# SIMON STEVIN AND THE RISE OF ARCHIMEDEAN MECHANICS IN THE RENAISSANCE

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ABSTRACT In this paper I will discuss the position of the Flemish mathematician and engineer Simon Stevin (1546–1620) in the rise of Archimedean mechanics in the Renaissance. Commandino represents the beginning of the Archimedean Renaissance in statics. The next steps were made by Guidobaldo Del Monte and Stevin. Del Monte and Stevin were contemporaries belonging to the generation preceding Galilei (1564–1642). Yet Stevin's work in mechanics is superior to Del Monte's. I will discuss the way in which Stevin's mechanical work, like Del Monte's, was influenced by the medieval science of weights. For example, the central notion "stalwicht" in Stevin's work, translated as "apparent weight' by the editors of Stevin's *Works*, clearly corresponds to the notion of positional weight (ponderis secundum situm) in the science of weights. I will also argue that while Del Monte remained caught in the conceptual framework of the science of weights the use of the Dutch language helped Stevin in liberating himself from those ideas. For Stevin the use of Dutch was part of his success. Finally I will discuss Stevin's work on windmills. Not only his original theoretical contributions to statics and hydrostatics but also the unity of theory and practice in Stevin's work make him in mechanics the first true successor of Archimedes in the Renaissance.

## 1. INTRODUCTION

In the past decades the Archimedean Renaissance in Italy has been studied by several authors (e.g. [9], [11] and [13]). In this particular context the work of Simon Stevin (1546–1620) has received less attention. At first sight Stevin appears to be a rather isolated figure. He seems not to belong to one of the Italian traditions. Yet he must be seen against the background of the mechanical work of the authors that preceded him. He seems isolated because we only have his mature work and we do not know its

S.A. Paipetis and M. Ceccarelli (eds.), *The Genius of Archimedes – 23 Centuries of Influence on Mathematics, Science and Engineering*, History of Mechanism and Machine Science 11,
DOI 10.1007/978-90-481-9091-1_7, © Springer Science+Business Media B.V. 2010 

genesis. There are few references to others in his work. Moreover, he wrote in Dutch, creating his own terminology and his own way of presenting the subject.

Yet Stevin is definitely part of the Archimedean Renaissance in mechanics. After Federigo Commandino of Urbino (1509–1575), the Archimedean Renaissance in statics continued with Guidobaldo Del Monte (1545–1607) and Simon Stevin (1548–1620). Del Monte knew Archimedes' work and he was familiar with a summary of Hero's *Mechanics* in the form it is given by Pappus in Book 8 of the *Collection*. Del Monte's contribution to the theory of machines consists of his *Mechanicorum liber* of 1577 and its Italian translation by Pigafetta which appeared in 1581. After having explained how useful mechanics is Del Monte formulates his goal to build mechanics up "from its foundation to its very top" ([7]. p. 246). In the text he starts with properties of the balance basing himself on Archimedes and then he proceeds to the Heronean core of mechanics: the five simple machines, in the order lever, pulley, wheel and axle, wedge and screw. Several historians have written about Del Monte's mechanics. See, for example, [11] and [21]. Duhem wrote:

> "sometimes erroneous, always mediocre, the *Mechanics* of Guido Ubaldo is often a regression from the ideas published in the writings of Tartaglia and Cardano" ([8], p. 226)

This is somewhat unfair and it is certainly unreasonable to put the writings of Tartaglia and Cardano so much higher than Del Monte's *Mechanicorum Liber.* On the other hand, although Del Monte's starting point was good, in the execution the problems that Del Monte could not solve dominated. In his treatment of the balance he lost himself in long discussions with the proponents of the science of weights. Del Monte left the problem of the inclined plane unsolved and in the Italian translation of his book on mechanics the erroneous solution of this problem by Pappus was included. In this paper I will argue that with original contributions to statics, hydrostatics and the theory of machines, Stevin was truly Archimedes' first successor in the Renaissance.

## 2. THE BACKGROUND

In the Renaissance there was a growing interest in machines and their theory: *mechanics*. The interest in machines is, for example, clearly reflected in the support of the French King for the publication, in 1571/72 by Jacques Besson (1540–1573), of one of the earliest theaters of machines.

The interest in the theory of machines is clear from the fact that several texts on mechanics from Antiquity and the Middle Ages were printed in the 16th century.

What did the theory of machines look like in the 1570s when Del Monte and Stevin were in their twenties? *Mechanical Problems* (Quaestiones Mechanicae) contained in the Aristotelian corpus, was available in print in Latin quite early in the century. It contains the oldest theory of machines usually ascribed to a follower of Aristotle, although parts of it may come from Archytas (Cf. [10]). In the 13th century, Jordanus de Nemore and his pupils had created a scholastic *science of weights*. Nicolo Tartaglia (1500?–1557) had access to some of the manuscripts and he published a version of this theory in his *Various Questions and Inventions of Niccolò Tartaglia of Brescia* (Quesiti ed inventioni diverse) of 1546. We will refer to this text as Tartaglia's *Quesiti*. The medieval Latin text that he used appeared in 1565 in Venice (See Figure 1).

Then there was Archimedes' work on statics and hydrostatics. In 1543 Tartaglia published the Latin translations of *On the Equilibrium of Planes* (Book I and II), *On the quadrature of the parabola*, *On the measurement of the circle* and *On floating bodies* (Book I only). Tartaglia's publication suggested that he had translated these texts himself. However, in 1881 it was discovered that they had been made by William of Moerbeke (circa 1215–1286). This translation left a lot to be desired. Actually there is no evidence that Tartaglia knew Greek and some that he did not ([2], pp. 555–556). In 1558 Federigo Commandino of Urbino (1509–1575) published a translation of several of Archimedes' works far superior to Moerbeke's translations. In 1565 Commandino published a translation of *On floating bodies*. Actually in *On floating bodies* Archimedes assumes properties without proof that led Commandino to publish his own *Book on the Center of Gravity of Solid Bodies* (Liber de centro gravitatis solidorum) in the same year ([3]).

This was not all. Hero's devices operated by water, air and steam were described in an encyclopedic work by Giorgio Valla printed in 1501. Hero's *Pneumatics* was published in Latin by Commandino in 1575. For the general public a Latin summary of Hero's *Mechanics* only became widely available in 1588 when Commandino's Latin translation of Pappus' *Collection* was published by Guidobaldo Del Monte ([7], p. 45). Commandino had made the translation before his death in 1575 and Del Monte had access to it. Pappus' summary of Hero's *Mechanics* in Book 8 of the *Collection* introduced Renaissance scholars to the idea that the five simple machines were the basic components of all machines. Pappus wrote, referring to Hero:

> "The names of these powers then are: the axle with a wheel turning on it; the lever; the compound pulley; the wedge; that which is called the endless screw" [14]

## 3. THE SCIENCE OF WEIGHTS: DEFINITIONS AND POSTULATES

Archimedes' work is well known. The science of weights is less known. Yet it is an essential part of the background of Stevin's work. That is why we will devote some attention to it.

