Chapter 3
Knowing and Identity: A Situated Theory of Mathematics Knowledge in Teaching

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A group of prospective secondary teachers are engaged in a school mathematics problem involving fractions: *why can you multiply to multiply, but not add to add?* All the prospective teachers are well qualified. In fact, several have doctorates in mathematics. All, of course, can add, subtract, multiply and divide fractions with ease. Yet, they are finding the problem of explanation exceedingly difficult.

How is it that such an apparently elementary problem can cause a group of mathematical experts such problems? Mathematically, the problem involves the algorithms for arithmetic involving fractions. One multiplies the numerators and the denominators to multiply fractions: $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$, but one does not add them to add: *in general*, $\frac{a}{b} + \frac{c}{d} \neq \frac{a+c}{b+d}$. Of course, the problem is that these prospective teachers have never been asked this question before. But the problem becomes more complex when posed in the context of teaching in that it is no longer simply a *mathematical* question (how to show the statement is true), but also a *pedagogical* question (how to enable others to see the statement is true). At the heart of these mathematical and pedagogical questions lie some of the "big ideas" of school mathematics: the notion of rational numbers as division of integers, the relationship between multiplication and addition and the ways in which rational number may be represented. The notion of pedagogical content knowledge (PCK) as developed by Shulman, Ball and others is one response to this complexity: mathematics teaching requires a specialist knowledge of mathematics for teaching that integrates a knowledge of mathematics and pedagogy. These approaches have been discussed in depth in Chapter 1 by Goulding and Petrou in this book. Yet, as Goulding and Petrou indicate, these approaches have downplayed the importance of context. In this chapter, I take this critique further. I examine this issue of context and argue that mathematics teacher knowledge is not simply *applied* within the context of teaching mathematics but is rather *situated* within the complex and social world of mathematics classrooms. In

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other words, to simply focus on application and context is to underplay the ways in which social structures support (or hinder) teacher knowledge and its use.

My analysis draws on what Lerman (2000) terms the 'social turn' in mathematics education. A key work in this social turn is Lave and Wenger's (1991) monograph examining the nature of learning as apprenticeship and re-casting knowledge in terms of situated cognition. Whilst this original work largely considered learning in informal settings outside formal education, it has nevertheless been influential in the formal context of mathematics education (Boaler, 2002; Greeno, 1998), particularly in relation to the perennial issue of how students use or transfer the mathematics learnt in school, into real world contexts. This notion of transfer – how knowledge learnt in one context can be used or applied in a different context – has in turn been the subject of much contentious debate (e.g., Anderson, Greeno, Reder, & Simon, 2000), although it is arguable that this debate has often been characterised more by misunderstandings than by genuine disagreement. As Putnam and Borko (2000, p. 12) argue,

> It is easy to misinterpret scholars in the situative camp as arguing that transfer is impossible—that meaningful learning takes place only in the very contexts in which the new ideas will be used. The situative perspective is not an argument against transfer, however, but an attempt to recast the relationship between what people know and the settings in which they know—between the knower and the known.

From this perspective, knowledge is social and contextualised rather than individual and general, whilst knowledge about mathematics teaching is less about general principles and more about 'intertwined collections of more specific patterns that hold across a variety of situations' (Putnam & Borko, 2000, p.13). It is a recognition of the similarities and differences between these patterns that enables the growth of a more abstract mathematical knowledge.

The Problem of Mathematics Teacher Knowledge

It appears self-evident that teachers should know about mathematics in order to teach it effectively. But teacher knowledge in mathematics is an area of some controversy. There is evidence that poor subject knowledge in mathematics has a negative impact on teaching (e.g., McDiarmid, Ball, & Anderson, 1989; Rowland, Martyn, Barber, & Heal, 2000). There is considerably less consensus on what constitutes the mathematical knowledge necessary for teaching. Some have argued that improving teachers' knowledge of mathematics per se will lead to better teaching (e.g., Alexander, Rose, & Woodhead, 1992). However, the evidence base in this area suggests otherwise. Several studies, for example, have found no link between teachers' mathematical knowledge, as measured in terms of academic mathematical qualifications, and effective teaching (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997; Begle, 1968). What is clear is that the connection between teacher knowledge and teaching outcomes is neither simple nor straightforward.

