Chapter 13
Learning to Teach Mathematics Using Lesson Study

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Introduction

In this chapter, we consider the use of Japanese lesson study in developing teaching practices and, in particular, the ways in which it is claimed to enhance mathematical knowledge for teaching. First, the general lesson study approach will be described with its key features outlined. The findings of a study carried out by Corcoran (2008) in Dublin will be reported and discussed, in order to examine the contribution of such an approach in the particular context of pre-service teacher education. In that study, engagement with peers in the lesson study enterprise transmuted students' negative attitudes to mathematics into a more positive, patient willingness to learn, and an optimism that they can go on learning mathematics in teaching. The report on the Dublin study will follow an overview of some of the published work reflecting claims made for the role of lesson study in focusing teachers on the knowledge for, and in, mathematics teaching.

Enhancement of Teaching Through Lesson Study

The lesson study approach is built on the collective development of teaching effectiveness through collaborative work and reflection on practice and thus appears to offer a great deal to enhance mathematics teaching. For example, in a National Research Council report, Kilpatrick, Swafford, and Findell (2001) suggest that through the lesson study approach to professional development, "... teachers engage in very detailed analyses of mathematics, of students' mathematical thinking and skill, of teaching and learning" (p. 395), thus bringing together subject and pedagogy in reflecting on and refining practice. Engagement in these analyses is firmly rooted in group responsibility and in particular classroom contexts and draws on a range of resources both internal and external to the particular context.

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Lesson study is said to be premised on the Confucian saying that, "seeing something once is better than hearing about it one hundred times" (Yoshida, 2005). Its ultimate purpose is to gain new ideas about teaching and learning based on a better understanding of children's thinking so the observation of actual research lessons is at the core of the lesson study process. Yet, the lesson study cycle encompasses much more than studying children's responses while observing a lesson. It requires time dedicated to intensive *kyozai kenkyu* – a process in which teachers collaboratively investigate all aspects of the content to be taught and instructional materials available – and to *jyugyo kentuikai* – the post-lesson review session (Takahashi, Watanabe, Yoshida, & Wang-Iverson, 2005). Its main feature is collaborative planning and reflection that does not shy away from a critique of practice focused on the results of the group's work rather than on any individual. In these ways, it appears to offer teachers an opportunity to pool their collective teaching skills in situ as they adopt research goals appropriate to a particular school context for their lesson study. This approach, in general, addresses the situated and social view of teacher knowledge (see Chapter 3 by Hodgen, this volume), in that the focus for study is the lessons taught in particular schools with particular local concerns. As teachers plan and reflect together in groups, knowledge development is social. The 'research' lesson is planned collaboratively and teachers spend time clarifying the mathematics. The knowledge is drawing on a variety of sources including the teachers themselves, published curricula, research studies and 'experts' such as university teachers in the role of 'knowledgeable others' (Watanabe & Wang-Iverson, 2005).

In Japan, where it is integral to schools, lesson study is often credited with the success of Japanese students in international comparisons of mathematical achievement (Stigler & Hiebert, 1999). Internationally, there has been an increase in cross-cultural study of ways of teaching mathematics, and a growing interest in using lesson study as a basis for improving teaching in a variety of other contexts, most notably in the US. Increasingly, lesson study is being adopted in diverse school systems as a means of developing innovative classroom teaching and learning of mathematics (Asia-Pacific Economic Cooperation Education Network, 2008). For the purpose of this chapter, the discussion will now turn to a brief overview of how lesson study operates in Japan, the main elements of which have been used in projects in the US.

While there are various ways in which lesson study is carried out, the model that has been adopted in the US is mainly focused on individual schools, though reports of the work are often disseminated more widely. In Japan, this model begins with a group of teachers in a school identifying a particular teaching problem in their own school context. They then plan a lesson together where the strong focus is on the thinking and likely responses of pupils, but great care is taken over aspects of the teaching such as questions, resources and examples to be used. The focus in teaching, according to Tall (2008, p. 6), is on the mathematical knowledge of the teachers, but also on the need for "deep experience of how children think as they learn mathematics". Teachers also investigate possible teaching materials in the process. They then observe children's responses as one of the group teaches the (usually videotaped) lesson and they reflect together afterwards on the mathematical

content of the lesson. The planning involves identifying the relevant mathematical knowledge and curriculum detail and although the focus is on one lesson, it is part of a sequence and progression in learning. The aim is not to produce perfect lessons to be offered as resources for others to use, but to be part of an ongoing process of deepening understanding of how teachers can bring about the meaningful learning of mathematics. This may include teaching specific methods (e.g. for calculation) as well as the solving of non-routine problems (Tall, 2008). The anticipated responses of pupils are an important aspect for discussion by teachers as are the potential difficulties that might be encountered. According to Fernandez (2005), the general approach is as follows. A group of teachers plans the detail of the lesson that one of the group will carry out in the classroom. The plan is written out in detail and, when the lesson is taught, other members of the study group (and sometimes invited visitors) observe what occurs. Feedback is then given after the lesson, usually starting with a reflection by the teacher who taught the lesson. After this, the lesson will be refined and other teachers may teach it again and follow this up with further analysis. The focus for reflection will be decided in advance, together with points for particular observation while the lesson is in progress.

