Chapter 11
The Cultural Dimension of Teachers' Mathematical Knowledge

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A Case for Considering Culture in Research on Teachers' Mathematical Knowledge

Much of the teacher knowledge literature has emerged from research programmes in North America. In one review of research on the topic, the educational philosopher Gary Fenstermacher identified four categories of teacher knowledge. Each category was linked to researchers based in the United States or Canada (Fenstermacher, 1994). The categories he identified were personal practical knowledge (associated with Jean Clandinin and Michael Connolly), knowledge developed from reflective practice (associated with Donald Schön), types of knowledge about teaching (associated with Lee Shulman), and knowledge generated by teacher-researchers (associated with Marilyn Cochran-Smith and Susan Lytle). Although this research originated in North America, it has influenced research on teacher knowledge elsewhere.

Take for example Shulman's work, which has inspired much research on teacher knowledge over the last two decades. In 1986, Shulman drew attention to the fact that researchers at the time were attending to generic aspects of teaching, such as classroom management and student reinforcement, whereas subject matter knowledge was being relatively neglected. Shulman's work inspired researchers to look more closely at the content preparation of teachers in all school subject areas and at all levels from primary school to college. In particular, his idea of pedagogical content knowledge (e.g., Shulman, 1986) captured the attention of many educators so that by now the term is taken for granted (Bullough, 2001). Although a huge amount of teacher knowledge research has taken place in the United States (e.g., Ball & Bass, 2003; Simon, 1993), researchers throughout the world have responded to the call to look at teacher content knowledge in several school subjects (e.g., Padilla, Ponce-de-León, Rembado, & Garritz, 2008; Rowland, Huckstep, &

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Thwaites, 2005). Whether the studies originated inside or outside the United States, a substantial number of them cite the work of Shulman.

The timing of Shulman's article was good, coming at a time when teachers and teacher educators in the United States were being criticised by several reports (see Bullough, 2001, for a discussion of these reports). But the popularity of the construct of pedagogical content knowledge inside and outside the United States is probably due to the way in which it brought together content knowledge and the practice of teaching. By combining content knowledge and the practice of teaching, the construct implied that a special kind of subject matter knowledge is unique to teaching. Although the idea of pedagogical content knowledge appealed to numerous researchers, many of them used the term in different ways and the construct needed additional specification, even in the United States (Ball, Thames, & Phelps, 2008). But even if a construct is well specified in a given country, it can be problematic if that construct is applied in a new setting where it may be interpreted differently. When a construct has different meanings in different settings, it is considered to lack conceptual equivalence (e.g., Harachi, Choi, Abbott, Catalano, & Bliesner, 2006).

The example of pedagogical content knowledge is an illustration of how a teacher knowledge construct developed in one country is assumed to apply universally. But assumptions of universality need to be treated with caution. In relation to intelligence tests, Straus (1969) made the following point:

> The items used in most standard intelligence tests contain many references to objects and events which would be outside the range of experience of a village child in Africa or India. Of course, some children would get the correct answer to these "culturally biased" items, but these are likely to be children who have had exposure to modern urban settings. Thus, children getting the highest scores will not necessarily be the brightest children, but rather the more "Westernized" (p. 234).

Although the construct of intelligence had been developed to the satisfaction of researchers in "westernized" settings, when tests based on the construct were transferred to another setting, the construct was different and students' scores on the tests, which were based on the construct, had little meaning in relation to the construct as originally conceived. Straus acknowledged that remedying such problems in research poses practical difficulties, including those of time and cost.

It is possible to understand why the cultural dimension of pedagogical content knowledge was not acknowledged when the construct was introduced. As Shulman himself noted about the teaching effectiveness studies which were popular when he proposed the idea of pedagogical content knowledge, "to conduct a piece of research, scholars must necessarily narrow their scope, focus their view, and formulate a question far less complex than the form in which the world presents itself in practice" (Shulman, 1986, p. 6). Shulman focused on specific school subjects at secondary school level and studied teachers in California. But can observations of teachers in one US state produce a construct that has the same meaning throughout the world? For example, Shulman asked about the knowledge needed by a teacher when presented with "flawed or muddled textbook chapters," and what "analogies, metaphors, examples, demonstrations, and rephrasings" the teacher can use to explain, represent or clarify ideas (Shulman, 1986, p. 8). Yet in some countries, flawed or muddled textbook chapters may be rare, reducing the necessity for teachers to possess such knowledge; and metaphors can be sensitive to national and organisational cultures (Gibson & Zellmer-Bruhn, 2001) so that knowing a useful metaphor in one setting may be unhelpful in another.

