The Critical Role of Task Development in Lesson Study

Brian Doig, Susie Groves and Toshiakira Fujii

Many teachers, researchers and systems have taken the lesson study process (and the related Japanese-style *structured problem-solving* lessons which typically form the basis of the research lesson) to heart, with lesson study groups blossoming around the world (see Hollingsworth and Oliver 2005; Marsigit 2007). While Lewis et al. (2006) claim that lesson study has "come of age" in the United States, underlying assumptions that shape classroom practice can pose obstacles to successfully implementing Japanese-style lessons (and hence also lesson study) in non-Japanese settings (see Doig et al. 2001; Sekiguchi 2005; Groves and Fujii 2008). So, for example, Perry and Lewis (2008) describe a long-term implementation of lesson study in California as "an 'existence proof' that lesson study can be practiced [*sic*], adapted, and sustained by US educators, [that] also highlights the persistent, extended learning by practitioners needed to adapt this form of teacher professional development to the US" (p. 23).

A significant focus for research into lesson study has been *neriage*—the 'kneading' stage of a lesson that allows students to compare, polish and refine solutions through the teacher's probing of student solutions (see Inoue 2008). Frameworks for effective teaching to support children's conceptual understanding also emphasize the need for tasks that are mathematically challenging and significant (Askew et al. 1997). Unfortunately, the report by Hollingsworth et al. (2003), on the *Third International Mathematics and Science Study* (TIMSS) 1999 video study, showed that about three-quarters of the problems set for Australian students were low in procedural complexity and repetitious—in sharp contrast to those problems set for students in higher achieving countries such as Japan.

In earlier research, Australian school principals, teachers and mathematics educators were found to strongly supported classrooms functioning as *communities of inquiry*, while believing that Australian practice falls far short of this goal, partly because the cognitive demands of typical lessons are low and do not challenge children, and partly because of the lack of conceptually focussed, robust tasks that are available to be used to support the development of sophisticated mathematical

B. Doig (✉)
Deakin University, Melbourne, Australia
e-mail: badoig@deakin.edu.au

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thinking (Doig et al. 2001; Groves and Doig 2002). They argued that a singular feature of a Japanese lesson they were shown that enabled the class to function as a community of inquiry was the nature of the task. This task, which focussed on the concept of a circle, is described later in this chapter.

We argue that Japanese educators place a strong emphasis on task design and selection and that this effort is largely ignored by non-Japanese adapters of lesson study, possibly because the effort involved may be almost invisible, in the way that about 90% of an iceberg is invisible, with all of our attention going to its visible tip.

In this chapter, we focus on *kyozaikenkyu*—the study of instructional materials—and the role of task development in lesson study. We identify and illustrate four types of tasks typically used as the basis for research lessons, discuss the role of lesson study in supporting teachers in their planning of structured problem-solving lessons, and conclude with recommendations for the role of lesson study in teacher education.

Lesson Study and Task Design

Despite records of successful implementation of lesson study as a form of professional development for teachers, we believe that the practice of lesson study outside Japan—although Huang and Li (2008) make a case for China—often tends to overlook the critical role played by the stimulus activity (the task or problem) and its presentation (the *hatsumon*) in creating the foundation of the research lesson, and that attention needs to be paid to "analyzing the topic carefully in accordance with the objective(s) of a lesson" (Shimizu 2002, p. 4). Takahashi (2006) argues that Japanese mathematics lessons emphasize the process of problem-solving activities and provide students with opportunities to re-invent mathematical ideas and concepts by themselves "and this is the reason that lessons employ carefully selected word problems and activities, and their cohesiveness" (p. 3). Further, he points out that "the selection of a problem for the problem-solving activity…is extremely critical for teachers when they plan a lesson" (p. 4). In Perry and Lewis' study (2008), teachers reported, among other changes, "increased use of tasks that elicit student thinking and support student exploration; [and] more experimentation with mathematical tasks before giving them to students, in order to understand task demands and anticipate student thinking" (p. 17).

In Japanese research lessons, the process of selecting the problem or task for the problem-solving activity comes about through *kyozaikenkyu*, which is the investigation of a large range of instructional materials, including textbooks, curriculum materials, lesson plans and reports from other lesson studies, as well as a study of students' prior understandings "which makes it possible for teachers to be able to anticipate students' reactions and solutions to the problems students study during lessons" (Research for Better Schools n.d.). While all teachers need to engage in *kyozaikenkyu* as part of their lesson planning, lesson study requires teachers to engage in it in much more depth.

