History of Mechanism and Machine Science 11

Stephanos A. Paipetis
Marco Ceccarelli
Editors

The Genius of Archimedes – 23 Centuries of Influence on Mathematics, Science and Engineering

Proceedings of an International Conference held at Syracuse, Italy, June 8–10, 2010

Springer

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HISTORY OF MECHANISM AND MACHINE SCIENCE
Volume 11

Series Editor
MARCO CECCARELLI

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PREFACE

The idea of a Conference in Syracuse to honour Archimedes, one of the greatest figures in Science and Technology of all ages, was born during a Meeting in Patras, Greece, dealing with the cultural interaction between Western Greece and Southern Italy through History, organized by the Western Greece Region within the frame of a EU Interreg project in cooperation with several Greek and Italian institutions. Part of the Meeting was devoted to Archimedes as the representative figure of the common scientific tradition of Greece and Italy. Many reknown specialists attended the Meeting, but many more, who were unable to attend, expressed the wish that a respective Conference be organized in Syracuse. The present editors assumed the task of making this idea a reality by co-chairing a World Conference on 'The Genius of Archimedes (23 Centuries of Influence on the Fields of Mathematics, Science, and Engineering)', which was held in Syracuse, Italy, 8–10 June 2010, celebrate the 23th century anniversary of Archimedes' birth.

The Conference was aiming at bringing together researchers, scholars and students from the broad ranges of disciplines referring to the History of Science and Technology, Mathematics, Mechanics, and Engineering, in a unique multidisciplinary forum demonstrating the sequence, progression, or continuation of Archimedean influence from ancient times to modern era.

In fact, most the authors of the contributed papers are experts in different topics that usually are far from each other. This has been, indeed, a challenge: convincing technical experts and historian to go further in-depth into the background of their topics of expertise with both technical and historical views to Archimedes' legacy.

We have received a very positive response, as can be seen by the fact that these Proceedings contain contributions by authors from all around the world. Out of about 50 papers submitted, after thorough review, about 35 papers were accepted both for presentation and publication in the Proceedings. They include topics drawn from the works of Archimedes, such as Hydrostatics, Mechanics, Mathematical Physics, Integral Calculus, Ancient Machines & Mechanisms, History of Mathematics & Machines, Teaching of Archimedean Principles, Pycnometry, Archimedean Legends and others. Also, because of the location of the Conference, a special session was devotyed to Syracuse at the time of Archimedes. The figure on the cover is taken from the the book 'Mechanicorum Liber' by Guidobaldo Del Monte, published in Pisa on 1575 and represents the lever law of Archimedes as lifting the world through knowledge.

The world-wide participation to the Conference indicates also that Archimedes' works are still of interest everywhere and, indeed, an in-depth knowledge of this glorious past can be a great source of inspiration in developing the present and in shaping the future with new ideas in teaching, research, and technological applications.

We believe that a reader will take advantage of the papers in these Proceedings with further satisfaction and motivation for her or his work (historical or not). These papers cover a wide field of the History of Science and Mechanical Engineering.

We would like to express my grateful thanks to the members of the Local Organizing Committee of the Conference and to the members of the Steering Committee for co-operating enthusiastically for the success of this initiative. We are grateful to the authors of the articles for their valuable contributions and for preparing their manuscripts on time, and to the reviewers for the time and effort they spent evaluating the papers. A special thankful mention is due to the sponsors of the Conference: From the Greek part, the Western Greece Region, the University of Patras, the GEFYRA SA, the Company that built and runs the famous Rion-Antirrion Bridge in Patras, Institute of Culture and Quality of Life and last but not least the e-RDA Innovation Center, that offered all the necessary support in the informatics field. From the Italian part, the City of Syracuse, the University of Cassino, the School of Architecture of Catania University, Soprintendenza dei Beni Culturali e Archeologici di Siracusa, as well as IFToMM the International Federation for the Promotion of Mechanism and Machine Science, and the European Society for the History of Science.

The Editors are grateful to their families for their patience and understanding, without which the organization of such a task might be impossible. In particular, the first of us (M.C.), mainly responsible for the preparation of the present volume, wishes to thank his wife Brunella, daughters Elisa and Sofia, and young son Raffaele for their encouragement and support.

Cassino (Italy) and Patras (Greece): January 2010

Marco Ceccarelli, Stephanos A. Paipetis, Editors
Co-Chairmen for Archimedes 2010 Conference

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AN ARCHIMEDEAN RESEARCH THEME: THE CALCULATION OF THE VOLUME OF CYLINDRICAL GROINS

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ABSTRACT Starting from Archimedes' method for calculating the volume of cylindrical wedges, I want to get to describe a method of 18th century for cilindrical groins thought by Girolamo Settimo and Nicolò di Martino. Several mathematicians studied the measurement of wedges, by applying notions of infinitesimal and integral calculus; in particular I examinated Settimo's *Treatise on cylindrical groins*, where the author solved several problems by means of integrals.

KEYWORDS: Wedge, cylindrical groin, Archimedes' method, G. Settimo.

1. INTRODUCTION

"Cylindrical groins" are general cases of cylindrical wedge, where the base of the cylinder can be an ellipse, a parabola or a hyperbole. In the Eighteenth century, several mathematicians studied the measurement of vault and cylindrical groins by means of infinitesimal and integral calculus. Also in the Kingdom of Naples, the study of these surfaces was a topical subject until the Nineteenth century at least because a lot of public buildings were covered with vaults of various kinds: mathematicians tried to give answers to requirements of the civil society who vice versa submitted concrete questions that stimulated the creation of new procedures for extending the theoretical system.

Archimedes studied the calculation of the volume of a cylindrical wedge, a result that reappears as theorem XVII of *The Method*:

If in a right prism with a parallelogram base a cylinder be inscribed which has its bases in the opposite parallelograms [in fact squares], and its sides [i.e., four generators] on the remaining planes (faces) of the

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prism, and if through the centre of the circle which is the base of the cylinder and (through) one side of the square in the plane opposite to it a plane be drawn, the plane so drawn will cut off from the cylinder a segment which is bounded by two planes, and the surface of the cylinder, one of the two planes being the plane which has been drawn and the other the plane in which the base of the cylinder is, and the surface being that which is between the said planes; and the segment cut off from the cylinder is one sixth part of the whole prism.

The method that Archimedes used for proving his theorem consist of comparing the area or volume of a figure for which he knew the total mass and the location of the centre of mass with the area or volume of another figure he did not know anything about. He divided both figures into infinitely many slices of infinitesimal width, and he balanced each slice of one figure against a corresponding slice of the second figure on a lever.

Using this method, Archimedes was able to solve several problems that would now be treated by integral and infinitesimal calculus.

The Palermitan mathematician Girolamo Settimo got together a part of his studies about the theory of vaults in his *Trattato delle unghiette cilindriche* (*Treatise on cylindrical groins*), that he wrote in 1750 about but he never published; here the author discussed and resolved four problems on cylindrical groins.

In his treatise, Settimo gave a significant generalization of the notion of groin and used the actual theory of infinitesimal calculus. Indeed, every one of these problems was concluded with integrals that were reduced to more simple integrals by means of decompositions in partial sums.

2. HOW ARCHIMEDES CALCULATED THE VOLUMES OF CYLINDRICAL WEDGES

The calculation of the volume of cylindrical wedge appears as theorem XVII of Archimedes' *The Method.* It works as follows: starting from a cylinder inscribed within a prism, let us construct a wedge following the statement of Archimedes' theorem and then let us cut the prism with a plane that is perpendicular to the diameter *MN* (see fig. 1.a). The section obtained is the rectangle *BAEF* (see fig. 1.b), where *FH'* is the intersection of this new plane with the plane generating the wedge, *HH'=h* is the height of the cylinder and *DC* is the perpendicular to *HH'* passing through its midpoint.

Then let us cut the prism with another plane passing through *DC* (see fig. 2). The section with the prism is the square *MNYZ*, while the section with the cylinder is the circle *PRQR'*. Besides, *KL* is the intersection between the two new planes that we constructed.

Let us draw a segment *IJ* parallel to *LK* and construct a plane through *IJ* and perpendicular to *RR'*; this plane meets the cylinder in the rectangle *S'T'I'T'* and the wedge in the rectangle *S'T'ST*, as it is possible to see in the fig. 3:

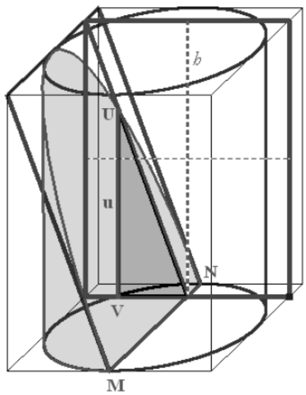

Fig. 1.a. Construction of the wedge.

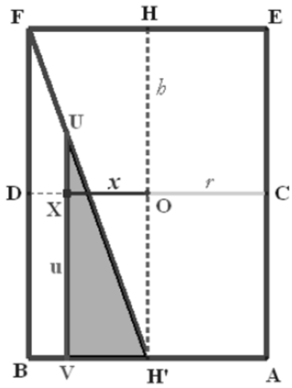

Fig. 1.b. Section of the cylinder with a plane perpendicular to the diameter MN.

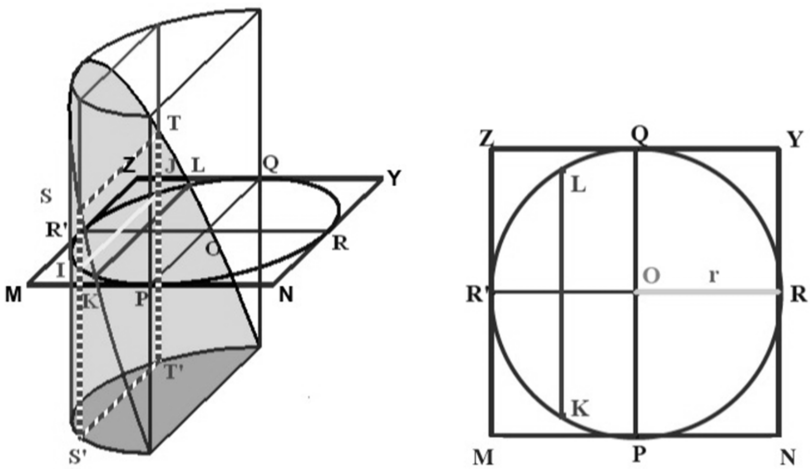

Fig. 2. Section of the cylinder with a plane passing through DC.

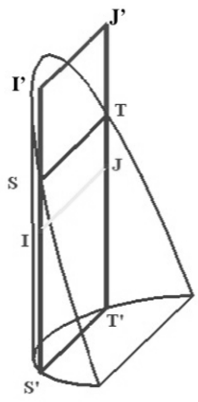

Fig. 3. Construction of the wedge.

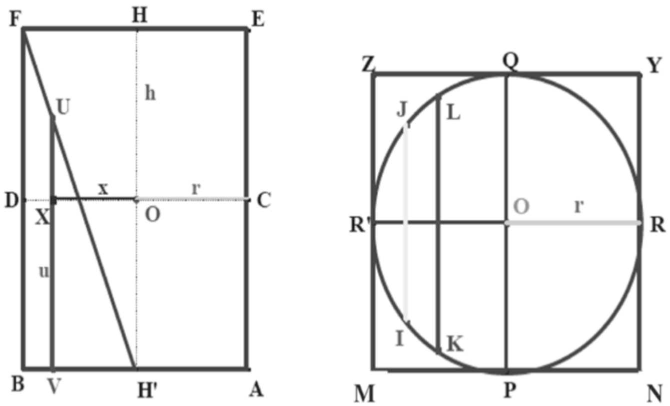

Fig. 4. Sections of the wedge.

Because *OH'* and *VU* are parallel lines cut by the two transversals *DO* and *H'F*, we have

$$DO : DX = H'B : H'V = BF : UV \text{ (see fig. 4)}$$

where $BF=h$ and *UV* is the height, u, of the rectangle *S'T'ST*. Therefore

$$DO : DX = H'B : H'V = BF : UV = h : u = (h \bullet IJ) : (u \bullet IJ).$$

Besides $H'B=OD$ (that is r) and $H'V=OX$ (that is x). Therefore

$$(FB \bullet IJ) : (UV \bullet IJ) = r : x, \text{ and } (FB \bullet IJ) \bullet x = (UV \bullet IJ) \bullet r.$$

Then Archimedes thinks the segment *CD* as lever with fulcrum in *O*; he transposes the rectangle *UV•IJ* at the right of the lever with arm *r* and the rectangle *FB•IJ* at the left with the arm *x*. He says that it is possible to consider another segment parallel to *LK*, instead of *IJ* and the same argument is valid; therefore, the union of any rectangle like *S'T'ST* with arm *r* builds the wedge and the union of any rectangle like *S'T'I'T'* with arm *x* builds the half-cylinder.

Then Archimedes proceeds with similar arguments in order to proof completely his theorem.

Perhaps it is important to clarify that Archimedes works with right cylinders that have defined height and a circle as the base.

3. GIROLAMO SETTIMO AND HIS HISTORICAL CONTEST

Girolamo Settimo was born in Sicily in 1706 and studied in Palermo and in Bologna with Gabriele Manfredi (1681–1761). Niccolò De Martino (1701–1769) was born near Naples and was mathematician, and a diplomat. He was also one of the main exponents of the skilful group of Italian Newtonians, whereas the Newtonianism was diffused in the Kingdom of Naples. Settimo and De Martino met each other in Spain in 1740 and as a consequence of this occasion, when Settimo came back to Palermo, he began an epistolar relationship with Niccolò. Their correspondence collects 62 letters of De Martino and two draft letters of Settimo; its peculiar mathematical subjects concern with methods to integrate fractional functions, resolutions of equations of any degree, method to deduce an equation of one variable from a system of two equations of two unknown quantities, methods to measure surface and volume of vaults[1].

One of the most important arguments in the correspondence is also the publication of a book of Settimo who asked De Martino to publish in Naples his mathematical work: *Treatise on cylindrical groins* that would have to contain the treatise *Sulla misura delle Volte* ("On the measure of vaults"). In order to publish his book, Settimo decided to improve his knowledge of infinitesimal calculus and he needed to consult De Martino about this argument.

In his treatise, Settimo discussed and resolved four problems: calculus of areas, volumes, centre of gravity relative to area, centre of gravity relative to volume of cylindrical groins. The examined manuscript of

[1] N. Palladino - A.M. Mercurio - F. Palladino, *La corrispondenza epistolare Niccolò de Martino-Girolamo Settimo. Con un saggio sull'inedito Trattato delle Unghiette Cilindriche di Settimo*, Firenze, Olschki, 2008.

Settimo, *Treatise on cylindrical groins,* is now stored at Library of *Società Siciliana di Storia Patria* in Palermo (Italy), *M.ss. Fitalia*, and it is included in the volume *Miscellanee Matematiche di Geronimo Settimo (M.SS. del sec. XVIII).*

4. GROINS IN SETTIMO'S TREATRISE

Settimo's *Treatise on cylindrical groins* relates four *Problems*. The author introduces every problem by *Definizioni*, *Corollari*, *Scolii* and *Avvertimenti*; adding also *Scolii*, *Corollari* and *Examples* after the discussion of it. On the whole, Settimo subdivides his manuscript into 353 *articles, Fig. 5.* The problems to solve are:

Problem 1: to determine the volume of a cylindrical groin;
Problem 2: to determine the area of the lateral surface of a cylindrical groin;
Problem 3: to determine the center of gravity relative to the solidity of a cylindrical groin;
Problem 4: to determine the center of gravity relative to the lateral surface of a cylindrical groin.

Settimo defines cylindrical groins as follows:

"*If any cylinder is cut by a plane which intersects both its axis and its base, the part of the cylinder remaining on the base is called a cylindrical groin*".

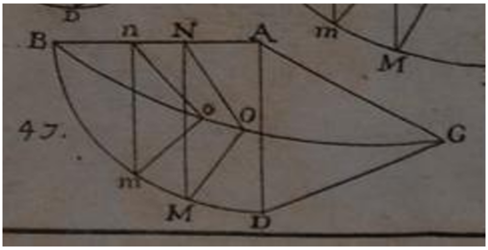

Fig. 5. Original picture by De Martino of cylindrical groin (in Elementi della Geometria così piana come solida coll'aggiunta di un breve trattato delle Sezioni Coniche, 1768).

Settimo concludes each one of these problems with integrals that are reduced to more simple integrals by means of decompositions in partial sums, solvable by means of elliptical functions, or elementary functions (polynomials, logarithms, circular arcs).

Settimo and de Martino had consulted also Euler to solve many integrals by means of logarithms and circular arcs[2].

Let us examine now how Settimo solved his first problem, *"How to determine volume of cylindrical groin"*.

He starts to build a groin as follows: let *AM* be a generic curve, that has the line *AB* as its axis of symmetry; on this plane figure he raises a cylinder; then on *AB* he drew a plane parallel to the axis of the cylinder; this plane is perpendicular to the plane of the basis (see fig. 6).

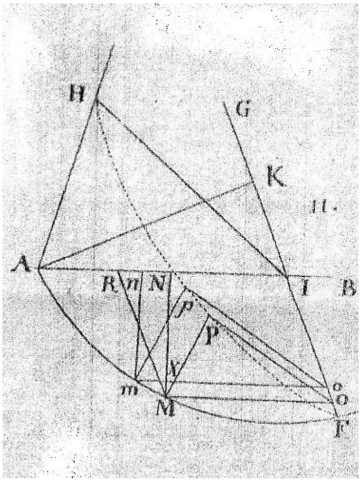

Fig. 6. Original picture of groin by Settimo.

Let *AH* be the intersection between this plane and the cylinder; *BAH* is the angle that indicates obliqueness of the cylinder; the perpendicular line from *H* to the cylinder's basis falls on the line *AB*.

Let's cut the cylinder through the plane *FHG*, that intersects the plane of basis in the line *FG*. Since we formed the groin *FAGH*, the line *FG* is the directrix line of our groin. If *FG* is oblique, or perpendicular, or parallel to *AB*, then the groin *FAGH* is "obliqua" (oblique), or "diretta" (direct), or "laterale" (lateral). To solve the problem:

1. firstly, Settimo supposes that the directrix *FG* intersects *AB* obliquely;
2. then, he supposes that *FG* intersects *AB* forming right angles;
3. finally, he supposes that *FG* is parallel to *AB*.

---

[2] In particular see L. Euler, *Introductio in analysin infinitorum*, Lausannae, Apud Marcum-Michaelem Bousquet & Socios, 1748 and G. Ferraro - F. Palladino, *Il calcolo sublime di Eulero e Lagrange esposto col metodo sintetico nel progetto di Nicolò Fergola*, Istituto Italiano per gli Studi Filosofici, Napoli, La Città del Sole, 1995.

The directrix *FG* and the axis *AI* intersect each other in *I*. On the line *FG* let's raise the perpendicular line *AK*. Let's put *AI=f, AK=g, KI=h*. From the generic point *M*, let's draw the distance *MN* on *AB* and then let's draw the parallel line *MR* to *FG*. Let us put *AN=x e MN=y*. Then, *NI* is equal to *f-x*. We have *AK:KI=MN:NR* and so $NR = \frac{hy}{g}$. Then, let's draw the parallel *MO* to *AB* and $MO = RI = f - x + \frac{hy}{g}$.

Let *Mm* be an *infinitely small arc*; let *mo* be parallel to *AB* and infinitely near *MO*; *mo* intersects *MN* in *X*. On *MO* let's raise the plane *MPO* and on *mo* let's raise the plane *mpo*, both parallel to *AHI*. *MPO* intersects the groin in the line *PO* and *mpo* intersects the groin in the line *po*.

The prism that these planes form is the "elemento di solidità" (*element of solidity*) of the groin. Its volume is the area of *MPO* multiplied by *MX* (where *MX=dy*). So, we are now looking for the area of *MPO*.

Let's put *AH=c*. Since *AHI* and *MPO* are similar, we have a proportion: *AI* is to *AH* as *MO* is to *MP*, and $MP = \frac{c}{f}\left(f - x + \frac{hy}{g}\right)$. The planes are parallel, *MP* is to the perpendicular line on *MO* from *P*, as radius is to sine of *BAH*. Let *r* be the radius and let *s* be the sine.

The dimension of the perpendicular is $MP = \frac{cs}{fr}\left(f - x + \frac{hy}{g}\right)$. Let us multiply it by $MO = f - x + \frac{hy}{g}$ and divide by 2. Therefore the area of the triangle is $\frac{cs}{2fr}\left(f - x + \frac{hy}{g}\right)^2$. Finally, we found the element of solidity of the groin multiplying by *dy*: $\frac{csdy}{2fr}\left(f - x + \frac{hy}{g}\right)^2$.

Since we know the curve of the groin, we can eliminate a variable in our equation $\frac{csdy}{2fr}\left(f - x + \frac{hy}{g}\right)^2$ and the element becomes "integrable".

Then, Settimo applies the first problem on oblique groins and on the elliptical cylinder

$$\frac{hy^2}{a} = bx - x^2 \Rightarrow x = \frac{b}{2} + \sqrt{\frac{b^2}{4} - \frac{hy^2}{a}}.$$

He writes the differential term like

$$\frac{csdy}{2rf}\left[p^2-2p\sqrt{\frac{b^2}{4}-\frac{hy^2}{a}}+\frac{b^2}{4}+\frac{hy^2}{a}+\frac{2phy}{g}-\frac{2hy}{g}\sqrt{\frac{b^2}{4}-\frac{hy^2}{a}}+\frac{h^2y^2}{g^2}\right]$$

and says that the problem of searching the volume of the groin is connected with the problem of squaring the ellipse.

At last, he talks about lateral groins, by analogous procedures.

In the second example, Settimo considers a hyperbolic cylinder and an oblique, direct or lateral groin. He says here that calculating volumes is connected with squaring hyperbolas. In the third example, he considers a parabolic cylinder and an oblique, direct or lateral groin, solving the problems of solidity for curves of equation $y^m=x$ that he calls "infinite parabolas".

We note that in the first problem, Settimo is able to solve and calculate each integral, but in the second problem, Settimo shows that its solution is connected with rectification of conic sections. He gives complicated differential forms like sums of more simple differentials that are integrable by elementary functions or connected with rectification of conic sections.

In the "first example" of the "second Problem", the oblique groin is part of an elliptical cylinder, where the equation of the ellipse is known; "the element of solidity" is the differential form:

$$\frac{c}{f}\left(f-x+\frac{hy}{g}\right)\sqrt{dy^2+\frac{s^2dx^2}{r^2}}\Rightarrow\frac{c}{f}\left(p+\sqrt{\frac{b^2}{4}-\frac{by^2}{a}}+\frac{hy}{g}\right)\frac{dy\sqrt{\frac{a^2}{4}-\frac{ay^2}{b}+\frac{s^2}{r^2}y^2}}{\sqrt{\frac{a^2}{4}-\frac{ay^2}{b}}}$$

that is decomposition of three differentials:

$$\frac{cp}{f}\frac{dy\sqrt{\frac{a^2}{4}-\frac{ay^2}{b}+\frac{s^2}{r^2}y^2}}{\sqrt{\frac{a^2}{4}-\frac{ay^2}{b}}}+\frac{bc}{af}dy\sqrt{\frac{a^2}{4}-\frac{ay^2}{b}+\frac{s^2}{r^2}y^2}+\frac{chydy}{fg}\frac{\sqrt{\frac{a^2}{4}-\frac{ay^2}{b}+\frac{s^2}{r^2}y^2}}{\sqrt{\frac{a^2}{4}-\frac{ay^2}{b}}}$$

Settimo starts studying the second differential: when he supposes the inequality $\frac{s^2}{r^2}<\frac{a}{b}$, he makes some positions and then makes a transformation on the differential that he rewrites like

$$\frac{bcm}{afr}\frac{\frac{1}{2}q^5du-\frac{1}{2}q^3u^2du}{\left(q^2+u^2\right)^2}+\frac{bcm}{afr}\frac{\frac{1}{2}q^5du+\frac{1}{2}q^3u^2du}{\left(q^2+u^2\right)^2}.$$

Settimo "constructs the solution", according to the classical method; i.e. he graphically resolves the arc that denotes the logarithm of imaginary numbers and shows that this solution solves the problem to search the original integral.

He calculates the integral of the first addend and transforms the second addend, but here he makes an important observation:

"*[this formula] includes logarithms of imaginary numbers [...]; now, since logarithms of imaginary numbers are circular arcs, in this case, from a circular arc the integral of the second part repeats itself. This arc, by 'il metodo datoci dal Cotes' [i.e. Cotes' method] has q as radius and u as tangent*".

Roger Cotes' method is in *Harmonia Mensurarum*[3]; there are also 18 tables of integrals; these tables let to get the "fluens" of a "fluxion" (i.e., the integral of a differential form) in terms of quantities, which are sides of a right triangle. Roger Cotes spent a good part of his youth (from 1709 to 1713) drafting the second edition of Newton's Principia. He died before his time, leaving incomplete and important researches that Robert Smith (1689–1768), cousin of Cotes, published in Harmonia Mensurarum, in 1722, at Cambridge.

In the first part of Harmonia Mensurarum, the Logometria, Cotes shows that problems that became problems on squaring hyperbolas and ellipses, can be solved by measures of ratios and angles; these problems can be solved more rapidly by using logarithms, sines and tangents. The "Scolio Generale", that closes the Logometria, contains a lot of elegant solutions for problems by logarithms and trigonometric functions, such as calculus of measure of lengths of geometrical or mechanical curves, volumes of surfaces, or centers of gravity.

We report here Cotes' method that Settimo uses in his treatise (see fig. 7).

Starting from the circle, let $CA=q$ and $TA=u$ the tangent*;* therefore $CT=\sqrt{q^2+u^2}$. Let's put $Tt=du$. Settimo investigates the arc that is the

3 R. Cotes, *Harmonia Mensurarum, sive Analysis & Synthesis per Rationum & Angulorum Mensuras Promotae: Accedunt alia Opuscula Mathematica: per Rogerum Cotesium. Edidit & Auxit Robertus Smith, Collegii S. Trinitatis apud Cantabrigienses Socius; Astronomiae & Experimentalis Philosophiae Post Cotesium Professor, Cantabrigiae,* 1722. See also R. Cotes, *Logometria*, «Philosophical Transactions of the Royal Society of London», vol. 29, n° 338, 1714.

logarithm of imaginary numbers and showed that this solution solves the problem of searching the original integral $\frac{bcm}{afr}\frac{\frac{1}{2}q^3du}{q^2+u^2}$.

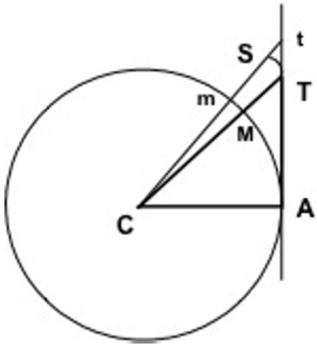

Fig. 7. Figure to illustrate Cotes' method.

The triangles *StT* and *ATC* are similar, therefore

$$Tt : TS = CT : CA \text{ and } TS = \frac{CA \cdot Tt}{CT} = \frac{qdu}{\sqrt{q^2+u^2}}.$$

CTS and *CMm* are also similar, therefore

$$TS : Mm = CT : CM \text{ and } Mm = \frac{TS \cdot CM}{CT} = \frac{q^2du}{q^2+u^2}.$$

Since the arc AM represents the integral of *Mm,* Cotes finds the original integral $\frac{bcm}{afr}\frac{\frac{1}{2}q^3du}{q^2+u^2}$. From $AM = \alpha q$ with $\alpha = \arctan\frac{u}{q}$, then

$$\frac{bcm}{afr}\frac{1}{2}q \times AM = \frac{bcm}{afr}\frac{1}{2}q^2 \arctan\frac{u}{q}$$

and its derivative is $\frac{bcm}{afr}\frac{\frac{1}{2}q^3du}{q^2+u^2}$.

Becoming again to Settimo's treatise, when Settimo supposes the inequality $\frac{s^2}{r^2} > \frac{a}{b}$, he solves the integral by means of logarithms of imaginary numbers, then (by using Cotes' method) with circular arcs.

Finally, Settimo shows problems on calculus of centre of gravity relative to area and volume of groins.

5. CONCLUSION

Various authors have eredited Archimedes, but we know that Prof. Heiberg found the Palimpsest containing *Archimedes' method* only in 1907, and therefore it is practically certain that Settimo did not know Archimedes' work.

Archimedes' solutions for calculating the volume of cylindrical wedges can be interpreted as computation of integrals, as Settimo really did, but both methods of Archimedes and Settimo are missing of generality: there is no a general computational algorithm for the calculations of volumes. They base the solution of each problem on a costruction determined by the special geometric features of that particular problem; Settimo however is able to take advantage of prevoious solutions of similar problems.

It is important finally to note that Settimo, who however has studied and knew the modern infinitesimal calculus (he indeed had to consult Roger Cotes and Leonhard Euler with De Martino in order to calculate integrals by using logarithms and circular arcs), considers the construction of the infinitesimal element similarly Archimedes.

Wanting to compare the two methods, we can say that both are based on geometrical constructions, from where they start to calculate infinitesimal element (that Settimo calls "elemento di solidità"): Archimedes' mechanical method was a precursor of that techniques which led to the rapid development of the calculus.

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ON ARCHIMEDEAN ROOTS IN TORRICELLI'S MECHANICS

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ABSTRACT In recent papers we analyzed the historical development of the foundations of the centres of gravity theory during the Renaissance. Using these works as a starting point, we shall briefly present a progression of knowledge with cultural and mathematical Archimedean roots in Torricelli's mechanics.

1. INTRODUCTION

Archimedes (287–212 B.C.) was a deeply influential author for Renaissance mathematicians according to the two main traditions. The humanistic tradition, adhering strictly to philological aspects, followed by Willem van Moerbeke (1215–1286), Regiomontanus (1436–1476) and Federigo Commandinus (1509–1575). The pure mathematical tradition followed by Francesco Maurolico (1694–1575), Luca Valerio (1552–1618), Galileo Galilei (1564–1642), Evangelista Torricelli (1608–1647).

The investigation into Archimedes's influence on Torricelli has a particular relevance because of its depth. Also it allows us to understand in which sense Archimedes' influence was still relevant for most scholars of the seventeenth century (Napolitani 1988). Besides there being a general influence on the geometrization of physics, Torricelli was particularly influenced by Archimedes with regard to mathematics of indivisibles. Indeed, it is Torricelli's attitude to confront geometric matter both with the methods of the ancients, in particular the exhaustion method, and with the indivisibles, so attempting to compare the two, as is clearly seen in his letters with Cavalieri (Torricelli 1919–1944; see mainly vol. 3). Torricelli, in particular, solved twenty one different ways the squaring a parabola (Heath 2002; *Quadrature of the parabola*, Propositio 17 and 24, p. 246;

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p. 251), a problem already studied by Archimedes: eleven times with exhaustion, ten with indivisibles. The *reductio ad absurdum* proof is always present.

Based on previous works (Pisano 2008) we can claim that the Archimedean approach to geometry is different from the Euclidean one. The object is different, because Archimedes mainly deals with metric aspects, which was quite new, also the aim is different, being more oriented towards solving practical problems. In addition, mainly the theory organization is different, because Archimedes does not develop the whole theory axiomatically, but sometimes uses an approach for problems, characterized by *reductio ad absurdum*. Moreover, the epistemological status of the principles is different. Archimedean principles are not always as self evident as those of the Euclidean tradition and may have an empirical nature. Some of the Archimedean *principles* have a clear methodological aim, and though they may express the daily feeling of the common man, they have a less cogent evidence then the principles of Euclidean geometry.

Knowledge of Archimedes' contribution is also fundamental to an historical study of Torricelli's mechanics. Archimedes was the first scientist to set *rational criteria* for determining centres of gravity of bodies and his work contains physical concepts formalised on mathematical basis. In *Book I* of the *On Plane Equilibrium* (Heath 2002) Archimedes, besides studying the rule governing the law of the lever also finds the centres of gravity of various geometrical plane figures (Heath 2002, Clagett 1964–1984; Heiberg 1881). By means of his *Suppositio* (principles) Archimedes (Heath 2002, pp. 189–202) is able to prove *Propositio* (theorems) (Heath 2002, pp. 189–202) useful in finding the centres of gravity of composed bodies. In particular, the sum of all the components may require the adoption of the method of exhaustion.

Archimedes's typical method of arguing in mechanics was by the use of the reduction *ad absurdum*, and Torricelli in his study on the centres of gravity resumes the same approach.

With regard to Torricelli's works, we studied mainly his mechanical theory (Capecchi and Pisano 2004; Idem 2007; Pisano 2009) in the *Opera geometrica* (Torricelli 1644), Table 1 and Fig. 1. We focused in detail on his discourses upon centres of gravity (Pisano 2007) where he enunciated his famous principle:

"It is impossible for the centre of gravity of two joined bodies in a state of equilibrium to sink due to any possible movement of the bodies".

The *Opera geometrica* is organized into four parts. Particularly, parts 1, 2, 3, are composed of *books* and part 4 is composed of an *Appendix*. Table 1 shows the index of the text:

Table 1. An index of *Opera geometrica* (Torricelli's manuscripts are now preserved at the Biblioteca Nazionale of Florence. Galilean Collection, n° 131–154).

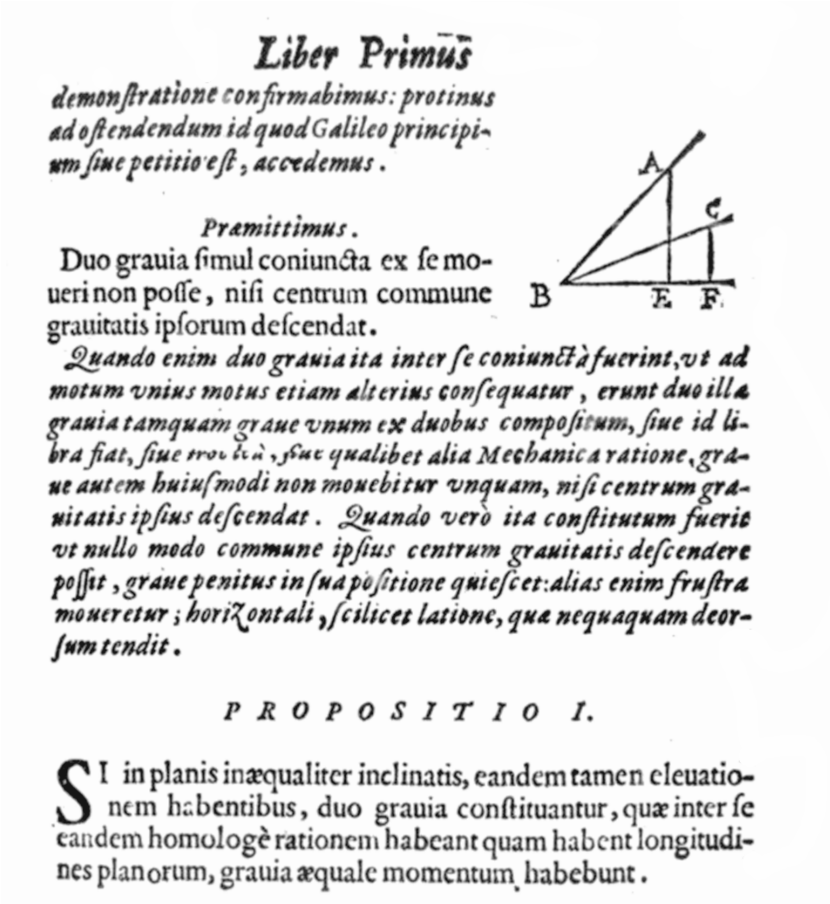
De sphaera et solidis sphaeralibus, Liber primus, 3–46; Liber secondus, 47–94.
De motu gravium naturaliter descendentium et proiectorum, Liber Primus, 97–153; Liber secundus, 154–243.
De dimensione parabolae Solidique Hyperbolici, 1–84.
Appendix: *De Dimensione Cycloidis*, 85–92.
De Solido acuto Hyperbolico, 93–135.
De Dimensione Cochlea, 136–150.

Fig. 1. The front page of Torricelli's Opera geometrica with the index of content.

Torricelli in his theory on the centre of gravity, following Archimedes' approach, uses

a) *Reductio ad absurdum* as a particular instrument for mathematical proof.
b) Geometrical representation of physical bodies: weightless beams and reference in geometrical form to the law of the lever.
c) Empirical evidence to establish principles.

We focused mostly upon the exposition of studies contained in *Liber primis. De motu gravium naturaliter descendentium,* where Torricelli's principle is exposed, Fig. 2 and 3. In Galileo's theory on dynamics, Torricelli present problems which, according to him, remain unsolved. His main concern is to prove a Galileo's supposition, which states: velocity degrees for a body are directly proportional to the inclination of the plane over which it moves:


Liber Primus

demonſtratione confirmabimus: protinus ad oſtendendum id quod Galileo principium ſiue petitio eſt, accedemus.

Præmittimus.

Duo grauia ſimul coniuncta ex ſe moueri non poſſe, niſi centrum commune grauitatis ipſorum deſcendat.

Quando enim duo grauia ita inter ſe coniunctà fuerint, vt ad motum vnius motus etiam alterius conſequatur, erunt duo illa grauia tamquam graue vnum ex duobus compoſitum, ſiue id libra fiat, ſiue troclea, ſiue qualibet alia Mechanica ratione, graue autem huiuſmodi non mouebitur vnquam, niſi centrum grauitatis ipſius deſcendat. Quando verò ita conſtitutum fuerit vt nullo modo commune ipſius centrum grauitatis deſcendere poſſit, graue penitus in ſua poſitione quieſcet: alias enim fruſtra moueretur; horizontali, ſcilicet latione, quæ nequaquam deorſum tendit.

PROPOSITIO I.

SI in planis inæqualiter inclinatis, eandem tamen eleuationem habentibus, duo grauia conſtituantur, quæ inter ſe eandem homologè rationem habeant quam habent longitudines planorum, grauia æquale momentum habebunt.

Fig. 2. Torricelli's principle. Opera Geometrica. De motu gravium naturaliter descendentium et proiectorum, p. 99.

The speeds acquired by one and the same body moving down planes of different inclinations are equal when the heights of these planes are equal (Galilei 1890–1909, Vol., VIII, p. 205)

Torricelli seems to suggest that this supposition may be *proved* beginning with a "theorem" according to which "the momentum of equal bodies on planes unequally inclined are to each other as the perpendicular lines of equal parts of the same planes" (Torricelli 1644, *De motu gravium naturaliter descendentium et proiectorum*, p. 99). Moreover, Torricelli also assumes that this theorem has not yet been demonstrated (Note, in the first edition of the Galileo's *Discorsi* in 1638, there is no proof of the "theorem". It was added only in 1656 to the *Opere di Galileo Galilei linceo*, (Galilei 1656). However Torricelli knew it, as is clear in some letters from Torricelli to Galileo regarding the "theorem"; Torricelli 1919–1944, Vol. III, p. 48, pp. 51, 55, 58, 61).

2. ARCHIMEDEAN THINKING

Torricelli frequently declares and explains his Archimedean background.

> Inter omnia opera Mathematics disciplinas pertinentia, iure optimo Principem sibi locum vindicare videntur Archimedis; quae quidem ipso subtilitatis miraculo terrent animos (Torricelli 1644, *Proemium*, p. 7).

Archimedes, in the *Quadratura parabolae*, first obtains results using the mechanical approach and then reconsiders the discourse with the classical methods of geometry to confirm in a rigorous way the correctness of his results (Heath 2002). Similarly, Torricelli, with the compelling idea of duplicating the procedure, devotes many pages to proving certain theorems on the "parabolic segment", by following, the geometry used in pre-history ancients (Torricelli (1644), *Quadratura parabolae pluris modis per duplicem positionem more antiquorum absoluta*, pp. 17–54)[1] and then proving the validity of the thesis also with the "indivisibilium" (Heath 2002, *Quadratura parabolae,* pp. 253–252; pp. 55–84; Torricelli 1644, *De solido acuto hyperbolico problema alterum*, pp. 93–135). In this respect, it is interesting to note that he underlines the "concordantia" (Torricelli 1644, *De solido acuto hyperbolico problema alterum*, p. 103) of methods of varying rigour.

> Hactenus de dimensione parabolae more antiquorum dictum sit; Reliquum est eandem parabolae mensuram nova quedam, sed mirabili ratione aggrediamur; ope scilicet Geometriae Indivisibilium, et hoc diversis modis: Suppositis enim praecipui Theorematib. antiquorum tam Euclidis, quam Archimedis, licet de rebus inter se diversissimis sint, mirum est ex unoquoque eorum quadraturam parabolae facili negotio elici posse; et vive versa. Quasi ea sit commune quoddam vinculum veritatis. […] Contra vero: supposita parabolae quadratura, praedicta omnia Theoremata facile demonstrari possunt. Quod autem haec indivisibilium Geometria novum penitus inventum sit equidem non ausim affirmare. Crediderim potius veteres Geometras hoc metodo usos in inventione Theorematum difficillimorum quamquam in demonstrationibus aliam viam magis probaverint, sive ad occultandum artis arcanum, sive ne ulla invidis detractoribus proferretur occasio contradicendi (Torricelli 1644, *Quadratura parabolae per novam indivisibilium Geometriam pluribus modis absoluta*, p. 55, *op. cit.*).

[1] In the original manuscripts of *Opera geometrica* there are some glosses to Eulid's *Elements*, to Apollonius' *Conic sections*, to Archimedes, Galileo, Cavalieri's works, et al., autograph by Torricelli.

From the previous passage there appears not only the desire to give the reader results and methods, but also to say that the indivisibles technique was not completely unknown to the ancient Greek scholars. Besides, Torricelli seems to hold onto the idea that the method of demonstration of the ancients, such as the Archimedes' method, was intentionally kept secret. He states that the ancient geometers worked according to a method "in invenzione" suitable "ad occultandum artis arcanum" (Torricelli 1644, *Quadratura parabolae per novam indivisibilium Geometriam pluribus modis absoluta*, p. 55).

However the Archimedean influence in Torricelli goes further. The well known books *De sphaera et solidis sphaeralibus* (Torricelli 1644, *Liber primus*, 3–46) present an enlargement of the Archimedean proofs of books I–II of *On the sphere and cylinder* (Heath 2002, pp. 1–90).

> […] In quibus Archimedis Doctrina de sphaera & cylindro denuo componitur, latius promovetur, et omni specie Solidorum, quae vel circa, vel intra, Sphaeram, ex conversione polygonorum regularium gigni possint, universalius Propagatur (Torricelli 1644, *De sphaera et solidis sphaeralibus*, p. 2).

In other parts Torricelli faces problems not yet solved by Archimedes, or by the other mathematicians of antiquity. With the same style as Archimedes, he does not try to arrive at the first principles of the theory and does not limit himself to a single way of demonstrating a theory.

> Veritatem praecedentis Theorematis satis per se claram, et per exempla ad initium libelli proposita confirmatam satis superque puto. Tamen ut in hac parte satisfaciam lectori etiam Indivisibilium parum amico, iterabo hanc ipsam demonstrationis in calce operis, per solitam veterum Geometrarum viam demonstrandi, longiorem quidem, sed non ideo mihi certiorem (Torricelli 1644, *De solido hyperbolico acuto problema secundum,* p. 116).

We note that the exposition of the mechanical argumentation present in Archimedes's *Method* was not known at Torricelli's time because Johan Heiberg only discovered it in 1906 (Heath 1912). Therefore, in Archimedes's writing there were lines of reasoning which, because a lack of justification, were labelled as mysterious by most scholars. Thus in such instances it was necessary to assure the reader of the validity of the thesis and also to convince him about the strictness of Archimedes' approaches, particularly exhaustion reasoning and *reductio ad absurdum*, by proving his results with some other technique.

> The appearance of approximation [in Archimedes's proofs] is surely a substantial innovation in the mathematical demonstrations and the difference between

Elements and Archimedes' work is a sign of a mentality more opened towards applications, and perhaps that the classical epoch of geometry was closed (Marchini 2005, pp. 189–190).

3. ON PROOFS

It is well known from the *Method* (Heiberg 1913) that Archimedes studied a given problem whose solution he anticipated by means of crucial propositions which were then proved by the *reductio ad absurdum* or by exhaustion. Indeed Archimedes's himself did not attribute the same amount of certainty to his *Method* proofs, as he attributes to classical mathematical proofs. His reasoning on *Quadratura parabolae* (Heath 2002, Proposition 24, p. 251) is exemplary. Addressing Eratosthenes (276–196 B.C.), Archimedes wrote at the beginning of his *Method*:

> […] I thought fit to write out for you and explain in detail in the same book the peculiarity of a certain method, by which it will be possible for you to get a start to enable you to investigate some of the problems in mathematics by means of mechanics. This procedure is, I am persuaded, no less useful even for the proof of the theorems themselves; for certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration. But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge (Heath 1913, p. 13).

One of the characteristics of Torricelli's proofs was the syntactic return to the demonstration approach followed by the ancient Greeks, with the explicit description of the technique of reasoning actually used. Besides the well known *ad absurdum* there were also the *permutando* and *the ex aequo*. In *De proportionibus liber* he defines them explicitly:

> Propositio IX. Si quatuor magnitudines proportionales fuerint, et permutando proportionales erunt. Sint quatuor rectae lineae proportionales *AB, BC, CD, DE*. Nempe ut *AB* prima ad *BC* secundam, ita sit *AD* tertia ad *DE* quartam. Dico primam *AB* ad tertiam *AD* ita esse ut secunda *BC* ad quartam *DE*. Qui modus arguendi dicitur permutando (Torricelli 1919–1944, *De Proportionibus liber*, p. 313)
>
> Propositio X. Si fuerint quotcumque, et aliae ipsis aequales numero, quae binae in eadem ratione sumantur, et ex aequo in eadem ratione erunt. Sint quotcumque magnitudines A, B, C, H, et aliae ipsis aequales numero D, E, F, I, quae in eadem ratione sint, si binae sumantur, nempe ut A ad B ita sit D ad E, et iterum ut B ad

C, ita sit E ad F, et hoc modo procedatur semper. Dico ex equo ita esse primam A ad ultimam H, uti est prima D ad ultimam I. Qui modus arguendi dicitur ex aequo (Torricelli 1919–1944, *De Proportionibus liber*, p. 314).

Torricelli seems to neglect the algebra of his time and adheres to the language of proportions. He dedicated a book to this language, *De Proportionibus liber* (Torricelli 1919–1944, pp. 295–327), where he only deals with the theory of proportions to be used in geometry. In such a way he avoids the use of the *plus* or *minus*, in place of which he utilizes the *composing* (Torricelli 1919–1944, p. 316) and *dividing* (Idem, p. 313). Such an approach allows him to work always with the ratio of segments. By following the ancients to sum up segments he imagines them as aligned and then translated and connected, making use of terms like "simul", "et" or "cum" (Torricelli 1919–1944, Prop. XV, p. 318). In what follows we present a table which summarizes the most interesting part of *Proportionibus liber* where Torricelli proves again theorems by referring to reasoning in the Archimedean manner, Table 2.

Table 2. Some Torricelli's Archimedean proofs in *Quadratura parabolae.*

Lemma II,V,VI, X–XI,XII–XIII, XVII – Propositio IV	Ad absurdum proofs
Lemma XIV	Ex aequo et dividendo et permutando
Lemma XVI, XVIII	Ex aequo
Lemma XIX	Ex aequo et Ad absurdum
Propositio III[2]	Componendo
Propositio V	Ad absurdum et Componendo
Propositio IX	Ex aequo et Ad absurdum

We notice that proofs by means of indivisibles are not *reductio ad absurdum*. This is so because these proofs are algebraic. Instead, in nearly all other proofs Torricelli uses the technique typical of proportions, *dividendo, permutando* and *ex aequo*.

[2] In proposition III Torricelli, referring to Luca Valerio, proves a Lemma differently from him: "Libet hic demonstrare Lemma Lucae Valerij, nostro tamen modo, diversisque penitus Mechanicae principijs. Ipse enim utitur propositione illa, qua ante demonstraverat centrum gravitatis hemisphereij. Nos autem simili ratione ac in praecedentibus [I–II], demonstrabimus et Lemma, et ipsam Valerij conclusionem" (Torricelli1644, *Quadratura parabolae pluris modis per duplicem positionem more antiquorum absoluta*, p. 33; Valerio 1604, book II, p. 12).

Ἐπιπέδων ἰσορροπιῶν ἢ κέντρα βαρῶν ἐπιπέδων α΄

α΄. Αἰτούμεθα τὰ ἴσα βάρεα ἀπὸ ἴσων μακέων ἰσορροπεῖν, τὰ δὲ ἴσα βάρεα ἀπὸ τῶν ἀνίσων μακέων μὴ ἰσορροπεῖν, ἀλλὰ ῥέπειν ἐπὶ τὸ βάρος τὸ ἀπὸ τοῦ μείζονος μάκεος.

β΄. εἴ κα βαρέων ἰσορροπεόντων ἀπό τινων μακέων ποτὶ τὸ ἕτερον τῶν βαρέων ποτιτεθῇ, μὴ ἰσορροπεῖν, ἀλλὰ ῥέπειν ἐπὶ τὸ βάρος ἐκεῖνο, ᾧ ποτετέθη.

γ΄. ὁμοίως δὲ καί, εἴ κα ἀπὸ τοῦ ἑτέρου τῶν βαρέων ἀφαιρεθῇ τι, μὴ ἰσορροπεῖν, ἀλλὰ ῥέπειν ἐπὶ τὸ βάρος, ἀφ' οὗ οὐκ ἀφῃρέθη.

δ΄. τῶν ἴσων καὶ ὁμοίων σχημάτων ἐπιπέδων ἐφαρμοζομένων ἐπ' ἄλλαλα καὶ τὰ κέντρα τῶν βαρέων ἐφαρμόζει ἐπ' ἄλλαλα.

ε΄. τῶν δὲ ἀνίσων, ὁμοίων δὲ τὰ κέντρα τῶν βαρέων ὁμοίως ἐσσεῖται κείμενα. ὁμοίως δὲ λέγομες σαμεῖα κεέσθαι ποτὶ τὰ ὁμοῖα σχήματα, ἀφ' ὧν ἐπὶ τὰς ἴσας γωνίας ἀγομέναι εὐθείαι ποιέοντι γωνίας ἴσας ποτὶ τὰς ὁμολόγους πλευράς.

Fig. 3. Archimedes' first suppositio: On plane equilibrium, Heiberg 1881, p. 142.

4. CONCLUSION

We focused on conceptual aspects of Archimedes' and Torricelli's studies of the centre of gravity theory based on previous investigations on Archimedes' *On the Equilibrium of Plane* and Torricelli's *Opera geometrica*. In the present work we have outlined some of the fundamental concepts common to the two scholars: the logical organization and the paradigmatic discontinuity with respect to the Euclidean technique. Indeed Archimedes' theory (mechanical and geometrical) does not appear to follow a unique pattern. It maintains two kinds of organization, one problematic the other axiomatic deductive.

In conclusion, to compare the science of Archimedes and Torricelli from an epistemological point of view, we resume in Table 3 the crucial aspects of their theory organization.

Table 3. Archimedes' and Torricelli's foundations of theory.

	Archimedes	Torricelli
Organization of the theory	– Problematic (mechanics) – Axiomatic (geometry)	– Problematic (mechanics) – Axiomatic (geometry)
Body systems	– Without explicating the type of connection	– Aggregate – Tied up way or untied
Foundational concept	– Centre of gravity	– Centre of gravity of Archimedes
Type of infinite	– Potential Infinitum – Toward Actual Infinitum	– Potential Infinitum – Actual Infinitum (indivisibles)
Central problem of the theory	– Criteria to determinate the centre of gravity for single and composed geometrical bodie	– Galileo's ballistic theory by means of Archimedean equilibrium theory
Techniques of arguing	– *Reductio ad Absurdum*	– *Reductio ad Absurdum*
Techniques of calculus	– Method of exhaustion	– Archimedes's method of exhaustion – Indivisible method

The breaking of the Euclidean paradigm, in the Khun sense (Khun 1962), by Archimedes could offer, with the limitation implicit in the concept of paradigm, a first possible lecture key. We pass from a normal science composed of axioms and self–evidence to a new science where proof also means also to find a field of applicability of a new theory, the centrobarica.