We have used the example of pedagogical content knowledge to urge caution in assuming that ideas about teacher knowledge which apply in one setting have universal application. Variations in how teacher knowledge is conceived matter because conceptions may be expressed similarly but understood differently in various countries. That is problematic for policy makers, researchers or educators who need to be explicit about the meaning of terms they use. Furthermore, some researchers are currently studying mathematical knowledge held by student teachers in several countries (Tatto et al., 2008). In order to interpret the findings of the mathematics known by student teachers, it is important to know what *kind* of mathematical knowledge they hold and why that knowledge is important in the particular countries in which they will teach.

Acknowledging the cultural dimension of teachers' mathematical knowledge is a relatively recent phenomenon (Ball et al., 2008; Delaney, 2008; Delaney, Ball, Hill, Schilling, & Zopf, 2008). One reason for the increased attention to the role of culture in teacher knowledge may be due to our growing understanding of the influence culture has in many aspects of life, from homicide rates (Nisbett, 1993) to safety on aeroplanes (Gladwell, 2008; Merritt, 2000). Although the work pilots do is similar from country to country, cultural attributes, such as taboos against questioning a more senior colleague, interact with their training and other factors to shape how they do their work. In her study of 9,400 pilots in 19 countries, Merritt (2000) concluded that "the effects of national culture can be seen over and above the professional pilot culture, and that one-size-fits all training is not appropriate" (p. 299).

A major reason for our interest in teacher knowledge is to inform the professional formation and development of teachers so that they in turn can help to raise the mathematical achievement of their students. If Stigler and Hiebert (1999) and others are correct that teaching is a cultural activity, then the knowledge teachers possess or need may depend on the culture in which they are working. Alternatively, if, like flying planes, teaching is largely the same from country to country[1] and teachers require the same knowledge wherever they teach, cultural attributes are likely to interact differently with teachers' acquisition of that knowledge from one country to another. In both cases, the cultural dimension of teacher knowledge needs to be considered. The four chapters in this section of the book add considerably to this discussion in relation to teachers' mathematical knowledge, and illustrate some of the avenues currently being pursued within the rapidly growing body of research that acknowledges and studies the cultural dimension of teachers' mathematical knowledge.

[1] For an overview of this argument, see Dale (2000).

In what follows, we review the four chapters with a focus on the interplay between the cultural context and mathematical knowledge for/in teaching. We make a distinction between "mathematical knowledge *for* teaching" and "mathematical knowledge *in* teaching." We use the former term to describe knowledge that can enable teachers to effectively support student learning of mathematics. In a sense, this kind of knowledge can be understood as being essential, or necessary, for successful teaching (as defined within a particular cultural setting). We use the latter term to describe knowledge that teachers use as they teach mathematics, i.e., teachers' knowledge as manifested in their practice. There is no suggestion about the capacity of this kind of knowledge to necessarily support a particular form of teaching (successful or not).

The review illuminates three different, but complementary, aspects of the cultural embedding of mathematical knowledge for/in teaching. The first aspect, which is represented by the chapters of Andrews and Pepin, situates mathematical knowledge *in* teaching in the context of different national educational systems. The second aspect, which is represented by the chapter of Adler and Davis, situates mathematical knowledge *for* teaching in the context of diverse teacher education programmes. The final aspect, which is represented by the chapter of Williams, situates mathematical knowledge *for* teaching in the culture of a "knowledge economy". In all cases, the identified context of mathematical knowledge for/in teaching denotes the main (rather than the exact or only) cultural locus of this knowledge as reflected in the chapters.

We acknowledge that the focus of our review on the cultural embedding of mathematical knowledge for/in teaching inevitably downplays some important contributions made by the chapters that did not fit directly within the scope of our review. We will allude to some of these contributions in the final section of our chapter where we will consider implications of the four chapters for teacher education research and practice.

