RATIONAL MECHANICS AND SCIENCE RATIONNELLE UNIQUE

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ABSTRACT We highlight the legacy of Simon Stevin and Gabriel Lamé and show how their work led to some of the most important recent developments in science, ultimately based upon the principles of balance and the act of weighing, virtual or real. These names are also important in the sense of a *unique rational science* and *universal natural shapes*.

1. INTRODUCTION

Since antiquity various geometers have strived to understand and expand the ideas and results obtained by Greek mathematicians. The foundations developed by Eudoxus, Euclid, Apollonius, Archimedes and many others

S.A. Paipetis and M. Ceccarelli (eds.), *The Genius of Archimedes – 23 Centuries of Influence on Mathematics, Science and Engineering*, History of Mechanism and Machine Science 11,
DOI 10.1007/978-90-481-9091-1_3, © Springer Science+Business Media B.V. 2010

were characterized by a pulsation between geometry and algebra. This remains so today, in our era of calculation and algorithms (Atiyah, 2000). Shiing-Shin Chern wrote (2000): "*While analysis and algebra provide the foundations of mathematics, geometry is at the core*".

More analytic than synthetic, contemporary differential geometry follows the ideas of Riemann and Helmholtz, for whom measurements should be given priority, in accordance with our abstraction of our perception of the world, and very much in line with Greek thoughts on commensurability. The Greek origin for the word geometry is –μετρεω. The root –μετρεω (also in the word συμμετρια = symmetry, proportion or right balance) means: to measure, to correspond. Like κοσμεω (ordering) in ancient Greek, symmetry also has a verb (συμμετρεω) meaning to measure, to correspond, to be commensurate (Vlastos, 2005).

A major task for geometers is to deepen the understanding of the legacy of the Greek geometers. Still much is to be learned from Bacon writings: "*Solomon saith: "There is no new thing upon the earth". So that as Plato had an imagination that all knowledge was but remembrance; so Solomon giveth his sentence, "that all novelty is but oblivion."*

For Klein *parabolic, elliptic* and *hyperbolic* had precisely the same geometric meaning as it had in the application of areas of the Pythagoreans or in the conics of Apollonius, namely *precise fitting, defect* and *excess* respectively. Indeed, science still revolves around the same questions that interested Greek scholars, such as the finite versus infinite or the discrete versus continuous (in doing mathematics all these dualities act simultaneously; Thurston, 1994). Geometry (and its applications in the natural sciences) is still about the notion of going straight. On recent developments on curvatures and intrinsic and extrinsic symmetries see: Haesen and Verstraelen, 2009 and Chen, 2007.

2. FROM RENAISSANCE TO THE ECOLE POLYTECHNIQUE

2.1. Simon Stevin's *Wonder en is gheen Wonder*

Simon Stevin was one of the greatest mathematicians of the Renaissance and the greatest mechanician of the long period extending from Archimedes to Galileo (Sarton, 1934; Bosmans, 1923, 1926). His works were translated and edited by Snellius and Albert Girard and were available in Dutch, French and Latin and known to, amongst others, Gregory Saint-Vincent and Descartes.

Essential in Stevin's work is the relation between *spiegeling* ("theory") and *daet* ("practice"). Besides the necessary theoretical approach there always should be an experimental one, either concrete, or through a thought experiment. In this way Stevin made valuable contributions in calculus, algebra, geometry, mechanics, hydrostatics, navigation, tides theory, fortification, the building of locks, economy, . . . On the theoretical side, he also solved the hydrostatic paradox and dropped two unequal weights from a tower in Leiden to prove that they would reach the ground level at the same time, well before Pascal and Galilei respectively.

In *De Thiende* (1585) Stevin systematically showed how all calculations with real numbers are reduced to the standard operations with natural numbers. The importance of the real numbers for science is clear, not in the least since this very same method was used by Newton in his Method of Fluxions. In 1586 in the books *De Beghinselen der Weegh-const (The art of weighing)* and *De Weeghdaet* (*The practice of weighing*) the foundations of the mathematical vector calculus were provided with the rule of the parallelogram for the addition of forces as concrete application in physics. He introduced the impossibility of a perpetuum mobile as a method of proof in physics with the famous Clootkrans proof.

