THE GOLDEN CROWN: A DISCUSSION

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ABSTRACT Archimedes's fame is universally more connected to his extraordinary inventions and to the legendary events that have been ascribed to him rather than to a deep and real knowledge of the historical personage and of his works. Systems of *levers* and *catapults*, *cochlea* and other mechanical or hydraulic contraptions, *water-clock*, *planetarium*, *heat rays*. Among these and further inventions, real or supposed to be, there is the episode of Hiero's Crown, Fig. 1. The episode of the apparent fraud goes generally around in two different versions; the first one, which is based on the volumetric comparisons, mentioned by the roman architect Vitruvius, the second one is anonymous, it is related by Priscian and it's essentially based on the hydrostatic balance. In this paper, we compare and discuss the two reconstructions, both of them to be considered plausible.

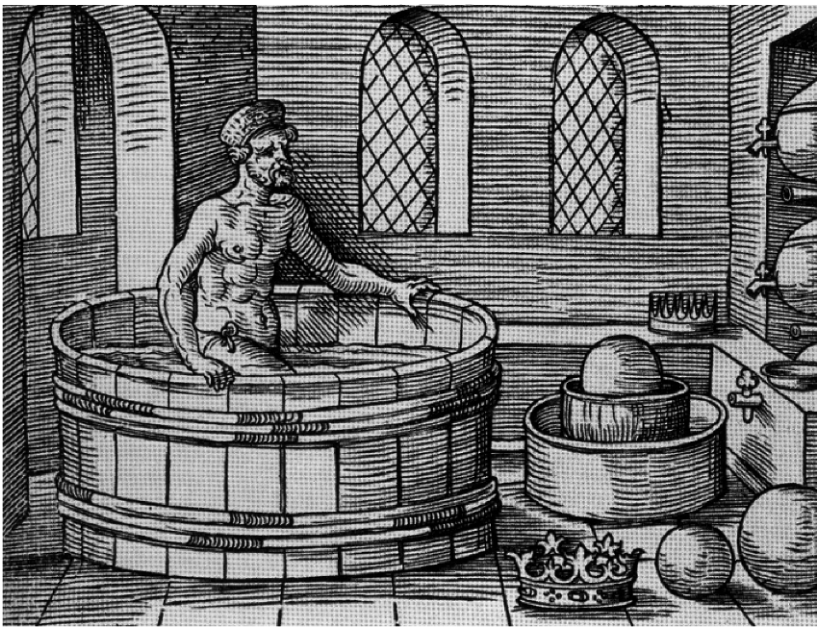

Fig. 1. Reproduction of wood engraving of the late Middle Ages.

S.A. Paipetis and M. Ceccarelli (eds.), *The Genius of Archimedes – 23 Centuries of Influence on Mathematics, Science and Engineering*, History of Mechanism and Machine Science 11,
DOI 10.1007/978-90-481-9091-1_15, © Springer Science+Business Media B.V. 2010

1. INTRODUCTION

Archimedes is *per antonomasia* the best known and revered scientist of the ancient times and, at the same time, the most underestimated and misunderstood. Many elements have contributed to this paradoxical result: his brief style and the objective originality of some of his results (hardly mentioned or even lacking proofs, references to non identified or missing works), (1) the troubled and sometimes risky vicissitudes of the works to which his thought has been committed, the contradictory and uncertainty of the various evidences which, through different centuries and cultures, have often left us a transformed and phantasmagorical version of this personage. In a word, Archimedes has become more an icon of scientific mythology and of novelized history, than an author whose works we know or a scientist whose results we understand. In his case, we're not only dealing about the usual smallness of biographical information and the fragmentary quality of the original sources, which characterized almost any scientist belonging to classical ancient times and, particularly, the «Hellenistic» ones. We're speaking about the modern removal of ancient people's scientific results, as if history were only progressive, and science an exclusivity of modern times. (2) Starting from his times, when, together with the refined Alexandrine culture, of wich he was a bright champion, Archimedes was swept away from the military preponderance of the Romans who couldn't understand him, and so couldn't (or wouldn't) translate him, and they just limited theirselves to tell his magnificent achievements, incidentally and not in a parallel to the *Life of Marcellus,* (3) beginning, in this way, to misinterpret and betray him. Coming to our times when the sloth of Italian publishing, together with the inattention of public, make us run the risk to loose his inheritance again. As a matter of fact, in comparison to the large number of the foreign editions of the complete works of Archimedes, the only, and late, Italian edition, though with the limits pointed out by several authors, is still the one edited by Frajese, (4) who, with the few copies still existing in libraries, keeps in life, in our country, an echo of the greatest genius' thought. Nevertheless, this is also a peculiar event in a larger and recurrent phenomenon: the arrogance of power, ever and ever dedicated to the defense of the *status quo* and of the «Reason of State», which loses sight of its best brains. In ancient times, through killing the genius engaged in his country's defense, yesterday forcing the best brains of all to emigrate, to escape from «concentration camps» and from extermination, and again today forcing to a «Brain Drain», away from one's Country, who is trying to build a better future.

