṛkprātiśākhya* ([20]), there is an example of such a stanza (stanza 7), which is given by Śaunaka. The stanza runs as follows:

indraḥ śacīpatir balena vīḷitaḥ
duścyavano vṛṣā samatsu sāsahiḥ

इन्द्रः शचीपतिर् बलेन वीळितः ।
दुश्च्यवनो वृषा समत्सु सासहिः ॥

This stanza is also found in the *Nidāna Sūtra (prapāṭhaka 1, Chando viciti)*([16]). A small part of this stanza occurs in the *Ṛgveda* (Eighth Maṇḍala, 19;20) namely:

yenā samatsu sāsahaḥ

येना समत्सु सासहः

(Yādava Prakāśa in his commentary of Piṅgala *Chandas Sūtra* notices this fact).

Similar forms of other metres are also discussed in this chapter. Mention is made of a class of those metres whose first and last verses have correct number of syllables, but whose middle verses have smaller number of syllables. Such metres are called *pip īlika madhya* that is, with a middle like that of an ant! For example, there are *gāyatrī* stanzas in which the first and the last *pādas* have eight syllables but whose middle *pāda* has only three syllables. A general rule states that the number of syllables in the first *pāda* determines the metre.

To the seven basic metres are sometimes associated the seven *svaras* of music : namely *ṣaḍja, ṛṣabha, gāndhāra, madhyama, pañcama, dhaivata* and *niṣāda* (respectively); also the following colours: *sita* (silvery), *sāraṅga* (variegated), *piśaṅga* (brown), *kṛṣṇa* (black), *nīla* (blue), *lohita* (red) and *gaura* (white); and to the seven rishis: *Agniveśya, Kāśyapa,* *Gautama, Āngīrasa, Bhāradvāja,Kauśika* and *Vāsiṣṭha*. These identifications are intended as alternate methods to identify these metres, in case there is a confusion!

In chapter 4, after discussing fifteen kinds of vedic metres from *utkṛti* to *jagatī*, Piṅgala introduces the cryptic statement 'from now on classical metres' and from then on, he deals only with classical metres till the end of the book. He in fact deals in the rest of this chapter with the so called *mātrā vṛttas*, that is those metres of classical Sanskrit based on the syllabic instants (a syllabic instant being the time taken to pronounce a short syllable: a long syllable takes twice as much time and is therefore said to constitute two syllabic instants). He discusses, in particular, the *Āryā* and the *Vaitālīya* metres. (We note, incidentally, that the *Āryabhaṭīya* of *Āryabhaṭa* is written in the *Āryā* metre.)

In the fifth chapter, Piṅgala discusses the so called *vṛtta chandas*. He classifies stanzas with four *pādas* into three types: *sama, ardhasama* and *viṣama*. *Samavṛttas* are those which consist of the same number of syllables in each *pāda*, while *ardhasamavṛttas* have the same number of syllables in the first and the third *pādas*, as well as in the second and the fourth *pādas*. *viṣama vṛttas* are those in which all the *pādas* have unequal number syllables.

The aim of the sixth chapter of Piṅgala's *Chandas Sūtra* is principally to define the notion of *yati* (*caesura*). The *sūtra* which describes *yati* is *yati vicchedah*. The word *vicchedaḥ* signifies 'resting place'. It is the mechanical pause introduced in the middle of the verse. As against the irregular pauses in the vedic metres like *triṣṭup* and *jagatī*, it is regularly admitted in classical metres. While the origin of *yati* can be traced to the need for the ease of recitation, it evolved into an art and ornamentation in classical poetry. The concept of *yati* has been discussed at length by all the later prosodists and has become a regular feature of classical *vṛttas*. The effectiveness of *yati* in classical Sanskrit poetry, is best illustrated in the beautiful verses of the exquisite *Meghadūta* of Kālidāsa (in the slow-moving, majestic metre of *mandākrāntā*, a classical metre, with seventeen syllables, with pauses at the end of the fourth and tenth syllables in each *pāda*).

