ON ARCHIMEDES' PURSUIT CONCERNING GEOMETRICAL ANALYSIS

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ABSTRACT Archimedes practices the heuristic method of analysis and synthesis only in Book II of his *On the Sphere and Cylinder*. This paper has a twofold objective. Firstly, the discussion of his analytical practice through the first problem of Book II, in relation to Pappus' study of the method of analysis and synthesis in Book VII of his *Mathematical Collection*. The conclusion of this discussion is that Archimedes applies the analytical method in a way, which does not substantially differ from Pappus' way. Secondly, the discussion about the missing part from the analysis of problem 4 of *On the Sphere and Cylinder*, II, combined with the above conclusion, lead us to advance a conjecture vis-à-vis a lost analytical treatise of Archimedes under the title *Book of Data*.

1. INTRODUCTION

It is widely accepted in the history of science that Greek mathematicians were very thorough in order to present a perfect form of their mathematical arguments in their writings through which they published their research. This goal, however, was being pursued at the expense of the reader's possibility of getting a faint idea of the method through which the result was obtained. Euclid's *Elements*, the most renowned work of Greek mathematics, is the most representative example of a book that follows this approach of Greek mathematicians.

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Archimedes is an exception to the aforementioned rule. In some of his works, Archimedes does not hesitate to register the method used to find the solution to the geometrical problems, before presenting their rigorous construction following the Euclidean model. Actually, in his *Method of Mechanical Theorems*, he presents the heuristic method he used in order to reach specific results, which are proved in a formal way elsewhere in his treatises. The value of this particular work has many times been exalted in recent historiography.

It should be mentioned, however, that the undoubtedly great importance of this work and the justifiable interest of scholars for it, sometimes contributed to the overlooking of the fact that the mechanical method was not the only method used by Archimedes in order to attain solutions to difficult problems or to prove theorems. It is widely known that Archimedes, as well as other Greek mathematicians of Plato's era and onwards, had also used the method of analysis and synthesis to this end. Thanks to Pappus we know that at least twelve works were written in antiquity on the subject of the heuristic method of analysis, while in recent literature it has been supported that analysis is behind the entire corpus of Greek geometry (Knorr 1993). Discussing the importance of the mechanical method in Archimedes' work, Dijksterhuis claims that: "In this exceptionally interesting document Archimedes therefore vouchsafes us a much more intimate glimpse of his mathematical workshop than was ever granted by any other Greek mathematician" (Dijksterhuis 1987, 315). However, taking into consideration the extent of the method of analysis in Greek geometry, this statement seems to be an exaggeration because like the mechanical method, analysis also reveals the mathematician's way of thinking while solving a problem. Moreover, there are extant analyses not only from Archimedes but also from geometers such as Apollonius, Euclid, Diocles, Pappus and others, whose work and examples also –to use Dijksterhuis' expression– vouchsafe us an intimate glimpse of their mathematical workshops.

In the extant work of Archimedes, the method of geometrical analysis is applied only in Book II from his work *On the Sphere and Cylinder*. Moreover it is well-known that the most thorough study of the method of analysis preserved from Greek antiquity is traced back to the period of Late Antiquity and is found in Book VII of the *Mathematical Collection* by Pappus of Alexandria. In this book, Pappus presents three theoretical descriptions of analysis, which have been greatly discussed and argued upon by scholars, and proceeds by applying the method in a number of geometrical problems.

This paper begins with a brief presentation of the basic principles of analysis, according to Pappus, followed by the discussion of an example from the analytical practice of Archimedes, and by conclusions arising from the comparative study of the analytical practice of the two geometers. Finally, through the discussion of the missing part from the analysis of problem 4 of *On the Sphere and Cylinder*, II, a conjecture is advanced according to which Archimedes had written a work on analysis which unfortunately no longer exists.