Although considerably less rigorous than Archimedes's work, unlike *Mechanical Problems,* the science of weights shows influence of the Greek deductive traditions. Definitions are followed by theorems and the geometry of the figures plays an actual role in the arguments.

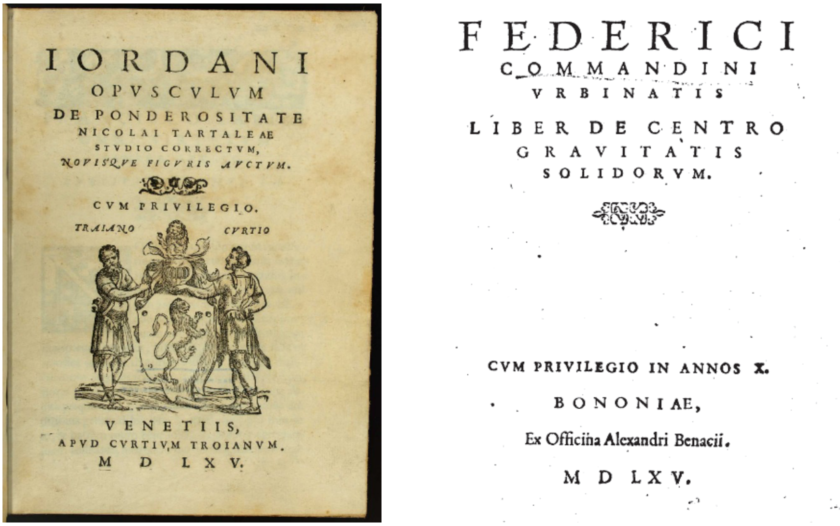
We will consider briefly some parts of the version of the theory that Tartaglia gave in his *Quesiti*. We will base ourselves on the English translation by Stillman Drake in [7]. The approach is deductive. *Definitions* and *petitions* (i.e. postulates) precede a series of *propositions* that are demonstrated on the basis of the definitions and petitions.

> Definition IX: Those bodies are said to be *simply equal in heav*iness which are actually of equal weight, even though of different material.
> Definition XIV: The heaviness of a body is said to be known when one knows the number of pounds, or other weight, that it weighs.
> Definition XIII: A body is said to be *positionally more or less heavy* than another when the quality of the place where it rests and is located makes it heavier [or less heavy] than the other, even though they are both simply equal in heaviness. ([7], p. 114, Italics are mine)

The distinction between the notions *simple heaviness* and *positional heaviness* is fundamental. Tartaglia relates positional heaviness to the *obliqueness of the descent (or ascent)* of the weight that takes place if the weight moves within the bounds of its mobility.

The notion of obliqueness is defined in

> Definition XVII: The descent of a heavy body is said to be more oblique when for a given quantity it contains less of the line of direction, or of straight descent toward the center of the world. ([7], p. 115)


IORDANI
OPVSCVLVM
DE PONDEROSITATE
NICOLAI TARTALEAE
STVDIO CORRECTVM,
NOVISQVE FIGVRIS AVCTVM.
CVM PRIVILEGIO.
TRAIANO CVRTIO
VENETIIS,
APVD CVRTIVM TROIANVM.
M D LXV.

FEDERICI
COMMANDINI
VRBINATIS
LIBER DE CENTRO
GRAVITATIS
SOLIDORVM.
CVM PRIVILEGIO IN ANNOS X.
BONONIAE,
Ex Officina Alexandri Benacii.
M D LXV.

**Fig. 1.**

The text has the form of a dialogue between Tartaglia and Mendoza, the imperial ambassador of Charles V at Venice. In the case of definition 17, Tartaglia exemplifies the definition with a reference to Figure 2.

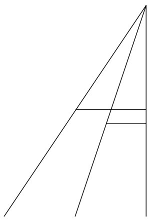


**Fig. 2.**

The descents AF and AE from the point A are oblique. Suppose that AF=AE. Then AH and AG are the vertical components of these descents, or, in Tartaglia's words, AH and AG are what the two descents contain of the *line of direction*, that is by definition the straight descent towards the center of the world. So AF is more oblique than AE, because AH is smaller than AG.

About positional heaviness Tartaglia says:

> Petition 4: Also we request that it be conceded that those bodies [bodies of equal simple weight – T. K.] are equally

> heavy positionally when their descents in such positions are equally oblique, and that will be the heavier [positionally – T. K.] which, in the position or place where it rests or is situated, has the less oblique descent. ([7], p. 119)

Clearly, a vertical line is not oblique. The positional weight in this case is equal to the simple weight. The positional weight of an object on an inclined plane depends on the slope: the more oblique the slope, the smaller the positional weight.

One notices that right from the start the problem of the inclined plane concerning the precise dependence of the positional weight on the steepness of the slope is implicitly present in the science of weights. As we will see below in the science of weights Jordanus and/or his pupils succeeded in precisely determining this relationship: they were the first ever to solve the problem of the inclined plane.

## 4. THE SCIENCE OF WEIGHTS: THE FIRST PROPOSITIONS

So far positional heaviness is determined by the simple weight plus the geometry of the situation. However, following the medieval science, Tartaglia relates positional heaviness to two other notions: *power* and *speed*. Essentially Tartaglia views *positional heaviness* as proportional to the *power a weight can exert* and in its turn this power is proportional to the *speed*, i.e. the distance covered in a certain period of time as a result of the power. Consider:

> Proposition 4: The ratio of the power of bodies simply equal in heaviness, but unequal in positional force, proves to be equal to that of their distances from the support or center of the scale. ([7], p. 123)

Tartaglia's proof is brief and from a modern point of view quite unsatisfactory. He looks at two bodies of equal simple weight positioned at unequal distances from the centre on the horizontal arm of a balance. When the arm moves the speeds or the distances covered by the weights are proportional to the distances of the weights to the center. Basically what Tartaglia is saying is the following: the power that a simple weight on the arm of a horizontal balance can exert is proportional to the length of the arm. He also implicitly assumes that with a particular arm length the power that a weight can exert is also proportional to the size of the weight.

Remarkable and revealing as for the problems that the in itself sound notion of positional weight brings about is the second part of

> Proposition 5: When a scale of equal arms is in a position of equality, and at the end of each arm there are hung weights simply equal in heaviness, the scale does not leave the said position of equality; and it happens that by some other weight [or the hand] imposed on one of the arms it departs from the said position of equality, then, that weight or hand removed, the scale necessarily returns to the position of equality ([7], p. 124).

He first part is demonstrated by remarking that on the basis of proposition 4 the bodies of equal simple weight put at equal arm length on a horizontal balance have equally oblique descents, which implies equal positional force. The second part is remarkable. See Fig. 3.