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RATIONAL MECHANICS AND SCIENCE RATIONNELLE UNIQUE

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ABSTRACT We highlight the legacy of Simon Stevin and Gabriel Lamé and show how their work led to some of the most important recent developments in science, ultimately based upon the principles of balance and the act of weighing, virtual or real. These names are also important in the sense of a *unique rational science* and *universal natural shapes*.

1. INTRODUCTION

Since antiquity various geometers have strived to understand and expand the ideas and results obtained by Greek mathematicians. The foundations developed by Eudoxus, Euclid, Apollonius, Archimedes and many others

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were characterized by a pulsation between geometry and algebra. This remains so today, in our era of calculation and algorithms (Atiyah, 2000). Shiing-Shin Chern wrote (2000): "*While analysis and algebra provide the foundations of mathematics, geometry is at the core*".

More analytic than synthetic, contemporary differential geometry follows the ideas of Riemann and Helmholtz, for whom measurements should be given priority, in accordance with our abstraction of our perception of the world, and very much in line with Greek thoughts on commensurability. The Greek origin for the word geometry is –μετρεω. The root –μετρεω (also in the word συμμετρια = symmetry, proportion or right balance) means: to measure, to correspond. Like κοσμεω (ordering) in ancient Greek, symmetry also has a verb (συμμετρεω) meaning to measure, to correspond, to be commensurate (Vlastos, 2005).

A major task for geometers is to deepen the understanding of the legacy of the Greek geometers. Still much is to be learned from Bacon writings: "*Solomon saith: "There is no new thing upon the earth". So that as Plato had an imagination that all knowledge was but remembrance; so Solomon giveth his sentence, "that all novelty is but oblivion."*"

For Klein *parabolic, elliptic* and *hyperbolic* had precisely the same geometric meaning as it had in the application of areas of the Pythagoreans or in the conics of Apollonius, namely *precise fitting, defect* and *excess* respectively. Indeed, science still revolves around the same questions that interested Greek scholars, such as the finite versus infinite or the discrete versus continuous (in doing mathematics all these dualities act simultaneously; Thurston, 1994). Geometry (and its applications in the natural sciences) is still about the notion of going straight. On recent developments on curvatures and intrinsic and extrinsic symmetries see: Haesen and Verstraelen, 2009 and Chen, 2007.

2. FROM RENAISSANCE TO THE ECOLE POLYTECHNIQUE

2.1. Simon Stevin's *Wonder en is gheen Wonder*

Simon Stevin was one of the greatest mathematicians of the Renaissance and the greatest mechanician of the long period extending from Archimedes to Galileo (Sarton, 1934; Bosmans, 1923, 1926). His works were translated and edited by Snellius and Albert Girard and were available in Dutch, French and Latin and known to, amongst others, Gregory Saint-Vincent and Descartes.

Essential in Stevin's work is the relation between *spiegeling* ("theory") and *daet* ("practice"). Besides the necessary theoretical approach there always should be an experimental one, either concrete, or through a thought experiment. In this way Stevin made valuable contributions in calculus, algebra, geometry, mechanics, hydrostatics, navigation, tides theory, fortification, the building of locks, economy, . . . On the theoretical side, he also solved the hydrostatic paradox and dropped two unequal weights from a tower in Leiden to prove that they would reach the ground level at the same time, well before Pascal and Galilei respectively.

In *De Thiende* (1585) Stevin systematically showed how all calculations with real numbers are reduced to the standard operations with natural numbers. The importance of the real numbers for science is clear, not in the least since this very same method was used by Newton in his Method of Fluxions. In 1586 in the books *De Beghinselen der Weegh-const (The art of weighing)* and *De Weeghdaet* (*The practice of weighing*) the foundations of the mathematical vector calculus were provided with the rule of the parallelogram for the addition of forces as concrete application in physics. He introduced the impossibility of a perpetuum mobile as a method of proof in physics with the famous Clootkrans proof.

For Stevin, when phenomena could be explained rationally, meaning geometrically, miracles where no longer miracles. Stevin's motto (and epitaph; Feynman, 1963) was *Wonder en is gheen wonder* (*Magic is no magic*; Devreese & Vanden Berghe, 2008). He was a great admirer of Archimedes and Stevin's vision was completely in line with rational mechanics, where balance and weighing are crucial. In *Beghinselen der Weeghconst*, he converted the method of weighing, which was a source of inspiration to Archimedes, into a method of proof, with the use of limits as culmination (figure 1 left; Bosmans, 1923; Sarton, 1936).

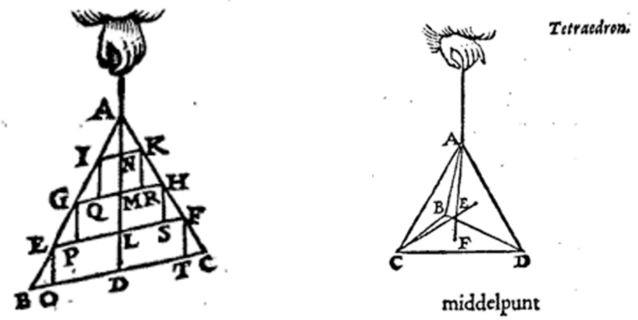

Fig. 1. Determining the centre of gravity of a triangle and a tetrahedron.

The use of limits in the elegant proofs of Stevin substituted for proofs using method of exhaustion involving a reductio ad absurdum by Archimedes. Stevin is thus an important link in the gradual transformation to modern methods of infinitesimal analysis in a chain involving Archimedes, Commandino, Stevin, Gregoire de Saint-Vincent, Boelmans, Tacquet, Pascal, Leibniz (Bosmans 1926: Sarton, 1934).

Being at the crossroads of algebra and geometry Stevin was first and foremost a geometer. With geometrical numbers he thought of powers in a very practical way. 2^3 is a cube with volume 8, and 2^4 is simply two cubes of volume 8. This pulsation of thinking both geometrically and algebraically and about cubes, numbers and roots in different ways, is an art, which should be practiced in our era of specialization.

2.2. From the Late Renaissance to Radical Enlightenment

In the Renaissance a number of exciting developments took place, forming the basis of contemporary science. These would be developed more fully during the Enlightenment, supposedly in Italy, France and England. It has been forcibly argued however, that Radical Enlightenment in the Northern Low Countries well predated the development of Enlightenment in other regions of Europe (Israel, 2005).

In the 17th century the Republic of the Northern Low Countries had become, under the patronage of Maurits van Oranje, a freehaven for science and religion. Many scholars from the Southern Low countries and France fled to the North and would provide the basis of the Golden Age (Struik, 1981). One of the foremost persons was Simon Stevin, who became the Prince's personal advisor. Stevin was co-founder of the Ingenieursschool in Leiden in 1600, where generations of Rekenmeesters (reckoning masters) were trained.

Stevin's early defense of the Copernican system was not appreciated by the clerics who ruled the universities. Stevin thus never held an academic position, but his influence on several generations of his "students" is very profound (Struik, 1981; Fig. 2). Among those we find, directly, Isaac Beeckman, Snellius (Sr. and Jr.) and Albert Girard, and indirectly Gregoire de Saint-Vincent, Descartes and Christiaan Huygens.

His legacy and influence on further developments was enormous, but he did not receive the proper recognition. George Sarton (1934) wrote: "*How could people truly admire one whom they do not understand, how could they consider great a man whose greatness they have not yet been educated to appreciate?*"

All in all, these developments in science in the first half of the 17th century would become the cornerstone of the Radical Enlightenment in the second half of that century, when the mathematicians De Witt and Hudde would also take important political positions (Israel, 2005). The political and religious freedom would allow for the development of Radical Enlightenment with Baruch de Spinoza as central figure (Van Bunge, 2001). His views were certainly influenced by the developments in mathematics and science in the first half of the 17th century in the Northern Low Countries, and his discourse was very much in line with Stevin's "*Wonder en is gheen Wonder.*"

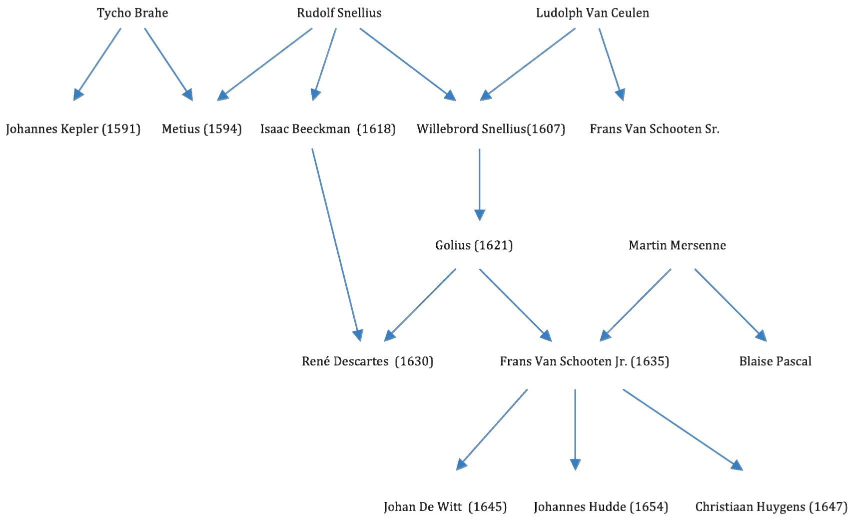

Fig. 2. Compiled from www.genealogy.ams.org.

The most important mathematician of the Low Countries in the late 17th century was Huygens, and the crucial encounter with Leibniz in Paris would be decisive for the development of science. The further developments in differential geometry initiated with Huygens and Leibniz would lead, through Basel, Berlin and Saint Petersburg and with the Bernoulli's, Euler and Lagrange as the main figures in the 18th century, to Paris in the second half of the 18th century. Paris became a leading center of mathematics in the 19th and 20th century. The trunk of the genealogical tree initiated in the Low Countries was continued and replanted in the Ecole Polytechnique EP in Paris, with brilliant teachers and students like Monge, Lagrange, Laplace, Fourier, Poisson, Legendre, Cauchy, Delaunay, Lamé, Clapeyron and Chasles, and in full agreement with the idea of Spiegeling and Daet or theory and practice.

3. SCIENCE RATIONNELLE UNIQUE & NATURAL SHAPES

3.1. Science Rationnelle Unique

Gabriel Lamé (1795–1860) entered the EP in 1813, graduated in 1817, and became a very famous *ingénieur savant*. Like Archimedes and Stevin before him he was both engineer and mathematician. Gauss praised Lamé as the most important French mathematician of his time, but in France he was considered too theoretical for engineers and too practical for mathematicians (Bertrand, 1878).

At the age of 21 he introduced equations of the type $x^n + y^n = 1$ in his book *Examen de differentes méthodes employées pour résoudre les problèmes de géometrie* (Lamé, 1818)[1] and noted that a special choice of exponents gave a uniform description of all conic sections. These Lamé curves gave the possibility of defining metrics based on powers other than two. This was also suggested by Riemann in his Habilitationsschrift (1856), which led to the development of Riemann-Finsler geometry (the metric structure of Finsler manifolds is given by a collection of convex symmetric bodies in the various tangent spaces; Berger, 2000).

During a decade in Saint Petersburg, Lamé and Clapeyron developed, amongst others, location theory (Franksen & Grattan-Guinness, 1989; Tazzioli, 1995; Gouzevitch & Gouzevitch, 2009). The development of the theory of optimal location was done with weights and balances based on machines that were used to demonstrate Stevin's parallelogram of forces. As engineers Lamé and Clapeyron used methods of weighing construction of bridges using funicular polygons (Tazzioli, 1993).

He returned to France to become professor of physics at the EP from 1832 onwards. Lamé's work on curvilinear coordinates was very influential (Struik, 1933) and his work was considered 'immortal' by Darboux (1878; "*les immortels travaux de Lamé sur les coordonnées curvilignes*"). This generalized the work of Euler on curves and of Gauss on surfaces. Elie Cartan (1931) considered Lamé as cofounder of Riemannian geometry, and his work opened the door for Ricci, Levi-Civita and Beltrami (Vincensini, 1972; Tazzioli, 1993). His influence on science continues to be most impressive (Guitart, 2009).

What connects all activities of Gabriel Lamé was his quest for a Unique Rational Science. Lamé foresaw "*l'avènement futur d'une science rationnelle unique*", of a unique rational science, which essentially is mathematical physics. His method used curvilinear coordinates designed to

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1 As a young student Gabriel Lamé's interest in geometry was aroused by Legendre's Géometrie. The profound impact of Legendre's educational books on the development of science is illustrated further by the influence of "Théorie des Nombres" on Riemann.

adapt a physical situation to a system of curvilinear coordinates. This model then provided the 'initial geometrical support' for a physical system. In this sense differences between phenomena would no longer be an arbitrary choice of a certain parameter, but *would organize itself to produce a natural intrinsic space* of the system.

Lamé thus envisaged that, from a mathematical point of view, the study of a physical system amounts to the study of a system of curvilinear coordinates, adapted to the given physical situation. The study of that physical problem, adapted with the appropriate system of curvilinear coordinates then becomes the characterization of the system of differential invariants or the calculation of the Laplacian in curvilinear coordinates. In his view this reduces to one equation only, namely the Poisson equation in curvilinear coordinates, with boundary conditions (Guitart, 2009).

3.2. Universal Natural Shapes

170 years after Lamé published his *Examens*, his writings on curvilinear coordinates and on Lamé curves have been united. Following attempts to describe natural shapes based on Lamé curves (Gielis, 1996) these curves were generalized as supershapes (Gielis, 2003; Equation (*); Fig. 3). This transformation can be applied to any planar function. Equation 0 in fact is a generalized Pythagorean Theorem, a conservation law for n-volumes.

$$\rho = \left(\left| \frac{\cos \frac{m_1}{4}\varphi}{A} \right|^{n_2} \pm \left| \frac{\sin \frac{m_2}{4}\varphi}{B} \right|^{n_3} \right)^{-\frac{1}{n_1}}$$

Equation (*): the Superformula with $m, n_2, n_3\ \psi, A, B, n_1\ \rho_0$

The names superformula and supershapes originate from the names superellipses and superquadrics. The name superformula was changed by mathematicians into Gielis Formula (Koiso and Palmer, 2008), and supershapes into Gielis' curves and surfaces (Verstraelen, 2004, 2009).

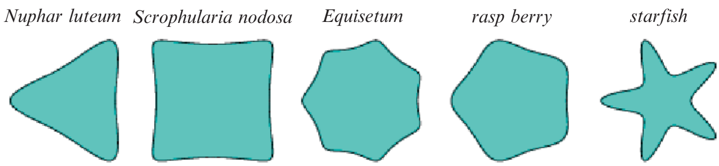

Fig. 3. Supershapes and natural analogues.

As these transformations provide for a one-step extension of conic sections to the description of natural shapes they were referred to as Universal Natural Shapes (Gielis et al., 2005). The shape coordinates make shapes commensurable. Shapes of starfish, flowers, and pyramids and a wide variety of natural shapes can now be described using one common yard stick, turning asymmetry (incommensurable) into symmetry (to make commensurable) or: "*From discord the fairest harmony*" (Heraclitus), thus expresses one of the most fundamental goals of mathematics.

Lamé-Gielis' curves and (hyper-) surfaces turn out to be the "*most natural curves and surfaces of Euclidean geometry.*" A wide range of shapes in the natural sciences can all be produced in this rather universal way: first, impose some "Euclidean" geometrical principle, and second, apply a Gielis transformation to the shapes resulting from these geometric principles (Verstraelen, 2008). Using tangents, and tangent spaces based on supershapes as length indicatrices, could unveil the geometrical meaning of all curvatures in Minkowski and Riemann-Finsler geometry, and the natural processes that are modeled in this way.

Shape description starting from a center using so-called Gielis curves and surfaces are in a natural way anisotropic, and induce a coordinate system of and on the surface, *adapted to the problem*. Generalized trigonometric or Fourier series can be defined (Gielis, 2009). This allows for strategies to develop computational tools, esp. those involving the Laplacian. Methods have been developed using stretched polar coordinates (Natalini et al., 2008; Caratelli et al., 2009), which allows for the use of Fourier series for boundary value problems combining the insights of Lamé and Fourier.

4. THE DIRICHLET PROBLEM FOR POISSON'S EQUATION IN A STARLIKE DOMAIN

Many applications of mathematical physics and electromagnetics are connected with the Laplacian (wave equation, heat propagation, Laplace, Helmholtz, Poisson and Schrödinger equations; Caratelli et al., 2009). Most boundary value problems (BVP) relevant to the Laplacian can be solved in explicit form only in domains with very special shapes or symmetries (Courant, 1950). The solution in more general domains can be obtained by using the Riemann theorem on conformal mappings, and the relevant invariance of the Laplacian.

The use of stretched co-ordinate systems allows the application of the classical Fourier method to a wide set of differential problems in complex two-dimensional normal-polar domains (Natalini et al., 2008; Caratelli et al., 2009). Such domains can be approximated as closely as desired by the above equations and numerical results are in good agreement with theoretical results of Lennart Carleson (Natalini et al., 2008). Here the solution of the Dirichlet problem for the Poisson equation in two-dimensional natural-shaped domains is presented. This differential problem is of great importance in different areas of scientific research, such as electrostatics, mechanical engineering and theoretical physics.

Let $D \subset R^2$ be an open, bounded, star-like domain, with boundary $\partial D \in C^1$ having outer normal unit vector $\nu = \nu(\rho)$. Then, a general representation formula for the solution of the Poisson equation:

$$-\Delta u(\rho) = f(\rho), \qquad \rho \in D, \tag{1}$$

subject to the Dirichlet boundary condition:

$$u(\rho) = g(\rho), \qquad \rho \in \partial D, \tag{2}$$

for given continuous functions $f(\rho)$, $g(\rho)$ can be easily obtained by using Green's function method. Under the assumption $u \in C^2(\overline{D})$, for any point $\rho \in D$ it is not difficult to show that:

$$\begin{aligned} u(\rho) = &\int_{\partial D} \left[\Phi(\rho' - \rho) \frac{\partial u}{\partial \nu}(\rho') - u(\rho') \frac{\partial \Phi}{\partial \nu}(\rho' - \rho) \right] dl' + \\ &- \int_D \Phi(\rho' - \rho) \Delta u(\rho') dS', \end{aligned} \tag{3}$$

where:

$$\Phi(\rho) = -\tfrac{1}{2\pi} \ln|\rho| \tag{4}$$

denotes the fundamental solution of the Laplace equation satisfying $-\Delta\Phi(\rho) = \delta(\rho)$, $\delta(\rho)$ being the Dirac measure on R^2 giving unit mass to the origin. As it can be noticed, formula (3) allows us to evaluate $u(\rho)$ once the values of $\Delta u(\rho)$ within D and the values of $u(\rho)$, $\partial u(\rho)/\partial \nu$ along ∂D are known. Hence, for the application to the Dirichlet problem for the Poisson equation (1)-(2), we must slightly modify (3) by removing the term involving the normal derivative of the unknown function $u(\rho)$ along the boundary ∂D. To achieve this, let us introduce for any fixed $\rho \in D$ the corrector function $\phi = \phi(\rho, \rho')$ solving the boundary-value problem for the Laplace equation:

$$\begin{cases} \Delta\phi(\rho,\rho')=0, & \rho'\in D, \\ \phi(\rho,\rho')=\Phi(\rho'-\rho), & \rho'\in\partial D. \end{cases} \tag{5}$$

Applying Green's formula readily yields:

$$\begin{aligned} -\int_D \phi(\rho,\rho')\Delta u(\rho')dS' &= \int_{\partial D}\left[u(\rho')\frac{\partial\phi}{\partial\nu}(\rho,\rho')+\right.\\ \left.-\phi(\rho,\rho')\frac{\partial u}{\partial\nu}(\rho')\right]dl' &= \int_{\partial D}\left[u(\rho')\frac{\partial\phi}{\partial\nu}(\rho,\rho')+\right.\\ &\left.-\Phi(\rho'-\rho)\frac{\partial u}{\partial\nu}(\rho')\right]dl'. \end{aligned} \tag{6}$$

As a consequence, Green's function for the Poisson equation (1) can be evaluated as follows:

$$G(\rho,\rho')=\Phi(\rho'-\rho)-\phi(\rho,\rho'),\;\; \rho,\rho'\in D,\;\; \rho\neq\rho'. \tag{7}$$

In fact, adding (6) to (3), we find:

$$\begin{aligned} u(\rho) &= -\int_{\partial D} u(\rho')\frac{\partial G}{\partial\nu}(\rho,\rho')dl' - \int_D G(\rho,\rho')\Delta u(\rho')dS' = \\ &- \int_{\partial D} g(\rho')\frac{\partial G}{\partial\nu}(\rho,\rho')dl' + \int_D f(\rho')G(\rho,\rho')dS', \end{aligned} \tag{8}$$

where:

$$\frac{\partial G}{\partial\nu}(\rho,\rho')=\nabla_{\rho'}G(\rho,\rho')\cdot\nu(\rho') \tag{9}$$

is the outer normal derivative of $G(\rho,\rho')$ with respect to the variable ρ' So, the solution of (1)-(2) can be derived by using (8), provided that we can construct Green's function for the given domain D. To this end, let us firstly introduce in the real plane the stretched curvilinear coordinate system:

$$\rho=(x,y),\quad \begin{cases} x=rR(\vartheta)\cos\vartheta, \\ y=rR(\vartheta)\sin\vartheta, \end{cases} \tag{10}$$

$R(\vartheta)$ denoting the polar equation of ∂D. Therefore, the domain D is described by the inequalities $0\le\vartheta\le 2\pi$, $0\le r\le 1$.

The following theorem provides an effective means to solve (5), and hence evaluate $G(\rho, \rho')$.

Theorem – Let:

$$\rho' = (x', y'), \qquad \begin{cases} x' = r'R(\vartheta)\cos\vartheta', \\ y' = r'R(\vartheta)\sin\vartheta', \end{cases} \tag{11}$$

and:

$$\Phi(\rho' - \rho) = \tilde{\Phi}(\vartheta', \rho) = \sum_{m=0}^{+\infty}\left[\alpha_m(\rho)\cos(m\vartheta') + \beta_m(\rho)\sin(m\vartheta')\right], \tag{12}$$

with $\rho' \in \partial D$, and:

$$\begin{Bmatrix} \alpha_m(\rho) \\ \beta_m(\rho) \end{Bmatrix} = \frac{\varepsilon_m}{2\pi}\int_0^{2\pi}\tilde{\Phi}(\vartheta', \rho)\begin{Bmatrix} \cos(m\vartheta') \\ \sin(m\vartheta') \end{Bmatrix} d\vartheta', \tag{13}$$

ε_m being the usual Neumann's symbol. Then, the boundary-value problem (5) admits a classical solution $\phi(\rho, \rho') \in L^2(D)$ such that the following Fourier-like series expansion holds:

$$\begin{aligned} &\phi(\rho,\rho') = \phi(\rho, r', \vartheta') = \\ &= \sum_{m=0}^{+\infty}\left[r'R(\vartheta')\right]^m\left[\mathrm{A}_m(\rho)\cos(m\vartheta') + \mathrm{B}_m(\rho)\sin(m\vartheta')\right]. \end{aligned} \tag{14}$$

The coefficients $\mathrm{A}_m(\rho)$, $\mathrm{B}_m(\rho)$ in (14) can be determined by solving the infinite linear system:

$$\sum_{m=0}^{+\infty}\begin{bmatrix} X^{+}_{n,m} & Y^{+}_{n,m} \\ X^{-}_{n,m} & Y^{-}_{n,m} \end{bmatrix} \cdot \begin{bmatrix} \mathrm{A}_m(\rho) \\ \mathrm{B}_m(\rho) \end{bmatrix} = \begin{bmatrix} \alpha_m(\rho) \\ \beta_m(\rho) \end{bmatrix}, \tag{15}$$

where:

$$X^{\{\pm\}}_{n,m} = \frac{\varepsilon_n}{2\pi}\int_0^{2\pi} R(\vartheta')^m \cos(m\vartheta')\begin{Bmatrix} \cos(n\vartheta') \\ \sin(n\vartheta') \end{Bmatrix} d\vartheta', \tag{16}$$

$$Y^{\{\pm\}}_{n,m} = \frac{\varepsilon_n}{2\pi}\int_0^{2\pi} R(\vartheta')^m \sin(m\vartheta')\begin{Bmatrix} \cos(n\vartheta') \\ \sin(n\vartheta') \end{Bmatrix} d\vartheta', \tag{17}$$

with $m, n \in N_0$.

Proof – In the stretched coordinate system (10)-(11), the domain D is transformed into the unit circle. Hence, we can use the eigenfunction method and separation of variables to solve the Laplace equation $\Delta\phi(\rho,\rho')=0$. In this way, it is straightforward to show that the elementary solutions of the problem are given by:

$$\phi_m(\rho,r',\vartheta')=\left[r'R(\vartheta')\right]^m\left[\mathrm{A}_m(\rho)\cos(m\vartheta')+\mathrm{B}_m(\rho)\sin(m\vartheta')\right], \quad (18)$$

with $m\in N_0$. So, enforcing the Dirichlet boundary condition $\phi(\rho,\rho')=\Phi(\rho'-\rho)$ ($\rho'\in\partial D$) and using the usual Fourier's projection method, equations (15)-(17) readily follow. □

Once the corrector function $\phi(\rho,\rho')$ for the assigned domain D is computed (Natalini et al., 2008), the solution of the boundary-value problem for the Poisson equation (1)-(2) can be obtained by applying suitable quadrature rules to Green's function representation (8).

5. OUTLOOK

The solution to the boundary-value problem for the Poisson equation is presented here. This particular problem was selected because of G. Lamé's preference. In the same way, canonical solutions to BVP of various types (also Neumann and Robin problems) can be obtained using Fourier series, avoiding cumbersome numerical techniques such as finite-difference or finite-element methods. It is also applicable in engineering since in three dimensions it allows for the development of computational solutions for mesh-free modeling, without the need for discretization in general. Almost 200 years after Fourier and Lamé, their original contributions to science are now united in the spirit of a Unique Rational Science.

We may go further, since from a purely geometrical point of view there is one and only one curve that can be expressed in a finite Fourier series only, and that is the circle itself, due to a theorem of B-Y Chen (1994). In the study of Riemannian submanifolds Chen introduced *finite type functions of k-type*. The circle is the only planar curve of finite type, namely of 1-type and any other curve is of infinite type (Verstraelen, 1991).

A generalized trigonometric series based on Eq. 1 can associate any term in the series with some anisotropic unit circle. It follows that all supershapes can be described in only one term (and in analogy with Chen finite type curves are of 1-type). As the set of Euclidean circles is a subset of the set of such unit circles, Fourier series reduce to a special case. Since anisotropic unit circles can have cusps or singularities, analysis based on pure shape description incorporates such singularities *a priori*.

In conclusion, with supershapes and Gielis transformations we are able to describe shape and development of a wide variety of basic shapes in nature using only pure numbers and we can begin to understand how other natural beings or objects "*geometrize their world*", with their own shapes as unit circles, and based on a generalized Pythagorean Theorem. We have called this program Universal Natural Shapes (Gielis et al., 2005).

An extension of Euclidean geometry, with a conservation law for n-volumes (or in a 3D world the act of weighing and equilibrium in the spirit of Archimedes, Stevin and Lamé), provides for a uniform description of natural and abstract shapes. It stimulates geometric research in the natural sciences and the development of new computational methods to address a variety of open problems in mathematical physics and mechanics.

"*La mécanique est la science des forces et du mouvement*" (Delaunay, 1856).

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ARCHIMEDES AND CAUSTICS: A TWOFOLD MULTIMEDIA AND EXPERIMENTAL APPROACH

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ABSTRACT In this work we present a twofold educational approach to the reflective properties of surfaces, starting from the historical context of Archimedes "burning mirrors". The properties of the emerging surface known as "caustic" of a given smoothly shaped mirror are illustrated by an interactive multimedia. An experimental device is also proposed to visualize the geometrical principles underlying the formation of caustics. The proposed didactical trail is intended also to contextualize the figure and work of Archimedes in a perspective tightly linked to modern technology, so to collect young learners' interest.

1. INTRODUCTION

The name of Archimedes is, among other things, associated with the defense of the ancient city of Syracuse from Romans during the Second Punic War in 212 B.C. According to tradition, in fact, to face the roman naval attack Archimedes would have used a burning mirror to set fire to the roman ships. Regardless of the authenticity of this story, Archimedes surely studied the geometric properties of the reflection of light from mirror surfaces. This is a very interesting topic from a technological point of view: just consider, as an example, the parabolic reflectors used in microwave transmission and receiving or the variously shaped reflecting surfaces employed to concentrate the light in artificial lighting devices.

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Despite their ubiquity, the geometric properties of reflecting surfaces are barely known, nor are they illustrated in standard scholar curricula.

In this context we present in the present work a twofold didactical approach to the reflective properties of surfaces, with particular regard to the emerging surface known as "caustic" of a given smoothly shaped mirror, i.e. the envelope of all *reflected* rays from it (properly speaking, this is the *catacaustic*; while the term *diacaustic* denotes the caustic obtained as envelope of *refracted* rays). The proposed approach is based both on a multimedia illustrating the properties of caustics, and an experimental device to produce them. The latter is built by employing easily found materials, such as laser diodes obtained from common laser pointers, and allows to explore caustics of most common reflecting conic profiles, i.e. circle, ellipse and parabola. The multimedia shows, in particular, the properties of the curve known as "cardioid", which is the circle's caustic.

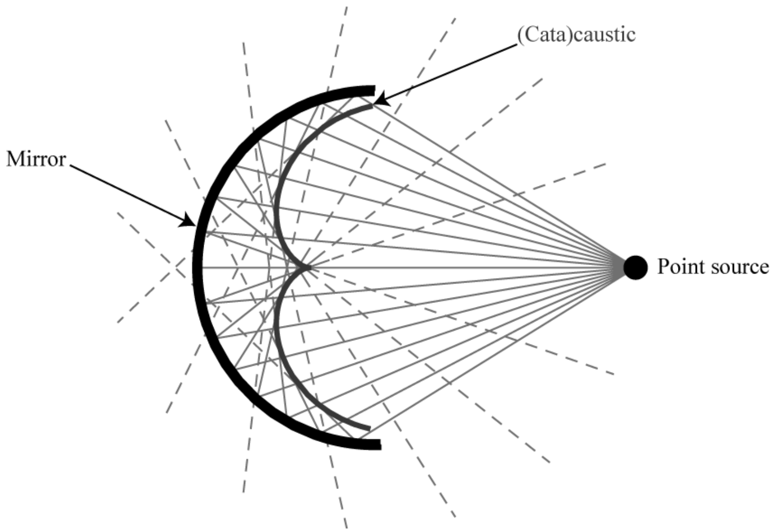

Fig. 1. Geometric construction of the caustic of a spherical mirror.

2. CAUSTICS

Let us consider a regularly shaped reflecting surface, such as a semi-spherical concave surface. To characterize the reflective behavior of such a mirror two cases of incident light rays are usually considered: a point source from which rays diverge isotropically or a beam of parallel rays (equivalent to the former one when the point source goes to infinity). The caustic of a surface is defined as the envelope of the rays reflected from that surface when the radiant point is put in front of it. Figure 1 shows the geometric construction of the caustic in the case of a spherical mirror with the radiant point on its symmetry axis at finite distance. Thanks to the axial symmetry of mirrors it is sufficient to consider a section of the rays' pattern; so the caustics illustrated in the drawings presented are not surfaces, but lines that generate the corresponding surface caustics by an axial rotation. So, properly speaking, those presented, for example in Fig. 2, are the *circle* caustics not the sphere ones. The last are obtained by an axial revolution of the former ones.

Figure 2 illustrates the circle caustics for various positions of the point source and have been obtained by employing Wolfram *Mathematica®* code freely available (Wesstein 2009).

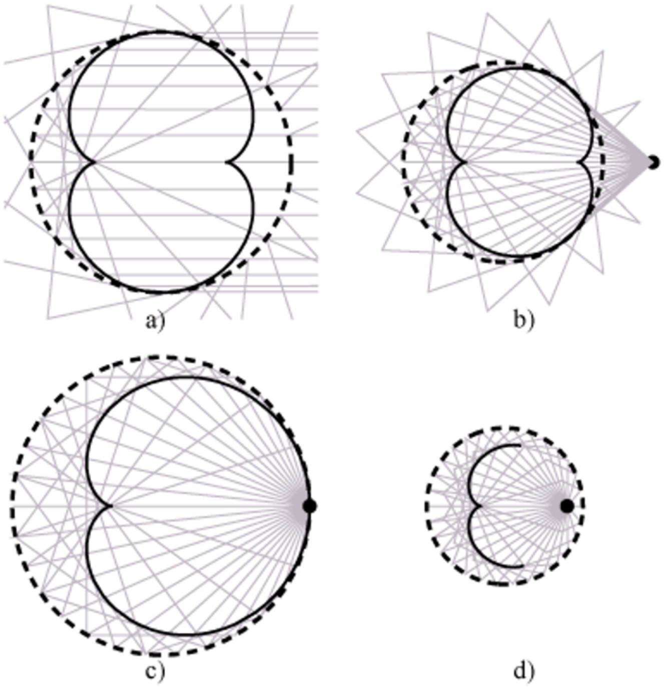

Fig. 2. Caustics of the circle for various positions of the point source: a) at infinite distance (parallel rays); b) outside the circle at finite distance; c) on the circle; d) inside the circle. The caustics obtained in cases a) and b) are named "nephroids" which literally means "kidney shaped". The caustic obtained in case c) is a curve named "cardioid".

3. COMPUTATIONAL CAUSTIC EXPLORATION

The caustic curves of a circle are different, depending on the distance *r* of the light point source from the circle's center, compared to the circle's radius *R*. In particular, if $r<R$, i.e. the source lies inside the circle, the caustic is a curve named *Pascal's limaçon*, if the light source is outside, i.e. $r>R$, the caustic is a *nefroid* (term which means "kidney-shaped"); while one obtains a *cardioid* as a caustic when the point source is just on the circle's perimeter. This last curve is a special case of the epicycloid in which the radius of the outer circle is the same as that of the inner circle.

The didactical use of the computer allows the simulation of mechanisms to trace the aforecited caustic curves beginning from their properties.

The cardiod, drawn by Roemer in 1674 while he was studying the form of a gear constituted by two toothed disks that can rotate one around the other, Fig. 3.

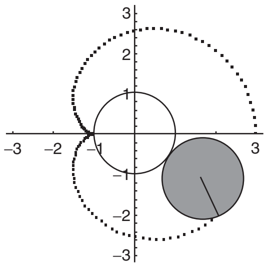

Fig. 3. Cardiod.

The name cardioid, was first used by de Castillon in *Philosophical Transactions of the Royal Society* in 1741 and derives from the Greek terms καρδί (heart) and έιδος (figure). Its arc length was found by P. de La Hire, architect, good mathematician disciple of Desargues, in 1708 (Lockwood 1978). The cardiod is a splendid curve that contains a lot of geometric properties, therefore, she can be treated through the various typologies of equations (Cartesian, parametric, polar).

The polar equation of the curve is:

$$r = a(1 - \cos\alpha)$$

While the Cartesian form is:

$$(x^2 + y^2 + ax)^2 = a^2(x^2 + y^2)$$

and the parametric equations are:

$$x = a\cos t(1-\cos t)$$
$$y = a\sin t(1-\cos t)$$

The cardioid has a cuspid at the origin, Fig. 4. The curve may also be generated as follows: *Draw a circle C and fix a point A on it. Now draw a set of circles centered on the circumference of C and passing through A. The envelope of these circles is then a cardioid* (Pedoe 1995).

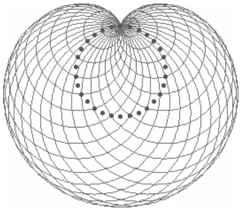

Fig. 4. Cardiod envelope.

Let the circle C be centered at the origin and have radius 1, and let the fixed point be A=(1,0). Then the radius of a circle centered at an angle α from (1,0) is:

$$r^2 = (0-\cos\alpha)^2 + (1-\sin\alpha)^2 = 2(1-\sin\alpha)$$

The cardiod, as envelop (Pedoe 1995), can be represented using the computer and an appropriate software tool, as the programming environment MatCos (Bonanno et al., 2006). Indeed, the following code reaches the purpose:

Code MCl

```
Q = punto;
c1 = circ(Q,80);
print("you choose a point A on the circumference");
A = punto;
per(i da 1 a 220) esegui;
o = puntoacaso_su(c1);
m = distanza(A,o);
Colorepenna(128,0,0);
c2 = circ(o,m);
fine;
```

If the fixed point A is not on the circle, then the resulting envelope is a limaçon instead of a cardioids (Fig. 5).

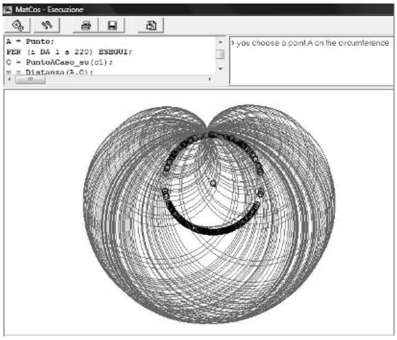

Fig. 5a. Output of Cardiod as envelope of circles.

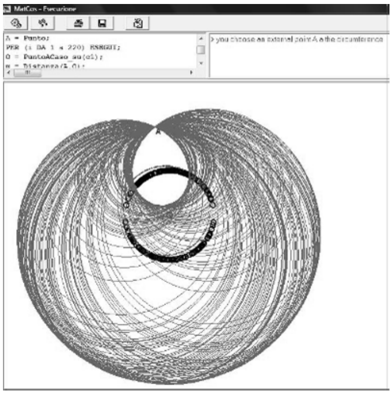

Fig. 5b. Output of limaçon as envelope of circles.

The nefroid is the catacaustic for rays originating at the cusp of cardioid and reflected by it. In addition, Huygens showed in 1678 that the nefroid is the catacaustic of a circle when the light source is at infinity, an observation which he published in his Traité de la Luminère in 1690 (MacTutor Archive).

The name nefroid ('kidney-shaped') was used for two- cusped epi-cycloids by Proctor in 1878; a year later, Freeth used the same name for a somewhat more elaborate curve. The curve Nefroid can be defined as a trace of a point fixed on a circle of radius *r/2* that rolls outside a fixed circle with radius *r*. Also, it is the trace of a point fixed on a circle of radius *(3/2) r* that rolls inside a fixed circle of radius *r* (by 'inside', it means the curvature of both circles at the contact point face the same direction), Fig. 6. The latter is known as double generation.

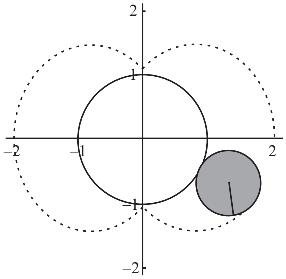

Fig. 6. Nefroid.

The nefroid is given by the polar equation

$$\rho^{\frac{2}{3}} = a^{\frac{2}{3}}\left(\cos^{\frac{2}{3}}\frac{\theta}{2}+\sin^{\frac{2}{3}}\frac{\theta}{2}\right)$$

While the Cartesian equation is:

$$(x^2 + y^2 - 4a^2)^3 = 108a^4 y^2$$

and the parametric equations:

$$x = a(3\cos t - \cos 3t) \qquad 0<t \leq 2\pi$$
$$y = a(3\sin t - \sin 3t)$$

The nefroid can be generated as the envelope of circles centered on a given circle and tangent to one of the circle's diameters (Wells 1991). This can be easily done using the programming environment MatCos, Fig. 7. Indeed, the following code reaches the purpose:

Code MCl

```
rifcart;
f=leggifunz("radiceq(4-x^2)");
g=leggifunz("-(radiceq(4-x^2))");
graficofunz(f);graficofunz(g);
A=punto(0,2);B=punto(0,-2); s=segmento(A,B);
A1=A.x; A2=A.y;
B1=B.x; B2=B.y;
x=legginum("valore -1.99<x<1.99");
x1=x;
```

```
ESEGUI FINQUANDO (x1<2);
  x1=x1+0.1;
  y1=valutafunz(f,x1);
  Q1=y1;
  r=radiceq((x1-A1)^2+(y1-Q1)^2);
  C1=Circ(Punto(x1,y1),r);
FINE;
ESEGUI FINQUANDO (x1>-2);
  x1=x1-0.1;
  y1=valutafunz(f,x1);
  Q1=y1;
  r=radiceq((x1-A1)^2+(y1-Q1)^2);
  C1=Circ(Punto(x1,y1),r);
FINE;
ESEGUI FINQUANDO (x1<2);
  x1=x1+0.1;
  y1=valutafunz(g,x1);
  Q1=y1;
  r=radiceq((x1-A1)^2+(y1-Q1)^2);
  C1=Circ(Punto(x1,y1),r);
FINE;
```

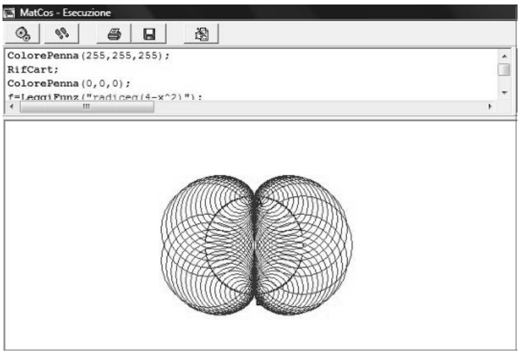

Fig. 7. Output of nefroid as envelope of circles.

4. EXPERIMENTAL CAUSTIC EXPLORATION

In this section an experimental device is presented aimed to really illustrate the way in which the emerging caustic curve is formed by the reflected rays. The proposed device is realized by employing easily found materials.

In fact, it is well known in the literature the learning potential either of experimental setup realized by means of common use stuffs that students know from everyday life or of informal learning (Jodl and Eckert 1998, Bosio et al., 1997, Michelini 2006).

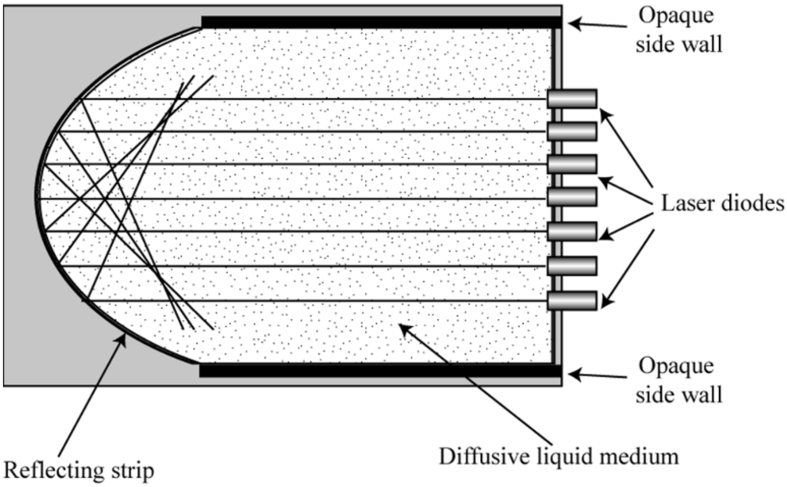

Fig. 8. Experimental device illustrating the geometry of the caustic formation. A parallel rays beam is obtained by using an array of small laser diodes. A flat box, having a reflecting profile (either circular, elliptic or parabolic) is filled with a diffusive liquid medium that allows to visualize the rays' pattern.

The device, illustrated in Fig. 8 consists in a flat transparent box (approximate dimensions 20 cm x 35 cm x 3 cm) filled with a slightly diffusive liquid. This last, prepared as a 2% aqueous solution of Maalox®, the well known syrup used to treat heartburn, has the property of visualizing the pattern of light rays passing through it. In low cost optics experiments the visualization of light rays is usually obtained by an aqueous suspension of powder milk; however, this solution has the disadvantage that after a certain time the milk goes bad and smells. This unpleasant side effect is eliminated by employing Maalox instead!

On one side of the flat box (on the left in Fig. 8) there is a reflective mirror strip fixed on a rigid support forged in a conical profile (either circular, elliptic or parabolic).

On the other side (right in Fig. 8) a lighting module is fixed, generating a parallel red rays beam. This beam generator (Fig. 10) is constituted by an array of small laser diodes obtained by breaking some laser pointer keychains (Fig. 9). In this way, when lasers are turned on, a parallel rays beam is visualized in the diffusive medium filling the box, together with the pattern of rays reflected from the mirror strip. Reflected rays tend to dispose following the strip profile caustic. In particular, in the case of parabolic mirror the reflected rays are nearly crossing through a single point: the focus. This last, in fact, is a degenerate case of caustic. We should emphasize a constructive detail: the narrow sides of the flat box are realized by opaque black PVC. This is to avoid further reflections from these walls, that otherwise would complicate the rays pattern in the liquid medium.

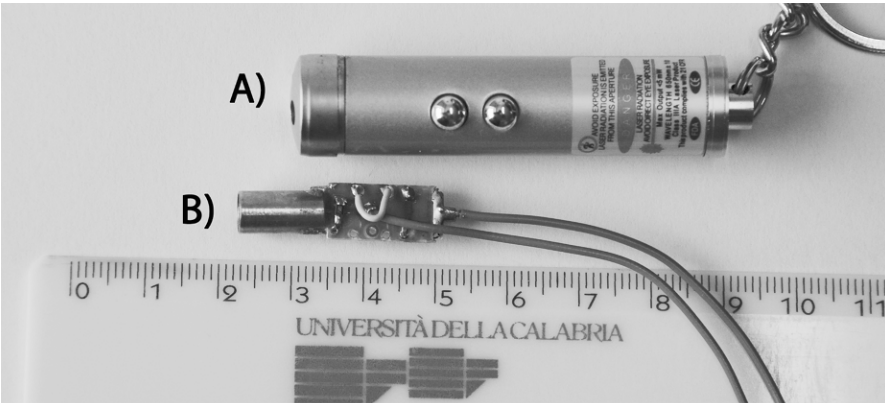

Fig. 9. A laser pointer keychain (A) from which a laser diode module (B) has been extracted. The diode case is about 1 cm long, 5 mm wide.

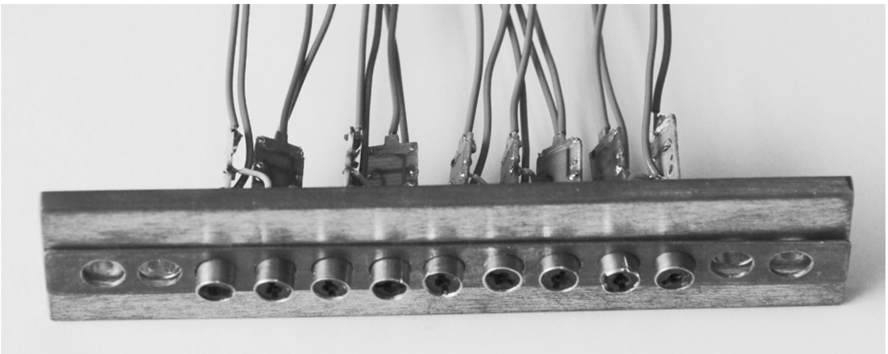

Fig. 10. A Laser diodes array for the generation of a beam of parallel light rays. The laser diodes, extracted from laser pointer keychains (Fig. 9) are fixed to a 12 cm long copper rod.

5. CONCLUSION

In this work we present a learning path on caustics, starting from the legendary Archimedean "burning mirrors". The proposed approach to the reflective properties of surfaces integrates the computational point of view with the experimental one. In particular, we propose an experimental device, realizable with low-cost and easy-to-find materials, that allows to visualize the geometrical principles underlying the formation of caustics.

The proposed didactical trail is also intended to contextualize the figure and work of Archimedes in a perspective tightly linked to modern technology, so to collect young learners' interest. In fact, whereas the Archimedean influence on present-day science and technology is well known as far as mechanics and hydrostatics are concerned, by far less known is the role of the Syracuse's thinker in other modern technological areas. We make reference in particular to all those optic and electronic devices whose functioning is essentially based on the principles of Archimedean "burning mirrors": radiotelescopes, reflecting optical telescopes, parabolic antennas for microwave-based telecommunications, and obviously the cars headlight reflectors. The study of the reflective properties of conic sections, together with the observation of the several technological contexts in which they are present, is useful to emphasize the Archimedes' influence on present-day technology.

As a concluding remark, we should note that the educational activity proposed is also aimed to highlight the role of Archimedes elaborations from the point of view of the history of mathematics. The Syracuse's thinker, in fact, has been variously acknowledged throughout centuries as a father of the topic proposed in this work (the reflective properties of conic sections), as testified among others by an influential treatise published in 1632 by the Italian mathematician Bonaventura Cavalieri (Cavalieri 1632), whose full title is: "The burning mirror, or a treatise on conic sections and some of their wonderful effects".

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ARCHIMEDES' QUADRATURES

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ABSTRACT In the paper we discuss the three methods that Archimedes employs to deal with the problem of the quadrature of a parabolic segment. We characterize the three approaches as heuristic, mechanical and geometric respectively. We investigate Archimedes' own attitude towards the three methods, and we conclude with a critical presentation of the prevalent views concerning the matter, which have been expressed in the past by historians of mathematics.

1. INTRODUCTION

Euclid (*fl. ca*. 300 B.C.) and Archimedes (*ca*. 287–212 B.C.) are the two most prominent figures in the history of mathematics of Antiquity. Both of them are part of that group of Greek mathematicians who, through their work, deeply influenced the development of early modern mathematics. The fact that they lived geographically and chronologically close to each other could lead one to infer that their scientific work, too, is completely analogous in breadth and content and that Archimedes also worked, at least up to a point, within the framework of the research program that Euclid had initiated a few decades earlier. Such a conclusion, however, is far from the truth. More specifically, as far as mathematics is concerned, the study of the work of the two men reveals that Euclid and Archimedes belong to two mathematical traditions which, although not entirely irrelevant to each other, are nevertheless distinct from each other, while both of them can be traced back to the times of the classical Greek mathematics.

The first was the "tradition of *stoicheiôsis* of mathematics." It was a tradition which focused on studying the logical structure of mathematical reasoning, ensuring the rigor and simplicity of mathematical proof and generally organizing and systematizing the structure of the mathematical edifice. Euclid was the main representative of this tradition and his *Elements* was the crowning achievement of the researches in the field.

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Alongside the tradition of *stoicheiôsis*, a second one was developed among the Greek mathematicians, more or less in the same period, which would be called the "metric tradition". Without being wholly unrelated to the former one, the metric tradition focused mainly on measuring geometric shapes, that is on discovering techniques to develop formulas, as we would say today, in order to measure the area or the volume for two- and three-dimensional shapes, as well as on developing arguments to prove these formulas. The metric tradition was established by Eudoxus (390–337 B.C. or 408–355 B.C.) although its origins can be traced even further back in time, in the second half of the 5th century, in the work of Democritus. Nevertheless, the tradition came to full fruition in the 3rd century, with the work of Archimedes. (Knorr 1993, 151–152)

Archimedes is the foremost representative of the metric tradition among the Greek mathematicians. An important part of his work is about quadrature and cubature, and indeed most of his treatises are devoted to such issues. These treatises could be characterized, by analogy with the tradition to which they belong, as "metric," and they are the following: *"Measurement of a Circle*, *Quadrature of the Parabola*, *On the Sphere and the Cylinder*, *On Conoids and Sheroids*, *On Spirals*, and *The Method of Mechanical Theorems*. Relevant to the above mentioned works, and more specifically to *The Method of Mechanical Theorems* and the *Quadrature of the Parabola*, is also the treatise *On the Equilibrium of Planes"*.