Watanabe et al. (2008) remind us that the purpose of lesson study is not just to improve a single lesson but to improve mathematical instruction in general, which involves careful attention to *kyozaikenkyu*, something that is not always attended to in non-Japanese lesson study. While the literal meaning of *kyozaikenkyu* is the study or investigation (*kenkyu*) of instructional materials (*kyozai*), the word *kyozai* means much more than textbooks or curriculum materials and needs to involve learning goals. According to Yokosuka (1990)

> It is important that *kyozai* and subject matter content (specific knowledge and procedures to be learned through lessons) are distinguished. It is possible to explore the same subject matter with different *kyozai*, or we can investigate different subject matter with the same *kyozai*. (p. 19, translation cited in Watanabe et al. 2008)

Furthermore, according to Watanabe et al. (2008), "*Kyozaikenkyu*, is the process to help teachers gain a deeper understanding of *kyozai*". It is

> the entire process of research activities related to *kyozai*, beginning with the selection/development, deepening the understanding of the true nature of a particular *kyozai*, planning a lesson with a particular *kyozai* that matches the current state of the students, culminating in the development of an instructional plan. (Yokosuka 1990, p. 73, translation cited in Watanabe et al. 2008)

This notion is also found in the Netherlands in the *Realistic Mathematics Education* (RME) approach (Freudenthal 1973; van den Heuvel-Panhuizen 2001) where a sequence of learning experiences, termed a hypothetical learning trajectory, is posited before tasks are constructed, although this has its difficulties (see Figueiredo et al. 2009).

While there may be insufficient attention to *kyozaikenkyu* in non-Japanese implementations of lesson study, there is limited yet increasing attention being paid to task design in Western contexts. For example, Wiliam (2008) emphasizes the need for mathematical tasks that are both engaging and 'contingent'. In his view, contingency is a key element for sustaining learning, which is not aided by the discrete, unrelated task of many classrooms. Further, Swan, a noted British educational designer, points out that design principles include focussing directly on significant conceptual obstacles and using tasks that are accessible (Swan 2008), while Teppo and van den Heuvel-Panhuizen (2008), presenting reasons for 'mathe-didactical' task designs in mathematics education research, claim that

> not only does an unpacked understanding of the mathematical possibilities (or lack thereof) inherent in the task increase the potential of the research to probe for rich mathematical activity, but this analysis also informs the nature of the inferences that are made related to observed behaviour. (p. 206)

This clearly relates to a lesson as much as to a research endeavour, particularly with respect to the inferences to be made from student behaviour.

Watson and Mason (2007), in a recent review of the ways in which mathematical tasks are used in (Western) mathematics teacher education, suggest that the task

> in the full sense includes the activity which results from learners embarking on a task, including how they alter the task in order to make sense of it, the way in which the teacher directs and redirects learner attention to aspects arising, and how learners are encouraged to reflect or otherwise learn from the experience of engaging in the activity initiated by the task. (p. 207)

This definition of *task* is similar to that of Mok (2004) in which what is "constituted by the interaction between the task and the discourse between the teacher and students or between students…[is] called a 'learning task' event" (p. 2). On the other hand, Herbst (2008) suggests, more succinctly, that the task is "a representation of the *mathematics* to be learned…[and that it is] an opportunity to *study* and *learn* [that] mathematics" (p. 126, italics in the original).

While researchers may disagree over the exact definition of a mathematical task, there has been agreement for some years that the task has a key role in the planning of a lesson (see Brousseau 1997; Doyle 1988). We likewise adopt the view of Japanese educators, where the problem selection, and its presentation to the class, is a distinct characteristic of a lesson (Takahashi 2006). The distinction between task and activity, made by Christiansen and Walther (1986, p. 247), is also worth noting. They also suggest that the "widespread use of ready-made tasks…serves to reduce the teacher's personal investigation to questions about accessibility for *his* students" (p. 249, italics in the original). And this was in the era before Internet lesson-planning sites became available!

Others, such as the influential National Council of Teachers of Mathematics (NCTM) in the United States, have expressed the centrality of the task in effective mathematics classroom practice. The NCTM's position was that "tasks convey messages about what mathematics is and what doing mathematics entails" (National Council of Teachers of Mathematics 1991, p. 24). Similarly, but earlier, Freudenthal (1973) argued for mathematics tasks that adhere to the principle of 'guided re-invention'; that is, tasks that help students to construct new mathematical ideas for themselves.

However, Henningsen and Stein (1997), who cite Doyle's earlier (1988) work on tasks, also point out that the task is necessary but not sufficient. They argue "that the mere presence of high-level mathematical tasks in the classroom will not automatically result in students' engagement in doing mathematics" (p. 527). They state that

> our findings suggest that there was a discernable set of factors influential in assisting students to engage at high levels. These included factors related to the appropriateness of the task for the students and to supportive actions by teachers, such as scaffolding and consistently pressing students to provide meaningful explanations or make meaningful connections. (p. 546)

Nonetheless, we argue that while the centrality of the task *per se* is beyond question, there are other factors that will mediate task effectiveness. For example, we concur with Tsur (2008, p. 141) when he says that the teacher's "facility with using the task as a pedagogical tool" is a factor to which one must pay due attention, thus implying that professional development of teachers is therefore also a vital factor.

Thus, with Doyle (1988), and others, we argue that teachers should pay careful attention to the extent and the way in which, mathematics is emphasized by the task. We characterize the careful attention of "analyzing…carefully" (Shimizu 2002, p. 4) as exploring an iceberg below the waterline, to understand the hidden support that makes it float. That is to say, we need to explicate all the mathematical concepts and understandings that make a particular task or problem 'float' mathematically.

