Chapter 15
The Computer Simulation of Shaping in Rotating Electrical Discharge Machining

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Abstract The effect of the tool electrode wear on the accuracy is very important problem in the Rotating Electrical Discharge Machining (REDM). Two mathematical models of REDM are presented: the first one considers machining with the face of the end tool electrode and the second one considers EDM with the lateral side of the electrode. The software for computer simulation of EDM machining with the side and face of the electrodes has been developed. This simulation model for NC contouring EDM using rotating electrode may also be applied for tool electrode path optimization. The experimental results confirm the validity of the proposed mathematical models and the simulation software.

1 Introduction

Today's manufacturing industries are facing challenges from advanced difficult-to-machine materials (tough super alloys, ceramics, and composites), stringent design requirements (high precision, complex shapes, and high surface quality), and machining costs. The greatly-improved thermal, chemical, and mechanical properties of the material (such as improved strength, heat resistance, wear resistance, and corrosion resistance) are making ordinary machining processes unable to machine them economically. The technological improvement of manufacturing attributes can be achieved by high efficiency Rotating Electrical Discharge Machining (REDM: Electrical Discharge Grinding-EDG or Electrical Discharge Milling), Abrasive Electrodischarge Grinding (AEDG), Rotary Electrochemical Arc/Discharge Machining – RECAM/RCDM or grinding using metallic bond diamond wheels. These machining processes use a rotating tool.

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The Electrical Discharge Machining process is widely used to machine complicated shapes with high accuracy on hard and advanced materials including hardened steels, super-alloys, tungsten carbides, electrically conductive engineering ceramics, polycrystalline diamonds etc. The material is removed by the thermal energy generated by a series of electrical discharges between the tool electrode and workpiece immersed in the dielectric. There is no direct contact between the tool and workpiece during the machining process. The workpiece can be formed, either by replication of a shaped tool electrode or by 3D movement of a simple electrode like in milling.

The fabrication of complex shaped multiple electrodes used in the die-sinking micro and macro Electrical Discharge Machining (EDM) is expensive and time consuming [1]. Moving a simple shaped electrode along designed tool paths has been proposed as a solution to some of these problems [2–5]. A numerical control system enables the workpiece or tool electrode to move a previously programmed path. In this way, very complex 2 or 3 dimensional shape according to the design requirement can be shaped.

However, the tool wear during machining adversely affects the accuracy of the machined components. The problem of tool wear in 3D micro and macro EDM using simple shaped electrodes has been addressed by applying the Uniform Wear Method to maintain the electrode shape unchanged and compensate for the longitudinal tool wear [5–8].

Tool wear in EDM is characterized by the relative tool-electrode wear (RTW) which is generally defined as ratio tool wear rate (*TWR*, which is the volume of tool material removed per unit time) to material removal rate (*MRR*, which is the volume of workpiece material removed per unit time):

$$v = \frac{TWR}{MRR} \tag{1}$$

Depending upon the operating parameters of REDM, the relative wear may be 0.01–2. Changes in dimensions of tool due to wear during machining is expected to reflect in the actual depth of cut and finally in the profile and dimensional accuracy of machined parts.