2. PAPPUS' DISCUSSION OF GEOMETRICAL ANALYSIS

Geometrical analysis, as described and practiced by Pappus in his *Mathematical Collection*, and as discussed in (Fournarakis, Christianidis 2006), comprises two parts, the analysis and the synthesis. In the first part, the analysis, which is the heuristic part of the method, the geometer intends to find a solution to the problem, but also to confirm that the solution is valid. In the second part, the synthesis, he presents the construction and the demonstration of the found solution, according to the Euclidean model.

The starting point of analysis is the admittance of the sought as if it were established and aims the noetic conception of the structure of the problem. In the aforementioned paper we argue that Pappian analysis includes two distinct parts. In each part, the analytical course follows a different main direction. The first part of analysis, that we called "hypothetical", is a course from the conclusion to the premises and therefore it is an upward movement. The second part of analysis, that we called "confirmatory," is a deductive process and therefore it is a downward movement. This confirmatory part is characterised by the use of the terms "*dothen*" and "*dedomenon*."

The hypothetical part begins from what one is seeking as if it were established, and aims to reach something that is true independently of the sought. The geometer, through this part, intends to arrive to something from which he *supposes* that the sought can be produced and the problem can be solved. The steps of this search have hypothetical character, since they are all based on the initial assumption that the sought has been accomplished. This search is not blind or exhaustive; it includes a number of upward noetic leaps, the results of which cannot be foreseen. The first of those leaps is the assumption that the sought has been accomplished. However, making one of these leaps and producing some of its consequences (which are also hypothetical), does not univocally and surely lead to finding the next leap, but it demands the combination of elements such as the researcher's knowledge, mental ability, experience and intuition. The last leap is the finding of something that is true independently of the sought. This is indeed a noetic leap because its admittance as the end of the hypothetical part includes two fundamental hypotheses: a) it can be produced independently of the sought, and b) it can be the starting point of a syllogism that will produce the sought. This is our interpretation of Pappus' understanding of the first part of analysis.

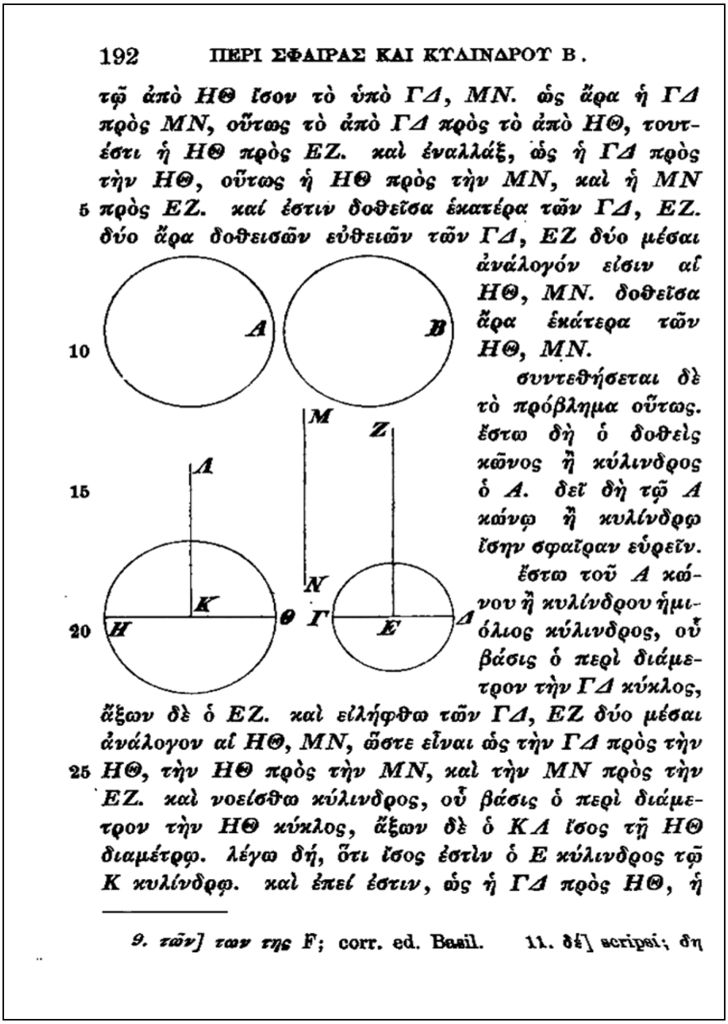
The confirmatory part, on the other hand, aims to assure that those contained in the hypothetical part, if taken in (somehow) reverse order, constitute a deduction which *can* solve the problem. It is an elaboration of the previous conceptions, in a course of valid deductions, which confirms the validity of the syllogism as to whether or not it *can* produce the necessity of the sought from the elements of the problem, as well as from the axioms and theorems of geometry. The confirmatory part does not concern concrete objects but "potential objects" that *can* be produced through the valid steps of the syllogism. The "potentiality" of these objects (or relations) is revealed by the use of the "given" (*dothen*-*dedomenon*) terminology. This terminology is used by Greek mathematicians only in the second part of the analytical process. This is how we interpret Pappus' understanding of the second part of analysis.