Four Types of Tasks Used in Research Lessons

In this section, we look at four types of tasks typically used in Japanese lesson study research lessons—tasks that

- directly address a concept;
- develop mathematical processes;
- have been chosen based on a rigorous examination of scope and sequence; and
- address known misconceptions.

These tasks have been selected to demonstrate not only different possible pedagogical foci, but also on what the detailed analysis and consideration of the tasks needs to be based—that is: an understanding of the mathematical content; its scope and sequence; children's mathematical understandings and hence their likely responses to the task; children's common misconceptions; and knowledge of a range of tasks and the possibilities the tasks offer to meet the teacher's goals.

The Circle Lesson: Directly Addressing a Concept

This lesson occurred in a Year-3 class of eight children at the Japanese School of Melbourne. According to the teacher, Mr. J, the main mathematics topic for the lesson was 'the concept of a circle'.

The lesson began with Mr. J producing a pole for a game of quoits, where a ring had to be thrown onto the pole from a distance. Mr. J placed the pole in the centre of an open space in the classroom and asked three children (shown as B1, G1 and B2 in Fig. 1) to stand at the three marked places on a red line at one side of the room. Children were concerned that the game would be unfair, but they focused mainly on the distance between children standing along the red line, rather than their distance from the pole.

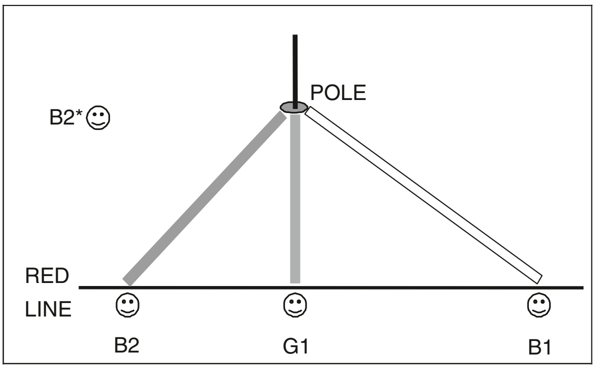

Fig. 1 B2's solution for making the quoits game fair

After a discussion on how to measure distances, with children using a metre ruler to measure their distance from the pole, Mr. J put different coloured, pre-cut strips of paper on the floor, as shown in Fig. 1, he then held them up together to show that their lengths, and hence the children's distances from the pole, were different. He then stated the question for the lesson (the *hatsumon*) as: "How can we make the game fair?" It took five more minutes, during which time children continued to try to find points to stand on the red line, before B2 came up with a way that two people could be the same distance from the pole—he moved the end of the G1's yellow strip to the point B2* on Fig. 1.

Mr. J then gave all the children a yellow strip and asked them to "think for themselves" and find somewhere to stand so that everyone was the same distance from the pole. Children were excited that no-one would be at a disadvantage. This segment took 20 minutes of the 45 minute lesson.

Mr. J then reproduced the situation on a large sheet of paper on the blackboard. He stuck a miniature pole on the paper and asked children to use sticky yellow paper strips and dots to represent their positions on the floor (see Fig. 2).

Mr. J: Look at the different positions—what do you notice?
G2: It's like a round circle [makes circle shape with hands].
G3: No—it's like a flower.
G1: If you follow the end of each yellow strip it will become a circle [traces large circle on the desk with her finger].
Mr. J: What if every student in the school took part? [adds more strips]…
B3: If there are many students standing round, maybe it's a circle.

Mr. J removed the pole and put another sheet of paper over the first with a circle drawn where the dots were and asked "How many yellow points would we need? A hundred? A million?" He then put the word circle on the paper and elicited names for the centre, radius and diameter from the children. The remaining 15 minutes of the lesson were taken up with the children working in pairs drawing circles. Initially, many children chose to use a compass, even though Mr. J told them that they

