THE MECHANICS OF ARCHIMEDES TOWARDS MODERN MECHANISM DESIGN

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ABSTRACT In this paper a relevant contribution of Archimedes is outlined as related to his developments in mechanics with application to mechanism design with a modern vision. He developed theoretical advances that were motivated and applied to practical problems with an enthusiastic behaviour with a modern spirit that can be summarized in his motto 'Give me a place to stand and I will move the earth'.

1. INTRODUCTION

Since Renaissance Archimedes and his mechanics have been reconsidered together with a new attention to Greek-Roman machine designs with the aim and result to develop an early approach for modern theory of mechanisms, as outlined in [1].

The works of Archimedes, mainly in the aspects of mechanism design, has been rediscovered and studied during Renaissance, as for example in [2-5], up to be used as fundamental background for the first developments of early TMM (Theory of Mechanisms and Machines) by Guidobaldo Del Monte, [6], and Galileo Galilei, [7]. Even at the beginning of the modern TMM in 19th century Archimedes' contribution was recognized in developing basic conceptual elements, like for example in [9, 10]. The modernity of Archimedes in MMS can be today still advised in his approach of classification for the variety of mechanism designs as function of a unique principle in the operation mechanics, as indicated in [11].

The mayor Archimedes' contributions in the field of modern MMS can be recognized in:

– identification and analysis of basic elements of machines and mechanisms, as pointed out in [8]
– analysis of machinery operation as function of a unique functionality of levers

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– application of theory to successful practical designs that since his time gave dignity of discipline and profession to machine design
– enthusiasm and optimism in mechanism design in developing technology for enhancing society and quality of life.

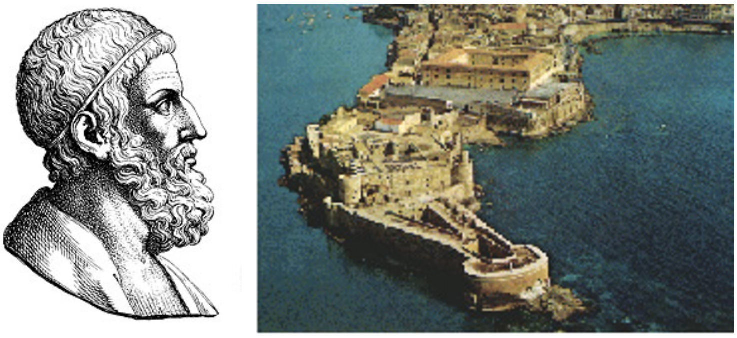
From historical viewpoint, it is never too much stressed the achievement in defining and using an early concept of pair of force and its equilibrium rule for the design and operation of mechanisms in machines.

In addition, the figure of Archimedes and his work have been investigated since Antiquity and still today they are of great interest in studies and investigations that are reported in publications and encyclopaedias, like for example in [12-20], and in congress discussions, like for example in [21, 22].

In this paper, attention is addressed in discussing the aspects and interpretation of legacy of Archimedes' work and personality in the modern MMS (Mechanism and Machine Science) not only for a historical assessment by also as an inspiration for future achievements.

2. ARCHIMEDES AND HIS WORKS

Archimedes (in classical Greek: Ἀρχιμήδης) (287 a. C.–212 a. C.), Fig. 1 a), was a Greek philosopher in the classical term as being mathematician, physicist, astronomer, inventor, and engineer, who lived in the core environment of Magna Grecia as Siracusa was, Fig. 1b).



a) b)

Fig. 1. a) Portrait of Archimedes; b) the peninsula of Ortygia as core of the ancient Siracusa with the Maniace Castle at its end.

Nevertheless little is known of his life. Very probably during his youth he spent a period of study in Alexandria, Egypt, where he had the chance to know Conon of Samos and Eratosthenes of Cyrene, since in his written works he cited them as friends. Archimedes was killed by a roman soldier, when a long siege of Siracusa during the Second Punic war (214–212 a. C.) was ended, although Marco Claudio Marcelo, the Roman commander, ordered to safe him. Cicero told that he saw the Archimedes' tomb where a sphere was drawn inside a cylinder, as an indication of the main achievement that Archimedes recognized to himself.