Among the works of Archimedes which survive in the Greek language, only *Sand-Reckoner, Floating Bodies* and *The Cattle Problem* are not related to the issue of quadrature and cubature, while the subject matter of his treatise *Stomachion* still remains uncertain, despite recent progress. It is worth noting, however, that even some of these works deal, in a way, with measurement issues. Thus, *Sand Reckoner* talks about the number of specks of sand which could fill the universe, *The Cattle Problem* is about determining the number of four sets of bulls and cows which satisfy certain conditions, while *Stomachion* is about the combinatory problem of the number of solutions to the problem of making a square out of the rearrangement of the 14 puzzle pieces into which the square had originally been divided. (Netz, Acerbi, Wilson 2005)

The statement that the mathematical work of Archimedes belongs to the "metric" tradition of Greek mathematics and that Archimedes is the leading representative of the tradition is by no means a new conclusion. It is common ground in the history of Greek mathematics nowadays, that Archimedes made use of the infinitesimal methods developed by Eudoxus, he refined and expanded them further and applied them skillfully to a great number of quadrature and cubature cases of geometric shapes.

Of the multitude of quadrature and cubature cases included in the works of Archimedes, two of them are especially interesting. The first such case is the quadrature of a parabolic segment. What is interesting about this quadrature is the fact that Archimedes deals with it three times, in two different treatises: in the *Quadrature of the Parabola* (hereafter, the *Quadrature*) and in *The Method of Mechanical Theorems* (hereafter, *The Method*). Indeed, the former is solely devoted to the subject of the quadrature of a parabolic segment, which Archimedes investigates using two different methods (a mechanical one and a geometric one), while the latter features the quadrature of the parabolic segment as an example of the application of the heuristic method, which Archimedes had devised in order to determine the surface area and the volume of geometric shapes, independently of the formal, rigorous proof of the conclusions he drew. The second such case appears in *The Method*. More specifically, in Propositions 12–15 of this treatise, Archimedes deals with the same problem of the cubature of a cylindrical segment (a "hoof") three times and in three different ways. So, in Propositions 12–13 he investigates the problem using a mechanical method, in Proposition 14 he employs the use of "indivisibles" (which, as we shall see, is an essential part of his heuristic method) and in Proposition 15 by a geometric method. (Saito 2006, 36 n° 3).

Obviously, the existence of multiple ways of handling these two problems in the works of Archimedes, even within the same work, poses a number of historiographical questions about the content, the role and the weight that the Syracusan mathematician placed on the various methods of quadrature that he employed. Later on in this paper, we shall draw on the example of the three ways which Archimedes employs to deal with the quadrature of a parabolic segment, as a means to investigate such questions raised by the historiographical research. Before doing so, however, it would be useful to mention a few facts concerning the two treatises in which Archimedes studies the quatradure of the parabolic segment.

2. THE *QUADRATURE* AND *THE METHOD*: SOME FEATURES OF THE TWO TREATISES

The *Quadrature* and *The Method* differ from each other both in style and use. The *Quadrature* has the style of a formal publication. It was written by Archimedes in order, as we would say, to have it published as a book addressing a reading public. *The Method's* form, on the other hand, gives the impression of a text extracted from Archimedes' personal records. It is attached to a letter to Eratostenes and it addresses, at most, a little circle of mathematicians associated with Eratosthenes. This work has a much stronger personal touch. It's more like an elaborate version of the notes a geometer keeps during his research project. Repeating a phrase by Dijksterhuis, we may say that, through this work, Archimedes allows us to have a look inside his mathematical study room. (Dijksterhuis 1987, 315) So, the two treatises have significant differences between them in form and intention.

The second point worth mentioning is that the writing of the *Quadrature* chronologically precedes *The Method*. This derives directly from a passage in Archimedes' letter to Eratosthenes, where he writes: "I now wish to describe the method in writing, partly, because I have already spoken about it before, that I may not impress some people as having uttered idle talk, partly because I am convinced that it will prove very useful for mathematics." (Dijksterhuis' translation) By saying that he has spoken in the past about it, Archimedes actually refers to a phrase in the preface of the *Quadrature* where he mentions that he first discovered the theorem about the square area of a parabolic segment by means of mechanics and then proved it by means of geometry. By juxtaposing the two extracts, it is concluded that Archimedes wrote the *Quadrature* before writing *The Method*. The same conclusion arises from the closing phrase of the first proposition of *The Method*, where, after stating that he found, by means of mechanics, the area of a segment of a parabola to be $^4/_3$ of the triangle which has the same base as the segment and equal height, Archimedes adds: "This has not therefore been proved by the above, but a certain impression has been created that the conclusion is true. Since we thus see that the conclusion has not been proved, but we suppose it is true, we shall mention the previously published geometrical proof, which we ourselves have found for it, in its appointed place." (Dijksterhuis' translation) In this extract Archimedes once more refers to the *Quadrature*, which leads to the conclusion that the writing of this work chronologically precedes the writing of *The Method*.

Of course, the chronological order of the writing of the two treatises does not coincide with the order in which Archimedes conceived what is included in them. On the contrary, the discovery of a theorem always precedes its formal, rigorous proof. The problem of the classification of Archimedes' works according to the chronological writing order, from the one hand, and according to the order they occupied in Archimedes' research agenda, on the other hand, is, indeed, a problem hard to solve, which still preoccupies the historians of Greek mathematics. (Knorr 1978; Vitrac 1992)

3. THE QUADRATURES OF THE PARABOLIC SEGMENT: A HISTORIOGRAPHICAL DISCUSSION

In this paper we shall not enter into the detailed technical presentation of the three methods Archimedes employs to deal with the problem of the quadrature of a parabolic segment. Such a presentation can be found in any book of the history of Greek mathematics, and, in particular, in (Dijksterhuis 1987), which is still considered the best review of Archimedes' complete works. We merely note that Archimedes investigates the problem in proposition 1 of *The Method*, by using an approach which can be characterized – anticipating in a way the discussion which will now follow– heuristic; in propositions 14 and 16 of the *Quadrature of the Parabola*, by using an approach which can be characterized as mechanical; and in proposition 24 of the same treatise, by using a geometric approach. We shall now examine Archimedes' own attitude towards the three methods, in other words, how he perceives the role and the importance of each one, so as to conclude the article with a critical presentation of the prevalent views concerning the matter, which have been expressed in the past by historians of mathematics. (Knorr 1982; 1993; 1996; Dijksterhuis 1987; Netz, Saito, Tchernetska 2001; Saito 2006)

Archimedes uses two pairs of terms to characterize, and distinguish from each other, the three methods which he employs in order to investigate the problem of the quadrature. These pairs could be rendered as: "heuristic" – "demonstrative", and "mechanical" – "geometric". It is true that not all of these words occur *en personne* in Archimedes' texts. The word "heuristic," for example, does not occur anywhere. Instead, other equivalent forms appear in expressions such as "*dia mechanikôn heurethen*," "*tou nun ekdidomenou theorêmatos tên heuresin*," "*fanentôn mêchanikôs*," "*fanen dia tôn mêchanikôn*." Similarly, instead of the word "mechanical," the phrase "*dia tôn mêchanikôn*," or the adverbial form "*mechanikôs*," occur. In the same way, variants of the other two words, "demonstrative" and "geometric," appear in the texts. For brevity's sake, from now on we shall render all the variants using the word pairs that we mentioned above.

Let us now see how Archimedes characterizes each method. The quadrature expounded in *The Method* is characterized as "heuristic" and "mechanical." Its heuristic character lies, he notes, in the fact that it gives one the ability to know in advance, by means of mechanics, some mathematical properties, a knowledge which is useful in finding proof for the relevant theorem. For "it is easier to supply the proof when we have previously acquired, by the method, some knowledge of the questions than it is to find it without any previous knowledge." (Dijksterhuis' translation). Nevertheless, Archimedes is keen to point out that the quadrature achieved by this method does not constitute proof of the conclusion. This is deduced from the clarification at the end of the proposition 1 of *The Method*, where he mentions: “This has not been proved by the above, but a certain impression has been created that the conclusion is true.” (Dijksterhuis’ translation). There is no doubt, therefore, that Archimedes considers the first method of quadrature, that is the quadrature expounded in his treatise *The Method*, as mechanical, heuristic, but not demonstrative.

Similarly clear is the way Archimedes treats the last of the two quadratures exposed in *Quadrature*. He characterizes it as geometric and demonstrative. This derives right from the following extract taken from the preface: “I have therefore written out the proofs (he is referring to the quadrature of the parabolic segment), and now send them, first as they were investigated by means of mechanics, and also as they may be proved by means of geometry.” (translation by Ivor Thomas) This extract is similar to another one taken from the same preface: “I set myself the task of communicating to you … a certain geometrical theorem, which had not been investigated before, but has now been investigated by me, and which I first discovered by means of mechanics and later proved by means of geometry.” (translation by Ivor Thomas). In both extracts Archimedes is referring to the geometric proof exposed in the second part of the *Quadrature of the Parabola*, in which the main theorem is proposition 24. The fact that, in the latter he uses the expression “exhibited (*epideichthen*) by means of geometry,” instead of the expression “proved (*apodeiknytai*) by means of geometry” used in the former, does not, in any way, change the conclusion that Archimedes considers the method he employs to deal with the problem of the quadrature of a parabolic segment in the second part of his treatise, as both geometric and demonstrative.

Now, as far as the method of quadrature expounded in the first part of the *Quadrature of the Parabola* is concerned, things are not so clear as in the previous two cases. Of course, there is no doubt about its mechanical nature. The problem lies in whether Archimedes believes that this reasoning constitutes a convincing and acceptable proof. As we shall see later on, this is exactly the point, which has led to disagreement among modern historians of mathematics. In our opinion, the extract quoted above (“I have therefore written out the proofs, and now send them, first as they were investigated by means of mechanics, and also as they may be proved by means of geometry”), in which Archimedes uses the plural form “proofs” to characterize both of the quadratures exposed in *Quadrature*, constitutes a piece of evidence which should not be overlooked.

However, the matters concerning this last mentioned method of quadrature are more complicated than what we presented earlier on, because the question is raised as to how valid and mathematically rigorous can a proof be which uses notions and reasonings taken from mechanics. The argument developed by Archimedes in this quadrature, is, in a way, a hybrid endeavor, which draws both on geometry and mechanics. It is true that the argument which is developed in the quadrature of *The Method* has similar characteristics. Archimedes, however, does not consider this quadrature as demonstrative; he considers it as heuristic, so, in this sense, no question of its demonstrative validity is raised. The quadrature, on the other hand, exhibited in the first part of *Quadrature*, does not have the characteristics of a heuristic procedure. Its development requires the conclusion to be known in advance. From the other hand, its mechanical character is indisputable and it consists in: a) considering the geometric magnitudes as physical (namely, as having weight), b) the use of the weighing balance, c) the application of the law of the lever, d) the use of properties concerning the centers of gravity. Is, however, the investigation of geometric properties, using arguments taken from mechanics, an acceptable method? To the pure mathematician, imbued with the Euclidean ideal of the rigor of proof, it would be unacceptable. Vitrac writes about this: "Pour un puriste ceci n'est pas recevable comme démonstration géométrique car il y a un problème par rapport aux principes de la démonstration utilisée." (Vitrac 1992, 75) Vitrac also refers to sections I, 6-7 from Aristotle's *Analytica posteriora*, adding that, "Une figure en mécanique a poids et grandeur; en géométrie elle a seulement une grandeur." (Vitrac 1992, 75 n^{o} 56) Indeed, according to Aristotle, principles taken from different scientific disciplines should not be used within the same proof; one should not enter a field of study using means and techniques belonging to another field. However, Archimedes himself, as it has been mentioned, calls, even if once, the quadrature demonstrative. Based on the above observations, it is clear that there are some open historiographical questions pertaining to the role and the character of the quadrature exhibited in the first part of the *Quadrature of the Parabola*. Let us now see how the historians of mathematics approached this subject.

The starting point of our investigation will be the quadrature set forth in *The Method*. As it has been mentioned, Archimedes always characterizes this method as heuristic, he adds that it does not constitute proof, and refers for its proof to his treatise *Quadrature of the Parabola*, which had been published earlier. The question which naturally arises is why the method employed in the quadrature of *The Method* is not viewed as sufficient in order for the conclusion to be considered valid and rigorously proved. Is there some lack of mathematical rigor in the method, and, if so, where exactly is this lack traced?

A decoding of the method reveals that there are two different types of arguments used in it:

1. Firstly, arguments taken from mechanics are used. The geometric magnitudes are considered as having weight, they are suspended from the beam of a hypothetical weighing balance, the law of the lever is applied to deduce relationships between the geometric magnitudes, and properties related to the centers of gravity are used.
2. Secondly, a plane figure is considered as made up of "all" the parallel segments of straight lines drawn along a given direction and whose endpoints lie on the perimeter of the figure. So, the figure may be decomposed into such parallel chords of a given length, and be reconstructed again by them. The "sum" of the segments of straight lines gives the area of the plane figure. We shall call these segments of straight lines "indivisibles." This notion can also be extended to solid figures, which can be decomposed into parallel cross-sections.

Taking into consideration these two different types of arguments involved in Archimedes' reasoning, let us now examine where, according to the historians of mathematics, the lack of mathematical rigor of this method of quadrature is located. The most frequently expressed view in the bibliography is the one formulated by E.J. Dijksterhuis. According to Dijksterhuis, the lack of mathematical rigor is due to the employment of the "indivisibles" and not, by any means, to the mechanical aspects of the method. (Dijksterhuis 1987, 319, 336) On the contrary, Dijksterhuis says that Archimedes assigned demonstrative validity to the mechanical aspects of the method, and this is inferred, first of all, by the fact that Archimedes himself had established mechanics (statics) as a demonstrative science in *On the Equilibrium of the Planes*, and secondly by the fact that, in the first part of *Quadrature*, he proves the conclusion about the parabolic segment applying mechanical considerations, but not indivisibles. In the following years, Dijksterhuis' point of view was adopted by other scholars and today it is considered as dominant.

Apart from the scholars who embraced Dijskterhuis' position, there were others who disagreed, claiming that the lack of rigor of the method is due not only to the use of indivisibles, but to its mechanical nature, as well. As a consequence, those scholars maintained that, because of its mechanical attributes, the quadrature in the first part of *Quadrature* does not constitute a valid and rigorous proof either, and that neither Archimedes considers it as such. An earlier historian who expressed such a view was Oskar Becker, from whom we quote the following extract: "Archimède ne tient pas cette méthode pour rigoureuse, tout d'abord à cause de considérations infinitésimales qui remontaient partiellement à Démocrite (B 155) …, mais aussi à cause de l'emploi de la Statique. Ainsi dans la *Quadrature de la parabole* il remplace dans ses considérations par exhaustion (le segment de parabole est décomposé non en un nombre infini de segments mais en un nombre d'éléments qui, fini à l'origine, est progressivement porté à l'infini). Cela conduit encore à une démonstration purement géométrique au cours de laquelle il utilise en même temps que certaines intégrales définies des séries infinies convergentes." (Becher, Hofmann 1956, 81–82) Becker repeated his view a year later, in a critique that he wrote on the English edition of Dijksterhuis' book. (Becker 1957)

The historian of mathematics, however, who most forcefully expressed his objections to the position of Dijksterhuis, was W.R. Knorr. (Knorr 1982; 1996) According to Knorr, the main weakness of Archimedes' method, as far as its mathematical rigor is concerned, is exactly its mechanical nature and not the indivisibles. As he notes, "in Archimedes's account the indivisibles are merely a secondary aspect; for the essence of his method lies in its appeal to mechanical principles." (Knorr 1982, 73) The arguments that Knorr appeals to in order to justify his view are the following: 1) Archimedes always refers to the heuristic method using the expression "*dia tôn mechanikôn*;" he never uses anything which would imply the indivisibles. 2) In the extracts 218.11–12 and 220.17–20 from *Quadrature* quoted above, Archimedes juxtaposes the "demonstrative" to the "mechanical." In this setting, when he mentions in *The Method* that some of the theorems he originally found by means of mechanics, he later proved by means of geometry, because "the investigation using this procedure does not constitute proof," he can only refer, Knorr claims, to the mechanical attributes of the method. 3) Finally, the inclusion of the geometric proof in the second part of the *Quadrature of the Parabola*, is due to Archimedes' wish to forestall possible objections raised in the name of pure mathematics about the legitimacy of the use of mechanical elements (such as weight, the weighing balance, equilibrium) in proofs which concern exclusively geometric properties of geometric figures, as in the case of the quadrature which takes place in the first part of the *Quadrature of the Parabola*.

Our comments on the above arguments are the following: Archimedes uses the expression "*dia tôn mechanikôn*" both when he describes the heuristic method, and when he refers to the mechanical quadrature of the *Quadrature of the Parabola*. In the fist case, however, he is careful to always add to this expression a participle such as "*heurethen*" or "*fanen*" (found), something he does not do in the second case. The juxtaposition, therefore, is not between the "demonstrative" and the "mechanical" as such, as Knorr claims, but between the "demonstrative" and the "found *dia tôn mechanikôn*", in other words between the "demonstrative" and the "heuristic." On the contrary, Archimedes by no means juxtaposes the "demonstrative" with the mechanical quadrature in *Quadrature of the Parabola*, since, as we have seen, he characterizes the latter as proof. Besides, the last argument does not state anything as to how Archimedes himself evaluated the mechanical quadrature of the *Quadrature of the Parabola*. The possibility that some mathematicians might raise objections over the legitimate use of mechanical elements in geometric proofs, does not mean that Archimedes himself shared their views. On the contrary, it is plausible to assume that Archimedes considered as legitimate and convincing the mechanical proof of the quadrature of the parabolic segment, which he himself invented, and that the inclusion of the geometric proof in the second part of the *Quadrature of the Parabola*, aimed at making the mechanical proof more easily acceptable by a public which might have had some disbelief and objections about the latter, but had no reservations whatsoever about the validity of the former.

The main issue in the preface to *Quadrature of the Parabola* is not to juxtapose the mechanical with the geometric treatment of quadrature problems, as Knorr claims. The main issue is to address the question of what kind of propositions should be taken as lemmas (axioms) in order for the proofs to be considered as valid. Archimedes notes that, in the past, there had been geometers who tried to solve (and to justify the solution, to prove) problems such as the quadrature of the circle, the quadrature of a segment of a circle, or the quadrature of an area bounded by an ellipse section and the chord at its ends, using lemmas which were not easily acceptable. The use of such lemmas had the result that the solutions proposed by those geometers were not recognized, by most of their colleagues, as having validity. However, nobody, Archimedes notes, had tried to square the parabolic segment, in the past. This problem was solved for the first time by himself, "and for the proof this lemma is assumed: given [two] unequal areas, the excess by which the greater exceeds the less can, by being added to itself, be made to exceed any given finite area." (translation by Ivor Thomas)

Archimedes explains that this lemma –often referred to as the "continuity axiom" in the bibliography–, had also been employed by earlier geometers, because, by its use or the use of similar lemmas, they showed several theorems that are included in Book XII of Euclid's *Elements*. By saying that Archimedes refers, from the one hand, to Eudoxus, and, from the other, to Proposition X, 1 of Euclid's *Elements*. So, after acknowledging the theorems which Eudoxus had proved, in the past, using a similar version to his continuity axiom, Archimedes adds the following critical phrase: "In the event, each of the aforesaid theorems has been accepted, no less than those proved without this lemma; and it will satisfy me if the theorems now published by me obtain the same degree of acceptance." (translation by Ivor Thomas).

The first conclusion drawn from this phrase is that the broader subject matter which occupies Archimedes in the preface to the *Quadrature* is proofs and their validity. Secondly, the question which preoccupies him is not the validity of the mechanical, as opposed to the purely geometric proofs, but the validity of the proofs which make use of the continuity axiom (independently of whether they are mechanical or purely geometric) as opposed to those which do not make use of this axiom. Thirdly, Archimedes states that he himself believes that the proofs which make use of the continuity axiom (in any version) are no less valid than the common geometric proofs which are carried out without the use of the aforementioned axiom.

In the context of the above discussion, Archimedes presents in the main body of his treatise two methods of treating the quadrature of a parabolic segment, a mechanical one and a purely geometric one, which he calls "proofs" and which, both of them, use the continuity axiom. Taking into consideration all of the above, we reach the conclusion that Archimedes included the mechanical treatment in the *Quadrature of the Parabola* as an entirely legitimate proof of the theorem of the quadrature of a parabolic segment, for which he only claims it to be considered as valid as the purely geometric proof, or proofs, of Eudoxus, which are included in the twelfth book of Euclid's *Elements*. Finally, as far as the quadrature of *The Method* is concerned, we are in accord with Dijksterhuis' view, namely, that its lack of rigor is due to the use of the indivisibles, and not to its mechanical aspects, and that this is the reason why Archimedes considers this method as heuristic and not demonstrative.

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ON ARCHIMEDES' PURSUIT CONCERNING GEOMETRICAL ANALYSIS

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ABSTRACT Archimedes practices the heuristic method of analysis and synthesis only in Book II of his *On the Sphere and Cylinder*. This paper has a twofold objective. Firstly, the discussion of his analytical practice through the first problem of Book II, in relation to Pappus' study of the method of analysis and synthesis in Book VII of his *Mathematical Collection*. The conclusion of this discussion is that Archimedes applies the analytical method in a way, which does not substantially differ from Pappus' way. Secondly, the discussion about the missing part from the analysis of problem 4 of *On the Sphere and Cylinder*, II, combined with the above conclusion, lead us to advance a conjecture vis-à-vis a lost analytical treatise of Archimedes under the title *Book of Data*.

1. INTRODUCTION

It is widely accepted in the history of science that Greek mathematicians were very thorough in order to present a perfect form of their mathematical arguments in their writings through which they published their research. This goal, however, was being pursued at the expense of the reader's possibility of getting a faint idea of the method through which the result was obtained. Euclid's *Elements*, the most renowned work of Greek mathematics, is the most representative example of a book that follows this approach of Greek mathematicians.

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Archimedes is an exception to the aforementioned rule. In some of his works, Archimedes does not hesitate to register the method used to find the solution to the geometrical problems, before presenting their rigorous construction following the Euclidean model. Actually, in his *Method of Mechanical Theorems*, he presents the heuristic method he used in order to reach specific results, which are proved in a formal way elsewhere in his treatises. The value of this particular work has many times been exalted in recent historiography.

It should be mentioned, however, that the undoubtedly great importance of this work and the justifiable interest of scholars for it, sometimes contributed to the overlooking of the fact that the mechanical method was not the only method used by Archimedes in order to attain solutions to difficult problems or to prove theorems. It is widely known that Archimedes, as well as other Greek mathematicians of Plato's era and onwards, had also used the method of analysis and synthesis to this end. Thanks to Pappus we know that at least twelve works were written in antiquity on the subject of the heuristic method of analysis, while in recent literature it has been supported that analysis is behind the entire corpus of Greek geometry (Knorr 1993). Discussing the importance of the mechanical method in Archimedes' work, Dijksterhuis claims that: "In this exceptionally interesting document Archimedes therefore vouchsafes us a much more intimate glimpse of his mathematical workshop than was ever granted by any other Greek mathematician" (Dijksterhuis 1987, 315). However, taking into consideration the extent of the method of analysis in Greek geometry, this statement seems to be an exaggeration because like the mechanical method, analysis also reveals the mathematician's way of thinking while solving a problem. Moreover, there are extant analyses not only from Archimedes but also from geometers such as Apollonius, Euclid, Diocles, Pappus and others, whose work and examples also –to use Dijksterhuis' expression– vouchsafe us an intimate glimpse of their mathematical workshops.

In the extant work of Archimedes, the method of geometrical analysis is applied only in Book II from his work *On the Sphere and Cylinder*. Moreover it is well-known that the most thorough study of the method of analysis preserved from Greek antiquity is traced back to the period of Late Antiquity and is found in Book VII of the *Mathematical Collection* by Pappus of Alexandria. In this book, Pappus presents three theoretical descriptions of analysis, which have been greatly discussed and argued upon by scholars, and proceeds by applying the method in a number of geometrical problems.

This paper begins with a brief presentation of the basic principles of analysis, according to Pappus, followed by the discussion of an example from the analytical practice of Archimedes, and by conclusions arising from the comparative study of the analytical practice of the two geometers. Finally, through the discussion of the missing part from the analysis of problem 4 of *On the Sphere and Cylinder*, II, a conjecture is advanced according to which Archimedes had written a work on analysis which unfortunately no longer exists.

2. PAPPUS' DISCUSSION OF GEOMETRICAL ANALYSIS

Geometrical analysis, as described and practiced by Pappus in his *Mathematical Collection*, and as discussed in (Fournarakis, Christianidis 2006), comprises two parts, the analysis and the synthesis. In the first part, the analysis, which is the heuristic part of the method, the geometer intends to find a solution to the problem, but also to confirm that the solution is valid. In the second part, the synthesis, he presents the construction and the demonstration of the found solution, according to the Euclidean model.

The starting point of analysis is the admittance of the sought as if it were established and aims the noetic conception of the structure of the problem. In the aforementioned paper we argue that Pappian analysis includes two distinct parts. In each part, the analytical course follows a different main direction. The first part of analysis, that we called "hypothetical", is a course from the conclusion to the premises and therefore it is an upward movement. The second part of analysis, that we called "confirmatory," is a deductive process and therefore it is a downward movement. This confirmatory part is characterised by the use of the terms "*dothen*" and "*dedomenon*."

The hypothetical part begins from what one is seeking as if it were established, and aims to reach something that is true independently of the sought. The geometer, through this part, intends to arrive to something from which he *supposes* that the sought can be produced and the problem can be solved. The steps of this search have hypothetical character, since they are all based on the initial assumption that the sought has been accomplished. This search is not blind or exhaustive; it includes a number of upward noetic leaps, the results of which cannot be foreseen. The first of those leaps is the assumption that the sought has been accomplished. However, making one of these leaps and producing some of its consequences (which are also hypothetical), does not univocally and surely lead to finding the next leap, but it demands the combination of elements such as the researcher's knowledge, mental ability, experience and intuition. The last leap is the finding of something that is true independently of the sought. This is indeed a noetic leap because its admittance as the end of the hypothetical part includes two fundamental hypotheses: a) it can be produced independently of the sought, and b) it can be the starting point of a syllogism that will produce the sought. This is our interpretation of Pappus' understanding of the first part of analysis.

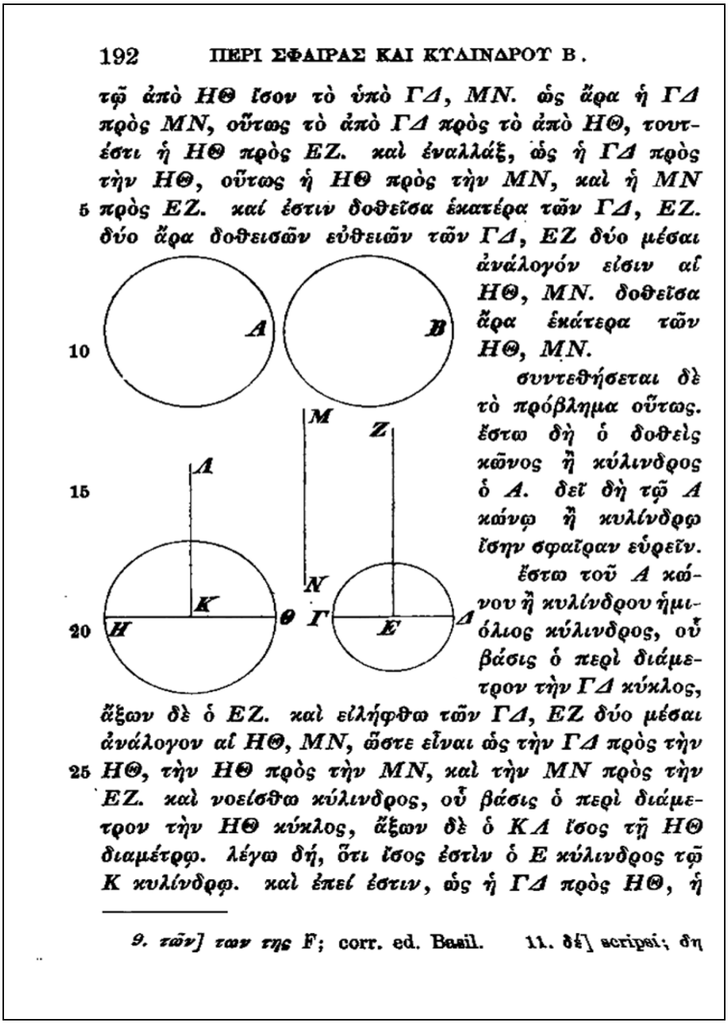
The confirmatory part, on the other hand, aims to assure that those contained in the hypothetical part, if taken in (somehow) reverse order, constitute a deduction which *can* solve the problem. It is an elaboration of the previous conceptions, in a course of valid deductions, which confirms the validity of the syllogism as to whether or not it *can* produce the necessity of the sought from the elements of the problem, as well as from the axioms and theorems of geometry. The confirmatory part does not concern concrete objects but "potential objects" that *can* be produced through the valid steps of the syllogism. The "potentiality" of these objects (or relations) is revealed by the use of the "given" (*dothen*-*dedomenon*) terminology. This terminology is used by Greek mathematicians only in the second part of the analytical process. This is how we interpret Pappus' understanding of the second part of analysis.

According to the account presented above, Pappus' analysis includes two directions, an "upward" and a "downward". The two directions do not pervade both parts of analysis, since the former is presented in the first part (the hypothetical) and the latter is presented in the second part (the confirmatory). In (Fournarakis, Christianidis 2006) we also show, by means of a specific and representative example, that using this account, one can adequately interpret how Pappus practices his analyses.

3. GEOMETRICAL ANALYSIS IN ARCHIMEDES

As previously mentioned, in the extant Archimedean corpus the method of geometrical analysis is used only in Book II of the work *On the Sphere and Cylinder*, Fig. 1. In this work, and more specifically in propositions 1, 3, 4, 5, 6, and 7, Archimedes applies the analytical method in a way, which does not differ substantially from Pappus' way. In fact, the practice of Archimedes includes the same elements previously remarked in the work of Pappus, specifically the admittance of the sought and the two parts of analysis, the hypothetical and the confirmatory. In addition, the second part of analysis can be identified, also in Archimedes, from the terms "*dothen*"–"*dedomenon*." However, as we will see further on in this paper, the practice of analysis by Archimedes also displays some specific features in comparison with the practice of Pappus.

We will corroborate the aforementioned claims through a representative example from the Archimedean analysis, and for this purpose we will discuss the first problem from *On the Sphere and Cylinder*, II. Our discussion is limited to the hypothetical and the confirmatory part of analysis, despite the fact that Archimedes presents in his text not only the analysis but also the synthesis of the problem. However, a discussion of the analytical part of the method is sufficient for our purpose. The Greek text can be found in (Heiberg 1910, I, 190–194; Stamatis 1970, A.2, 164–166) and for the English translation (Netz 2009) was used.



192 ΠΕΡΙ ΣΦΑΙΡΑΣ ΚΑΙ ΚΥΛΙΝΔΡΟΥ Β.

τῷ ἀπὸ ΗΘ ἴσον τὸ ὑπὸ ΓΔ, ΜΝ. ὡς ἄρα ἡ ΓΔ πρὸς ΜΝ, οὕτως τὸ ἀπὸ ΓΔ πρὸς τὸ ἀπὸ ΗΘ, τουτέστι ἡ ΗΘ πρὸς ΕΖ. καὶ ἐναλλάξ, ὡς ἡ ΓΔ πρὸς τὴν ΗΘ, οὕτως ἡ ΗΘ πρὸς τὴν ΜΝ, καὶ ἡ ΜΝ πρὸς ΕΖ. καί ἐστιν δοθεῖσα ἑκατέρα τῶν ΓΔ, ΕΖ. δύο ἄρα δοθεισῶν εὐθειῶν τῶν ΓΔ, ΕΖ δύο μέσαι ἀνάλογόν εἰσιν αἱ ΗΘ, ΜΝ. δοθεῖσα ἄρα ἑκάτερα τῶν ΗΘ, ΜΝ.

συντεθήσεται δὲ τὸ πρόβλημα οὕτως. ἔστω δὴ ὁ δοθεὶς κῶνος ἢ κύλινδρος ὁ Α. δεῖ δὴ τῷ Α κώνῳ ἢ κυλίνδρῳ ἴσην σφαῖραν εὑρεῖν. ἔστω τοῦ Α κώνου ἢ κυλίνδρου ἡμιόλιος κύλινδρος, οὗ βάσις ὁ περὶ διάμετρον τὴν ΓΔ κύκλος, ἄξων δὲ ὁ ΕΖ. καὶ εἰλήφθω τῶν ΓΔ, ΕΖ δύο μέσαι ἀνάλογον αἱ ΗΘ, ΜΝ, ὥστε εἶναι ὡς τὴν ΓΔ πρὸς τὴν ΗΘ, τὴν ΗΘ πρὸς τὴν ΜΝ, καὶ τὴν ΜΝ πρὸς τὴν ΕΖ. καὶ νοείσθω κύλινδρος, οὗ βάσις ὁ περὶ διάμετρον τὴν ΗΘ κύκλος, ἄξων δὲ ὁ ΚΛ ἴσος τῇ ΗΘ διαμέτρῳ. λέγω δή, ὅτι ἴσος ἐστὶν ὁ Ε κύλινδρος τῷ Κ κυλίνδρῳ. καὶ ἐπεί ἐστιν, ὡς ἡ ΓΔ πρὸς ΗΘ, ἡ

9. τῶν] των της F; corr. ed. Basil. 11. δὲ] scripsi; δη

Fig. 1. Page 192 of the first volume of Heiberg's edition of Archimedes' Opera omnia, with his drawing of the diagram of proposition 1 of On the Sphere and Cylinder.

The problem is the following: Given a cone or a cylinder, to find a sphere equal to the cone or to the cylinder.

The hypothetical part of this analysis includes the following steps:

H.1 Let a cone or a cylinder be given, A, and let the sphere B be equal to A,
H.2 and let a cylinder be set out, ΓZΔ, half as large again as the cone or cylinder A, and <let> a cylinder <be set out>, half as large again as the sphere B, whose base is the circle around the diameter HΘ, while its axis is: KΛ, equal to the diameter of the sphere B;
H.3 therefore the cylinder E is equal to the cylinder K. [But the bases of equal cylinders are reciprocal to the heights];
H.4 therefore as the circle E to the circle K, that is as the <square> on ΓΔ to the <square> on HΘ so KΛ to EZ.
H.5 But KΛ is equal to HΘ [for the cylinder which is half as large again as the sphere has the axis equal to the diameter of the sphere, and the circle K is greatest of the <circles> in the sphere];
H.6 therefore as the <square> on ΓΔ to the <square> on HΘ, so HΘ to EZ.
H.7 Let the <rectangle contained> by ΓΔ, MN be equal to the <square> on HΘ;
H.8 therefore as ΓΔ to MN, so the <square> on ΓΔ to the <square> on HΘ, that is HΘ to EZ,
H.9 and alternately, as ΓΔ to HΘ, so (HΘ to MN) and MN to EZ.

The confirmatory part of the analysis includes the following steps:

C.1 And each of <the lines> ΓΔ, EZ is given;
C.2 therefore HΘ, MN are two mean proportionals between two given lines, ΓΔ, EZ;
C.3 therefore each of <the lines> HΘ, MN are given.

The hypothetical part of the analysis presented above, starts with the supposition that the problem has been solved. Accordingly, all the steps of this mental route have a hypothetical character, since they are all based on the initial assumption that the sought has been accomplished. The assumption (H.1) means that a sphere B, equal to the cone or cylinder A (see Fig. 2), is found [so $V_A = V_B$]. The next hypothetical step (H.2) is the hypothetical construction of two cylinders: E, which is equal to $\frac{3}{2}$ A, and K, which is equal to $\frac{3}{2}$ B. It is a noetic leap because Archimedes sees that if we could make another cylinder K equal to the cylinder E but such that its height EZ is equal to the diameter of its base HΘ, then the problem

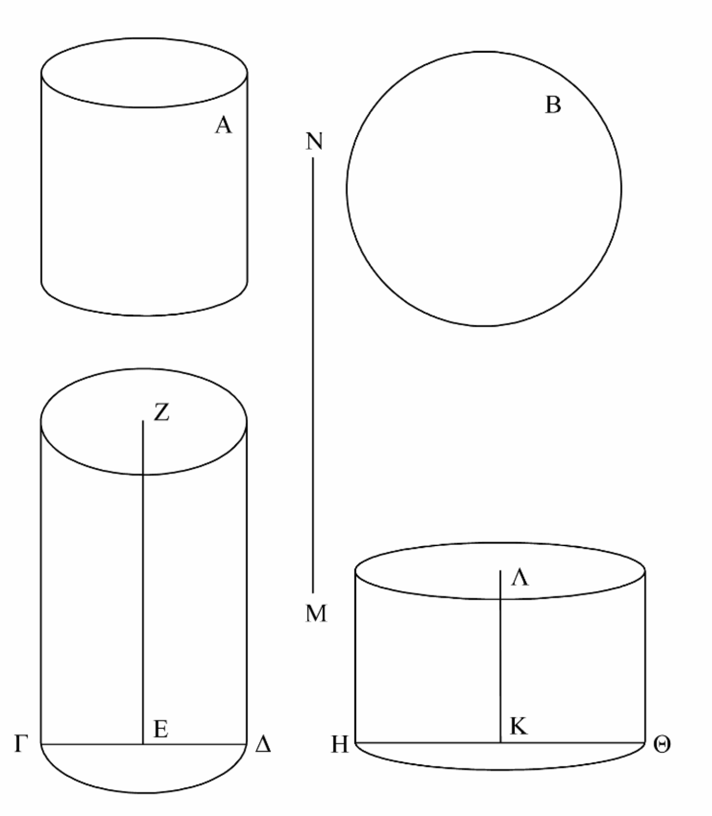


Fig. 2.

would be solved because this cylinder K would be equal to $\frac{3}{2}$ V_A, and the sphere whose diameter is equal to the diameter of the base HΘ of the same cylinder would be the required sphere (according to I.34 of *On the Sphere and Cylinder*). (H.3) declares the obvious equality of the cylinders E and K, but it leads the geometer to think the proposition XII.15 of the *Element*s: the bases of equal cylinders are reciprocal to the heights. It is exactly this relation, that, is, the proportion sq (ΓΔ) : sq (HΘ) :: KΛ : EZ, that is "hypothetically produced" in (H.4). But (H.5) reminds that KΛ was taken equal to HΘ because the cylinder K, which is $\frac{3}{2}$ of the sphere B, was hypothesized with both its height and the diameter of its base equal to the diameter of the sphere B, and the circle K is greatest of the circles in the sphere. So in (H.6) the latter proportion is hypothesized as

$$\text{sq}(\Gamma\Delta) : \text{sq}(\text{H}\Theta) :: \text{H}\Theta : \text{EZ}. \quad (1)$$

Then comes the next noetic leap (H.7), that is, the supposition of MN as the one side of a rectangle (whose other side is ΓΔ) which equals the square on HΘ:

$$\text{sq (H}\Theta) = \text{rec (}\Gamma\Delta\text{, MN).} \tag{2}$$

This supposition "produces hypothetically" in (H.8) the proportion ΓΔ : MN :: sq (ΓΔ) : sq (HΘ), and then the

$$\Gamma\Delta : \text{MN} :: \text{H}\Theta : \text{EZ.} \tag{3}$$

The last supposition if combined with (2) "produces hypothetically" in (H.9) the proportions ΓΔ : HΘ :: HΘ : MN :: MN : EZ. So the diameter of the required sphere B is the first of the two mean proportionals between ΓΔ and EZ.

This relation signals the end of the hypothetical part of the analysis but not the end of the analytical research. It is also a noetic leap as the geometer assumes, on the one hand, that he has reached something that is true independently of the sought and, on the other hand, it can be the starting point of a syllogism that will produce the sought. He still cannot answer whether his efforts were successful because the whole of it is based on the assumption that the sought has been accomplished, thus it is not deductive reasoning. Although in some parts of the hypothetical course the consequences of certain noetic leaps are produced, even this production is hypothetical as well, since it stands only if the sought is admitted to be true. But Archimedes assumes that it *can* evolve to a deduction, ending with the confirmation of the sought as "given". If the claim in (H.8) and (H.9) was already deductively derived, as one might maintain by observing the beginning of it using the adverb "therefore", there would be no reason for Archimedes to produce its result again in the steps C.1–C.3, using "potential" objects. If the last reached (H.9) in the hypothetical part, that is, the diameter HΘ of the required sphere was produced as the first of two mean proportionals between ΓΔ and EZ, then the synthesis of the problem would be clear and must have started at exactly this point. But Archimedes goes on with three more steps (C.1–C.3), characterized by the "given" terminology, in order to confirm that HΘ *can* be produced with logical necessity. Only after that will Archimedes start the synthesis of the problem, as he declares after the confirmatory part of analysis.

From the above discussion, we conclude that the Archimedean analysis includes the three elements described by Pappus, namely, the admittance of the sought, the hypothetical part and the confirmatory part. The latter is also formulated, like in Pappus' analysis, with the "given" terminology.

This analysis of Archimedes raises a question in respect to the rather concise form of presentation of the confirmatory part. In fact, the confirmatory part confirms only the last proportion of the hypothetical part and not the entire hypothetical part or at least the major part of it (in reverse order), as in the case of Pappian analysis. One could propose that this is a specific feature of Archimedes that differentiates him from Pappus. However, a closer examination of the confirmatory part of the Archimedean analysis shows that nothing is missing from the essence of a confirmatory part of an analysis.

Indeed, in the problem discussed above Archimedes reduces the initial problem to the problem of finding two mean proportionals between two given lines (*apagôgê*). The confirmatory part of this analysis also ends with step C.3, because of Archimedes' confirmation that the diameter of the required sphere is the first of the two mean proportionals between two given lines (C.2), the first of which is given by the problem while the second, being the $\frac{3}{2}$ of the first, can also be considered as given.

Therefore, the confirmation of the potential construction, with logical necessity, of the mean proportionals, can fully produce the sought of the problem.

From the aforementioned analysis, we can also infer that Archimedes uses as "given" (*dedomena*) propositions that are not included in Euclid's *Data* (i.e. the problem of two mean proportionals). This observation is of great significance since it leads to the assumption that perhaps there were other works in antiquity with context similar to Euclid's *Data*. As we will see in the last section of this paper, a work of this kind is attributed to Archimedes by an Arabic source.

Another issue relative to the aforementioned analysis of the first problem of *On the Sphere and Cylinder*, II, but also of the fourth and the fifth problems, is whether propositions that include conic sections can be considered as "givens." Archimedes' response to this issue is without doubt that: conic sections can be used in analyses exactly like the propositions included in Euclid's *Data*.

4. A CONJECTURE ON THE MISSING ANALYSIS OF PROBLEM 4 OF ARCHIMEDES' *ON THE SPHERE AND CYLINDER*, II

In problem 4 of *On the Sphere and Cylinder*, Book II, Archimedes solves the problem of dividing a sphere into two segments that have to each other a given ratio. The analysis of this problem presents certain characteristics that are not found in other analyses. More specifically, at the end of the confirmatory part, Archimedes uses as "given" a proposition, which has not been proved in a previous work of his, neither was it obtained as prefabricated data from any other work known to us. Instead, he announces that he will deal with this proposition analytically and synthetically "*at the end*." This statement was interpreted as referring to a lost *addendum* at the end of problem 4. However, this cannot be confirmed from the known manuscripts of the works of Archimedes neither from the copies of *On the Sphere and Cylinder* owned by Dionysodorus and Diocles, two geometers posterior to Archimedes by only a few decades. In fact, both geometers elaborated a different analysis of problem 4 from the start but do not deal with the proposition that Archimedes promises to present "at the end." Eutocius, in the 6th century AD, claims to have discovered this analysis of Archimedes in an old book in deplorable condition, without revealing any other information about the identity or the origin of this book.

Another feature of the analysis of problem 4 is that –according to the reconstruction of Eutocius– Archimedes does not treat the missing part *per se*, but he does so through the analysis of a more general construction problem, a special case of which is the missing analysis of problem 4. The way that Archimedes deduces the special case to be used in the solution of problem 4 from the analysis of a more general problem, presents a similarity to the way in which Pappus uses Euclid's *Data* (which also includes analyses of a more general nature). Note that *Data* includes prefabricated geometrical analyses which are used by Pappus, stating when required the necessary limiting condition, in order to solve, using the method of analysis, the various problems that he deals with in his *Mathematical Collection*. This remark leads us to examine from a new point of view certain historiographical issues as regards the problem 4 of *On the Sphere and Cylinder*, II.

The analytical procedure that Archimedes follows in the more general problem is a complete (according to Pappus) analysis which includes the three basic elements that constitute the analysis of a geometrical problem: the admittance of the sought, the hypothetical part and the confirmatory part; the latter is accomplished using the terms "*dothen*" and "*dedomenon*." Also, conic sections are used in this analysis. Furthermore, the hypothetical and confirmatory parts of this analysis are fully carried out, in a way that reminds us of Pappus' analyses as well as of the confirmatory parts found in Euclid's *Data*.

In order to solve problem 4, Archimedes does not need to use the analysis and synthesis of the general problem but only a part derived from the confirmatory part of the analysis, and moreover under certain conditions. However, in order to use this part, the complete analysis of the general problem should be presented first. This presentation could not be made in the middle of the analysis of problem 4 of *On the Sphere and*

Cylinder, II, since it is, in fact, the analysis of a completely different problem. Therefore, he announces that the analysis and synthesis of the latter problem will be presented "*at the end*."

However, is it certain that the phrase "*at the end*" refers to the end of Proposition 4? Or at least, that it refers to the end of Book II of *On the Sphere and Cylinder*? In other words, was the analysis that was discovered and restored by Eutocius, a lost *addendum* in *On the Sphere and Cylinder*? Currently, there is no evidence that verifies this hypothesis. On the contrary, there is evidence, which can make us skeptical about this hypothesis. First of all, the alleged addendum was not included in the copies of *On the Sphere and Cylinder* that Dionysodorus and Diocles owned, a few decades after Archimedes. Therefore, if it existed it should have been lost shortly after the death of Arhimedes. Furthermore, Eutocius reports having found the lost analysis in an obscure old book, partially written in Dorian dialect, without mentioning whether this book was Archimedes' *On the Sphere and Cylinder*. Netz claims that this book was "totally independent of the *On the Sphere and Cylinder*" (Netz 2009, 206). Moreover, we know that Archimedes used to announce his propositions to his colleagues first, and to present the complete proofs of them in a later time. For example, in the preface of the *Method of Mechanical Theorems* Archimedes, referring to the complete proofs of some propositions that had been announced in the past, uses a similar expression: "At the end of the book we give the geometrical proofs of the theorems the propositions of which we sent you on an earlier occasion" (*epi telei de tou bibliou grafomen tas geômetrikas apodeixeis ekeinôn tôn theôrêmatôn, hôn tas protaseis apesteilamen soi proteron*) (Heiberg 1910–1915, II, 430.23–26). In this extract Archimedes makes clear that he refers to the specific book he introduces, and he also uses present tense. On the other hand, the similar expression in problem 4 of *On the Sphere and Cylinder* is: "And these will be, each, both analyzed and synthesized at the end" (*hekatera de tauta epi telei analythêsetai te kai syntethêsetai*) (Heiberg 1910–1915, I, 214.25–26). Here Archimedes does not clarify that he intends to write the analysis and synthesis he omits in the same book, and he also uses future tense. These differences in expression are in our view indicative of his intentions.

All the above lead us to advance the conjecture that the analysis discovered by Eutocius was not published as an *addendum* in *On the Sphere and Cylinder* but in a different work by Archimedes. Additionally, we could further extend this conjecture by supposing that this analysis was included on a book of analysis written by Archimedes, similar to Euclid's *Data*. This conjecture is no more arbitrary than the hypothesis on the existence of an *addendum*. If the book *Fihrist* (Catalogue) of the 10th century Arab bio-bibliographer al-Nadim, which mentions that Archimedes has written a book called *Book of Data*, is a reliable source, then the aforementioned conjecture should be further investigated.

5. CONCLUSION

In this paper we discussed the analytical practice of Archimedes through the first problem of Book II of his *On the Sphere and Cylinder*, in relation to Pappus' study of the method of analysis and synthesis in Book VII of his *Mathematical Collection*. The conclusion of this discussion is that Archimedes applies the analytical method in a way which does not substantially differ from Pappus' way. In fact, the practice of Archimedes includes the same elements as those of Pappus', that is, the admittance of the sought, and the two parts of analysis, the hypothetical and the confirmatory. In addition, the second part of analysis can be identified, also in Archimedes as in Pappus, from the terms "*dothen*"–"*dedomenon*." Furthermore, Archimedes uses Euclid's *Data* propositions, under certain conditions, to solve problems with the method of analysis, like Pappus does, but he also uses propositions, which have to do with conic sections, and are not included in Euclid's *Data*, as data. The above reached conclusions would of course be better argued if the whole of Archimedes' and Pappus' practices of analysis were discussed here.

Secondly, the discussion of the missing part from the analysis of problem 4 of *On the Sphere and Cylinder*, II, and the various solutions of this problem that are preserved, combined with the above reached conclusions and the remark that Archimedes writes the analysis of the missing part of problem 4 in a general way, all these lead us to advance a reasoned conjecture vis-à-vis a lost analytical treatise of Archimedes written in the way Euclid's *Data* is written. This treatise could be the *Book of Data* that the Arab bio-bibliographer of the 10th century al-Nadim attributes to Archimedes.

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2. LEGACY AND INFLUENCE IN ENGINEERING AND MECHANISMS DESIGN

SIMON STEVIN AND THE RISE OF ARCHIMEDEAN MECHANICS IN THE RENAISSANCE

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ABSTRACT In this paper I will discuss the position of the Flemish mathematician and engineer Simon Stevin (1546–1620) in the rise of Archimedean mechanics in the Renaissance. Commandino represents the beginning of the Archimedean Renaissance in statics. The next steps were made by Guidobaldo Del Monte and Stevin. Del Monte and Stevin were contemporaries belonging to the generation preceding Galilei (1564–1642). Yet Stevin's work in mechanics is superior to Del Monte's. I will discuss the way in which Stevin's mechanical work, like Del Monte's, was influenced by the medieval science of weights. For example, the central notion "stalwicht" in Stevin's work, translated as "apparent weight' by the editors of Stevin's *Works*, clearly corresponds to the notion of positional weight (ponderis secundum situm) in the science of weights. I will also argue that while Del Monte remained caught in the conceptual framework of the science of weights the use of the Dutch language helped Stevin in liberating himself from those ideas. For Stevin the use of Dutch was part of his success. Finally I will discuss Stevin's work on windmills. Not only his original theoretical contributions to statics and hydrostatics but also the unity of theory and practice in Stevin's work make him in mechanics the first true successor of Archimedes in the Renaissance.

1. INTRODUCTION

In the past decades the Archimedean Renaissance in Italy has been studied by several authors (e.g. [9], [11] and [13]). In this particular context the work of Simon Stevin (1546–1620) has received less attention. At first sight Stevin appears to be a rather isolated figure. He seems not to belong to one of the Italian traditions. Yet he must be seen against the background of the mechanical work of the authors that preceded him. He seems isolated because we only have his mature work and we do not know its

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genesis. There are few references to others in his work. Moreover, he wrote in Dutch, creating his own terminology and his own way of presenting the subject.