For Stevin, when phenomena could be explained rationally, meaning geometrically, miracles where no longer miracles. Stevin's motto (and epitaph; Feynman, 1963) was *Wonder en is gheen wonder* (*Magic is no magic*; Devreese & Vanden Berghe, 2008). He was a great admirer of Archimedes and Stevin's vision was completely in line with rational mechanics, where balance and weighing are crucial. In *Beghinselen der Weeghconst*, he converted the method of weighing, which was a source of inspiration to Archimedes, into a method of proof, with the use of limits as culmination (figure 1 left; Bosmans, 1923; Sarton, 1936).

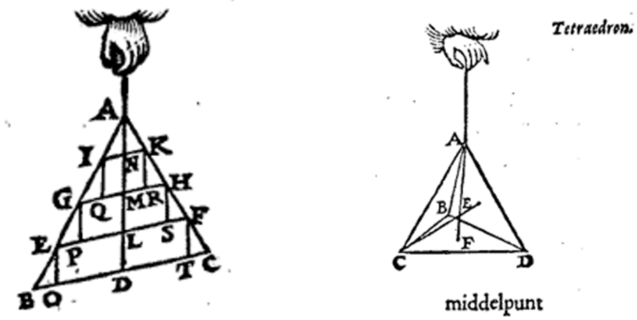

Fig. 1. Determining the centre of gravity of a triangle and a tetrahedron.

The use of limits in the elegant proofs of Stevin substituted for proofs using method of exhaustion involving a reductio ad absurdum by Archimedes. Stevin is thus an important link in the gradual transformation to modern methods of infinitesimal analysis in a chain involving Archimedes, Commandino, Stevin, Gregoire de Saint-Vincent, Boelmans, Tacquet, Pascal, Leibniz (Bosmans 1926: Sarton, 1934).

Being at the crossroads of algebra and geometry Stevin was first and foremost a geometer. With geometrical numbers he thought of powers in a very practical way. 2^3 is a cube with volume 8, and 2^4 is simply two cubes of volume 8. This pulsation of thinking both geometrically and algebraically and about cubes, numbers and roots in different ways, is an art, which should be practiced in our era of specialization.

2.2. From the Late Renaissance to Radical Enlightenment

In the Renaissance a number of exciting developments took place, forming the basis of contemporary science. These would be developed more fully during the Enlightenment, supposedly in Italy, France and England. It has been forcibly argued however, that Radical Enlightenment in the Northern Low Countries well predated the development of Enlightenment in other regions of Europe (Israel, 2005).

In the 17th century the Republic of the Northern Low Countries had become, under the patronage of Maurits van Oranje, a freehaven for science and religion. Many scholars from the Southern Low countries and France fled to the North and would provide the basis of the Golden Age (Struik, 1981). One of the foremost persons was Simon Stevin, who became the Prince’s personal advisor. Stevin was co-founder of the Ingenieursschool in Leiden in 1600, where generations of Rekenmeesters (reckoning masters) were trained.

Stevin’s early defense of the Copernican system was not appreciated by the clerics who ruled the universities. Stevin thus never held an academic position, but his influence on several generations of his “students” is very profound (Struik, 1981; Fig. 2). Among those we find, directly, Isaac Beeckman, Snellius (Sr. and Jr.) and Albert Girard, and indirectly Gregoire de Saint-Vincent, Descartes and Christiaan Huygens.

His legacy and influence on further developments was enormous, but he did not receive the proper recognition. George Sarton (1934) wrote: “*How could people truly admire one whom they do not understand, how could they consider great a man whose greatness they have not yet been educated to appreciate?*”

All in all, these developments in science in the first half of the 17th century would become the cornerstone of the Radical Enlightenment in the second half of that century, when the mathematicians De Witt and Hudde would also take important political positions (Israel, 2005). The political and religious freedom would allow for the development of Radical Enlightenment with Baruch de Spinoza as central figure (Van Bunge, 2001). His views were certainly influenced by the developments in mathematics and science in the first half of the 17th century in the Northern Low Countries, and his discourse was very much in line with Stevin's "*Wonder en is gheen Wonder.*"