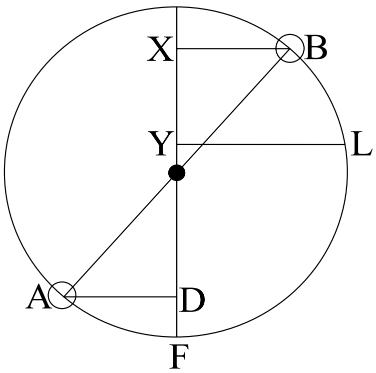



**Fig. 3.**

We have two weights simply equal in heaviness in A and B. Suppose the arcs BL and AF are equal. The projections of the arcs on the vertical line are unequal. XY is bigger than DF. That is why Tartaglia concludes that the descent of B is more oblique that the decent of A, so B is positionally heavier than A and that is why he feels that the balance will return to its horizontal position. One notices that the obliqueness of a decent is measured by projecting the descent on a vertical line in accordance with Definition XVII. Clearly we do no longer except this result as correct. If Tartaglia had considered *infinitesimal* displacements, he would have drawn a different conclusion.

## 5. THE SCIENCE OF WEIGHTS: THE LAW OF THE LEVER AND THE LAW OF THE INCLINED PLANE

The central results in the science of weights are the law of the lever and, more importantly, the law of the inclined plane. The law of the lever is phrased by Tartaglia as follows:

> Proposition 8: If the arms of the balance are proportional to the weights imposed on them, in such a way that the heavier weight is on the shorter arm, then those bodies or weights will be equally heavy positionally. ([7], pp. 132–134)

The proof is based on Proposition 4, which is applied as saying:

Positional heaviness on a balance = Simple weight x Length of arm.

(Nota bene: Tartaglia cannot express it in this way, constrained as he is by Eudoxus' theory of proportions, which was at the time generally excepted. In Eudoxus' theory only ratios of quantities of the same kind can be considered: ratios of weights can be equal to ratios of lengths, but weights and lengths cannot be multiplied.)

It is highly remarkable that in the science of weights Jordanus and his pupils succeeded in solving the problem of the inclined plane. Tartaglia almost literally follows Jordanus proof. Consider

> Proposition 15: If two heavy bodies descend by paths of different obliquities, and if the proportions of inclinations of the two paths and of the weights of the two bodies be the same, taken in the same order, the power of both the said bodies in descending will also be the same. ([7], p. 141)

See Figure 4. It is clear that one can imagine the two heavy bodies E and H, on the slopes DC and DA respectively, connected by a rope EDH. The proposition says: We have equilibrium if

$$\text{Weight E} : \text{Weight H} = DC : DA$$

We consider that situation and we imagine a weight G equal to E on slope DK which has the same tilt as DC. Suppose now that E and H "are not in the same power" and let us suppose that E descends as far as point L. Then H ascends as far as M. Assuming GN equal to LE, we have also GN equal to HM, and one can easily prove by means of similarity considerations that

$$MX : NZ = DK : DA.$$

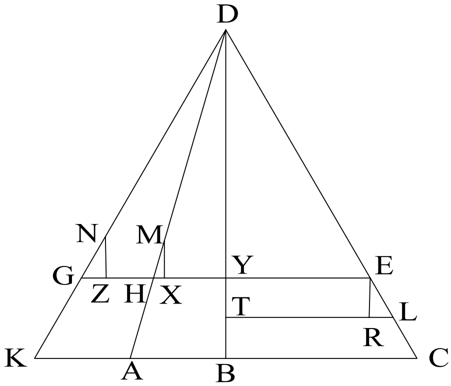



**Fig. 4**

We also have

DK : DA = Weight G : Weight H

Then

MX : NZ = Weight G : Weight H.

And Tartaglia concluded:

> Therefore, by however much the body G is simply heavier than the body H, by so much does the body H become heavier by positional force that the body G, and thus they become to be equal in force or power. ([7], p. 142)

From a modern point of view what Tartaglia is basically doing is applying this rule with respect to an inclined plane:

Positional heaviness = Simple weight x Obliqueness

(Again Tartagia cannot put it in this way because he uses Eudoxus' theory of proportions. For the modern reader this expression is somewhat more transparent.)

Obliqueness is measured by means of the vertical component of an arbitrary constant descent along the plane. Weight G and Weight H are simple weights. From a modern point of view we have:

Positional weight G in its present position = Weight G x NZ
Positional weight H in its present position = Weight H x MX

Clearly the equality of these two positional weights implies equilibrium.
Q. E. D.

## 6. STEVIN: FROM THE SCIENCE OF WEIGHTS TO *THE PRINCIPLES OF THE ART OF WEIGHING* AND *THE PRACTICE OF WEIGHING*

In 1581 the Flemish engineer and mathematician Simon Stevin (1548–1620) settled in Leiden a city in Holland, not very far from Amsterdam. He studied at the University of Leiden for two years matriculating in 1583. In 1586 he published three books that would bring him everlasting fame:

*The Principles of the Art of Weighing (De Beghinselen der Weeghconst)*
*The Practice of Weighing (De Weeghdaet)*
*The Principles of the Weight of Water (De Beghinselen des Waterwichts).*

With these books Stevin wanted to develop mechanics along strict Archimedean lines and he wanted, which implied from his point of view a further development of what we nowadays call statics plus its application to actual machines. He had read Aristotle's *In Mechanicis* ([15], pp. 508–509) as Stevin called the book *Quaestiones Mechnicae*. As we will see below he must have been familiar with ideas from the medieval science of weights as well, but we do not know how. Cardano's *Opus novum de proportionibus etc*. Basilae 1578, is mentioned twice in Stevin's works ([15], pp. 508–511).

Stevin had also read Archimedes' mechanical works and Commandino's book on centers of gravity. Stevin refers to Pappus' definition of the centre of gravity before Commandino's translation of Pappus's *Collection* had even appeared. Because Commandino quotes Pappus' definition in Greek at the beginning of chapter 1 of his book on centers of gravity, Stevin probably has it from there. I think it is improbable that Stevin had read book 8 of Pappus' *Collection*. So if I am right, Stevin was unaware of Hero's notion of simple machines (Duhem hesitates at this point Cf. [8], p. 143). I think the fact that Stevin does not treat the screw at all is revealing. Had he known about the five simple machines, he would have treated them. This supposition implies that Stevin in 1586 had not had access to Del Monte's work.

Had he known about Pappus' erroneous treatment of the inclined plane, the wedge and the screw, he would at least have shown the correct treatment of the screw. He did not. The only incorrect treatment of the inclined plane that he criticizes is Cardano's. Cardano had argued that the force needed to move a weight upwards on an inclined plane is proportional to the angle that between the slope and the horizon, the maximum value being reached when the plane is vertical (Cardano, *Opus Novum*, Propositio LXXII, Basilea, 1570, p. 63).