The Interplay Between the Cultural Context and Mathematical Knowledge for/in Teaching

The Cultural Embedding of Mathematical Knowledge in Teaching in the Context of National Educational Systems

Andrews and Pepin both located teachers' mathematical knowledge in the national cultural discourse in which mathematics teaching and learning occur, and considered teacher knowledge as a social construction that is shaped by the particular national educational system wherein it functions. They argued that consideration of the characteristics of different educational systems (curricular expectations, typical teaching practices, etc.) can offer useful insight into explaining the variation observed in the ways teachers' mathematical knowledge is manifested in teaching practices of these systems. Accordingly, the two chapters acknowledged the importance, and examined the role, of the cultural embedding of mathematical knowledge *in* teaching in the study of mathematics teachers' practices in different countries. The cross-national comparative aspect of the chapters became, then, a means by which the authors understood and described mathematical knowledge in teaching in their selected countries.

Having outlined in general terms the chapters' common position on the cultural embedding of mathematical knowledge in teaching, we will now consider the development of this position in each chapter separately.

Andrews criticised existing frameworks on teachers' mathematical knowledge in that they tend to consider this knowledge as a personal construct, paying insufficient attention to its cultural embedding in the context of national educational systems that have their own systemic imperatives and didactic folklore. In an attempt to contribute to the development of existing frameworks, Andrews proposed a complement to these frameworks, a tripartite classification of what he called "idealised", "received", and "intended" curricula. This classification considers teacher knowledge as a social construction that is located in the classification's constituent and culturally dependent curricula: one that describes teachers' personal and articulable perspectives on mathematics teaching and learning (idealised curriculum), one that describes hidden and inarticulable aspects of teachers' practices that are taken for granted within an educational system (received curriculum), and a third one that describes systemically defined expectations of learning outcomes that often reflect societal or historical values (intended curriculum).

Andrews used this classification as an analytic tool to examine mathematical knowledge *in* teaching as manifested in two lesson sequences on linear equations taught by a Flemish teacher and a Hungarian teacher to grade 8 students in their respective countries. The findings of the examination, which had a comparative cross-national nature, suggested the utility of the classification in revealing culturally relevant aspects of mathematical knowledge in teaching. The importance of the findings lies in that the revealed aspects could remain tacit, or defy explanation, under alternative examinations that would use existing frameworks on teachers' mathematical knowledge. For example, Andrews discussed the case of the Flemish teacher who seemed reluctant to deviate from her planned lesson activities, an attribute of her practice that could be construed as a low level "contingency" (see Rowland et al., 2005). However, Andrews observed that it is difficult for one to determine whether this teacher's reluctance suggests a deficit in her pedagogical practice or whether it actually reflects a conscious decision on the part of the teacher not to deviate from well-articulated procedures. Andrews noted, then, that the terms "intended" and "received" curricula offer a useful language for one to describe the teacher's observed behaviours: these behaviours set the teacher apart both from systemic expectations of the Flemish educational system (intended curriculum) and from practices shared among her colleagues (received curriculum).

Pepin began from the premise that the practice of "listening" is central to mathematics teaching (and thus an important element of mathematical knowledge *for* teaching) and examined how mathematical knowledge with respect to listening is manifested in the teaching practices of English, French, and German teachers. Specifically, she used a socio-cultural approach to examine mathematical knowledge *in* teaching from the point of view of listening, using data from interviews and lesson observations with 42 teachers (14 in each country). The cross-national nature of Pepin's examination illuminated, like Andrews' study did, culturally relevant aspects of mathematical knowledge in teaching that seemed to be shaped by the national educational contexts in which the teaching practices were embedded.

In particular, Pepin's examination showed that teachers' listening (and, by implication, teachers' knowledge with respect to listening) took different forms in the three countries and that this variation might be explained in terms of different aims, values, or school types in place in each country's educational system. In England, an aspect of teachers' listening was its individualistic nature, which might be explained with reference to one of the aims of the English educational system to provide students with the individual support they need to make progress in their studies. Contrary to what was observed in England, teachers' listening in France tended to attend to the group as a whole; this aspect of French teachers' listening might be attributed to the fact that the French educational system values whole-class discussions of mathematical problems. Finally, the considerable variation that was observed among German teachers' listening might be explained in terms of the different school types where the teachers worked. For example, German teachers who worked in secondary modern schools (Hauptschulen), which are considered to be educationally challenging working environments for teachers, tended to listen more for the correctness of students' contributions and less for the logical underpinnings of these contributions, which was one of the characteristics of the listening practices of their colleagues who worked in the local grammar schools (Gymnasien).

To conclude, the two chapters reinforced, extended, and further exemplified an important point made by prior comparative research: the cultural aspects of national educational systems not only influence what mathematics teaching looks like in these systems and students' learning outcomes (e.g., Cogan & Schmidt, 1999; Hiebert et al., 2003), but also the nature and manifestation of teachers' mathematical knowledge in teaching practice.