Yet Stevin is definitely part of the Archimedean Renaissance in mechanics. After Federigo Commandino of Urbino (1509–1575), the Archimedean Renaissance in statics continued with Guidobaldo Del Monte (1545–1607) and Simon Stevin (1548–1620). Del Monte knew Archimedes' work and he was familiar with a summary of Hero's *Mechanics* in the form it is given by Pappus in Book 8 of the *Collection*. Del Monte's contribution to the theory of machines consists of his *Mechanicorum liber* of 1577 and its Italian translation by Pigafetta which appeared in 1581. After having explained how useful mechanics is Del Monte formulates his goal to build mechanics up "from its foundation to its very top" ([7]. p. 246). In the text he starts with properties of the balance basing himself on Archimedes and then he proceeds to the Heronean core of mechanics: the five simple machines, in the order lever, pulley, wheel and axle, wedge and screw. Several historians have written about Del Monte's mechanics. See, for example, [11] and [21]. Duhem wrote:

> "sometimes erroneous, always mediocre, the *Mechanics* of Guido Ubaldo is often a regression from the ideas published in the writings of Tartaglia and Cardano" ([8], p. 226)

This is somewhat unfair and it is certainly unreasonable to put the writings of Tartaglia and Cardano so much higher than Del Monte's *Mechanicorum Liber.* On the other hand, although Del Monte's starting point was good, in the execution the problems that Del Monte could not solve dominated. In his treatment of the balance he lost himself in long discussions with the proponents of the science of weights. Del Monte left the problem of the inclined plane unsolved and in the Italian translation of his book on mechanics the erroneous solution of this problem by Pappus was included. In this paper I will argue that with original contributions to statics, hydrostatics and the theory of machines, Stevin was truly Archimedes' first successor in the Renaissance.

2. THE BACKGROUND

In the Renaissance there was a growing interest in machines and their theory: *mechanics*. The interest in machines is, for example, clearly reflected in the support of the French King for the publication, in 1571/72 by Jacques Besson (1540–1573), of one of the earliest theaters of machines.

The interest in the theory of machines is clear from the fact that several texts on mechanics from Antiquity and the Middle Ages were printed in the 16th century.

What did the theory of machines look like in the 1570s when Del Monte and Stevin were in their twenties? *Mechanical Problems* (Quaestiones Mechanicae) contained in the Aristotelian corpus, was available in print in Latin quite early in the century. It contains the oldest theory of machines usually ascribed to a follower of Aristotle, although parts of it may come from Archytas (Cf. [10]). In the 13th century, Jordanus de Nemore and his pupils had created a scholastic *science of weights*. Nicolo Tartaglia (1500?–1557) had access to some of the manuscripts and he published a version of this theory in his *Various Questions and Inventions of Niccolò Tartaglia of Brescia* (Quesiti ed inventioni diverse) of 1546. We will refer to this text as Tartaglia's *Quesiti*. The medieval Latin text that he used appeared in 1565 in Venice (See Figure 1).

Then there was Archimedes' work on statics and hydrostatics. In 1543 Tartaglia published the Latin translations of *On the Equilibrium of Planes* (Book I and II), *On the quadrature of the parabola*, *On the measurement of the circle* and *On floating bodies* (Book I only). Tartaglia's publication suggested that he had translated these texts himself. However, in 1881 it was discovered that they had been made by William of Moerbeke (circa 1215–1286). This translation left a lot to be desired. Actually there is no evidence that Tartaglia knew Greek and some that he did not ([2], pp. 555–556). In 1558 Federigo Commandino of Urbino (1509–1575) published a translation of several of Archimedes' works far superior to Moerbeke's translations. In 1565 Commandino published a translation of *On floating bodies.* Actually in *On floating bodies* Archimedes assumes properties without proof that led Commandino to publish his own *Book on the Center of Gravity of Solid Bodies* (Liber de centro gravitatis solidorum) in the same year ([3]).

This was not all. Hero's devices operated by water, air and steam were described in an encyclopedic work by Giorgio Valla printed in 1501. Hero's *Pneumatics* was published in Latin by Commandino in 1575. For the general public a Latin summary of Hero's *Mechanics* only became widely available in 1588 when Commandino's Latin translation of Pappus' *Collection* was published by Guidobaldo Del Monte ([7], p. 45). Commandino had made the translation before his death in 1575 and Del Monte had access to it. Pappus' summary of Hero's *Mechanics* in Book 8 of the *Collection* introduced Renaissance scholars to the idea that the five simple machines were the basic components of all machines. Pappus wrote, referring to Hero:

> "The names of these powers then are: the axle with a wheel turning on it; the lever; the compound pulley; the wedge; that which is called the endless screw" [14]

3. THE SCIENCE OF WEIGHTS: DEFINITIONS AND POSTULATES

Archimedes' work is well known. The science of weights is less known. Yet it is an essential part of the background of Stevin's work. That is why we will devote some attention to it.

Although considerably less rigorous than Archimedes's work, unlike *Mechanical Problems,* the science of weights shows influence of the Greek deductive traditions. Definitions are followed by theorems and the geometry of the figures plays an actual role in the arguments.

We will consider briefly some parts of the version of the theory that Tartaglia gave in his *Quesiti*. We will base ourselves on the English translation by Stillman Drake in [7]. The approach is deductive. *Definitions* and *petitions* (i.e. postulates) precede a series of *propositions* that are demonstrated on the basis of the definitions and petitions.

> Definition IX: Those bodies are said to be *simply equal in heav*iness which are actually of equal weight, even though of different material.
> Definition XIV: The heaviness of a body is said to be known when one knows the number of pounds, or other weight, that it weighs.
> Definition XIII: A body is said to be *positionally more or less heavy* than another when the quality of the place where it rests and is located makes it heavier [or less heavy] than the other, even though they are both simply equal in heaviness. ([7], p. 114, Italics are mine)

The distinction between the notions *simple heaviness* and *positional heaviness* is fundamental. Tartaglia relates positional heaviness to the *obliqueness of the descent (or ascent)* of the weight that takes place if the weight moves within the bounds of its mobility.

The notion of obliqueness is defined in

> Definition XVII: The descent of a heavy body is said to be more oblique when for a given quantity it contains less of the line of direction, or of straight descent toward the center of the world. ([7], p. 115)


IORDANI
OPVSCVLVM
DE PONDEROSITATE
NICOLAI TARTALEAE
STVDIO CORRECTVM,
NOVISQVE FIGVRIS AVCTVM.
CVM PRIVILEGIO.
TRAIANO CVRTIO
VENETIIS,
APVD CVRTIVM TROIANVM.
M D LXV.

FEDERICI
COMMANDINI
VRBINATIS
LIBER DE CENTRO
GRAVITATIS
SOLIDORVM.
CVM PRIVILEGIO IN ANNOS X.
BONONIAE,
Ex Officina Alexandri Benacii.
M D LXV.

Fig. 1.

The text has the form of a dialogue between Tartaglia and Mendoza, the imperial ambassador of Charles V at Venice. In the case of definition 17, Tartaglia exemplifies the definition with a reference to Figure 2.

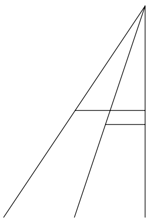

Fig. 2.

The descents AF and AE from the point A are oblique. Suppose that AF=AE. Then AH and AG are the vertical components of these descents, or, in Tartaglia's words, AH and AG are what the two descents contain of the *line of direction*, that is by definition the straight descent towards the center of the world. So AF is more oblique than AE, because AH is smaller than AG.

About positional heaviness Tartaglia says:

> Petition 4: Also we request that it be conceded that those bodies [bodies of equal simple weight – T. K.] are equally

> heavy positionally when their descents in such positions are equally oblique, and that will be the heavier [positionally – T. K.] which, in the position or place where it rests or is situated, has the less oblique descent. ([7], p. 119)

Clearly, a vertical line is not oblique. The positional weight in this case is equal to the simple weight. The positional weight of an object on an inclined plane depends on the slope: the more oblique the slope, the smaller the positional weight.

One notices that right from the start the problem of the inclined plane concerning the precise dependence of the positional weight on the steepness of the slope is implicitly present in the science of weights. As we will see below in the science of weights Jordanus and/or his pupils succeeded in precisely determining this relationship: they were the first ever to solve the problem of the inclined plane.

4. THE SCIENCE OF WEIGHTS: THE FIRST PROPOSITIONS

So far positional heaviness is determined by the simple weight plus the geometry of the situation. However, following the medieval science, Tartaglia relates positional heaviness to two other notions: *power* and *speed*. Essentially Tartaglia views *positional heaviness* as proportional to the *power a weight can exert* and in its turn this power is proportional to the *speed*, i.e. the distance covered in a certain period of time as a result of the power. Consider:

> Proposition 4: The ratio of the power of bodies simply equal in heaviness, but unequal in positional force, proves to be equal to that of their distances from the support or center of the scale. ([7], p. 123)

Tartaglia's proof is brief and from a modern point of view quite unsatisfactory. He looks at two bodies of equal simple weight positioned at unequal distances from the centre on the horizontal arm of a balance. When the arm moves the speeds or the distances covered by the weights are proportional to the distances of the weights to the center. Basically what Tartaglia is saying is the following: the power that a simple weight on the arm of a horizontal balance can exert is proportional to the length of the arm. He also implicitly assumes that with a particular arm length the power that a weight can exert is also proportional to the size of the weight.

Remarkable and revealing as for the problems that the in itself sound notion of positional weight brings about is the second part of

> Proposition 5: When a scale of equal arms is in a position of equality, and at the end of each arm there are hung weights simply equal in heaviness, the scale does not leave the said position of equality; and it happens that by some other weight [or the hand] imposed on one of the arms it departs from the said position of equality, then, that weight or hand removed, the scale necessarily returns to the position of equality ([7], p. 124).

He first part is demonstrated by remarking that on the basis of proposition 4 the bodies of equal simple weight put at equal arm length on a horizontal balance have equally oblique descents, which implies equal positional force. The second part is remarkable. See Fig. 3.

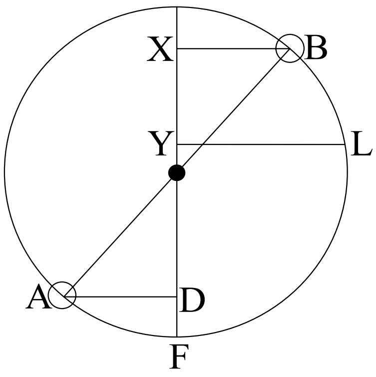

Fig. 3.

We have two weights simply equal in heaviness in A and B. Suppose the arcs BL and AF are equal. The projections of the arcs on the vertical line are unequal. XY is bigger than DF. That is why Tartaglia concludes that the descent of B is more oblique that the decent of A, so B is positionally heavier than A and that is why he feels that the balance will return to its horizontal position. One notices that the obliqueness of a decent is measured by projecting the descent on a vertical line in accordance with Definition XVII. Clearly we do no longer except this result as correct. If Tartaglia had considered *infinitesimal* displacements, he would have drawn a different conclusion.

5. THE SCIENCE OF WEIGHTS: THE LAW OF THE LEVER AND THE LAW OF THE INCLINED PLANE

The central results in the science of weights are the law of the lever and, more importantly, the law of the inclined plane. The law of the lever is phrased by Tartaglia as follows:

> Proposition 8: If the arms of the balance are proportional to the weights imposed on them, in such a way that the heavier weight is on the shorter arm, then those bodies or weights will be equally heavy positionally. ([7], pp. 132–134)

The proof is based on Proposition 4, which is applied as saying:

Positional heaviness on a balance = Simple weight x Length of arm.

(Nota bene: Tartaglia cannot express it in this way, constrained as he is by Eudoxus' theory of proportions, which was at the time generally excepted. In Eudoxus' theory only ratios of quantities of the same kind can be considered: ratios of weights can be equal to ratios of lengths, but weights and lengths cannot be multiplied.)

It is highly remarkable that in the science of weights Jordanus and his pupils succeeded in solving the problem of the inclined plane. Tartaglia almost literally follows Jordanus proof. Consider

> Proposition 15: If two heavy bodies descend by paths of different obliquities, and if the proportions of inclinations of the two paths and of the weights of the two bodies be the same, taken in the same order, the power of both the said bodies in descending will also be the same. ([7], p. 141)

See Figure 4. It is clear that one can imagine the two heavy bodies E and H, on the slopes DC and DA respectively, connected by a rope EDH. The proposition says: We have equilibrium if

$$\text{Weight E} : \text{Weight H} = DC : DA$$

We consider that situation and we imagine a weight G equal to E on slope DK which has the same tilt as DC. Suppose now that E and H "are not in the same power" and let us suppose that E descends as far as point L. Then H ascends as far as M. Assuming GN equal to LE, we have also GN equal to HM, and one can easily prove by means of similarity considerations that

$$MX : NZ = DK : DA.$$

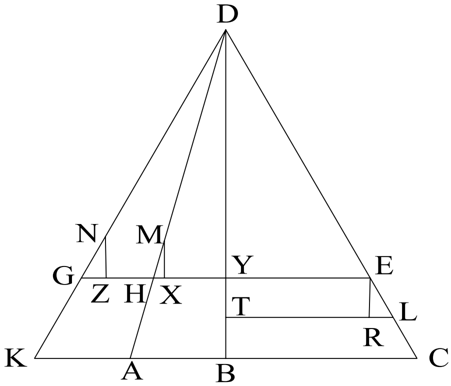

Fig. 4

We also have

DK : DA = Weight G : Weight H

Then

MX : NZ = Weight G : Weight H.

And Tartaglia concluded:

> Therefore, by however much the body G is simply heavier than the body H, by so much does the body H become heavier by positional force that the body G, and thus they become to be equal in force or power. ([7], p. 142)

From a modern point of view what Tartaglia is basically doing is applying this rule with respect to an inclined plane:

Positional heaviness = Simple weight x Obliqueness

(Again Tartagia cannot put it in this way because he uses Eudoxus' theory of proportions. For the modern reader this expression is somewhat more transparent.)

Obliqueness is measured by means of the vertical component of an arbitrary constant descent along the plane. Weight G and Weight H are simple weights. From a modern point of view we have:

Positional weight G in its present position = Weight G x NZ
Positional weight H in its present position = Weight H x MX

Clearly the equality of these two positional weights implies equilibrium.
Q. E. D.

6. STEVIN: FROM THE SCIENCE OF WEIGHTS TO *THE PRINCIPLES OF THE ART OF WEIGHING* AND *THE PRACTICE OF WEIGHING*

In 1581 the Flemish engineer and mathematician Simon Stevin (1548–1620) settled in Leiden a city in Holland, not very far from Amsterdam. He studied at the University of Leiden for two years matriculating in 1583. In 1586 he published three books that would bring him everlasting fame:

The Principles of the Art of Weighing (De Beghinselen der Weeghconst)
The Practice of Weighing (De Weeghdaet)
The Principles of the Weight of Water (De Beghinselen des Waterwichts).

With these books Stevin wanted to develop mechanics along strict Archimedean lines and he wanted, which implied from his point of view a further development of what we nowadays call statics plus its application to actual machines. He had read Aristotle's *In Mechanicis* ([15], pp. 508–509) as Stevin called the book *Quaestiones Mechnicae*. As we will see below he must have been familiar with ideas from the medieval science of weights as well, but we do not know how. Cardano's *Opus novum de proportionibus etc*. Basilae 1578, is mentioned twice in Stevin's works ([15], pp. 508–511).

Stevin had also read Archimedes' mechanical works and Commandino's book on centers of gravity. Stevin refers to Pappus' definition of the centre of gravity before Commandino's translation of Pappus's *Collection* had even appeared. Because Commandino quotes Pappus' definition in Greek at the beginning of chapter 1 of his book on centers of gravity, Stevin probably has it from there. I think it is improbable that Stevin had read book 8 of Pappus' *Collection*. So if I am right, Stevin was unaware of Hero's notion of simple machines (Duhem hesitates at this point Cf. [8], p. 143). I think the fact that Stevin does not treat the screw at all is revealing. Had he known about the five simple machines, he would have treated them. This supposition implies that Stevin in 1586 had not had access to Del Monte's work.

Had he known about Pappus' erroneous treatment of the inclined plane, the wedge and the screw, he would at least have shown the correct treatment of the screw. He did not. The only incorrect treatment of the inclined plane that he criticizes is Cardano's. Cardano had argued that the force needed to move a weight upwards on an inclined plane is proportional to the angle that between the slope and the horizon, the maximum value being reached when the plane is vertical (Cardano, *Opus Novum*, Propositio LXXII, Basilea, 1570, p. 63).

7. THE GENESIS OF STEVIN'S STATICS

Stevin must have been familiar with the science of weights in some form. On the title page of the *Practice of Weighing* (See Figure 6, right) in small letters there is written in Latin "praxis artis ponderaria". Obviously Stevin saw his art of weighing as a sequel to the science of weights. Stevin was familiar with Archimedes' works as well. He may have used Tartaglia's edition of *On the Equilibrium of Planes* (Book I and II). I propose the following speculative genesis of Stevin's books on statics.

i) Stevin realized that an Archimedean approach to the science of weights implied that all considerations concerning motion had to be dropped. Moreover, Archimedes' treatment of the balance rigorously solves the problem of positional weight for what Stevin called vertical weights, but not for oblique weights.

ii) Right from the start Stevin was thinking of situations suggested by actual machines. If one does so, the importance of forces (or weights) not acting vertically but obliquely, is obvious (See Figures 7 and 9). Stevin realized that the distinction between simple weight and positional weight made sense, but that it had to be preceded by the distinction between vertical weights and oblique weights.

iii) While studying positional weight on an inclined plane Stevin found the most beautiful proof of his life: the key to the treatment of oblique weights. This discovery determined the structure of *The Principles of the Art of Weighing*. It consists of two books. Book I consists of a part 1 on vertical weights with the law of the balance as central result and a part 2 on oblique weights with the law of the inclined plane as central result. Book II is devoted to the centers of gravity of solids, taking Commandino's book on the subject as a starting point.

iv) Stevin decided to devote a separate book to the application of the theory to real machines: *The Practice of Weighing*. In Stevin's work theory and practice are developed separately, but the unity of theory and practice is a central dogma.

8. STEVIN'S TREATMENT OF THE INCLINED PLANE: THE CRUCIAL PROOF

Let us consider some details. From Commandino Stevin took Pappus' definition of the center of gravity. It is worded by Stevin as follows:

> The center of gravity of a solid is the point through which any plane divides the solid into parts of equal positional weight ([15], pp. 100–101)

This definition is perfectly in accordance with Archimedes. By the way, the editors of Stevin's works translated the word "euestaltwichtigh", that Stevin uses, with "having equal apparent weight". This hides the relation with the science of weights. I think that the word "stalt" (or elsewher "ghestalt") must in this context be translated with "position" and "euestaltwichtigh" then becomes "of equal positional weight". See in the etymological dictionary [17] the lemma on "stede" and "stee".

Dijksterhuis interpreted "staltwicht" as "the component of an acting force which is actually exerting an influence" ([19], p. 52). This seems simply wrong to me. When Stevin uses the word "staltwicht" with respect to weights on a horizontal balance, this interpretation holds no water.

Pappus' definition of the center of gravity obviously implies that the second part of Proposition 5 in Tartaglia's treatment of the science of weights cannot be correct. We discussed it above. Stevin does not even mention such errors. He simply dropped in the science of weights everything that contradicted Archimedes.

Consider, for example, the role of motion in the science of weights. Stevin wrote an appendix to *The Art of Weighing* in which he gave the following argument:

> That which hangs motionless does not describe a circle.
> Two (bodies) of equal positional weight hang motionless
> Conclusion: Two (bodies) of equal positional weight do not describe circles ([15], pp. 508–509)

This is why Stevin rejects the view that the cause of the law of the balance resides in the fact that the extremities of the arms describe circles. More generally: the real causes of equilibrium are not to be found in the mobility of the weights involved. By the way, this argument has been criticized, for example, by Duhem. The point, however, is not whether the argument in itself is valid. The point is that without completely rejecting the notion of motion in statics Stevin would not have been able to liberate himself from the confusions that vexed his predecessors.

As we have seen in the *Quesiti* in the science of weights equilibrium is linked to mobility via the Aristotelian view that the power that a weight can exert is proportional to the speed that is reached if the power can be exerted. Stevin rejected this element as well. Several years before Galilei possibly dropped the two weights from the tower of Pisa, Stevin executed a similar experiment in the city of Delft with two spheres of lead from a height of 30 feet. They reached the ground at the same moment. That Aristotle was wrong had in 1562 already been argued by Jean Taisnier. Stevin had read Taisnier's book and he took a radical course here as well. All considerations concerning speed in the science of weights had to be ignored

Before turning to Stevin's treatment of the inclined plane, let us briefly look at the core idea of Archimedes' proof of the law of the lever. Archimedes' takes the validity of the law in the symmetric case with equal masses and equal arms as obvious.

See Figure 5 left. Suppose we have in A 6 white units of weight and in B 4 grey units of weight. The unit of weight should be chosen in such a way that the two numbers are even. Suppose, moreover, that for the arms we have OA and OB are, respectively, equal to 4/2=2 units of length and 6/2=3 units of length.

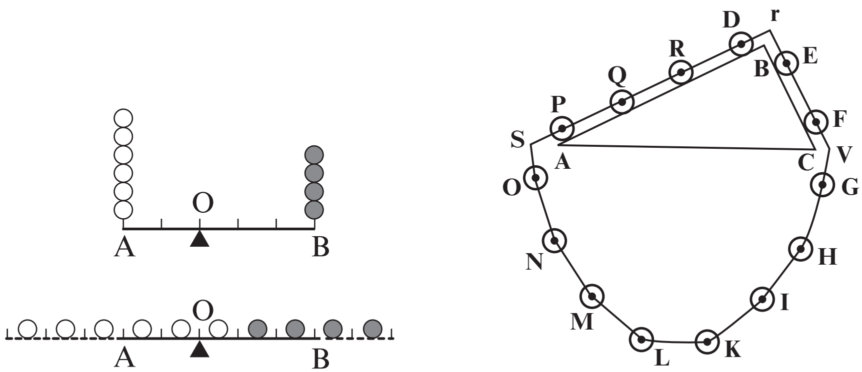

Fig. 5.

In this way we have created a situation in which the weights are inversely proportional to the corresponding arms. Archimedes now extends the arms: OA is extended with the length of OB and OB is extended with the length of OA. We then divide the units of weight over the units of length on the extended balance as shown in the figure. The result is that the center of gravity of the white units remains in A and the center of gravity of the grey units in B. At the same time the center of gravity of the whole is in O. So we have equilibrium. The core idea of Stevin's proof of the lever is similar.

Stevin's proof of the law of the inclined plane is also based on splitting the two weights in a number of units. See Figure 5 right. If Stevin knew the answer that the science of weights had given – there exists equilibrium if the two weights are proportional to the lengths of the two inclined planes -, which is from my point of view probable, splitting the two weights into numbers of units proportional to the length of the inclined planes would have been a rather natural move.

The crucial idea must have come to Stevin suddenly. One considers the units as beads on a chain and one closes the chain by adding a lower part. The lower part is symmetrical and it will not disturb the relation of the positional weights on the inclined planes. That the positional weights are equal is shown as follows by contradiction. Stevin assumes that the positional weight on the left hand side is bigger. Then the chain will start to rotate. All the time the chain of balls as a whole has the same position as before ("den crans der clooten sal alsucken ghestalt hebben als sy te voren dede" [15], pp. 178–179). Stevin concludes: "so the spheres will out of themselves perform a perpetual motion" ("ende de clooten sullen uyt haer selven een eeuwich roersel maken" [15], pp. 178–179). This Stevin finds impossible and he draws the conclusion that the chain will not start to rotate. We know that Stevin was extremely proud of this proof and he used the corresponding figure basically as his logo, accompanied by the text "The miracle is no miracle" (Wonder en is geen wonder). See Figure 6 with the frontispieces of *The Principles of the Art of Weighing* and *The Practice of Weighing*.

It has been assumed that Stevin rejected perpetual motion in the general sense of the word. Duhem has argued that Stevin must have read Cardano's work. Cardano was apparently influenced by Da Vinci, whose manuscripts were at the time kept by Menzi in his villa close to Cardano's hometown Milano. Cardano rejected the existence of a perpetuum mobile basically on Aristotelian grounds. Aristotle had indeed assumed that in order to maintain motion a constant force is needed. If Duhem is correct, this would be ironic, because Stevin rejected the Aristotelian views. However, another interpretation is possible.


DE
BEGHINSELEN
DER WEEGHCONST
BESCHREVEN DVER
SIMON STEVIN
van Brugghe.

WONDER EN IS GHEEN WONDER

TOT LEYDEN,
Inde Druckerye van Christoffel Plantijn,
By Françoys van Raphelinghen.
cIↃ. IↃ. LXXXVI.

DE
WEEGHDAET
BESCHREVEN DVER
SIMON STEVIN
van Brugghe.

WONDER EN IS GHEEN WONDER

TOT LEYDEN,
Inde Druckerye van Christoffel Plantijn,
By Françoys van Raphelinghen.
cIↃ. IↃ. LXXXVI.

Fig. 6. *The Principles of the Art of Weighing* (left) and *The Practice of Weighing* (right).

Stevin wrote that eternal motion *starting spontaneously* was absurd. Such a motion under the influence of gravity would from a modern point of view necessarily be accelerated and excluding friction, which Stevin explicitly does, the circular character of the motion would imply the possibility of the spontaneous occurrence of an infinitely accelerated circular motion. This in itself does not imply inconsistency but it is something one would definitely want to exclude from one's theory. See [18] for a subtle analysis of Stevin's rejection of a perpetuum mobile by Van Dyck.

9. *THE PRINCIPLES OF THE ART OF WEIGHING*

Stevin's book is characterized by extreme clarity. His approach is Euclidean, but there is a certain similarity with Tartaglia's *Quesiti*. Where definitions and postulates in the *Quesiti* are often accompanied by explanations directed at Tartaglia's partner in the dialogue, Stevin adds extensive explanations as well to his definitions and postulates, although the work is not in the form of a dialogue.

It is striking that *The Principles of the Art of Weighing* is preceded by a long introduction on the superiority of the Dutch language. There is more to it than that Stevin is part of an international trend to replace Latin by the vernacular and that he may have found it easier to express himself in Dutch. The Dutch language enabled Stevin to develop his ideas using his own Dutch technical terminology, thus liberating himself completely from the different terminologies that his predecessors had used. Certainly for Stevin the use of Dutch was part of his success.

In part 1 of Book I some fundamental definitions are

> Definition II: The heaviness of a solid is the power of its descent in a given place.
> Definition III: A known heaviness is expressed in terms of a known *weight*.

Compare with Definitions 9 and 14 in Section 3. A known weight is, for example, a pound or an ounce. Definition XII introduces the notions *lifting weight* and *lowering weight*. It is basically Stevin's way to handle the positive or the negative effect of a weight. Definition XIV contains the fundamental distinction between *vertical weight* and *oblique weight.* The fundamental notion *positional weight* is not introduced in a separate definition. It occurs first in the explanation following the definition of the center of gravity: The center of heaviness is the point through which any plane divides the solid into parts of *equal positional weight*.

See Figure 7 left for examples of vertical and oblique lifting weights. The Figure concerns Proposition XX, which says: "Like vertical-lift-line is to oblique-lift-line, so is vertical-lift-weight to oblique-lift-weight" The figure shows three examples of a prism supported in a point E. The prism can be kept in its position by vertical lifting weights G that apply at point F. However, G being taken away, equilibrium can also be established by means of the oblique weights H. Stevin concluded that in the three situations of Figure 7 we have

Weight G : Weight H = Vertical lifting line IF : Oblique lifting line FK.

This is correct and one notices that Stevin is here very close to the parallelogram of forces: segment IK represents the force that must be added to FI in order to get FK; it is the support by point E along the axis EC of the prism.

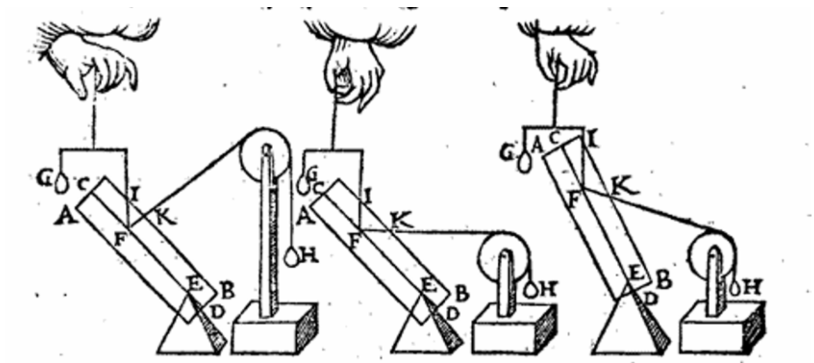

Fig. 7. *The Art of Weighing* Book I [15], pp. 196–197).

Actually Stevin was aware of the validity of the parallelogram of forces, as we will see below. With his work the principles of the statics of vertical and oblique forces had been defined. Others would elaborate on them and reformulate then, but the basis was there. Stevin brought considerable conceptual clarity to the subject by means of these notions. The ease by means of which he could phrase his new conceptual franework in Dutch led him to believe in the superiority of Dutch. For example, in the dedication he refers to words like "Evestaltwichtich", "Rechthefgewicht", "Scheefdaellinie", that literally stand for, respectively, "Equal-position-weight-ly" (means: of equal positional weight), "Vertical-lift-weight" and "Oblique-lowering-line" ([15], pp. 84–85). He wrote about them: "[These words] do not exist [in other languages – T. K.], Nature has specially designed Dutch for it". In the same vein he refers to his Proposition XX, "Ghelijck rechtheflini tot scheefheflini, also rechthefwicht tot scheefhefwicht", which means as we have seen "Like vertical-lift-line is to oblique-lift-line, so is vertical-lift-weight to oblique-lift-weight". He wrote about it:

> "Such secrets have been hidden hitherto in all other languages. Let them try to do something similar in another language. You can safely promise them a cake and I assure you that you will get away without damage". ([15], pp. 90–91)

Stevin had a very clear mind. His exposition is admirable, but he confused the superiority of his concepts and approach with the alleged superiority of the Dutch language.

Stevin called the science of weights an art, because he put it on the same level as arithmetic and geometry. In his dedication to the Holy Roman Emperor Rudolph II that precedes the *Principles of the Art of Weighing* he wrote, with an implicit reference to the *Book of Wisdom*: "number, magnitude and weight are in all things inseparable" ([15], pp. 54–55). Arithmetic (rekenconst) and geometry (meetconst) were established arts (art=const in Stevin's Dutch). The principles of the art of weighing, however, had according to the text of the dedication remained hidden from his predecessors. The law of the lever with respect to vertical weights was indeed known but, according to Stevin, incorrectly explained. Moreover, according to Stevin in the dedication preceding the art of weighing the theory of oblique weights was completely unknown ([15], pp. 54–55). As we have seen this was not quite correct because the law of the inclined plane had been correctly derived in Jordanus' school. Although it is clear to me that Stevin must have had some knowledge of the science of weights, this remark suggests the possibility that he had not seen or not understood Jordanus' result on the inclined plane.

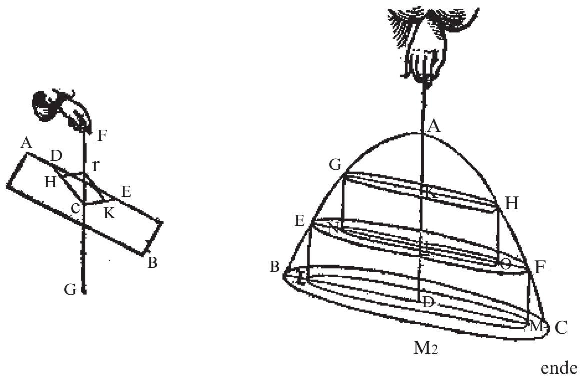

Fig. 8.

It is striking that Stevin feels no need to strengthen his own arguments with a criticism of his predecessors. Not bothered by the mistakes of his predecessors, without hesitations and completely sure of himself, he proceeds to a treatment of his own ideas.

This paper does not allow a further discussion of Stevin's work on statics. In a *Supplement to the Art of Weighing* he has the parallelogram of forces. See Figure 8 left ([15], pp. 532–533). This should not be surprising considering the results expressed by Figure 7. Figure 8 right ([15], pp. 276–277) illustrates the major result in Book II of *The Principles of the Art of Weighing* on the center of gravity of a segment of a paraboloid: it is on the axis AD in a point I which is such that AI is equal to twice ID. Although his derivations are somewhat different, Stevin did not add anything substantial on centers of gravity to those of Archimedes and Commandino.

10. A REMARK ON *THE PRACTICE OF WEIGHING*

With Del Monte Stevin has in common the intention to combine Archimedean mechanics with a theory of actually existing machines. Stevin solved the problem of the gap between theory and practice by writing two volumes. The *Practice of Weighing* contains the application of *The Principles of the Art of Weighing* to machines.

Figure 9 is from *The Practice of Weighing (De Weeghdaet)*. The figure shows Stevin's design of a machine he called the *Almighty* (Almachtich). Stevin refers at this point to Besson who had put a drawing in his book of the machine that Archimedes allegedly used to pull a ship from the shore into the sea, the *Charistion* (called polyspaston by others). Besson's machine had at least one screw.

Stevin said about his own design:

> "[it] is more suited to such work, for the following reasons: sturdier and more durable construction; of lower cost; by which is done more in shorter time, and (like the Charistion) of infinite power, that is to say: potentially, not actually". ([15], pp. 354–355)

Stevin's calculation of the mechanical advantage of the gear train is essentially based on a repeated application of the law of the lever, but he actually calculates the ratio of the number of revolutions of the crank DLMN and the axle S. Below we will se the same approach in Stevin's analysis of windmills.

This leads him to the conclusion that with a force of 25 pounds (he assumes that one man can exert such a force) a force of 5400 pounds can be exerted. Because he assumes that the simple weight of the ship is 6 times its positional weight, a ship weighing 32400 pounds can be pulled up the inclined plane ([14], pp. 358–365).

It is questionable whether at the time a really reliably functioning *Almighty* could have been built. Yet Stevin meant business as for the application of his Art of Weighing. In order to see this it is god to look at his work on windmills.

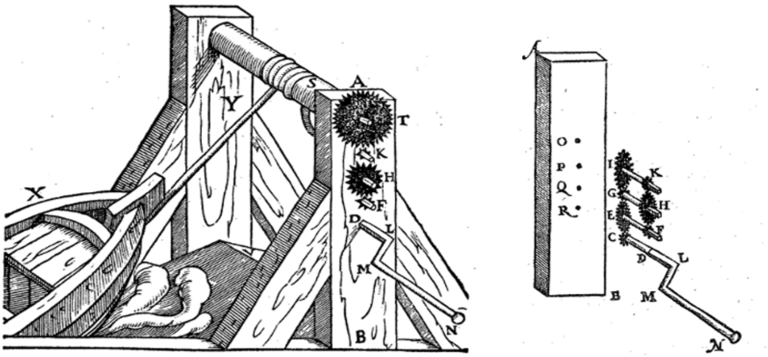

Fig. 9. Stevin's Almighty.

11. STEVIN'S ANALYSIS OF WINDMILLS

The third important book that Stevin published in 1586 is called *The Principles of the Weight of Water* (De Beghinselen des Waterwichts). In the preface in which Stevin congratulates the States of the Unites Netherlands he remarks that because the Netherlands are permanently dealing with water, knowledge of the statical properties of water can yield great advantage ([15], pp. 380–381). We will see below that this was more than rhetoric; Stevin meant it. A discussion of this book falls beyond the scope of this paper. However, one of the original results in the book concerns the pressure that water exerts on a vertical rectangular wall: the force is equivalent to the weight of a volume of water equal to ½ times the area of the wall times the height of the wall, exerted horizontally at 1/3 of the height of the wall. See [15], pp. 420–423; Figure 10 shows the accompanying image.

As we will see, this result played a crucial role in Stevin's work on windmills. A volume that could have had the title *The Practice of the Weighing of Water* and would have contained the application of the content of *The Principles of the Weight of Water* was certainly planned, but with the exception of a few pages (that contain among some other results the hydrostatic paradox) never appeared. The hydrostatic part of his work on drainage windmills that we will discuss now, could easily have been included in such a volume.

Already in 1586 and 1588 Stevin obtained patents on windmill designs. Stevin also actually built such mills. They were drainage mills meant to lift water by means of a scoop wheel from a basin with a low water level to a basin with a higher water level. Particularly interesting is the case of the mill he built in a polder near the city of IJsselstein, south of the city of Utrecht. The contract was signed on April 8, 1589 by Stevin's business partner, Jan Hugo Cornets de Groot, with representatives of the polder (the polder Leege Biesen, Achtersloot, Meerloo and the Brouck in the land of IJsselstein).

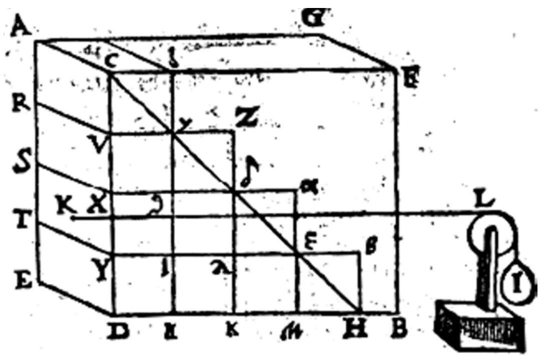

Fig. 10. The pressure exerted by water on a wall.

Stevin promised to build a mill of wood and iron for 630 Carolus Florins ([16], p. 324). The mill, that "would draw as much water as two of the best mills of thereabouts could do", would be ready in the fall of 1589. It soon became clear that the project was vexed with problems. In the end, after the polder refused to pay the last installment, De Groot and Stevin appealed to Princess Maria of Nassau, who while her brother was in captivity in Spain, was responsible for the barony of IJsselstein. After years a settlement was reached. The case is interesting because while Stevin accused the board of the polder of mismanagement, the representatives of the polder accused Stevin of mistakes in the design of the mill.

We know a lot about Stevin's ideas on windmills because Stevin left a manuscript called *On Mills* (Van de Molens) which contains calculations concerning both mills of the traditional type and mills of a different type based on Stevin's new design. He also left a manuscript on the design of gear wheels: *On the most perfect cogs and staves* (Van Aldervolmaackste

Cammen en Staven, [16], pp. 48–63). The mill near IJsselstein was built on the basis of Stevin's new design.

Stevin's considerations are based on an abstract kinematical model of the classical Dutch drainage mill See Figure 11 left. This model consists of the following structure reduced to certain fundamental geometrical parameters:

1. An oblique windshaft B turned by the sails.
2. A vertical upright shaft K with an upper gear wheel S driven by a gear wheel C on the windshaft.
3. A horizontal scoop-wheel shaft W with on it a gear wheel O, driven by a lower gear wheel N on the upright shaft, and a scoop-wheel R.
4. A tower with a movable cap on top of it. The oblique windshaft B was fixed inside this cap. The cap could be turned to make the sails face the wind. Both upright shaft K and scoop-wheel shaft W were fixed inside the tower.
5. The windshaft drives with its gear wheel C the upper gear wheel S of the upright shaft K and the upright shaft drives with its lower gear wheel N the scoop-wheel shaft W.

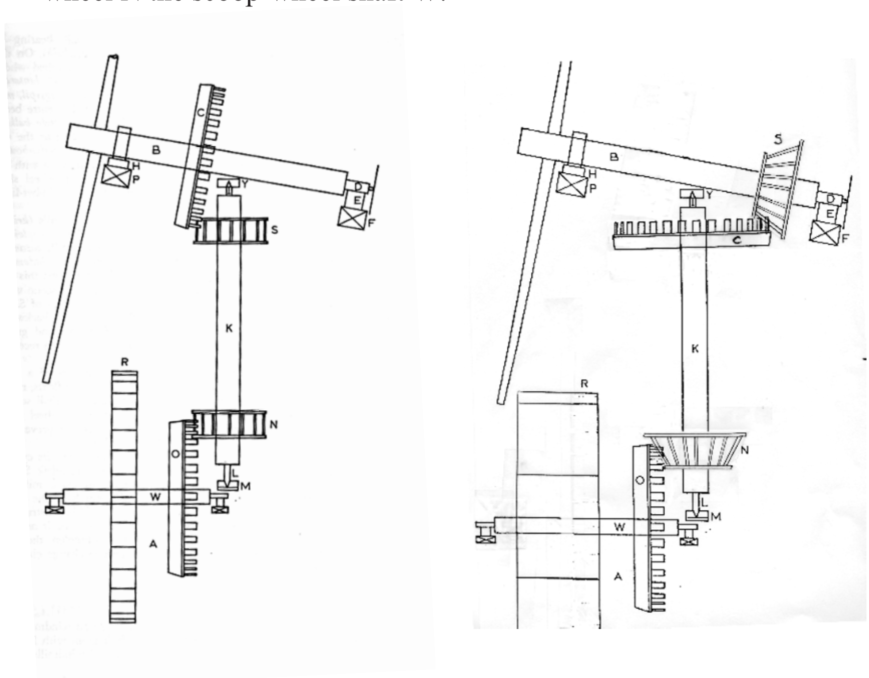

Fig. 11. The old design (left) and Stevin's design (right). Nota bene: the names of the wheels in the text refer to the old design.

He treats the gear trains in exactly the same way he had treated them in the *Almighty*. The dimensions of the mechanism and the number of teeth of the gears are the determining geometrical parameters of the kinematical model and he superimposes a chain of forces on the kinematical model: an input force brought about by the wind, transmission forces and an output force exerted on the water to be lifted. Subsequent forces are related to each other by means of the law of the lever.

Stevin's calculations all concern specific numerical cases and he does not give us general algebraic formulae. Yet he is fully aware of the generality of his method. Suppose that the wind exerts a force F_{wind} perpendicular to the wing and the wind shaft at a distance from the shaft equal to half the length of the wings, i.e. ½ $Radius_{Wings}$. (In modern terms the force causes a moment about the shaft of F_{wind} x ½ $Radius_{Wings}$). The gear train then is a chain of levers and given the dimensions by repeatedly applying the law of the lever we could determine the force F_{water} exerted on the scoops (at, for example a distance ½ $Radius_{Wings}$ from the axle) needed to have equilibrium.

Yet Stevin's calculations are not based on this approach. For Stevin the numbers of teeth of the gears and the numbers of revolutions they bring about are the parameters he calculates with. Let the numbers of teeth of respectively C, S, N and O be N_C, N_N, N_S and N_O. Then we have for the number of revolutions $R_{windshaft}$ of the windshaft and the number of revolutions $R_{scoopwheelshaft}$ of the scoop wheel shaft the following relation:

$$R_{scoopwheelshaft}/R_{windshaft} = (N_C.N_N)/(N_S.N_O).$$

This is Stevin's way to deal with the transmission of force in gears. He argues as follows. If the wings would rotate exactly as fast as the scoop-wheel, we would have equilibrium if F_{water} exerted by the water on the scoops (at a distance ½ $Radius_{Wings}$) would be equal to F_{Wind}.

However, in general in a situation of equilibrium we have the following relation between F_{Wind} (exerted on the wings at ½ $Radius_{Wings}$) and F_{Water} (exerted on the scoops at ½ $Radius_{Wings}$)

$$F_{wind} = (N_C.N_N)/(N_S.N_O) \text{ x } F_{water}$$

I will call this the *Fundamental relation*. It is remarkable that this is a kinematical relation, while Stevin refers for its proof of it to a result in statics: the law of the lever. Yet it shows that he was aware of the validity of the principle of the conservation of work.

12. CALCULATION OF THE WIND PRESSURE WITHOUT KNOWLEDGE OF AERODYNAMICS

Stevin's originality with respect to windmills does not lie in his insight in the fundamental relation. It lies in what he did with it. The authors who wrote on the subject, Dijksterhuis and Forbes, agreed on the originality of Stevin's approach, but as for how good Stevin's designs actually were, they seem to be hesitant. The remark "Stevin tried to calculate the minimum wind pressure needed to move his scoop wheel, but he failed to relate the wind velocity to the energy available on the scoop-wheel shaft, for in his days there were no means of measuring the speed of the wind" ([16], pp. 319–320) suggests that Stevin failed somewhere in his analysis. From our point of view such a criticism is unjustified. It is true that Stevin was not capable of deducing F_{wind} on the basis of, for example, aerodynamic considerations. However, the originality of Stevin lies firstly in the fact that he realized that the *Fundamental Relation* can be used to determine F_{wind} in a completely different way. He first measured and counted the fundamental geometrical parameters of several existing and functioning windmills. Then he used his original hydrostatic results to determine F_{water} for those windmills. And finally he applied the *Fundamental Relation* to calculate F_{wind} for those windmills.

In order to determine the force F_{water} (exerted on the scoop) at a distance ½ $Radius_{Wings}$) he models the scoop of the scoop wheel as a vertical rectangular board that separates high level water from low level water. His hydrostatical results enabled him to determine the moments exerted by the pressure of the high and the low level water. F_{water} is the force needed at the distance ½ $Radius_{Wings}$ to create equilibrium with the high and low level pressures. In this way Stevin determined for all mills that he investigated the force F_{water} and by means of the fundamental relation he calculated F_{wind}.

Actually in *On Mills,* for all mills Stevin divides F_{wind} by the area of the four wings together. He finds answers like 2 480/1336 ounces per square foot of sail (for the Zuyt Nootdorp Mill) and 4 536/1230 ounces per square foot of sail (for the Pynacker Mill at the bridge) or 3 44/1020 ounces per square foot (for the Cralingen Marsh Mill).

In passing Stevin also calculated in the case of the Zuyt Nootdorp Mill the force that the teeth of lower gears N and O exert upon each other by means of the law of the lever, in the way described above.

His answer is: $F_{lowergears}$= 1193 pounds. Without giving us the calculation he writes that the force between the teeth of the uppergears, $F_{uppergears}$, can be found by means of the relation:

$$F_{lowergears}/F_{uppergears} = Radius_S/Radius_N$$

Of course Stevin does not use this formula. He writes: "I say: as the radius of the driven wheel against the radius of the wallower, so the pressure above to that below" ([16], pp. 338–339).

13. A BRILLIANT NEW DESIGN?

It is clear from Stevin's work that the calculation of F_{wind} was only a means to design a more efficient windmill. Figure 11 (right) shows us one version of Stevin's new design. The basic new element of Stevin's design is a much bigger scoop wheel. As a result the resistance of the water that must be conquered is consequently much higher. In his calculations Stevin uses the following data as a starting point: length and width of the wings, the radius of the scoop wheel, the width of the scoop-wheel, the immersion of the spoons and the difference between the high-water and the low-water level. Moreover, he assumes that the wind yields a pressure of 3 ounces per square foot. This value is somewhat below the values he determined for the existing mills.

By means of his hydrostatics Stevin calculated F_{water} for his new design and used his model to calculate the dimensions of the gear wheels such that the force that the wind can apparently yield on the basis of his earlier calculations is enough to resist the pressure of the water on these big spoons. One of the consequences of the new design is that while in the traditional mill the transmission from the upper axis to the central axis speeds up the velocity of rotation and the transmission from the central axis to the lower axis slows it down again, in the new mill the big force needed to move the big spoons makes it necessary to use both transmissions to slow down the rate of rotation. In the traditional design the gear wheels on the central axis are both lantern wheels and the two other gear wheels, on respectively the upper shaft and the scoop-wheel shaft, are crown wheels. The need to slow down the rate of rotation immediately made it necessary to put the upper lantern wheel on the upper axis and the upper crown wheel on the central axis: the wheels S and C change places. In the new design the forces that the teeth of the gear wheels exert on each other are bigger than in the case of the traditional mills. That is why it is understandable that Stevin gave special attention to the position and shape of the teeth in *On the most perfect cogs and staves*.

14. HOW SUCCESSFUL WAS THE NEW DESIGN?

Understandably Dijksterhuis and Forbes give considerable attention the problems that Stevin encountered in the case of the IJsselstein mill. They studied the files in the IJsselstein Archive and there conclusion is the following: "The main point seems to have been that the upright shaft was made of too soft a timber and thus the thrust journal ('onderijzer' in Dutch) penetrated into the timber and the smooth turning in the thrust bearing was endangered." ([16], p. 325). Dijksterhuis and Forbes add a second point concerning the greater forces that were generated in his design: "Stevin in increasing the size of the scoop wheel caused heavier load on the pit wheel (the diameter of which remained the same) and thus greater stresses on the cogs and staves of this wheel and the crown wheel. He was not able to solve this difficulty mechanically nor to cope with the greater stresses in other parts of the machinery" ([16], p. 325). These remarks all suggest that Stevin's new design was a failure. Moreover Dijksterhuis and Forbes add: "Stevin encountered similar trouble in the case of the Kralingen mills" ([16], p. 326). In this case the trouble concerned the pit wheel, a crown wheel on the scoop-wheel shaft, which was originally not strong enough.

It is interesting that in the case of the IJsselstein mill Stevin felt it necessary to prove that his design worked well in other parts of the country and obtained a series of testimonials. The counsel of the polder may have felt that such testimonials were written by "disciples of Mr. Stevin", it is a fact that they contain a very positive report on other mills built by Stevin ([17], pp. 386–391). There is also a very positive report in which authorities from Kralingen declare their satisfaction with Stevin's mills. It is remarkable that Dijksterhuis and Forbes believe that this positive testimonial should be regarded as a conciliatory gesture ([17], p. 327) concerning the rebuilding and strengthening of the above-mentioned pit wheel, thereby suggesting doubt concerning the reliability of the testimonial. This is particularly strange, because the negotiations had ended with a contract in May 21, 1593 and the testimonial was accorded to Stevin in June 1594. Why would a conciliatory gesture be necessary more than a year after agreement had been reached?

Yet, whatever the causes, the conclusion can only be that Stevin's new design was not a big success.

15. CONCLUSION

The essence of the Archimedean Renaissance in mechanics is the attempt to study mechanics or the science of machines in an Archimedean way. Because none of Archimedes' works on mechanics had survived Renaissance scholars had to bridge the gap between, on the one hand, the highly theoretical treatises of Archimedes, in which he had turned statics and hydrostatics into pure, strictly deductive sciences, and, on the other hand, the real machines.

Del Monte saw the problem clearly. In his *Mechanicorum Liber* he derived the law of the balance or lever in an Archimedean way and then attempted to explain the functioning of the five simple machines on the basis of this law. Del Monte defined the problem but did not go far beyond what his predecessors had reached.

Very probably without having read Del Monte Stevin was much more successful. With some new highly original contributions to statics and hydrostatics and an approach in which the unity of theory and practice was a central dogma Stevin showed that an Archimedean science of machines going beyond what had been reached in Antiquity was quite possible. Stevin thought and wrote in Dutch. I have argued that this probably helped him in his new and fresh approach to mechanics. It also meant a disadvantage. His work was not immediately noticed. Only in 1634 some of his important works were translated into French [20]. The importance of Stevin is still sometimes underestimated. In an extensive paper on the emergence of Archimedean mechanics in the Late Renaissance published in 2008 [11] Stevin is not even mentioned.

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ARCHIMEDES' CANNONS AGAINST THE ROMAN FLEET?

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ABSTRACT In the paper is discussed the possibility tat Archimedes built and used against the Roman fleet a steam cannon.

It is well-known that Archimedes, during the siege of Syracuse, designed and built several war machines to fight against the Romans. Among these war machines, the legend about the large concave mirrors that concentrated the sun rays burning the Roman ships is rather interesting. On this topic are also interesting some drawings by Leonardo Da Vinci where a steam cannon is described and attributed to Archimedes.

Starting from passages by ancient Authors (mainly Plutarchos, Petrarca and Da Vinci), the author investigates on the possibility that Archimedes built a steam cannon and used it to hit the Roman ships with incendiary proiectiles.

1. INTRODUCTION

Everybody knows the legend telling that Archimedes, during the siege of Syracuse (214–212 B.C.), designed and built several war machines to fight against the Roman fleet. Among these war machines, the legend about the burning mirrors is rather interesting. According to the legend, these burning mirror consisted in large concave mirrors that concentrated the sun rays in a point, a Roman ship, burning it; a scheme is reported in figure 1.