Results of investigation of EDM with rotating electrode reported in studies [9, 10] show a slope curvilinear profile of bottom surface of machined groove due to the wear of disk electrode. Controlling the path of electrode can reduce shape error. In the paper [3] preliminary analysis of shape error indicates that one of the main factors leading to shape errors is wheel wear. More extended study of this problem based on mathematical modeling and experiments is reported in articles [11, 12], where the general differential equation described relationship between tool wear, initial and final shape of machined surface has been derived. Effect of wheel wear on dimensional accuracy of grinding is known from theory of tolerances. During machining of parts loaded together in a pocket, and in one pass of the rotary tool, height of parts achieved is random variable in the machined set. Based on the
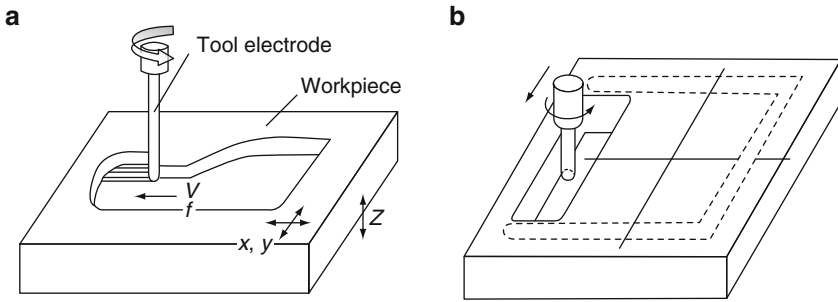
Fig. 1 Example of EDM using rotating tool electrode: (**a**) machining with the face of end electrode, (**b**) machining by the side of rotating electrode

Fig. 2 Principal scheme for mathematical modeling of REDM by end tool electrode
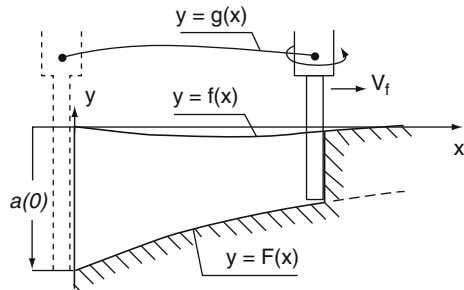
assumption of constant wear rate, uniform probability density function (PDF) has been obtained for height of machined parts [11].

Two cases of machining operations are taken for mathematical modeling: the first one considers machining with the face of the tool electrode and the second one considers EDM with the side of the electrode (Fig. 1).

In first case, the technique of integrating Uniform Wear Method with CAD/CAM software has been successful in generating very complex 3D cavities, it involves a time consuming, empirical approach for selecting tool paths and machining parameters. Therefore, it is necessary to develop a theoretical model which accounts for the effect of tool wear on the surface profile generation during each pass. As the number of passes can be very large, a corresponding computer simulation software simulation also needs to be developed.

2   Mathematical Modelling of Redm Shaping by End Tool Electrode

The principal scheme of shaping using the end tool electrode process is presented in Fig. 2. The initial profile of workpiece is given by function $y = f(x)$. The electrode is controlled to move along the tool head path $y = g(x)$. However, the

longitudinal tool wear results in the profile of machined surface $y = F(x)$, which is different from $g(x)$.

The purpose of this mathematical modeling and computer simulation is to determine surface profile $y = F(x)$, taking into account the change in tool length which occurs due to the wear of electrode. The final profile depends on input parameters, such as depth of cut a_0, initial profile $y = f(x)$, diameter of tool d_0 and tool head path $y = g(x)$.

Let us consider the case of machining presented in Fig. 2, when the initial surface is $y = f(x)$ and $g(x) = $ constant.

In determining the profile of machined surface, the following assumptions are made:

– Changes in tool shape are neglected because the Uniform Wear Method is applying.
– The gap between tool electrode and workpiece is neglected.
– Material removal rate *MRR* is equal:

$$MRR = (f(x) - F(x)) \cdot d_0 \cdot V_f \tag{2}$$

– Tool wear rate *TWR*, defined as the volume of tool material removed per unit time, for $g(x) = const.$ is:

$$TWR = \frac{\pi \cdot d_0^2}{4} \frac{dF}{dt} \tag{3}$$

or

$$TWR = \frac{\pi \cdot d_0^2}{4} \frac{dF}{dx} \frac{dx}{dt} = \frac{\pi \cdot d_0^2}{4} \frac{dF}{dx} V_f \tag{4}$$

where V_f is feed rate.

After substituting (2) and (4) in (1) for v, the equation describing the profile of machined surface takes the following form:

$$\frac{dF}{dx} + mF = mf(x) \tag{5}$$

with initial condition $F(0) = -a(0)$ (Fig. 2), where $m = 4v/\pi \cdot d_0$ is the wear factor.