The Cultural Embedding of Mathematical Knowledge for Teaching in the Context of Diverse Teacher Education Programmes

Adler and Davis examined the constitution of mathematical knowledge *for* teaching in various teacher education cultures, which were shaped by the broader, socio-economically diverse South African context. Adler and Davis argued that descriptions of the constitution of mathematical knowledge for teaching in teacher education would be incomplete without serious consideration of how mathematics teaching was *modelled* in it, i.e., the images of the mathematics teacher and, by implication, of mathematics teaching, presented to pre- or in-service teachers in teacher education programmes. Accordingly, Adler and Davis studied how mathematics teaching was modelled in teacher education programmes as a means of describing the kinds of learning opportunities teachers are afforded in these programmes to develop mathematical knowledge for teaching.

In analysing how mathematics teaching is modelled, and thus in interpreting the mathematical knowledge for teaching constituted, in teacher education programmes, Adler and Davis used a methodology that built on the works of Bernstein (1996) and Davis (2001). The methodology was premised on the assumption that pedagogic practice entails continuous evaluation, with every evaluative act, a form of pedagogic judgement, appealing to an authorising ground (being mathematics, mathematics education, teaching experience, etc.) in order to legitimise the pedagogic judgement. Adler and Davis applied this methodology in case studies of three teacher education programmes for in-service teachers in South Africa, and derived three different models of mathematics teaching (one for each programme).

The models of mathematics teaching, and the corresponding kinds of learning opportunities for teachers to develop mathematical knowledge for teaching that these support or imply, are referred to in the chapter as (1) "look at my practice", (2) "look at your own practice", and (3) "look at mathematics teaching practice". In the first model, developing mathematical knowledge for teaching is by emulation of the practice (performance) of the teacher educator who aims to provide teachers with an experiential base of the (reform-oriented) practice teachers are expected to enact in their classrooms. In the second model, developing mathematical knowledge for teaching is by systematic reflection on teachers' own practices as part of an action-research paradigm to teacher professional development. In the third model, developing mathematical knowledge for teaching is by interrogation of records of classroom practice, using analytic tools derived from the field of mathematics (teacher) education.

In light of their findings of how mathematical knowledge is constituted in teacher education, Adler and Davis raised questions about, and set a foundation for future investigations of, the role of teacher education in redressing or reproducing socio-economic inequality in South Africa and elsewhere. *Who* has access to *what* learning opportunities in teacher education for developing mathematical knowledge for teaching? How does teachers' acquired knowledge in teacher education shape teachers' capacity to teach mathematics and, by implication, the learning opportunities that the teachers ultimately offer to students in schools in different areas and of different socio-economic status? For example, Adler and Davis observed that the first model of mathematics teaching (look at my practice) was promoted in a teacher education programme for teachers from rural and socio-economically disadvantaged schools. To what extent, then, does this particular teacher education orientation to mathematics teaching account for, or contribute to, the generally low student learning outcomes in these schools?

The Embedding of Mathematical Knowledge for Teaching in a "Knowledge Economy" Culture

Williams situates his chapter in the culture of the "knowledge economy". One feature of a knowledge economy is a political requirement to audit services and service providers in order to establish a cost-benefit analysis for expenditure in particular areas. Teachers' mathematical knowledge *for* teaching has not been immune from this societal preoccupation with auditing. The goal of auditing teacher knowledge is to establish an “exchange value” so that if it is found to be satisfactorily present, resources will continue to flow towards teachers who possess such knowledge or towards the teacher educators who help teachers develop such knowledge. In contrast to auditing, evaluating mathematical knowledge for teaching is concerned with determining its “use-value”, or how it can be used in the practice of teaching.