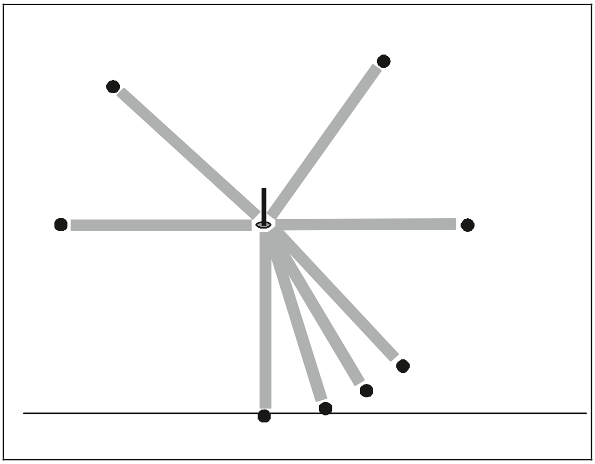

Fig. 2 Paper representations of children's positions

had not yet learned how to use one, and encouraged one girl who said that she could use a yellow strip of paper or a plastic circle to trace round, to show him how. After about 7 minutes, Mr. J asked children to find a way to draw a circle without a compass. A few minutes later Mr. J said: "Now everyone is tracing—is there another way?" Children tried various ways while Mr. J pivoted one of the yellow strips of paper around an end held by his finger. B2 excitedly cried out that he could do it and demonstrated drawing a circle by holding the middle of one end of his pencil case and tracing a circle with his finger in the hole at the other end. Children applauded and Mr. J demonstrated B2's method at the front of the class. The lesson finished with a few minutes of suggestions from children how to fix one end, culminating in the use of a drawing pin. Mr. J summed up by saying: "As you suggested, there are other ways of drawing a circle than just using a compass."

Both in the lesson and in his responses to a questionnaire, it was clear that Mr. J chose the task to enable him to directly address the concept of a circle and that this was consistent with his goals for the lesson. He stated his aims as "children have the *concept* of a circle and find *real* circular objects' [emphasis in original transcript]". According to Mr. J, the most important aspect of the lesson in terms of children's learning was that they understood that the circle is a locus. The purpose of working in groups (in this case pairs) was "to facilitate discussions while working", while the purpose of the whole class discussion was for children to "share ideas and strategies for solutions [demonstrating that] there are many different ways of thinking which reach the same conclusions". Mr. J further described his mathematics lessons in general as follows:

> Introductory lessons [to a topic] use materials. So this was typical. The introduction is very important and takes a lot of time. After that there is much practice, then we go to calculations—a series of 3 or 4 lessons [per topic].

Mr. J concluded with the comment that "Mathematics should be part of children's daily lives." In the 40 minute quoit activity, Mr. J embedded the concept of a circle in a rich, intriguing, intrinsically motivating, problematic framework by asking: "How can we make the game fair?"

This lesson highlights two important features of some Japanese lessons and the tasks that underpin them: the highly conceptual nature of the goal for the lesson and the need for sophisticated mathematical understanding. By way of contrast, in Australia, the concept of a circle as the locus of a set of points would not be regarded as part of the scope of primary school mathematics. Instead, there is considerable emphasis on descriptions and categorization of shapes.

Snow White and the Seven Dwarfs: Developing Mathematical Processes

This lesson took place in a preparatory class in Melbourne, Australia in the middle of the five- and six-year-old children's first year at school. Children sat on the floor while the teacher, Mrs. B, reminded the class that they had heard the story of *Snow White and the Seven Dwarfs* the previous day. She then put out a sheet of paper to represent a ‘long table’ at which Snow White and the seven dwarfs sat for their dinner. She said that Snow White always sat at the head of the table, while the dwarfs sat at the two long sides, with a different number of dwarfs sitting on each side each day. Seven counters were used to represent the dwarfs. One child was asked to illustrate a possible way—she placed one counter on one side and the remaining six on the other side. The teacher then presented the problem for the day on the board as shown below.

How might the 7 dwarfs sit at the table?

Did you find all the ways?

How do you know?

Children were told they could paste coloured rectangles onto paper to represent the table and draw “quick maths drawings”[1]—not ones with the dwarfs’ “hair and hats and eyelashes”—as well as write numbers. Alternatively, they could use concrete materials and jotters, where they could record their answers because “your job is to find as many ways as possible”.

The children worked individually or in pairs at their tables or on the floor for about 15 minutes, after which the teacher called the children back to the floor for a discussion of the different solutions.

Mrs. B commented that one child had said he had found seven ways, while another had found six. She reminded them that they wanted to find all the possible ways. As individual children contributed different ways, she wrote their solutions on a piece of card which she attached to a whiteboard as shown in Fig. 3.

The teacher commented that it was very difficult to see whether all the ways were represented, and suggested that they look at an ordering that had been used by one of the girls, Melody (see Fig. 4). When asked to try to find a pattern, the children replied that “it’s just the opposite”.

After some discussion with Melody as to whether she actually moved the counters around (which she had done) or just wrote the numbers ‘the other way around’, the teacher noted that there was “another pattern we could make”. When none of the children volunteered such a pattern, she showed the class the second pattern that Melody had made—see Fig. 5.

[1] In this description, quotes are from the video recording of the lesson. All children’s names are pseudonyms.