2. ARCHIMEDES, WHO'S THAT?

Among scientists is generally accepted his relief, for someone even *"Archimedes is the most important scientist ever existed"* [...] *The most*

certain general characteristic of european scientific tradition is that itself consists of a series of additional notes to Archimedes's work." [5] Also among common people he is generally famous, though in a more disputed and paradoxical way. Surely he came to limelight in collective imaginary, embodying well the figure of a bizarre genius, in the shoes of *Disney's Gyro Gearloose* [1] but *"What does the modern cultured man know about him?* [...] *He just remembers that he did strange things: he ran naked along the streets shouting Eureka! He dipped crowns into the water, he drew geometrical figures while he was being killed and so on. The children anecdotes* [...] *equalize Archimedes more to the legendary and the mythological characters than to other thinkers. The result is that we remember him, but we do as a legendary character, completely out of history"* [2] As for as the historical personage, only his death date is sure: 212 BC, because he died in the fault of Syracuse, in consequence of the roman siege, an epochal event related in the *Annales*. It's uncertain, on the contrary, his birth date, though it's supposed to be usually placed in 287 BC this deduction is based upon this note (reported by the Bizantine philosopher Ioannes Tzetzes about fifteen centuries! after): he died *"at an elderly age: 75 years old"*. [5] In any case, it's fairly probable that he was an old man for his times, as reported by our most reliable source about the Punic Wars, [6] whose The Siege of Syracuse is a part. As for as Archimedes as a thinker and his works, which survived fortunately and often in a *rocambolesque* way, [5] many writings are now available, but it is still lacking an updated critical edition written in italian language: *nemo profeta in patria* we could say. According to Plutarco's version, Archimedes despised every technical activity and *"lived continually enchanted by geometry,* [...] *a mermaid who was to him family and domestic, so far as to forget even to eat and to take care of his own body."* [3] It's still very doubtful the truthfulness of this opinion, that's based only upon the *"opinion of a scientist who, more than three centuries since Archimedes was dead, ascribes to him gratuitously his own Platonic inclinations."* [2] Whatever was the relief that Archimedes himself gave to the different aspects of his own multiform intelligence, or if he made any preference between his «geometrical works» and his «mechanical works», today he is considered above all as a grand mathematical, a forecaster of the infinitesimal and combinatorial calculus while, in his times, he was considered above all as an engineer, a technologist, an inventor of wonderful and frightening machines, which memory caused the oblivion of his mathematical *corpus* which few people could really understand.