According to the account presented above, Pappus' analysis includes two directions, an "upward" and a "downward". The two directions do not pervade both parts of analysis, since the former is presented in the first part (the hypothetical) and the latter is presented in the second part (the confirmatory). In (Fournarakis, Christianidis 2006) we also show, by means of a specific and representative example, that using this account, one can adequately interpret how Pappus practices his analyses.

3. GEOMETRICAL ANALYSIS IN ARCHIMEDES

As previously mentioned, in the extant Archimedean corpus the method of geometrical analysis is used only in Book II of the work *On the Sphere and Cylinder*, Fig. 1. In this work, and more specifically in propositions 1, 3, 4, 5, 6, and 7, Archimedes applies the analytical method in a way, which does not differ substantially from Pappus' way. In fact, the practice of Archimedes includes the same elements previously remarked in the work of Pappus, specifically the admittance of the sought and the two parts of analysis, the hypothetical and the confirmatory. In addition, the second part of analysis can be identified, also in Archimedes, from the terms "*dothen*"–"*dedomenon*." However, as we will see further on in this paper, the practice of analysis by Archimedes also displays some specific features in comparison with the practice of Pappus.

We will corroborate the aforementioned claims through a representative example from the Archimedean analysis, and for this purpose we will discuss the first problem from *On the Sphere and Cylinder*, II. Our discussion is limited to the hypothetical and the confirmatory part of analysis, despite the fact that Archimedes presents in his text not only the analysis but also the synthesis of the problem. However, a discussion of the analytical part of the method is sufficient for our purpose. The Greek text can be found in (Heiberg 1910, I, 190–194; Stamatis 1970, A.2, 164–166) and for the English translation (Netz 2009) was used.


192 ΠΕΡΙ ΣΦΑΙΡΑΣ ΚΑΙ ΚΥΛΙΝΔΡΟΥ Β.

τῷ ἀπὸ ΗΘ ἴσον τὸ ὑπὸ ΓΔ, ΜΝ. ὡς ἄρα ἡ ΓΔ πρὸς ΜΝ, οὕτως τὸ ἀπὸ ΓΔ πρὸς τὸ ἀπὸ ΗΘ, τουτέστι ἡ ΗΘ πρὸς ΕΖ. καὶ ἐναλλάξ, ὡς ἡ ΓΔ πρὸς τὴν ΗΘ, οὕτως ἡ ΗΘ πρὸς τὴν ΜΝ, καὶ ἡ ΜΝ πρὸς ΕΖ. καί ἐστιν δοθεῖσα ἑκατέρα τῶν ΓΔ, ΕΖ. δύο ἄρα δοθεισῶν εὐθειῶν τῶν ΓΔ, ΕΖ δύο μέσαι ἀνάλογόν εἰσιν αἱ ΗΘ, ΜΝ. δοθεῖσα ἄρα ἑκάτερα τῶν ΗΘ, ΜΝ.