In many cases it is possible to assume, that relative wear during machining is constant ($m = const.$), i.e. it is not dependent on the actual depth of cut. For this condition and for $f(x) = 0$, the solution of (5) becomes:

$$F(x) = -a_0 \exp(-mx) \tag{6}$$

While considering accuracy, the profile is conveniently described in coordinates relative to required allowance, $-a_0$ i.e.:

$$h(x) = F - (-a_0) = a_0[1 - \exp(-mx)] \tag{7}$$

For more universal application of (6) a non-dimensional form of notations is used. The non-dimensional variables are defined as $\bar{h} = h/a_0$ and $\bar{x} = x/L$, where L is the length of workpiece. In non-dimensional form, (7) can be written as:

$$\bar{h} = 1 - \exp(-A \cdot \bar{x}) \tag{8}$$

where $A = m \cdot L = 4v\, L/(\pi \cdot d_0)$.

Profiles of machined surface obtained for different values of A are presented in Fig. 3. As result of wear, the depth of cut is decreasing and the tool wear rate (*TWR*) is changing during machining. For comparison in Fig. 3 the dashed line (for $A = 1$) represents the profile neglecting changes in *TWR* during machining.

In general case when $g = g(x)$, mathematical model of REDM process is described as follows:

$$\frac{dF}{dx} + mF = mf(x) + \frac{dg}{dx} \tag{9}$$

with initial condition $F(0) = -a_0$.

The machining accuracy may be improved by control motion of tool electrode. For example, the tool wear can be compensated by moving tool or workpiece along y-axis, to obtain flat surface. In the case when initial surface is flat $f(x) = 0$ and the depth of cut is equal a_0, the required flat shape is $F(x) = -a_0$. Therefore, based on solution (9) the linear path of tool is needed, which can be described as follows:

$$y = g(x) = H_0 - a_0 \cdot m \cdot x \tag{10}$$
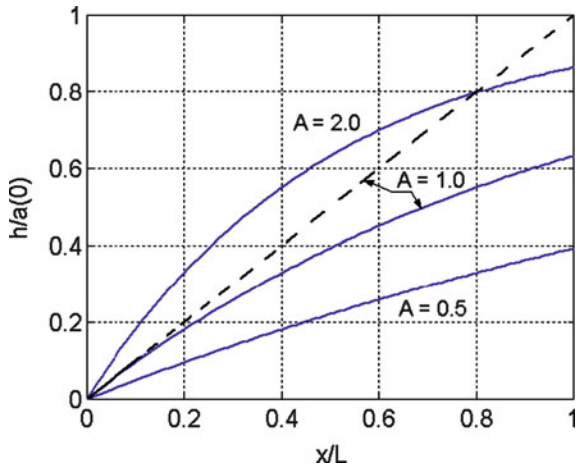
where H_0 is initial position of tool head.

Fig. 3 Profile of machined surfaces for different values of A

For machining with constant feed rate V_f along x-axis, the compensation of the tool wear can be obtained by adding the relative motion of the tool/workpiece with constant feed rate V_y along y-axis equal:

$$V_y = -a_0 \cdot m \cdot V_f = -\frac{4v \cdot a_0}{\pi \cdot d_0} V_f \qquad (11)$$

This theoretical conclusion about linear path of tool electrode for compensation of wear has been confirmed by the experiments [5].

3 Mathematical Modelling of REDM Shaping by Lateral Surface of Tool Electrode

The principal scheme of shaping process using the lateral side of tool electrode is presented in Fig. 4. The purpose of this mathematical modeling and computer simulation is to determine the profile of generated surface $y = F(x)$, taking into account the change in tool diameter which occurs due to the tool wear of rotary tool during machining (Fig. 4). The final surface profile depends on the input data, such as depth of cut a_0, initial surface profile $y = f(x)$, diameter of the tool d_0 and the curvilinear path of the center of tool $y = g(x)$.