There is not any doubt that a parabolic mirror can burn a piece of wood as it was demonstrated by a Greek engineer (Joannis Stakas) in 1974 [1]; in addition such devices are commonly used nowadays in applications of the solar energy. In particular, by modern linear mirrors, it is possible to heat a fluid mix of salts flowing in a pipe (that is located in the locus of the

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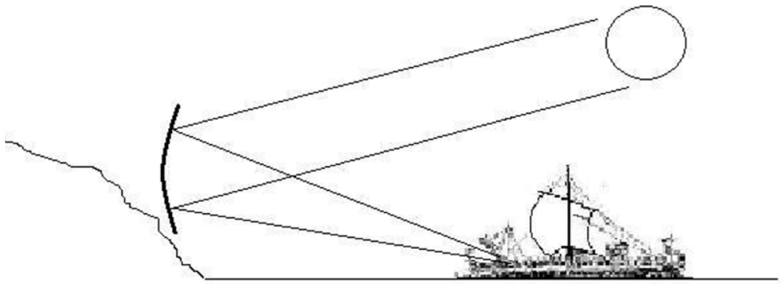

Fig. 1. Working principle of a burning mirror.

parabola's foci) up to 600°C. Nevertheless, the use of such mirrors as a weapon against ships is rather difficult to believe [2–5]. The described device, in fact, can work only if the ship's wood is located in the focus of the parabola. Hence if the ship moves forwards or backwards respect to this point, the only way to "adjust" the device could be to change the mirror's curvature. As far as this aspect is concerned, some Authors suggested the use of a device that consisted in a "composite burning mirror" made up by a number of (plain) mirrors that could be adjusted in order to concentrate the sun rays at different distances from the device itself. Experiments with such devices were carried on by some scientists (see e.g. [6,7]).

Nevertheless, even if a point of the ship (made of rather wet wood) was lighted, it is quite impossible to concentrate in this point enough energy to sustain the fire; in addition, the fire could be easily put out by few bucket of water. This aspect has been already remarked by other Authors (see e.g. [4,5]).

At the end of the XV century, Leonardo Da Vinci drew a steam cannon that he ascribed to Archimedes and, for a tribute to Archimedes, was called "architronito" (Tunder of Archimedes); the drawing is reported in figure 2.

On the same folio is reported also the working principle:

Architronito è una macchina di fine rame, invenzione di Archimede, e gitta ballotte di ferro con grande strepito e furore. E usasi in questo modo. La terza parte dello strumento istà in fra gran quantità di foco di carboni, e quando sarà bene da quelle infocata, serra la vite d, ch'è sopra al vaso dell'acqua abc; e nel serrare di sopra la vite e' si distopperà di sotto, e tutta l'acqua discenderà nella parte infocata dello strumento, e lì subito si convertirà in tanto fumo che parirà maraviglia, e massime a vedere la furia e sentire lo strepido.

Questa cacciava una ballotta, che pesava un talento, sei stadi. …

Architronito is a machine of pure copper, invented by Archimedes, and throws iron balls with great noise and fury. It is operated as follows. The third part of the device is located in a big quantity of fire by coal, and when it is well made red-hot by it (coal), the valve d is closed, that is on the water reservoire abc; and by closing the valve above e' it will be stopped below, and all the water will go down in the heated part of the device, and there suddenly will be converted in so much smoke (vapour) that it will appeas as astonishing, and even more by seeing the fury and hearing the noise.

This (device) threw a ball weighting one talent (≈26÷38 kg), (with a range of) six stadia (≈1100 m). ...

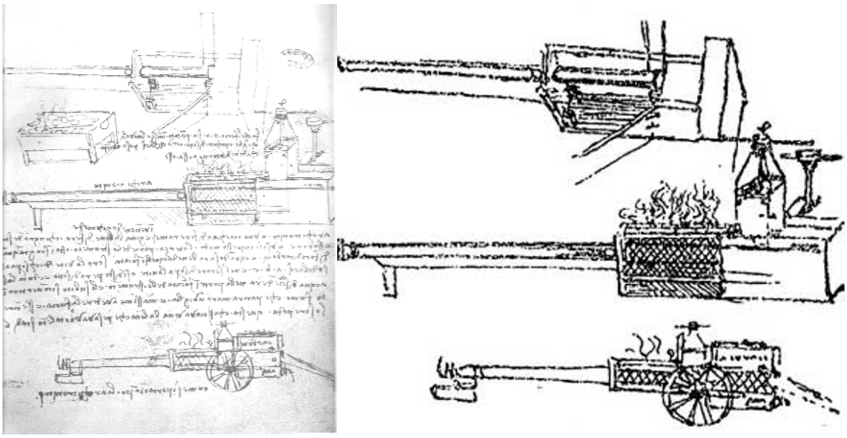

Fig. 2. Drawings by L. Da Vinci (Ms. B, f. 33 v) of the architronito.

Several authors also described similar devices; among them we can cite Francesco Petrarca (1304–1374) that, in a minor work (De Remediis Utriusque Fortunae) describes a steam cannon about one century before Da Vinci:

Straordinario, se non anche le palle di bronzo, che vengono scagliate con tuono orribile. Non era abbastanza l'ira di Giove che tuonava dal cielo, se il piccolo uomo (o crudeltà unita alla superbia) non avesse tuonato anche dalla terra: la violenza umana ha imitato il non imitabile fulmine, come dice Virgilio. E quello che di solito è scagliato dalle nuvole, e mandato con uno strumento sì di fuoco, ma infernale. Ed alcuni ritengono che questo sia stato inventato da Archimede, nel tempo in cui Marcello assediava Siracusa. Per la verità lo escogitò per difendere la libertà dei suoi cittadini, sia per allontanare sia per differire la rovina della patria; e voi vene servite, invece, per opprimere i popoli liberi o col giogo o con la distruzione. Questa peste non molto tempo fa rara, ora siccome gli animi sono succubi alle cose più malvagie, è comune come qualsiasi genere di armi.

It is extraordinary, if not only the bronze balls, that are thrown with orrible thunder. It was not enough the anger of Jove that thundered from the sky if the little man (oh cruelty of the haughtiness) had not thundered also from the heart: the human violence imitated the non imitable lightning, as Virgil says. And what usually is thrown by the clouds, is (now) thrown by a device that is also made by fire but hellfire. Some people believe that this (device) was invented by Archimedes when Marcellus besieged Syracuse. In truth he invented it to defend the freedom of his fellow citizens and to retard and defer the ruin of its Country; you, instead, use it to oppress free people with yoke or destruction. This plague not many time ago was rather rare, now, since the minds are dominated by the most wicked things, is common like any other kind of weapon.

It must also be considered that parabolic mirrors were used during the Renaissance for brazing the copper. In addition, nowadays, parabolic mirrors are used to obtain energy from the sun; in some application a fluid mix of salts is heated (in a pipe located in the locus of the foci of a parabolic linear mirror) up to 600°C.

The Greek historian Plutarchos (later Roman citizen as Lucius Mestrius Plutarchus ≈ A.D. 46–120), in his *Vite parallele,* vol. II, *Pelopida e Marcello* 14-15, tells that, during the siege of Syracuse, when the Romans saw something that was similar to a pole protruded from the walls ran away shouting :"Archimedes is going to throw something on us now". Now, let us consider that no ancient throwing machine (such as onager, ballista or catapult) looks like a pole [8]. In the appendix, some examples of the main pieces of the Roman artillery are reported.

Very interesting is also a piece cited by Simms [3]: in it, it is reported that Niccolò Tartaglia (Italian mathematician, about 1499–1557) wrote that Valturius (Roberto Valturio, Italian engineer and literary man 1405–1475) in his treatise De re militari, *"... States that ... there are many references to Archimedes having designed a device made from iron out of which he could shoot, against any army, very large and heavy stones with an accompanying loud report."*

Finally, as it was already remarked by several investigators, no mention about burning mirrors was made by the historians of the Greek-Roman era but this legend appears only during the middle age.

For the all the reasons above reported it seems plausible to suppose that Archimedes used burning mirrors to heat the breech of steam cannons. In the next paragraphs the possibility to use a such device is investigated.

2. A RECONSTRUCTION OF THE ARCHIMEDES' STEAM CANNON

In figure 3 is reported a possible scheme of parabolic mirror fitted to heat the breech of a cannon.

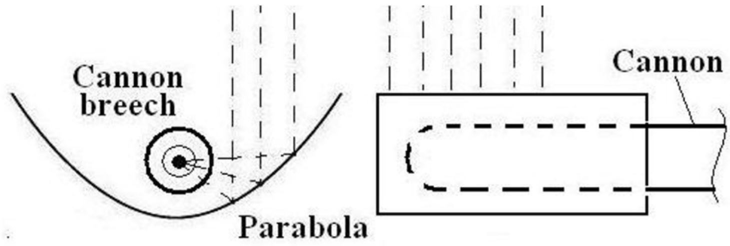

Fig. 3. Possible scheme of parabolic mirror heating the breech.

In figure 4 is reported a scheme of the device that was probably used to inject the water in the breech. It is mainly based on the drawings and the brief description by Da Vinci's manuscript and the steam cannon model built by I. Sakas [9]. It must be said that, the Sakas' reconstruction of the Archimedes' steam cannon, the ball was constrained down in the barrel by two wooden stick: a first stick is put inside the barrel, a second stick is put at the muzzle, orthogonal to the barrel axis and hooked to the muzzle by two rings. When the pressure was high enough to break the second stick, the ball could start. In this way rather high steam pressures and thereby high muzzle velocities could be achieved. This solution or similar ones invented by several modern steam cannon builders are rather dangerous and they could not be easily adopted by a military weapon of the age of Archimedes.

Since neither in the drawings by L. Da Vinci nor in any other bibliographical source the author could found any evidence of the equipment used by Sakas, in the following it was not considered.

A proper amount of water is put in the reservoir A, then the valve B_1 is opened and the water fills the tank C. Next the valve B_1 is closed and the valve B_2 is opened: the water flows in the chamber of the cannon and vaporizes. Through the pipe D, the pressure in the tank C is equalized to the one in the chamber of the cannon. The steam pressure throws the ball E outside the barrel.

It must be pointed out what follows:

1. By a burning mirror and the described working cycle, it is difficult to achieve high energy and hence high ball muzzle velocities.
2. In order to shoot at a (moving) ship from a city wall it is necessary that the cannon ball has a rather flat trajectory; otherwise it is rather difficult to hit the target.

Naturally, low muzzle energies could carry to a low muzzle velocity if the ball mass was about 30 kg as described by Da Vinci. This would permit only a parabolic trajectory that was unsuitable to hit a moving ship.

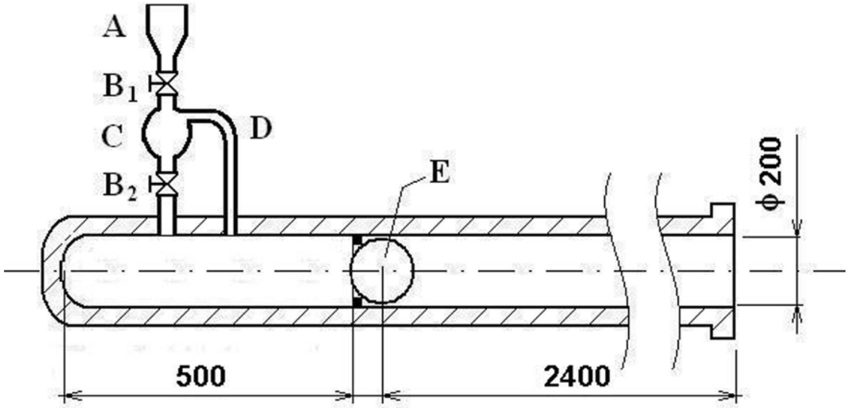

Fig. 4. Scheme of the Archimedes' steam cannon.

On the other side, we must remember that, up to the middle of the XIX century, the calibre of a gun was given as a weight; the latter indicated a barrel which diameter was the one of a ball (made of cast iron for cannons and of lead for guns) having that weight. Also for the Archimedes' steam cannon it could be the same.

In this case, we can suppose that the cannon could throw rather lightweight hollow balls made of clay and filled by incendiary mixture that was well-known by the Greeks. These balls could reach an higher muzzle velocity and hence a flatter trajectory and, when hit the ship, they broke off spreading the incendiary mixture, setting fire on the ship. The possibility that the roman ships were burned by Archimedes by means of somewhat like the famous "Greek fire" is also suggested by Simms; in [4], in fact, it is reported that Galen (Aelius Galenus or Claudius Galenus or Galen of Pergamum 129–216) in his De Temperamentis says that "… Archimedes sat on fire the enemy triremes by means of πυρεια." Now this word, in ancient Greek indicates something used to light fire or can be translated as "brazier" but can not be translated as "burning mirror".

As for the incendiary mixture known as "Greek fire" it has to be said that its exact composition is unknown; nevertheless, the main components very probably were sulphur, liquid bitumen, pitch and calcium oxide. It is also well-known the use of a mixture that could burn underwater or even be ignited by water (that the Byzantines named marine fire or Roman fire) and even the use of flamethrowers for sea warfare in the Greek-Roman era [10].

It is also known the use incendiary projectiles (*vasa fictilia*) that consisted in *"clay containers filled with flax soaked in a mixture of liquid bitumen, pitch and sulphur, with a sulphonated fuse. They were hurled using special machines. When they fell, the vase broke and the incendiary composition came into contact with the object it struck. These types of projectiles are mentioned by: Appiano, Dionysius of Halicarnassus, and Frontino. They were widely used in many locations, especially by Demeritus during his naval attack against Rhodes (304 B.C.), and in the naval battles that took place during the second Punic wars. They also launched porous rocks after filling their cavities with flammable material and setting them on fire"*[10].

In figure 5 is reported a possible incendiary projectile made by hollow clay ball that was filled by incendiary mixture. From the proposed dimensions, that are reported in figure, the mass of such a ball could be round 6 kg; this could bring to reasonably flat trajectories, as it will be shown in the following paragraph.

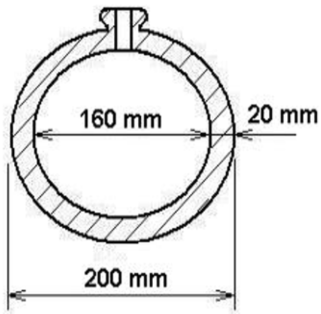

Fig. 5. Hollow clay ball.

Balls like the one described are shown in figure 6. The one on the top left is from the fortress of Chania (X–XII Century) and presently are at the National Historical Museum, Athens, Greece, the picture at the top right is reported a representation of a gun (a fire lance) and a grenade (upper right), from the cave murals at Dunhuang, c. 950 A.D., those in the lower part of the image are ceramic bombs found on the 1281 shipwreck of the fleet who attempted to invade Japan. In the figure it is possible to observe an hole from which the incendiary mixture was filled and that was closed by a cork bringing the fuse.

Fig. 6. Incendiary projectiles.

3. ROUGH EVALUATION OF ENERGIES AND TRAJECTORIES

In this paragraph the possibility that the device described before could be effective against a ship is roughly evaluated. It must be pointed out that all the assumptions and the computations are rough since the main purpose is to asses whether such a device, conceptually, could "work" or not.

3.1. The Projectile Muzzle Energy

As it was told before, it was supposed that the projectile diameter was 200 mm and it's mass was 6 kg; moreover, the barrel length covered by the ball was 2,4 m. In the following paragraph it will be shown that a suitable ball's muzzle velocity is 60 m/s. So, from these assumption and supposing that the ball's acceleration in the barrel is constant, it is easy to obtain:

Ball's muzzle energy Eo = 10,8 kJ
Ball's time to cover the barrel length t = 0.08 s

Now lat us assume that:

- the water temperature, when introduced in the breech, was 30°C,
- the mean breech temperature during the vaporisation process was 430°C (i.e. mean $\Delta t = 400°C$),
- the surface wetted by the water was half the breech inner surface,
- the heat transfer coefficient between the breech and the water spray can be assumed, very conservatively, K = 10 kJ/m^2/s/°K [6], then the heat that was transferred from the breech inner surface to the water is: $Q \approx 53$ kJ.

Now, it seems reasonable that 20% of this energy was transferred to the ball; this means a ball's muzzle energy Eo = 10,6 kJ and a ball's muzzle velocity Vo = 59,44 m/s.

It must be observed that in the scheme reported in fig. 4, the ratio between the barrel length and its diameter is only 12 (very little if compared to modern cannons and near to the ratio of the I WW howitzers) while from the table by L. Da Vinci it is possible to observe a ratio of about 30. This suggests that in ancient devices, probably, the time required by the ball to cover the barrel length was comparably higher and the steam worked more efficiently.

3.2. The Projectile Trajectory

In order to evaluate the projectile energy, because of it's low speed, it was considered a simple model for the drag force R due to the air:

$$R = \tfrac{1}{2}\rho\, V^2 A \tag{1}$$

Where:

ρ is the mass density of the air = 1,225 kg/m^3,
V is the speed of the object relative to the air,
A is the area of the projectile's cross section.

The equations of motion:

$$\begin{aligned} -m\ddot{x} + R &= 0 \\ -m\ddot{y} - mg \pm R &= 0 \end{aligned} \tag{2}$$

were solved numerically. It must be observed that, naturally, the sign of R in the second of the equations (2) depends on the sign of the vertical component of the velocity.

In figure 7 is reported a simple scheme showing the gun position on the sea level, the gun elevation β and the range.

In figure 8 is reported a trajectory that was computed by assuming a muzzle velocity Vo = 60 m/s, an elevation angle β = 10° and that the gun was 10 m above the sea level.

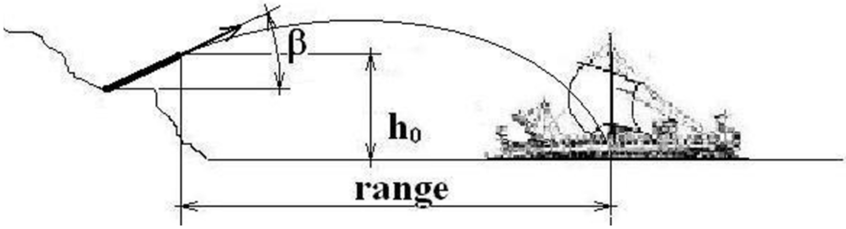

Fig. 7. Scheme of the cannon shooting.

From the figure it is possible to observe that the range is about 150 m and the trajectory is rather flat (the scales of the axes in fig. 7 are isometric); that is to say the maximum elevation of the projectile over the line of sight is very small if compared to the range. The range seems to be adequate to the use of the device while the rather flat trajectory is important for the anti-ship fire.

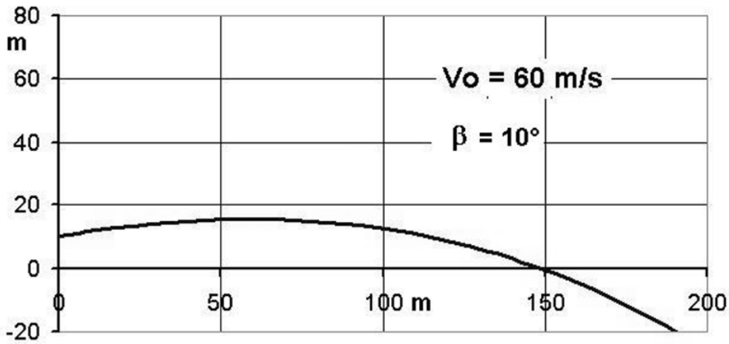

Fig. 8. Projectile's trajectory.

In figure 9 are reported some other trajectories, near the target, with different muzzle velocities and gun's elevation, all hitting the target.

The latter is represented by a 6 m wide and 3 m high silhouette (approximately the dimensions of a Roman trireme's cross section), placed at a distance of 100 m in the plane of the ball's trajectory.

It is possible to observe that if the muzzle velocity is Vo = 60 m/s the target is hit with elevations ranging from β = 3.1° to β = 5.1°, while if the elevation is fixed to β = 4°, the target is hit with muzzle velocities ranging from Vo = 57 m/s Vo = 64 m/s. This means that at those ranges, it was not necessary a very high accuracy in the pointing neither was necessary a very high repeatability of the muzzle velocity.

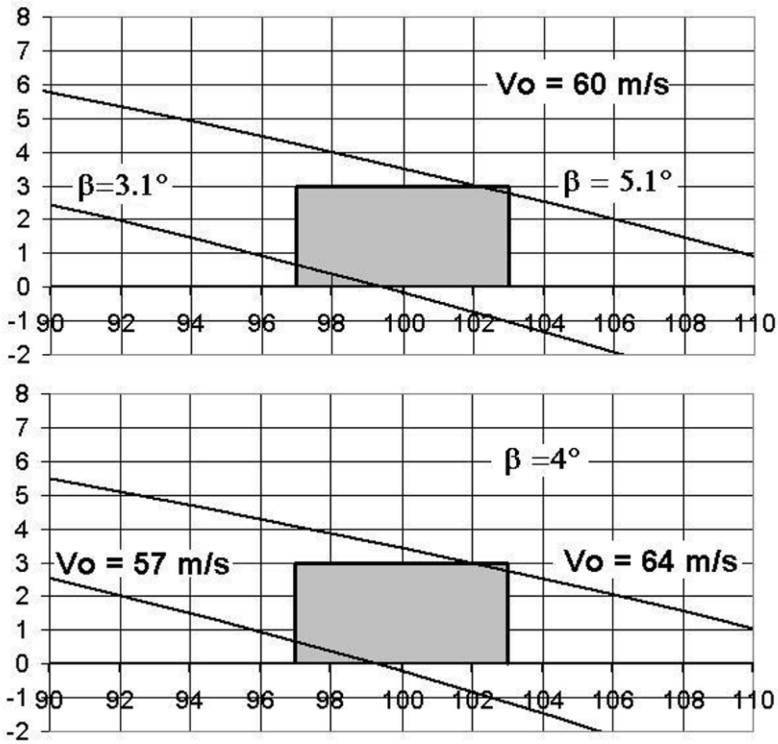

Fig. 9. Different trajectories hitting the target.

It must be observed that, by assuming the target silhouette shown in fig. 9, it was supposed that the ship moved in a direction orthogonal to the plane of the projectile's motion; this condition is the one in which the ship offers the smaller section in the plane of the projectile's motion. This is shown in fig. 10, where a ship is represented in it's plane of motion that is orthogonal to the plane of the projectile's motion.

In the figure the dashed dotted lines are the intersections between the projectile plane of motion with the ship's plane of motion hence the lines A-B or A'-B' represent the width of the silhouette reported in fig. 8. It is evident that if the ship moves in a direction non orthogonal to the cannon's barrel, the "apparent width" of the target increases.

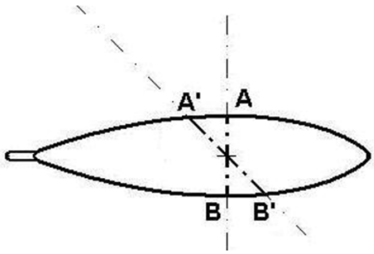

Fig. 10. Ship's silhouette in it's plane of motion.

4. CONCLUSIONS

A possible reconstructions of the performances of a steam gun by Archimedes was proposed.

Computations and assumptions are rather rough since the main aim is to asses whether the device was capable to hit and burn a Roman ship or not. Naturally, if it was possible, this (alone) doesn't mean that it happened.

Nevertheless, it seems reasonable to believe that the only possibility that Archimedes had to burn Roman ships by mirrors was to use the described device for two main reasons:

I) First of all is rather difficult to built a bursting mirror suitable for those applications; in fact a concave mirror having a diameter of (say) 4 meters that concentrates the sunrays at a distance of (say) 100 meters has a concavity of few millimetres. In some experiments (1973 Sakas and Stamatis and 2005 MIT) were used a number of plane mirrors and little boats or mock-ups were really burned; nevertheless a practical use of such a device during a battle seems not very realistic. In fact it must be considered that in the experiment by Sakas and Stamatis about 50 sailors of the Greek Navy were necessary to point the mirrors and in the experiment at the MIT 300 mirrors were used; in addition, in both cases, the target was absolutely motionless. Very different conditions take place during a battle, hence it is difficult to believe that a big number of mirrors can be pointed on a moving target efficiently.

Then it must be considered that a fire lighted in this way could be extinguished very easily. Really the wood starts to burn at about 250°C and at temperatures a little higher than the latter burns with flames even without any further external supply of heat; but smoke and flames were clearly visible and, also, were the main threat for wooden ships. So, it is difficult to understand the reason why nobody had extinguished those initial fires, just by few buckets of water.

II) The second reason is that, as already told, there are no historical sources in the Archimedes' age telling about the use of burning mirrors for warfare. Silius Italicus (~25-101 A.D.), about 3 centuries after the siege of Syracuse, in his poem "Punica" does not tell about any mirror but mentions a tower from which Archimedes threw incendiary projectiles against the Roman ships. Valerius Maximus (Factorum et dictorum memorabilium libri IX ~31 A.D.) is probably the first who mentions burning mirrors. Later, Lucian of Samostata (~125 – after 180) refers about Roman ships burned but without indicating how the fire was set on them. Finally, as already mentioned, Galen of Pergamum says that Archimedes burned some Roman ships but the term he used can not be translated as "burning mirror". The use of a set of articulated plain mirrors is supposed for the first time by Anthemius of Tralles (~474 – before 558 A.D.) in his treatise "*Perì paradòxon mesantmaton"* (On the paradoxes of the Mechanics) [5,12].

As for the steam cannon, it must also be remarked that the described technology (valves, pipes etc.) was available in those ages [10]. Also, steam cannons where used till in the XIX century [10]. Finally, a number of writings (e.g. Plutarchos, Francesco Petrarca, Leonardo Da Vinci etc.) strongly suggests that Archimedes built and used such a device.

5. APPENDIX

In the introduction a piece by Plutarchus has been cited in which the Roman Soldiers were frightened by a weapon, similar to a pole, that Archimedes used against them. Since, as already told, no heavy weapons looked like a pole, it could be interesting a very brief review on some examples of the main Greek-Roman artillery pieces. Since the Roman artillery was almost "copied" by Greek designs, some drawings of Roman artillery pieces will be shown.

First of all it must be pointed out that in the III century B.C., thanks to Greek engineers, the motor of the throwing machines was mostly the torsion motor that was made generally by women's hair or horse air [8, 10, 15-18] as shown in figure 11.

The main pieces were the catapult (and the scorpio that was a little catapult), the ballista and the onager (in Latin: onagrum), all powered, as told, by torsion spring motors.

It must be pointed out that during the Roman Empire the word "catapult" (probably from the ancient Greek katà pelte = through the shield) was used for a machine that throws darts, while the word ballista (that also comes from the Greek word βαλλω (ballo = I throw) was used for a machine that throws balls. During the Middle Age the words were used with the opposite meaning : ballista for a dart throwing machine and catapult for a ball throwing one.

In figure 12 is reported a pictorial reconstruction of a Greek-Roman catapult [10].

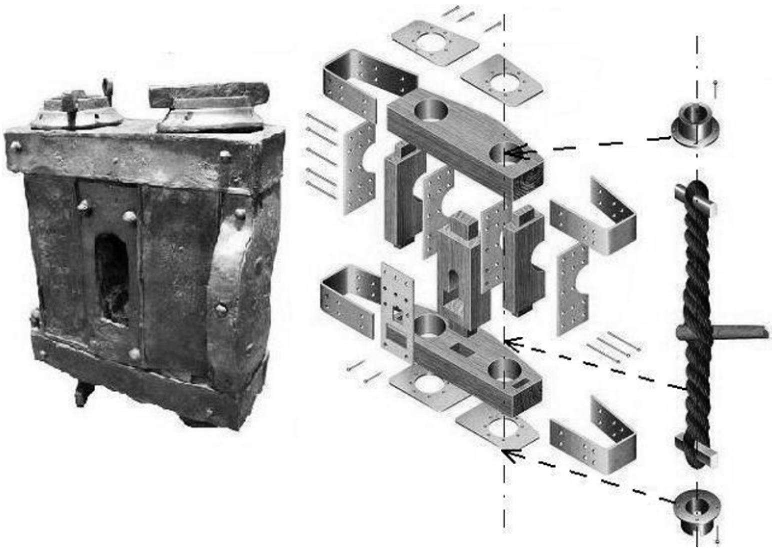

Fig. 11. Torsion motor: find (left) and reconstruction (right).

Fig. 12. Pictorial reconstruction of a catapult.

In figure 13 are reported a pictorial reconstruction of an eutyntonon ballista on the left and a pictorial reconstruction of the large ballista found at Hatra (palintonon ballista) on the right [8].

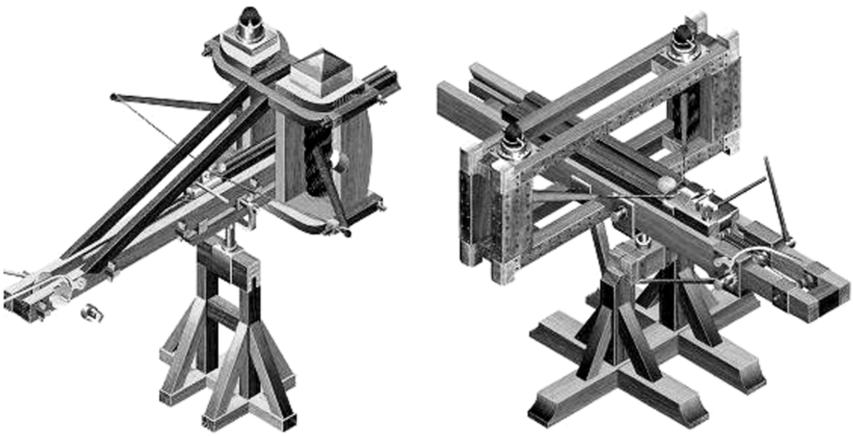

Fig. 13 a). Pictorial reconstruction of ballistae: eutyntonon (left) and palintonon (right).

It must be observed that while in the eutyntonon ballista the arms, during the run, are always in the same half-plane respect the frame, in the palintonon ballista the arms pass through the frame as shown in the sceme in figure 13. This permitted to the arms to rotate by a larger angle and, hence, an higher efficiency of the palintonon [8, 19, 20].

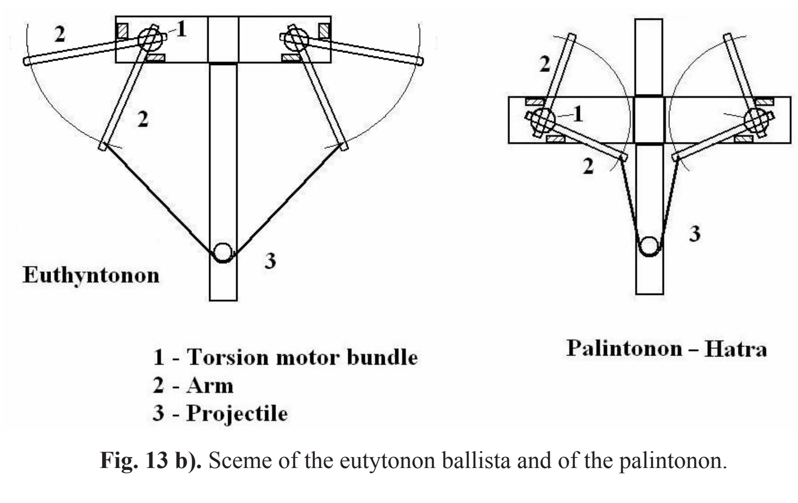

Fig. 13 b). Sceme of the eutytonon ballista and of the palintonon.

The studies on the throwing machines technology was certainly carried on till the I century B.C. when a repeating catapult [15, 16, 21, 22] was developed The device was described by Philon of Bizantium and attributed to Dionysius of Alexandria and, apparently, it was used around the I century B.C.; it was a part of the arsenal of Rhodes that may be considered as a concentration of the most advanced mechanical kinematic and automatic systems of the time, many of which show working principles and a conceptions that still can be considered as "modern". A pictorial reconstruction is shown in figure 14 and a cinematic reconstruction of the automatisms can be found in [23].

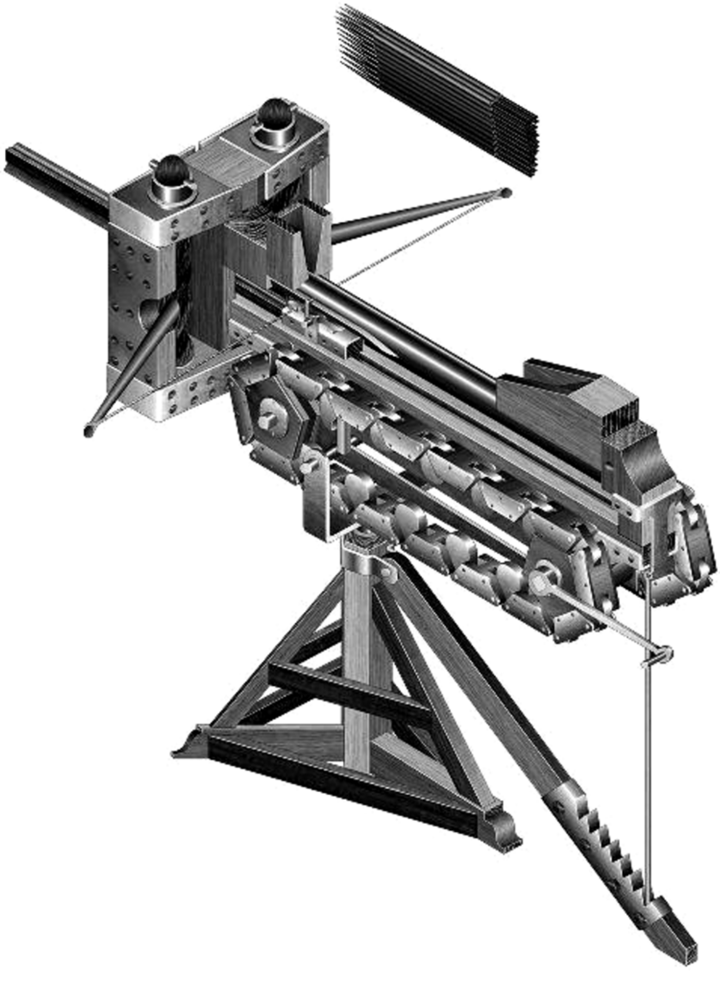

Fig. 14. A pictorial reconstruction of the repeating catapult [23].

In figure 15 is reported a drawing of the fully automatic mechanism, as it was proposed in the reconstruction by the author [23].

From this figure, that is based on previous studies [15, 16, 23, 22] and on the author's study of the description given by Philon of Bizantium, it is easy to understand the "modernity" of the Greek weapon designers.

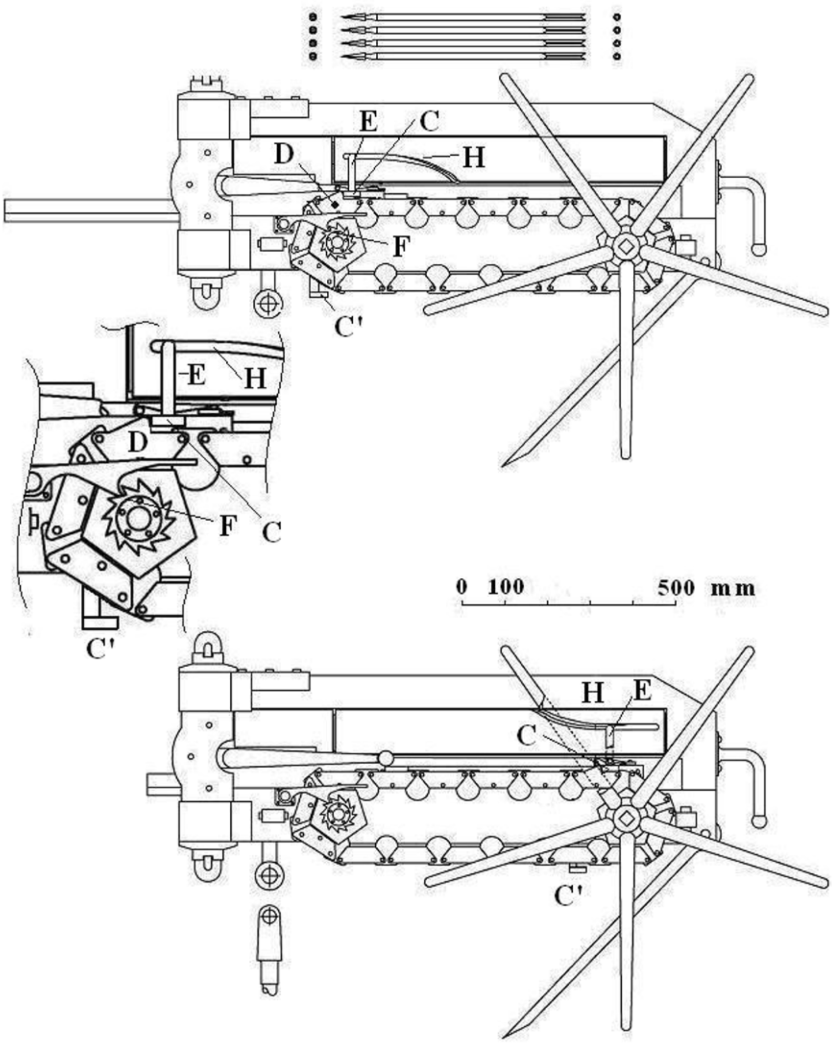

Fig. 15. The mechanism of the repeating catapult.

Finally, in figure 16 is reported a pictorial reconstruction of an onager. From the figure it is easy to understand that the projectiles (stones or similar round objects) thrown by this weapon could describe only parabolic trajectories like those of an howitzer and not flat ones.

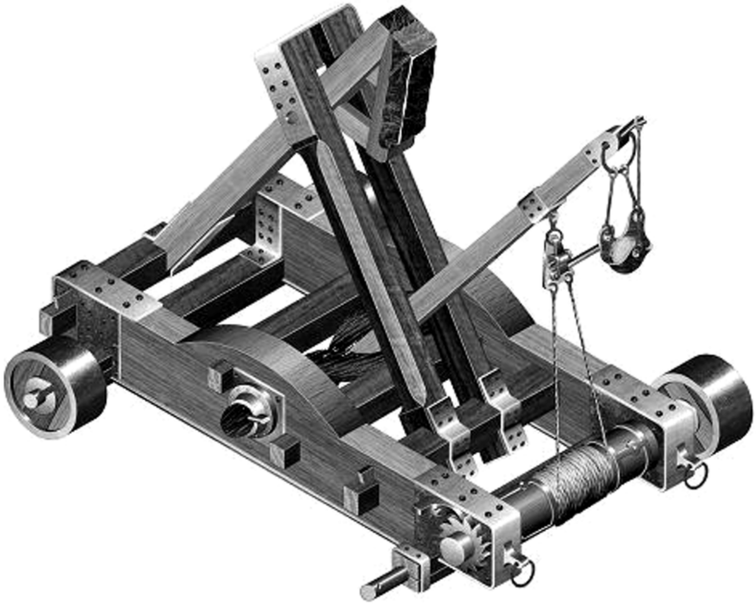

Fig. 16. Pictorial reconstruction of an onager.

From the brief notes reported above, it is evident that the steam cannon was something of very different either for what the shape is concerned and (even more) from a conceptual point of view. As far as the latter aspect is concerned, the extraordinary modernity of the Archimedes' cannon is evident.

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V-BELT WINDING ALONG ARCHIMEDEAN SPIRALS DURING THE VARIATOR SPEED RATIO SHIFT

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ABSTRACT Starting from a previous model for the shift mechanics of rubber belt variators, this lecture elaborates practical design formulas for the torque and the axial thrust making use of the very close resemblance of the belt path to a linear spiral of Archimedes along a large part of the arc of contact. In addition, as an alternative to the modern calculus tools, it is shown how the drive variables can be equally calculated applying some propositions of Archimedes' classical treatise *περὶ ʽελίκων* (On Spirals).

1. INTRODUCTION

The usual operation of the continuously variable transmissions (CVT) for vehicles or motorcycles consists in a random continuous change of the speed ratio, where a gross radial motion of the belt toward the inside or outside of the groove is superimposed to the circular motion. Only a few approaches to the CVT transient mechanics can be found in the literature (see references in [1,2]) and a practical formulary is still missing. The present paper resumes the theory of [1] and constructs useful design formulas for the torque and the axial thrust. The full equation system is strongly non linear and its exact solution requires complex numerical procedures. Attempts at approximate solutions were carried out for some applicative cases [2], achieving a very fine agreement with experiments. Here, a much simpler formulation will be developed, taking advantage of the Archimedean spiral shape of the instantaneous belt line.

The involvement of Archimedes in the mechanics of machines was quite relevant and several ingenious devices are to be ascribed to him, both for the civil and military application, though such inventions arose more from practical occasional requirements than from his intimate disposition to this kind of activity [3,4]. Among many other interests, he was also concerned with the cable and belt mechanics and for example, his compound-pulley tackles (*πολύσπαστα*) for the launch of very big ships are

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recorded by the historians. Nevertheless, he preferred the speculative aspects of the theoretical mechanics and the investigation on several mathematical and geometrical problems regarding plane and solid figures [3-7]. His treatise on spirals, though characterized by a limited divulgation in the past, may be recognized of a great modernity after more than two thousand years and may still yield alternative methods for the solution of today's mechanical problems, as will be shown also in the following.

The history of belt mechanics is traced by Gerbert in ref. [8]. Despite the extensive use of cable and rope devices in the antiquity, the theoretical analysis of the belt drives originates only in recent centuries, starting from the well known capstan formula of Euler-Eytelwein and proceeding with the fundamental distinction made by Grashof between the adhesive arc (*Ruhebogen*) and the sliding arc (*Gleitbogen*). The V-shaped belts were introduced by John Gates at the beginning of the 20th century. Their analysis and their use in variable speed drives date from more recent times (Lutz, Worley, Dittrich, Gerbert) and also the author of the present paper has been working in this research area during the last years.

2. BELT-PULLEY COUPLING

A scheme of the belt element with the wall forces is represented in Fig. 1a, while Fig. 1b shows some details about the geometry and kinematics of the belt path. These figures may be used as a reference for the notation.

Putting $\theta = \theta(t)$ and $r = r[t,\theta(t)]$ along the trajectory of a belt element, one has $dr/dt = \dot{r} + \dot{\theta}\, r'$, where dots and primes indicate the differentiation with respect to t and θ respectively. Moreover, letting $x = (r_\infty - r)/r_\infty$ be the dimensionless elastic penetration of the belt, where r_∞ is the nominal radius for infinite transverse stiffness of the belt, the self-evident geometrical relationship $r' = -r\tan\chi$ gives

$$x' = (1 - x)\tan\chi \tag{1}$$

The above formulas give rise to the relationship $v\sin\delta = v\cos\delta\tan\chi - \dot{r}$, while the triangle of velocities points out that $v\cos\delta - \omega r - v\sin\delta\tan\gamma = 0$, and such two relationships yield $v\cos\delta(1 - \tan\chi\tan\gamma) = \omega r(1 - \rho\tan\gamma)$, where $\rho = \dot{r}/(\omega\cdot r) \cong \dot{r}_\infty/(\omega\cdot r_\infty)$ is the dimensionless shift speed. Therefore, if $\rho = \tan\chi$, then $v\cos\delta = \omega r$ and $v\sin\delta = 0$, i.e. there is adhesion between the belt and the pulley and one has $x' = (1 - x)\rho$ by Eq. (1), so that the belt has the shape of a logarithmic spiral. Nevertheless, as $x \ll 1$, the instant path can be confused with a linear spiral of Archimedes.

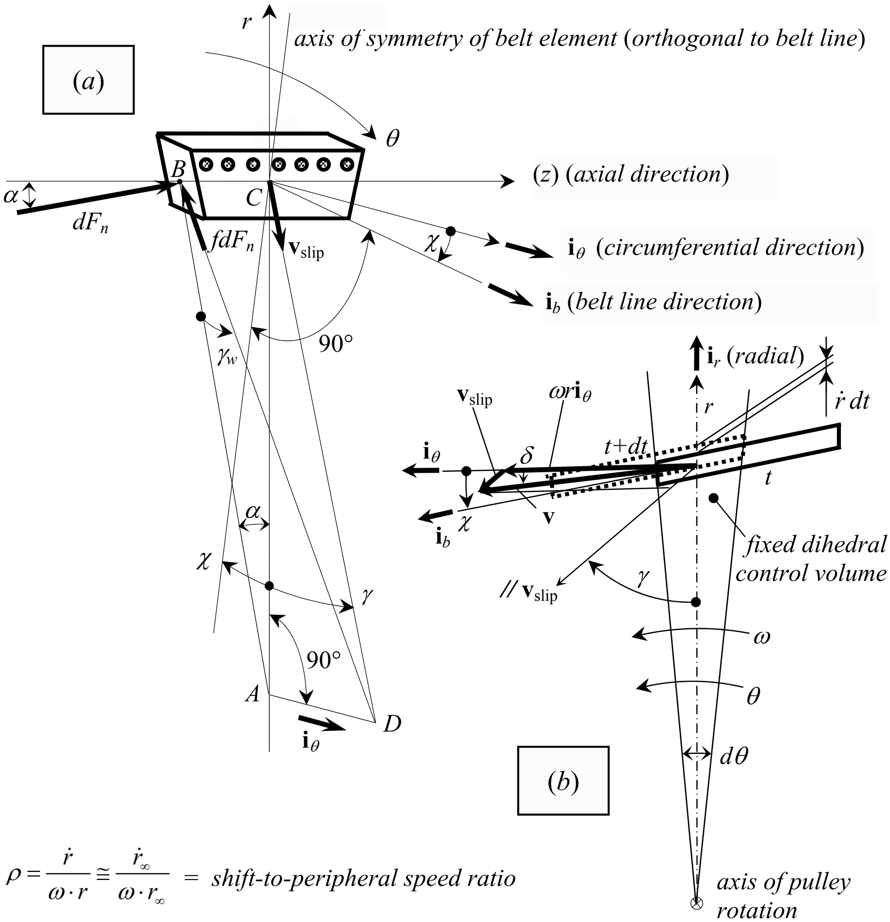


Fig. 1a. Belt-pulley interaction. Tetrahedron of rotational (ACD), meridian (ABC), sliding (BCD) and wall-tangent (ABD) planes.
Fig. 1b. Control volume. Triangle of velocities.

NOTATION

dF_n *and* fdF_n *= normal and frictional elementary wall forces*

α *= groove half-angle,* δ *= belt velocity angle,* χ *= belt penetration angle*

γ *= sliding angle in plane of rotation,* γ_w *= sliding angle on pulley wall* ($\tan\gamma_w = \cos\alpha \tan\gamma$)

r *= belt radius,* θ *= angular coordinate*

$\mathbf{v}$ *= belt velocity,* $\mathbf{v}_{\text{slip}}$ *= slip velocity in rotation plane,* ρ *= dimensionless shift speed,* ω *= pulley angular velocity*

Indicating the longitudinal elongation with $\varepsilon = T/S_l$, T and S_l being the belt force and the longitudinal stiffness, the usual order of magnitude of x, χ, ε and ρ is of a few thousandths. Then, combining the Eulerian and Lagrangian formulations of the mass conservation condition with reference to the dihedral control volume of Fig. 1b and neglecting small terms, it is possible to arrive, as in [1], at the relationship

$$u' = (1+u)\left[\frac{\varepsilon'}{1+\varepsilon} + \tan\chi(1-\chi')\right] - \rho \tag{2}$$

where u is the dimensionless slip velocity in the circumferential direction:

$$u = \frac{v_{\text{slip}}}{\omega \cdot r}\sin\gamma = \left(\frac{\tan\chi - \rho}{1-\tan\gamma\tan\chi}\right)\tan\gamma \;\rightarrow\; \tan\gamma = \frac{u}{(1+u)\tan\chi - \rho} \tag{3a,b}$$

Neglecting small terms, equilibrium yields $(T_d\,\mathbf{i}_b)' + \mathbf{F}'_w \cong 0$, where $T_d = T - \mu v_b^2$ is a "dynamic" belt force, μ the belt mass per unit length, μv_b^2 the momentum flux along the belt and $\mathbf{F}'_w$ the resultant wall force per unit angle. Likewise, it is possible to define the "dynamic" elongation $\varepsilon_d = T_d/S_l$. Splitting the vectorial equilibrium equation in the directions tangential and normal to the belt, defining the transverse elastic modulus E_z and the transverse stiffness parameter $S_t = 2\tan\alpha E_z h r_\infty^2/w$ (h and w: belt height and width), formulating a transverse constitutive equation, one gets as a whole

$$S_l\, d\varepsilon_d = 2\left[\sin\alpha\sin\chi + f(\cos\gamma_w\cos\alpha\sin\chi + \sin\gamma_w\cos\chi)\right] dF_n \tag{4}$$

$$S_l\varepsilon_d(1+\chi')d\theta = 2\left[\sin\alpha\cos\chi + f(\cos\gamma_w\cos\alpha\cos\chi - \sin\gamma_w\sin\chi)\right] dF_n \tag{5}$$

$$dF_z = (\cos\alpha - f\cos\gamma_w\sin\alpha)\, dF_n = S_t x(1-x)\, d\theta/\cos\chi \tag{6}$$

Eliminating dF_n from Eqs. (4-5) and introducing the belt elastic parameter $k = 2\tan\alpha\, S_t / S_l$, the momentum balance leads to

$$\varepsilon'_d = \frac{kx(1-x)(\tan\beta + \tan\chi)}{1 - \dfrac{\tan\beta}{\cos^2\alpha\tan\gamma}} \tag{7}$$

$$\chi' = \frac{kx(1-x)(1-\tan\beta\tan\chi)}{\varepsilon_d\left(1 - \dfrac{\tan\beta}{\cos^2\alpha\tan\gamma}\right)} - 1 \tag{8}$$

where $\tan\beta = f\sin\gamma/(f\cos\gamma + \sin\alpha\sqrt{1+\tan^2\alpha\cos^2\gamma})$.

We have thus collected four 1st order differential equations, (1), (2), (7), (8), and one parametric equation, (3 a or b), in the five variables x, ε_d, χ, u and γ, where the first four are very small, while γ may range between $-\pi$ and $+\pi$. This differential system is "degenerescent", as its order "degenerates" from four to three when neglecting all smaller terms, including χ' in Eq. (8). The problem is then of the boundary layer type, so that a rather smooth variation of the variables is expected along most of the contact, but with sharp gradients near the boundaries, and the equations must be applied in their unabridged form to match all boundary conditions. The numerical solution shows a short "seating" region at the contact entrance, where the belt slides inward ($\gamma \cong 0$), and a short "unseating" region at the exit, where it slides outward ($\gamma \cong \pm\pi$). The belt force is nearly constant in both of them by Eq. (7), but the elastic penetration is subject to rapid changes, together with the belt curvature. Putting $\gamma \cong 0$ at the seating region exit, one may obtain $x_{in} \cong \varepsilon_{d,in} / k_1$ by Eq. (8), where $k_1 = k \tan(\alpha + \arctan f)/\tan\alpha$ and this expression of x_{in} gives a good approximation for the penetration at the start of the inner main region of contact.

ADHESIVE SUB-REGION. As proven in [1], a wide adhesive region where $\tan\chi = \rho$ must develop inside the arc of contact of the closing pulleys ($\rho > 0$), both driver and driven, next to the seating region and bounded by the endpoints U and D (Upstream and Downstream). Here, all the previous relationships hold, except that f must be replaced by a variable adhesion factor $f_a \leq f_s$, where f_s is the coefficient of static friction, and γ by the angle γ_a of the resultant adhesion force in the plane of rotation. The adherence limit is reached when $f_a = f_s$. The adhesive conditions, $\tan\chi = \rho$, $u = 0$, imply the constancy of the belt force, as $\varepsilon'_d = 0$ by (2), while Eq. (1) gives $x = (r_\infty - r) / r_\infty = 1 - (1 - x_U) \exp[-\rho (\theta - \theta_U)]$, where $x_U \cong \varepsilon_{in}/k_1$,. Then, at a fixed time instant, the belt coils along a logarithmic spiral, which, since x and $|\rho|$ are << 1, may be roughly confused with a linear spiral of Archimedes

$$x \cong x_U + \rho\,(\theta - \theta_U) \quad \rightarrow \quad r \cong r_U - r_\infty\,\rho\,(\theta - \theta_U) \tag{9}$$

This spiral develops very slightly inward in the motion direction as $\rho > 0$ and $\rho << 1$, but the belt radius increases at each fixed angular position due to the pulley rotation.