συντεθήσεται δὲ τὸ πρόβλημα οὕτως. ἔστω δὴ ὁ δοθεὶς κῶνος ἢ κύλινδρος ὁ Α. δεῖ δὴ τῷ Α κώνῳ ἢ κυλίνδρῳ ἴσην σφαῖραν εὑρεῖν.

ἔστω τοῦ Α κώνου ἢ κυλίνδρου ἡμιόλιος κύλινδρος, οὗ βάσις ὁ περὶ διάμετρον τὴν ΓΔ κύκλος, ἄξων δὲ ὁ ΕΖ. καὶ εἰλήφθω τῶν ΓΔ, ΕΖ δύο μέσαι ἀνάλογον αἱ ΗΘ, ΜΝ, ὥστε εἶναι ὡς τὴν ΓΔ πρὸς τὴν ΗΘ, τὴν ΗΘ πρὸς τὴν ΜΝ, καὶ τὴν ΜΝ πρὸς τὴν ΕΖ. καὶ νοείσθω κύλινδρος, οὗ βάσις ὁ περὶ διάμετρον τὴν ΗΘ κύκλος, ἄξων δὲ ὁ ΚΛ ἴσος τῇ ΗΘ διαμέτρῳ. λέγω δή, ὅτι ἴσος ἐστὶν ὁ Ε κύλινδρος τῷ Κ κυλίνδρῳ. καὶ ἐπεί ἐστιν, ὡς ἡ ΓΔ πρὸς ΗΘ, ἡ

9. τῶν] των της F; corr. ed. Basil. 11. δὲ] scripsi; δη

Fig. 1. Page 192 of the first volume of Heiberg's edition of Archimedes' Opera omnia, with his drawing of the diagram of proposition 1 of On the Sphere and Cylinder.

The problem is the following: Given a cone or a cylinder, to find a sphere equal to the cone or to the cylinder.

The hypothetical part of this analysis includes the following steps:

H.1 Let a cone or a cylinder be given, A, and let the sphere B be equal to A,

H.2 and let a cylinder be set out, ΓΖΔ, half as large again as the cone or cylinder A, and <let> a cylinder <be set out>, half as large again as the sphere B, whose base is the circle around the diameter ΗΘ, while its axis is: ΚΛ, equal to the diameter of the sphere B;

H.3 therefore the cylinder E is equal to the cylinder K. [But the bases of equal cylinders are reciprocal to the heights];

H.4 therefore as the circle E to the circle K, that is as the <square> on ΓΔ to the <square> on ΗΘ so ΚΛ to ΕΖ.

H.5 But ΚΛ is equal to ΗΘ [for the cylinder which is half as large again as the sphere has the axis equal to the diameter of the sphere, and the circle K is greatest of the <circles> in the sphere];

H.6 therefore as the <square> on ΓΔ to the <square> on ΗΘ, so ΗΘ to ΕΖ.

H.7 Let the <rectangle contained> by ΓΔ, ΜΝ be equal to the <square> on ΗΘ;

H.8 therefore as ΓΔ to ΜΝ, so the <square> on ΓΔ to the <square> on ΗΘ, that is ΗΘ to ΕΖ,

H.9 and alternately, as ΓΔ to ΗΘ, so (ΗΘ to ΜΝ) and ΜΝ to ΕΖ.

The confirmatory part of the analysis includes the following steps:

C.1 And each of <the lines> ΓΔ, ΕΖ is given;

C.2 therefore ΗΘ, ΜΝ are two mean proportionals between two given lines, ΓΔ, ΕΖ;

C.3 therefore each of <the lines> ΗΘ, ΜΝ are given.