For the modeling purpose the following assumptions were made:

– The inter electrode gap s is constant and included to the effective radius of tool i.e. $R = R(electrode) + s$.
– Actual depth of cut, a, is determined by the position of points A and B, which are point tangency of the tool electrode to the generated profile and intersection point of the tool and the initial profile $y = f(x)$, respectively (Fig. 4).
– Feed rate along axis y is significant lower in compare to the feed rate V_f i.e. the value of $dg/dx \ll 1$.
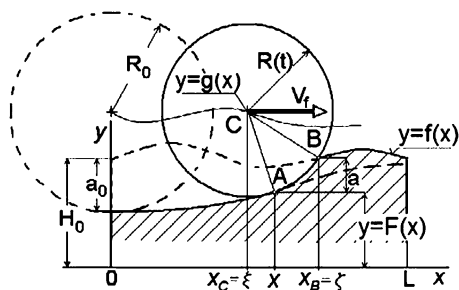– Changes in the tool shape along the axis of rotation are neglected.

Fig. 4 Scheme of machining with curvilinear path of rotating tool

After machining for certain time t, the center of the tool has reached the coordinate $x_C = \xi$, the feed rate is $V(t) = V_f$ and the effective tool radius due to the wear is $R(t)$.

The material removal rate MRR at $dg/dx \ll 1$ is:

$$MRR = a \cdot b \cdot V_f = [(f(\zeta) - F(x)] \cdot b \cdot V_f \tag{12}$$

where b is width of workpiece and ξ is function of x, $\xi = \xi(x)$.

The coordinates of intersection point $B(\xi, f(\xi))$ can be calculated from the relation $AC = BC$ (Fig. 4):

$$(x - \xi)^2 + [F(x) - g(\xi)]^2 = (\varsigma - \xi)^2 + [f(\varsigma) - g(\xi)]^2 \tag{13}$$

The tool wear rate TWR, is given by:

$$TWR = -2\pi \cdot b \cdot R(t) \cdot \frac{dR}{dt} \tag{14}$$

Since

$$\frac{dR}{dt} = \frac{dR}{d\xi}\frac{d\xi}{dt} = \frac{dR}{d\xi}V_f \tag{15}$$

and according to the relation from Fig. 4,

$$R^2(t) = [g(\xi) - F(x)]^2 + (x - \xi)^2 \tag{16}$$

the rate of change of tool radius can be expressed as following:

$$\frac{dR}{dt} = \frac{1}{R}[g(\xi) - F(x)]\left(\frac{dg}{d\xi} - \frac{dF}{dx}\frac{dx}{d\xi}\right) + (x - \xi)\left(\frac{dx}{d\xi} - 1\right) \tag{17}$$

Substituting (12) and (17) into (1) and performing transformation, the profile of machined surface can be described by (18):

$$\frac{dF}{dx} + \frac{v}{2\pi}\frac{F}{[g(\xi) - F]}\frac{d\xi}{dx} =$$
$$= \frac{v}{2\pi}\frac{f(\varsigma)}{[g(\xi) - F]}\frac{d\xi}{dx} + \frac{dg}{dx} - \frac{(x - \xi)}{[g(\xi) - F]}\left(1 - \frac{d\xi}{dx}\right) \tag{18}$$

with initial condition $F = F(0) = g(0) - R(0)$.

Generally, the value of relative tool wear is depending on the depth of cut $a = f(\xi) - F(x)$ i.e. $v = v(a)$, and this function can be only determined experimentally.

An additional equation to (18) and (13) can be derived from the condition for tangency of the tool to the profile machined surface at the point A as follows:

$$[g(\xi) - F(x)]\frac{dF}{dx} = x - \xi \tag{19}$$

The described above mathematical model can be used in following processes: REDM/EDG (side EDM milling), AREDM, RECAM/RCDM and grinding (in this case from definition of the G-ratio, $v = 1/G$).