In the seventh chapter, Piṅgala describes and discusses metres *atijagatī, śakvarī, atiśakvarī, aṣṭi, atyaṣṭi, dhṛti, atidhṛti, kṛti, prakṛti, ākṛti, vikṛti, saṃkṛti, abhikṛti* and *utkṛti* which are the so called *atichan-*

das (hyper metres) containing 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100 and 104 syllables respectively. At the end of the chapter, he also explains the metre *daṇḍaka.*

The eighth chapter which is the concluding chapter of Piṅgala's book begins with the *sūtra, atrānuktaṃ gāthā*; Piṅgala's idea is to include in this chapter those metres which had not been mentioned in the earlier chapters.

The last fifteen *sūtras* of this chapter (*sūtras* 20 till 35) are the most interesting ones from the point of view of mathematics and deal with binary arithmetic and combinatorial questions arising out of the study of prosody. We shall discuss these in the next section.

§8. Piṅgala's sūtras and binary arithmetic

Since prosody deals with two symbols l and g and their repetitions, it is rather an easy matter for us (who live in this computer age) to guess[1] that the study of prosody should naturally lead to questions on binary arithmetic. Indeed, the study of prosody did lead the ancient Indian mathematicians to binary arithmetic and combinatorics, as is evidenced by the *sūtras* 20-35 in the eighth chapter. As is usual with Piṅgala, these *sūtras* are cryptic to the point of being obscure. However, as is customary with the ancient Indian system of preserving knowledge, the later commentators of Piṅgala's *Chandas sūtra* have provided ample explanations of the *sūtras* ([17], [22]).

The *sūtras* 20-23 deal with the construction of the so called *prastāra* of a metre, which can be translated roughly into English as a matrix or an array of syllables. The *laghus* and *gurus* in a metre of a given length are listed horizontally as rows (or lines) in a *prastāra.* This device of a *prastāra* can be thought of as a table written either on the ground or on a board. The rules for the construction of a *prastāra*, for metres of length one, two or three are given in these *sūtras*. For example, the *prastāra* for a metre of length 1 is obtained by first writing the symbol g (for *guru*) and beneath it the symbol l (for *laghu*). The *prastāra* for

[1]One remembers the words of Schiaparelli, the Italian historian of Early Greek Astronomy who wrote in the introductory section of his paper on the work of Eudoxus on Astronomy: "Tutto il nostro merito sta nell'esser venuti al mondo piu tardi": 'Our sole merit consists in having come to the world a little later'.

a metre of length 2 starts with a horizontal row with two *gurus*: $g \quad g$. We begin the next row , by writing l (for a *laghu*) below the first entry g of the first row and write g below the second entry g of the first row, so that this row reads $l \quad g$. In the third row, we begin with a g and write a l beneath the next entry g of the second row so that, the third row reads $g \quad l$. We begin the fourth row with an l and write a l below the the next entry l of the third row. The *prastāra* for a metre with two syllables is now complete and is the array of four horizontal rows

g	g	1
l	g	2
g	l	3
l	l	4

consisting of two syllables each. The general rules for constructing the *prastāras* of metres of a given length n are similar and explained by the *sūtras*. Namely, we start with a horizontal row consisting entirely of n *gurus*. The rest of the rows of the *prastāra* are constructed by using the following rule: Start any row and continue filling the row with *gurus until* we see for the *first* time a *guru* in the previous row. Then write a *laghu* as the entry for this row below this *guru* and from then on, copy the rest of the syllables from the previous row. We continue filling rows this way until we reach a row consisting of all *laghus*, where we stop. [2] This method applied to two syllables gives obviously the *prastāra* of two syllables we have written above. The *prastāra* for a metre of three syllables, using the rule described above gives the table for the eight *gaṇas* (trisyllables) we wrote down in the beginning of the previous section.

The *sūtra* 23 reads *vasavastrikaḥ*, which simply enumerates the number of trisyllables as eight! (there are eight *vasus* according to the vedic lore!)

[2]As has been kindly pointed out by Professor M.G. Nadkarni, this rule applied to infinite sequences of zeros and ones ($g = 0$, $l = 1$) gives rise to a transformation on the space of sequences of zeros and ones. It is a very basic object in ergodic theory called dyadic adding machine or odometer transformation, and when viewed as a transformation of the unit interval, it is called von Neumann transformation, a name given by Kakutani. This transformation plays a very important role in orbit-equivalance theory and related areas.

Let us also add one more fact regarding the construction of the *prastāra*. We number the rows of a *prastāra* serially with the first row of the *prastāra* consisting of all *gurus* being numbered as 1.