In general, Archimedes is reputed for his contributions in Mechanics and Hydrostatics. He is also a reference personality in the developments of Mathematics because of his calculations and theorems for figure volumes and the number pi. Since Antiquity, he is also considered a unique inventor of innovative machines, as applying his mathematical results to practical problems. The most celebrated ones are the screw pump and the war machines that Syracusans used against the Romans.

However, although the inventions of Archimedes were known over the time, his written works were forgotten even in the last part of Antiquity and completely ignored during the Middle Ages. A first attempt to collect all his works was made by Isidor of Milete (c. 530 d. C.). During Renaissance those works were reconsidered, like in the Latin translation by Jacobus Cremonensis in 1458 from a collection made by Eutocius in the 6th century. Only in 1906 a palimpsest was discovered with seven works by Archimedes in a better version than previously known.

The works of Archimedes, that are today known even through a interpreted text, like for example in [20], are:

- On the Equilibrium of Planes
- Measurement of a Circle
- On the Sphere and Cylinder
- On Spirals
- On Conoids and Spheroids
- Quadrature of the Parabola
- On Floating Bodies
- Stomachion
- Cattle Problem
- The Sand Reckoner
- The Method of Mechanical Problems

In particular in the two-volume *On the Equilibrium of Planes* Archimedes introduced the Law of Levers by stating, "Magnitudes are in equilibrium at distances reciprocally proportional to their weights." Archimedes used it to calculate the areas and centers of gravity of several geometric figures.

His short work *Measurement of a Circle* consists of three propositions by which he computed number pi by approximations.

The treatise *On Spirals* deals with study of curves and particularly spirals.

In *On the Sphere and Cylinder* he computed volumes and areas of spheres and cylinders. A sculptured sphere and cylinder were placed on the tomb of Archimedes at his request, to remind of what he considered as his best achievement. In the fact that a sphere has a volume and surface area two-thirds that of a cylinder.

In the treatise *On Conoids and Spheroids* Archimedes calculated the areas and volumes of cones, spheres, and paraboloids.

In Quadrature of the Parabola he calculated the area of geometrical figures by using the concept of a geometrical series.

In his work *On Floating Bodies* Archimedes introduced the law of the equilibrium of the fluids and used it to demonstrate the shape of water volumes. Then he calculated the equilibrium positions of sections of paraboloids floating in the water. Archimedes' principle of buoyancy is expressed as: Any body wholly or partially immersed in a fluid experiences an upthrust equal to, but opposite in sense to, the weight of the fluid displaced.

Stomachion is a study for dissection of a puzzle by computing the area of pieces that can be assembled to obtain a square.

In the *Cattle Problem*, Archimedes attached the problem to count the numbers of cattle in the Herd of the Sun by solving a number of simultaneous Diophantine equations.

In *The Sand Reckoner*, Archimedes calculated the number of grains of sand that can be inside the universe, along with considerations on Astronomy.

The Method of Mechanical Problems introduced concepts of infinitesimal calculus by using geometric description of how to calculate areas and volumes by summing the small parts of a partitioning of the figures.

Although is can be thought that Archimedes did not indeed invent the lever and its use, as many researchers point out, nevertheless he was the first in giving a rigorous explanation and formulation of the mechanics equilibrium law under which it operates. According to Pappus of Alexandria, while discussing the lever mechanics Archimedes commented the famous sentence 'Give me a place to stand and I will move the earth' (in Greek: δῶς μοι πᾶ στῶ καὶ τὰν γᾶν κινάσω). This sentence can be considered also a summarized thought on the optimism that Archimedes relied in the mechanism design.