The small variables x, ε_d, χ and u are obviously continuous when entering/leaving the adhesion region, while γ and f are always discontinuous with γ_a and f_a at D, but are continuous at U if $f_a(U) = f$.

ADHESIVE-LIKE SUB-REGION. No adhesive contact may develop in the opening pulleys, but the growth of sufficiently large regions of contact requires the presence of adhesive-like regions, where the adhesion condition $\tan\chi = \rho$ is just approached but not fulfilled. Here, the slip velocity $\mathbf{v}_{slip}$ and the sliding angle γ are rather small, whence we get $\varepsilon'_d \cong \rho - \tan\chi \cong$ constant and $\varepsilon_d \cong k_1 x$ by Eqs. (2) and (8). Moreover, as the trend of χ appears rather flat inside the adhesive-like region, the second derivative χ'' must tend to vanish as well. Then, differentiating Eq. (8) and retaining only the dominant terms, it is possible to get $\chi'' \cong [\tan\chi - (\rho - \tan\chi) / k_1] / x \cong 0$, whence $\tan\chi \cong \rho / (1 + k_1)$ and the approximate gradients $x' \cong \rho / (1 + k_1)$ and $\varepsilon'_d \cong k_1 \rho / (1 + k_1)$ are obtainable. These results permit constructing approximate solutions for the adhesive-like sub-regions:

$$x = \frac{\varepsilon_{d,in}}{k_1} + \frac{\rho}{1+k_1}\left(\theta - \theta_{in}\right) \quad \rightarrow \quad r = r_{in} - \frac{r_\infty \rho}{1+k_1}\left(\theta - \theta_{in}\right)$$

$$\varepsilon_d = \varepsilon_{d,in} + \frac{k_1 \rho}{1+k_1}\left(\theta - \theta_{in}\right) \tag{10a,b}$$

Summing up, spiral-shaped paths grow up in the closing and opening shift phases. In the former ones, the belt force keeps constant due to adhesion, while a slight tension variation occurs in the last ones due to a small creep motion. Moreover, it is noteworthy that the sum of the gradients of x and ε_d is roughly equal to ρ in both the shift operations. Figure 2 shows two belt paths schematically for a shift up and a shift down phase respectively. The belt velocity vectors are such that the belt radius increases or decreases in the closing or opening pulley respectively.

3. NUMERICAL RESULTS

The shift model can be dealt with as an initial value problem, starting the integration from one of the contact endpoints, e.g. the exit point E, where of course the belt penetration x_E must be zero, and moving backward until x vanishes again and a complete solution has been achieved. A great care must be put in the control of the integration step, reducing its width on approaching the adhesive or adhesive-like regions to avoid numerical instability. An iterative procedure must be followed, correcting successively the starting values by a sort of shooting technique, until all the external boundary conditions are fulfilled, i.e. for the contact width $\Theta = \theta_{exit} - \theta_{entrance}$, the applied torque $(\varepsilon_{d,exit} - \varepsilon_{d,entrance})\, S_l\, r_\infty$ and the axial thrust $F_z = \int_{\text{wrap arc}} dF_z$. Moreover, in the case of a closing pulley, the backward integration proceeds until the condition $\tan\chi = \rho$ is attained at the downstream adhesion boundary D, continues along the adhesive arc, according to the previous adhesive model until the condition $f_a = f$ is attained, assuming equal coefficients of static and sliding friction, and goes on upstream in the seating region. In general: 1) a decrease of the exit angle χ_E, which is always negative, produces an increase of the contact width; 2) a small increase of the sliding angle γ_E, which must be very close to $\pm\pi$, tends to change the pulley behaviour from driven to driver; 3) an increase of the belt elongation $\varepsilon_{d,E}$ produces an increase of the axial thrust.

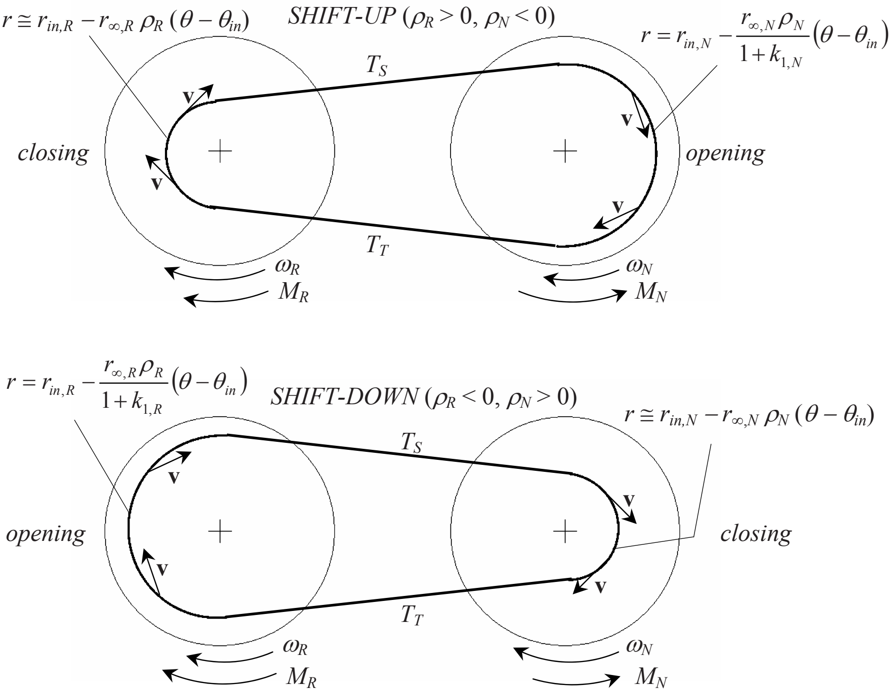
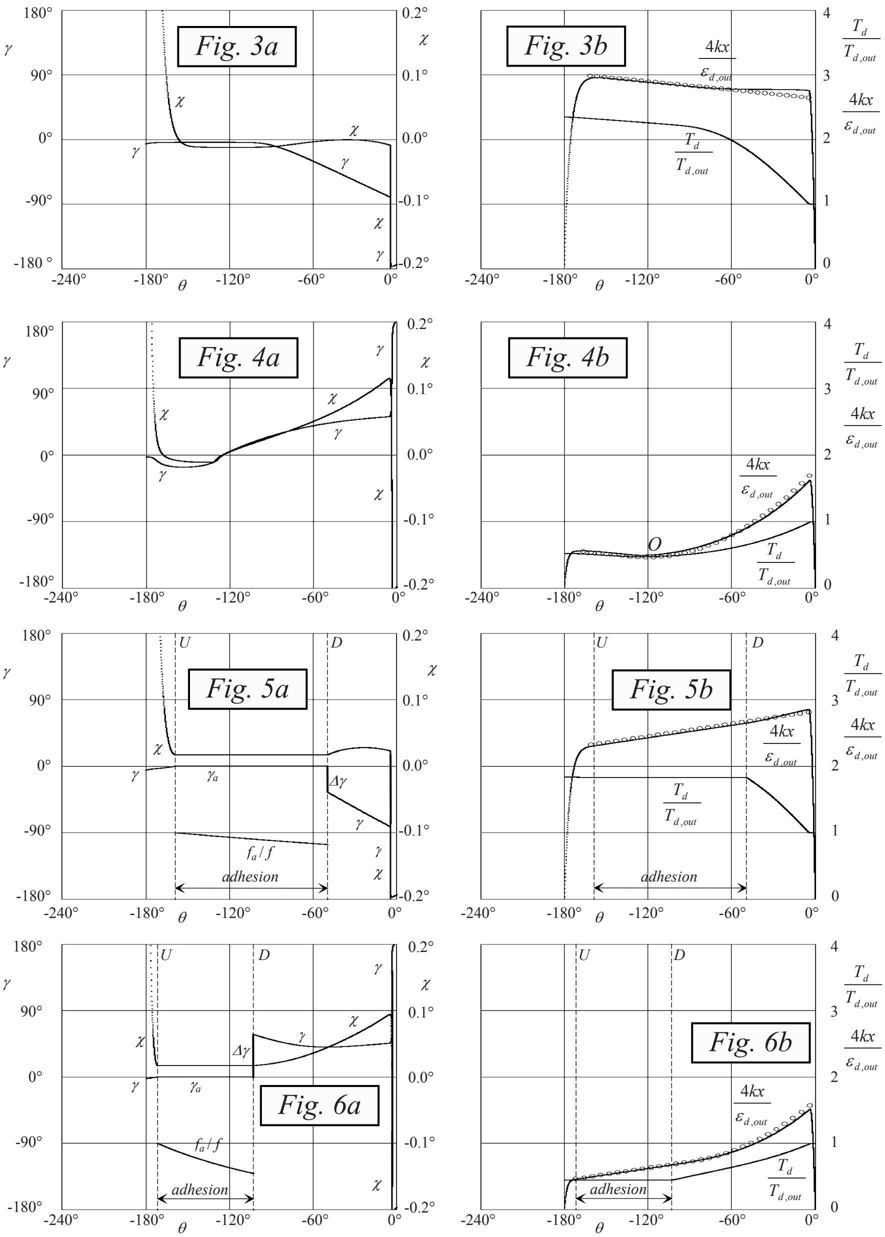


Fig. 2. Examples of shift-up and shift-down phases with magnified spiral shape of the belt path. $|\rho_R| \cong |\rho_N| \cong 0.05$, $k_{1,R} \cong k_{1,N} \cong 0.5$. Subscripts R and N: driveR and driveN pulleys.

Figures 3 to 6 show examples of numerical results for the four possible operative conditions of a pulley: driver/driven, opening/closing. The solutions were obtained fixing the shift-to-peripheral speed ratio ρ, the “centrifugal



Figs. 3-6. Solutions from unabridged equations (continuous line) and interpolating functions (dots).

$k = 0.15$, $\alpha = 13°$, $f = 0.4$, $\Theta = 180°$, $\rho = 0.0003$, $\mu v_b = 0.0001$.

3) opening driver p., $\gamma_E = 186°$; 4) opening driven p., $\gamma_E = 182°$;

5) closing driver p., $\gamma_E = 186°$; 6) closing driven p., $\gamma_E = 183°$.

elongation" $\mu v_b^2/S_l$ and the exit values of γ_E and $\varepsilon_{d,E} = (T_E - \mu v_b^2)/S_l$, where T_E is the belt tension in the free strand downstream. The third initial value χ_E (< 0) was corrected systematically to get a pre-fixed wrap width.

The ratio f_a/f is also reported for the adhesive case (closing p.) and a jump $\Delta\gamma$ is observable at the downstream boundary, due to the jump from f_a to f.

The belt angle χ is quite small everywhere, save in the seating and unseating regions, where it is affected by sharp negative gradients. Moreover, the sliding angle γ is close to 0 and to $\pm\pi$ in these short boundary regions, where thus the belt tension is nearly constant. Nevertheless, a sharp variation of γ occurs when passing from the main internal region of contact into the unseating region. Therefore, indicating with the subscripts $\dots_{in}$ and $\dots_{out}$ the ends of the wide inner region, the previous initial relationship $\varepsilon_{d,in} \cong k_1 x_{in}$ is valid, but a similar relationship cannot be written at the end.

Observing the diagrams of the closing pulleys, the belt tension T_d is constant in the adhesive sub-region, where the elastic penetration x varies in practice linearly with θ according to Eq. (9). Likewise, linear trends of T_d and x may be observed in the adhesive-like regions of the opening pulleys, according to Eqs. (10a,b). This suggests approximate solutions.

4. PRACTICAL FORMULARY

Neglecting the short seating and unseating regions, the linear trend of x in the adhesive or adhesive-like sub-regions may be conveniently used for the approximate integration of Eq. (6) for the axial thrust $F_z = \int_{\text{wrap arc}} dF_z \cong \int_{\text{wrap arc}} S_t x d\theta$.

Considering the driver pulley and observing several solutions, this linear trend may be approximately prolonged inside the downstream main sliding region as far as its endpoint, both for the closing and opening phases. Since only the area under this plot is of interest and not the exact shape of the locus, this approximation may give an extraordinary tool for the practical calculation of the driver pulley performance:

$$F_{z,R} = \frac{\left(T_T - \mu v_b^2\right)\Theta_R}{2\tan\left(\alpha + \arctan f\right)} + \frac{\rho_R S_{t,R} \Theta_R^2}{2 + k_{1,R}\left[1 - \text{sgn}\left(\rho_R\right)\right]} \tag{11}$$

where the subscripts $\dots_R$ and $\dots_T$ refer to the drive$R$ behaviour and to the tight strand. Notice that the belt parameters vary during the shift phase and, in particular, $S_{t,R}$ and k_{1R} vary with the square of the radius.

As regards the driven pulleys, it is better to divide the region of contact in two sub-regions: a first adhesive or adhesive-like sub-region, where Eqs. (9) or (10a) hold, and a second main sliding sub-region, where the solutions may be constructed according to the following reasoning, based on the observation that the penetration-to-elongation ratio x/ε_d and their differential ratio $dx/d\varepsilon_d$ tend roughly to the same "asymptotic" value on approaching the endpoint: $dx_{out}/d\varepsilon_{d,out} \to x_{out}/\varepsilon_{d,out} \to$ constant $= m$.

Neglecting χ, χ' and putting $1 - x \cong 1$, Equations (7) and (8) change into $\varepsilon'_d \cong \varepsilon_d \tan\beta \cong \cos^2\alpha \tan\gamma(\varepsilon_d - kx)$ and one may solve for γ_{out} and $\varepsilon'_{d,out}$

$$\tan\gamma_{out} \cong \frac{\sqrt{\left(1-f^2\tan^2\alpha\right)\left(k_1 m-1\right)\left(1-k_2 m\right)}}{\cos\alpha\left(1-km\right)} \tag{12}$$

$$\varepsilon'_{d,out} \cong \varepsilon_{d,out}\cos\alpha\sqrt{\left(1-f^2\tan^2\alpha\right)\left(k_1 m-1\right)\left(1-k_2 m\right)} \tag{13}$$

where the new parameter $k_2 = k\tan(\alpha - \arctan f)/\tan\alpha$ has been introduced, which is generally negative for rubber belts. Then, indicating the boundary point between the two sub-regions with O ($O \equiv D$ if the first sub-region is adhesive), u_O can be equated to zero because either $\gamma_O \cong 0$ (opening p.) or $\tan\chi_O = \rho$ (closing p.) and, integrating Eq. (2) from θ_O to θ and accounting for Eqs. (1) and (3), one obtains $x' \cong \rho + [\varepsilon_d - \varepsilon_{d,O} + x - x_O - \rho(\theta - \theta_O)]/\tan\gamma$. Hence, considering that $x_{in} = \varepsilon_{d,in}/k_1$ and $\varepsilon_{d,O} + x_O = \varepsilon_{d,in} + x_{in} + \rho(\theta_O - \theta_{in})$ for both the adhesive and adhesive-like cases and using Eq. (12), one has

$$x'_{out} \cong \rho + \varepsilon_{d,out}\cos\alpha\left(1-km\right)\frac{1+m-\dfrac{\varepsilon_{d,in}}{\varepsilon_{d,out}}\left(1+\dfrac{1}{k_1}\right)-\dfrac{\rho}{\varepsilon_{d,out}}\Theta}{\sqrt{\left(1-f^2\tan^2\alpha\right)\left(k_1 m-1\right)\left(1-k_2 m\right)}} \tag{14}$$

Minding that $x'_{out}/\varepsilon'_{d,out} = m$, the division of Eq. (14) by Eq. (13) yields

$$\begin{aligned} & m\left(1-f^2\tan^2\alpha\right)\left(k_1 m-1\right)\left(1-k_2 m\right)= \\ & =\frac{\rho}{\varepsilon_{d,out}\cos\alpha}\sqrt{\left(1-f^2\tan^2\alpha\right)\left(k_1 m-1\right)\left(1-k_2 m\right)}+ \\ & +\left(1-km\right)\left[1+m-\frac{\varepsilon_{d,in}}{\varepsilon_{d,out}}\left(1+\frac{1}{k_1}\right)-\frac{\rho}{\varepsilon_{d,out}}\Theta\right] \end{aligned} \tag{15}$$

which is a sixth degree algebraic equation for m, that can be easily solved by a few iterations, in dependence on the drive data, T_{in}, T_{out} and Θ.

Expressing the solution for x in the main sliding sub-region by a simple parabolic form $x = x_{out} + x'_{out}(\theta - \theta_{out}) + x''_{out}(\theta - \theta_{out})^2/2$, one may impose the exit conditions $x_{out} = m\varepsilon_{d,out}$, $x'_{out} = m\varepsilon'_{d,out}$ and the connection at point O with the upstream adhesive or adhesive-like solution with the same slope $x'_{in} = \rho/\{1 + 0.5k_1[1 - \mathrm{sgn}(\rho)]\}$. Such continuity conditions yield $\theta_O = \theta_{out} - 2(m\varepsilon_{d,out} - x'_{in}\Theta - x_{in})/(x'_{out} - x'_{in})$, $x''_{out} = (x'_{out} - x'_{in})/(\theta_{out} - \theta_O)$ and, using the subscript $\ldots_N$ for the drive$N$ pulleys, the axial thrust $F_{z,N} \cong \int_{\text{wrap arc}} S_t x d\theta$ turns out to be

$$F_{z,N} = \frac{\left(T_S - \mu v_b^2\right)\Theta_N}{2\tan\left(\alpha + \arctan f\right)} + \frac{x'_{in,N} S_{t,N} \Theta_N^2}{2} + \frac{2S_{t,N}}{3S_l} \frac{\left[m\left(T_T - \mu v_b^2\right) - \dfrac{T_S - \mu v_b^2}{k_{1,N}} - x'_{in,N}\Theta_N S_l\right]^2}{m\dfrac{\varepsilon'_{d,out}}{\varepsilon_{d,out}}\left(T_T - \mu v_b^2\right) - x'_{in,N} S_l} \tag{16}$$

where the ratio $\varepsilon'_{d,out}/\varepsilon_{d,out}$ is given by Eq. (13).

Figures 3 to 6 report the above analytical approximations by dots. Their agreement with the solutions of the full equations is quite acceptable, also in consideration that what is more significant is the whole area under the diagrams and not the local elastic penetration along the arc of contact.

The last equation for completing the formulary is the torque equation. Curtailing the torque values on the driver and driven sides of the torque losses in the bearings if the torque pickups are external to the housing, and averaging them in order take into account the inelastic bending stiffness of the belt, it is possible to write

$$T_T - T_S = \frac{1}{2}\left(\frac{M_R}{r_{\infty R}} + \frac{M_N}{r_{\infty N}}\right) \tag{17}$$

The given operative data of a V-belt variator are generally the transmitted torque, the speed and the axial thrust on one of the two pulleys, exerted for example by a spring load. According to which axial thrust is given, on the driver or driven side, one has to associate Eq. (17) with either Eq. (11) or Eq. (16) and calculate the unknown belt forces T_T and T_S on the tight and slack strands. In the case of known driver load, Equation (11) gives the tighter tension directly and then Equation (17) permits calculating the slacker tension. If on the contrary the known axial load is on the driven side, eliminating one of the two tensions, e.g. T_S, from Eqs. (16–17), one obtains a quadratic equation for the other tension, T_T:

$$a\left(T_T-\mu v_b^2\right)^2+b\left(T_T-\mu v_b^2\right)+c=0\ ,\quad \text{where putting } \Delta T=T_T-T_S\ ,$$

$$a=\frac{2S_{t,N}}{3S_l}\left(m-\frac{1}{k_{1,N}}\right)^2+\frac{m\Theta_N}{2\tan(\alpha+\arctan f)}\frac{\varepsilon'_{d,out,N}}{\varepsilon_{d,out,N}}$$

$$\begin{aligned}b=&\frac{4S_{t,N}}{3S_l}\left(m-\frac{1}{k_{1,N}}\right)\left(\frac{\Delta T}{k_{1,N}}-x'_{in,N}\Theta_N S_l\right)+\\&+m\frac{\varepsilon'_{d,out,N}}{\varepsilon_{d,out,N}}\left[\frac{S_{t,N}x'_{in,N}\Theta_N^2}{2}-\frac{\Theta_N\Delta T}{2\tan(\alpha+\arctan f)}-F_{z,N}\right]-\frac{\Theta_N x'_{in,N}S_l}{2\tan(\alpha+\arctan f)}\end{aligned}\qquad(18)$$

$$c=\frac{2S_{t,N}}{3S_l}\left(\frac{\Delta T}{k_{1,N}}-x'_{in,N}\Theta_N S_l\right)^2-\frac{S_{t,N}S_l x'^2_{in,N}\Theta_N^2}{2}+\frac{\Theta_N x'_{in,N}S_l\Delta T}{2\tan(\alpha+\arctan f)}+F_{z,N}x'_{in,N}S_l$$

and the driven load can be easily treated as well.

5. AXIAL THRUST CALCULATION USING THE PROPOSITIONS 24–27 OF ARCHIMEDES' TREATISE "*ΠΕΡΙ ΕΛΙΚΩΝ*"

It is quite interesting that the axial thrust on the pulley can be alternatively obtained avoiding the integral calculus and using the findings of Archimedes about the areas enclosed by the various branches of a spiral line (see [5,6]). These areas were calculated by the Syracusan scientist through very elaborate procedures based on his exhaustion method, as described in the palimpsest of "The Method" (see [7]).

According to the Latin version, the proposition n° 24 states: *Spatium comprehensum spirali prima circumactione descripta et linea prima earum, quae in principio circumactionis sunt, tertia pars est circuli primi.* Therefore, if $R=a\theta$ is the polar equation of the spiral, $R_j=2\pi ja$ the value of the radius after the j^{th} revolution and A_j the area enclosed by the spiral branch between the points with angular coordinates $\theta=2\pi(j-1)$ and $\theta=2\pi j$ and by the segment of the axis $\theta=0$ between the points with radial coordinates R_{j-1} and R_j, we have $A_1=\pi R_1^2/3=4\pi^3a^2/3$.

The following proposition n° 25 says: *Spatium comprehensum spirali secunda circumactione descripta et linea secunda earum, quae in principio circumactionis sunt, ad circulum secundum eam habet rationem, quam 7:12,*

... (*omissis*). In practice, the result is that $A_2 = 7\pi R_2^2/12 = 28\pi^3 a^2/3$ and $A_2 - A_1 = 6A_1$, as also ascertained in the discussion of proposition 27, where the difference $A_2 - A_1$ is the area enclosed by the first and the second branches of the spiral line and by the segment between the points R_1 and R_2 of the axis $\theta = 0$.

Furthermore, the proposition 27 states: *Spatiorum comprehensorum spiralibus et lineis, quae in circumactione sunt, tertium duplo maius est secundo, quartum vero triplo maius, quintum vero quadruplo maius, et semper deinceps insequens spatium toties multiplex erit, quam spatium secundum, quoties indicant numeri ordine sequentes, primum autem spatium sexta pars est secundi.* This means in practice that the areas enclosed by two subsequent spiral branches and the axis $\theta = 0$ are given by the recursion formula $A_j - A_{j-1} = (j - 1)(A_2 - A_1) = 8(j - 1)\pi^3 a^2$, which result is valid starting from $j = 2$.

Summing A_1 and all the area differences from $j = 2$ to $j = n$, it is thus possible to calculate the total area A_n: $A_n = 4\pi^3 a^2[1/3 + 2(1 + 2 + 3 + \dots + n - 1] = 4\pi^3 a^2(1/3 + n^2 - n)$. The difference of the n^{th} circle and A_n is a curved triangular stripe, whose area is $\Delta A_n = 4\pi^3 n^2 a^2 - 4\pi^3 a^2(1/3 + n^2 - n) = 4\pi^3 a^2(n - 1/3)$ and, for very small slope a and very large n, as in the V-belt winding, this area is approximately equal to $\Delta A_n \cong 4\pi^3 a^2 n$.

Since the radial width of this curved triangular area ΔA_n increases linearly with the distance from its vertex, the area of a segment of angular extension Θ is $\Delta A_{n\Theta} = \Delta A_n \Theta^2/4\pi^2 \cong \pi\Theta^2 a^2 n = R_n \Theta^2 a/2$. Considering that $R_n \cong r_\infty$ and that the spiral slope is $a = r_\infty \rho/[1 + 0.5 \times k_1(1 - \text{sgn}\rho)]$ for the belt-pulley coupling, nearly in the whole arc of contact for driver pulleys and in a large part of it for driven pulleys, we get $\Delta A_{n\Theta} \cong r_\infty^2 \Theta^2 \times \rho/[2 + k_1(1 - \text{sgn}\rho)]$.

Tracing a circle of radius $R_n \pm \Delta r$, where Δr is the radial penetration at the beginning of the main arc of contact downstream of the small seating region and the positive or negative signs are valid for the closing or opening phases respectively, the absolute value of the radial distance between this circumference and the spiral line gives the local radial penetration, variable along the wrap region.

As the axial push per unit length between the belt and the pulley is obtainable multiplying the radial penetration by the compression-to-penetration ratio $2\tan\alpha$ and by the axial elastic stiffness $E_z h/w$ of the belt, the total axial thrust is given by the product of the above curved trapezoidal area, between the spiral and the circumference $R_n \pm \Delta r$, and these two quantities:

$$F_z = 2\tan\alpha E_z \frac{h}{w}\left[r_\infty \Theta \times \Delta r + \frac{r_\infty^2 \Theta^2 \rho}{2 + k_1(1 - \operatorname{sgn}\rho)} \right] \tag{19}$$

Minding that $\Delta r = (T_{\text{entrance}} - \mu v^2)\, r_\infty \,/\, (S_l k_1)$, Equation (19) leads exactly to the same result of Eq. (11), valid for the driver pulleys. Clearly, some discrepancy appears in the case of a driven pulley, where a part of the winding arc is not spiral-shaped.

6. CONCLUSIONS

A very simple model may be derived from the results of a previous analysis of the author on the ratio shift of V-belt variators, whose findings were characterized by a very fine accordance with the experimental results.

The observation of several numerical solutions points out the Archimedean spiral shape of the instantaneous belt path along an extended part of the winding region, with adhesive or adhesive-like conditions for the closing or opening pulleys respectively. In particular, the elastic belt penetration increases in the motion direction for the former and decreases for the latter, independently of the working condition, of driver or driven pulley, that may at most affect the trends of the belt force, of the penetration and of the sliding direction in the following sliding portion of the wrap arc, downstream of the adhesive/adhesive-like sub-region. An easy-to-use formulary has been reported, which may be very useful for design purposes, permitting the evaluation of the axial forces exerted by the pulley walls or else the tension level produced by a given axial thrust on the loaded half-pulley.

It is shown how these calculation may be worked out without recourse to the modern integral calculus, by simply using some propositions of the classical Archimedean treatise On Spirals, as evidence of the up-to-dateness of Archimedes' thought.

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ANCIENT MOTORS FOR SIEGE TOWERS

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ABSTRACT In the paper are proposed some mechanical systems, all certainly used in the Classic Age, that could be easily adopted to power the siege towers, devices invented by Greek engineers and called Helepolis. These ancient motors are made up by capstans, tread wheels like those used for Greek-Roman cranes and counterweight motors, all installed into the helepolis.

The proposed motors are also analyzed from a mechanical point of view in order to examine, at least theoretically, their effectiveness in such applications.

1. INTRODUCTION

Among the siege devices and engines that were used in ancient times the siege towers or helepolis are particularly interesting. The term "helepolis" (ἑλέπολις ≈ "taker of cities") probably comes from the ancient greek words elein (ελειν from the verb αιρεω = to take, to conquer) and polis (πολις=city). These machines were widely described from ancient times by many authors, see. e.g. Diodorus Siculus (I century B.C.) [1], Publius Flavius Vegetius Renatus (IV-V century A.D.) [2], Julius Caesar [3] and others and were commonly used till the Middle Age. The first documented use dates back to the siege of Rhodes (305 B.C.) when the machines built by Demetrius I of Macedon (337–283 B.C.), called Poliorchetes were used: incidentally, the word "poliorchetes" (πολιορκητης) can be etymologically translated as "besieger or town conqueror".

For the reconstructions of the helepolis, we started from several classics; among them it seems interesting to report a piece from the "Epitoma Rei Militari" (Liber IV, par. XVII), written by Publius Flavius Vegetius

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Renatus among the end of the IV century and the first half of the V century A.D. in which these machines are well described. Generally the siege towers were mainly made by wood and higher than the walls of the besieged town; an average high of about 30 m can hence be considered, but much higher towers were also described. The base was rectangular or square with sides length equal to about 1/5–1/3 of the tower height; the structure was generally tapered at the upper part. Inside the tower some stairs permitted to reach the intermediate floors and the loft. The front side and perhaps also the lateral ones were covered by metallic plates (Diodorus Siculus) to protect the tower from the projectiles thrown by the defenders; the "armour" was completed by a curtain of not tanned and wet leathers held loosen that defended the tower from the incendiary projectiles. Under the machine some wheels were installed.

As for the helepolis moving, probably the ground was prepared by putting on it a track made by wooden boards.

Several authors also think that the helepolis were pushed or pulled by oxen or by a system of ropes and pulleys, the latter were installed on poles that were ram down at the base of the town's walls. We think that any system that pulled the tower (ropes, oxen etc.) was extremely vulnerable to the defenders' fire and hence very few effective. With regards to this aspect we can remember a piece by the Byzantine historian Procopius of Caesarea (about 500–565 A.D.) that tells about the unsuccessful siege of Rome from the Goths: Vitige, the king of the Goths (Wittigeis, ? – 540 A.D.) used wooden siege towers that were pulled by oxen; the defenders, however, easily killed the oxen making of no use the towers. Moreover is also difficult to think that so wide and heavy machines could be moved by pushing them from their back.

We think that external systems to move the towers could be used probably in the Middle Ages, but in the Classic Age more advanced systems were used. In fact, in the Classic Age, many knowledge about the Mechanics (and not only) were much more advanced than those of the Middle Ages. To this end, we can consider a piece from the De Bello Gallico (liber II, par. XXX and XXXI [3]), in which Caesar describes the siege at a town of the Gauls Atuatuci. From this piece, we understand that the Gauls were surprised when they saw very big machines that moved without any external source. Hence, it seems to us reasonable that the old helepolis were moved by "motors" fitted inside themselves.

In the following paragraphs some possible mechanical systems for the ancient helepolis propulsion are presented.

2. POSSIBLE INTERNAL "MOTORS" FOR THE HELEPOLIS

In this paragraph we propose some mechanical systems that were commonly used in the Classic Age or were perfectly compatible with the knowledge of the Mechanics of that age.

In any case we suppose that the torque (from the motor) was applied to the wheel by a rope that was rolled on a drum connected to the wheel axle and was pulled by one of the devices described later. This system was certainly used in many lifting devices in those ages and is schematically shown in figure 1.

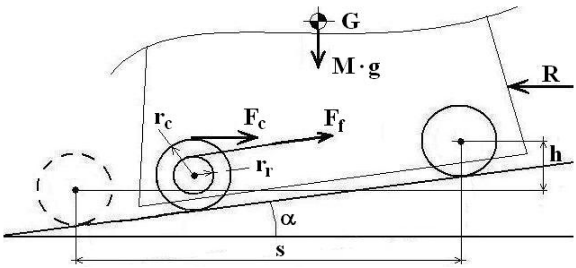


Fig. 1. Scheme of the device to apply the motor torque to the wheels axle.

2.1. The Force Required for the Traction

In order to evaluate the force required for the traction of an helepolis, we considered a machine of average dimensions having the following technical characteristics:

Helepolis height from the ground: 30 m;
Full helepolis' mass: 40000 kg,
Radius of the wheel rim: $r_c = 1.5$ m:
Radius of the drum connected to the wheel axle on which is rolled the rope: $r_r = 0.8$ m;
Slope: 2%;
Coefficient of friction between helepolis and ground: f = 0.02.

As for the data reported above, it must be pointed the followings:

- The slope value was fixed to represent an almost level ground with some local bottomlands;
- As for the coefficient of friction, it was considered wooden wheels on hard ground; it is evident that, if we had considered a track made by wooden boards, the friction would be rather lower.

With the above reported data it is easy to compute the force required to move the helepolis; it is given by the friction force and by the force required to climb the height difference.

$$R = M \times g \times (f + h/s) = 40000 \times 9.81 \times (0.02 + 0.02) \tilde{} 16000 \text{ N} \tag{1}$$

This force, naturally, is the force that must be exerted on the wheel rim to move the helepolis at constant speed; hence, on the drum it is necessary to exert a force:

$$F_c = R \times r_c / r_r = 16000 \times 1.5/0.8 = 30000 \text{ N} \tag{2}$$

A good rope made by hemp having 48 mm diameter made nowadays has a tensile strength higher than 150000 N (British Standard), that is to say 5 times higher. Obviously an high safety factor must be considered because it must be taken onto account both the rope wear and that 2000 years ago the ropes were not manufactured as well as now. The latter aspect plays a less important role than it could be thought: the British Standards of the middle of the XX century for naval ropes, cited before, give the same tensile strength for ropes made by stationary stranding-machine and for ropes made on the rope work train; the latter manufacturing technique is very similar to the one used from the age of Egyptians for medium and large ropes.

So, it seems reasonable to assume that, on the drum, a rope having 50 mm diameter was rolled. The force required to unroll the rope on a pulley can be computed by means of the following empirical equation [4]:

$$F_{av} = 0.02 \, F \, d^2 / D \tag{3}$$

If a rope diameter d = 50mm and a drum diameter D = 2 r_r = 1600 mm are considered, by using the units of eq. (3), we obtain:

$$F_{av} = 0.02 \times 30000 \times (502/1600) = 937{,}5 \text{ N} \tag{4}$$

That can be neglected since, for our purposes, the computing can be rather rough. Hence, it will be assumed that the force that must be exerted on the drum is the one given by eq. (2).

In the following paragraphs the possible mechanical systems to exert this traction on the rope rolled on the drum will be presented.

2.2. Capstan Motor

The capstan is such a simple and well-known machine that it is not necessary to report any historical reference for it. The working principle is shown in figure 2. In the figure are indicated:

F^1 the force exerted on each of the capstan bars;
F_c the traction on the rope;
F_2 the force exerted on the other rope's end, essentially in order to obtain the necessary friction between the rope and the capstan;
b_1 the distance from the capstan axis where the force F_1 is exerted;
b_2 the radius of the rolled rope on the capstan.

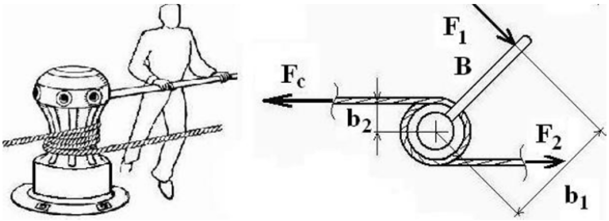


Fig. 2. Scheme of the capstan.

If we assume $b_1 = 1.5$ m, $b_2 = 0.3$ m and if we neglect the force F_2, the force that is necessary to apply to the bars in order to obtain the force F_c given from eq. (2) is:

$$F_1 = F_c \times r_2/r_1 = 30000 \times 0.3/1.5 = 6000 \text{ N}$$

If we assume that a man can exert on the bar a continuous force of 200 N average, we obtain that almost 30 men were necessary; this means that, for instance, we must suppose the presence of 2 capstan with 8 bars each and 2 men on each bar, that is to say 32 men. Since in the analysis we did not consider neither the force to unroll the rope nor the friction on the winch drum, the average force exerted by each one of the 32 men should be higher; this was possible but it seems not so easy.

In figure 3 is reported an our possible pictorial reconstruction of the propulsion system by capstans.

2.3. Tread Wheel

The tread wheel (or tread mill) is a device used since the Greek-Roman era to power lifting machines such as cranes etc. and is very similar (but obviously much bigger) to a squirrel cage. In figure 4 are reported a drawing from a bas-relief found at Capua (Italy) showing a crane of Hellenistic age, powered by a tread wheel, and the working principle of the latter.

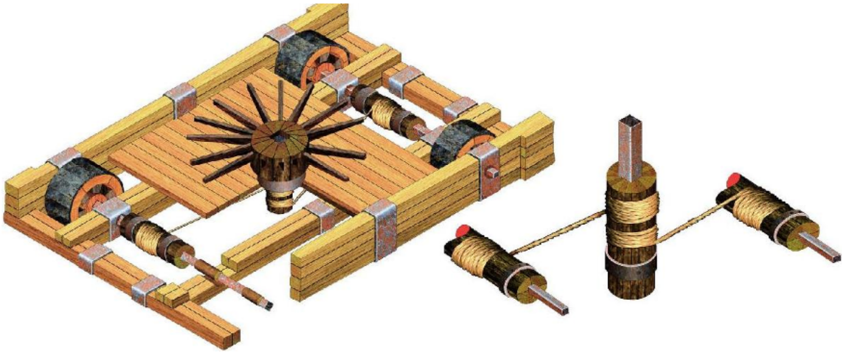


Fig. 3. Authors' pictorial reconstruction of the propulsion by capstans.

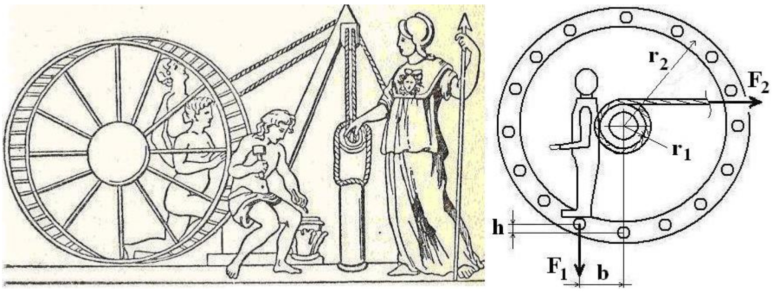


Fig. 4. Tread wheel.

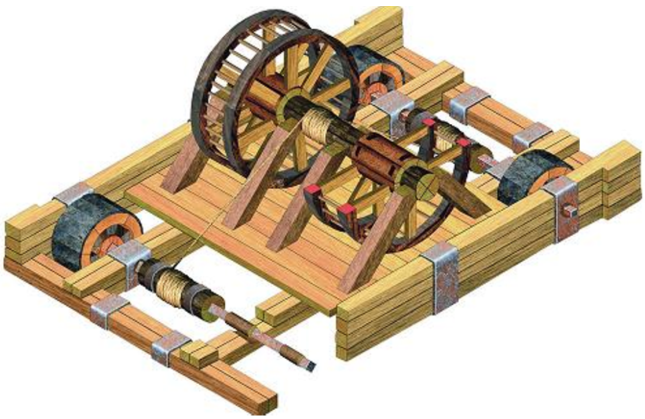


Fig. 5. Pictorial reconstruction of tread wheels for Helèpolis' propulsion.

In order to compute, roughly, the traction force on the rope that is possible to exert, let us assume the following data:

Mass of a man (in those ages): m = 65 kg, hence: $F_1 \approx 650$ N;
Mean radius of the rolling of the rope: $r_1 = 0.3$ m;
Mean radius of the tread wheel: $r_2 = 3$ m;
Mean level at which a man acts from the bottom: h = 0.5 m.
From figure 4 it is:

$$b = \sqrt{r_2^2 - (r_2 - h)^2} = 1.66\,\mathrm{m}$$

Hence:

$$F_2 = F_1\, b/r_1 \approx 3600\,\mathrm{N}$$

Obviously F_2 represents the force exerted on the rope by each of the men in the wheel. Hence, in order to obtain the traction computed by eq. (2), 30000/3600 ≈ 8 men were necessary. So, it is possible to suppose the presence of 2 tread wheels, each one with 4 men, disposed as in our pictorial reconstruction reported in figure 5. This reconstruction seems more realistic than the previous one.

2.4. Counterweight Motor

The counterweight motor, as will be illustrated, seems to be the more effective motor, from many points of view, for the helepolis' propulsion.

2.4.1. Historical references

The use of counterweight motors is documented in the Roman age for several applications like to move the curtains in the theatres [5]. It is also well-known that Heron of Alexandria, in the I century A.D., used counter-weight motors to move figurines representing animals in a sort of theatre in which the actors were automata moved by counterweight motors and a device that permitted, among other things, to program the law of motion of the automaton itself [5-8]. To this end it could be interesting to report the following piece from the Heron's treatise Perì Automatopoietiches (Περι αυτοματοποιητικης = about automatics) [9, 11] in which figurines mechanically moved in an automata's theatre are described:

ς´δύνανται δὲ καὶ ἕτεραι κινήσεις ὑπὸ τὸν πίνακα γίγνεσθαι, οἷον πῦρ ἀνάπτεσθαι ἢ ζώδια ἐπιφαίνεσθαι πρότερον μὴ φαινόμενα καὶ πάλιν ἀφανίζεσθαι. καὶ ἁπλῶς, ὡς ἄν τις ἕληται δυνατόν ἐστι κινεῖν μηδενὸς προσιόντος τοῖς ζωδίοις.

Also other movements under the platform (of the theatre) can be present, like to light a fire or figurines representing animals that before were not visible suddenly appear and then disappear again. And simply, like one could touch them, it is possible that they move without anyone approaches to the figurines representing animals.

In this one and in other pieces are described automata that move without any action from outside.

The treatise by Heron was translated during the Renaissance from Berardino Baldi, abbot of Guastalla, (Urbino, 1553–1617) [6]; in this work are described, among others, some examples of mobile automata, moved by a counterweight motor. In figure 6 are reported drawings from Baldi's work; on the left the working principle of the counterweight motor is evident since the counterweight, the rope linked to the latter and rolled on the wheels axle are clearly observable. In the figure it is possible to observe also the third wheel that is idle and the counterweight that is located in a tank filled with millet or mustard seeds in order to regulate the counterweight motion.

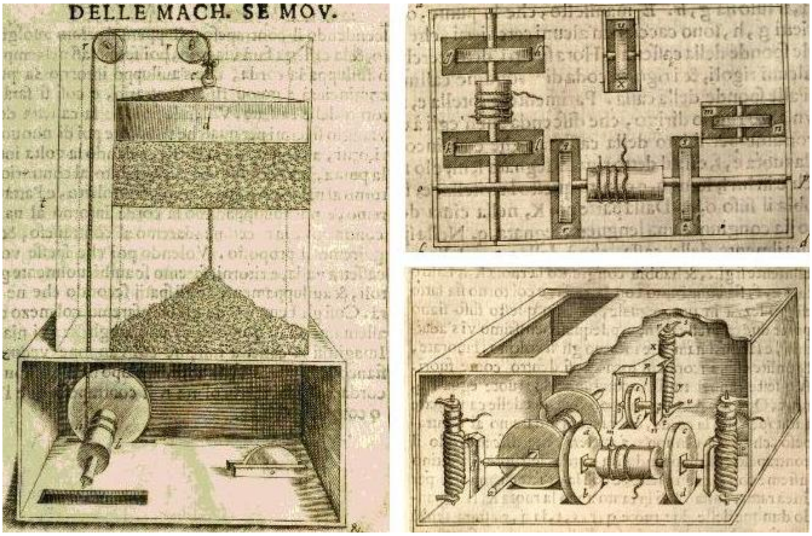
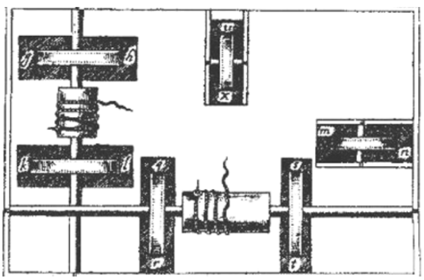


Fig. 6. Counterweight motor and mechanism to change direction [6].

Also very interesting are the systems, invented by Heron and described by Baldi, to change the cart's direction. In figure 6, on the right, are reported two Baldi's drawings in which is shown a first system used to change the cart's direction: in the drawing above it can be observed that two driving axles are used, each one is perpendicular to the other one; in the same way, also the axes of the idle wheels are perpendicular.

During the running, two driving wheels and an idle wheel lean on the ground while the other wheels (which axes are orthogonal to the first ones) are lifted up. By means of screw jacks, shown in the lower drawing in figure 8, that are also operated by ropes, it is possible to take down the wheels which axis is orthogonal to the ones' that lean on the ground. After this manoeuvre the chart will lean on these latter wheels and will move in a direction that is orthogonal to the previous one. To this end it is interesting to observe that the castle (or rook or tower) of the chessboard (that probably symbolize a siege tower) move on the chessboard just in the same way; it is well-known that the chess is a very ancient game that is described in Indian writings of the first centuries A.D.

Another system to change direction seems even more interesting because it uses the programmability of motion concept; this system also is attributed to Heron and is described by Baldi. In figure 7, on the left, a drawing from the work by Baldi is reported. The axle of the driving wheels is divided in two axle shafts that are independent one from the other; on each one of the latter a rope is rolled. If the rope is rolled on one of the axle shaft in a different way from the other axle shaft, when the counterweight goes down pulling the rope, one of the two driving wheels will rotate in different way from the other one. Moreover, it is also possible that, during the counterweight's run, one of the wheel stops while the other rotates; this is obtained by wrapping a piece of the rope in an hank like shown in figure 7, on the right.



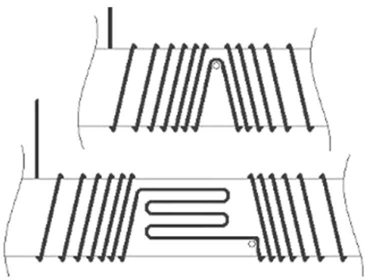


Fig. 7. Traction with independent axle shafts (left); scheme of rope rolling to program the motion (right).

During the time in which the hank unleashes that axle shaft is stopped. It is also possible to obtain that one of the axle shafts rotates in the opposite sense respect the other one; this is simply obtained by rolling the rope on one axle in the opposite sense respect the other one. Finally even a programming of the motion can be obtained by putting some knobs on the axle shaft like shown in figure 7; by means of these knobs it is possible to modify the rolling of the rope, as described before, in order to obtain different laws of motion for each wheel.

Some scholars (see e.g. [8,10]) have built, quite recently, models of charts moved by counterweight motors based on the works by Heron; they demonstrated, practically, the possibility of programming the motion.

2.4.2. The proposed reconstruction

In figure 8 is reported a scheme of our reconstruction.

In order to verify, conceptually, the possibility that a counterweigh motor could move an helepolis, we assumed the following data:

Counterweight mass = 1000 kg;
Radius of the helepolis' wheels: $r_c = 1.5$ m;
Radius of the drum that is the axle shaft: $r_r = 0.8$ m;
Block and tackle with 5 pulleys (Pentaspaston, described by Vitruvius in I century B.C.);
With the data above, it is easy to compute that if the counterweight goes down 20 m, the helepolis will go ahead: 20/5·1.5/0.8 = 7.5 m.

This amount seems reasonable with respect to the speed of a siege machine.

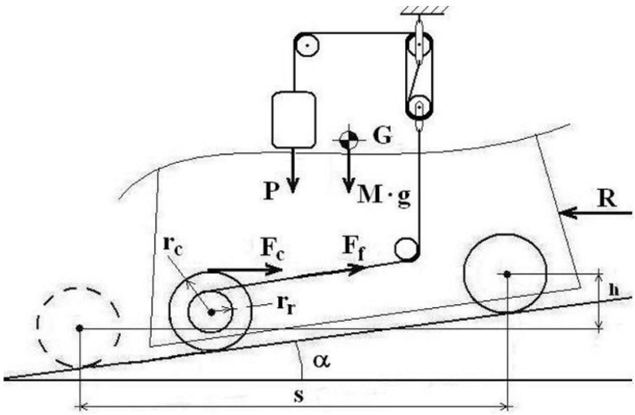


Fig. 8. Scheme of the helepolis' counterweight.

It can also be supposed that the force F_c that must be exerted on the wheels ring to move the helepolis at a constant speed is the one computed by eq. (1) and that the force F_f that must be exerted on the drum is that given by eq. (2).

Since the Block and tackle has 5 pulleys, and also two more pulleys are present as transfer case, if we suppose that manufacturing of pulleys and shaft was not very accurate, we can compute [12] an efficiency $\eta \approx 0,7$.

So, the counterweight that exerts a force of about 10000 N, through the block and tackle will pull the rope rolled on the drum with a force:

$$F = 1000 \cdot 9.807 \cdot 5 \cdot 0,7 = 34335 \text{ N} > F_f$$

Therefore, conceptually, such a motor could be able to move an helepolis which mass is 40000 kg.

It must be also observed that a counterweight, which mass is 1000 kg, can be easily made by a tank having a capacity of 1 m^3, filled with water; the tank could be unloaded when it reached the lower end of its run, then brought empty at the top and there filled by water with a chain of buckets. In addition, at those ages, suitable reciprocating water pumps were available (see e.g. [5]). The presence of water on the helepolis was documented in a piece (XX, 851) by Diodorus Siculus.

The study of the helepolis' movement with a counterweight motor can be carried on by means of a simulation software; in figure 9 is reported the simulation model and the results of a dynamical 2-dimensional simulation made by Working Model 2D ™.

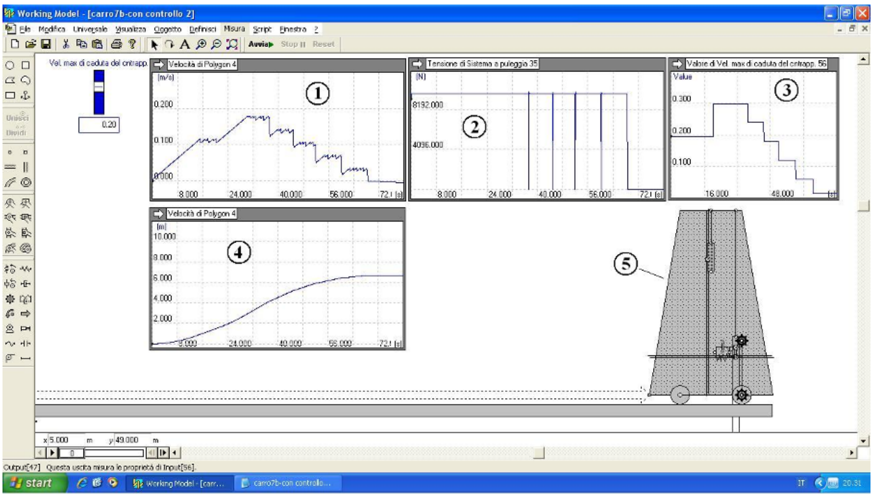


Fig. 9. Helepolis' with counterweight motor WM2D model.

In the figure are reported: the velocity of the helepolis 1, the stress on the pulley system 2, the velocity of the counterweigh 3, the displacement of the helepolis 4 and the model of the helepolis 5.

As for the modelling, the following can be observed:

1) The tower was modelled with a polygonal rigid body; the counterweight (1000 kg mass), running downwards, lifts a body (having negligible mass) that is pressed against a cylinder which diameter is 1.6 m. The spring that presses the body on the cylinder is pre-charged and, in parallel, a damper was added to avoid that the body could bounce on the cylinder. The latter is moved, hence, by friction. The cylinder moves the wheel of the chart through a transmission having a gear ratio 0.2, in order to simulate the 5 pulleys block and tackle (pentaspaston).