Before discussing the rest of the *sūtras*, it is perhaps convenient to introduce a stanza which lists the various techniques, termed as *pratyayas*, by which some arithmetic questions related to metres can be analysed. This stanza is found in text books on prosody subsequent to Piṅgala's work. For instance, it is found in Kedāra Bhaṭṭa's *vṛtta ratnākara*, ([25]) Yādava Prakāśa's commentary of Piṅgala's *Chandas Sūtra*, Hemacandra's *chandonuśāsana*, and in many other works on prosody. The stanza in question ([25], p. 187) reads as follows:

prastāro naṣṭamuddiṣṭam ekadvayādi lagakriyā
saṅkhyā caivādhvayogaśca ṣaḍete pratyayāḥ smṛtāḥ

प्रस्तारो नष्टमुद्दिष्टं एकद्वयादि लगक्रिया ।
सङ्ख्या चैवाध्वयोगश्च षडेते प्रत्यया: स्मृता: ॥

As we said, the above stanza enumerates the various components of some of the arithmetic aspects of prosody, namely: (*i*) *prastāra* (whose meaning we just now explained), (*ii*) *naṣṭam*, (*iii*) *uddiṣṭam*, (*iv*) *ekadvayādilagakriyā*, (*v*) *saṅkhyā*, (*vi*) *adhva yoga*.

The following stanza ([25], p. 188) summarises what we said already about the way a *prastāra* is constructed:

pāde sarvagurāvādyāllaghuṃ nyasya guroradhaḥ
yathopari tathāśeṣaṃ bhūyaḥ kuryādamuṃ vidhim
ūne dadyād gurūneva yāvat sarvalaghurbhavet
prastāro'yaṃ samākhyātaḥ chandovicitivedibhiḥ

पादे सर्वगुरावाद्याल्लघुं न्यस्य गुरोरध: ।
यथोपरि तथाशेषं भूय: कुर्यादमुं विधिम् ॥
ऊने दद्याद् गुरूनेव यावत्सर्वलघुर्भवेत् ।
प्रस्तारोऽयं समाख्यात: छन्दोविचितिवेदिभि: ॥

We shall describe now each of the other aspects listed above related to the arithmetic of prosody. *Sūtras* 24 and 25 of chapter eight of Piṅgala's *Chandas Sūtra*, which read:

(24) *l'ardhe*
and
(25) *saike g*

refer to the process of *naṣṭam* and this word means 'vanishing' or 'disappearance'. Suppose that the *prastāra* of the metre (which is usually written on the sand) has been erased by mistake. The process described shows how to recover the metre only through the knowledge of the number of the row in which the particular metre had appeared. This process is illustrated by the following example: Suppose that we know that a certain metre with a fixed number of syllables say 6, occurs as the 44th row in the *prastāra*, how does one write down the corresponding metre? The answer is given by the two *sūtras* above as elaborated further by the following stanza in *Vṛtta ratnākara* ([25], p.192), which explains the process *naṣṭam*. (There are similar explanations of these *sūtras* in Halāyudha's and Yādava Prakāśa's commentaries.)

naṣṭasya yo bhavedaṅkastasyārdhe'rdhe same ca laḥ
viṣame caikamādāya tasyārdhe'rdhe gururbhavet

नष्टस्य यो भवेदङ्कस्तस्यार्धेऽर्धे समे च लः ।
विषमे चैकमाधाय तस्यार्धेऽर्धे गुरुर्भवेत् ॥

The procedure indicated is best explained by applying it to the example mentioned above: the number 44 being divisible by 2, we write an l (for *laghu*) and divide 44 by 2 to get 22. The number 22 being still divisible by 2, we append an l to the earlier *laghu* and divide 22 by 2, to get 11. Up to this point, the procedure is indicated by *sūtra* 24 which says *if it is possible to halve, then an l*. When we, however, hit the number 11 which is odd, *sūtra* 25 takes over and it says *otherwise add* 1 *and a g*. Now add 1 to 11 and write a g (a *guru*). The number now is 12, which is divisible by 2; and we divide by 2 to get 6. Now 6 being divisible by 2, *sūtra* 24 applies, we write an l and divide 6 by 2. We get 3 which is odd. *Sūtra* 25 tells us that we should add 1 and write a g. We now get 4, which we divide by 2 to get 2. We write an l and divide 2 by 2 to get 1 as quotient and we stop here (since the metre has 6 syllables) and the metre we are looking for is

$$l \quad l \quad g \quad l \quad g \quad l$$

This is also the general rule given in the *sūtras* 24 and 25 (and explained in the stanza) for writing down a metre, given the number of its row in the *prastāra*.