## 7. THE GENESIS OF STEVIN'S STATICS

Stevin must have been familiar with the science of weights in some form. On the title page of the *Practice of Weighing* (See Figure 6, right) in small letters there is written in Latin "praxis artis ponderaria". Obviously Stevin saw his art of weighing as a sequel to the science of weights. Stevin was familiar with Archimedes' works as well. He may have used Tartaglia's edition of *On the Equilibrium of Planes* (Book I and II). I propose the following speculative genesis of Stevin's books on statics.

i) Stevin realized that an Archimedean approach to the science of weights implied that all considerations concerning motion had to be dropped. Moreover, Archimedes' treatment of the balance rigorously solves the problem of positional weight for what Stevin called vertical weights, but not for oblique weights.

ii) Right from the start Stevin was thinking of situations suggested by actual machines. If one does so, the importance of forces (or weights) not acting vertically but obliquely, is obvious (See Figures 7 and 9). Stevin realized that the distinction between simple weight and positional weight made sense, but that it had to be preceded by the distinction between vertical weights and oblique weights.

iii) While studying positional weight on an inclined plane Stevin found the most beautiful proof of his life: the key to the treatment of oblique weights. This discovery determined the structure of *The Principles of the Art of Weighing*. It consists of two books. Book I consists of a part 1 on vertical weights with the law of the balance as central result and a part 2 on oblique weights with the law of the inclined plane as central result. Book II is devoted to the centers of gravity of solids, taking Commandino's book on the subject as a starting point.

iv) Stevin decided to devote a separate book to the application of the theory to real machines: *The Practice of Weighing*. In Stevin's work theory and practice are developed separately, but the unity of theory and practice is a central dogma.

## 8. STEVIN'S TREATMENT OF THE INCLINED PLANE: THE CRUCIAL PROOF

Let us consider some details. From Commandino Stevin took Pappus' definition of the center of gravity. It is worded by Stevin as follows:

> The center of gravity of a solid is the point through which any plane divides the solid into parts of equal positional weight ([15], pp. 100–101)

This definition is perfectly in accordance with Archimedes. By the way, the editors of Stevin's works translated the word "euestaltwichtigh", that Stevin uses, with "having equal apparent weight". This hides the relation with the science of weights. I think that the word "stalt" (or elsewher "ghestalt") must in this context be translated with "position" and "euestaltwichtigh" then becomes "of equal positional weight". See in the etymological dictionary [17] the lemma on "stede" and "stee".

Dijksterhuis interpreted "staltwicht" as "the component of an acting force which is actually exerting an influence" ([19], p. 52). This seems simply wrong to me. When Stevin uses the word "staltwicht" with respect to weights on a horizontal balance, this interpretation holds no water.

Pappus' definition of the center of gravity obviously implies that the second part of Proposition 5 in Tartaglia's treatment of the science of weights cannot be correct. We discussed it above. Stevin does not even mention such errors. He simply dropped in the science of weights everything that contradicted Archimedes.

Consider, for example, the role of motion in the science of weights. Stevin wrote an appendix to *The Art of Weighing* in which he gave the following argument:

> That which hangs motionless does not describe a circle.
> Two (bodies) of equal positional weight hang motionless
> Conclusion: Two (bodies) of equal positional weight do not describe circles ([15], pp. 508–509)

This is why Stevin rejects the view that the cause of the law of the balance resides in the fact that the extremities of the arms describe circles. More generally: the real causes of equilibrium are not to be found in the mobility of the weights involved. By the way, this argument has been criticized, for example, by Duhem. The point, however, is not whether the argument in itself is valid. The point is that without completely rejecting the notion of motion in statics Stevin would not have been able to liberate himself from the confusions that vexed his predecessors.

As we have seen in the *Quesiti* in the science of weights equilibrium is linked to mobility via the Aristotelian view that the power that a weight can exert is proportional to the speed that is reached if the power can be exerted. Stevin rejected this element as well. Several years before Galilei possibly dropped the two weights from the tower of Pisa, Stevin executed a similar experiment in the city of Delft with two spheres of lead from a height of 30 feet. They reached the ground at the same moment. That Aristotle was wrong had in 1562 already been argued by Jean Taisnier. Stevin had read Taisnier's book and he took a radical course here as well. All considerations concerning speed in the science of weights had to be ignored

Before turning to Stevin's treatment of the inclined plane, let us briefly look at the core idea of Archimedes' proof of the law of the lever. Archimedes' takes the validity of the law in the symmetric case with equal masses and equal arms as obvious.

See Figure 5 left. Suppose we have in A 6 white units of weight and in B 4 grey units of weight. The unit of weight should be chosen in such a way that the two numbers are even. Suppose, moreover, that for the arms we have OA and OB are, respectively, equal to 4/2=2 units of length and 6/2=3 units of length.

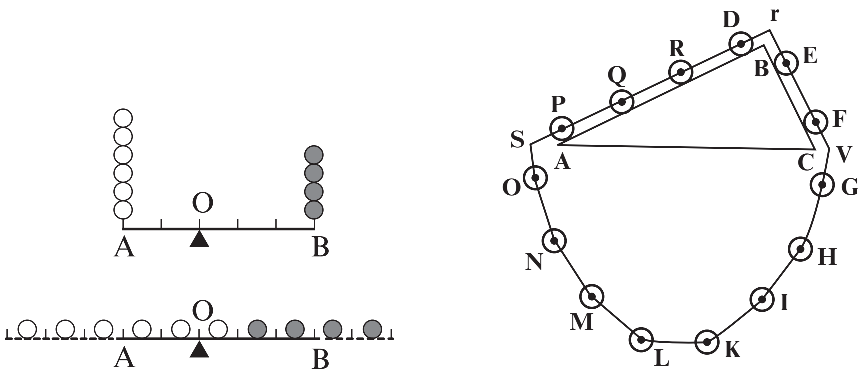



**Fig. 5.**

In this way we have created a situation in which the weights are inversely proportional to the corresponding arms. Archimedes now extends the arms: OA is extended with the length of OB and OB is extended with the length of OA. We then divide the units of weight over the units of length on the extended balance as shown in the figure. The result is that the center of gravity of the white units remains in A and the center of gravity of the grey units in B. At the same time the center of gravity of the whole is in O. So we have equilibrium. The core idea of Stevin's proof of the lever is similar.

Stevin's proof of the law of the inclined plane is also based on splitting the two weights in a number of units. See Figure 5 right. If Stevin knew the answer that the science of weights had given – there exists equilibrium if the two weights are proportional to the lengths of the two inclined planes -, which is from my point of view probable, splitting the two weights into numbers of units proportional to the length of the inclined planes would have been a rather natural move.

The crucial idea must have come to Stevin suddenly. One considers the units as beads on a chain and one closes the chain by adding a lower part. The lower part is symmetrical and it will not disturb the relation of the positional weights on the inclined planes. That the positional weights are equal is shown as follows by contradiction. Stevin assumes that the positional weight on the left hand side is bigger. Then the chain will start to rotate. All the time the chain of balls as a whole has the same position as before (“den crans der clooten sal alsucken ghestalt hebben als sy te voren dede” [15], pp. 178–179). Stevin concludes: “so the spheres will out of themselves perform a perpetual motion” (“ende de clooten sullen uyt haer selven een eeuwich roersel maken” [15], pp. 178–179). This Stevin finds impossible and he draws the conclusion that the chain will not start to rotate. We know that Stevin was extremely proud of this proof and he used the corresponding figure basically as his logo, accompanied by the text “The miracle is no miracle” (Wonder en is geen wonder). See Figure 6 with the frontispieces of *The Principles of the Art of Weighing* and *The Practice of Weighing*.