Although audit and evaluation may often be in conflict, common to both endeavours is the need for tools to audit and evaluate teacher knowledge. According to Williams, developers of such tools face two major challenges. One is that knowledge is sensitive to the tool that is used to audit or evaluate it. Propositional knowledge is removed from practice and often found in research on teaching, whereas case knowledge and strategic knowledge are more directly connected to classroom practices; a tool designed to evaluate one type of knowledge may be ineffective in evaluating another type. The second challenge identified by Williams is that teacher knowledge is distributed; rather than being held “in the head” of any individual teacher, such knowledge is held by teachers collectively – in a school or in the profession. Given these challenges, any tool designed to evaluate or audit teacher knowledge involves compromise. Furthermore, Williams contends, the tools that are shaped will ultimately shape our conception of teacher knowledge.[2]

An analysis of culture is central to Williams’s thesis. When people are immersed in a culture, they can be unaware of how it shapes their thoughts and actions. But when features of a culture are highlighted or contrasted with other cultures, biases and orientations become apparent. By attending to contemporary Western society’s bias towards audit, and the potential for evaluation to be conflated with audit, it is possible to consider how such an orientation affects our understanding of teacher knowledge.

Implications for Teacher Education

Implications for teacher education of the cultural dimension of *mathematics* and *mathematics education* (the values inherent in them, their historical and social bases, etc.) have been considered elsewhere (see, e.g., Bishop, 1988; Gerdes, 1998). The four chapters in this section add to these implications via another route: that of acknowledging the cultural dimension of *teachers’ mathematical knowledge.* In this section, we discuss implications that derive from this route and concern how teacher education is, or might be, influenced by the cultural dimension of teachers’ mathematical knowledge.[3]

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[2]In interpreting and describing Williams’s view, we paraphrased an expression attributed to McLuhan (1964, 1994) in Lewis H. Lapham’s introduction to the 1994 edition of *Understanding Media*: “we shape our tools and then our tools shape us” (p. xi). McLuhan’s actual quotation seems to be that “the *beholding* of idols, or the use of technology conforms men to them” (p. 45).

[3]We use “teacher education” broadly to include both the initial training of pre-service teachers and the continued professional development of in-service teachers.

An important concern of mathematics teacher education is to articulate a curriculum that will help teachers develop mathematical knowledge that is useful in teaching. Some of the research that is available to inform such a curriculum has evaluated the knowledge – often the absence of knowledge – held by teachers and student teachers (e.g., Ball, 1990; Borko et al., 1992; Ma, 1999; Stein, Baxter, & Leinhardt, 1990). The methods used in this research included teacher observation, asking teachers to respond to mathematics teaching scenarios, and categorising mathematical objects. But Williams would advocate adopting a sceptical approach to the findings of such research because in at least one case described by him the knowledge teachers were deemed to hold was sensitive to the methodology used to audit it. Although such methods have yielded compelling data to inform, by implication, teacher education curricula, Williams's chapter cautions against complacency with existing tools for evaluating teacher knowledge and advocates the need to be mindful of the cultural-boundedness of any tools used. Future research into teachers' mathematical knowledge, therefore, would benefit from using multiple and innovative means to study mathematical knowledge in and for teaching. This would ensure that mathematics teacher educators have rich and diverse data about teacher knowledge to draw on when designing and delivering teacher education curricula.

Another area influenced by the cultural dimension of teachers' mathematical knowledge concerns the role of teacher educators. The chapters by Andrews and Pepin both made the point that the manifestation of teachers' mathematical knowledge in teaching practice is shaped by cultural aspects of the national educational systems wherein teachers work. In cases where these cultural aspects are aligned with visions of effective teaching and learning of mathematics in the respective educational systems, a potentially important element of the role of teacher educators would be to facilitate teachers' acculturation to the existing systems. Part of this process of acculturation would happen naturally anyway, assuming that prospective teachers were themselves educated in those systems in which they will be employed.

Yet, in several educational systems nowadays, new visions of effective teaching and learning are introduced in the context of curricular reforms. These new visions deviate from some previous systemically defined expectations of learning outcomes or how to achieve those outcomes, thereby creating a need for teacher educators to acculturate teachers to novel (from the point of view of a given educational system) conceptions of teaching and learning mathematics. The "apprenticeship-of-observation" (Lortie, 1975) would be a major obstacle to the process of acculturation to novel conceptions, as "it is an ally of continuity [of existing practices] rather than of change" (p. 67).[4] Accordingly, a different potentially important element of teacher educators' role would be to help teachers become more aware of, and reflect critically on, their "cultural scripts for teaching" (see Stigler & Hiebert, 1999) with

[4]The apprenticeship-of-observation is a process through which students internalise (in the most part unconsciously) the practices of their own teachers. Lortie (1975) commented on the apprenticeship-of-observation: "[T]he apprenticeship-of-observation undergone by all who enter teaching begins the process of socialization in a particular way; it acquaints students with the tasks of the teacher and fosters the development of identifications with teachers" (p. 67).

an eye towards developing new conceptions of teaching and learning that better meet the goals of curricular reforms in the respective educational systems.[5] The method, however, by which teacher educators can acculturate teachers to new forms of teaching and learning remains unclear and is a fertile direction for future research.