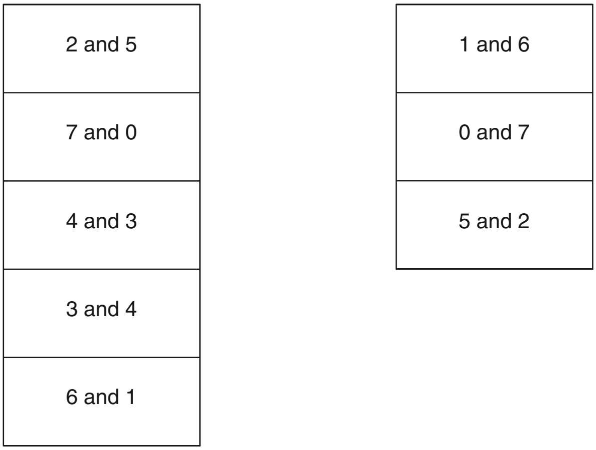

Fig. 3 Children's different seating arrangements for the seven dwarfs

Mrs. B then reproduced Melody's ordering on the board by writing the first two arrangements on the board and asking different children to supply the remainder: "She's making a pattern—0 and 7, 1 and 6.… Have a look what's happening—0, 1—what do you think comes next?" When the children got to 3 and 4, the teacher asked Ivy, "What is the pattern on this [right] side?" and Ivy replied, "The pattern is 7, 6, 5, 4—like counting backwards…from 7." After asking another child for the pattern on the left hand side, the teacher and children completed the list of arrangements. Mrs. B asked the children whether there was anything different they could have done and, just to make sure, she told the children to go back to their tables and tell her if they had an arrangement that was missing from the list—but to make sure there were still seven dwarfs! Some children thought they had different arrangements, but of course none really did.

Mrs. B then told the children to come back to the floor and asked, "Have we found all the ways?" to which the children chorused, "Yes!" She continued: "Nobody else got any more at their tables for seven…but how do we *know* we've got all the ways?" One boy replied: "We've used all the numbers." The teacher confirmed this, discussed with the children why there could not be more, and asked them again how many ways there were for seven dwarfs. She then asked what would happen if

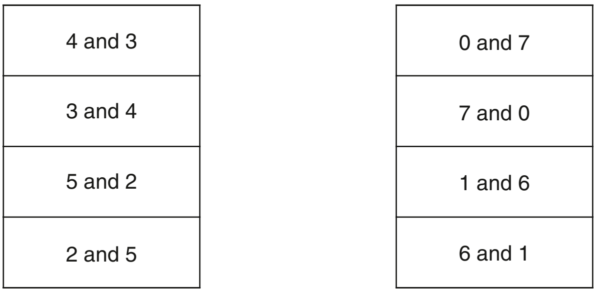

Fig. 4 Melody's first ordering of the seating arrangements

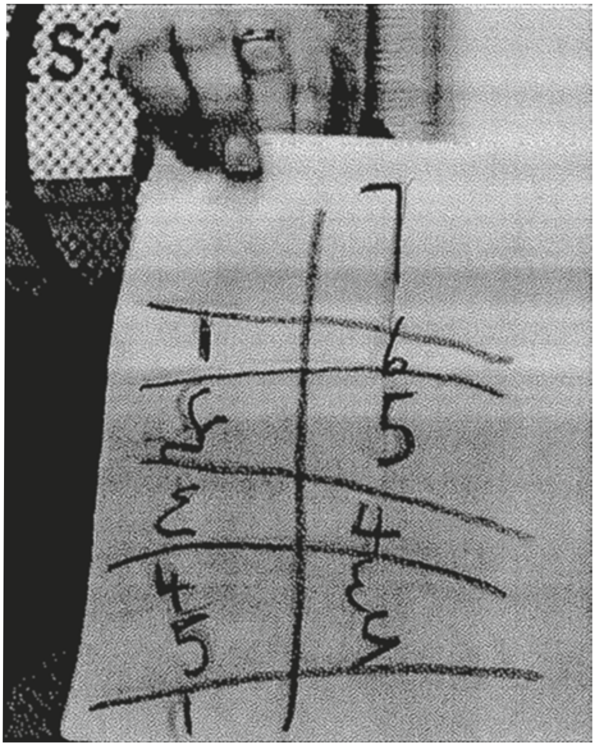

Fig. 5 Melody's second ordering of the seating arrangements

"at the three pig's house" there were eight people—how many different ways could they sit? After Ivy answered 9, the teacher asked what if there were 10 visitors and Caitlin replied 11 ways. "So what if we had all 24 children in the class sitting at a very long table?" Ivy answered 25. The discussion continued:

Mrs. B:	What is the pattern? How did you know each time without doing it? When there were 7 people there were 8 ways. When there were 8 people there were 9 ways. When there were 10 people there were 11 ways. When there were 24 there were…25 ways. What's the pattern?… How did you know without doing it each time?… What if there 100 people?
Child:	101.
Mrs. B:	What if there were 300 people?
Children:	400, 500, 104.
Caitlin:	301.

Children came to the board to try to write 301—the fourth child wrote it correctly after the first three wrote 1ε1, 3001, and 131, and the lesson ended with a discussion about which of these was correct. In total, discussion of the children's solutions, how they knew they had found all the ways, and the discussion of the number of ways for different numbers of people, took approximately 25 minutes.

When asked about the goals for this lesson, Mrs. B focussed on developing problem-solving processes, such as working systematically, as well as looking for patterns. In Japan, such a lesson might be termed a ‘jump-in lesson’ to indicate that it could take place at many different points in the curriculum sequence.