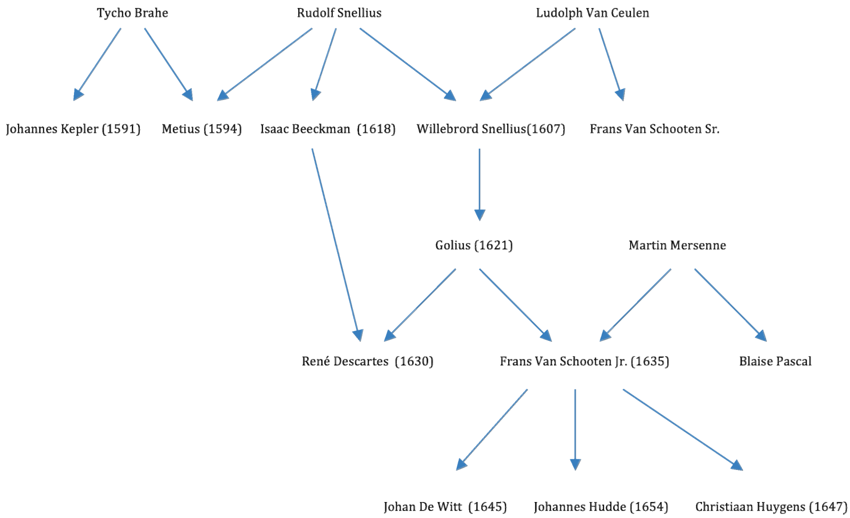

Fig. 2. Compiled from www.genealogy.ams.org.

The most important mathematician of the Low Countries in the late 17th century was Huygens, and the crucial encounter with Leibniz in Paris would be decisive for the development of science. The further developments in differential geometry initiated with Huygens and Leibniz would lead, through Basel, Berlin and Saint Petersburg and with the Bernoulli's, Euler and Lagrange as the main figures in the 18th century, to Paris in the second half of the 18th century. Paris became a leading center of mathematics in the 19th and 20th century. The trunk of the genealogical tree initiated in the Low Countries was continued and replanted in the Ecole Polytechnique EP in Paris, with brilliant teachers and students like Monge, Lagrange, Laplace, Fourier, Poisson, Legendre, Cauchy, Delaunay, Lamé, Clapeyron and Chasles, and in full agreement with the idea of Spiegeling and Daet or theory and practice.

3. SCIENCE RATIONNELLE UNIQUE & NATURAL SHAPES

3.1. Science Rationnelle Unique

Gabriel Lamé (1795–1860) entered the EP in 1813, graduated in 1817, and became a very famous *ingénieur savant*. Like Archimedes and Stevin before him he was both engineer and mathematician. Gauss praised Lamé as the most important French mathematician of his time, but in France he was considered too theoretical for engineers and too practical for mathematicians (Bertrand, 1878).

At the age of 21 he introduced equations of the type $x^n + y^n = 1$ in his book *Examen de differentes méthodes employées pour résoudre les problèmes de géometrie* (Lamé, 1818)[1] and noted that a special choice of exponents gave a uniform description of all conic sections. These Lamé curves gave the possibility of defining metrics based on powers other than two. This was also suggested by Riemann in his Habilitationsschrift (1856), which led to the development of Riemann-Finsler geometry (the metric structure of Finsler manifolds is given by a collection of convex symmetric bodies in the various tangent spaces; Berger, 2000).

During a decade in Saint Petersburg, Lamé and Clapeyron developed, amongst others, location theory (Franksen & Grattan-Guinness, 1989; Tazzioli, 1995; Gouzevitch & Gouzevitch, 2009). The development of the theory of optimal location was done with weights and balances based on machines that were used to demonstrate Stevin's parallelogram of forces. As engineers Lamé and Clapeyron used methods of weighing construction of bridges using funicular polygons (Tazzioli, 1993).