3. CROWN OR WREATH

Then, starting from his death, his fame spread universally, and it was linked to amazing inventions and events, more than to his works. On the

occasion of the launching of the famous ship *Syracuse* (later called *Alexandris*), the largest vessel of ancient times, it's told that Archimedes himself, alone, launched the heaviest ship, making it slip sweetly to the sea, using a *polyspaston*, (3) that's to say a two blocks tackle with a large number of mobile pulleys. This event is linked to the famous phrase *"give me a fulcrum, and I'll lift the World up.* (7) According to another testimony, during his Egyptian visit, he invented the *cochlea*, a spire pump, that's called in fact «Archimedes's screw», able to lift water up, in a very efficient way, and with little effort. (8) Another very admired technical result was the *planetarium*, described by the consul Caius Sulpicius Gallus in one of his works, received by his colleague Marcus Claudius Marcellus, a nephew of the plunderer of Syracuse. (9) In an Arabian manuscript is contained the description of a particularly ingenious *water-clock* he invented. (10) In his treatise *The Sand Reckoner*, Archimedes himself describes the *dioptre*, an instrument used in order to measure the apparent size of Sun. Also *"the history of astronomy is a debtor to The Sand Reckoner: in this work we found, as a matter of fact, the most ancient attestation of the «heliocentric system» by Aristarcus of Samos."* (4) Amazing and frightening were, finally, the war devices (*iron claw and heat rays*), designed and used by Archimedes in order to defend Syracuse from the roman siege, burning and sinking the roman ships. The event is reported nor by Polibius, nor by Lyvy, nor Plutarch, but it is related only by several late sources as Galen, Dio Cassius and more authors among which the mentioned learned Byzantine man. The *heat rays* are here described as composed of a series of conveniently oriented flat mirrors, able to focus sun rays in a single point: the wooden roman ships to be burnt out in Syracuse sea. The structure was probably formed by at least 24 large flat mirrors, disposed in a hexagonal shape over a grate, which spun over a pole fixed to the ground: the central mirror was used to direct sun rays on the target, while the side mirrors were focused with a belt system. A history or a legend? This episode has always been considered extremely unlikely, maybe impossible, but an experiment, realized by MIT, showed for the first time it was at least practicable. (11) Anyhow, the episode that excited most common imagination is the Golden crown of Hiero II. *"It was, more correctly, a golden wreath (**στέφανος**), and not a crown, as everybody usually says.* [...] *The difference is not so accessory, because the wreath was a sacred object, and could be altered in no way"*. (8)

4. IN THE MANNER OF VITRUVIUS

Plutarch scarcely mentions this event, while Vitruvius reports it extensively. *"Hiero after gaining the royal power in Syracuse, resolved, as a consequence of his successful exploits, to place in a certain temple a*

*golden crown which he had vowed to the immortal gods. He contracted for its making at a fixed price, and weighed out a precise amount of gold to the contractor. At the appointed time the latter delivered to the king's satisfaction an exquisitely finished piece of handiwork, and it appeared that in weight the crown corresponded precisely to what the gold had weighed. But afterwards a charge was made that gold had been abstracted and an equivalent weight of silver had been added in the manufacture of the crown. Hiero, thinking it an outrage that he had been tricked, and yet not knowing how to detect the theft, requested Archimedes to consider the matter. The latter, while the case was still on his mind, happened to go to the bath, and on getting into a tub observed that the more his body sank into it the more water ran out over the tub. As this pointed out the way to explain the case in question, he jumped out of the tub and rushed home naked, crying with a loud voice that he had found what he was seeking; for he as he ran he shouted repeatedly in Greek, «**Εὕρηκα, εὕρηκα!**»* Vitruvius says. According to him, Archimedes *"he made two masses of the same weight as the crown, one of gold and the other of silver. After making them he filled a large vessel with water to the very brim, and dropped the mass of silver into it. As much water ran out as was equal in bulk to that of the silver sunk in the vessel. Then, taking out the mass, he poured back the lost quantity of water, using a pint measure, until it was level with the brim as it had been before. Thus he found the weight of silver corresponding to a definite quantity of water. After this experiment, he likewise dropped the mass of gold into the full vessel and, on taking it out and measuring as before, found that not so much water was lost, but a smaller quantity: namely, as much less as a mass of gold lacks in bulk compared to a mass of silver of the same weight. Finally, filling the vessel again and dropping the crown itself into the same quantity of water, he found that more water ran over the crown than for the mass of gold of the same weight. Hence, reasoning from the fact that more water was lost in the case of the crown than in that of the mass, he detected the mixing of silver with the gold, and made the theft of the contractor perfectly clear."* (12) According to an eminent American scientist, *"Vitruvius' method compares the volume (V_c) of a date weight (P_c) crown with the volume (V_o) of an equal weight (P_c) of gold and with the volume (V_a) of the same weight (P_c) of silver. So the relationship between the unknown weights of gold (P_o) and silver (P_a) is immediately given (knowing their sum P_c) by the following proportionality:"* (13)