The process *uddiṣṭam* is indicated by two *sūtras* of the Chapter 8 of Piṅgala's *Chandas Sūtra* which read:

(26) pratilomagaṇam dvirlādyam
and
(27) tatogyekaṃ jahyāt

and expanded upon in the following couplet of Kedāra Bhaṭṭa ([25], p.194):

uddiṣṭaṃ dviguṇādyadyuparyaṅkānsamālikhet
laghusthā ye tu tatrāṅkāstaiḥ saikairmiśritairbhavet

उद्दिष्टं द्विगुणाद्याद्युपर्यङ्कान्समालिखेत् ।
लघुस्था ये तु तत्राङ्कास्तैः सैकैर्मिश्रितैर्भवेत् ।

These *sūtras*, as interpreted by the couplet above, answer the following question: Suppose that one is given a metre with a certain number of syllables what is the number of the row representing this metre in the *prastāra*?

The process *uddiṣṭam* is thus the converse of *naṣṭam* and can be translated as 'determination'; it gives a method of determining the number of the row representing a metre with a certain number of syllables.

The answer, as given by the couplet is the following: We make the number 1 correspond to the first syllable from the left and from then on, make powers of 2, namely $2, 4, 8, \ldots$ correspond to each succeeding syllable. Ignoring the powers of 2 corresponding to the *gurus* of the metre and adding the powers of 2 corresponding only to the *laghus* of the metre and increasing this sum by 1 gives the requisite number of the line in the *prastāra*. (Put in the mathematical language, one thinks of the metre as a mnemonic for a dyadic expansion by thinking of the *laghus* as representing 1 and the *gurus* as representing 0!) Let us consider as an example, the metre

$$l \quad l \quad g \quad l \quad g \quad l,$$

as above. Then the number in question is $1 + 2 + 8 + 32 = 43$ increased by 1, that is 44.

It should be remarked at this point that the *sūtras* 26 and 27 as stated by Piṅgala *do not* suggest the above procedure outlined by the couplet. The *sūtras* themselves have been interpreted by Halāyudha in a different way and this interpretation is also found in the commentary of Piṅgala's *Chandas sūtra* by Yādava Prakāśa. We shall discuss this presently. But before doing this, let us note that the processes of *uddiṣṭam* and *naṣṭam* described above, together give a one to one correspondence between non-negative integers and their dyadic expansions, via, metres. In fact, given any metre we get an integer by the process described above, (by assigning the value 1 to a *laghu* and 0 to a *guru* and assigning the value 2^{i-1} to the syllable which occurs at the ith position from the left; summing these numbers and adding 1 to it we get the number of the row corresponding to this metre in the *prastāra*). Conversely, *sūtras* 24 and 25 (explained further by the process of *naṣṭam*) assign to every integer a metre. These two processes are obviously inverses of each other. We further note that the metre which consists only of *gurus* corresponds to the dyadic expansion of 0 and since this is the first row of the *prastāra*, the number of the row corresponding to any metre is one more than the number given by the corresponding dyadic expansion.

We shall now give the interpretations of Halāyudha and Yādava Prakāśa of the *sūtras* 26 and 27 of Piṅgala, which give a very interesting method of computing the number represented by a dyadic expansion.

We shall explain this principle now, mainly through examples, and then state the general principle without proof (the proof is easy to establish).

Consider for example the string of syllables:

$$l \quad g \quad l$$

The rule given by the *sūtras* (and explained by Halāyudha and Yādava Prakāśa in their commentaries), applied to the above metre says that we first look at the syllable on the extreme right. Noting that this syllable is a *laghu*, we attach the value 2 to it. We next look at the syllable to its immediate left. Noting that this is a *guru*, we attach to it the value 3 which is *one less than twice* the number 2, attached to the previous syllable. We then look at the next syllable to the left which is a *laghu*. To this we attach twice the value attached to the previous syllable and this is 6. The number of the row representing this metre in the *prastāra* is 6!