Based on the presented mathematical model, computer simulation of evolution of workpiece profile can be carried out by using in-house developed software and two main tasks can be formulated:

1. The tool path, $g(x)$, and initial shape of surface, $f(x)$, are known but the resulting shape of the machined surface, $F(x)$, and needs to be predicted.
2. For a required shape of the machined surface, $F(x)$, the tool path, $g(x)$, needs to be determined in order to compensate for the tool wear.

These tasks have been solved numerically using the Finite Difference Method and iterative procedure.

In many cases the changes in the radius and the differences between coordinates x and ξ (for points A and B, respectively) are usually small when compared to the initial radius. Therefore $d\xi/dx \cong 1$ and (18) will evolve in:

$$\frac{dF}{dx} + \frac{v}{2\pi}\frac{F}{[g(x) - F]} = \frac{v}{2\pi}\frac{f(x)}{[g(x) - F]} + \frac{dg}{dx} \tag{20}$$

This simplified mathematical model can be successfully used to plan REDM operations and in solving the mentioned above two main tasks.

4   Software for Computer Simulation

The developed software supports process design for the REDM. The software is easy to use, menu driven application allowing evaluating the effect of various parameters on process performance.

Programming of kinematics of the tool electrode can be achieved by supplying program with tool path in x direction described. Geometry of the surface at the beginning of machining can be defined in two ways:

–   By supplying software with the surface equation, $y = f(x)$
–   By supplying software with coordinates of the surface (for example, a surface just obtained a machining process cycle can be used as a starting surface for the next machining operation)

The important feature of the software is the capability to define relative tool wear as function of depth of cut.

Input date is inserted in the overlap Data. The windows for input data are shown in Fig. 5.

Results of simulation can be viewed in two ways: tables of coordinates of points of the workpiece surface and 2D graphs.

Fig. 6 shows an example of a graph of REDM machined surface defined by function $y = a/(1 + bsin(2x/)$. The tool path was given by $g(x) = H_0 - a_0 + R_0 =$ constant (in Fig. 7 is shown shifted tool path to axis x).
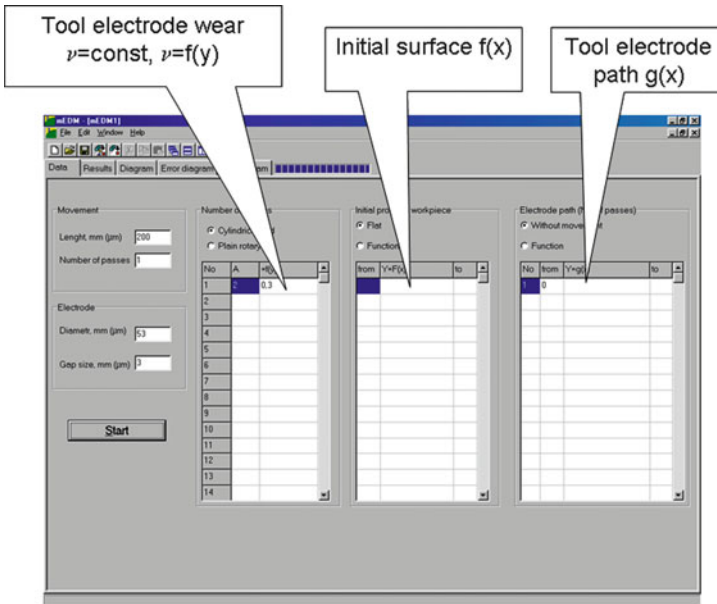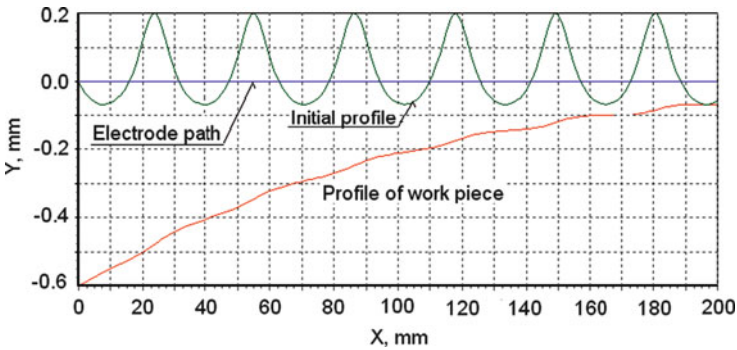