Lesson Study Appraised

In her work in the US, Fernandez (2005) was interested in the potential of the lesson study approach to support teachers in learning about mathematics for teaching, and also what the constraints might be for teachers whose own subject knowledge was limited. She emphasized that her interest was in "what lesson study has to offer, not on what teachers actually make of it" (p. 268). This is the central question being addressed in this chapter – what claims can be made for the contribution of lesson study to the development of mathematics for teaching? Elsewhere, Fernandez, Cannon, and Chokshi (2003) reviewed an initiative where Japanese teachers worked with teachers in the US to develop their work through lesson study. They suggest that through experience of lesson study teachers will draw on three critical lenses used by Japanese teachers to enhance their teaching of mathematics – the perspectives of researcher, of curriculum developer and of the pupils. The authors also report some of the difficulties that US teachers had in adopting these 'lenses' as ways of examining their teaching practice.

The *researcher* perspective requires teachers to observe the responses of pupils in a focused way and to gather specific and concrete evidence of those responses. US teachers' evaluations, according to Fernandez et al., were general and this might have resulted from the fact that the teachers appeared to find it difficult to be observers rather than teachers in this context. In the first lesson observation, the US teacher group tended to act as an extra pair of hands in the class while the Japanese teachers acted as an extra pair of eyes. In fact, had it not been for the presence and intervention of the Japanese teachers, much less would have been learned by the US teachers and, perhaps, it was their presence that had more influence on teacher learning than the participation in lesson study itself. There did, however, appear to

be a potential for the addressing of mathematical teacher knowledge through the *curriculum developer* lens. Fernandez et al. suggest that the US teachers were not accustomed to discussing different ways of organising and sequencing elements of the curriculum, but rather tended to accept the authority of textbooks. Adopting this perspective, then, opens up possibilities for discussion and critical evaluation of how and why mathematical ideas might be arranged in order to maximize pupil understanding. It is claimed generally that the dynamic and interactive nature of the lesson study process offers participants multiple opportunities to deepen their knowledge of mathematics and of mathematics teaching. However, the work of Fernandez et al. suggests these will only be taken up in a context where teachers are enabled to position themselves as critical reflectors on their practice, who take ownership of their mathematical learning. In a later study of one school's lesson study work, Fernandez (2005) identifies the questions that arise for teachers in planning and implementing lessons which did result in discussion about mathematical knowledge for teaching, some resulting from children's difficulties and some from the unexpected ways children used to tackle activities, thereby giving an example of the 'opportunities' lesson study can offer. While she suggests that analysis of issues arising from the act of teaching can support the development of strategies for future teaching, at the same time she recognizes that it is not always possible to predict exactly what will occur next in teaching, so the matter of the development of mathematical knowledge is more vexed.

The Role of Knowledgeable Other(s)

Fernandez (2005) observes that the US teachers were rather limited, at times, in their deliberations because of their own understanding about some of the connections in mathematics, in this case the relationship between fractions and division, and what 'whole' is referred to in fraction problems. However, she suggests positively that the cycle of planning and reflecting and the related analyses allowed space for teachers to expose and begin to address areas where they lacked confidence. Consequently, some teachers identified the need to develop their own understanding of mathematical ideas in order to discuss them fruitfully with pupils. According to Fernandez, it is the type of help and the manner in which it is given that will be crucial, that it "does not ask teachers to relinquish control of their work and [that it] does not overwhelm, alienate, or discourage teachers" (p. 285). Likewise, a study of Highlands school, also in the US, by Lewis, Perry, Hurd, and O'Connell (2006) found that, over time, teachers' observations became more focused and oriented to discussion of the detail of the mathematics they taught and the mathematical learning they analysed. Another change that they observed was the move towards using external sources of knowledge such as a wider range of texts, for comparison, and also research articles. Like the teachers in Fernandez's study, knowledge needs were identified through the study of local problems in their context.

While there are many positive commentaries on the potential for developing mathematical knowledge for teaching through lesson study, there have also been notes of caution. In particular, questions of the likely success of transferring an approach from one culture to another have been raised. Tall (2008, p. 1) suggests that it may be possible to learn from and use practices originating in another culture, if "we think reflectively about what it is we are trying to do in teaching mathematics." Teaching aims are central in decisions about approaches to practice and to teacher development and, without an examination of whether change in practice is required or possible, change may simply occur on the fringes of what happens in classrooms. One key feature of the way in which lesson study is described in the studies referred to here is that it is a 'bottom-up' rather than a 'top-down' model of teacher development. However, there are potential dangers in over-emphasising a localised, school-based approach. While knowledge can be seen as social and situated and, in the studies discussed, groups of teachers have been observed learning and developing confidence in recognizing what else they need to know, access to knowledge and expertise beyond the local context allows teachers to draw on a wider range of alternative views and to make informed, critical decisions to support the development of mathematical teaching in their own context. In relation to this, the role of knowledgeable other(s) who can provide such support is one which requires further exploration. In the next part of this chapter, the first author describes some of the findings from her research into a lesson study approach used with pre-service student teachers in Dublin.