We note that according to our earlier computation, the above row of syllables represents the number $1 + 4$ increased by 1 which is 6 again!

As we shall remark presently, the above process applies, in general, to all metres of a given length n and the number associated to the first syllable is indeed the number of the row of the given metre in the *prastāra* of metres of length n.

We look at the *gāyatrī* metre, considered earlier, as another example.

$$l \quad l \quad g \quad l \quad g \quad l$$

We assign the value 2 for the *laghu* on the extreme right, the value $4 - 1 = 3$, for the next syllable on its left which is a *guru* and then 6 for the next which is a *laghu*, then 11 for the next syllable which is a *guru* and 22 for the next syllable which is a *laghu* and finally 44 for the first syllable on the extreme left which is a *laghu*. This is the number for the *gāyatrī* row in the *prastāra* for a metre of six syllables!

The general rule can now be formulated: If we take a metre of any length, and wish to find out what its number is as a row in the *prastāra* of metres of this length, we start by giving the value 2 or $2-1 = 1$ to the syllable on the extreme right, according as it is a *laghu* or a *guru*. We multiply this number by 2 and attach this number to the next syllable on its left, if it happens to be a *laghu* or attach this number decreased by 1 if this syllable happens to be a *guru*. Keep on repeating this procedure till we reach the beginning syllable of the metre. The number attached to this syllable is the number of the row in the *prastāra*.

It is easily verified that the number obtained by the procedure indicated above coincides with the number given by the dyadic expansion (by assigning the value 1 to a *laghu*, 0 to a *guru. increased by 1*).

Thus, the above is another method of finding a number through its dyadic expansion and this does not use *addition* of terms (as the earlier one did) and is more algorithmic, suited to the computer. In this sense, this ancient method is as modern as that of the computer! Actually, the *sūtra* 26 says that we first reverse the metre and carry the process from left to right.

We now turn to the *sūtras* 28 to 32 and 34, 35 of Piṅgala, which deal with the combinatorics given rise to by the study of metres. The *sūtras* in question are:

(28) dvirardhe;

(29) rūpe śūnyam;

(30) dvi śūnye;

(31) tāvadardhe tadguṇitam;

(32) dvirdyūnaṃ tadantānām;

(34) pare pūrṇam;
and

(35) parepūrṇamiti.

The questions asked and answered are: *How many metres with a given length have gurus ocurring once, twice etc? How many metres are there with a given length?* These questions which naturally arise in the study of prosody, obviously deal with the theory of permutations and combinations. We shall see that in this connection, the so called *Pascal triangle*, from which one can read off the binomial coefficients was already constructed by the ancient prosodists of India.

These topics are covered under the headings *ekadvayādilagakriyā* and *saṅkhyā* by the later prosodists like Kedāra Bhaṭṭa and others. (As a matter of fact, Piṅgala's *Chandas Sūtra* deals with these topics in the reverse order.) Piṅgala's *sūtras* 28-32 treat *saṅkhyā* and 34 and 35 with the computation of number of metres of a given length with prescribed number of *gurus* and *laghus* in it, through the combinatorics of what is now known as the *Pascal triangle.*) The two verses in Kedāra Bhaṭṭa's work ([25], p.196) which describe the first process is the following:

varṇān vṛttabhavān saikān auttarādharya taḥ sthitān
ekādikramataścaitānuparyupari nikṣipet
upāntyato nivarteta tyajennekaikamūrdhvataḥ
uparyādyāt gurorevamekadvayādilagakriyā

वर्णान् वृत्तभवान् सैकान् औत्तराधर्यतः स्थितान् ।
एकादिक्रमतश्चैतानुपर्युपरि निक्षिपेत् ॥

उपान्त्यतो निवर्तेत त्यजेन्नेकैकमूर्ध्वतः ।
उपर्याद्यात् गुरोरेवमेकद्व्यादिलगक्रिया ॥

The method to find the number of metres of length n in which *gurus* and *laghus* occur once, twice etc, as suggested in the above verse, is the following:

We start with a row of length $n+1$ consisting of the number 1. (In what follows we assume for simplicity that $n = 6$ and the next figure illustrates the procedure for $n = 6$.)