To deal with this problem, research has focused on exploring the nature of teacher knowledge in mathematics. One strand of this research has been to link mathematical knowledge for teaching to ways of knowing in the discipline of mathematics. Lampert (1986), for example, distinguishes between *procedural* and *principled* knowledge of mathematics. Procedural knowledge is a rule-guided 'knowing that' and concerns mathematical procedures and their use to compute correct answers. Principled knowledge, on the other hand, is a wider and more conceptual 'knowing how' and includes the knowledge of mathematical concepts that enable the construction of procedures for solving mathematical problems. Lampert's distinction has similarities to Skemp's (1976) distinction between instrumental and relational understandings, Prestage and Perks (2001) learner-knowledge and teacher-knowledge, and Thompson, Philipp, Thompson, and Boyd's (1994) calculational and conceptual orientations.

Increasingly, researchers have argued that mathematical knowledge for teaching is distinct and different to the knowledge necessary to practice mathematics. As I have already noted, a key starting point for much of this work is Shulman's (1986) notion of pedagogical content knowledge which 'goes beyond the subject per se to the dimension of subject knowledge *for teaching* . . . the particular form of content knowledge that embodies the aspects of content most germane to its teachability' (p. 9, original emphasis). The nature of pedagogical content knowledge is itself, however, something of a contested idea within the education research community. McNamara (1991), for example, argues that there is no clear distinction between subject knowledge and pedagogical content knowledge. Indeed, Corbin and Campbell (2001) argue that pedagogical content knowledge is most useful as a metaphor that locates teacher knowledge as embedded within the complex and unpredictable practice of teaching. Another critique is epitomised by Brown and McIntyre (1993), who argue that much of teachers' knowledge is tacit, craft knowledge that cannot be codified as theoretical abstract knowledge. For Brown and McIntyre, the knowledge of an expert teacher is more intuitive and, in a very real sense, less explicit than that of a novice.

Taking this notion of tacit knowledge further, situated theorists problematise the very nature of knowledge, arguing that teachers' mathematical knowledge, like any other form of knowledge, is located in social practice (Greeno, 1998; Putnam & Borko, 2000). Hence, in a development of Lave and Wenger's (1991) work, Adler (1998) refers to a dynamic, contextualized and active process of 'knowing' rather than the more static, abstract and passive notion of 'knowledge'. Thus, teacher knowledge is *embedded* in the practices of teaching and any attempt to describe this knowledge abstractly is likely to fail to capture its dynamic nature.

A Case Study from Primary Mathematics: Alexandra's Knowledge of the Multiplication and Division of Fractions

In this section, I discuss the case of Alexandra,[1] a primary teacher, and her knowledge of proportional reasoning. I contrast Alexandra's knowledge of

[1] Alexandra is a pseudonym.

proportional reasoning in the context of developing lessons and leading professional development sessions with her knowledge in the context of a structured mathematics interview.

This case study is drawn from a 4-year longitudinal study into the professional change of six teachers involved as teacher-researchers in the Primary Cognitive Acceleration in Mathematics Education (CAME) Project research team (Johnson, Hodgen, & Adhami, 2004). This team consisted of four researchers, four teacher-researchers and the Local Education Authority mathematics advisor. During the school year 1997/1998, the team met fortnightly to develop Thinking Maths lessons specifically for primary children aged 9–11. During the second phase of the project, over the school years 1998/1999 and 1999/2000, a further cohort of teachers joined the project to implement the Thinking Maths lessons more widely. In Phase 2, the teacher-researchers led professional development sessions aimed at enabling this new cohort of teachers to teach the Thinking Maths lessons in their own classes.