Plutarch described how Archimedes designed a capstan system as based on the lever operation to help the harbour workers to lift heavy loads in the ships. Furthermore, an anecdote reports how Archimedes designed and built a capstan system to move a large ship from the harbour in order to gain a cultural challenge that the tyrant Hieron proposed to him. Thus, by means of a system with several capstans and pulley systems, while seated on a chair, Archimedes could lift the ship with great surprise of the people present.

Additionally, Archimedes is credited to have increased power and precision of catapult machines used during the Punic war against the Romans. He also invented an odometer in a chariot by counting the miles with the number of small balls collected after a certain distance.

Pappus of Alexandria mentioned that Archimedes wrote a treatise on how to build mechanisms for planetary models and for the construction of spheres, an important account of the technological skills of the time. Several such devices have been recently discovered, like for example the Antikethyra mechanism that is equipped with several differential gears, as a proof of technical achievements thought of as belonging to modern times only.

The most relevant invention of Archimedes can be considered the screw pump, still known as *Archimedes screw*. Although there is some evidence that the pump already existed in some forms in Ancient Egypt, it is important that Archimedes based its design on theoretical principles, permitting a rational use of it.

3. MECHANICS OF MACHINERY AND MECHANISM DESIGN

The contribution of Archimedes in Mechanics of Machinery towards a development of an early Theory of Mechanisms can be recognized in the following aspects:

- a theoretical study with a mathematical approach that was useful both for analysis and design of mechanisms and machines;
- interest for an experimental activity in the theory and application of the mechanics of machinery;
- application to engineering with invention of new machines.

In addition, in all his works Archimedes expressed a strong believe and optimism in a practical application of science as demonstrated by the many inventions that are attributed to him. This is summarized in his motto *‘Give me a place to stand on, and I will move the Earth’*, as it was ascribed to him since Antiquity and it was considered the basis for the study of the mechanics of the machinery in Renaissance, with great emphasis as for example in the cover of the book by Guidobaldo Del Monte in 1577.

The theoretical mathematical approach by Archimedes is developed in depth but not only for pure mathematical interest. The study and formulation, that can be even deduced in modern terms, were deduced for their application both in explaining the attached problems and in providing suitable solutions for them. For this reason, the Archimedes's treatises can be considered simultaneously the basis and complete of his activity as machine inventor and builder. This permits to link the theoretical activity and the mathematician approach to his practice of experiments in mechanics as well as to his machine ingenuity. As it can be understood from the treatises, Archimedes considered as the basis of his study the observation of natural phenomena and the operation of existing systems. Once both the generalities and peculiarities of real events were understood, Archimedes studied and proposed a logic reasoning and theoretical development that he could use not only to forecast operations and events but even to deduce new ideas and possibilities in structures and functionalities of machines. From this perspective it is quite amazing how Archimedes was able to identify and formulate a unique principle in order to explain the operation of a large variety of machines that were already existing at his time. Thus, it is not only the identification of elementary machines (or mechanisms) by which it is possible to classify all the machine components, but even the identification of the lever mechanics (better known as the law of levers) as operation principle for any mechanism and machine. Through this study, Archimedes expressed first the equilibrium of rigid bodies as related to the momentum of forces.

Relevant is also how he examined the mechanical phenomenon and its application in other existing devices by extrapolating the specific mechanics from the general principle and formulation. In Egypt, there were scales for measuring and comparing weights as it is also documented in several documents and even artistic representations like the one in Fig. 2, from which it is evident the lever mechanics was used in somehow conscious way. Most probably, Archimedes could know and appreciate the Egyptian technique in scales of several types. But he was the one who examined and formulated the mechanics of levers with deep insights and generality in his work 'On the Equilibrium of Planes' by assuming seven postulates that he identified for a careful analysis of experimental phenomena even in the diary experience and with other systems. Once assumed the natural evidence of the seven postulates, also in terms of determination of the center of gravity of rigid bodies, Archimedes formulated the static equilibrium as due to conditions related to the weight and distance of the body from a point, about which the body can rotate. Thus, he indicated the equilibrium as function of torques of forces that are due to the weights from both sides of a lever. A graphical representation of the analysis process from observation is reported in the scheme in Fig. 3a) that can lead to the modern mechanical-mathematical model in Fig. 3b). Fig. 3a) is taken from the book by Del Monte and Fig. 3b) is a modern interpretation of the law of levers by referring to the parameters used by Archimedes. The law of levers was used by Archimedes, but even more important is that it was extended in the books that rediscovered the Archimedes's works during Renaissance, as mainly in the works by Del Monte and Galilei with the aim to determine a rational classification of the operation of machines.