It has been assumed that Stevin rejected perpetual motion in the general sense of the word. Duhem has argued that Stevin must have read Cardano’s work. Cardano was apparently influenced by Da Vinci, whose manuscripts were at the time kept by Menzi in his villa close to Cardano’s hometown Milano. Cardano rejected the existence of a perpetuum mobile basically on Aristotelian grounds. Aristotle had indeed assumed that in order to maintain motion a constant force is needed. If Duhem is correct, this would be ironic, because Stevin rejected the Aristotelian views. However, another interpretation is possible.


**Fig. 6.** *The Principles of the Art of Weighing* (left) and *The Practice of Weighing* (right).

Stevin wrote that eternal motion *starting spontaneously* was absurd. Such a motion under the influence of gravity would from a modern point of view necessarily be accelerated and excluding friction, which Stevin explicitly does, the circular character of the motion would imply the possibility of the spontaneous occurrence of an infinitely accelerated circular motion. This in itself does not imply inconsistency but it is something one would definitely want to exclude from one's theory. See [18] for a subtle analysis of Stevin's rejection of a perpetuum mobile by Van Dyck.

## 9. *THE PRINCIPLES OF THE ART OF WEIGHING*

Stevin's book is characterized by extreme clarity. His approach is Euclidean, but there is a certain similarity with Tartaglia's *Quesiti*. Where definitions and postulates in the *Quesiti* are often accompanied by explanations directed at Tartaglia's partner in the dialogue, Stevin adds extensive explanations as well to his definitions and postulates, although the work is not in the form of a dialogue.

It is striking that *The Principles of the Art of Weighing* is preceded by a long introduction on the superiority of the Dutch language. There is more to it than that Stevin is part of an international trend to replace Latin by the vernacular and that he may have found it easier to express himself in Dutch. The Dutch language enabled Stevin to develop his ideas using his own Dutch technical terminology, thus liberating himself completely from the different terminologies that his predecessors had used. Certainly for Stevin the use of Dutch was part of his success.

In part 1 of Book I some fundamental definitions are

> Definition II: The heaviness of a solid is the power of its descent in a given place.
> Definition III: A known heaviness is expressed in terms of a known *weight*.

Compare with Definitions 9 and 14 in Section 3. A known weight is, for example, a pound or an ounce. Definition XII introduces the notions *lifting weight* and *lowering weight*. It is basically Stevin's way to handle the positive or the negative effect of a weight. Definition XIV contains the fundamental distinction between *vertical weight* and *oblique weight.* The fundamental notion *positional weight* is not introduced in a separate definition. It occurs first in the explanation following the definition of the center of gravity: The center of heaviness is the point through which any plane divides the solid into parts of *equal positional weight*.

See Figure 7 left for examples of vertical and oblique lifting weights. The Figure concerns Proposition XX, which says: "Like vertical-lift-line is to oblique-lift-line, so is vertical-lift-weight to oblique-lift-weight" The figure shows three examples of a prism supported in a point E. The prism can be kept in its position by vertical lifting weights G that apply at point F. However, G being taken away, equilibrium can also be established by means of the oblique weights H. Stevin concluded that in the three situations of Figure 7 we have

Weight G : Weight H = Vertical lifting line IF : Oblique lifting line FK.

This is correct and one notices that Stevin is here very close to the parallelogram of forces: segment IK represents the force that must be added to FI in order to get FK; it is the support by point E along the axis EC of the prism.

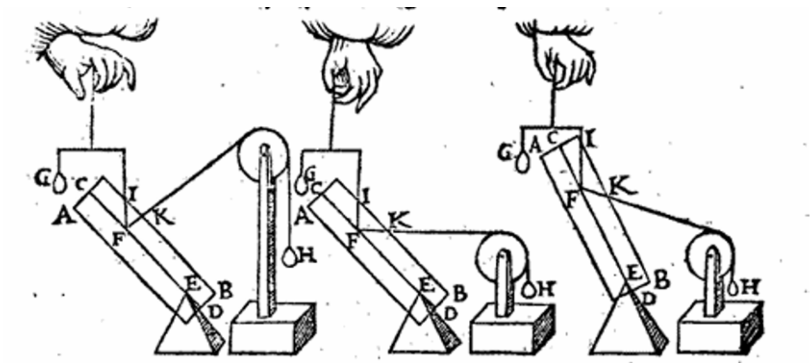



**Fig. 7.** *The Art of Weighing* Book I [15], pp. 196–197).

Actually Stevin was aware of the validity of the parallelogram of forces, as we will see below. With his work the principles of the statics of vertical and oblique forces had been defined. Others would elaborate on them and reformulate then, but the basis was there. Stevin brought considerable conceptual clarity to the subject by means of these notions. The ease by means of which he could phrase his new conceptual franework in Dutch led him to believe in the superiority of Dutch. For example, in the dedication he refers to words like "Evestaltwichtich", "Rechthefgewicht", "Scheefdaellinie", that literally stand for, respectively, "Equal-position-weight-ly" (means: of equal positional weight), "Vertical-lift-weight" and "Oblique-lowering-line" ([15], pp. 84–85). He wrote about them: "[These words] do not exist [in other languages – T. K.], Nature has specially designed Dutch for it". In the same vein he refers to his Proposition XX, "Ghelijck rechtheflini tot scheefheflini, also rechthefwicht tot scheefhefwicht", which means as we have seen "Like vertical-lift-line is to oblique-lift-line, so is vertical-lift-weight to oblique-lift-weight". He wrote about it:

> "Such secrets have been hidden hitherto in all other languages. Let them try to do something similar in another language. You can safely promise them a cake and I assure you that you will get away without damage". ([15], pp. 90–91)

Stevin had a very clear mind. His exposition is admirable, but he confused the superiority of his concepts and approach with the alleged superiority of the Dutch language.

Stevin called the science of weights an art, because he put it on the same level as arithmetic and geometry. In his dedication to the Holy Roman Emperor Rudolph II that precedes the *Principles of the Art of Weighing* he wrote, with an implicit reference to the *Book of Wisdom*: "number, magnitude and weight are in all things inseparable" ([15], pp. 54–55). Arithmetic (rekenconst) and geometry (meetconst) were established arts (art=const in Stevin's Dutch). The principles of the art of weighing, however, had according to the text of the dedication remained hidden from his predecessors. The law of the lever with respect to vertical weights was indeed known but, according to Stevin, incorrectly explained. Moreover, according to Stevin in the dedication preceding the art of weighing the theory of oblique weights was completely unknown ([15], pp. 54–55). As we have seen this was not quite correct because the law of the inclined plane had been correctly derived in Jordanus' school. Although it is clear to me that Stevin must have had some knowledge of the science of weights, this remark suggests the possibility that he had not seen or not understood Jordanus' result on the inclined plane.