The Dublin Study

The Dublin study proposed to introduce Japanese lesson study to an Irish context and had as a primary goal the trialling of lesson study as a means of developing student teachers' mathematical content knowledge for primary teaching. Lewis et al. (2006, p. 5) offer two conjectures as to how lesson study might work to bring about the improvement of teaching. Conjecture 1 posits, "lesson study improves instruction through the refinement of lesson plans", while conjecture 2 proposes that "lesson study strengthens three pathways to instructional improvement; teachers' knowledge, teachers' commitment and community, and learning resources." This study was based on conjecture 2. The research project, therefore, is located in a theory of social practice, which conceptualises learning as *legitimate peripheral participation* (Lave & Wenger, 1991), and the student participants and I, as course facilitator/researcher, forming a *community of practice* (Wenger, 1998), where "membership [...] translates into an identity as a form of competence" (p. 153). The notion of identity formation as learning in practice and the possibility of mathematics knowledge for teaching arising from engagement in an enterprise dedicated to developing good mathematics teaching makes lesson study an attractive and potentially powerful tool for mathematics teacher development.

Overview of the Lesson Study Elective Course

The lesson study research spanned the academic year 2006–2007 and took place in the context of a newly-offered elective module in education – Learning to Teach Mathematics Using Lesson Study – in an Irish college of education. Student teachers commonly pursue a concurrent model of teacher education there leading to an honours bachelor's degree in education (B. Ed) and including a single academic subject studied to degree level. Six third-year B. Ed student teachers participated. The lesson study protocols of collaborative lesson preparation and post-lesson collaborative reflection were adopted to further our goal of learning to teach the primary mathematics curriculum well. Each member of the elective group was involved in planning, teaching, analysing and revising mathematics lessons intended to promote children's mathematical reasoning. The lesson study elective course revolved around these mathematics lessons and extended over three cycles of lesson study. Research lessons were taught at two different school sites (see Table 13.1 for details). Because the student teachers came to the schools to teach the research lessons only, these are known as 'dive-in' lessons. As such, they lacked some of the rich potential for learning about their pupils available to class teachers working on lesson study within their own schools, yet the act of teaching and observing the research lessons for different age groups of children in widely different school settings constituted a valid and valuable lesson study experience for the student participants. A theme was chosen by the group for the research lessons in each cycle and each student teacher volunteered to teach particular lessons. On the research lesson days, the group divided into two with some members accompanying each 'teacher'.

Table 13.1 Lessons taught during each lesson study cycle

Lesson study cycle	School	Class/ages	Topic	Student teacher pseudonyms
Cycle 1	St Peter's	4th/9–10 years	Weight	Treasa
	St Paul's*	4th/9–10 years	Weight	Finola
Cycle 2	St Peter's*	5th/10–11 years	Fractions	Bríd
	St Paul's	3rd/8–9 years	Fractions	Ethna
Cycle 3	St Peter's	3rd/8–9 years	Division	Róisín
	St Paul's*	5th/10–11 years	Fractions	Nóirín

*Researcher present.

Three distinct aspects of the lesson study elective course emerged, and these were used to frame analysis. First, students participated in the course by engaging with the group in preparing, teaching and reflecting on lessons, i.e. by 'doing' lesson study. Secondly, participants also engaged with the elective course by 'doing' mathematics together, regularly. This aspect of engagement with interesting mathematics was for the students themselves and independent of mathematics to be taught in lessons. Thirdly, students participated in the elective by 'being' lesson study elective

students, where engagement meant pursuing activities related to the elective enterprise but not essential to lesson study, for example, watching DVDs about lesson study and writing reflective journals.

Data Analysis

Each of the six lessons was observed, recorded, transcribed and analysed using the Knowledge Quartet (KQ) framework (Rowland, Huckstep, & Thwaites, 2005). The KQ is a four-dimensional, practice-based framework for mathematics lesson observation and analysis developed inductively from analysis of videotaped lessons taught by novice teachers. The four dimensions are termed *Foundation, Transformation, Connection* and *Contingency*. Foundation includes teachers' knowledge, beliefs and understanding of mathematics and mathematics pedagogy, acquired before and during teacher preparation; this dimension is seen as underpinning the other three. Transformation encompasses the ways in which the teacher's own knowledge is transformed to make it accessible to the learner, especially through the use of representations and examples. Connection pertains to knowledge displayed when teachers make connections between and among mathematical ideas; it includes issues of sequencing and judgements about conceptual complexity. Finally, Contingency is manifested in the ways that a teacher responds to unanticipated events as they emerge during instruction. This could be described as 'thinking on your feet'. For further details of the KQ, see Chapter 12 by Turner and Rowland (this volume).