Fig. 2. An artistic representation of an Egyptian scales in a tomb of 1250 B.C. in Thebes.

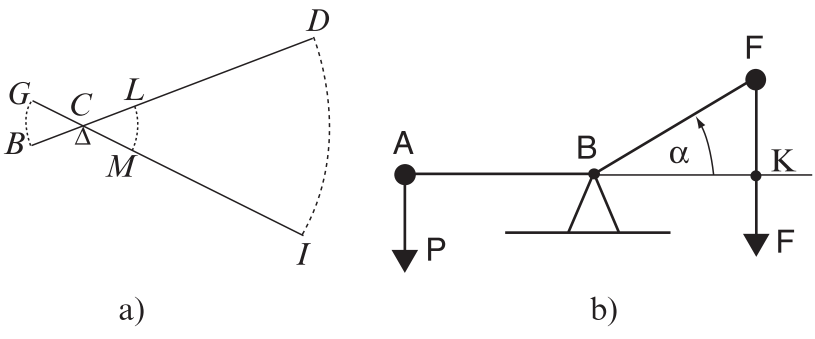


Fig. 3. The law of levers by Archimedes as viewed by Del Monte and Galilei: a) a scheme; b) a modern interpretation.

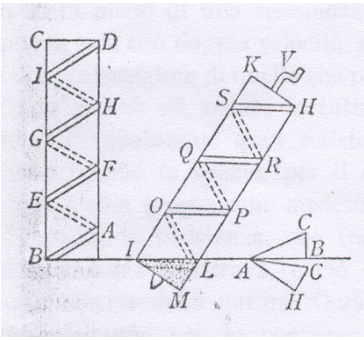
The law of levers was a great achievement as introduced and used by Archimedes both from theoretical and practical viewpoints in rational mechanics and machinery mechanics. Relevant is the introduction of the concept and use of the pair of a force, even if not yet in a clear form, to explain the static equilibrium but even dynamic rotational operation towards it. Archimedes used the law of levers in a descriptive form but with mathematical means both to explain the machine operation as well as to conceive new inventions.

This was the case of the screw of Archimedes, Fig. 4, as Del Monte and Galilei described clearly as based on the law of levers in agreement with the scheme in Fig. 4a). The invention of the screw of Archimedes, as used extensively (even today) and widespread as fluid pump, shows also the experimental approach that Archimedes uses in developing new machines for practical applications still by using the law of levers. In general, it is used to elevate water, floor, or cereals. It is based on a screw that rotates within a hollow cylinder that is installed on an inclined plane so that the water in the bottom is elevated to the top of the screw.

The scheme in Fig. 4 b) is an explanation of the pumping of the screw as an alternating inclined plane operating as mobile lever. This explanation with experimental and practical insights has attracted the interest of scientists, even for teaching purposes. Fig. 4b) shows an experimental setup of the 18th century for laboratory practices to teach the screw operation.

Indeed the study of the screw as in Fig. 4a) shows a clear example how Archimedes (and later Del Monte and Galilei) used the law of levers as a general principle to understand and explain the operation of a large variety of machines, also with the aim to provide a simple means of study and comparison of the operation and design of machines. This practical vision permitted Archimedes, who was motivated by the needs of his time and local circumstances (as even requested by the tyrant of Siracusa), to solve practical problems and to conceive new machine inventions. Thus, the studies of Archimedes were fundamental for the development of science and its engineering application in many fields. Relevant is the case of the Hydraulics in which Archimedes defined the law of buoyancy, that he used successfully in the design of large ships, as ascribed to him like for example in [22].