1	1	1	1	1	1	1
1	2	3	4	5	6	
1	3	6	10	15		
1	4	10	20			
1	5	15				
1	6					
1						

We start the second row with a 1. For the next position we take the sum of the number which precedes it in the row (which is 1 in our example) and the number of the previous row in the position above it (which is 1 again in our example), and the sum here is 2. We choose the next number of the row to be once again the sum of the two numbers, one which is in the preceding position in the row and the number in the position above it in the previous row.

Hence, in this case, we take 2+1=3 as the next number in the second row. The third number in the second row is chosen similarly and we continue this procedure, and *end* the second row with the number of entries one less than that of the first row. In our example, the second row has therefore 6 entries, the last entry being $5 + 1 = 6$. We start the third row once again with a 1; choose the number for the second position of the row the sum of the number in the row in the position preceding it which is 1 in our case and the number in the position above it in the second row which is 2 so that we take 1+2=3 as the second number of the third row. We stop this row once again with the number of its entries one less than the second row, which is five in our example, the last entry being 10+5=15. We continue this process until we stop with the $(n+1)$th row which has just one entry namely 1.

The number of metres with n syllables in which *guru* appears only once is given by the last number of the second row, which is n. This number is obviously also the number of metres of length n, in which the *laghus* appear $n-1$ times. The number of metres of length n in which *guru* appears exactly twice is given by the last number of the third row which is seen to be $n(n-1)/2$. More generally, the number of metres of length n in which the *guru* appears i-times is given by the last term of the $(i+1)$th row, and which is $\binom{n}{i}$.

Thus, the array constructed with the specifications of the two verses above gives a computation for the binomial coefficients and is the so called *Pascal triangle*, (with its base tilted by 45 degrees) which was constructed by Pascal in 1654. This device had however been used by the Indian prosodists, under the name *meru prastāra*, at least two thousand years earlier, in connection with the study of metres.

It is interesting to note that Bhāskarācārya II, the mathematician, who lived in the 12th century A.D, in his famous book of problems called *Līlāvatī*, has the following verse ([8]) which asks for the number of metres with a prescribed length and with a specified number of *gurus* or *laghus* (and the commentary provides a very simple algorithm for finding these.) The verse in question (for the *gāyatrī* metre), for example, is the following:

prastāre mitra gayatryāḥ syuḥ pāde vyaktayaḥ kati
ekādi guravaścāśu kathyatāṃ tatpṛthak pṛthak?

प्रस्तारे मित्र ! गायत्र्याः स्युः पादे व्यक्तयः कति ।
एकादिगुरवश्चाशु कथ्यतां तत्पृथक् पृथक् ?

The figure below gives the solution: We begin a row with the length of the metre as its first entry. The succeeding entries of the row are those gotten by decreasing this number successively by one at a time, the last entry of the row being 1. Below this row, we start a new row beginning with 1, the succeeding numbers in this row being those obtained by increasing the numbers successively by one, the last entry of this row being the length of the metre. We fill in a new row above these two rows by the following numbers. The first entry in the new row shall be the number obtained by multiplying the first entries of the two rows below, so that we get as the first entry of the row above as $1 \times 6 = 6$. The next entry of the new row is obtained by multiplying the first two entries of the first row and dividing it by the product of the first two entries of the second row, so that we get in our example, the number in the new row to be $\frac{6\times 5}{1\times 2} = 15$. The third entry in the new row shall be the product of the first three entries of the first row divided by the product of the first three entries of the second row, which for our example is the number $\frac{6\times 5\times 4}{1\times 2\times 3} = 20$.

We continue the process, till we get the last entry of the new row which is 1.

6	15	20	15	6	1
6	5	4	3	2	1
1	2	3	4	5	6

More generally, for any metre with length n, we get, as the first entry of the new row, the number $n = n \cdot 1 = \binom{n}{1}$, the second entry to be $\frac{n\cdot(n-1)}{1\cdot 2} = \binom{n}{2}$ and, more generally, for the ith entry the number $\frac{n\cdot(n-1)\cdots(n-i+1)}{1\cdot 2\cdots i} = \binom{n}{i}$.

These, as we know, give the number of metres of length n, in which the *guru* (and similarly the *laghu*) occurs exactly once, twice, $\ldots, i$-times. In particular,in the example of the *gāyatrī* metre, the numbers are $6, 15, 20, 15, 15, 6$.