Lesson study community members were all encouraged to think of aspects of their mathematics lessons in terms of the four dimensions of the KQ, and the negotiation of the meaning of the framework as a language to describe mathematics teaching contributed to the *shared repertoire* (Wenger, 1998) of the community. As initially understood by the group, the KQ appeared linear in its exposition of the four dimensions of the mathematics knowledge required for teaching. Engagement in lesson study, however, brought about a reordering of the KQ components. By starting with a focus on children's learning of mathematics, a strong emphasis was placed on the Contingency dimension of teachers' mathematical knowledge in teaching, followed by the Connection dimension, which when contextualised by studying particular research lessons gave rise to revisiting the Transformation and Foundation dimensions. In lesson study cycle one, the lesson preparation and post-lesson reflection meetings were audio-recorded. In lesson study cycle two, these sessions were video-recorded. In lesson study cycle three, I was not present at the preparation meeting and the students' journals are the only record, although two post-lesson reflection meetings were audio-recorded and transcribed. For the purpose of coding these records, at first, I drew on concepts of *participation* and *identity* borrowed from Wenger (1998), but gradually the data analysis became more inductive, as various fresh indicators of mathematics teacher development were generated from the data rather than determined by reference to the literature or my own preconceptions. Since two research lessons were taught simultaneously on each occasion,

I made personal observation notes for the one in which I was present. Students wrote a reflective journal for seven/eight of the lesson study sessions, and these were also important data. As well as describing some elements, which arose from analysis of the entire lesson study elective course, I shall illustrate here the crafting of a mathematics teacher identity by one student participant, Bríd, over the three cycles of lesson study.

'Doing' Lesson Study

The dialectical nature of the learning of persons in activity presents a rich tapestry of interactions and interpretations of how that learning occurs. Rogoff, Matusov, and White contend, "learning involves transformation of participation in collaborative endeavour" (1996, p. 388). Learning to teach requires participation in and the collaboration of a group of people, and I sought evidence of this in my analysis of the lesson study community of practice. An agreed goal of each lesson study cycle was to establish what mathematical ideas or concepts the student teacher wanted pupils to engage with as a result of the particular lesson being planned, and to study children's responses to the mathematical task(s) during the lesson with a view to assessing the kind of mathematical thinking in evidence in the class. This became the shared enterprise of the community of practice.

Preparing the Lessons: Cycle One

When the lesson study group turned to planning the lessons to be taught on weight, a tension emerged for some group members. What some students perceived as straying from the objective – "at times we could wander from that and begin including less relevant things" (Noirín's reflective journal entry 2) – was from another perspective, a process of exploration of the teaching resource materials available, discussion of the meanings of 'mass' and 'weight', leading to agreement to focus on the attribute of 'weight'. As a group, we engaged in study of resource materials which is meant to support the teacher of the lesson. I considered that my role as knowledgeable other (Watanabe & Wang-Iverson, 2005) was to collaborate with the team in order to enhance content knowledge, guide the thinking about pupil learning and support the team's work. A handbook of lesson study protocols advises that:

> Discussing the content, scope and sequence of curriculum helps teachers to be clear about where they are going with the lesson they are preparing and what outcomes they are looking for from the students. (Yoshida, 2005, p. 7)

As can be expected of student teachers, there was very little experience among them of 10-year-old children's current state of mathematics learning, or what they might be expected to know. Nor was there much evidence of theoretical underpinnings of pedagogy, which raised issues about these student teachers' Foundation knowledge for teaching primary mathematics. As a consequence, the effort expended in deciding which specific learning outcomes we were planning for extended the student teachers considerably. During this planning session and in later

sessions, student teachers worked together to align content objectives from the curriculum with suitable contexts in which to base problems and activities designed to promote children's reasoning about the mathematical ideas underpinning each teaching objective chosen.

Research Lessons: Cycle Two

In lesson study cycle two, the student teachers wanted to do something different and what they perceived as more difficult. They chose fractions as the topic to be taught. The main goal of these lessons was to provide children with an experience of exploring fractions in a realistic context while affording the student teachers an opportunity to study what the children already knew about fractions with a view to developing that understanding. In this instance, the school textbooks were put aside and the two lessons were planned as a fractions investigation activity.