2) Naturally, the counterweight motion would be an uniformly accelerated motion. In order to adjust the counterweight motion a force proportional to the counterweight speed F=k·v was added. Since the weight is 9077 N, the constant k is simply: k=F/v=9807/v; so, if a speed of 0.2 m/s is required, it will be k=49035Ns/m;

3) The force against the motion (due to friction between wheels and ground and to a 2% ground slope) were considered by applying a resistance force R = 16000N (proportional to the tower's weight) at the tower's base. This force acts only if the towers goes forwards and is null if the tower stops.

The scheme of the applied forces is reported in figure 8. These represent, from top left clockwise, the helepolis' speed, the strain of the block and tackle rope, the counterweight speed and the helepolis' displacement. It must be observed that, in the presented simulation, we supposed that the counterweight motion was controlled. In the small counterweight motors (small self-propelled automata and similar devices) this control was obtained by putting the counterweight itself in a cylinder filled with millet or mustard seed and by regulating the seed's flow by a valve, as described by Heron and reported by Baldi. Such a device, although theoretically possible, was not suitable for an helepolis but we can imagine that a brake could be installed on one of the mechanism's ropes. In the reported simulation's results we supposed that the counterweight maximum speed was set at 0.2 m/s at the run's beginning, then was increased to 0.3 m/s and finally was decreased at 0.06 m/s till the stop. In figure 9 the instants of time when the manoeuvres to adjust the speed are made are clearly visible because in those instants the strain of the block and tackle rope becomes zero for a very short time.

3. CONCLUSIONS

Some possible reconstructions of motors that could have been used for the helepolis' motion were examined. Among these, the one that seems more suitable and effective is the counterweight motor.

We must admit that, while from the historical sources it clearly comes that the helepolis were self-propelled by internal motors (in which certainly mechanical devices were present), the “proofs” that counterweight motors were adopted for the helepolis are mostly circumstantial. It is sure, in fact, that such motors were adopted, rather widely, in ancient times to move self-propelled automata and charts; nevertheless as far as their use in the helepolis is concerned, we did not found, still, a deciding proof.

Nevertheless it seems quite certain that the siege towers were self propelled; as for this aspect is concerned, it is also interesting to report the following piece from Julius Caesar (the De Bello Gallico, liber II, par. XXX and XXXI [3]), in which describes the siege at a town of the Atuatuci Gauls:

XXX – …Ubi vineis actis aggere exstructo turrim procul constitui viderunt, primum inridere ex muro atque increpitare vocibus, quod tanta machinatio a tanto spatio instrueretur: quibusnam manibus aut quibus viribus praesertim homines tantulae staturae - nam plerumque omnibus Gallis prae magnitudine corporum suorum brevitas nostra contemptui est - tanti oneris turrim in muro posse conlocare confiderent?

XXXI – Ubi vero moveri et adpropinquare moenibus viderunt, nova atque inusitata specie commoti legatos ad Caesarem de pace miserunt, qui ad hunc modum locuti: non se existimare Romanos sine ope divina bellum gerere, qui tantae altitudinis machinationes tanta celeritate promovere et ex propinquitate pugnare possent, se suaque omnia eorum potestati permittere dixerunt.

XXX – … As soon as (the Gauls) saw that, having we pushed on the vinea (mobile roofs) and built an embankment, we started to built a tower, at first they derided and insulted us because a so big device was built so far (the walls): on what hands and on what force could ever the Romans rely, small as they were, in order to bring near the walls a so heavy tower? All the Gauls, in fact, scorn our height if compared with their large bodies.

XXXI – As they saw that the tower was moved and was approaching their walls, frightened by the unusual sight, (the Gauls) sent ambassadors to Caesar to negotiate the peace; they said that they think the Roman make war with the help of the goods since they can move such big machines so fast, (hence) the put themselves and all their wealth under the power of Caesar.

This study, anyway, demonstrates that the use of counterweight motors for the propulsion of the helepolis was certainly possible and probably the most effective.

Finally this also is an example that shows how, in order to correctly understand the past, it is necessary a wider cooperation between scholars having humanistic knowledge and scholars having technical knowledge.

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FROM ARCHIMEDEAN SPIRALS TO SCREW MECHANISMS – A SHORT HISTORICAL OVERVIEW

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ABSTRACT Mathematics forms the common roof for Archimedean spirals on the one side and screw mechanisms on the other side. Moreover, Archimedes was a genius of mechanics and mechanisms and was famous for solving mathematical and mechanical problems. There is also a historical justification for the title of the present paper, because the technical notion "Archimedean water-screw" is well-known to those mechanical engineers who are fond of looking back to the ideas and inventions of some famous protagonists and forerunners in the past and still today want to learn from their successes and failures.

1. INTRODUCTION

Did *Archimedes* (287–212 BC) actually invent the screw, one of the known five "mechanical abilities" or "simple machines" of Antiquity? Apart from the screw we still have the lever, the wedge, the roll or wheel, and the pulley as shown for example by *Guidobaldo del Monte* (1545–1607), Fig. 1 (left). On the right side of Fig. 1 we look at different types of screws (del Monte 1577).

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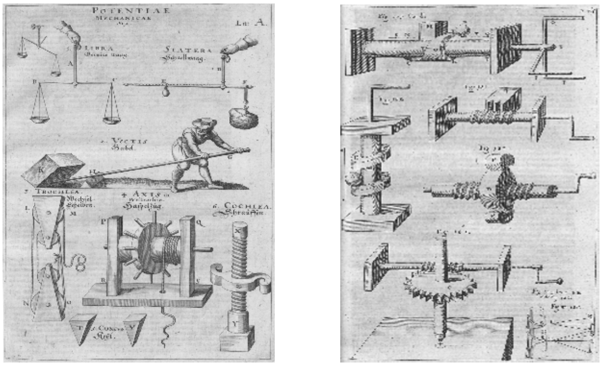

Fig. 1. The five "mechanical abilities" of Antiquity (left) and different types of screws by G. del Monte (right).

T. Beck (Beck 1899) points to the fact that *Aristotle* (384–322 BC) mentions the "mechanical abilities", but the screw is missing. On the other hand, *Heron of Alexandria* (ca. 10–85 AD) describes in chapter 34 of his book "Dioptra" an odometer, a chariot device used for indicating travel distances. It is based on a series of worm gears and may have been inventted by Archimedes during the 1st Punic War, but the reference to Archimedes is missing. So, there is a real chance for Archimedes in the time between to have invented the screw. *Salomon de Caus* (1576–1626) offered a compromise by drawing the two famous protagonists in a common picture, Fig. 2 (de Caus 1615).

Fig. 2. Archimedes (left below) and Heron of Alexandria (right below).

However, in accordance with *W. Treue* (Treue 1954) the historian *T. Koetsier* (Koetsier 1999; Koetsier, Blauwendraat 2004) declares that there is no reliable proof of the claim “Archimedes invented the screw”. But Archimedes undoubtedly knew the screw as a cylindrical helix (a wedge twisted around a cylinder) and also knew its mechanical properties. Furthermore, we surely know that the genius of mechanics and mechanisms (Chondros 2009), mathematics and geometry, namely Archimedes, intensively dealt with spirals and their mathematical properties (Czwalina-Allenstein 1922). He developed numerous theorems and equations for the type of spirals that were named after him, i.e. Archimedean spirals. The Archimedean spiral in the present form is a planar curve and follows the equation

$$x = r \cdot \cos\varphi,\ y = r \cdot \sin\varphi \tag{1}$$

in Cartesian x-y coordinates, where $\varphi = \omega \cdot t$ (time t), $p = v/\omega$, and $r = p \cdot \varphi$; its curve is generated by a point that moves with constant linear velocity v along a semi-ray starting at the origin O of the coordinate system; the semi-ray itself turns around O with constant angular velocity ω. How can we derive the spatial screw curve from the planar spiral curve? We succeed by adding the third dimension z, i.e.

$$x = r \cdot \cos\varphi,\ y = r \cdot \sin\varphi,\ z = p \cdot \varphi \tag{2}$$

taking any constant radius r and any constant pitch p. The sign of p marks either a left-hand screw ($p < 0$) or a right-hand screw ($p > 0$).

For technical purposes the screw is applied in form of a regular surface or with simple geometrical cross-sections, as rectangle, trapezium, circle etc., twisted around a cylinder, and thus giving the pitch p, the translation along the screw axis during one full screw rotation. Screws for technical purposes can be roughly classified into “motion screws” and “fastening screws”. In the present paper we shall concentrate on motion screws and neglect fastening screws. Dealing with motion screws reminds the kinematician that screw mechanisms exist and that screw mechanisms once belonged to the group of elementary mechanisms in machinery at the beginning of the period of mechanization in mechanical engineering more than 100 years ago, independent from the fact whether Archimedes actually invented the screw or not. But first we will have a look back to the Renaissance period and its artist engineers.

2. MOTION SCREWS IN THE MACHINE BOOKS OF THE RENAISSANCE PERIOD

The Roman architect and engineer *Marcus Vitruvius Pollio* wrote around 25 BC his ten books under the main title "De architectura" (Beck 1899). From his work we take that the Romans did not only learn architecture and arts from the Greeks, but also mechanics and mechanical engineering. In the 6th chapter of his 10th book about machines Vitruvius describes very precisely the construction and function of a water-helix (Fig. 3, left), however, without mentioning Archimedes. The reference to Archimedes occurs later in the machine books of the Renaissance and the baroque period (15th to 17th century) (Hilz 2008): *Agostino Ramelli* (1530–1590), Italian military engineer from Ponte Tresa, presents a triple Archimedean screw in his famous illustrated book (Ramelli 1588) (Fig. 3, right) which set standards for machine books in this time. The German artist engineer *Georg Andreas Böckler* (1648–1685) demonstrates in a similar way how to use a triple Archimedean screw for the lifting of water (Böckler 1661) (Fig. 3, middle).

Such a water-screw is driven by human muscle power (treading) or by water power, for example by means of water wheels. The early historian *Diodor* (1st century BC) from Sicily also mentions like Vitruvius that water-screws or water-helices were used in Egypt for water supply in cities and military camps near the river Nile. Very probably Archimedes came to know water-screws when travelling through Egypt.

A very interesting survey of the development of Archimedean water-screws is given by *J. Hennze* (Hennze 1992) in the catalogue of the Museum for Screws and Threads of the worldwide acting Würth Screw Wholesaling Company in Künzelsau (Germany).

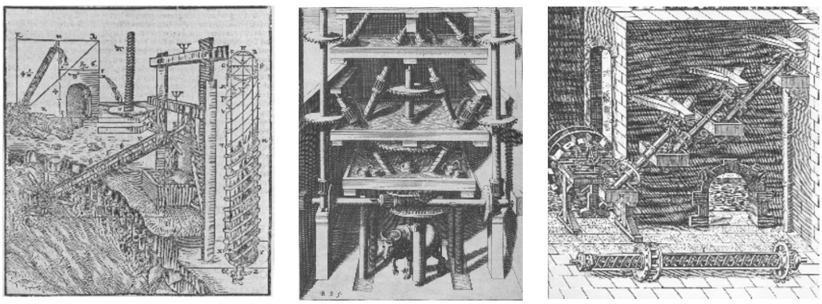

Fig. 3. Archimedean water-screws as recorded by Walter Ryff (German translator of Vitruvius´ books, 1548), Böckler and Ramelli.

Francesco di Giorgio Martini (1439–1501) of Siena was a very gifted designer of machines, architecture and fortifications. In the 7th book of his main treatise "Trattato di Architettura" we also find screws and worm gears (Moon 2007). The second famous artist engineer, contemporary to di Giorgio, *Leonardo da Vinci* (1452–1519) certainly owned a copy of di Giorgio's work. He used the endless screw (helix) as an input unit in numerous mechanical prototypes because of the profitable mechanical reinforcement, often combined with a worm or pin wheel (Fig. 4, left) (Leonardo 1493). But Leonardo also drew machines taking screws as output units, e.g. for setting upright heavy columns (Fig. 4, right) (Hoepli 1894–1904).

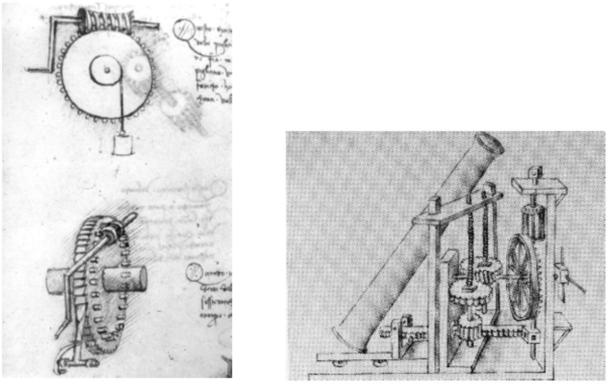

Fig. 4. Screw input units (left) and output units (right) by Leonardo da Vinci.

The idea of using a screw in combination with a lever or in combination with a lever and a gear wheel, single or multiple in series, is taken up by many artist engineers in the Renaissance. The lifting of loads was a very important task in a time of building monuments and fortresses and also of sea trading. *Jacques Besson* (1500–1569), professor of mathematics and natural philosophy at the University of Orléans (France) and one of the successors of Leonardo as engineer and consultant at the French court, presents for example a screw-driven crane (Fig. 5, left) in his famous book "Theatrum instrumentorum et machinarum" and also a lathe for the manufacture of screws (Fig. 5, right) (Hartenberg, Denavit 1956).

Besson also describes a press with three screws in parallel order for pressing grapes, clothes or lather (Fig. 6) (Treue 1954). Variants of presses with one or two motion screws were well-known since Antiquity and spread over in countries around the Mediterranean Sea.

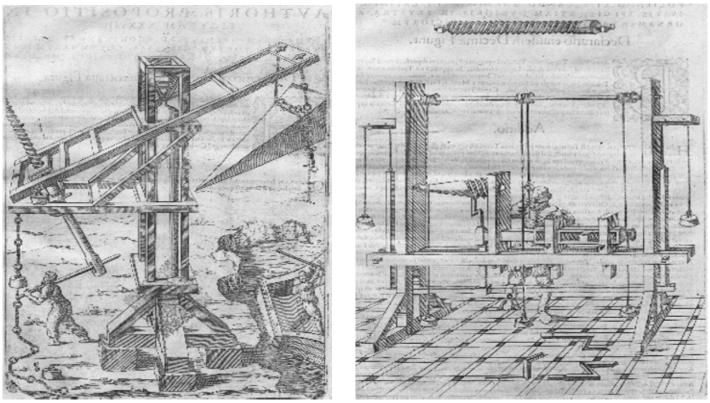

Fig. 5. Screw-driven crane and screw manufacturing lathe described by J. Besson.

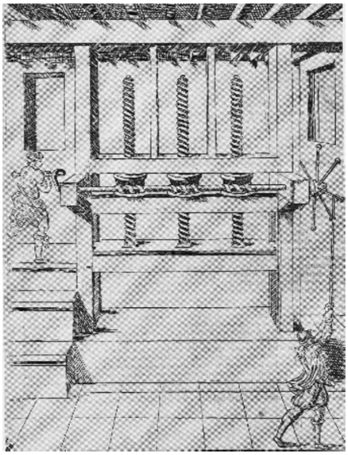

Fig. 6. Press with three parallel screws described by J. Besson.

The invention of the book press in the first half of the 15th century augmented the supply of presses based on screws. The book press mainly originates from *Johannes Gutenberg* (1397–1468) from Mainz (Germany). In Fig. 7 (left) we look at a book press made in Padova (Italy) with screw and long nut, drawn by *Vittorio Zonca* (1568–1602) in his machine book published only in 1621 (Zonca 1621). However, already Leonardo had designed a press for printing wood engravings (Fig. 7, right) (Hoepli 1894–1904). When the upper press board is lifted, the press table rolls down an inclined plane and comes out, so that the printed sheet can be removed easily and another one put on the table. When the screw is turned in opposite direction, the sheet goes back under the press table.

Heinrich Zeising (?–1613) wrote four volumes of a machine book titled "Theatrum machinarum" which were published between 1607 and 1613 (Mauersberger 1993). He took up again an idea of Ramelli concerning a big machine for the horizontal moving of heavy loads by means of multiple pulleys in parallel order (Fig. 8). One or two men are able to operate such a machine using a wheel-screw-combination.

Simpler and also more practicable seems to be a solution from Zeising for the lifting of trunks with a spindle which is operated by two men (Fig. 9).

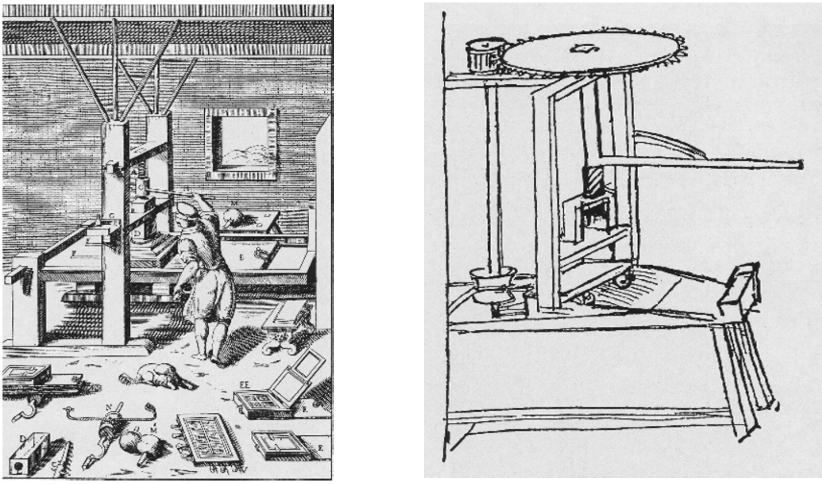

Fig. 7. Book presses by V. Zonca (left) and Leonardo (right).

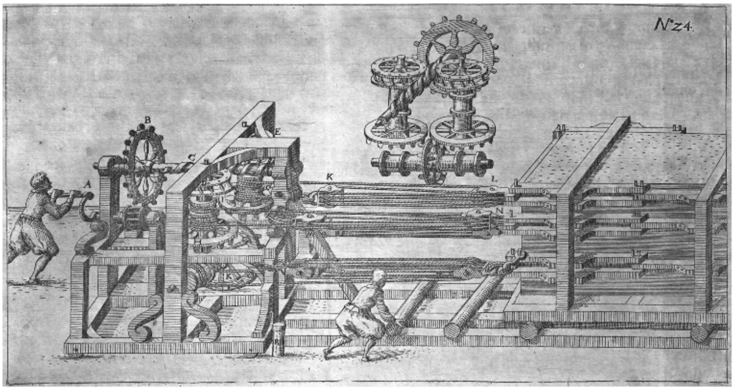

Fig. 8. Machine for horizontal moving of heavy loads by H. Zeising/A. Ramelli.

Fig. 9. Spindle hoist by H. Zeising.

In 1586 Pope Sixtus V called for an engineering congress and invited experts in mechanics to discuss about the following problem: Which mechanical solution could be set into practice for the dislocation of a fragile obelisque of some hundred tons of weight on St. Peter´s place in Rome? This meant an enormous challenge for contemporary engineers. The winner became *Domenico Fontana* (1543–1607), chief architect of the pope. He proposed to use ground winches and pulleys (Fig. 10, left) (A). Other variants are also shown in this figure: Floating bodies (B) as proposed by *Francesco Masini* (1530–1603) following the buoyancy principle of Archimedes, with screws (H), etc.

Years later, the German architect and artist engineer *Joseph Furttenbach* (1591–1667) from Leutkirch also made a proposal to erect an obelisque by means of numerous screws in serial order and of a lever. Fig. 10 (right) shows the mechanically extravagant solution taken from his book “Mannhafter Kunst-Spiegel” (Furttenbach 1663). Viewing and evaluating the variants of solutions in Fig. 10 the fantastic approaches concerning the pure as well as combined applications of the “mechanical abilities” of Antiquity are very surprising. But the mechanical engineer of today immediately discovers that there is a gap between the (virtual) models presented and a possible realization (Kerle et al., 2009). Proper materials were missing; time had not yet come in order to test original machines by means of physical models of minor scales. And there was hardly experience with mechanical energy losses caused by friction and wear, a

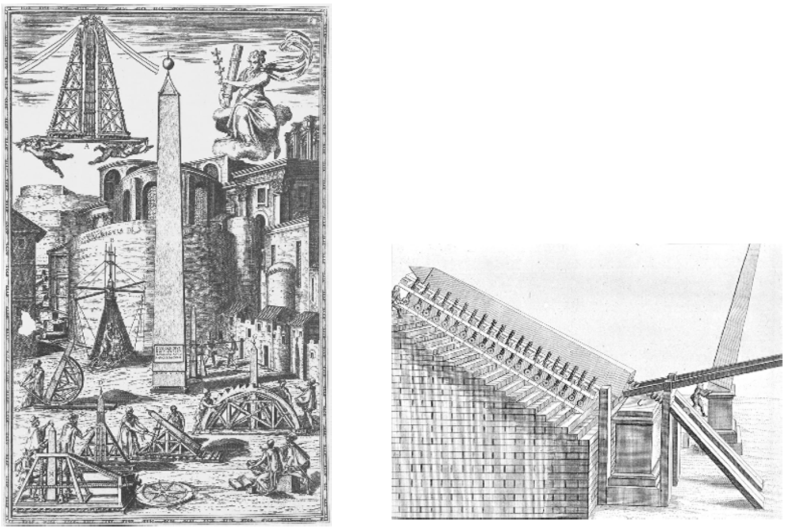

Fig. 10. Solution variants for erecting an obelisque.

severe handicap for many traditional gigantic drafts of machines and mechanisms.

Therefore machinery evolution was waiting for *Jacob Leupold* (1674–1727) from Leipzig, who wrote a ten-volume machine book titled “Theatrum machinarum” (Hartenberg, Denavit 1956). Not every machine in Leupold’s books was his own invention, but he added special views of machines whose functions and purposes he had studied and understood. He also drew details of machine parts and machine elements that could help to explain and to build the machine (Fig. 11). Leupold belonged to a new generation of mechanics who did not only want to describe a machine, but tried to dismantle it into its different parts of function and design. Leupold rejected Ramelli’s machine for moving heavy loads as shown in Fig. 8, mainly because of the considerable amount of friction between moving parts in counter directions. For the same reasons Leupold was also not fond of Furttenbach’s idea (cf. Fig. 10, right) to erect an obelisque by means of a multitude of screws.

Therefore, we can take Jacob Leupold as the last artist engineer of the Renaissance who developed a more modern view of mechanical engineering combining theory with practice and thus opened the door to the pre-industrial age.

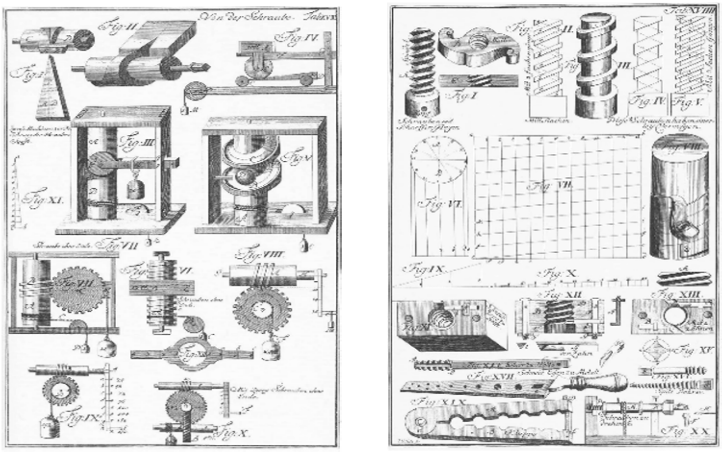

Fig. 11. Some machine element groups with screws by J. Leupold.

3. FROM MOTION SCREWS TO SCREW MECHANISMS

In the 2^{nd} volume of his famous book "Lehrbuch der Kinematik" the pioneer kinematician *F. Reuleaux* (1829–1905) enumerates six typical groups of mechanism drives for use in machines (Reuleaux 1900): Screw, crank, gear, roll, cam and ratchet drives. From the systematic point of view simple (planar) screw mechanism with one DOF consist of three links and three joints/pairs; the complete version of such a screw mechanism consequently has three screw pairs with coaxial screw axes. In Fig. 12 taken from Reuleaux the links are designated by the letters a, b, c and the pairs by the digits 1, 2, 3. Link a is the input link, link b the output link, and link c belongs to the frame or fixed link.

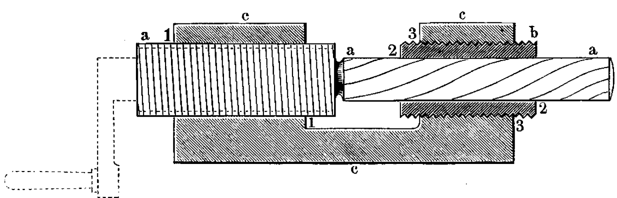

Fig. 12. Reuleaux's three-link coaxial screw mechanism with three screw pairs.

With a screw pair the rotation around the screw axis is coupled to the linear displacement by the pitch p, i.e. the dof of the pair yields one. Screw mechanisms can be treated systematically like planar wedge-slider or prism mechanisms (Beyer 1958).

Instead of actual screw pairs with a finite negative or positive pitch value ($p \neq 0$) it is possible to insert simple turning pairs ($p \equiv 0$) or sliding pairs ($1/p \equiv 0$). Thus, six different simple three-link screw mechanisms with one, two or three screw pairs are developed, Fig. 13 (Rabe 1958a, 1958b).

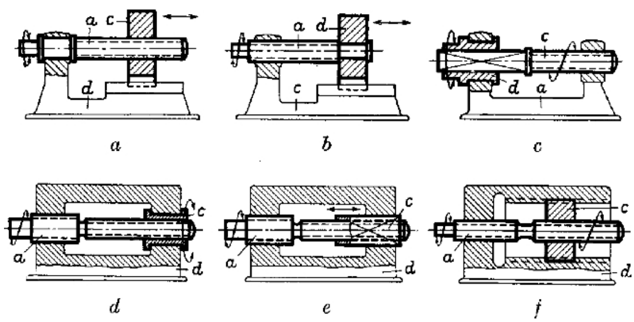

Fig. 13. Basic forms of three-link coaxial screw mechanisms: *a, b, c* one screw pair; *d, e* two screw pairs; *f* three screw pairs.

Most of the machines (and instruments) based on screws and screw motions can be derived from the mechanisms systematically represented in Fig. 13.

In the days of Reuleaux the fastening screw was a very well-known machine element. Screws of bigger size were made on forge machines; especially the heads of the screws being in a red-hot condition were jolted with the help of spindle-friction presses, Fig. 14 (left) (Georg, Ripke 1920). Another example with a crank press for metal parts is given by Reuleaux himself (Reuleaux 1900), Fig. 14 (right). The distance between the punch and its counterpart with the workpiece between can be continuously varied by a screw mechanism of the type *e* in Fig. 13 with two screws having the same pitch, but with different sign.

The following last examples are taken from *A. Widmaier's* catalogue (Widmaier 1954), a collection of mechanical solutions for the generation of motion on the base of mechanism theory. The catalogue was published at the beginning of a new industrialization period in Germany after World War II and was meant to be a practical source of knowledge in kinematics for the designer of machines and machine components, a knowledge that was created by generations of mechanical engineers before.

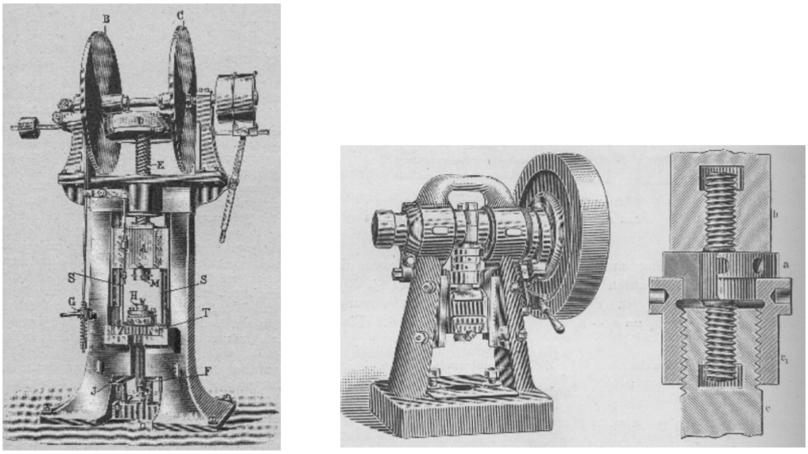

Fig. 14. Spindle-friction press (left) and crank press with a double screw-pair unit for adjustment (right).

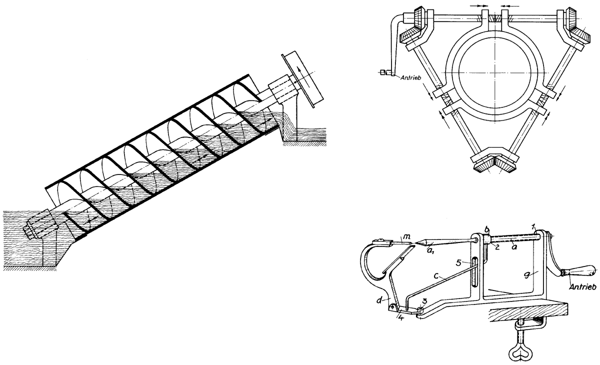

Fig. 15. Three modern mechanism examples based on screws by A. Widmaier.

On the left side of Fig. 15 we find again a screw for lifting fluids (or also powder or grain), it is a double start helix and in contrast to the Archimedean screw a screw surface is used for transportation of the fluid (cf. also Rauh 1939). On the right side of Fig. 15 (above) we look at a clamping device for tubes or cylinders with six screws having the same pitch, but with different signs two by two; below there is a simple machine for peeling potatoes or fruit on a table in the kitchen. The potato is pinned onto the needle a_1 which is turned manually by the crank (Antrieb). During crank rotation the nut b is moved along the screw axis a and makes rotate the knife m around the axis 3.

4. CONCLUSIONS

The screw for transforming rotary motion into linear motion in combination with an input torque around and an output force along the screw axis was one of the five "mechanical abilities" in Antiquity. Motion screws of this kind were used in presses and hoists and could be operated by means of human or animal muscle power. The machine books of the Renaissance partly present motion screws with gigantic dimensions. Most of those drafts of machines never could be set into practice because of energy losses due to friction and wear. Only Jacob Leupold at the end of the 17th century regarded and classified screws as parts of mechanisms (motion screws) and as machine elements (fastening screws) and proposed ways of proper design. His studies were introductory for the later development of compact screw mechanisms in modern machinery inspiring following mechanical engineers like Franz Reuleaux.

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THE MECHANICS OF ARCHIMEDES TOWARDS MODERN MECHANISM DESIGN

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ABSTRACT In this paper a relevant contribution of Archimedes is outlined as related to his developments in mechanics with application to mechanism design with a modern vision. He developed theoretical advances that were motivated and applied to practical problems with an enthusiastic behaviour with a modern spirit that can be summarized in his motto 'Give me a place to stand and I will move the earth'.

1. INTRODUCTION

Since Renaissance Archimedes and his mechanics have been reconsidered together with a new attention to Greek-Roman machine designs with the aim and result to develop an early approach for modern theory of mechanisms, as outlined in [1].

The works of Archimedes, mainly in the aspects of mechanism design, has been rediscovered and studied during Renaissance, as for example in [2-5], up to be used as fundamental background for the first developments of early TMM (Theory of Mechanisms and Machines) by Guidobaldo Del Monte, [6], and Galileo Galilei, [7]. Even at the beginning of the modern TMM in 19th century Archimedes' contribution was recognized in developing basic conceptual elements, like for example in [9, 10]. The modernity of Archimedes in MMS can be today still advised in his approach of classification for the variety of mechanism designs as function of a unique principle in the operation mechanics, as indicated in [11].

The mayor Archimedes' contributions in the field of modern MMS can be recognized in:

– identification and analysis of basic elements of machines and mechanisms, as pointed out in [8]
– analysis of machinery operation as function of a unique functionality of levers

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- application of theory to successful practical designs that since his time gave dignity of discipline and profession to machine design
- enthusiasm and optimism in mechanism design in developing technology for enhancing society and quality of life.

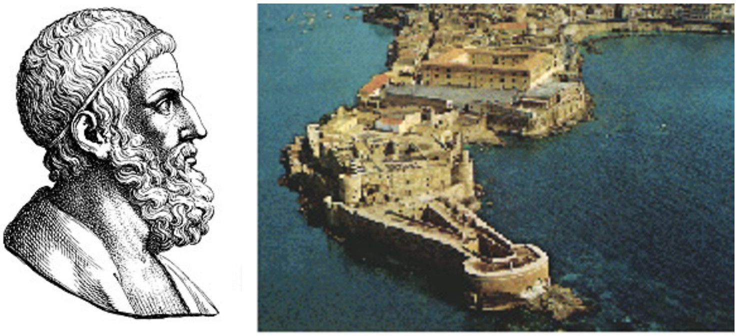
From historical viewpoint, it is never too much stressed the achievement in defining and using an early concept of pair of force and its equilibrium rule for the design and operation of mechanisms in machines.

In addition, the figure of Archimedes and his work have been investigated since Antiquity and still today they are of great interest in studies and investigations that are reported in publications and encyclopaedias, like for example in [12-20], and in congress discussions, like for example in [21, 22].

In this paper, attention is addressed in discussing the aspects and interpretation of legacy of Archimedes' work and personality in the modern MMS (Mechanism and Machine Science) not only for a historical assessment by also as an inspiration for future achievements.

2. ARCHIMEDES AND HIS WORKS

Archimedes (in classical Greek: Ἀρχιμήδης) (287 a. C.–212 a. C.), Fig. 1 a), was a Greek philosopher in the classical term as being mathematician, physicist, astronomer, inventor, and engineer, who lived in the core environment of Magna Grecia as Siracusa was, Fig. 1b).



a) b)

Fig. 1. a) Portrait of Archimedes; b) the peninsula of Ortygia as core of the ancient Siracusa with the Maniace Castle at its end.

Nevertheless little is known of his life. Very probably during his youth he spent a period of study in Alexandria, Egypt, where he had the chance to know Conon of Samos and Eratosthenes of Cyrene, since in his written works he cited them as friends. Archimedes was killed by a roman soldier, when a long siege of Siracusa during the Second Punic war (214–212 a. C.) was ended, although Marco Claudio Marcelo, the Roman commander, ordered to safe him. Cicero told that he saw the Archimedes' tomb where a sphere was drawn inside a cylinder, as an indication of the main achievement that Archimedes recognized to himself.

In general, Archimedes is reputed for his contributions in Mechanics and Hydrostatics. He is also a reference personality in the developments of Mathematics because of his calculations and theorems for figure volumes and the number pi. Since Antiquity, he is also considered a unique inventor of innovative machines, as applying his mathematical results to practical problems. The most celebrated ones are the screw pump and the war machines that Syracusans used against the Romans.

However, although the inventions of Archimedes were known over the time, his written works were forgotten even in the last part of Antiquity and completely ignored during the Middle Ages. A first attempt to collect all his works was made by Isidor of Milete (c. 530 d. C.). During Renaissance those works were reconsidered, like in the Latin translation by Jacobus Cremonensis in 1458 from a collection made by Eutocius in the 6th century. Only in 1906 a palimpsest was discovered with seven works by Archimedes in a better version than previously known.

The works of Archimedes, that are today known even through a interpreted text, like for example in [20], are:

– On the Equilibrium of Planes
– Measurement of a Circle
– On the Sphere and Cylinder
– On Spirals
– On Conoids and Spheroids
– Quadrature of the Parabola
– On Floating Bodies
– Stomachion
– Cattle Problem
– The Sand Reckoner
– The Method of Mechanical Problems

In particular in the two-volume *On the Equilibrium of Planes* Archimedes introduced the Law of Levers by stating, "Magnitudes are in equilibrium at distances reciprocally proportional to their weights." Archimedes used it to calculate the areas and centers of gravity of several geometric figures.

His short work *Measurement of a Circle* consists of three propositions by which he computed number pi by approximations.

The treatise *On Spirals* deals with study of curves and particularly spirals.

In *On the Sphere and Cylinder* he computed volumes and areas of spheres and cylinders. A sculptured sphere and cylinder were placed on the tomb of Archimedes at his request, to remind of what he considered as his best achievement. In the fact that a sphere has a volume and surface area two-thirds that of a cylinder.

In the treatise *On Conoids and Spheroids* Archimedes calculated the areas and volumes of cones, spheres, and paraboloids.

In Quadrature of the Parabola he calculated the area of geometrical figures by using the concept of a geometrical series.

In his work *On Floating Bodies* Archimedes introduced the law of the equilibrium of the fluids and used it to demonstrate the shape of water volumes. Then he calculated the equilibrium positions of sections of paraboloids floating in the water. Archimedes' principle of buoyancy is expressed as: Any body wholly or partially immersed in a fluid experiences an upthrust equal to, but opposite in sense to, the weight of the fluid displaced.

Stomachion is a study for dissection of a puzzle by computing the area of pieces that can be assembled to obtain a square.

In the *Cattle Problem*, Archimedes attached the problem to count the numbers of cattle in the Herd of the Sun by solving a number of simultaneous Diophantine equations.

In *The Sand Reckoner*, Archimedes calculated the number of grains of sand that can be inside the universe, along with considerations on Astronomy.

The Method of Mechanical Problems introduced concepts of infinitesimal calculus by using geometric description of how to calculate areas and volumes by summing the small parts of a partitioning of the figures.

Although is can be thought that Archimedes did not indeed invent the lever and its use, as many researchers point out, nevertheless he was the first in giving a rigorous explanation and formulation of the mechanics equilibrium law under which it operates. According to Pappus of Alexandria, while discussing the lever mechanics Archimedes commented the famous sentence 'Give me a place to stand and I will move the earth' (in Greek: δῶς μοι πᾶ στῶ καὶ τὰν γᾶν κινάσω). This sentence can be considered also a summarized thought on the optimism that Archimedes relied in the mechanism design.

Plutarch described how Archimedes designed a capstan system as based on the lever operation to help the harbour workers to lift heavy loads in the ships. Furthermore, an anecdote reports how Archimedes designed and built a capstan system to move a large ship from the harbour in order to gain a cultural challenge that the tyrant Hieron proposed to him. Thus, by means of a system with several capstans and pulley systems, while seated on a chair, Archimedes could lift the ship with great surprise of the people present.

Additionally, Archimedes is credited to have increased power and precision of catapult machines used during the Punic war against the Romans. He also invented an odometer in a chariot by counting the miles with the number of small balls collected after a certain distance.

Pappus of Alexandria mentioned that Archimedes wrote a treatise on how to build mechanisms for planetary models and for the construction of spheres, an important account of the technological skills of the time. Several such devices have been recently discovered, like for example the Antikethyra mechanism that is equipped with several differential gears, as a proof of technical achievements thought of as belonging to modern times only.

The most relevant invention of Archimedes can be considered the screw pump, still known as *Archimedes screw*. Although there is some evidence that the pump already existed in some forms in Ancient Egypt, it is important that Archimedes based its design on theoretical principles, permitting a rational use of it.

3. MECHANICS OF MACHINERY AND MECHANISM DESIGN

The contribution of Archimedes in Mechanics of Machinery towards a development of an early Theory of Mechanisms can be recognized in the following aspects:

- a theoretical study with a mathematical approach that was useful both for analysis and design of mechanisms and machines;
- interest for an experimental activity in the theory and application of the mechanics of machinery;
- application to engineering with invention of new machines.

In addition, in all his works Archimedes expressed a strong believe and optimism in a practical application of science as demonstrated by the many inventions that are attributed to him. This is summarized in his motto *'Give me a place to stand on, and I will move the Earth'*, as it was ascribed to him since Antiquity and it was considered the basis for the study of the mechanics of the machinery in Renaissance, with great emphasis as for example in the cover of the book by Guidobaldo Del Monte in 1577.

The theoretical mathematical approach by Archimedes is developed in depth but not only for pure mathematical interest. The study and formulation, that can be even deduced in modern terms, were deduced for their application both in explaining the attached problems and in providing suitable solutions for them. For this reason, the Archimedes's treatises can be considered simultaneously the basis and complete of his activity as machine inventor and builder. This permits to link the theoretical activity and the mathematician approach to his practice of experiments in mechanics as well as to his machine ingenuity. As it can be understood from the treatises, Archimedes considered as the basis of his study the observation of natural phenomena and the operation of existing systems. Once both the generalities and peculiarities of real events were understood, Archimedes studied and proposed a logic reasoning and theoretical development that he could use not only to forecast operations and events but even to deduce new ideas and possibilities in structures and functionalities of machines. From this perspective it is quite amazing how Archimedes was able to identify and formulate a unique principle in order to explain the operation of a large variety of machines that were already existing at his time. Thus, it is not only the identification of elementary machines (or mechanisms) by which it is possible to classify all the machine components, but even the identification of the lever mechanics (better known as the law of levers) as operation principle for any mechanism and machine. Through this study, Archimedes expressed first the equilibrium of rigid bodies as related to the momentum of forces.

Relevant is also how he examined the mechanical phenomenon and its application in other existing devices by extrapolating the specific mechanics from the general principle and formulation. In Egypt, there were scales for measuring and comparing weights as it is also documented in several documents and even artistic representations like the one in Fig. 2, from which it is evident the lever mechanics was used in somehow conscious way. Most probably, Archimedes could know and appreciate the Egyptian technique in scales of several types. But he was the one who examined and formulated the mechanics of levers with deep insights and generality in his work 'On the Equilibrium of Planes' by assuming seven postulates that he identified for a careful analysis of experimental phenomena even in the diary experience and with other systems. Once assumed the natural evidence of the seven postulates, also in terms of determination of the center of gravity of rigid bodies, Archimedes formulated the static equilibrium as due to conditions related to the weight and distance of the body from a point, about which the body can rotate. Thus, he indicated the equilibrium as function of torques of forces that are due to the weights from both sides of a lever. A graphical representation of the analysis process from observation is reported in the scheme in Fig. 3a) that can lead to the modern mechanical-mathematical model in Fig. 3b). Fig. 3a) is taken from the book by Del Monte and Fig. 3b) is a modern interpretation of the law of levers by referring to the parameters used by Archimedes. The law of levers was used by Archimedes, but even more important is that it was extended in the books that rediscovered the Archimedes's works during Renaissance, as mainly in the works by Del Monte and Galilei with the aim to determine a rational classification of the operation of machines.

Fig. 2. An artistic representation of an Egyptian scales in a tomb of 1250 B.C. in Thebes.

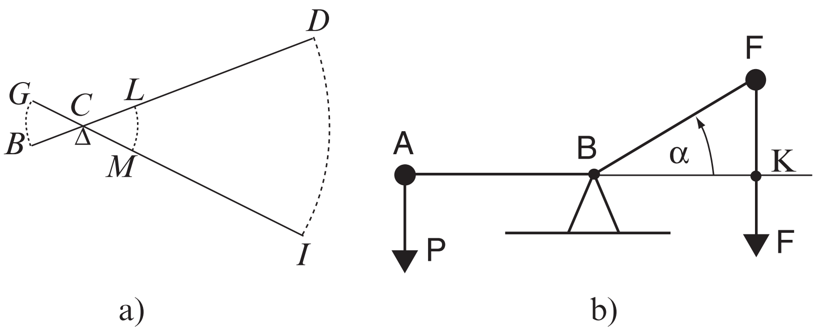


Fig. 3. The law of levers by Archimedes as viewed by Del Monte and Galilei: a) a scheme; b) a modern interpretation.

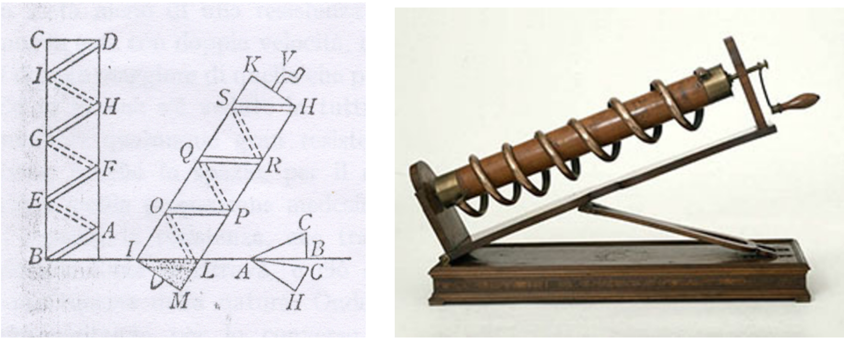
The law of levers was a great achievement as introduced and used by Archimedes both from theoretical and practical viewpoints in rational mechanics and machinery mechanics. Relevant is the introduction of the concept and use of the pair of a force, even if not yet in a clear form, to explain the static equilibrium but even dynamic rotational operation towards it. Archimedes used the law of levers in a descriptive form but with mathematical means both to explain the machine operation as well as to conceive new inventions.

This was the case of the screw of Archimedes, Fig. 4, as Del Monte and Galilei described clearly as based on the law of levers in agreement with the scheme in Fig. 4a). The invention of the screw of Archimedes, as used extensively (even today) and widespread as fluid pump, shows also the experimental approach that Archimedes uses in developing new machines for practical applications still by using the law of levers. In general, it is used to elevate water, floor, or cereals. It is based on a screw that rotates within a hollow cylinder that is installed on an inclined plane so that the water in the bottom is elevated to the top of the screw.

The scheme in Fig. 4 b) is an explanation of the pumping of the screw as an alternating inclined plane operating as mobile lever. This explanation with experimental and practical insights has attracted the interest of scientists, even for teaching purposes. Fig. 4b) shows an experimental setup of the 18th century for laboratory practices to teach the screw operation.

Indeed the study of the screw as in Fig. 4a) shows a clear example how Archimedes (and later Del Monte and Galilei) used the law of levers as a general principle to understand and explain the operation of a large variety of machines, also with the aim to provide a simple means of study and comparison of the operation and design of machines. This practical vision permitted Archimedes, who was motivated by the needs of his time and local circumstances (as even requested by the tyrant of Siracusa), to solve practical problems and to conceive new machine inventions. Thus, the studies of Archimedes were fundamental for the development of science and its engineering application in many fields. Relevant is the case of the Hydraulics in which Archimedes defined the law of buoyancy, that he used successfully in the design of large ships, as ascribed to him like for example in [22].

In this paper the focus is on Archimedes's ingenuity in machine design. Several machines are ascribed to Archimedes like chariot odometers, cranes, screw, war machines, most of which were later perfected during the Roman Empire. All these inventions are based on the law of levers and on experiential activity from a previous mathematical study.

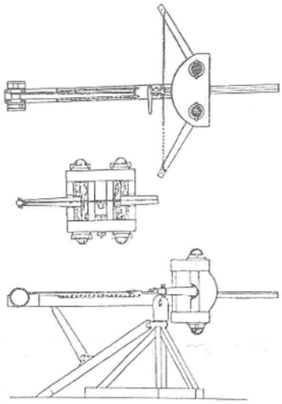


a) b)

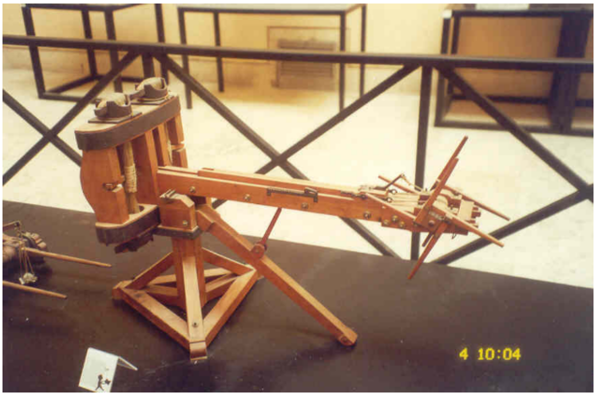
Fig. 4. The screw design by Archimedes: a) according to Del Monte and Galilei; b) as a machine model for lab activity in 18th century.

Archimedes designed machines by merging mechanisms with pulley systems in cranes of different sizes. Some of them were used to build the Siracusa fortress and some others for moving ships in maintenance works, as examples of versatility of the design ingenuity of Archimedes.

In the field of war machines, an example is shown in Fig. 5 where a catapult (as correctly named in ancient Greek) is illustrated for launching arrows to long distance. As shown in the scheme of Fig. 5a), the structure of this war machine is a combination of different components in an anti litteram mechatronic design, namely gears, prismatic slider, pulleys, and flexible bars. A capstan is used to tension the arc, made of flexible elements, to accumulate the elastic energy as in the launching motor for the two rods fixed in the machine frame. The platform on the prismatic guide allows a practical feeding of the arrows and a suitable moving body for the launching. An improved mechanism has been found that permitted even a repeated launching at high rates like a riffle, as pointed out in [23]. In the reconstruction in Fig. 5b) the design is emphasized as made of materials of common use in antiquity, namely woods, cables of natural fibres, hairs for the torsion elastic motor (even human hairs), so that they permitted a rapid construction, usage, and maintenance of the war machine, as a result of a careful attention of the technology of available materials in a war campaign. This again shows the practical vision of Archimedes in designing and developing machines by combining mechanical theory, experimental experience, knowledge of technology with an approach that is today typical of modern MMS (Mechanism and Machine Science).



a)



b)

Fig. 5. Roman catapult as an invention of Archimedes: a) a scheme; b) reconstruction in the Museum of Roman Culture in Rome.

4. CONCLUSIONS

In this paper, the modernity of the personality and work of Archimedes has been discussed as related to the mechanics of machinery, viewed from the engineering viewpoint of MMS. The modern relevance of Archimedes contributions can be recognized in a determination of theoretical bases, even through experimental activity, for a formulation of the mechanics of machines that was useful not only for a general theory but even for practical inventions of new machines of durable interest. In addition, his enthusiastic approach to apply science to practical engineering reveals Archimedes as a modern source of inspiration for future MMS researchers.

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ARCHIMEDEAN MECHANICAL KNOWLEDGE IN 17TH CENTURY CHINA

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ABSTRACT From the end of the 16th century to the beginning of the 18th century, Jesuit missionaries introduced Chinese the Western scientific knowledge and technology. In 1612, Archimedean-screw was introduced by Sabbathinus de Ursis and Xu guangqi. Since then, Part of Archimedean mecanical knowledge was transmitted into China. In this paper we will present an account about this transimission. The main points are as the following: in the 17th century, parts of both theoritical were introduced to China; Chinese paid more attention to the practical knowledge then, Theoretical knolwedge was only studied becase that it is the base of making useful devices.

1. THE CONTEXT OF TRANSMISSION OF WESTERN MECHANICAL KNOWLEDGE INTO CHINA

From the end of 16^{th} century to 18^{th} century, Jesuit missionaries played a major role in the transmission of knowledge between the Europe and China. The reason why Jesuit missionaries decided to transmit European scientific and technological knowledge is two-fold. First, scientific and technological knowledge could be quoted to prove the validity of Christian theories. Second, scientific and technological knowledge could help in attracting the attention of Chinese scholars. The latter played a more important role in the transmission of Archimedean Mechanical Knowledge into China.

In Matteo Ricci (1552–1610)'s own word, he "sought to make himself all things to all men, in order to win them all to Christ"[1]. He realized that literati played an important role and held a high standing in Chinese society, hence, converting literati became his major concern. As for literati, mechanical knowledge, especially practical mechanical knowledge was

[1] Matthew Ricci. 277.

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especially significant to them. At that time, there was an urgent need for devices used for irrigation and flood defense, while internal rebellions and external threats meant that there was a dire need for military devices. Further more, the calendar used in China failed to give exact prediction of special celestial phenomena, and Chinese astronomers worked in Imperial astronomical bureau were incapable in reform of calendar. In this context, Ricci and his fellow missionaries showed their ability in solving such problems to attract Chinese.

In 1607, after finishing the translation of the first six volumes of Euclid's *Elements*, Ricci showed his cooperator, Xu Guangqi (1562–1633), some models of European machines of irrigation made by Sabbathinus de Ursis (1575–1620) and suggested Xu to consult Ursis about such devices. In 1612, Xu and Ursis translated Western text into Chinese and compiled a book the *Taixi Shuifa (Hydraulic Methods of the Great West, Hydraulic Methods)*. It depicted some water-lifting devices such as Archimedean screw and devices similar to pump invented by Hero of Alexandria (See: Zhang).