He returned to France to become professor of physics at the EP from 1832 onwards. Lamé's work on curvilinear coordinates was very influential (Struik, 1933) and his work was considered 'immortal' by Darboux (1878; "*les immortels travaux de Lamé sur les coordonnées curvilignes*"). This generalized the work of Euler on curves and of Gauss on surfaces. Elie Cartan (1931) considered Lamé as cofounder of Riemannian geometry, and his work opened the door for Ricci, Levi-Civita and Beltrami (Vincensini, 1972; Tazzioli, 1993). His influence on science continues to be most impressive (Guitart, 2009).

What connects all activities of Gabriel Lamé was his quest for a Unique Rational Science. Lamé foresaw "*l'avènement futur d'une science rationnelle unique*", of a unique rational science, which essentially is mathematical physics. His method used curvilinear coordinates designed to

[1] As a young student Gabriel Lamé's interest in geometry was aroused by Legendre's Géometrie. The profound impact of Legendre's educational books on the development of science is illustrated further by the influence of "Théorie des Nombres" on Riemann.

adapt a physical situation to a system of curvilinear coordinates. This model then provided the ‘initial geometrical support’ for a physical system. In this sense differences between phenomena would no longer be an arbitrary choice of a certain parameter, but *would organize itself to produce a natural intrinsic space* of the system.

Lamé thus envisaged that, from a mathematical point of view, the study of a physical system amounts to the study of a system of curvilinear coordinates, adapted to the given physical situation. The study of that physical problem, adapted with the appropriate system of curvilinear coordinates then becomes the characterization of the system of differential invariants or the calculation of the Laplacian in curvilinear coordinates. In his view this reduces to one equation only, namely the Poisson equation in curvilinear coordinates, with boundary conditions (Guitart, 2009).

3.2. Universal Natural Shapes

170 years after Lamé published his *Examens*, his writings on curvilinear coordinates and on Lamé curves have been united. Following attempts to describe natural shapes based on Lamé curves (Gielis, 1996) these curves were generalized as supershapes (Gielis, 2003; Equation (*); Fig. 3). This transformation can be applied to any planar function. Equation 0 in fact is a generalized Pythagorean Theorem, a conservation law for n-volumes.

$$\rho = \left(\left| \frac{\cos \frac{m_1}{4} \varphi}{A} \right|^{n_2} \pm \left| \frac{\sin \frac{m_2}{4} \varphi}{B} \right|^{n_3} \right)^{-\frac{1}{n_1}}$$

Equation (*): the Superformula with m, n_2, n_3 ψ, A, B, n_1 ρ_0

The names superformula and supershapes originate from the names superellipses and superquadrics. The name superformula was changed by mathematicians into Gielis Formula (Koiso and Palmer, 2008), and super-shapes into Gielis’ curves and surfaces (Verstraelen, 2004, 2009).

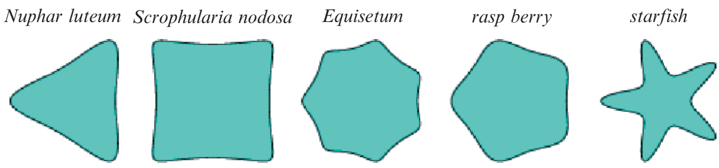

Fig. 3. Supershapes and natural analogues.

As these transformations provide for a one-step extension of conic sections to the description of natural shapes they were referred to as Universal Natural Shapes (Gielis et al., 2005). The shape coordinates make shapes commensurable. Shapes of starfish, flowers, and pyramids and a wide variety of natural shapes can now be described using one common yard stick, turning asymmetry (incommensurable) into symmetry (to make commensurable) or: "*From discord the fairest harmony*" (Heraclitus), thus expresses one of the most fundamental goals of mathematics.

Lamé-Gielis' curves and (hyper-) surfaces turn out to be the "*most natural curves and surfaces of Euclidean geometry.*" A wide range of shapes in the natural sciences can all be produced in this rather universal way: first, impose some "Euclidean" geometrical principle, and second, apply a Gielis transformation to the shapes resulting from these geometric principles (Verstraelen, 2008). Using tangents, and tangent spaces based on supershapes as length indicatrices, could unveil the geometrical meaning of all curvatures in Minkowski and Riemann-Finsler geometry, and the natural processes that are modeled in this way.