In the research study, data collection was qualitative using multiple methods, including observations of seminars, lessons and professional development sessions, interviews with individuals and groups, and structured mathematical interviews (adapted from Millett, Askew, & Simon, 2004).[2] Here, my focus is on the mathematics interview, which took place in December 2000. During this interview, Alexandra was asked to solve several problems and to suggest models, stories, or diagrams to use when teaching the ideas to children. The questions themselves largely related to two aspects underlying the elementary mathematics curriculum: rational number and multiplicative reasoning. I focus on three related questions from this interview:

> How would you solve these problems? What would be a good story, diagram or model for them?

$$0.5 \times 0.2 \quad 3 \div 0.75 \quad 1\frac{3}{4} \div \frac{1}{2}$$

I was particularly interested in the extent to which the teachers could generate a variety of appropriate and pedagogically useful illustrations, and in the range of different meanings of multiplication and division that they drew upon. Ma (1999), for example, describes three models of division: measurement, partitive, and factors and product. These broadly relate the understandings of multiplication in terms of repeated addition, scaling and arrays (and areas). There is extensive research evidence to suggest that the area model is used in only limited ways in UK primary (and secondary) mathematics classrooms (Nunes, 2001).

As a Primary CAME teacher-researcher, Alexandra was involved in the development of a number of lessons addressing students' misconceptions in collaboration with other teacher-researchers and academics. Specifically, she, together with another teacher, developed two lessons focusing on fractions: 'Share an Apple', and

[2] This drew on previous work at King's (Askew et al., 1997), which in turn drew on a range of sources. An item on division of fractions, $1\frac{3}{4} \div \frac{1}{2}$, for example, was drawn from Ball (1990) work and is also discussed in Ma (1999).

'Halving and Thirding' (Johnson et al., 2003). In Share an Apple, the focus is on representations and comparisons of fractions. So, for example, children are asked to consider various ways of representing and comparing the magnitude of simple fractions of everyday objects. In Halving and Thirding, the focus is on developing and connecting different representations for the multiplication of fractions, including repeated multiplication by $^1/_2$ and $^1/_3$, with a particular focus on developing the area model for multiplication and linking this to other representations. Alexandra herself suggested this focus on the area model based on her experiences of team-teaching the lesson. She also led the professional development sessions introducing these lessons and had contributed to an academic paper on their development.

The research reported here took place in the context of the National Numeracy Strategy (NNS) in England, a national initiative focused on primary mathematics pedagogy (Brown, Millett, Bibby, & Johnson, 2000). One feature of the NNS was the appointment of several hundred Numeracy Consultants. The role of these local primary mathematics specialists was to support teachers and to deliver professional development to them. Throughout much of her participation in Primary CAME, Alexandra was also a Numeracy Consultant, whose responsibilities included delivering training aimed at enhancing primary teachers' subject knowledge of mathematics. In this mathematics educator role, I observed her teach several National Numeracy training sessions on both fractions and multiplication, during which she appeared to be fluent with a variety of both techniques and representations. In addition, in response to her perceptions of weaknesses in these training materials and in collaboration with another Numeracy Consultant, she developed a further session for teachers in which she focused on the use of the area model of multiplication in relation to fractions together with the concept of equivalence.

Given these experiences, I had expected Alexandra to demonstrate a sophisticated understanding of multiplication in the mathematics interview. Yet, her knowledge appeared to be very significantly weaker in this setting: in the interview, she appeared to know 'less' and to know it less securely. Alexandra could successfully answer all the questions performing most of the necessary calculational procedures correctly, although on several questions this took a considerable amount of time and whilst solving the problems, she made several mistakes which she corrected during the interview. At one point, she indicated some awareness of her limited understanding referring to division by fractions [$1^3/_4 \div {}^1/_2$] as follows: "If I was doing that the way I was taught to do it, I would just turn that all upside down. And I have real problems with this idea of division by fractions." However, she was unable to carry out this procedure and solved the question by converting to decimals mentally, and then using a calculator. To solve 0.5×0.2, she used a standard multiplication algorithm, as in Fig. 3.1.

$$\begin{array}{r} 0.5 \\ \times\ 0.2 \\ \hline 1\ 0 \\ \hline 0\ 0\ 0 \\ \hline 0.1\ 0 \\ \hline \end{array}$$

Fig. 3.1 Alexandra's procedure for solving 0.5×0.2

As she carried out the algorithm, she commented on how she knew where to place the decimal point in the product: "There are two decimal places in the question, so there must be two decimal places in the answer." This, together with her inclusion of the multiplication by zero, strongly suggests that her understanding of this method is certainly heavily reliant on procedural knowledge.