The hypothetical part of the analysis presented above, starts with the supposition that the problem has been solved. Accordingly, all the steps of this mental route have a hypothetical character, since they are all based on the initial assumption that the sought has been accomplished. The assumption (H.1) means that a sphere B, equal to the cone or cylinder A (see Fig. 2), is found [so $V_A = V_B$]. The next hypothetical step (H.2) is the hypothetical construction of two cylinders: E, which is equal to $^3/_2$ A, and K, which is equal to $^3/_2$ B. It is a noetic leap because Archimedes sees that if we could make another cylinder K equal to the cylinder E but such that its height EZ is equal to the diameter of its base ΗΘ, then the problem

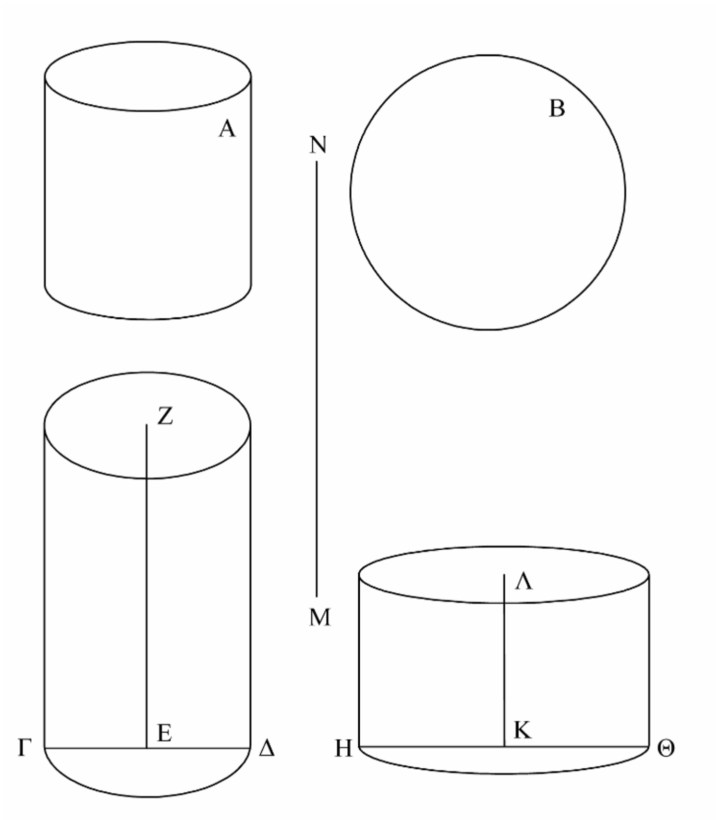


Fig. 2.

would be solved because this cylinder K would be equal to $\frac{3}{2}\, V_A$, and the sphere whose diameter is equal to the diameter of the base HΘ of the same cylinder would be the required sphere (according to I.34 of *On the Sphere and Cylinder*). (H.3) declares the obvious equality of the cylinders E and K, but it leads the geometer to think the proposition XII.15 of the *Element*s: the bases of equal cylinders are reciprocal to the heights. It is exactly this relation, that, is, the proportion sq (ΓΔ) : sq (HΘ) :: KΛ : EZ, that is “hypothetically produced” in (H.4). But (H.5) reminds that KΛ was taken equal to HΘ because the cylinder K, which is $\frac{3}{2}$ of the sphere B, was hypothesized with both its height and the diameter of its base equal to the diameter of the sphere B, and the circle K is greatest of the circles in the sphere. So in (H.6) the latter proportion is hypothesized as

$$\text{sq}(\Gamma\Delta) : \text{sq}(H\Theta) :: H\Theta : EZ. \qquad (1)$$

Then comes the next noetic leap (H.7), that is, the supposition of MN as the one side of a rectangle (whose other side is ΓΔ) which equals the square on HΘ:

$$\text{sq (H}\Theta\text{)} = \text{rec (}\Gamma\Delta\text{, MN).} \tag{2}$$

This supposition "produces hypothetically" in (H.8) the proportion ΓΔ : MN :: sq (ΓΔ) : sq (HΘ), and then the

$$\Gamma\Delta : \text{MN} :: \text{H}\Theta : \text{EZ}. \tag{3}$$

The last supposition if combined with (2) "produces hypothetically" in (H.9) the proportions ΓΔ : HΘ :: HΘ : MN :: MN : EZ. So the diameter of the required sphere B is the first of the two mean proportionals between ΓΔ and EZ.