Learning Takes Time

Full participation in the community of practice was proving challenging, however, as this excerpt from one student teachers' journal attests:

> Initially I thought that by working together on a lesson we would work quicker but as we got more experience at lesson study, we began to spend longer discussing our intentions and really getting behind the mathematics and what we wanted the children to learn. It was now not a case of devising fun activities to enhance the lesson but a matter of questioning the mathematics and how best to teach it. (Nóirín's journal entry 4)

This spending of time on lesson study was proving to be personally demanding, yet inherently rewarding. We came back to 'doing' mathematics again with renewed interest and fresh eyes when, for the last hour of the preparation session, the agreed 'teachers' for lesson study cycle two took turns to practise their research lessons on the group.

'Doing' Lesson Study: Cycle Three

Student participants conducted the planning for research lessons in cycle three in the absence of a knowledgeable other. Their success, as evidenced in the research lessons taught, shows that lesson study belongs to the participants, and that a knowledgeable other, while an integral part of the process, need not be centrally involved at every stage of the lesson study cycle. Using the Knowledge Quartet framework, there is strong evidence of learning along the Connection dimension in the data here, prompted by efforts to interpret the primary mathematics curriculum. Transformation issues were explored by Bríd, who reported:

> We also spent a lot of time debating whether or not to supply counters for the children to work with. Would they hinder or help them in their problem solving and would the distribution of them take time from the maths? Through our discussion we felt it best not to use them as they might distract the children from the actual problem. (Brid's journal entry 6)

The focus had shifted to actual mathematical details of the lesson and suitability of context, choice of example (3 as a divisor), whether or not to use counters, *et cetera* had become more central in the planning. All six student teachers' journal entries corroborate this engagement with the details of planning a successful lesson on division. However, the group also engaged in planning a lesson on percentages where the difficulties encountered by the group resulted in their abandoning the task until they could get 'expert' help. After outlining suggestions made to teach a lesson on percentages, which linked with fractions but was not aligned with what the community by now considered good practice, the embryonic lesson plan was shelved, in favour of a variation on Bríd's lesson. Nóirín balked at teaching the proposed lesson on percentages, because she did not know how it would relate to the class's current understandings of fractions. The student teachers' connection of the two mathematical topics raised is indicative of the presence of the second perspective characteristic of Japanese teachers – the curriculum developer lens (Fernandez et al., 2003). These students were becoming aware of complexity in the mathematical connections teachers are required to make in teaching the curriculum well.

'Doing' Mathematics

Each of the lesson study sessions included some element of exploring mathematical ideas by the participants. The mathematical tasks presented in the first session were intended as an introduction to thinking about primary school mathematics in other than the traditional algorithmic terms. One illustrative example is offered:

> We then worked out some maths problems, in pairs. I was surprised at the simplicity some children (sic) worked out theirs in comparison to mine. The problem of the bus: 328 people to be transported in a forty-seater bus. I divided 40 into 328 directly to get my answer. My partner drew out circles of forty, until she had enough . . . that was how she realised she had enough buses. We both had the correct answer. This opened my mind and I realised that there is no right way of solving a maths problem. (Róisín's journal entry 1)

Reflective journals all referred in emotive terms to this element of the first session with one theme emerging strongly – the student teachers' differing relationships with mathematics were all "both complicated and powerful" (Mendick, 2006, p. 156). Each student reflected on her own responses to the problems in terms which ranged from comments on emotions like "fear . . . panicked and confused," through perceived personal deficiencies, "I always doubt my ability to do it," to the more measured "very interesting," and realisation of "how indoctrinated we are" (journal entry 1 of Ethna, Bríd, Nóirín and Treasa). Doing mathematics ourselves became an essential element of the lesson study elective and, while not explicitly part of the Japanese lesson study protocol, can be subsumed under the "purposeful learning" of the goal-driven pre-lesson planning phase of each cycle (Fernandez & Chokshi, 2005, p. 73). If the lesson study community of practice were to direct its research gaze on how children respond to mathematical tasks, then it had to direct its research gaze on members' own doing of mathematics also.

These student teachers accepted that to pose realistic problems and to focus on children's responses were aspects of good mathematics teaching practices that were challenging for them because of their own fragile relationships with mathematics.

Treasa: That's what I'm afraid of . . . cos I'm very . . .
I get very, I'm very insecure about maths. If they could say something and I'm standing there like an idiot saying 'God I don't know what to do next'. (Planning session lesson study cycle 1)

Making the lesson study elective sessions a safe place to question one's own and each other's mathematical ideas became an important element of the process. The role of the course tutor as knowledgeable other developed in the selection of interesting mathematics to engage the group, and in drawing pedagogical inferences from events in the group setting. I also sought to establish communication norms within the community of practice, which facilitated the expression of mathematical thinking. When working in community, all members had the responsibility to the enterprise and knowing when to use Contingent opportunities – by deviating from the planned agenda, or when to allow the discourse to continue uninterrupted – presented an occasional dilemma. The different approaches to mathematics were discussed and celebrated within the group and tended to mask the fact that some students were quicker and surer in proffering solutions than others. By focusing on improving pedagogy and constantly making connections between intentions and actions of the teacher and imagined and actual responses of the children, this aspect of difference within the group was minimised. Nonetheless, challenges in the communication of ideas between members emerged as a theme requiring further exploration.