In this paper the focus is on Archimedes's ingenuity in machine design. Several machines are ascribed to Archimedes like chariot odometers, cranes, screw, war machines, most of which were later perfected during the Roman Empire. All these inventions are based on the law of levers and on experiential activity from a previous mathematical study.



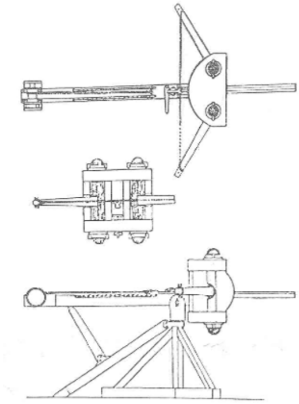
a)

b)

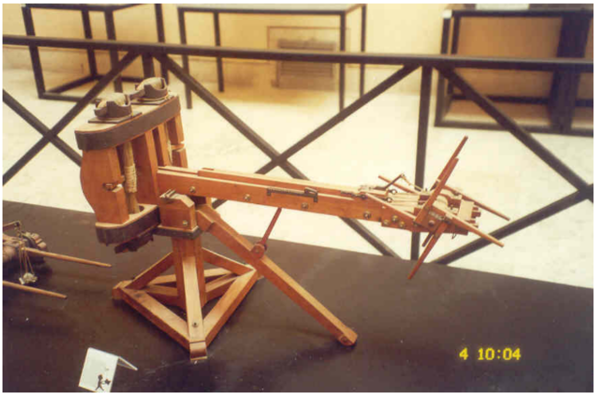
Fig. 4. The screw design by Archimedes: a) according to Del Monte and Galilei; b) as a machine model for lab activity in 18th century.

Archimedes designed machines by merging mechanisms with pulley systems in cranes of different sizes. Some of them were used to build the Siracusa fortress and some others for moving ships in maintenance works, as examples of versatility of the design ingenuity of Archimedes.

In the field of war machines, an example is shown in Fig. 5 where a catapult (as correctly named in ancient Greek) is illustrated for launching arrows to long distance. As shown in the scheme of Fig. 5a), the structure of this war machine is a combination of different components in an anti litteram mechatronic design, namely gears, prismatic slider, pulleys, and flexible bars. A capstan is used to tension the arc, made of flexible elements, to accumulate the elastic energy as in the launching motor for the two rods fixed in the machine frame. The platform on the prismatic guide allows a practical feeding of the arrows and a suitable moving body for the launching. An improved mechanism has been found that permitted even a repeated launching at high rates like a riffle, as pointed out in [23]. In the reconstruction in Fig. 5b) the design is emphasized as made of materials of common use in antiquity, namely woods, cables of natural fibres, hairs for the torsion elastic motor (even human hairs), so that they permitted a rapid construction, usage, and maintenance of the war machine, as a result of a careful attention of the technology of available materials in a war campaign. This again shows the practical vision of Archimedes in designing and developing machines by combining mechanical theory, experimental experience, knowledge of technology with an approach that is today typical of modern MMS (Mechanism and Machine Science).



a)



b)

Fig. 5. Roman catapult as an invention of Archimedes: a) a scheme; b) reconstruction in the Museum of Roman Culture in Rome.

4. CONCLUSIONS

In this paper, the modernity of the personality and work of Archimedes has been discussed as related to the mechanics of machinery, viewed from the engineering viewpoint of MMS. The modern relevance of Archimedes contributions can be recognized in a determination of theoretical bases, even through experimental activity, for a formulation of the mechanics of machines that was useful not only for a general theory but even for practical inventions of new machines of durable interest. In addition, his enthusiastic approach to apply science to practical engineering reveals Archimedes as a modern source of inspiration for future MMS researchers.

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