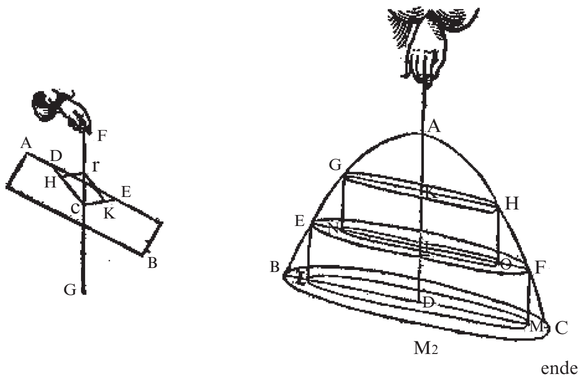



**Fig. 8.**

It is striking that Stevin feels no need to strengthen his own arguments with a criticism of his predecessors. Not bothered by the mistakes of his predecessors, without hesitations and completely sure of himself, he proceeds to a treatment of his own ideas.

This paper does not allow a further discussion of Stevin's work on statics. In a *Supplement to the Art of Weighing* he has the parallelogram of forces. See Figure 8 left ([15], pp. 532–533). This should not be surprising considering the results expressed by Figure 7. Figure 8 right ([15], pp. 276–277) illustrates the major result in Book II of *The Principles of the Art of Weighing* on the center of gravity of a segment of a paraboloid: it is on the axis AD in a point I which is such that AI is equal to twice ID. Although his derivations are somewhat different, Stevin did not add anything substantial on centers of gravity to those of Archimedes and Commandino.

## 10. A REMARK ON *THE PRACTICE OF WEIGHING*

With Del Monte Stevin has in common the intention to combine Archimedean mechanics with a theory of actually existing machines. Stevin solved the problem of the gap between theory and practice by writing two volumes. The *Practice of Weighing* contains the application of *The Principles of the Art of Weighing* to machines.

Figure 9 is from *The Practice of Weighing (De Weeghdaet)*. The figure shows Stevin's design of a machine he called the *Almighty* (Almachtich). Stevin refers at this point to Besson who had put a drawing in his book of the machine that Archimedes allegedly used to pull a ship from the shore into the sea, the *Charistion* (called polyspaston by others). Besson's machine had at least one screw.

Stevin said about his own design:

> "[it] is more suited to such work, for the following reasons: sturdier and more durable construction; of lower cost; by which is done more in shorter time, and (like the Charistion) of infinite power, that is to say: potentially, not actually". ([15], pp. 354–355)

Stevin's calculation of the mechanical advantage of the gear train is essentially based on a repeated application of the law of the lever, but he actually calculates the ratio of the number of revolutions of the crank DLMN and the axle S. Below we will se the same approach in Stevin's analysis of windmills.

This leads him to the conclusion that with a force of 25 pounds (he assumes that one man can exert such a force) a force of 5400 pounds can be exerted. Because he assumes that the simple weight of the ship is 6 times its positional weight, a ship weighing 32400 pounds can be pulled up the inclined plane ([14], pp. 358–365).

It is questionable whether at the time a really reliably functioning *Almighty* could have been built. Yet Stevin meant business as for the application of his Art of Weighing. In order to see this it is god to look at his work on windmills.

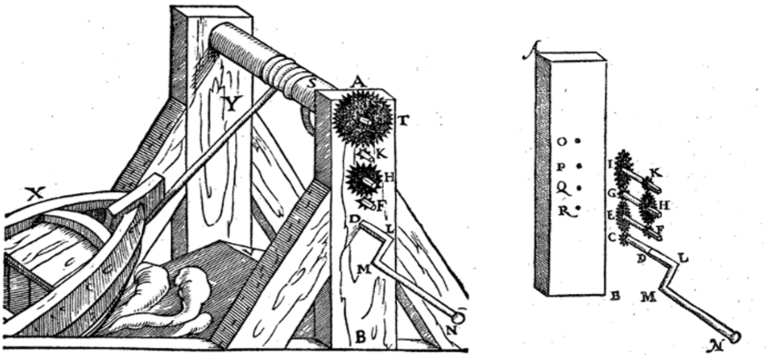



**Fig. 9.** Stevin's Almighty.

## 11. STEVIN'S ANALYSIS OF WINDMILLS

The third important book that Stevin published in 1586 is called *The Principles of the Weight of Water* (De Beghinselen des Waterwichts). In the preface in which Stevin congratulates the States of the Unites Netherlands he remarks that because the Netherlands are permanently dealing with water, knowledge of the statical properties of water can yield great advantage ([15], pp. 380–381). We will see below that this was more than rhetoric; Stevin meant it. A discussion of this book falls beyond the scope of this paper. However, one of the original results in the book concerns the pressure that water exerts on a vertical rectangular wall: the force is equivalent to the weight of a volume of water equal to ½ times the area of the wall times the height of the wall, exerted horizontally at 1/3 of the height of the wall. See [15], pp. 420–423; Figure 10 shows the accompanying image.

As we will see, this result played a crucial role in Stevin's work on windmills. A volume that could have had the title *The Practice of the Weighing of Water* and would have contained the application of the content of *The Principles of the Weight of Water* was certainly planned, but with the exception of a few pages (that contain among some other results the hydrostatic paradox) never appeared. The hydrostatic part of his work on drainage windmills that we will discuss now, could easily have been included in such a volume.

Already in 1586 and 1588 Stevin obtained patents on windmill designs. Stevin also actually built such mills. They were drainage mills meant to lift water by means of a scoop wheel from a basin with a low water level to a basin with a higher water level. Particularly interesting is the case of the mill he built in a polder near the city of IJsselstein, south of the city of Utrecht. The contract was signed on April 8, 1589 by Stevin's business partner, Jan Hugo Cornets de Groot, with representatives of the polder (the polder Leege Biesen, Achtersloot, Meerloo and the Brouck in the land of IJsselstein).

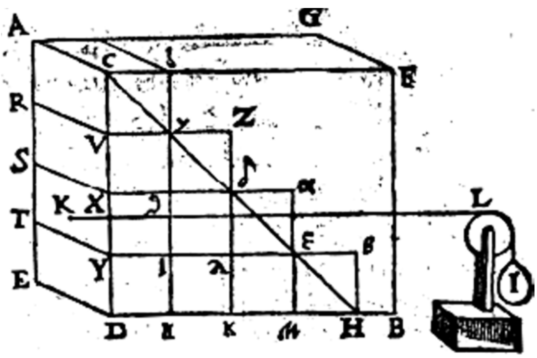



**Fig. 10.** The pressure exerted by water on a wall.

Stevin promised to build a mill of wood and iron for 630 Carolus Florins ([16], p. 324). The mill, that "would draw as much water as two of the best mills of thereabouts could do", would be ready in the fall of 1589. It soon became clear that the project was vexed with problems. In the end, after the polder refused to pay the last installment, De Groot and Stevin appealed to Princess Maria of Nassau, who while her brother was in captivity in Spain, was responsible for the barony of IJsselstein. After years a settlement was reached. The case is interesting because while Stevin accused the board of the polder of mismanagement, the representatives of the polder accused Stevin of mistakes in the design of the mill.