'Being' in the Lesson Study Elective Community of Practice

Each cycle of the lesson study elective course had a component that was not directly related to the preparation, teaching and reflecting on lessons taught. In cycle two, the student teachers engaged in watching two DVDs. These were representative of lesson study as practised by American teachers – *How Many Seats*? (Mills College Lesson Study Group, 2005), and as practised by a Japanese teacher – *To Open a Cube* (Mills College Lesson Study Group, 2003). The group members overtly identified with the lesson study process while owning their own practice. Reflections which followed this session were deeply insightful and focused on the student teachers' own learning about mathematics teaching from observing the two different settings for learning on the lesson study videos. Bríd's concluding remarks are salient:

> Overall, watching the DVDs gave me more insight into how kids think mathematically and ideas that I can use and take into account when planning. They also made me consider my own problem-solving abilities and realise that I am only as effective as my own level of thinking. This scares me a bit because my mathematical ability may prevent better learning by hindering rather than helping the pupils. I think that lesson study is vital to do with my colleagues so that I can both challenge my own thinking and receive support when planning for maths lessons. (Bríd's journal entry 6)

Discussion

The research lessons on weight in the first cycle provided rich examples of the complexity of enacting the task of teaching for prospective teachers. The research lessons on fractions in the second cycle appeared to have taken on a research perspective, which arose from the goals of the lesson study enterprise. This was akin to the 'researcher perspective' used by Japanese teachers to enhance their teaching of mathematics (Fernandez et al., 2003). The two research lessons in lesson study cycle three were characterised by a more 'improvisational' approach to children's learning of mathematics and were influenced by an attempt to use an interpretation of the critical lenses applied to student learning and curriculum development. We will now explore emergent themes suggested by the lesson study process relating to the development of mathematical knowledge for teaching.

Identity in Terms of Learning to Teach Mathematics

By identity, we mean the learning that occurs while individuals are mutually engaged in a worthwhile enterprise (Lave, 1996). In this case, the student teachers pursued learning to teach mathematics together. By identity-work, we also mean the narratives people share while participating in such a community of practice. Identities are formed through participation and identification with the goals of the enterprise and as such are socially formed. These students' "knowing and knowledgeability" (Roth & Lee, 2006) of and for good mathematics teaching was exhibited through their belonging to a community of practice dedicated to developing this very knowing and knowledgeability. Participation in the enterprise of studying mathematics teaching by engaging in actual teaching, and then reflecting critically on it as a group of individuals who are all similarly engaged, contributes to the identity of an individual engaged with learning to teach mathematics and to the community of practitioners building knowledge of and through the enterprise of lesson study dedicated to mathematics teaching. By putting the spotlight on the practice of the lesson study community as a whole, it is possible to illustrate, largely from the student teachers' journals, how one participant re-positioned herself, through identity work as a pre-service primary teacher learning to teach mathematics well, thereby increasing her mathematical knowledge in teaching.

The Case of Bríd

In lesson study cycle two, Bríd engaged with preparing a mathematically productive lesson on fractions, which emerged from practice of 'doing' fractions work and reading articles on good practice in teaching fractions. While acknowledging the contribution of the group, Bríd embraced her role as teacher, despite her feelings of inadequacy:

> While this session was of huge benefit to me, I am aware that I need to become more comfortable with fractions and I can only do this by immersing myself in them and engaging with them in a meaningful manner. I am hesitant about going into classrooms with them, but I have a funny feeling that it will be the children teaching me about fractions rather than me teaching them! Perhaps it is better to say it will be a joint sharing of learning. (Bríd's journal entry 4)

Descriptive Synopsis of Bríd's Lesson

Bríd's lesson was focused on studying children's understanding of fractions as a designated number of equal parts of a whole and proposed using a pizza context. She set the scene by asking the class to imagine that it was one boy's birthday and that he had invited seven friends to share six pizzas with him. Children were given a teacher-made handout showing six identical circles on an A4 page, which they were invited to think of as pizzas. After some time working in pairs, the class was called to attend while some pairs were invited to the board to explain what they had done. Next, the children addressed a second similar problem, involving six pizzas divided between ten people, and later, other pairs of children were invited to explain their work, which Bríd illustrated on the board. A whole-class session concluded the lesson where the teacher talked the children through the process of adding a half and a quarter and finally elicited why the size of the unit was an important element in dealing with fractions (Fig. 13.1).