Fig. 5 Overlap data

Fig. 6 The profiles from software screen ($R_0 = 10$ mm, $a_0 = 0.6$, $v = 0.3$)

In Fig. 7 is shown simulation of REDM process with compensation of tool wear. An effect of periodical changes in the depth on final shape is appeared on machined surface.

5 Experimental Verification

The theoretical model and simulation results were verified in Rotary Electrical Discharge Machining (REDM) on M35K Mitsubishi, EDM – NC machine tool using electrode with diameter of 8.2 and 18 mm. The range for workpiece movement was 0–160 mm for x-axis. Experiments were performed on tool steel material P233330 (HRC 56) with copper electrode using the following process parameters: Pulse voltage of U: 40 V, Pulse current I: 55 and 120 A, Pulse on time t_p = pulse off
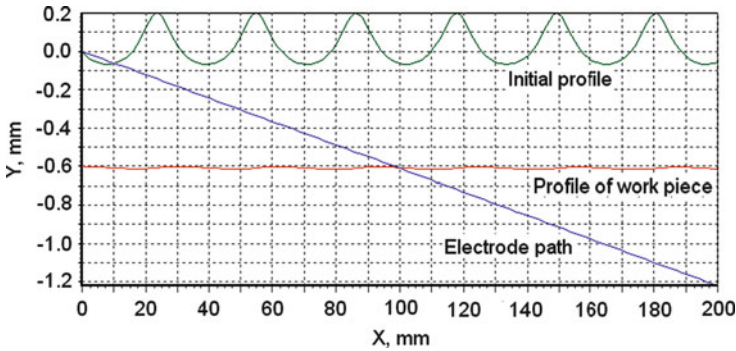
Fig. 7 The surface profile and tool path at EDM with compensation of tool wear (R_0 = 10 mm, a_0 = 0.6, v = 0.3)
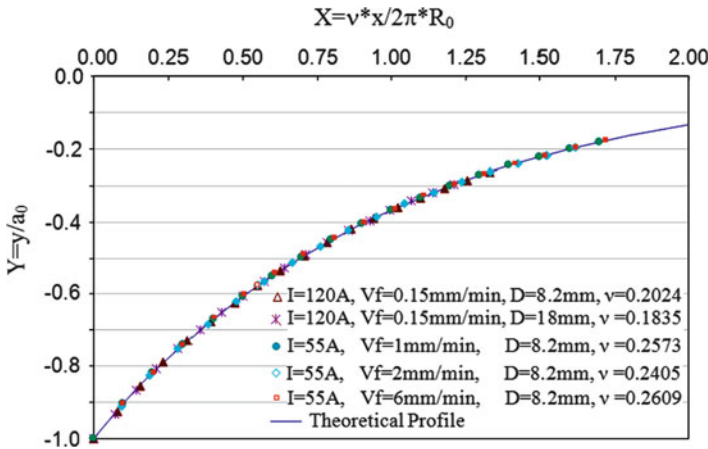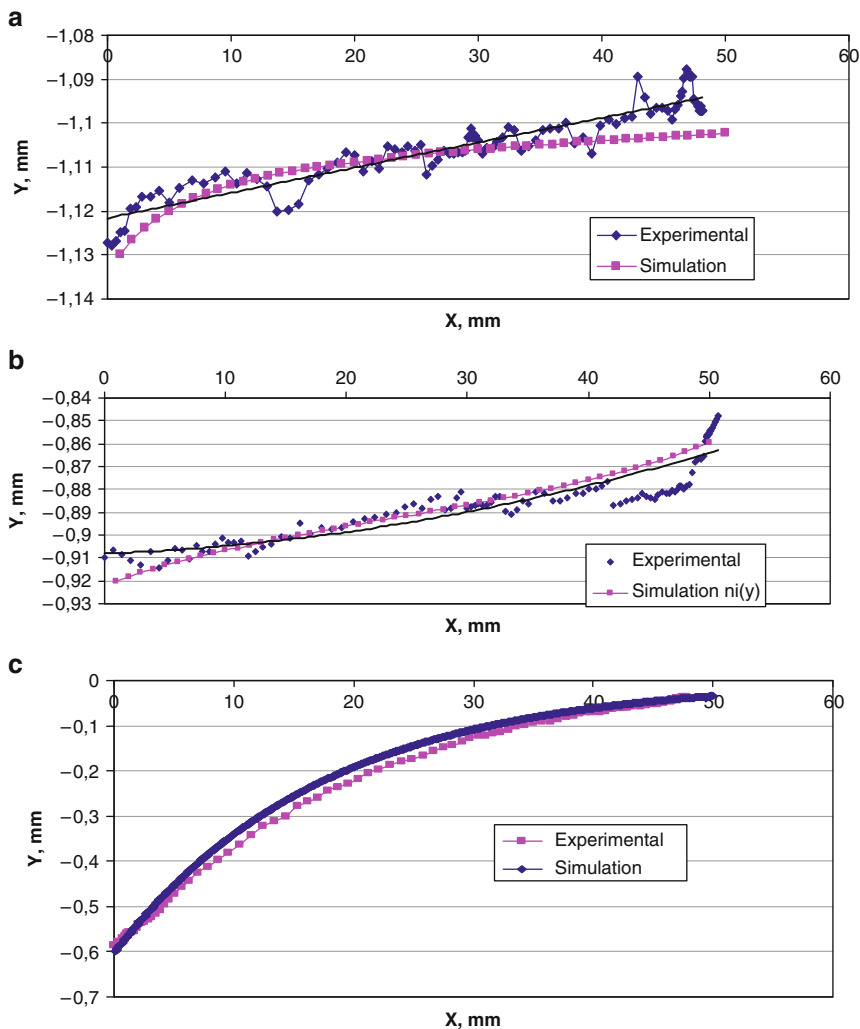
Fig. 8 Comparison of theoretical and experimental results in non-dimensional system coordinates. Setting parameters: $t_p = t_0 = 120$ μs, $n = 200$ rpm