Shape description starting from a center using so-called Gielis curves and surfaces are in a natural way anisotropic, and induce a coordinate system of and on the surface, *adapted to the problem*. Generalized trigonometric or Fourier series can be defined (Gielis, 2009). This allows for strategies to develop computational tools, esp. those involving the Laplacian. Methods have been developed using stretched polar coordinates (Natalini et al., 2008; Caratelli et al., 2009), which allows for the use of Fourier series for boundary value problems combining the insights of Lamé and Fourier.

4. THE DIRICHLET PROBLEM FOR POISSON'S EQUATION IN A STARLIKE DOMAIN

Many applications of mathematical physics and electromagnetics are connected with the Laplacian (wave equation, heat propagation, Laplace, Helmholtz, Poisson and Schrödinger equations; Caratelli et al., 2009). Most boundary value problems (BVP) relevant to the Laplacian can be solved in explicit form only in domains with very special shapes or symmetries (Courant, 1950). The solution in more general domains can be obtained by using the Riemann theorem on conformal mappings, and the relevant invariance of the Laplacian.

The use of stretched co-ordinate systems allows the application of the classical Fourier method to a wide set of differential problems in complex two-dimensional normal-polar domains (Natalini et al., 2008; Caratelli et al., 2009). Such domains can be approximated as closely as desired by the above equations and numerical results are in good agreement with theoretical results of Lennart Carleson (Natalini et al., 2008). Here the solution of the Dirichlet problem for the Poisson equation in two-dimensional natural-shaped domains is presented. This differential problem is of great importance in different areas of scientific research, such as electrostatics, mechanical engineering and theoretical physics.

Let $D \subset R^2$ be an open, bounded, star-like domain, with boundary $\partial D \in C^1$ having outer normal unit vector $\nu = \nu(\rho)$. Then, a general representation formula for the solution of the Poisson equation:

$$-\Delta u(\rho) = f(\rho), \qquad \rho \in D, \tag{1}$$

subject to the Dirichlet boundary condition:

$$u(\rho) = g(\rho), \qquad \rho \in \partial D, \tag{2}$$

for given continuous functions $f(\rho)$, $g(\rho)$ can be easily obtained by using Green's function method. Under the assumption $u \in C^2(\overline{D})$, for any point $\rho \in D$ it is not difficult to show that:

$$\begin{aligned} u(\rho) = \int_{\partial D} \left[\Phi(\rho' - \rho) \frac{\partial u}{\partial \nu}(\rho') - u(\rho') \frac{\partial \Phi}{\partial \nu}(\rho' - \rho) \right] dl' + \\ - \int_D \Phi(\rho' - \rho) \Delta u(\rho') dS', \end{aligned} \tag{3}$$

where:

$$\Phi(\rho) = -\tfrac{1}{2\pi} \ln|\rho| \tag{4}$$

denotes the fundamental solution of the Laplace equation satisfying $-\Delta\Phi(\rho) = \delta(\rho)$, $\delta(\rho)$ being the Dirac measure on R^2 giving unit mass to the origin. As it can be noticed, formula (3) allows us to evaluate $u(\rho)$ once the values of $\Delta u(\rho)$ within D and the values of $u(\rho)$, $\partial u(\rho)/\partial \nu$ along ∂D are known. Hence, for the application to the Dirichlet problem for the Poisson equation (1)-(2), we must slightly modify (3) by removing the term involving the normal derivative of the unknown function $u(\rho)$ along the boundary ∂D. To achieve this, let us introduce for any fixed $\rho \in D$ the corrector function $\phi = \phi(\rho, \rho')$ solving the boundary-value problem for the Laplace equation:

$$\begin{cases} \Delta\phi(\rho,\rho') = 0, & \rho' \in D, \\ \phi(\rho,\rho') = \Phi(\rho'-\rho), & \rho' \in \partial D. \end{cases} \tag{5}$$

Applying Green's formula readily yields:

$$\begin{aligned} -\int_D \phi(\rho,\rho')\Delta u(\rho')dS' &= \int_{\partial D}\Bigg[u(\rho')\frac{\partial\phi}{\partial\nu}(\rho,\rho') + \\ -\phi(\rho,\rho')\frac{\partial u}{\partial\nu}(\rho')\Bigg]dl' &= \int_{\partial D}\Bigg[u(\rho')\frac{\partial\phi}{\partial\nu}(\rho,\rho') + \\ -\Phi(\rho'-\rho)\frac{\partial u}{\partial\nu}(\rho')\Bigg]dl'. \end{aligned} \tag{6}$$