This relation signals the end of the hypothetical part of the analysis but not the end of the analytical research. It is also a noetic leap as the geometer assumes, on the one hand, that he has reached something that is true independently of the sought and, on the other hand, it can be the starting point of a syllogism that will produce the sought. He still cannot answer whether his efforts were successful because the whole of it is based on the assumption that the sought has been accomplished, thus it is not deductive reasoning. Although in some parts of the hypothetical course the consequences of certain noetic leaps are produced, even this production is hypothetical as well, since it stands only if the sought is admitted to be true. But Archimedes assumes that it *can* evolve to a deduction, ending with the confirmation of the sought as "given". If the claim in (H.8) and (H.9) was already deductively derived, as one might maintain by observing the beginning of it using the adverb "therefore", there would be no reason for Archimedes to produce its result again in the steps C.1–C.3, using "potential" objects. If the last reached (H.9) in the hypothetical part, that is, the diameter HΘ of the required sphere was produced as the first of two mean proportionals between ΓΔ and EZ, then the synthesis of the problem would be clear and must have started at exactly this point. But Archimedes goes on with three more steps (C.1–C.3), characterized by the "given" terminology, in order to confirm that HΘ *can* be produced with logical necessity. Only after that will Archimedes start the synthesis of the problem, as he declares after the confirmatory part of analysis.

From the above discussion, we conclude that the Archimedean analysis includes the three elements described by Pappus, namely, the admittance of the sought, the hypothetical part and the confirmatory part. The latter is also formulated, like in Pappus' analysis, with the "given" terminology.

This analysis of Archimedes raises a question in respect to the rather concise form of presentation of the confirmatory part. In fact, the confirmatory part confirms only the last proportion of the hypothetical part and not the entire hypothetical part or at least the major part of it (in reverse order), as in the case of Pappian analysis. One could propose that this is a specific feature of Archimedes that differentiates him from Pappus. However, a closer examination of the confirmatory part of the Archimedean analysis shows that nothing is missing from the essence of a confirmatory part of an analysis.

Indeed, in the problem discussed above Archimedes reduces the initial problem to the problem of finding two mean proportionals between two given lines (*apagôgê*). The confirmatory part of this analysis also ends with step C.3, because of Archimedes' confirmation that the diameter of the required sphere is the first of the two mean proportionals between two given lines (C.2), the first of which is given by the problem while the second, being the $^3\!/_2$ of the first, can also be considered as given.

Therefore, the confirmation of the potential construction, with logical necessity, of the mean proportionals, can fully produce the sought of the problem.

From the aforementioned analysis, we can also infer that Archimedes uses as "given" (*dedomena*) propositions that are not included in Euclid's *Data* (i.e. the problem of two mean proportionals). This observation is of great significance since it leads to the assumption that perhaps there were other works in antiquity with context similar to Euclid's *Data*. As we will see in the last section of this paper, a work of this kind is attributed to Archimedes by an Arabic source.

Another issue relative to the aforementioned analysis of the first problem of *On the Sphere and Cylinder*, II, but also of the fourth and the fifth problems, is whether propositions that include conic sections can be considered as "givens." Archimedes' response to this issue is without doubt that: conic sections can be used in analyses exactly like the propositions included in Euclid's *Data*.