Although Alexandra read the answer correctly as 0.1 and used the same form as in the question, she did not notice that this could be read as a tenth or that the calculation was equivalent to either of the relatively simple 'half of two tenths' or 'half of a fifth'. Hence, she appeared to have no strategy to check, or make sense of the result of this calculation procedure. Indeed, she could not generate an illustration of this problem. Whilst she did not get this problem 'wrong', her knowledge did appear to be partial and limited.

Alexandra found the generation of any models extremely difficult and required considerable support and prompting to tackle these questions. Indeed, she asked me, with apparent disbelief, if I could do it. She provided a single story for just two of the three problems. Reflecting her preference for decimal fractions, she found $3 \div 0.75$ relatively straightforward, after I had suggested thinking about contexts involving measures: "how many lots of 75 pence can you get from three pounds." However, she had considerable difficulty with $1^3/_4 \div {}^1/_2$, eventually producing the following story:

> If you said that was one, and that was three quarters you'd get three halves and half a half out of it. But that's not very helpful is it? . . . One, OK, that's one and three quarters, so you can get one, two, three. Three halves out of it. And half of a half.

The example is more 'helpful' than Alexandra suggests. Repeated addition does provide one satisfactory explanation for the answer. The 'pure' mathematics context of numbers is, in this case, rather more helpful than the commonplace use of pizzas or cakes to illustrate problems involving fractions. Yet, whilst Alexandra's subject knowledge here appears stronger than any of the US teachers in Ma's (1999) study, this example does highlight a problematic issue. Alexandra had developed the two fractions lessons with the specific aim of enabling children to develop a range of models for the representation of fractions. The Halving and Thirding lesson had used both measurement and area representations for the multiplication of fractions, an aspect of the lesson which she herself had highlighted several times during the lesson simulation to Phase 2 teachers. It is somewhat surprising that, given these fairly intense lesson development experiences together with her experiences as a mathematics educator, she was not able in the interview to draw on the area model to division by fractions, or more significantly, to the multiplication of decimal fractions. Indeed, she was unable to provide an illustration of 0.5×0.2. More surprising still is her reaction to being asked to think of models, given that I had observed her emphasise different meanings of multiplication and division (including repeated subtraction/addition and the area/array models) and the need to understand children's different ways of seeing mathematical relationships when leading training sessions. Of course, this does not mean that she did *not know* other models. However, the difficulty that she encountered generating these stories does suggest that she lacked an intuitive familiarity with these and different models of multiplication/division. Alexandra's failure to draw on her experiences of developing the fractions lessons suggests that her knowledge was highly contextualised.

This case presents a dilemma. This is not a case where transfer has failed. Faced with these interview problems in other situations, Alexandra 'knew' more and performed 'better', and thus, in these different contexts, her mathematics appeared 'good enough'. Certainly, her knowledge *in context* appeared stronger than her knowledge *out of context.* In order to make sense of this, Lave and Wenger's insights about situated cognition are helpful. Alexandra's mathematics knowledge for teaching had developed in large part within the context of teaching, teacher education and curriculum development. Contrary to common wisdom, this knowledge was *situated*; it was 'known' in the context of teaching.

In the context of teaching, Alexandra 'knew', for example, about different models for the multiplication of fractions in the context of lesson development and, as a tutor during INSET sessions, when such knowledge was explicitly part of her role. Significantly, these were the settings where she was working in collaboration with others, and she had access to lesson or course guidance. She was not simply a passive participant in these contexts, nor was she simply 'delivering' the pre-prepared course materials. In fact, Alexandra's knowledge appeared to be relatively strong in both settings, and at least as strong as that of Numeracy Consultants in general[3]: it allowed her not only to participate in the discussions within the research team, but also to respond authoritatively to teachers' questions. However, Alexandra's knowledge in teaching did not simply derive from a more general individual mathematics knowledge. Rather, her knowledge in teaching was supported by the social communities and relationships in which she acted as an expert. These communities provided the cognitive and discursive tools with which Alexandra could be knowledgeable mathematically. It was distributed in the sense that it was 'stretched over' (Lave, 1988) and supported by other individuals and artefacts, in particular, lesson materials and structures. In other words, in being situated, her knowledge was both *social* and *distributed* (Putnam & Borko, 2000).