As a consequence, Green's function for the Poisson equation (1) can be evaluated as follows:

$$G(\rho,\rho') = \Phi(\rho'-\rho) - \phi(\rho,\rho'),\ \ \rho,\rho' \in D,\ \ \rho \neq \rho'. \tag{7}$$

In fact, adding (6) to (3), we find:

$$\begin{aligned} u(\rho) &= -\int_{\partial D} u(\rho')\frac{\partial G}{\partial\nu}(\rho,\rho')dl' - \int_D G(\rho,\rho')\Delta u(\rho')dS' = \\ &- \int_{\partial D} g(\rho')\frac{\partial G}{\partial\nu}(\rho,\rho')dl' + \int_D f(\rho')G(\rho,\rho')dS', \end{aligned} \tag{8}$$

where:

$$\frac{\partial G}{\partial\nu}(\rho,\rho') = \nabla_{\rho'}G(\rho,\rho')\cdot\nu(\rho') \tag{9}$$

is the outer normal derivative of $G(\rho,\rho')$ with respect to the variable ρ' So, the solution of (1)-(2) can be derived by using (8), provided that we can construct Green's function for the given domain D. To this end, let us firstly introduce in the real plane the stretched curvilinear coordinate system:

$$\rho = (x,y), \quad \begin{cases} x = rR(\vartheta)\cos\vartheta, \\ y = rR(\vartheta)\sin\vartheta, \end{cases} \tag{10}$$

$R(\vartheta)$ denoting the polar equation of ∂D. Therefore, the domain D is described by the inequalities $0 \le \vartheta \le 2\pi$, $0 \le r \le 1$.

The following theorem provides an effective means to solve (5), and hence evaluate $G(\rho,\rho')$.

Theorem – Let:

$$\rho' = (x', y'), \qquad \begin{cases} x' = r'R(\vartheta)\cos\vartheta', \\ y' = r'R(\vartheta)\sin\vartheta', \end{cases} \tag{11}$$

and:

$$\Phi(\rho' - \rho) = \tilde{\Phi}(\vartheta', \rho) = \sum_{m=0}^{+\infty}\left[\alpha_m(\rho)\cos(m\vartheta') + \beta_m(\rho)\sin(m\vartheta')\right], \tag{12}$$

with $\rho' \in \partial D$, and:

$$\begin{Bmatrix} \alpha_m(\rho) \\ \beta_m(\rho) \end{Bmatrix} = \frac{\varepsilon_m}{2\pi}\int_0^{2\pi} \tilde{\Phi}(\vartheta', \rho)\begin{Bmatrix} \cos(m\vartheta') \\ \sin(m\vartheta') \end{Bmatrix} d\vartheta', \tag{13}$$

ε_m being the usual Neumann's symbol. Then, the boundary-value problem (5) admits a classical solution $\phi(\rho,\rho') \in L^2(D)$ such that the following Fourier-like series expansion holds:

$$\begin{aligned} &\phi(\rho,\rho') = \phi(\rho, r', \vartheta') = \\ &= \sum_{m=0}^{+\infty}\left[r'R(\vartheta')\right]^m \left[\mathrm{A}_m(\rho)\cos(m\vartheta') + \mathrm{B}_m(\rho)\sin(m\vartheta')\right]. \end{aligned} \tag{14}$$