The 13 – 9 Subtraction Lesson: A Rigorous Examination of Scope and Sequence

The 13 – 9 subtraction lesson described in this section was part of a sequence of lessons on subtraction with regrouping in a Year-1 class in Japan conducted by a ‘veteran teacher’ Mr. N. The lesson began with the problem, “There are 13 persimmons. I have eaten □ of them, how many are left?” The teacher started with the number eaten as 2, then 3, then moved on to 9, at which point some children responded that they could not subtract 9 from 3. The teacher stated that the problem for the day was 13 – 9. Children worked individually on the problem for about 10 minutes, during which time the teacher identified three different solution strategies used by the children to be discussed in the *neriage* phase of the lesson. The first strategy was counting down. The second was *subtraction-subtraction* (see Fig. 7) and the third was *subtraction-addition* (see Fig. 6). During this part of the lesson, individual children came up to the blackboard at the front of the class to

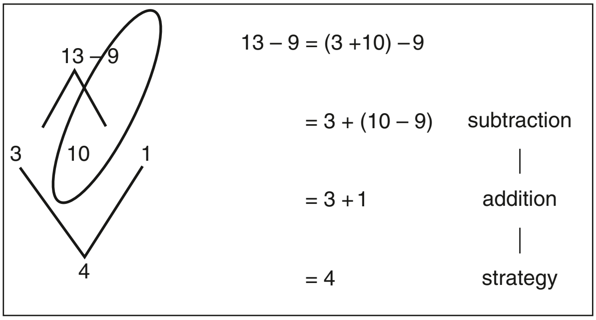

Fig. 6 The subtraction-addition strategy

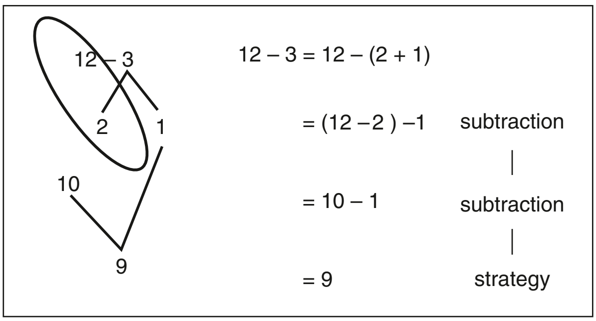

Fig. 7 The subtraction-subtraction strategy

explain their own, or another child's, strategy or to demonstrate it with magnetic blocks.

At the end of the explanation of the subtraction-addition method, the teacher asked each child to use their own blocks to demonstrate this method step-by-step. Although all three methods were explained, the lesson emphasized the subtraction-addition method by letting children experience this method with concrete materials. During the last part of the *neriage* phase the teacher asked children the similarity between the last two methods. Children responded that both methods used 10 as a unity. Children also said that they could use their previous knowledge.

Japanese first-grade textbooks contain a unit concerning subtraction of one-digit numbers from two-digit numbers, with regrouping, which is regarded as an important area of content to learn. There are a total of 36 such possible examples: 18 – 9, 17 – 9, 17 – 8, 16 – 9, 16 – 8, 16 – 7,…, 11 – 3 and 11 – 2. In Japan, which of the 36 tasks should be the first for children to learn is hotly contested. There are six companies publishing textbooks for elementary mathematics in Japan, and all textbooks start the unit with either 13 – 9 or 12 – 9. The second task is 14 – 8, and the third one is 12 – 3. Non-Japanese readers might be surprised that every textbook uses almost the same sequence of tasks, and also that, according to the teachers' guide, one hour should be spent on each of these three tasks.

Why should 13 – 9 or 12 – 9 be the first task for children to learn when they meet subtraction with regrouping for the first time? The reason given is that these tasks naturally lead to the subtraction-addition strategy. The subtraction-addition strategy, illustrated in Fig. 6, refers to subtracting 9 from 10 first, then adding 1 to 3, so the sequence of operations is subtracting first, then adding.

To subtract 9 from 10, we need to see 13 as 10 and 3 from our knowledge of the base ten notation. As we use the base ten system, children have learned to compose, or decompose, numbers with a 10, and in the first-grade textbooks this occurs just before the unit on subtraction with regrouping. The subtraction-addition strategy is usually dominant when solving a problem such as 13–9, because 9 is close enough to 10 for children to naturally relate it to 10, and see 9 and 1 become 10. Therefore the dominant strategy for 13 – 9 becomes (10 + 3) – 9 = (10 – 9) + 3 – that is the subtraction-addition strategy.

On the other hand, for the problem 12 – 3, the dominant strategy used by children is the *subtraction-subtraction* strategy. Here, as shown in Fig. 7, the 3 becomes 2 and 1, then 12 – 3 = 12 – (2 + 1) = 12 – 2 – 1 = 9. This strategy does not depend on place value notation. In this sense, it does not make full use of the base ten system. And also, the subtraction-subtraction strategy has the disadvantage that children often make mistakes in the tens or hundreds places if they try to adopt this strategy for larger numbers.