It is important to recognise that the interview was something of a 'testing' and artificial situation. The problems posed were deliberately 'tricky', and the situation raised issues of mathematics anxiety. Alexandra certainly seemed to perceive the interview as something of a threat to her professional identity. It is quite possible that in normal classroom contexts, Alexandra would be less unsettled. However, an important aspect of teacher knowledge is that it can act as a resource to enable a teacher to act in an unpredicted or unexpected situation. Thus, the situated, social

[3]My evidence here is partly based on my own observations and partly based on evidence gathered for the Leverhulme Numeracy Research Programme, an extensive 5-year longitudinal study of primary mathematics covering the period of the introduction of the NNS and the appointment of Numeracy Consultants (Millett, Brown, & Askew, 2004b).

and distributed nature of a teacher's mathematical knowledge for teaching may hinder the teacher's ability to respond appropriately in novel contexts, for which the teacher does not have an instant recourse to support her knowledge.

Is this just an Issue for Primary Teaching?

The literature on teacher knowledge is dominated by research in primary/elementary education, and one could be forgiven for concluding that the problem of teacher knowledge is primarily an issue in this sector. In a sense, this emphasis is unsurprising since the problem of teacher knowledge is brought into sharp focus in a sector where the majority of teachers are generalists[4] and primary teachers generally have considerably less formal education in mathematics. As a result, their mathematical knowledge is likely to be weaker and more influenced by contextual factors. Certainly, most *mathematically trained* secondary teachers' mathematical knowledge is likely to be rather more secure than that of most primary teachers, particularly when it comes to solving school mathematical problems of the sort Alexandra was asked to solve.[5]

The evidence, whilst less extensive, suggests that secondary teachers' knowledge is no less situated. Thompson and Thompson (1994), for example, describe a middle-school specialist teacher whose knowledge of rate and speed was strong and fluent: he himself could solve classroom problems with ease. Yet this very fluency was a barrier to teaching. When observed teaching a student one-to-one, the teacher conceived, albeit implicitly, of speed in terms of the covariance of distance and time, whilst the student's understanding was additive and discrete. The student did not have an image of motion as the simultaneous accumulation of distance and time (i.e. direct proportion). The teacher's own connections between representational structures and 'calculational' procedures for solving the problem were so strong that, when working with a student, he "*saw* (i.e. imputed) appropriate reasoning any time [the student] employed an appropriate calculation" (p. 299, emphasis in original). In a later analysis, they argue that the teacher's understandings of division and proportionality were so "packed" that they were "insensitive to conceptual subtleties in the situations" (Thompson & Thompson, 1996, p. 4).

One aspect of the power of mathematics lies in this "packed" and abbreviated nature. A fluent mathematician can choose the most appropriate representation for solving a problem irrespective of whether this representation is actually appropriate for modelling this particular problem. The essence of teacher knowledge involves an *explicit* recognition of this – "unpacking" the mathematical ideas (Ball & Bass,

[4] I recognize that there are a number of educational systems internationally (e.g. in Israel) in which there are specialists teachers of elementary mathematics. Nevertheless, the generalist remains the norm.

[5] Not all secondary teachers of mathematics are mathematically trained, of course. In England, for example, a significant proportion of them have weak mathematics qualifications, particularly those teaching lower secondary mathematics (Johnston-Wilder et al., 2003).

2000). On the other hand, doing mathematics only requires an *implicit* recognition of this. Indeed, fluency in mathematics arguably involves developing such implicit understandings. To a competent mathematician, the nuances of meaning inherent in these different pedagogical representations of mathematics can seem trivial and unimportant.[6] Hence, it may be that there is a tension for many secondary teachers of mathematics in that some aspects of mathematics knowledge for teaching run counter to the habits and norms of mathematics as a discipline.

This is not to argue that mathematics knowledge does not matter, but rather that mathematical knowledge is not sufficient in isolation. Lloyd and Wilson (1998) discuss how a teacher's sophisticated understanding of functions enabled him to implement an innovatory reform-focused curriculum. Lloyd and Wilson's teacher had previously taught a traditional curriculum for 14 years. They argue that the teacher's rich and well-articulated mathematical knowledge enabled innovation, but only in the context of curriculum materials and a related professional development programme that supported the innovation. Like Alexandra, Lloyd and Wilson's teacher's mathematical knowledge for teaching was supported by artefacts and social structures. Unlike Alexandra, his knowledge was also supported by a rich understanding of mathematics.