The coefficients $\mathrm{A}_m(\rho)$, $\mathrm{B}_m(\rho)$ in (14) can be determined by solving the infinite linear system:

$$\sum_{m=0}^{+\infty}\begin{bmatrix} X_{n,m}^{+} & Y_{n,m}^{+} \\ X_{n,m}^{-} & Y_{n,m}^{-} \end{bmatrix} \cdot \begin{bmatrix} \mathrm{A}_m(\rho) \\ \mathrm{B}_m(\rho) \end{bmatrix} = \begin{bmatrix} \alpha_m(\rho) \\ \beta_m(\rho) \end{bmatrix}, \tag{15}$$

where:

$$X_{n,m}^{\{\pm\}} = \frac{\varepsilon_n}{2\pi}\int_0^{2\pi} R(\vartheta')^m \cos(m\vartheta')\begin{Bmatrix} \cos(n\vartheta') \\ \sin(n\vartheta') \end{Bmatrix} d\vartheta', \tag{16}$$

$$Y_{n,m}^{\{\pm\}} = \frac{\varepsilon_n}{2\pi}\int_0^{2\pi} R(\vartheta')^m \sin(m\vartheta')\begin{Bmatrix} \cos(n\vartheta') \\ \sin(n\vartheta') \end{Bmatrix} d\vartheta', \tag{17}$$

with $m, n \in N_0$.

Proof – In the stretched coordinate system (10)-(11), the domain D is transformed into the unit circle. Hence, we can use the eigenfunction method and separation of variables to solve the Laplace equation $\Delta\phi(\rho,\rho')=0$. In this way, it is straightforward to show that the elementary solutions of the problem are given by:

$$\phi_m(\rho,r',\vartheta')=\left[r'R(\vartheta')\right]^m\left[\mathrm{A}_m(\rho)\cos(m\vartheta')+\mathrm{B}_m(\rho)\sin(m\vartheta')\right], \quad (18)$$

with $m\in N_0$. So, enforcing the Dirichlet boundary condition $\phi(\rho,\rho')=\Phi(\rho'-\rho)$ ($\rho'\in\partial D$) and using the usual Fourier's projection method, equations (15)-(17) readily follow. □

Once the corrector function $\phi(\rho,\rho')$ for the assigned domain D is computed (Natalini et al., 2008), the solution of the boundary-value problem for the Poisson equation (1)-(2) can be obtained by applying suitable quadrature rules to Green's function representation (8).

5. OUTLOOK

The solution to the boundary-value problem for the Poisson equation is presented here. This particular problem was selected because of G. Lamé's preference. In the same way, canonical solutions to BVP of various types (also Neumann and Robin problems) can be obtained using Fourier series, avoiding cumbersome numerical techniques such as finite-difference or finite-element methods. It is also applicable in engineering since in three dimensions it allows for the development of computational solutions for mesh-free modeling, without the need for discretization in general. Almost 200 years after Fourier and Lamé, their original contributions to science are now united in the spirit of a Unique Rational Science.

We may go further, since from a purely geometrical point of view there is one and only one curve that can be expressed in a finite Fourier series only, and that is the circle itself, due to a theorem of B-Y Chen (1994). In the study of Riemannian submanifolds Chen introduced *finite type functions of k-type*. The circle is the only planar curve of finite type, namely of 1-type and any other curve is of infinite type (Verstraelen, 1991).

A generalized trigonometric series based on Eq. 1 can associate any term in the series with some anisotropic unit circle. It follows that all supershapes can be described in only one term (and in analogy with Chen finite type curves are of 1-type). As the set of Euclidean circles is a subset of the set of such unit circles, Fourier series reduce to a special case. Since anisotropic unit circles can have cusps or singularities, analysis based on pure shape description incorporates such singularities *a priori.*

In conclusion, with supershapes and Gielis transformations we are able to describe shape and development of a wide variety of basic shapes in nature using only pure numbers and we can begin to understand how other natural beings or objects "*geometrize their world*", with their own shapes as unit circles, and based on a generalized Pythagorean Theorem. We have called this program Universal Natural Shapes (Gielis et al., 2005).

An extension of Euclidean geometry, with a conservation law for n-volumes (or in a 3D world the act of weighing and equilibrium in the spirit of Archimedes, Stevin and Lamé), provides for a uniform description of natural and abstract shapes. It stimulates geometric research in the natural sciences and the development of new computational methods to address a variety of open problems in mathematical physics and mechanics.

"*La mécanique est la science des forces et du mouvement*" (Delaunay, 1856).

ACKNOWLEDGMENTS

J.G. would like to thank P. Verstraelen and T. Gerats for many stimulating discussions.

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