$$\frac{P_o}{P_a}=\frac{V_a-V_c}{V_c-V_o} \qquad (0)$$

This «volumetric method» is described, though in a generic way, in the anonymous *Carmen de ponderibus et mensuris*, where also the «method hydrostatic» appears, about which we're going to speak largely in the next section. Here appears then *"the definition of specific gravity (maybe the first, surely the plainest among the descriptions of density contained in the most ancient Latin works)."* (13) This method is also further developed in *De insidentibus in humidum* by the Pseudo Archimedes and in *Quadripartitum numerorum* by Jean de Murs. Commenting to this work, where the Parisian astronomer applied the volumetric method to the wreath problem, an eminent studious of mechanical medieval science warned readers about this method, because on his opinion it was founded on the premise *"that the volume of mixed materials is equal to the sum of the volumes of their components,* [while] *in mixtures (alloys included) there are often volumetric variations; therefore the assumption above is often a not very careful approximation"* (13) Anyhow, Vitruvius' version soon appeared to be suspect and, in Galilee's words, *"a crude thing, far from scientific precision; and it will seem even more so to those who have read and understood the very subtle inventions of this divine man in his own writings; from which one most clearly realizes how inferior all other minds are to Archimedes's and what small hope is left to anyone of ever discovering things similar to his* [discoveries]*"*. (14) So, starting from late ancient times, and then also during the Middle Ages, it was resolved that this reconstruction «in the manner of Vitruvius» was not based at all *«On Floating Bodies»* and soon begun to spread alternative reconstructions, largely based on hydrostatic principles. Before inspecting these alternative ideas, it would be appropriate to verify if the relation (0) is really so far from the «buoyancy». It will be anyhow interesting to discuss, being it an «exact» relationship, how it could be obtained. Granted that, according to Frajese, *"there's not, in Archimedes, a term which literally corresponds to our «specific weight» or «density» but* [that] *the concept is with no doubt present* [...] *almost as in ours."* (4) Therefore, if the crown had really contained some silver mixed to gold, making a comparison in the water between it and a one with the same weight (P_c) in gold, because of the difference between the specific gravity of gold (γ_o) and the one of silver (γ_a), there'd have been a difference between the volume of crown (V_c) and the volume of gold (V_o). If we made a comparison with the volume of an equal weight in gold (P_c), the crown's volume would be:

$$V_c = \frac{P_a}{\gamma_a} + \frac{P_c - P_a}{\gamma_o} = \frac{\gamma_o \cdot P_a + \gamma_a \cdot (P_c - P_a)}{\gamma_a \cdot \gamma_o} \qquad (1)$$

where we see the unknown amount of silver $(P)_a$ while the difference between the volume of the crown and the one of the equal weight (P_c) in gold would be:

$$V_c - V_o = \frac{\gamma_o \cdot P_c + (\gamma_a - \gamma_o) \cdot P_a}{\gamma_o \cdot \gamma_a} - \frac{P_c}{\gamma_o} = \frac{(\gamma_o - \gamma_a) \cdot P_c + (\gamma_a - \gamma_o) \cdot P_a}{\gamma_a \cdot \gamma_o} \quad (2)$$