The Contribution of Situated Theories: What Does This Mean for Teacher Knowledge?

There is no doubt that Shulman's (1986) pedagogical content knowledge and the work of Ball and others provide a very significant contribution to understanding teacher knowledge. However, the analysis that I have presented here strongly suggests that mathematics teacher knowledge is very much more deeply embedded in practice than the PCK literature generally acknowledges. Whilst subsequent work has emphasised the aspects of Shulman's work that attempt to codify teacher knowledge, it is often overlooked that he did examine the forms of knowledge. This neglected area of Shulman's work relates to the way teacher knowledge is 'held' and used in teaching. Shulman conceives of knowledge as involving propositional, case and strategic aspects. These are discussed in some depth by Goulding and Petrou in Chapter 2. The case and strategic aspects of knowledge do certainly go some way towards recognizing the interrelationship between knowledge and its use. Shulman conceives of teaching "*theory* through cases" (p. 11). Further, he suggests that the strategic may be better captured as a process of "knowing" rather than the more static "knowledge" (p. 14) and argues that this "comes into play as the teacher confronts particular situations or problems whether theoretical, practical or moral, where principles collide and no simple solution is possible" (p. 13). This aspect

[6] See Saunders (1999) for an example in which a professional mathematician rejects the pedagogical distinction between fractions as operators and quantities as "playing tricks" (p. 3) and indicative of the de-professionalisation of teachers.

of Shulman's work provides many insights, particularly regarding the application and use of teacher knowledge. Nevertheless, it is a largely individual conception of knowledge. One consequence is a negative focus on the problem of teacher knowledge in terms of finding and fixing individual deficits (Askew, 2008). However, some aspects of mathematics teachers' subject knowledge are more difficult than others to pin down and codify. Almost inevitably, the focus on knowledge is concentrated on the more easily describable ideas (e.g. number facts) with much less emphasis placed on the more ephemeral but equally important ideas that Yackel and Cobb (1996) term socio-mathematical norms such as symmetry.

In viewing knowledge as situated, social and distributed, the situated perspective presents a significant advance. A major contribution is that this approach places much greater emphasis on the communities in which mathematics teachers are engaged rather than on individual knowledge. In principle, it is certainly desirable for teachers to 'possess' a sophisticated knowledge of mathematics for teaching that is evident in a variety of contexts, both inside and outside the classroom. I have little doubt, for example, that, were the gaps in Alexandra's mathematics knowledge to be addressed, her knowledge of mathematics in teaching – and her teaching of mathematics – would also improve. However, it is also important to bear in mind that the key setting in which teachers 'use' and 'apply' their mathematics knowledge is in the classroom. In Alexandra's case, there certainly were significant gaps in her knowledge of rational number, as evident in the mathematics interview. But ultimately, the quality of a teacher's mathematical knowledge in interview situations does not matter in itself, except possibly for research purposes. What does matter is that a teacher's mathematical knowledge as situated in teaching contexts is sufficient for successful learning to occur. The evidence presented here suggests that classroom knowledge is not a straightforward contextualisation or application of a more abstract and general a priori mathematical knowledge.

A second contribution relates to the nature of learning. Adler (1998) argues that becoming a mathematics teacher involves learning to talk both *within* and *about* mathematics teaching and learning, rather than simply learning new knowledge. In their study involving a group of mathematics teachers from one middle school, Stein, Silver, and Smith (1998) similarly highlight the importance of story and narrative in restructuring and reworking knowledge about mathematics teaching. They see this restructuring of existing knowledge and experience as more important than the acquisition of new knowledge – echoing Askew et al.'s (1997) findings about the importance of teachers' beliefs about mathematics in the teaching of numeracy. Stein et al. (1998) place these notions of story and narrative in the context of teachers' professional identities, arguing that teacher learning is best conceived of as a process of identity change.