4. A CONJECTURE ON THE MISSING ANALYSIS OF PROBLEM 4 OF ARCHIMEDES' *ON THE SPHERE AND CYLINDER*, II

In problem 4 of *On the Sphere and Cylinder*, Book II, Archimedes solves the problem of dividing a sphere into two segments that have to each other a given ratio. The analysis of this problem presents certain characteristics that are not found in other analyses. More specifically, at the end of the confirmatory part, Archimedes uses as "given" a proposition, which has not been proved in a previous work of his, neither was it obtained as prefabricated data from any other work known to us. Instead, he announces that he will deal with this proposition analytically and synthetically "*at the end*." This statement was interpreted as referring to a lost *addendum* at the end of problem 4. However, this cannot be confirmed from the known manuscripts of the works of Archimedes neither from the copies of *On the Sphere and Cylinder* owned by Dionysodorus and Diocles, two geometers posterior to Archimedes by only a few decades. In fact, both geometers elaborated a different analysis of problem 4 from the start but do not deal with the proposition that Archimedes promises to present "at the end." Eutocius, in the 6th century AD, claims to have discovered this analysis of Archimedes in an old book in deplorable condition, without revealing any other information about the identity or the origin of this book.

Another feature of the analysis of problem 4 is that –according to the reconstruction of Eutocius– Archimedes does not treat the missing part *per se*, but he does so through the analysis of a more general construction problem, a special case of which is the missing analysis of problem 4. The way that Archimedes deduces the special case to be used in the solution of problem 4 from the analysis of a more general problem, presents a similarity to the way in which Pappus uses Euclid's *Data* (which also includes analyses of a more general nature). Note that *Data* includes prefabricated geometrical analyses which are used by Pappus, stating when required the necessary limiting condition, in order to solve, using the method of analysis, the various problems that he deals with in his *Mathematical Collection*. This remark leads us to examine from a new point of view certain historiographical issues as regards the problem 4 of *On the Sphere and Cylinder*, II.

The analytical procedure that Archimedes follows in the more general problem is a complete (according to Pappus) analysis which includes the three basic elements that constitute the analysis of a geometrical problem: the admittance of the sought, the hypothetical part and the confirmatory part; the latter is accomplished using the terms "*dothen*" and "*dedomenon*." Also, conic sections are used in this analysis. Furthermore, the hypothetical and confirmatory parts of this analysis are fully carried out, in a way that reminds us of Pappus' analyses as well as of the confirmatory parts found in Euclid's *Data*.

In order to solve problem 4, Archimedes does not need to use the analysis and synthesis of the general problem but only a part derived from the confirmatory part of the analysis, and moreover under certain conditions. However, in order to use this part, the complete analysis of the general problem should be presented first. This presentation could not be made in the middle of the analysis of problem 4 of *On the Sphere and*

Cylinder, II, since it is, in fact, the analysis of a completely different problem. Therefore, he announces that the analysis and synthesis of the latter problem will be presented "*at the end*."

However, is it certain that the phrase "*at the end*" refers to the end of Proposition 4? Or at least, that it refers to the end of Book II of *On the Sphere and Cylinder*? In other words, was the analysis that was discovered and restored by Eutocius, a lost *addendum* in *On the Sphere and Cylinder*? Currently, there is no evidence that verifies this hypothesis. On the contrary, there is evidence, which can make us skeptical about this hypothesis. First of all, the alleged addendum was not included in the copies of *On the Sphere and Cylinder* that Dionysodorus and Diocles owned, a few decades after Archimedes. Therefore, if it existed it should have been lost shortly after the death of Arhimedes. Furthermore, Eutocius reports having found the lost analysis in an obscure old book, partially written in Dorian dialect, without mentioning whether this book was Archimedes' *On the Sphere and Cylinder*. Netz claims that this book was "totally independent of the *On the Sphere and Cylinder*" (Netz 2009, 206). Moreover, we know that Archimedes used to announce his propositions to his colleagues first, and to present the complete proofs of them in a later time. For example, in the preface of the *Method of Mechanical Theorems* Archimedes, referring to the complete proofs of some propositions that had been announced in the past, uses a similar expression: "At the end of the book we give the geometrical proofs of the theorems the propositions of which we sent you on an earlier occasion" (*epi telei de tou bibliou grafomen tas geômetrikas apodeixeis ekeinôn tôn theôrêmatôn, hôn tas protaseis apesteilamen soi proteron*) (Heiberg 1910–1915, II, 430.23–26). In this extract Archimedes makes clear that he refers to the specific book he introduces, and he also uses present tense. On the other hand, the similar expression in problem 4 of *On the Sphere and Cylinder* is: "And these will be, each, both analyzed and synthesized at the end" (*hekatera de tauta epi telei analythêsetai te kai syntethêsetai*) (Heiberg 1910–1915, I, 214.25–26). Here Archimedes does not clarify that he intends to write the analysis and synthesis he omits in the same book, and he also uses future tense. These differences in expression are in our view indicative of his intentions.