Consistent with the sequence of tasks for subtraction, the addition part of the textbook uses 9 + 4 as the first task.

When children solve 14 – 8, the two strategies, subtraction-addition and subtraction-subtraction tend to both occur in approximately equal numbers. Therefore, teachers who wish to promote argumentation in their classes sometimes use 14 – 8 as the first task for children, while textbook companies adopt a more conservative stance based on their desire to make it easy for teachers to anticipate student responses and to be sure that there will be enough children who use the subtraction-addition strategy.

Area and Perimeter: Addressing Misconceptions

Space does not permit a full discussion here of the fourth category of tasks, addressing a common misconception. An example of such a misconception, that exists among both Japanese and Western students, is that shapes with the same perimeter have the same area, and so, for example, if we were trying to measure the approximate area of a puddle we could carefully place a string around its perimeter and then deform its shape to that of a rectangle to calculate its area.

The first lesson on area in a fourth-grade Japanese textbook (Hironaka et al. 2006, pp. 22–23) shows children trying to decide which of the two shapes representing two 'newsletters' being compared has the larger area. Shapes have been chosen deliberately to have the same perimeter but different areas. After introducing the idea that area can be measured using square centimetres, a 'maths story' is used to further illustrate the fact that not all shapes with equal perimeters have equal area (see also Takahashi 2006). This addresses the common misconception of there being a unique relationship between area and perimeter for all shapes.

The Role of Lesson Study

While none of the Japanese lessons described here arose directly from lesson study, they are nonetheless the products of lesson study. That is to say, like many lessons in Japanese schools, these lessons owe their focus on mathematics, and their pedagogy of implementation, to teachers who have been imbued with the philosophy and practice of lesson study. This level of professional support, offered to Japanese teachers through lesson study, contrasts dramatically with that available to Australian and American teachers.

Firstly, most Japanese teachers have experienced lesson study themselves. Moreover, they do not have to develop structured problem-solving lessons by themselves—instead they are encouraged, like Mr. J did, to adapt lessons from the textbook, or draw on published articles such as the 'Study on teaching materials' or 'Practical study' sections of the *Journal of Japan Society of Mathematical Education*. In the case of research lessons for lesson study (and at other times too, presumably) teachers can usually also draw on the expertise of an expert mathematics teacher in their school. All of these support teachers in their *kyozaikenkyu.*

In Australia there have been a number of projects which have resulted in the publication in hard copy or online of so-called exemplary lessons (see Lovitt and Clarke 1985). However, for all but the most competent teachers, there remain problems with using such "exemplary" materials. Many of the lessons are designed to be highly flexible and capable of being adapted for a wide range of year levels. Often links with the 'regular' curriculum are not obvious and they are more of the Japanese 'jump-in' type. When asked about the source of her tasks, Mrs. B (the Australian teacher in the Snow White example) said she used a wide variety of sources for tasks, including books and the Internet, and frequently adapted tasks or developed her own. She stated that "There's no one particular place I get things from, I wish there was because it would make it a lot easier."

As with other researchers (see Swan 2008; Takahasi 2006, for example), Mrs. B's main criteria for suitable tasks were that all the children

- can start [the task] and that the solution is not immediately obvious;
- are interested, that they are engaged with the story, or whatever [the task was];
- have choice with whatever they use to help them solve the problem.

For Mrs. B, the selection of task is influenced by contextual factors as well as the mathematics. She is a well-known, highly experienced and highly respected teacher, with an excellent understanding of the scope and sequence of mathematics, particularly in the first few years of school, but without the support of a tradition of lesson study she is very much left to work on her own in carrying out *kyozaikenkyu.*

Regarding the subtraction lesson, Western observers are often astonished not only by the thought of entire lessons being spent on single tasks like these, but also by the order of presentation being the subject of so much study and debate. However, Japanese lesson study is frequently used to investigate sequences of tasks that are different from those traditionally used. For example, while one of the authors was in Japan recently, she observed several of a sequence of eight or more research lessons designed to explore the effect of introducing quotitive (measurement) division before partition (sharing) division to a Year-3 class. Thus, Japanese lesson study involves continuing efforts to examine and improve approaches: it is not seeking merely to transmit a single 'best' approach to all teachers.

The area and perimeter lesson, mentioned only briefly here, illustrates the use of extensive research into finding and using tasks that directly address common student misconceptions. Teachers need to be aware of the research literature on children's understandings, and use this knowledge when planning lessons and selecting tasks.

The 'iceberg' metaphor reminds us that there are many aspects of mathematics tasks to be considered when planning a lesson. For example, as described above, selecting a suitable task, or activity, in the lesson study context is not simply a matter of finding a task that carries the required mathematical content. The fact that different versions of the task may provoke different responses from the children necessitates a careful choice to be made. This is evident in the lesson introducing subtraction 'across ten', 13 – 9. Clearly it is imperative for a teacher to know the common strategies that children use, in order to orchestrate the discussion of a range of strategies. According to Killen (2009), being prepared for a variety of responses is a key point in any lesson involving discussion.