One criticism of the situated learning literature, and in particular the work of Lave and Wenger, is that the context is conceived of as relatively static and fixed. Hence, individual learning can appear as following fixed and predictable trajectories of learning. Holland, Lachicotte, Skinner, and Cain's (1998) conceptualisation of identity in terms of agency and social structure provides a way of understanding the unexpected and surprising nature of learning. In an analysis of students' mathematical identities, Boaler and Greeno (2000) relate Holland et al.'s (1998) conception of identity to Belenky, Clinchy, Goldberger, and Tarule's (1986) notions of authority and knowing. They link procedural knowing to an acceptance of external authority in mathematics; and conceptual or principled knowing, to a more questioning and critical stance – the need to 'know why'. Similarly, Povey, Burton, Angier, and Boylan (1999) discuss how developing an authorial stance towards mathematics enables teachers to develop such a critical stance. Hodgen and Johnson (2004) examine teacher motivation and the reasons why teachers participate (or do not participate) in learning about mathematics education, arguing that the motivation to change is inextricably linked to teachers' identities and the social context in which they are located. Focusing on the aforementioned case of Alexandra, they discuss how the context of a school mathematics lesson prompted her to make an explicit connection between spatial and numerical representations (seeing the Cartesian system as "like a 2D number line", p. 236) and to 'see' the mathematical nature of diagrams and representations of fractions. Clearly, these are key components of mathematical knowledge for teaching, but Hodgen and Johnson conceive of her learning as an *authorial* choice in response to the particular demands of circumstance. This focus on identity highlights part of the difficulty of teacher learning. Bartholomew (2006), for example, uses the notion of the 'defended self' to highlight how mathematics teachers may resist learning because they perceive it as a threat to their being. Hodgen and Askew (2007) suggest that imagination plays a key role in overcoming such threats, thus developing and transforming teachers' relationships with and knowledge of (school) mathematics.

A third contribution relates to the analysis of learning settings. The situative perspective is often seen as providing a critique of current practices in schooling rather than offering an alternative vision (Lerman, 2000).[7] Greeno's (1998) work, however, provides a useful method of analysing learning situations. He highlights the importance of understanding the constraints and affordances: constraints that enable participants (teachers and learners) to predict and anticipate activities and outcomes; affordances that provide opportunities for participants to draw on practices from elsewhere. Boaler (2000) highlights the importance of the social context of learning. In a re-analysis of her study of open-ended and traditional approaches to school mathematics (2002), she describes how the students, who experienced the open-ended approach, more easily related school mathematics to out-of-school contexts in part because of the similarities in the way mathematics was practiced. In a similar vein, Lave (1992) argues that much problem-solving in schools is not authentic: in contrast to the messy and complex problems of the real world, school mathematics problems tend to be straightforward and routine. But Putnam and Borko (2000, pp. 4–5) argue that the problem of authenticity is related to the authenticity of learning rather than necessarily to the authenticity of problems themselves: "Authentic activities foster the kinds of thinking and problem-solving skills that are important in out-of-school settings, whether or not the activities themselves mirror what practitioners do". This highlights the two-fold problem of authenticity in mathematics.

[7] See, for example, Lave and Wenger's (1991) rather brief and simplistic critique of school education.

Mathematics teaching involves two stages of re-contextualisation of mathematics knowledge: a re-contextualisation of teachers' own mathematics learner knowledge for the classroom to enable students to re-contextualise this classroom mathematics for out-of-school contexts.

Implications for the Practices of Teaching, Teacher Education and Development

Recognising the situated nature of mathematics knowledge suggests that focusing exclusively on mathematics knowledge in isolation from the classroom context is unlikely to be effective. In developing strategies directed at improving teacher knowledge, there is a need to examine the contextual constraints and affordances which help or hinder teachers to act knowledgably in the classroom (Greeno, 1998). A crucial issue is to examine how collective knowledge can be harnessed to support an individual teacher's mathematical knowledge in the classroom.