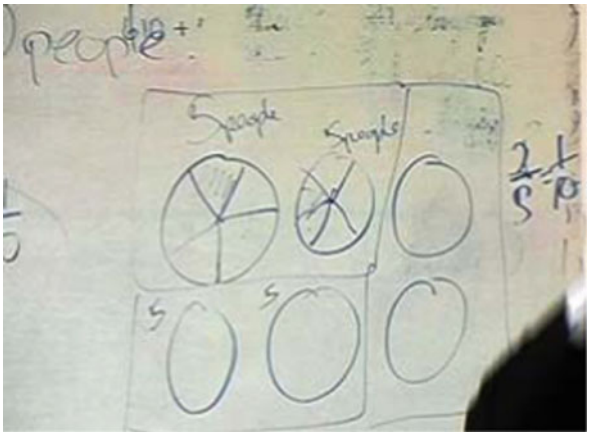

Fig. 13.1 An innovative way of dividing six pizzas among 10 people?

Learning from Teaching

Bríd's lesson plan had anticipated alternative answers and her strengthening mathematics teacher identity led to an engaging and challenging lesson. While she had

planned the lesson to be about dividing pizzas, because this was a 'dive-in' lesson she did not know the children. Bríd invited a boy to tell her his name (Cathal) and then asked the class to imagine it was Cathal's birthday and that he was having a pizza party. This tactic demonstrated strong pedagogical content knowledge on Bríd's part. When Bríd's lesson was analysed by the group, her opening scenario became one of the stories of the practice which was adopted in later lessons. It illustrates how learning about teaching occurs through engagement with the enterprise.

Despite the comprehensive planning, which focused on the mathematical content of the lesson, Bríd was confronted twice, in the act of teaching, with more sophisticated thinking on the part of some pupils than she had anticipated or was prepared for. In connection with the second problem, a child at the board divided half a pizza into five equal parts, which Bríd incorrectly called "fifths". This was remedied on the spot, when one of her colleagues observing the lesson alerted Bríd to the error. In the second, two children suggested taking two pizzas together and giving each person a fifth of one pizza, resulting in 10 people getting a fifth each, from each of three sets of two pizzas. Bríd appeared confused by this innovative approach and while the children had articulated their thinking clearly, she wrote (incorrectly) on the board: $\frac{2}{5} = \frac{1}{10}$.

Aware of her own confused thinking, but unable to clarify it on the spot, Bríd quickly erased the 'solution' (without acknowledging that the children's strategy led to $\frac{3}{5}$ of a pizza per person) and moved on with the rest of the lesson. Her knowledge of fractions was inadequate for the teaching activity she had set herself, and this mismatch between her plan to encourage children's disparate ways of thinking and her ability to recognise the validity of all ideas, caused Bríd to consider the need to expand her facility with equivalence of fractions further. Her experience of enactment of the teaching role she had adopted as being less powerful than she had hoped and planned could have caused Bríd to be less adventurous in future, and stick to the textbook or to teaching by telling what she knew. However, such was Bríd's engagement with the act of learning through teaching and the strength of her belonging to the collective enterprise that her lapse into mathematical misinformation on that occasion became a motivational force to learn more about fractions for teaching. This embarrassing episode also became a story for the community of practice and gave rise to considerable further mathematical work on fractions within the group. The meaning of the Knowledge Quartet dimension of Contingency was expanded for all participants in the community of practice through this challenge to Bríd's knowledge of fractions and everyone's Foundation mathematical understanding was expanded by her lesson. The case of Bríd is used here to exemplify how these students' attitudes to the enterprise of teaching mathematics changed over the course. From feelings as individuals of inadequacy and fear, they had moved to a collective research orientation into how mathematics can be taught well, and into building the mathematical knowledge required to do so.

Mathematics Teaching and Matters of Interpretation

Where mathematics is only one of many subjects to be taught by generalist primary teachers, learning how to teach mathematics can become largely a matter of interpretation of the language used and the meanings intended by teacher educators. Evidence from the lessons taught by the student teachers in this study indicates that they often experience difficulty in interpreting what is meant by contested terms like 'problem-based teaching' or 'realistic mathematics.' Mathematical process skills, such as 'communicating and expressing mathematical ideas', are widely interpreted in Ireland to mean the more generic notion of [teacher] 'talk and discussion'. Curricular guidelines on mathematics pedagogy, for example, the optimal use of materials and mathematical representations, are filtered in the light of past experiences. A community of practice by definition functions as an *economy of meaning*, which suggests that some meanings do achieve superior status (Wenger, 1998, p. 198). The role of a knowledgeable other is crucial in this economy. The lesson study community of practice became an important site where meanings of mathematical practices and mathematics teaching were negotiated through *engagement, imagination* and *alignment* (Wenger, 1998). Alignment with a reform interpretation of the mathematics curriculum, with good teaching practices, with recent research findings was critical to the lesson study enterprise, together with alignment of the lesson study community of practice with the other communities of practice with which it interacted. Accountability to the enterprise begets negotiations of meaning in a highly reflexive manner and participation in the lesson study community of practice involved negotiating and renegotiating meanings for an increasing number of mathematical ideas and practices.