All the above lead us to advance the conjecture that the analysis discovered by Eutocius was not published as an *addendum* in *On the Sphere and Cylinder* but in a different work by Archimedes. Additionally, we could further extend this conjecture by supposing that this analysis was included on a book of analysis written by Archimedes, similar to Euclid's *Data*. This conjecture is no more arbitrary than the hypothesis on the existence of an *addendum*. If the book *Fihrist* (Catalogue) of the 10th century Arab bio-bibliographer al-Nadim, which mentions that Archimedes has written a book called *Book of Data*, is a reliable source, then the aforementioned conjecture should be further investigated.

5. CONCLUSION

In this paper we discussed the analytical practice of Archimedes through the first problem of Book II of his *On the Sphere and Cylinder*, in relation to Pappus' study of the method of analysis and synthesis in Book VII of his *Mathematical Collection*. The conclusion of this discussion is that Archimedes applies the analytical method in a way which does not substantially differ from Pappus' way. In fact, the practice of Archimedes includes the same elements as those of Pappus', that is, the admittance of the sought, and the two parts of analysis, the hypothetical and the confirmatory. In addition, the second part of analysis can be identified, also in Archimedes as in Pappus, from the terms "*dothen*"–"*dedomenon*." Furthermore, Archimedes uses Euclid's *Data* propositions, under certain conditions, to solve problems with the method of analysis, like Pappus does, but he also uses propositions, which have to do with conic sections, and are not included in Euclid's *Data*, as data. The above reached conclusions would of course be better argued if the whole of Archimedes' and Pappus' practices of analysis were discussed here.

Secondly, the discussion of the missing part from the analysis of problem 4 of *On the Sphere and Cylinder*, II, and the various solutions of this problem that are preserved, combined with the above reached conclusions and the remark that Archimedes writes the analysis of the missing part of problem 4 in a general way, all these lead us to advance a reasoned conjecture vis-à-vis a lost analytical treatise of Archimedes written in the way Euclid's *Data* is written. This treatise could be the *Book of Data* that the Arab bio-bibliographer of the 10th century al-Nadim attributes to Archimedes.

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REFERENCES

Dijksterhuis E.J., Archimedes, Princeton University Press, Princeton, NJ, 1987.

Fournarakis P. and Christianidis J., Greek geometrical analysis: A new interpretation through the "Givens"-terminology, Bollettino di Storia delle Scienze Matematiche, Vol. XXVI/1, 2006, pp. 33–56.

Heiberg J.L., Archimedis opera, 3 vols, Teubner, Leipzig, 1910–1915. (Reproduction: Teubner, Stuttgart, 1972).
Knorr W.R., The Ancient Tradition of Geometric Problems, Dover, New York, 1993.
Netz R., The Two Books On the Sphere and Cylinder, Cambridge University Press, Cambridge, 2009.
Stamatis E.S., Archimêdous Hapanta, 3 vols (in 4 parts), Technikon Epimelêtêrion tês Hellados, Athens, 1970–1974.