Further, selecting the exact wording of the *hatsumon*, or question posed to the children to solve, is also a critical step in the planning of a lesson. To many, the question of how to make the quoit game fair could seem a long way from the lesson intention of exploring the concept of a circle. However, as we have described, this outcome was achieved in a very engaging and, we would hope, memorable manner. Again, in the Snow White example, the question posed was intriguing and appropriate for the age group. Moreover, it was an opportunity to allow children to explore and use a mathematical process, as well as to experience, in school, the unusual situation of there being a variety of correct answers.

This is not to argue that other aspects of the Japanese lesson are unimportant: the discussion, polishing and refining of children's strategies (*neriage*), observing differences in children's work (*kikan-shido*), and summarizing (*matome*) are also indispensable supports to an effective lesson (Shimizu 1999). However, we wish to concentrate, in this chapter, on the more 'hidden' facets of lesson study, ones that support the iceberg: these we wish to place in the forefront of our thinking, in order to gain the greatest benefit for those involved in the practice of lesson study in countries outside Japan.

The development of practice that already has some local currency is thought to be the most effective way in which to change teacher practice (following the I'Ching argument that going with the river's current rather than against it, is more effective). While the precise manner in which these practices would be developed, and culturally mediated, is still currently a work-in-progress, our research on *Communities of Mathematical Inquiry* (see Doig et al. 2001; Groves et al. 2000; Groves and Doig 2004), suggests that many teachers are aware of deficiencies in current practice, and are eager for professional development that addresses these. The situation in the United States is more advanced than Australia, with more than 500 active lesson study groups. Thus, it is suggested that following the example of the United States, of mathematics educators working closely with small groups of interested teachers, offers opportunities for creating a cadre of effective teachers with clear understandings of not only what to do, but also how to achieve effective mathematics classroom learning: teachers who are attentive to the tasks that they use, and capable of "analyzing…carefully" (Shimizu 2002, p. 4) these tasks.

Agreeing with Zaslavsky (2007, p. 435) that "effective MTEs [mathematics teacher educators] engage MTs [mathematics teachers] in carefully crafted tasks in order to…construct what they need to know about teaching school mathematics", we believe that research into "carefully crafted" tasks suitable for teacher education is a priority no less than is research on lesson study tasks for classroom teachers. There are at least two key reasons for this priority. First, this research would provide the information needed to change and improve the learning of mathematics pedagogy. Second, such research would provide tasks for mathematics teacher educators that would allow them to model the type of classroom pedagogy that we believe our mathematics teachers should be using. Although examples of suitable tasks have existed for some time (see Lovitt and Clarke 1985), their use has suffered to date, in Tsur's (2008) terms, from teachers' inability to use tasks effectively as a pedagogical tool.

Clearly, we need to heed Hiebert and Stigler's (2000) argument that "improving teaching does not depend on eventually perfecting 182 lessons but rather on engaging intensively with the issues involved in teaching any lesson" (p. 16). Such intensive engagement, in both teacher pre- and in-service education, we believe, should provide future teachers with practical experience of effective mathematics pedagogy, as well as a better understanding of the foundations of this pedagogy and the mathematics that they are preparing to teach: lesson study has the potential to do this.

However, if the key elements of lesson study are to be effective more widely as Professional Development for teachers, a re-conceptualization of curriculum and textbooks is needed, to assist in re-orientating teachers, and researchers, to the need for coherent sequences of lessons which are focussed on the mathematical tasks *per se*. While examples of curriculum materials with such an orientation exist, they are not widely seen outside the Netherlands (see van Galen et al. 2008) or Japan.

Conclusion

We firmly believe that the practice of lesson study, either as part of initial teacher education, or later professional development, has the potential to increase the number of effective mathematical learning experiences enjoyed by children. But lesson study is not achieved without effort: it is no 'silver bullet'. The reason for this is that the strengths of lesson study rest on two significant bases. The first is the detailed planning of lessons, which, in turn, is based on deep reflection on the mathematics and the pedagogy. While many lesson study groups outside Japan focus attention on the mathematics, often this is at the expense of the pedagogy, or *vice versa*. It is critical that a balance be maintained.

The second basis of lesson study is cultural, including both the classroom culture and the wider professional culture of teachers. In an ideal lesson study classroom, the 'didactic contract' assures that every student willingly engages in the set tasks, contributes to discussion, and knows that their contributions are valued. This is not always the case in non-Japanese classroom cultures. Additionally, in the broader professional culture of teachers, lesson study requires a culture where being open to other perspectives on teaching and critical commentaries on a lesson are seen as positive contributions to pedagogical knowledge and understanding. For many non-Japanese teachers this is not an easy attitude to achieve.

However, despite such difficulties, cultural and pedagogical, we believe that lesson study, in its full sense, has the potential to make learning mathematics an enjoyable and positive experience for students, as well as a rewarding professional experience for teachers. Further, we hope that the examples and discussion provided in this chapter will help to reveal possibilities and encourage teachers to consider exploring beneath the iceberg.

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