Findings

Learning to teach mathematics is hard work, and the six young women in the Dublin study faced the negative aspects of their different relationships with mathematics and worked collaboratively, with considerably energy to forge a new path for making sense of the primary mathematics curriculum and meaningful ways to teach it. By whole-hearted participation in the lesson study elective course, they validated the potential of lesson study as a means of learning to teach mathematics. But lesson study is not a panacea for mathematics teacher education or development. Rather, it is a process, which by design allows teachers to augment their mathematical and pedagogical skills for teaching the mathematics curriculum, by refining their goals and focusing on what and how children learn mathematics as a result of their practice. These outcomes do not result from individual effort, but from participation in practice. Japanese teachers have long realised that lesson study is a powerful means for teacher development and curricular change. Lesson study is, in essence, a road map for socio-cultural learning about mathematics in teaching. A road map is a

useful, even necessary, tool with which to get from one place to another in unfamiliar territory. But, of itself, lesson study is not enough, no more than a road map is of use to the person who does not know how to read it, does not know where s/he is on the map, does not know the landmarks to look out for, and/or is not sure of the destination s/he is travelling towards. A second more specific set of directions is also required. The Knowledge Quartet is such a framework of mathematics knowledge in teaching that provides signposts to help answer the intermediate questions. Where are we now? In what direction are we heading? Why this representation or that example?

The student teachers in the Dublin study began by focusing on children's responses to the mathematics lessons they had planned and taught. From there, their attention moved in two directions: towards the Connection and Transformation dimensions of their mathematics teaching. The need for both dimensions – for example, the ability to make connections between mathematical ideas and procedures, to sequence material conducive to learning and to make optimal choices of representations and examples – becomes obvious when one studies Contingency opportunities which naturally arise in the course of any lesson. However, mathematics teaching is not a static activity. Rather, like the discipline of mathematics itself and the art of teaching, it is a dynamic cultural pursuit and the above three dimensions of the Knowledge Quartet arise from, are informed by and in turn transform the primary dimension which has been called Foundation knowledge. Findings from this study indicate that Foundation knowledge for mathematics teaching expands with participation in lesson study. There were marked changes in how these students approached planning for, and teaching of, mathematics. There was evidence in them, of a growing awareness of the depth and connectedness of mathematical ideas. They have developed a much more focused eye on how children build mathematical thinking and have expended considerable energy in designing opportunities for children to do so. The student participants in this study have all grown in self-confidence, a self-confidence that recognises personal agency and thrives on communal support. These student teachers and I have come to view mathematical knowledge for teaching primary mathematics in new ways, and in consequence, think differently about how mathematical knowledge can be developed or stifled by classroom experiences.

Lesson Study as a Tool for Developing Mathematical Knowledge in Teaching

Viewing knowledge as situated in social contexts and constructed through social interaction requires a different view of how the development of mathematics teacher knowledge may or may not be fostered. The focus shifts from the individual to the communities in which mathematics teachers are engaged and the extent to which these communities support teacher learning and induct teachers into teaching practices. The scope for teachers to have ownership of, and to play an active part in developing their knowledge and expertise is also central to enabling the production of critically reflective practitioners who are better able to deal with the challenges

faced in engaging with new knowledge, or knowledge constructed in the variety of teaching contexts they will experience. They will be able to deal with the discomfort that will inevitably be felt in having to revise their own mathematical knowledge in order to teach if the community in which they are learning to teach mathematics encourages a more collective responsibility where it is possible to be open about questions about mathematical subject knowledge as well as ways to teach it. Such communities of mathematics teachers are identified by Ma (1999) as contributing to what she calls the 'profound understanding of fundamental mathematics' characteristic of Chinese elementary school teachers. Recognising that knowledge is not simply located in the individual teacher but distributed over people and resources implies that the responsibility for development of mathematical knowledge for teaching is not an individual but a collective one, which participation in the practice of lesson study appears to meet.

Lesson study fosters the collective development of mathematical knowledge. Engagement with lesson study also enriches the personal knowledge base on which individual teachers draw in developing his or her own practice. Fernandez's work (2005) underlines the need for a knowledgeable other to act as a catalyst and to properly challenge accustomed ways of working. The Dublin study confirms that the practice of collectively studying teaching, in the immediacy of a research lesson, designed for a specific context, can then be extended and tested against an even wider knowledge and research base through working with a knowledgeable other. Lesson study, as a practice, accepts that it is always possible to improve one's teaching and to continually develop mathematical knowledge in the process. By recognising the importance of the knowledgeable other role, teachers are reminded of the importance of investigating multiple sources in preparing for teaching, while ownership of the practice remains firmly with the teachers themselves. The Dublin study leads us to conclude that engagement in a lesson study community with the purpose of learning about mathematics teaching also develops mathematical knowledge for teaching in the process.

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