The chapter by Adler and Davis informs this issue with its discussion of how mathematics teaching is modelled in teacher education. It would seem that traditional discourses in teacher education would advocate the importance of the "look at my practice" model of mathematics teaching on the basis that teachers might not be expected to enact reform-oriented teaching without having first experienced for themselves this kind of teaching from the learners' point of view. Notwithstanding this argument in favour of the "look at my practice" model when acculturating teachers to new forms of teaching and learning, Adler and Davis's chapter suggests that an effective teacher education practice would incorporate a variety of models of mathematics teaching, for each model would underpin different kinds of learning opportunities for teachers to develop mathematical knowledge for teaching.

Another question to be asked by teacher educators is the extent to which the unit to be concerned with is the individual teacher. Williams argues that the knowledge that matters for teaching is distributed across the system and that strengths and shortcomings are located not in individuals but across textbooks, school plans, assessment, professional development and so on. Take, for example, the issue of textbooks: "educative curriculum materials" (Davis & Krajcik, 2005), which are concerned not only with student learning but also with teacher learning, are more likely to complement teachers' knowledge in ways that will have a positive impact on their teaching than other kinds of materials. Yet, existing curriculum materials, even those which are reform-oriented, fall short of meeting key expectations for being considered "educative" (Stylianides, 2007). But even the availability of educative curriculum materials does not imply by itself that teachers use these materials productively to enhance their knowledge for teaching (Castro, 2006). Consequently, teacher educators need to study the wider context in which individual teachers' knowledge interacts with other aspects of the system. This study can inform teacher educators' understanding of the mathematical knowledge required of individual teachers, of specialists within a school, of teachers collectively in a school, and of the entire teaching profession.

Williams argues further that the systems across which teachers use their mathematical knowledge differ to such an extent that auditing the knowledge of individual teachers is futile. If Williams is correct about the distributed nature of teacher knowledge and its cultural specificity, how should teacher educators plan and assess their courses? What are the distinctive features of different pedagogical contexts

[5] According to Stigler and Hiebert (1999), people within an educational system share a mental picture of what teaching is like, that is, they share a "cultural script for teaching." A major factor involved in the development of teachers' cultural scripts for teaching is the apprenticeship-of-observation. Indeed, Stigler and Hiebert (1999) argued that "we learn how to teach indirectly, through years of participation in classroom life, and that we are largely unaware of some of the most widespread attributes of teaching in our own culture" (p. 11).

that affect the mathematical knowledge required by teaching? Is it feasible to offer prospective teachers individualised programmes to prepare them specifically for the diverse contexts in which they might teach and to evaluate every teacher's knowledge in the teacher's unique teaching environment? Implementing such a system in many contemporary models of teacher education would pose practical and economic difficulties. But, is it possible to adapt existing models of mathematics teacher education to take on board the idea of distributed mathematical knowledge of mathematics for teaching? Research that would directly address such questions may yield fruitful answers to inform teacher education.

For example, one way in which teacher educators can incorporate the distributed knowledge assumption of mathematical knowledge for teaching is to review how they plan for and organise learning in their courses. Hewitt and Scardamalia (1998) identified six strategies for distributed learning processes which could be modified for and used in the teacher education context. These strategies are to:

1. support educationally effective peer interactions,
2. integrate different forms of discourse,
3. focus students on communal problems of understanding,
4. promote awareness of participants' contributions,
5. encourage students to build on each others' work, and
6. emphasise the work of the community.

Such strategies for developing teacher knowledge recognise the distributed nature of knowledge for teaching mathematics at the level of teacher education. The central goal in using the strategies could be to create a "Knowledge-Building Community" where the focus would be on advancing knowledge through "reading relevant resource materials, posing questions, offering theories, conducting experiments, and generally working with peers to make sense of new ideas" (Hewitt & Scardamalia, 1998, p. 82). By applying the work of Hewitt and Scardamalia to teachers' acquisition of mathematical knowledge, and being mindful of the cultural dimension of that knowledge, the possibility is opened for the chapters in this section to enhance our understanding of how teachers acquire, or can be supported in acquiring, mathematical knowledge for teaching.

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