Table 1 The machining conditions for experiments carried out on EDIOS-16

Setting parameters							
Test	t (min)	a_0 (mm)	I (A)	U (V)	n (rev/min)	TWR (mm³/min)	MRR (mm³/min)
a	25	1	6	90	600	0,033	9,077
b	19	1,5	10	50	2,700	0,254	2780
c	18	0,5	5	100	6,000	1,954	3,262

Fig. 9 Comparison of theoretical and experimental results

time t_0: 60 and 120 µs, Depth of cut a_0: 1.2 and 1.5 mm, Feed rate V_f: 0.15, 1.0, 2.0, and 6.0 mm/min, Rotation speed n: 200 rpm. For different combination of setting parameters for verification used non-dimensional system coordinates $X = v \cdot x/(2\pi \cdot R_0)$ and $\bar{Y} = y/a_0$. The results are presented in Fig. 8.

Further experiments were carried out on EDIOS-16 machine tool using copper electrode with diameter of 4 mm, pulse on time 160 µs pulse off time 10 µs and the maximal travel distances in x axis were 0–50 mm.

The machining conditions are shown in the Table 1.

Experimental results are compared with simulation prediction in Fig. 9.

Experimental verifications show high accuracy of developed mathematical model and computer simulation. An overall average of 6% (of the initial depth of cut) deviation was found between simulation and experimental results.

6   Conclusion

The study showed a good agreement of theoretical and experimental results of modeling of REDM process. The developed software can be useful for analysis of the REDM process, parameter optimization and surface prediction. Computer simulation of REDM has a significant potential to be used in industry.

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