And if, instead, the crown should be compared in the water with an equal weight (P_c) in silver, its volume would be expressed by this formula:

$$V_c = \frac{P_o}{\gamma_o} + \frac{P_c - P_o}{\gamma_a} = \frac{\gamma_a \cdot P_o + \gamma_o \cdot (P_c - P_o)}{\gamma_o \cdot \gamma_a} \quad (3)$$

and we'd find a difference between the volume of the crown and the one of the equal weight (P_c) in silver, which is

$$V_a - V_c = \frac{P_c}{\gamma_a} - \frac{\gamma_a \cdot P_c + (\gamma_o - \gamma_a) \cdot P_o}{\gamma_o \cdot \gamma_a} = \frac{(\gamma_o - \gamma_a) \cdot P_c + (\gamma_a - \gamma_o) \cdot P_o}{\gamma_o \cdot \gamma_a} \quad (4)$$

where we find, instead, the unknown amount of gold (P_o). If we divide the expression (4) with the expression (2), and we simplify some terms, we find the same expression we had at the beginning (0).

$$\frac{P_o}{P_a} = \frac{V_a - V_c}{V_c - V_o} \quad (0)$$

In the process above, we can notice that, arranging the terms of the former relationship, we obtain anyway:

$$\frac{V_a - V_c}{P_o} = \frac{V_c - V_o}{P_a} \approx \frac{1}{23{,}3} \quad (5)$$

a constant value that's exactly equal to the difference in terms of weight in water between the unit weight in gold and silver, that's to say:

$$\frac{1}{23{,}3} \approx \frac{1}{\gamma_a} - \frac{1}{\gamma_o} \quad (6)$$

And this difference in weight between silver and gold is due exactly to the different «hydrostatic thrust» that they get from water, as a consequence of their specific gravity. It's so possible to gather that, at least in principle, the reconstruction, «in the manner of Vitruvius» wasn't extraneous to the spirit of Archimedes. It's necessary, anyway, to warn that all the former reconstruction is a guess, because it has been run with actual logics and notes. For a Greek scientist, in fact, the relationship between different amounts as weight and volume wouldn't have had any meaning. Furthermore, a proof by Archimedes would be based on the «theory of proportions» among amounts of the same species and would have been developed through the «method of exhaustion». Making anyhow the hypothesis (taken from an exercise of an actual manual of physics) that the wreath would weight 5 kg and that it was made of gold (70%) and of silver (30%), the differences in terms of volume would have been expressed in deciliters, and so surely detectable by Archimedes who, among his many inventions, was also an improver of a water-clock. Sure, if we consider that the wreath for a «big head» had to be much larger than the little blocks of silver and gold of equal weight, the experiment could have been realized only in a vase of 20 cm diameter. In this case the differences in level would be surely narrow, but not paltry. It is suggestive to think, *"the great discoverer of volumes and areas determined with mathematical accuracy, is here forced to deal with this problem, using the practical measurement (necessarily imprecise)* [...] *of the amount of water displaced."* (4)

5. IN THE MANNER OF PRISCIAN

We resume the words of an authoritative Italian scientist to introduce another way to solve this crown problem, here improperly called «in the manner of Priscian». *"The general process, as Vitruvius writes it down, is not considered to be the one the great scientist from Syracuse used, and it's completely different the story we can read in a poem which was for a long time ascribed to Priscian; in this freely translated version we read that Archimedes took a pound of gold, and one of silver, and he put them on the plates of a balance, where they were of course in equilibrium; then he dipped them into the water, but as they lost their equilibrium for the overflow of gold, he decided to add some weight of silver, for example three drachms, to restore it, and from this he noticed that one pound and three dramms of silver equalized one pound of gold when they were in the water. After this, he weighted the crown, which had to be completely made of gold, and when he discovered that it weighted, for instance, six pounds, he took six more pounds of silver, and put them on the balance together with the first ones, dipping all of them into the water. If the crown had been*

really completely made of gold, eighteen drachms of silver, added to the former six, should have been enough to put the plates in equilibrium, but any drachm less than the eighteen proofed the existence in the crown of one third of pound of silver" [15] The process mentioned above was related in the anonymous *Carmen de ponderibus* dated V century AC, which is present in several codes by the Latin grammarian Priscian. [13] In a section of the short poem we find two methods to solve the wreath problem, the former is essentially based on the principle by Archimedes. The technique is symbolically expressed by the following formula:

$$\frac{P_o}{P_a} = \frac{\sigma_a - \sigma_c}{\sigma_c - \sigma_o} \qquad (7)$$

where can be noticed the losses of weight in water respectively of the crown (σ_c), of an equal weight in gold (σ_o) of the same weight in silver (σ_a), caused by the different push that different objects get from the water. The first «modern» presentation on this method, entirely based on the laws of lever and of floating, is related in the book *Magia naturalis* published in London by the Italian scholar Giambattista Della Porta. About in the same years also Galileo strongly criticized the reconstruction made by Vitruvius and, in his juvenile work called «La bilancetta», exposed *"a method came to my mind which very accurately solves our problem. I think it probable that this method is the same that Archimedes followed, since, besides being very accurate, it is based on demonstrations found by Archimedes himself."* [14] Also the author of a beautiful children's book dedicated to the genius from Syracuse [9] opts for the same version, explaining later the reasons which brought him to this choice: *"(1) I don't think the volumetric measure can be performed with great precision.* [...] *If the wreath has a complex structure (that's to say a high ratio surface/volume, how it seems to be likely), it needs to be dipped in a vase which has a much larger volume than hers, and consequently with a wide surface, which makes very small the height difference to be compared and consequently increases more and more the chance of a mistake.* [...] *In any case, in order to compare the overflowed waters, it would be better to weigh them. Why, then, don't we use a weight approach? (The mistake in measuring here, unlike volumetric measures, doesn't depend on the dimension of the vase where the objects are dipped; this seems to me the point!)* [...] *(3) Vitruvius presents the episode when he writes about Archimedes hydrostatics. The volume measures don't have, in this case, any connection with the object of discussion, while the measures done with the «hydrostatic balance» would be a perfect introduction to it. All that lets us think that Vitruvius source exposed a measure based on a hydrostatic push*

and that Vitruvius thought to «simplify» the argument (ignoring the precision of the measure itself).” (16) If we wanted, anyhow, subject the same wreath of the former example (5 kg) to a control, using the above-mentioned method, we could have noticed a difference in weight of about 65 g. A clue definitely more important and much easier to grasp.

6. CONCLUSION

And finally, «pycnometer», «hydrostatic balance»? Currently this episode goes on circulating in both versions and with a certain confusion, as if they were antithetical one another. It’s instead proved that both reconstructions are based on the same principles: the former measuring volume differences, the latter weight ones. In short, let’s consider that between the two wreaths really found, similar to Hiero’s and belonging to the same age, none is heavier than 1 kg. The golden wreath of Verginia, in Macedonia (IV century BC), for example, is just 700 g heavy. If we made a comparison between a 1 kg wreath, and a sample made of gold of equal weight, the «volumetric method» would show a difference of half a millimeter, while the «hydrostatic method» a difference of 13 g. This comparison allows us to say that the «hydrostatic method» is more careful and the easiest to be carried out. It seems therefore the most plausible but there's no chance to decide surely which was the process Archimedes really used, supposing that the episode called «The Golden Crown» is really reliable. This writer likes to think that this anecdote really happened and that, as many time before (for example the *Quadrature of the Parabola,* which was obtained and with a geometrical process, and with a mechanical one) Archimedes came to this result in a double way: the first time with a «volumetric method», the second one with a «hydrostatic method». There are no proofs of this, but it’s nice to think that a careful scientist can find, one day, under some miniature, the traces of another work, which, in the Doric Greek of Archimedes, reveals us some other aspects of his wonderful genius.

ACKNOWLEDGEMENTS

It’s due for me to thank all the authors I quoted in this discussion: without their teaching, to which I made a continuous and textual reference, these pages wouldn’t have been written. To all of them, as to anyone I should accidentally have forgotten, I’m enormously grateful. A special thanks to Lucio Russo, who made me share his point of view and to Marco Ceccarelli, who encouraged me a lot to publish these notes. Obviously, any eventual but unavoidable mistake is due just to me and to my lack of skill. Finally, I want to acknowledge my partner, our children’s mother, to have patiently stood my long immersions in books.

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