One productive strategy is to provide tools that focus on the use of teacher knowledge in the practice of teaching mathematics. One constructive tool of this kind, the Knowledge Quartet, is described in some depth elsewhere in this book, particularly in the Chapter 12 by Turner and Rowland. To date, like much of the literature on mathematics teacher knowledge, this approach has focused on primary or elementary mathematics teaching, although there is every reason to suggest that this approach could be useful in secondary teaching and teacher education. In particular, key aspects of the Knowledge Quartet resonate with active research topics in secondary mathematics, including the choice and construction of mathematical examples.

A second implication relates explicitly to the social aspect of teacher knowledge. If teacher knowledge is supported by social structures and relationships, then it is likely to be productive to focus on developing shared expertise rather than individual 'knowledge'. The efficacy of collaborative approaches to mathematics teacher education is well-established (e.g. Clarke, 1994) and the situated perspective lends further theoretical weight to such approaches. Millett, Brown and Askew (2004a) highlight the importance of the professional community of teachers in a school and find that some primary schools appear to be able to successfully 'share' mathematics knowledge and expertise amongst a group of teachers through a mathematics co-ordination team.

A third implication concerns lesson materials, textbooks and, more broadly, the distributed aspects of teacher knowledge. There is certainly an urgent need to examine how textbooks and other materials can best support teacher knowledge in the practice of teaching. However, there is a great deal of evidence that materials on their own are insufficient (e.g. Askew, 1996). Spillane (1999) argues that for professional change of any significance, mathematics teachers need social spaces in which they have access to "rich deliberations about the substance . . . a practising of reform ideas with other teachers and reform experts includ[ing] material resources or artefacts that support [these] deliberations" (p. 171). Looking at Alexandra's subject knowledge development, one of the significant features was her engagement in lesson development (Hodgen & Johnson, 2004; Johnson et al., 2004). There has been a great deal of focus on translating the Japanese practice of lesson study to a Western context. But actually, this may be a misguided attempt to transfer a very contextualised cultural practice. What made a difference for Alexandra was not lesson study per se, but rather the more general practice of lesson development carried out in collaboration with others: constructing pedagogic strategies, examples, tasks, etc. that enable students to do and learn mathematics. Key to this is that lesson development is not merely a pedagogic exercise; it necessitates the investigation and exploration of topics from school mathematics, as described in Chapter 5 by Watson and Barton. That such apparently simple and elementary topics can challenge mathematical experts, is clear from the example of prospective secondary teachers cited at the beginning of this chapter.

A fourth implication relates to identity, care and relationships. For many primary teachers, the problem of maths anxiety is well-documented (Bibby, 1999). However, simply reducing anxiety and enabling teachers to 'feel better' about mathematics can lead to complacency (Askew, 1996). Askew and I have argued that teachers' knowledge of mathematics is both intellectual and emotional (Hodgen & Askew, 2007). The motivation to do mathematics – or to teach mathematics – is both individual and social. This is as true for well-qualified and knowledgeable secondary teachers, as it is for primary teachers. However, interventions related to teachers' knowledge of mathematics have generally focused on cognitive and pedagogic issues: teachers' mathematics subject knowledge, how children learn and teaching approaches. These issues are, of course, important, but the importance of identity in coming to know as suggested by the situated perspective, implies that such an approach is doomed to failure unless placed within an affective frame in which teachers have space to question and enjoy mathematics and mathematics teaching. In analysing mathematics subject knowledge, for example, Askew (2008) presents a convincing case for a focus on the big ideas – or sociomathematical norms – of precision and generalization, as well as the romance of the subject.

Finally, there are implications for research into mathematics teacher knowledge. There is an increasing interest in the measurement of teachers' mathematics knowledge and the relationship with student learning (Hill, Rowan, & Ball, 2005). However, the situated perspective suggests that problem goes beyond this issue of codification in that teachers' knowledge is not only situated but also social and distributed. The testing of individual teachers is likely to focus on de-contextualised mathematics knowledge which, as in the case of Alexandra above, may be very different from their classroom knowledge. Nevertheless, the issue of how mathematics teacher knowledge is enacted and the relationship with classroom practice remains poorly understood, and research in this area, like the research in mathematics teacher education generally (Adler, Ball, Krainer, Lin, & Novotna, 2005), is largely limited to small scale studies. Given the analysis above, approaches that focus on the notion of re-contextualisation (Adler & Davis, 2006) may offer insights in this area.

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