We know a lot about Stevin's ideas on windmills because Stevin left a manuscript called *On Mills* (Van de Molens) which contains calculations concerning both mills of the traditional type and mills of a different type based on Stevin's new design. He also left a manuscript on the design of gear wheels: *On the most perfect cogs and staves* (Van Aldervolmaackste

Cammen en Staven, [16], pp. 48–63). The mill near IJsselstein was built on the basis of Stevin's new design.

Stevin's considerations are based on an abstract kinematical model of the classical Dutch drainage mill See Figure 11 left. This model consists of the following structure reduced to certain fundamental geometrical parameters:

1. An oblique windshaft B turned by the sails.
2. A vertical upright shaft K with an upper gear wheel S driven by a gear wheel C on the windshaft.
3. A horizontal scoop-wheel shaft W with on it a gear wheel O, driven by a lower gear wheel N on the upright shaft, and a scoop-wheel R.
4. A tower with a movable cap on top of it. The oblique windshaft B was fixed inside this cap. The cap could be turned to make the sails face the wind. Both upright shaft K and scoop-wheel shaft W were fixed inside the tower.
5. The windshaft drives with its gear wheel C the upper gear wheel S of the upright shaft K and the upright shaft drives with its lower gear wheel N the scoop-wheel shaft W.

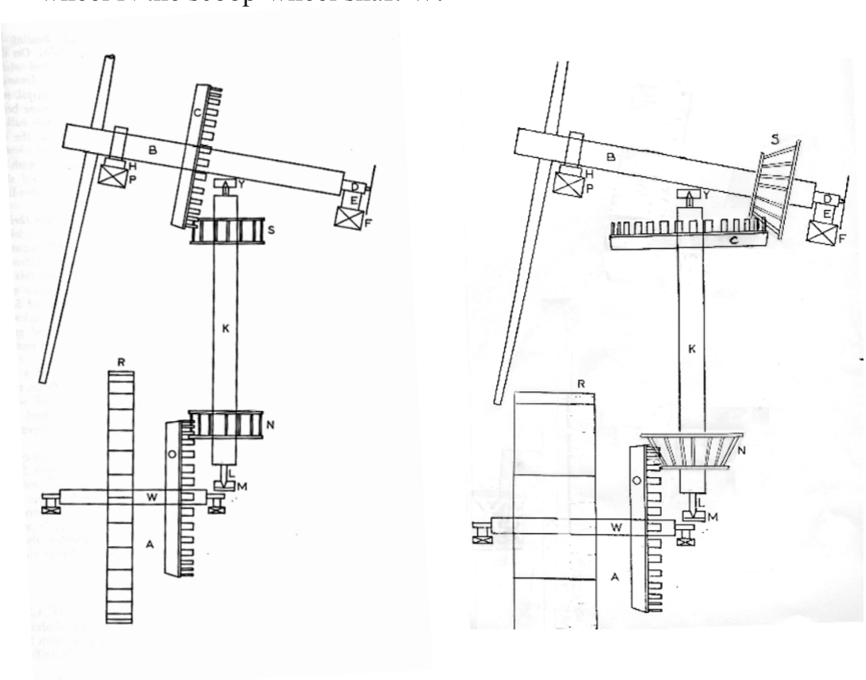



**Fig. 11.** The old design (left) and Stevin's design (right). Nota bene: the names of the wheels in the text refer to the old design.

He treats the gear trains in exactly the same way he had treated them in the *Almighty*. The dimensions of the mechanism and the number of teeth of the gears are the determining geometrical parameters of the kinematical model and he superimposes a chain of forces on the kinematical model: an input force brought about by the wind, transmission forces and an output force exerted on the water to be lifted. Subsequent forces are related to each other by means of the law of the lever.

Stevin's calculations all concern specific numerical cases and he does not give us general algebraic formulae. Yet he is fully aware of the generality of his method. Suppose that the wind exerts a force $F_{wind}$ perpendicular to the wing and the wind shaft at a distance from the shaft equal to half the length of the wings, i.e. ½ $Radius_{Wings}$. (In modern terms the force causes a moment about the shaft of $F_{wind}$ x ½ $Radius_{Wings}$). The gear train then is a chain of levers and given the dimensions by repeatedly applying the law of the lever we could determine the force $F_{water}$ exerted on the scoops (at, for example a distance ½ $Radius_{Wings}$ from the axle) needed to have equilibrium.

Yet Stevin's calculations are not based on this approach. For Stevin the numbers of teeth of the gears and the numbers of revolutions they bring about are the parameters he calculates with. Let the numbers of teeth of respectively C, S, N and O be $N_C$, $N_N$, $N_S$ and $N_O$. Then we have for the number of revolutions $R_{windshaft}$ of the windshaft and the number of revolutions $R_{scoopwheelshaft}$ of the scoop wheel shaft the following relation:

$$R_{scoopwheelshaft}/R_{windshaft} = (N_C.N_N)/(N_S.N_O).$$

This is Stevin's way to deal with the transmission of force in gears. He argues as follows. If the wings would rotate exactly as fast as the scoop-wheel, we would have equilibrium if $F_{water}$ exerted by the water on the scoops (at a distance ½ $Radius_{Wings}$) would be equal to $F_{Wind}$.

However, in general in a situation of equilibrium we have the following relation between $F_{Wind}$ (exerted on the wings at ½ $Radius_{Wings}$) and $F_{Water}$ (exerted on the scoops at ½ $Radius_{Wings}$)

$$F_{wind} = (N_C.N_N)/(N_S.N_O) \text{ x } F_{water}$$

I will call this the *Fundamental relation*. It is remarkable that this is a kinematical relation, while Stevin refers for its proof of it to a result in statics: the law of the lever. Yet it shows that he was aware of the validity of the principle of the conservation of work.

## 12. CALCULATION OF THE WIND PRESSURE WITHOUT KNOWLEDGE OF AERODYNAMICS

Stevin's originality with respect to windmills does not lie in his insight in the fundamental relation. It lies in what he did with it. The authors who wrote on the subject, Dijksterhuis and Forbes, agreed on the originality of Stevin's approach, but as for how good Stevin's designs actually were, they seem to be hesitant. The remark "Stevin tried to calculate the minimum wind pressure needed to move his scoop wheel, but he failed to relate the wind velocity to the energy available on the scoop-wheel shaft, for in his days there were no means of measuring the speed of the wind" ([16], pp. 319–320) suggests that Stevin failed somewhere in his analysis. From our point of view such a criticism is unjustified. It is true that Stevin was not capable of deducing $F_{wind}$ on the basis of, for example, aero-dynamic considerations. However, the originality of Stevin lies firstly in the fact that he realized that the *Fundamental Relation* can be used to determine $F_{wind}$ in a completely different way. He first measured and counted the fundamental geometrical parameters of several existing and functioning windmills. Then he used his original hydrostatic results to determine $F_{water}$ for those windmills. And finally he applied the *Fundamental Relation* to calculate $F_{wind}$ for those windmills.