Līlāvatī ([8], Appendix p. 48) has another problem on the determination of the number of *sama, ardhasama* and *viṣama vṛttas* in the metre *anuṣṭbh*, which is preceded by a general rule valid for any metre, given below.

pādākṣaramitagacche guṇavargaphalañjaye dviguṇe

samavṛttānāṃ saṅkhyā tadvargo vargavargaśca
svasvapadonau syātāmardhasamānāñca viṣamāṇām

पादाक्षरमितगच्छे गुणवर्गफलज्ञये द्विगुणे ।
समवृत्तानां सङ्ख्या तद्वर्गो वर्गवर्गश्च ।
स्वस्वपदोनौ स्यातामर्धसमानाञ्च विषमाणाम् ॥

The number of syllables in the four verses of the *vṛtta* in anuṣṭup, being 32 (the anuṣṭup, has 8 syllables), the number of arrangements of the long and short syllables of all the *pādas* is $2^{32} = 4294967296$. Evidently, the number of arrangements where the *pādas* are all alike is the number of arrangements of the syllables in a single *pāda* and is hence $2^8 = 256$. The number of arrangements occuring as *ardhasama vṛttas* is $2^{16} - 256 = 65280$. The number of *viṣama vṛttas* which is the total number of all the arrangements 'minus' the number of possibilities where two *pādas* are alike is $2^{32} - 2^{16} = 4294967296 - 65536 = 4294901760$.

We discuss finally *saṅkhyā*. As we remarked earlier, this has been discussed in the Piṅgala's *sūtras* 28-32. Kedāra Bhaṭṭa ([25], p.201), on the other hand, has the following verse describing *saṅkhyā*.

lagakriyāṅkasandohe bhavetsaṅkhyā vimiśrite
uddiṣṭānkasamāhāraḥ saiko vā janayedimām

लगक्रियाङ्कसन्दोहे भवेत्संख्या विमिश्रिते ।
उद्दिष्टाङ्कसमाहारः सैको वा जनयेदिमाम् ॥

As the verse says, there are two methods of computing the number of metres of length n. One can either sum up the numbers (obtained by the process of *lagakriyā*) which count the number of metres in which the *gurus* occur once, twice, etc. (These numbers are the entries on the extreme right in the *meru prastāra* we constructed earlier.) In other words, one is here summing up all the binomial coefficients of n and therefore one gets $\binom{n}{0} + \binom{n}{1} \cdots + \binom{n}{n} = 2^n$, which is obviously the number of metres of length n.

Otherwise, one can use the process of *uddiṣṭam*. We note that the required number is the number of metres in the *prastāra* of the metre.

The length of the metre being n, as we have remarked earlier, the last row of the *prastāra* consists of n *laghus*, and then we know that it corresponds to the dyadic expansion $1 + 2^1 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$, and the number of rows of the *prastāra* is gotten by adding 1 to it, so that one obtains 2^n.

Piṅgala's *sūtras*, mentioned above, give a somewhat elaborate method of arriving at this number, which we shall not discuss, since, in any case, it is simple combinatorics.

The *sūtra* 33 of Piṅgala reads *ekona adhvā*, which deals with the space required on the sacrificial ground for writing the *prastāra* of a metre of a given length. Since there is no mathematics involved in it, we shall pass over this *sūtra* and its explanation given by Kedāra Bhaṭṭa in his *Vṛtta Ratnākara.*

§9. Concluding remarks

To summarise,our aim in this article has been to highlight the contributions of Piṅgala to Sanskrit prosody with a special emphasis on the combinatorial aspects. As we mentioned earlier, the influence of Piṅgala on the later prosodists has been profound. Particularly interesting is the development of Prakrit prosody with emphasis on *mātra vṛttas*. One of the greatest of the later prosodists is Hemacandra whose name we already have mentioned. The construction of the *prastāra* and the other devices mentioned in the earlier section can be extended to *mātra vṛttas* too, as has been explained by Kedāra Bhaṭṭa [25]. The work of Hemacandra ([8], [2]) has a complete chapter on the combinatorics of prosody with special reference to *mātra vṛttas* (in particular, to the *Ārya* metre). We do not discuss these here.

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