In order to determine the force $F_{water}$ (exerted on the scoop) at a distance ½ $Radius_{Wings}$) he models the scoop of the scoop wheel as a vertical rectangular board that separates high level water from low level water. His hydrostatical results enabled him to determine the moments exerted by the pressure of the high and the low level water. $F_{water}$ is the force needed at the distance ½ $Radius_{Wings}$ to create equilibrium with the high and low level pressures. In this way Stevin determined for all mills that he investigated the force $F_{water}$ and by means of the fundamental relation he calculated $F_{wind}$.

Actually in *On Mills,* for all mills Stevin divides $F_{wind}$ by the area of the four wings together. He finds answers like 2 480/1336 ounces per square foot of sail (for the Zuyt Nootdorp Mill) and 4 536/1230 ounces per square foot of sail (for the Pynacker Mill at the bridge) or 3 44/1020 ounces per square foot (for the Cralingen Marsh Mill).

In passing Stevin also calculated in the case of the Zuyt Nootdorp Mill the force that the teeth of lower gears N and O exert upon each other by means of the law of the lever, in the way described above.

His answer is: $F_{lowergears}$= 1193 pounds. Without giving us the calculation he writes that the force between the teeth of the uppergears, $F_{uppergears}$, can be found by means of the relation:

$$F_{lowergears}/F_{uppergears} = Radius_S/Radius_N$$

Of course Stevin does not use this formula. He writes: "I say: as the radius of the driven wheel against the radius of the wallower, so the pressure above to that below" ([16], pp. 338–339).

## 13. A BRILLIANT NEW DESIGN?

It is clear from Stevin's work that the calculation of $F_{wind}$ was only a means to design a more efficient windmill. Figure 11 (right) shows us one version of Stevin's new design. The basic new element of Stevin's design is a much bigger scoop wheel. As a result the resistance of the water that must be conquered is consequently much higher. In his calculations Stevin uses the following data as a starting point: length and width of the wings, the radius of the scoop wheel, the width of the scoop-wheel, the immersion of the spoons and the difference between the high-water and the low-water level. Moreover, he assumes that the wind yields a pressure of 3 ounces per square foot. This value is somewhat below the values he determined for the existing mills.

By means of his hydrostatics Stevin calculated $F_{water}$ for his new design and used his model to calculate the dimensions of the gear wheels such that the force that the wind can apparently yield on the basis of his earlier calculations is enough to resist the pressure of the water on these big spoons. One of the consequences of the new design is that while in the traditional mill the transmission from the upper axis to the central axis speeds up the velocity of rotation and the transmission from the central axis to the lower axis slows it down again, in the new mill the big force needed to move the big spoons makes it necessary to use both transmissions to slow down the rate of rotation. In the traditional design the gear wheels on the central axis are both lantern wheels and the two other gear wheels, on respectively the upper shaft and the scoop-wheel shaft, are crown wheels. The need to slow down the rate of rotation immediately made it necessary to put the upper lantern wheel on the upper axis and the upper crown wheel on the central axis: the wheels S and C change places. In the new design the forces that the teeth of the gear wheels exert on each other are bigger than in the case of the traditional mills. That is why it is understandable that Stevin gave special attention to the position and shape of the teeth in *On the most perfect cogs and staves*.

## 14. HOW SUCCESSFUL WAS THE NEW DESIGN?

Understandably Dijksterhuis and Forbes give considerable attention the problems that Stevin encountered in the case of the IJsselstein mill. They studied the files in the IJsselstein Archive and there conclusion is the following: "The main point seems to have been that the upright shaft was made of too soft a timber and thus the thrust journal ('onderijzer' in Dutch) penetrated into the timber and the smooth turning in the thrust bearing was endangered." ([16], p. 325). Dijksterhuis and Forbes add a second point concerning the greater forces that were generated in his design: "Stevin in increasing the size of the scoop wheel caused heavier load on the pit wheel (the diameter of which remained the same) and thus greater stresses on the cogs and staves of this wheel and the crown wheel. He was not able to solve this difficulty mechanically nor to cope with the greater stresses in other parts of the machinery" ([16], p. 325). These remarks all suggest that Stevin's new design was a failure. Moreover Dijksterhuis and Forbes add: "Stevin encountered similar trouble in the case of the Kralingen mills" ([16], p. 326). In this case the trouble concerned the pit wheel, a crown wheel on the scoop-wheel shaft, which was originally not strong enough.

It is interesting that in the case of the IJsselstein mill Stevin felt it necessary to prove that his design worked well in other parts of the country and obtained a series of testimonials. The counsel of the polder may have felt that such testimonials were written by "disciples of Mr. Stevin", it is a fact that they contain a very positive report on other mills built by Stevin ([17], pp. 386–391). There is also a very positive report in which authorities from Kralingen declare their satisfaction with Stevin's mills. It is remarkable that Dijksterhuis and Forbes believe that this positive testimonial should be regarded as a conciliatory gesture ([17], p. 327) concerning the rebuilding and strengthening of the above-mentioned pit wheel, thereby suggesting doubt concerning the reliability of the testimonial. This is particularly strange, because the negotiations had ended with a contract in May 21, 1593 and the testimonial was accorded to Stevin in June 1594. Why would a conciliatory gesture be necessary more than a year after agreement had been reached?

Yet, whatever the causes, the conclusion can only be that Stevin's new design was not a big success.

## 15. CONCLUSION

The essence of the Archimedean Renaissance in mechanics is the attempt to study mechanics or the science of machines in an Archimedean way. Because none of Archimedes' works on mechanics had survived Renaissance scholars had to bridge the gap between, on the one hand, the highly theoretical treatises of Archimedes, in which he had turned statics and hydrostatics into pure, strictly deductive sciences, and, on the other hand, the real machines.

Del Monte saw the problem clearly. In his *Mechanicorum Liber* he derived the law of the balance or lever in an Archimedean way and then attempted to explain the functioning of the five simple machines on the basis of this law. Del Monte defined the problem but did not go far beyond what his predecessors had reached.

Very probably without having read Del Monte Stevin was much more successful. With some new highly original contributions to statics and hydrostatics and an approach in which the unity of theory and practice was a central dogma Stevin showed that an Archimedean science of machines going beyond what had been reached in Antiquity was quite possible. Stevin thought and wrote in Dutch. I have argued that this probably helped him in his new and fresh approach to mechanics. It also meant a disadvantage. His work was not immediately noticed. Only in 1634 some of his important works were translated into French [20]. The importance of Stevin is still sometimes underestimated. In an extensive paper on the emergence of Archimedean mechanics in the Late Renaissance published in 2008 [11] Stevin is not even mentioned.

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