In 1623, Italian missionary Giulio Aleni (1582–1649), assisted by Chinese scholar Yang Tingjun, compiled *Zhifang Wai Ji (Areas Outside the Concern of the Chinese Imperial Geographer, ZFWJ*), which introduced geography, climate, situation of the people, celebrities, products and dextrous devices in the world. In which Archimedes and his inventions were mentioned (See: Aleni, pp. 76, 87).

Calendar has a special significant in Chinese history. In Confucianism, it is believed that celestial phenomena have direct connection with the government and society. The Emperor, who was regarded as the son of the Heaven, should act as a coordinator between the heaven and the earth. An accurate calendar indicated the validity of the emperor's governing. Thus, Chinese emperor paid great attention to calendar[2]. When the Imperial astronomers failed to give exact prediction of the special celestial phenomena, Ricci showed that the European calendar is superior to the Chinese one. After realizing the fact that astronomy could be a great aid for the Christianization of China, Ricci suggested that Jesuits astronomers should be sent to China. In 1619, such experts arrived China, among the, there are Johann Terrenz (1576–1630), Johann Adam Schall von Bell (1592–1666) and Jacques Rho, (1590–1638), who later played great role in calendar reform in China.

The European machines attracted a number of Chinese scholars, Wang Zheng (1571–1644) is one among them. Wang Zheng took an interesting

[2] An emperor was looked upon as "the son of Heaven". Emperor's government organizes its astronomers to make an accurate calendar in order to indicate the validity of his regime. Mathematics, which was close related to astronomy, was rated as one of "six arts".

in machine making from a young age. Round about the winter of 1615 or the Spring of 1616, he got acquainted with a missionary de Pantoja, and was converted by them afterwards (See: Song). He read Aleni's *ZFWJ* and became interested in the machines mentioned in the book. Round about December, 1626 or January, 1627, Wang Zheng came to Beijing and met Nicclo Longobardi (1559–1654), Terrenz and Adam Schall von Bell. He consulted to them to have a good understanding of machines depicted in *ZFWJ*. They showed him the European books on machines. Even Wang could not read the text, he admired the machines contained in them, and he proposed to compose a book to introduce such machines to China. According to Wang's record, Terrenz indicated connection between machines and such theoretical knowledge as mathematics (See: Terrenz & Wang, p. 603)

After mastering the basic mathematical knowledge in a few days, Terrenz and Wang Zheng began to compose a book in mechanics and machines. Wang preferred to translate knowledge about "the most important, simplest and most ingenious" machines to serve Chinese people. In February or March, 1627, they finally compiled a 3-volumed book, *Yuanxi Qiqi Tushuo Luzui* (the Record of the Best illustrations and Descriptions of Extraordinary Devices of the Far West, For short, *Extraordinary Machines*), which was first printed in Yangzhou in the summer of 1628.

The *Extraordinary machines* was the first book specially contributed to Western mechanical knowledge in Chinese. In the guide to its use, the authors expressed that man must study such disciplines as *zhongxue* (learning of weight), *gewu qiongli zhi xue* (a study to investigate things to attain knowledge, especially natural philosophy), arithmetic, geometry and perspective before he studies the art of making machine. They also wrote an introduction to discussed the nature and usage of mechanics.

Following the introduction, there are three chapters that selectively expounded Western mechanical knowledge and machines from Archimedean time to the early 17th century. The first chapter contained the definition of the center of gravity and the way of finding the center of gravity of different geometric shapes, the definition of specific gravity, the numerical relation of the weight, specific gravity and volume of a thing, as well as the proportional relation among weights and volumes of things with different specific gravity, etc., and some theory concerned with fluids and buoyancy. The second chapter, named as *Qi Jie* (explanations of implements), discussed the principles of simple machines, such as balance, steelyard, lever, pulley, wheel, screw and so on. The third chapter is composed of illustrations and descriptions of 54 kinds of whole set of machines. We do not intend to deal with the content in detail, but we need to stress that the character of the two first chapters is in accordance with that of a European scholarly mechanical work. We believe that such a structure was designed by the

European compiler, Terrenz. Even all the mathematical proofs in the European sources are all deleted, Terrenz's stress on mathematics indicated that his understanding of mechanics was in according with Archimedean tradition. In the 17th century Europe, Archimedean tradition of mechanics, which stress the mathematical proof of mechanical theories is prevalent in scholarly studies. The main sources of *Extraordinary machines*, Simon Stevin's *Hypomnemata Mathematica* (1605–1608) and Quidobaldo del Monte's *Mechanicorum liber* are all following Archimedean tradition. Therefore, we may say that the theoretical part of the first mechanical book in Chinese was influence by Archimedean tradition (See: Zhang, Baichun. Tian Miao. Matthias Schemmel. Juergen Renn. Peter Damerow. 92–120). We will return to this book again in the detail discussion concerning the content of Archimedean mechanical knowledge transmitted to China in this paper.

We already mentioned above that calendar making has special political significant in China. In 1629, a lunar eclipse occurred, which the Imperial Astronomical Bureau had failed to forecast exactly, the imperial astronomers confessed that they did not have the ability to reform the calendar effectively. Xu Guangqi, vice-minister of the Board of Rites at the time and a convert of Catholicism, presented a memorial to the throne recommending that the Jesuits reform the calendar. Besides working out a practical and comparatively exact calendar, Johannes Terrenz, Giacono Rho, and Adam Schall von Bell also translated a magnum collection of books concerning the creation of a new calendar, *Chongzhen Lishu* (Calendar books of Chongzhen Reign, completed in 1634). Some branches of mathematical and mechanical knowledge concerned with calendar-making were also translated into Chinese in this work[3]. After a series of debates and examinations about whether a calendar based on the European method could be used, in 1644, the Chongzhen emperor (r. 1628–1644) finally decided from 1645 on that it should be used[4]. However, the Ming dynasty did not survive beyond the first half year of 1644, and the Qing dynasty began.

The change of the ruling family did not bring misfortune to the Western missionaries. After providing an exact forecast of a solar eclipse, on, 30 October, the *Shixian Calendar*, based on Western astronomy was promulgated as the statutory calendar in China. Schall became the effective director of the Imperial Astronomical Bureau, and a personal acquaintance of the Shunzhi emperor. Thus, with the help of astronomical instruments, the Jesuits made a good start in the new court. From this time to the beginning of the 18th century, except a short period of interruption for Yang

3 An introduction about the content of *Chongzhen Lisu*, see: Hashimoto. 1988.
4 For the debates and the examination on the calendar, see: Xu Guangqi and Li Tianjing.

Gangxian's successful accusation of European missionaries, the European calendar making system was adopted by the court. In 1674, In order to fulfill the demands of the emperor, Belgian Jesuit Ferdinand Verbiest (1623–1688) compiled textbooks on the usage of the 6 astronomical and mathematical instruments he made for the court, *Xinzhi Lingtai Yixiang Zhi* (*XZLTYXZ*, A Record of the New-built Astronomical Instruments of Observatory). In this book, Verbiest use mechanical knowledge to explain the style and the structure of the instruments to show Chinese that the European construction is based on rational base, and further more, the missionaries are scholars rather than craftsmen[5]. In doing so, he introduced new mechanical knowledge, including parts of Galilean mechanics into China. (See: Golvers, pp. 112–114, 117–123, Chen Yue.).

On the 16th October, 1683, Verbiest presented Emperor Kangxi a book, entitled *Qiong Li Xue* (*a Thorough Inquiry into the Reason*), in which, he collected some of the former works translated into Chinese by Jesuit missionaries and formulated a system of reasoning. Verbiest hoped that the emperor could be taken as a part of the curriculum for the Chinese state examination system. To his disappointment, the book was refused, hence not be published. Beside Aristotle philosophy, the book also included mathematical and mechanical content, the great part of the latter was derived from translated books, mainly from *Extraordinary machines* (See: Wang Bing, pp. 88–101. Ad Dudink. Nicolars Standaert.)

Even Jesuit missionaries continually worked in the court until the beginning of the 18th century, and they introduced new mathematical and astronomical knowledge into China as well as constructed different kinds of devices for the Emperor Kang Xi it seems that there is no systemic attempt made in transmission mechanical knowledge to China after Verbiest. In the following parts, we will focus on the detail of the transmission of Archimedean mechanics into China.

2. THE INTRODUCTION OF ARCHIMEDES

Even Archimedean-screw was introduced in the *Hysraulic Methods*, Archimedean's name was not mentioned. It is in the early 1620s, Aleni mentioned Archimedes (287–212 B.C.) as an astronomer in the section concerning the introducing of Sicilia in Italy of chapter 2 of *ZFWJ*:

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[5] Some of mechanical knowledge in *XZLTYXZ* may be regarded as a supplement. For example, there are some discourse on center of gravity and its use. Verbiest wrote: so-called center of gravity was a point in object. Two weights around this point has the same weights (See: Verbiest.1674, p. 98).

There was a famous astronomer Archimedes (*Yaerjimode1)*. He had three unique skills. A few hundred ships of an enemy state once reached his island. His compatriots could do nothing about it. He made a large bronze mirror. When he focused sunlight with it on the enemy ships, they were emblazed. All ships were soon burned. Withal, the king ordered him to build a very large seagoing vessel. After it had been built, it should be launched. However, the vessel could not be moved although the country exerted itself to the utmost, namely applied tens of thousands of cattle, horses and camels to tow it. Archimedes (*Jimode*) invented an ingenious skill so that the vessel was launched just as a hill moving according the order of the king. In addition, he made an automatic armillary sphere with 12 overlapping rings that corresponding to the sun, the moon and 5 planets. It could accurately demonstrate the movement of the sun, the moon, five planets and constellations. This transparent instrument was made of glass. Indeed, it was a rare treasure." (See: Aleni, p. 87) Here, Archimedes' name was phonetically translated as *Yaerjimode1* and in a short form *jimode*.

In the section concerning Egypt in Chapter 3 of *ZFWJ*, Archimedes was mentioned again.

A king once sought a measure to combat a waterlogging. He found *Yaerjimode*, a clever and deft man, who invented a water-lifting device. It provided people with incomparable facilities. It is named *longweiche* (water-screw) now. The people in this country were very tactful. Many of them studied the learning of *a* thorough Inquiry into the Reason through the study of thing (Natural Philosophy) and were also accomplished in astronomy." (See: Aleni, p. 110)

Yaerjimode1 and *yaerjimode2* were denoted by two different set of Chinese characters, any of which was made from five Chinese characters. As two transliterations of Archimedes, these two words have the same pronunciation although their correspond different Chinese characters. We have no idea why Aleni gave different translation of the same person, Archimedes, in two different way. There could be two reasons for that, one is possible that Aleni or his Chinese assistant regarded Archimedes as two persons, the other could be Chinese just denote the names according to Aleni's pronunciation and Alein did not noticed the difference.

The authors of *Extraordinary machines* retailed the story about Archimedes' building seagoing vessel and armillary sphere in *ZFWJ* and mentioned another inventor:

There was a great man who was named_*Yaximode*. He invented water-screw, small screw and other devices. He was able to depict the principles of all kinds of machines." (See: Terrenz & Wang, p. 611) *Yaximode* here again can only be Archimedes.

the *Extraordinary machines* also mentioned the crown problem, named Archimedes *Yaximode*:

Archimedes was once asked to solve a problem concerning the substitution of gold with silver, but he could not make it. While he had a bath, he suddenly thought out the reason. (See: Terrenz & Wang, pp. 614–615).

This paragraph followed by a detailed narration about the crown problem.

In the 17th century, the last volume of *Ouluoba Xijing Lu* (*XJL*, Records of the European Written Calculation) repeated this story, but the crown was changed to a cooking vessel (*ding*). The author named Archimedes *Yaerribaila* (See: Anonym, 17th century, p. 302)[6]. It is interesting that the fifth volume of *Celiang Quanyi* (*CLQY*, On Astronomical Surveying) named Archimedes "*ajimide*, a great personage in antiquity" or "*Mode*, a personage in antiquity"[7]. Afterward, *Ajimide* became a popular Chinese translation of "Archimedes".

Based on the world map composed by Ricci and Aleni, Verbiest wrote *Kunyu Tushuo* (*KYTS*, An Explanation of the Map of the Earth) in order to explain his *Kunyu Quan*tu (*KYQT*, Complete map of the earth). *KYTS* was printed in 1674. This book repeated the stories of Archimedes in *ZFWJ* (See: Verbiest, pp. 6260, 6261, 6263).

3. THE INTRODUCTION OF ARCHIMEDEAN MECHANICAL KNOWLEDGE

3.1. Archimedean-screw

Archimedes is associated with the invention of water-screw (Oleson, pp. 291–301). In the first volume of *Hyderulic Methods*, de Ursis introduced the water-screw into China. Xu Guangqi, the Chinese partner in translation of the book vividly named it *longwei che* (vehicle with the shape as dragon's tail)[8]:

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6 So far, no printed edition of *XJL* has been found. We can only read a handwritten copy of it. A great part of is similar to *TWSZ*, but some of terms in two books are different. It is possible that their contents were derived from same book or books, and *XJL* was written not long after *TWSZ* had been finished.

7 *CLQY* was translated by Rho in collaboration with Xu Guangqi and Li Zhizao and was first printed in 1631. The authors explained how Archimedes had calculated area of circularity, area of ellipse and so on.

8 In Chinese, *long* means dragon; *wei* means tail; *che* means machine. *Long wei che* means a machine that is like the tail of a dragon. Since 1612, *long wei che* has been a special Chinese translation of water-screw, and was inherited by authors of *ZFWJ*, *WONDERFUL MACHINES* and other books.

Following an description of the main parts of Archimedean Scew,, the shaft, the wall, the tube, the pivot, wheel and underprop, de Ursis and Xu Guangqi give further explaination about these parts, as well as their materials they are making of, the way of manufacture and usage. (See: de Ursis & Xu, vol. 1) They presented the explanations of water-screw with five illustrations (figure 1).

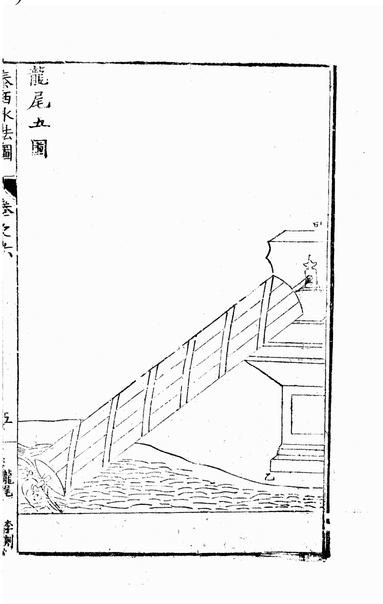

Fig. 1.

Terrenz and Wang not only explained the structure and usage of water-screw in the third chpater of *Extraordinary machines*, but also analyzed its geometrical principle in the second volume. They used the words such as vine line, the implement as vine line, snail line and the tail of dragon to denote helix, screw or spiral. Section 82 indicates the relation between a helix and a inclined plane: "an inclined plane wraps a cylinder, as a result, a helix or a screw comes into being"; sections 83–85 indicate relation between lead angle of helix and transmission effect of force (See: Terrenz & Wang, pp. 649–650). Sections 89–92 give a few examples to illustrate how force or lead angle is calculated. Section 74 emphasizes that *tengxian qi* has many advantages and usage; therefore Archimedes often used this kind of wonderful implement. Man can easily make all kinds of machines if he understand the why and wherefores of this implement (See: Terrenz & Wang, p. 648). This clearly shows that the Terrenz and Wang clearly know that the screw they are introduced to China comes from Archimedes.

3.2. Mechanical Propositions

In *Extraordinary machines*, Archimedes' theory concerning floating body, equilibrium and related calculations, as well as Archimedean propositions were also introduced into China.

Section 36 of the first chapter: "Water floats along with surface of the earth, which is in large round shape. Water adheres to the earth, so water's surface is round too." (See: Terrenz & Wang, p. 625) This is similar to the Archimedean proposition 2 in *On Floating Bodies* (See: Archimedes, p. 254).

Section 40 of the first chapter: "There is an object. If its specific gravity is equal to water's, it will neither sink nor float. Its top is at the same level as water's surface." (See: Terrenz & Wang, p. 626) It almost repeats the Archimedean proposition 3 in *On Floating Bodies* (See: Archimedes, p. 255).

Section 41 of the first chapter: "There is an object. If its specific gravity is lighter than water's, it will not totally sink; one part of it will be above water, while the other will be in water." "Because water is heavier than the object, water may lift it up." (See: Terrenz & Wang, p. 626) This is equivalent to the Archimedean proposition 4 in *On Floating Bodies* (See: Archimedes, p. 256).

Section 43 of the first chapter: "There is an object. If its specific gravity is lighter than water's, the weight of total object will be equal to the weight of water, the volume of which is the same as the volume of the part of object that sinks in water." (See: Terrenz & Wang, p. 626) It is close to the Archimedean proposition 5 in *On Floating Bodies* (See: Archimedes, p. 257).

Section 42 of the first chapter: "There is an object. If its specific gravity is heavier than water's, it will not stop until it sinks to the bottom." (See: Terrenz & Wang, p. 626) Section 46 of the first volume: "Solid in water is lighter than it in air. The difference is equal to the weight of the water of the volume equal to the part occupied by the solid." (See: Terrenz & Wang, p. 627) It is almost equal to the Archimedean proposition 7 in *On Floating Bodies* (See: Archimedes, p. 258).

Sections 44–61 of the first chapter explain how to calculate weight and volume of an object in water, and discuss pressure of water. These calculations were scarce in ancient China.

Section 19 of the second chapter (explanations of the steelyard): "There are two weights that are in a state of balance. The proportion between the large weight and the small weight is equal to the proportion between the length of the long section and the length of the short section of the beam that is in level position. Likewise, the proportion between the large weight and the short section's length is equal to the proportion between the long section's length and the small weight." "This is the most cardinal principle of 'study of weight' (mechanics). All calculations are based on it." (figure 2)[9] (See: Terrenz & Wang, p. 636)

Section 36 of the second chapter (explanations of the lever) almost repeats that principle: "A lever is horizontally supported by a fulcrum. There is a weight at its head. A force acts on its handle. The proportion between the weight and the force is equal to the proportion of length between two sections of lever." (figure 3) (See: Terrenz & Wang, p. 639) Here and in other sections, the authors used such concepts as force or capacity frequently, both of them are interchangeable (See: Terrenz & Wang, pp. 641–651). After section 36, this principle is applied to analyses and calculations of the pulley and wheel as well as other devices.

The principle narrated in section 19 and section 36 actually is lever principle, namely Archimedean proposition 6 and proposition 7 in *On the Equilibrium of Planes* (See: Archimedes, p. 192).

Section 16 of the first chapter: "There is a rectangle, center of gravity of which is at the midpoint of any radial line of two midpoints of subtenses." (See: Terrenz & Wang, p. 621) This may be regarded as a special example of Archimedean proposition 9 in *On the Equilibrium of Planes* (See: Archimedes, p. 194).

Section 12 of the first chapter: "There is a triangle. Draw a line from an angle to the midpoint of its subtense, well then the center of gravity of the triangle must be at the line." Section 13: "There is a triangle. Its center of gravity is the same point as its geometrical center." (See: Terrenz & Wang, p. 620) These sentences should be equal to Archimedean proposition 13 in *On the Equilibrium of Planes* (See: Archimedes, p. 198).

Section 14 of the first chapter: "The method to find the center of gravity of a triangle is as following: drawing a line from the midpoint of any side to its corresponding angle. The center of gravity is at the point of intersection of two lines." (See: Terrenz & Wang, p. 621) This is Archimedean proposition 14 in *On the Equilibrium of Planes* (See: Archimedes, p. 201).

Section 18 of the first chapter: "The geometrical center of circle or ellipse is the same as it's center of gravity." (See: Terrenz & Wang, p. 621) This is identical with Archimedean proposition 6 in *The Method* (See: Archimedes, supplement, p. 27).

Section 20 of the first chapter: "The center of gravity of any regular prism is at its axis." (See: Terrenz & Wang, p. 622) This is similar to

[9] After section 23, some sections of the second volume of *Wonderful Machines* takes a beam for a weight, namely the weight of the beam has to be calculated.

Archimedean proposition 7 in *The Method* (See: Archimedes, supplement, p. 30).

The way of expression in *Extraordinary machines* is in some extent different from the original proposition in Archimedes' works. The reason is that mainly because Terrenz and Wang used some books printed and published in Europe in the 16–17th centuries instead of Archimedes' works. In addition, they did not translate European texts literally. H. Verhaeren identified that the first two chapters of *Extraordinary machines* are derived mainly from Simon Steven's *Hypomnemata Mathematica…Mauritius, Princeps Auraicus, Comes Nassoviac…*, (1608)[10]. (See: Verhaeren)

3.3. The Crown Problem

The first Chinese book that introduced the crown problem was *Tongwen Suanzhi* (Rules of Arithmetic Common to Cultures, 1614. TWSZ), but the problem was transformed to an arithmetic problem:[11]:

"Question: one hundred *jin* of gold is used to make a golden *lu*"[12]. When it has been finished, man doubted that a craftsman stole gold and substituted partgold with silver. He dreaded the economic losses if he test it damage it to test it, but feared economic losses. How to find how much silver has been mixed into gold?

The book *XJL* related the similar problem to Archimedes:

A monarch ordered a craftsman to use pure gold of 100 (*jin*) to make a cooking vessel (*ding*). The craftsman stole some of gold and mixed silver into gold. After the vessel had been finished, it was presented to the monarch. He noticed the gold's colour was lighter [than it was expected], whereupon he order an astronomer Archimedes to calculate how much gold was stolen. The answer: gold of sixteen and two-thirds *jin* was stolen, gold of eighty-three and one-third *jin* remains. (See: Anonym, 17th century, p. 302)

The calculating method in *XJL* is the same as in *TWSZ*.

Section 29 and section 30 of the first volume of *Wonderful Machines* introduced the theoretical base of the crown problem: "There are two

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[10] Iwo Amelung thought that Terrenz may have been aware of the original Dutch version of Stevin's work, which had been published as *De Beghinselen der Weeghconst beschreven duer Simon Stevin von Brugghe* (Elements of the Science of Weighting described by Simon Stevin from Bruges) in 1586 and was probably also available in the Jesuit's library in Beijing. It is possible that Terrenz, who was of Swiss origin and thus probably able to read Dutch, had consulted it when preparing the work. *Zhongxue* is a loan translation for *scientia ponderibus* (or for that matter a slightly modified loan translation for Stevin's *weeghconst*) (See: Amelung).

[11] *Lu* is an estrade, on which a drinking vessel in a drinkery is emplaced.

[12] *Jin* was a unit of weight in China, Now one *jin* is equal 0.5 kilogram.

objects. Becaues they have the same weight and the same volume, they must be the same kind of weight." "The same kind of weights has the same specific gravity." (See: Terrenz & Wang, p. 624)

4. THE INFLUENCE OF ARCHIMEDEAN MECHANICAL KNOWLEDGE ON CHINA

Being introduced as one of the greatest astronomer and engineer, Archimedes certainly roused the interest of Chinese scholars. At the end of the 19th century, Ruan Yuan (1764–1849) compiled *Chouren Zhuan* (Collection of biographies of astronomers and mathematicians), which is the first Chinese book specially focusing on mathematicians and astronomers. A biography of Archimedes was also included, which is based on the information concerning Archimedes in *CLQY* (See: Ruan, p. 507).

According to the previous discussion, in the 17th century, parts of practical and theoretical knowledge of Archimedean mechanics were transmitted to China. The book contained such knowledge, such as *Hydraulic Methods* and *Extraordinary machines* were both reprinted several times before the end of the 19th century. From the study of such books, some Chinese get acquainted with Archimedean mechanics.

Generally, Chinese scholars paid more attention to the practical part of Archimedean mechanics, because that such knowledge was useful for the society, and serving the society was one major concern of Confucians. Nevertheless, the theoretical knowledge was also studied by Chinese scholars. In his *Lixue Huitong* (Integrated Calendrical Studies, *LXHT*), Xue Fengzuo (1600–1680) reconstructed the first two chapters of *Extraordinary machines*, as he think that this part provided the reason for the construction and invention of machines. Thus, it is important for scholars, as by learning this, one can invent and construct machines himself. Therefore, he selected part of the content from these two chapters including the above mentioned Archimedes' knowledge concerning balance, lever and screw. (Tian Miao, Zhang Baichun).

But from Xue Fengzuo's example we may also conclude why he thinks that the mechanical theories were important is that it is the base of practical knowledge. He does not interested in philosophical inquire of mechanical knowledge or the system of the mechanics. He only chose the part of knowledge relating to machine making. The machines are really appreciated by Chinese people.

Both *Hydraulic Methods* and *Wondrful Machines* were included in the *Siku Quanshu* (*SKQS*, Complete Library in Four Branches of Literature), which was compiled according to the order of the Qianlong emperor and many important and famous scholars attended the compilation. The introduction of each book in the *Siku Quanshu* shows the common official and scholarly attitude toward the book. The introduction of *HYDRAULIC METHODS* provides us such information:

The best part of Western learning is measurement and mathematics, and the (study of) extraordinary machines take the second place. Among the extraordinary machines, their method of irrigation is the very important to people.

People who deal with irrigation must use (this book). (Anonym. 18th century, introduction to *HYDRAULIC METHODS*, 1.)

Similar idea was expressed in the introduction of *Wonderful Machines*:

Both *Biaoxing Yan* and *Biaode Yan* exaggerated marvellousness of those methods. (In fact,) most of them are absurd and unrestrained, and were not worth deep examining. But the method of machine construction in the book is actually the most ingenious. (See: Anonym. 18th century. Introduction of *Extraordinary machines)*

The common attitude of Chinese scholars is that the European machines are very useful. But the theoretical knowledge, which usually mean European philosophy and religion are absurd. As Wang Zheng praised the religion in his *Biaoxing Yan* and *Biaode Yan*, these two parts were criticized.

There were some accounts of manufacturing and use of the water-screw in 18th and 19th centuries. Xu Chaojun, a descendants of the 5th generation of Xu Guangqi, had a good grasp of astronomy and clock-making. A book, which printed before 1911, told us that he constructed a water-screw that could be driven by a child to irrigate crop in 1809. A procurator ordered some people to print the illustration of the water-screw to popularize it in a few counties (See: Group, p. 213).

Qi Yanhuai (1774–1841), who first held office as a county magistrate in Jinkui and afterwards as a prefect in Suzhou, also made a water-screw and a pump on the basis of *HYDRAULIC METHODS*. He believed that one water-screw is analogous to five square-pallet chain-pump (See: Group, pp. 205–206). Lin Zexu (1785–1850), a dignitary in Jiangsu province[13], asked Qi to made one a screw for him, and afterward Lin suggested that this kind of machine should be spread in the countryside, but he failed.

For craftsmen and farmers, the construction and using of water-screw also bring some inconvenience. Qian Yong told us a short story. A water-screw was made in Suzhou in 18th century. It may irrigate cropland of thirty or forty *mu* (a traditional unit of area) every day. However, its very expensive in construction. And if it was damaged, it could not be repaired

[13] Suzhou was and is a part of Jiangsu province, which was and is one of the most developed provinces in China.

and be used again. A majority of farmers was so poor that they were not able to make it (See: Group, p. 209). Zheng Guangzu recorded that a water-screw was made in an area nearby the Great Canal in Jiangsu province in 1836. It was so large that it needed one hundred people to carry it. People can use it to irrigate cropland very efficiently, but again, the construction and using of such a machine is too expansive for them. It was not only expensive but also delicate (See: Group, pp. 209–210). Craftsmen and farmers could skillfully manufacture, operate and repair traditional Chinese water-lifting devices that were actually not much inferior to the water-screw in function. Without an absolute advantage in function and efficient over the old ones, a new device is very difficult to be accepted in a mature technical tradition.

In general, part of Archimedean mechanical knowledge exerted a limited influence on China in 17th century. It had partly been studied and practiced by Chinese by the mid-19th century.

5. CONCLUSION

In the 17th century, Jesuit missionaries introduced part of Archimedean mechanics into China. They and their Chinese partner selected and translated the water-screw, part of Archimedean mechanical propositions and relating calculations from western language into Chinese. Generally, very few Chinese are interested in the theoretical content of mechanics, and even some of them study the theoretical propositions.

Chinese scholars and officials attached importance to Western machines. Some of them even made water-screws. But due to the inconvenience and high cost in construction and using, the water-screw was not popularly used in China.

The transmission of European science and technology into China in the 17th and 18th century was shaped by multifold factors. The aim and think of the transmitter and the accepter played a major role in it. Concerning the transmission of Archimedean mechanics, the transmitter, the Jesuit missionaries, aim at using such knowledge in gaining Chinese scholars. While as the accepter, Chinese scholars are interested in practical knowledge which is useful to the society. The selection and the function of presentation of knowledge transmitted are shaped by the needs and interest of these two parts. After the knowledge was already transmitted to China, the social and technique condition decided whether it could be accepted or which part of it can be accepted and flourished. The transmission of Archimedean mechanics can serve as a case study of the research on the transmission of science and technology across culture boundaries, and the relation between social context and the function of development and content of the knowledge.

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ARCHIMEDES ARABICUS. ASSESSING ARCHIMEDES' IMPACT ON ARABIC MECHANICS AND ENGINEERING

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ABSTRACT Archimedes is an author who is frequently quoted in Arabic texts in relationship with mathematics and mechanics, including hydraulic devices such as water-clocks. The present study traces transmission paths and evidence for an assessment of the impact of the Archimedean works on the Arabic tradition of mechanics and hydraulics.

1. INTRODUCTION

The impact of Archimedes of Syracuse (ca. 287 – ca. 212 BCE) on medieval mathematics, sciences and engineering has been discussed in several overviews (e.g. I. Schneider 1979) as well as in comprehensive studies (e.g. Clagett 1964) published during the last decades. The actuality of the Archimedean mechanical concepts and the contribution of Archimedes to what is called in modern terms design of mechanisms has been a frequent subject of current studies (e.g. Chondros 2009).

Concerning the perception and tradition of the Archimedean works in the Arabic literature, scholar assessments and interpretations diverge considerably. A first, although rather superficial, attempt to assess the influence of the works and the fame of Archimedes on the Arabic-Islamic science could be based on some kind of statistics with respect to (a) mentioning the name or the treatises of Archimedes, as well as to (b) copying, quoting, commenting or compiling the work of the Syracusian scholar in the Arabic grammatology. However, pseudepigrapha, corrupted texts, lost links in the tradition and difficulties in reconstructing the treatises scattered in several manuscripts or codices demand for more thorough and balanced considerations. Most of the studies focus on the tradition of mathematics – perhaps

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due to the fact that the study of the cultural transfer of mathematics from Greek into Arabic as well as the development of the science itself in the Arabic environment has a longer and broader scholar tradition. The present study will focus on Archimedean mechanics and engineering with a major goal consisting in revisiting and elucidating the relations among Greek, Arabic, Iranian, and Byzantine traditions or influences with respect to Arabic treatises attributed to Archimedes.

2. ARCHIMEDES IN ARABIC TRANSLATIONS

The most frequent and common transcription of Archimedes' name in Arabic is *Aršimīdis*. In Arabic texts *Aršimīdis* is mentioned not as a certain author (somebody vaguely known under this foreign name), but as an author who is expected to be well-known to the reader. A conventional way of listing the Arabic translations of Archimedes' works is given by Clagett (1964, pp. 3–4) in respect to the names of the treatises as known in modern scholarship:

a) Works known also in Greek (Byzantine) tradition: "*On the sphere and the cylinder*" (with at least a portion of Eutocius' commentary), "*The measurement of the circle*" (with perhaps Eutocius' commentary), a fragment of "*On floating bodies*".
b) Some indirect material from the treatise "*On the equilibrium of planes*" found in mechanical works of other authors translated into Arabic (e.g. the "*Mechanics*" of Heron, nowadays extant only in Arabic).
c) Works (perhaps) based on original treatises of Archimedes which are not extant in Greek: "Lemmata", "Book of triangles", "On the seven-part division of the circle", "On touching circles", "On parallel lines", "On data", "On properties of a right-angled triangle" and the treatise "On the construction of water-clocks" which will be discussed below.

Controversial on its philological and interpretative background is the last category – depending on the focus, the method of examining the extant manuscripts and the evidence available to the scholars. Folkerts (2009), for example, mentions only "*Lemmata*" and "*On touching circles*", whereas a the treatise known under the title "*On the construction of water-clocks*" has been classified as pseudepigraphic or, at least, as a compilation of pseudepigraphic and Archimedean parts (s. below).

3. CATEGORIES OF IMPACT ON THE ARABIC-ISLAMIC SCIENCE

In the 1990s Roshdi Rashed summarised the impact of Archimedes on the Arabic-Islamic science by proposing a threefold categorisation. The first group comprises the "inspiring legends", to which Rashed counts the military triumph of the catoptrics marked by the story about Archimedes putting fire on the Roman fleet attacking Syracuse by means of gigantic mirrors, as well as the story about Hieron's golden crone. These as well as similar stories were reported in Greek and Roman *literary* texts (e.g. Plutarch, Cicero), sources of minor interest for the Muslims in comparison to the higher estimated texts of purely scientific content. However, they should have undoubtedly accompanied and inspired Arab and Iranian scholars such as al-Kindī (9th century CE), Ibn Sahl (10th century CE) and al-Birūnī (10th century CE) in research activities on optics (catoptrics), mechanics and hydrostatics.

The second category of scientific works comprises a large amount of Arabic treatises ascribed in the Arabic bibliography to Archimedes. These treatises are most probably – partial or complete – pseudepigrapha, occasionally considered erroneously as translations of lost Greek versions of Archimedean treatises. The most representative treatise of this category is the treatise on hydraulic clocks. Another group of texts of this category are related to cosmological models quoted by authors of the (Greek) late antiquity in the field of alchemy. In this context Archimedes was quoted as author of a treatise on "*Pneumatics*", e.g. in texts ascribed to the 4th century CE alchemist Zosimos of Panopolis (Lippmann 1919, p. 85), an author who was very "popular" in Arabic alchemical and occult treatises.

The third category comprises Arabic translations from the Greek of Archimedean treatises such as "*The measurement of the circle*" or "*On the sphere and the cylinder*", which influenced considerably the Arabic geometry and mechanics. This influence has been demonstrated in the case of "*The knowledge of the measurement of plane and spherical figures*" of the brothers Banū Mūsā (9th century CE), as well as in the works of Thābit ibn Qurra (9th century CE) and the numerology of the I<u>kh</u>wān aṣ-Ṣafā', a group of Muslim scholars whose encyclopaedic activities are dated to the 10th century CE in 'Abbasid Basra in Iraq (Bafioni 1997).

A close study of the specific traditions, however, renders the limiting lines among the above categories less visible. In the following some "interaction" modes among the above influence paths around works on mechanical devices will be presented.

4. ARCHIMEDES IN THE ARABIC TRADITION OF MECHANICS

The admiration of Muslim (Arab, Persian etc.) mathematicians for Archimedes may mislead our modern expectation concerning the alleged perception of the mechanical treatises of Archimedes in the Arabic-Islamic environment. Whereas he was designed by the title of "leader" (*al-imām*) in mathematical sciences (Abattouy 1997, p. 13, no. 18), his reputation among the Muslim scholars in the field of mechanics is still controversial. Tracing his role in the Arabic tradition of mechanics depends not only on our knowledge about the direct translation of relevant Archimedean treatises into Arabic, but also on the compilation of excerpts from his works together with other texts – whereas original Arabic or translations from other Greek treatises. It is characteristic in this context that in the Arabic text of Heron's "*Mechanics*", a treatise extant only in the Arabic translation, Archimedes is quoted several times. Typical subjects in conjunction with these quotations are questions on equilibrium, of distribution of loads and of centres of gravity (Abattouy 1997, p. 12, no.16). Heron of Alexandria quotes in the Arabic text the unknown "*Kitāb al-qawā'im (Book of the supports)*", in which Archimedes should have treated these questions. Heron provides also the reader with more references of Archimedean treatises under unknown names, e.g. "*Kutub al-amḥāl (Books of the levers)*"– perhaps a problem of translating titles, of lost Greek originals, or of pseudepigrapha.

On the other hand Arabic bibliographers of mechanical treatises do not mention relevant Archimedean treatises – although Arabic works on the balances, a typical subject of Arabic mechanics, contain parts that leave little doubt about the acquaintance of the authors (e.g. al-Khāzinī) with the Archimedean ideas (Abattouy 1997, p. 12).

The perception of Archimedean mechanical concepts (e.g. levers) in Arabic is closely correlated with similar questions in the Arabic perception of the famous "*Problemata mechanica (Mechanical questions)*", a Greek treatise ascribed to Aristotle. Although there is no extant Arabic translation, recent research has established relationship between this Greek pseudo-Aristotelian text and works of Thābit ibn Qurra, especially his "*Kitāb fī'l-qarasṭūn*", a treatise on the steelyard lever, as well as the Arabic text of Heron's "*Mechanics*" (Abattouy 1997, p. 10). The establishing of the interdependence between this corpus and the translations of Archimedean treatises by Thābit ibn Qurra is a topic of current research (see also Abattouy 2002).

A final remark concerns the relevance of the mechanical subjects for the corpus of Arabic treatises on ingenious mechanical devices (*ḥiyal*). These devices correspond to the Greek *automata* (e.g. those described by Heron in his "*Pneumatica*"). In the Arabic tradition the most common ones are hydraulic clocks, one of which was ascribed by several authors to Archimedes.

5. THE WATER-CLOCK OF (PSEUDO-) ARCHIMEDES

In 1891 Carra de Vaux published an Arabic treatise attributed to Archimedes concerned with what the editor called "clepsydra". Actually, the text of the edited, translated and commented manuscript treats a hydraulic clock. The treatise was re-examined and translated into German under the title "*The clock of Archimedes*" in 1918 by E. Wiedemann shortly after a treatment of the work in the more general context of clocks in the medieval Islamic world (Wiedemann 1915). The treatise was systematically collated by means of more manuscripts and published under the title "*On the construction of water-clocks*" by D. Hill (1976; 1981).

One result of Hill's study is that the treatise almost certainly contains Hellenistic and Byzantine material, as well as material from the Arabic-Islamic tradition. The mechanisms described in the first two chapters, a water machinery and a ball-release mechanism for marking the hours, are, according to D. Hill, essentially the same as those presented by two other authors of Arabic treatises on water-clocks at the beginning of the 13rd century CE. Riḍwān describes in a treatise dated 1203 CE the water-clock built by his father in Damascus. In his treatise on ingenious mechanical devices (*ḥiyal*) completed in Diyār Bakr in 1206 CE al-Jazarī describes – among others – similar hydraulic-mechanical machines. Hill suggests that the basic machinery should be an invention of the historical Archimedes (or, at least, it should go back to the Greek/Hellenistic tradition under this name). He points out that the outlook of the clock follows Iranian and Indian styles, but also Syrian-Byzantine construction models as reflected at the hydraulic-mechanic clock of Gaza described by Procopius (5th century CE).

6. CONCLUSION

Combining functional elements of hydraulics and mechanics of one cultural tradition with aesthetic models of another according to a plan of a third one in the case of the water-clock of Archimedes resulted presumably to a device for which the name of Archimedes offers more than a famous affiliation. This hypothesis about the design practice could also explain the patchwork composition of some manuscripts, in which the pseudo-Archimedean treatise is just one part of a group of device descriptions and, eventually, also just one of several theoretical considerations. The assessment of the impact of the Archimedean tradition on Arabic mechanics still remains an on-going quest, in which new evidence on contributions to the development of the Arabic tradition of mechanics and hydraulic clocks inevitably contain new aspects of the connection of this development with the Arabic tradition under the name of Archimedes.

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3. LEGACY AND INFLUENCE IN HYDROSTATICS

THE GOLDEN CROWN: A DISCUSSION

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ABSTRACT Archimedes's fame is universally more connected to his extraordinary inventions and to the legendary events that have been ascribed to him rather than to a deep and real knowledge of the historical personage and of his works. Systems of *levers* and *catapults*, *cochlea* and other mechanical or hydraulic contraptions, *water-clock*, *planetarium*, *heat rays*. Among these and further inventions, real or supposed to be, there is the episode of Hiero's Crown, Fig. 1. The episode of the apparent fraud goes generally around in two different versions; the first one, which is based on the volumetric comparisons, mentioned by the roman architect Vitruvius, the second one is anonymous, it is related by Priscian and it's essentially based on the hydrostatic balance. In this paper, we compare and discuss the two reconstructions, both of them to be considered plausible.

Fig. 1. Reproduction of wood engraving of the late Middle Ages.

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1. INTRODUCTION

Archimedes is *per antonomasia* the best known and revered scientist of the ancient times and, at the same time, the most underestimated and misunderstood. Many elements have contributed to this paradoxical result: his brief style and the objective originality of some of his results (hardly mentioned or even lacking proofs, references to non identified or missing works), [(1)] the troubled and sometimes risky vicissitudes of the works to which his thought has been committed, the contradictory and uncertainty of the various evidences which, through different centuries and cultures, have often left us a transformed and phantasmagorical version of this personage. In a word, Archimedes has become more an icon of scientific mythology and of novelized history, than an author whose works we know or a scientist whose results we understand. In his case, we're not only dealing about the usual smallness of biographical information and the fragmentary quality of the original sources, which characterized almost any scientist belonging to classical ancient times and, particularly, the «Hellenistic» ones. We're speaking about the modern removal of ancient people's scientific results, as if history were only progressive, and science an exclusivity of modern times. [(2)] Starting from his times, when, together with the refined Alexandrine culture, of wich he was a bright champion, Archimedes was swept away from the military preponderance of the Romans who couldn't understand him, and so couldn't (or wouldn't) translate him, and they just limited theirselves to tell his magnificent achievements, incidentally and not in a parallel to the *Life of Marcellus,* [(3)] beginning, in this way, to misinterpret and betray him. Coming to our times when the sloth of Italian publishing, together with the inattention of public, make us run the risk to loose his inheritance again. As a matter of fact, in comparison to the large number of the foreign editions of the complete works of Archimedes, the only, and late, Italian edition, though with the limits pointed out by several authors, is still the one edited by Frajese, [(4)] who, with the few copies still existing in libraries, keeps in life, in our country, an echo of the greatest genius' thought. Nevertheless, this is also a peculiar event in a larger and recurrent phenomenon: the arrogance of power, ever and ever dedicated to the defense of the *status quo* and of the «Reason of State», which loses sight of its best brains. In ancient times, through killing the genius engaged in his country's defense, yesterday forcing the best brains of all to emigrate, to escape from «concentration camps» and from extermination, and again today forcing to a «Brain Drain», away from one's Country, who is trying to build a better future.

2. ARCHIMEDES, WHO'S THAT?

Among scientists is generally accepted his relief, for someone even *"Archimedes is the most important scientist ever existed"* [...] *The most* *certain general characteristic of european scientific tradition is that itself consists of a series of additional notes to Archimedes's work."* (5) Also among common people he is generally famous, though in a more disputed and paradoxical way. Surely he came to limelight in collective imaginary, embodying well the figure of a bizarre genius, in the shoes of *Disney's Gyro Gearloose* (1) but *"What does the modern cultured man know about him?* [...] *He just remembers that he did strange things: he ran naked along the streets shouting Eureka! He dipped crowns into the water, he drew geometrical figures while he was being killed and so on. The children anecdotes* [...] *equalize Archimedes more to the legendary and the mythological characters than to other thinkers. The result is that we remember him, but we do as a legendary character, completely out of history"* (2) As for as the historical personage, only his death date is sure: 212 BC, because he died in the fault of Syracuse, in consequence of the roman siege, an epochal event related in the *Annales*. It's uncertain, on the contrary, his birth date, though it's supposed to be usually placed in 287 BC this deduction is based upon this note (reported by the Bizantine philosopher Ioannes Tzetzes about fifteen centuries! after): he died *"at an elderly age: 75 years old"*. (5) In any case, it's fairly probable that he was an old man for his times, as reported by our most reliable source about the Punic Wars, (6) whose The Siege of Syracuse is a part. As for as Archimedes as a thinker and his works, which survived fortunately and often in a *rocambolesque* way, (5) many writings are now available, but it is still lacking an updated critical edition written in italian language: *nemo profeta in patria* we could say. According to Plutarco's version, Archimedes despised every technical activity and *"lived continually enchanted by geometry,* [...] *a mermaid who was to him family and domestic, so far as to forget even to eat and to take care of his own body."* (3) It's still very doubtful the truthfulness of this opinion, that's based only upon the *"opinion of a scientist who, more than three centuries since Archimedes was dead, ascribes to him gratuitously his own Platonic inclinations."* (2) Whatever was the relief that Archimedes himself gave to the different aspects of his own multiform intelligence, or if he made any preference between his «geometrical works» and his «mechanical works», today he is considered above all as a grand mathematical, a forecaster of the infinitesimal and combinatorial calculus while, in his times, he was considered above all as an engineer, a technologist, an inventor of wonderful and frightening machines, which memory caused the oblivion of his mathematical *corpus* which few people could really understand.

3. CROWN OR WREATH

Then, starting from his death, his fame spread universally, and it was linked to amazing inventions and events, more than to his works. On the occasion of the launching of the famous ship *Syracuse* (later called *Alexandris*), the largest vessel of ancient times, it's told that Archimedes himself, alone, launched the heaviest ship, making it slip sweetly to the sea, using a *polyspaston*, (3) that's to say a two blocks tackle with a large number of mobile pulleys. This event is linked to the famous phrase *"give me a fulcrum, and I'll lift the World up.* (7) According to another testimony, during his Egyptian visit, he invented the *cochlea*, a spire pump, that's called in fact «Archimedes's screw», able to lift water up, in a very efficient way, and with little effort. (8) Another very admired technical result was the *planetarium*, described by the consul Caius Sulpicius Gallus in one of his works, received by his colleague Marcus Claudius Marcellus, a nephew of the plunderer of Syracuse. (9) In an Arabian manuscript is contained the description of a particularly ingenious *water-clock* he invented. (10) In his treatise *The Sand Reckoner*, Archimedes himself describes the *dioptre*, an instrument used in order to measure the apparent size of Sun. Also *"the history of astronomy is a debtor to The Sand Reckoner: in this work we found, as a matter of fact, the most ancient attestation of the «heliocentric system» by Aristarcus of Samos."* (4) Amazing and frightening were, finally, the war devices (*iron claw and heat rays*), designed and used by Archimedes in order to defend Syracuse from the roman siege, burning and sinking the roman ships. The event is reported nor by Polibius, nor by Lyvy, nor Plutarch, but it is related only by several late sources as Galen, Dio Cassius and more authors among which the mentioned learned Byzantine man. The *heat rays* are here described as composed of a series of conveniently oriented flat mirrors, able to focus sun rays in a single point: the wooden roman ships to be burnt out in Syracuse sea. The structure was probably formed by at least 24 large flat mirrors, disposed in a hexagonal shape over a grate, which spun over a pole fixed to the ground: the central mirror was used to direct sun rays on the target, while the side mirrors were focused with a belt system. A history or a legend? This episode has always been considered extremely unlikely, maybe impossible, but an experiment, realized by MIT, showed for the first time it was at least practicable. (11) Anyhow, the episode that excited most common imagination is the Golden crown of Hiero II. *"It was, more correctly, a golden wreath (**στέφανος**), and not a crown, as everybody usually says.* [...] *The difference is not so accessory, because the wreath was a sacred object, and could be altered in no way".* (8)

4. IN THE MANNER OF VITRUVIUS

Plutarch scarcely mentions this event, while Vitruvius reports it extensively. *"Hiero after gaining the royal power in Syracuse, resolved, as a consequence of his successful exploits, to place in a certain temple a* *golden crown which he had vowed to the immortal gods. He contracted for its making at a fixed price, and weighed out a precise amount of gold to the contractor. At the appointed time the latter delivered to the king's satisfaction an exquisitely finished piece of handiwork, and it appeared that in weight the crown corresponded precisely to what the gold had weighed. But afterwards a charge was made that gold had been abstracted and an equivalent weight of silver had been added in the manufacture of the crown. Hiero, thinking it an outrage that he had been tricked, and yet not knowing how to detect the theft, requested Archimedes to consider the matter. The latter, while the case was still on his mind, happened to go to the bath, and on getting into a tub observed that the more his body sank into it the more water ran out over the tub. As this pointed out the way to explain the case in question, he jumped out of the tub and rushed home naked, crying with a loud voice that he had found what he was seeking; for he as he ran he shouted repeatedly in Greek,* ***«Εὕρηκα, εὕρηκα!»*** Vitruvius says. According to him, Archimedes *"he made two masses of the same weight as the crown, one of gold and the other of silver. After making them he filled a large vessel with water to the very brim, and dropped the mass of silver into it. As much water ran out as was equal in bulk to that of the silver sunk in the vessel. Then, taking out the mass, he poured back the lost quantity of water, using a pint measure, until it was level with the brim as it had been before. Thus he found the weight of silver corresponding to a definite quantity of water. After this experiment, he likewise dropped the mass of gold into the full vessel and, on taking it out and measuring as before, found that not so much water was lost, but a smaller quantity: namely, as much less as a mass of gold lacks in bulk compared to a mass of silver of the same weight. Finally, filling the vessel again and dropping the crown itself into the same quantity of water, he found that more water ran over the crown than for the mass of gold of the same weight. Hence, reasoning from the fact that more water was lost in the case of the crown than in that of the mass, he detected the mixing of silver with the gold, and made the theft of the contractor perfectly clear."* (12) According to an eminent American scientist, *"Vitruvius' method compares the volume (V_c) of a date weight (P_c) crown with the volume (V_o) of an equal weight (P_c) of gold and with the volume (V_a) of the same weight (P_c) of silver. So the relationship between the unknown weights of gold (P_o) and silver (P_a) is immediately given (knowing their sum P_c) by the following proportionality:"* (13)

$$\frac{P_o}{P_a} = \frac{V_a - V_c}{V_c - V_o} \qquad (0)$$

This «volumetric method» is described, though in a generic way, in the anonymous *Carmen de ponderibus et mensuris*, where also the «method hydrostatic» appears, about which we're going to speak largely in the next section. Here appears then *"the definition of specific gravity (maybe the first, surely the plainest among the descriptions of density contained in the most ancient Latin works)."* (13) This method is also further developed in *De insidentibus in humidum* by the Pseudo Archimedes and in *Quadripartitum numerorum* by Jean de Murs. Commenting to this work, where the Parisian astronomer applied the volumetric method to the wreath problem, an eminent studious of mechanical medieval science warned readers about this method, because on his opinion it was founded on the premise *"that the volume of mixed materials is equal to the sum of the volumes of their components,* [while] *in mixtures (alloys included) there are often volumetric variations; therefore the assumption above is often a not very careful approximation"* (13) Anyhow, Vitruvius' version soon appeared to be suspect and, in Galilee's words, *"a crude thing, far from scientific precision; and it will seem even more so to those who have read and understood the very subtle inventions of this divine man in his own writings; from which one most clearly realizes how inferior all other minds are to Archimedes's and what small hope is left to anyone of ever discovering things similar to his* [discoveries]*".* (14) So, starting from late ancient times, and then also during the Middle Ages, it was resolved that this reconstruction «in the manner of Vitruvius» was not based at all *«On Floating Bodies»* and soon begun to spread alternative reconstructions, largely based on hydrostatic principles. Before inspecting these alternative ideas, it would be appropriate to verify if the relation (0) is really so far from the «buoyancy». It will be anyhow interesting to discuss, being it an «exact» relationship, how it could be obtained. Granted that, according to Frajese, *"there's not, in Archimedes, a term which literally corresponds to our «specific weight» or «density» but* [that] *the concept is with no doubt present* [...] *almost as in ours."* (4) Therefore, if the crown had really contained some silver mixed to gold, making a comparison in the water between it and a one with the same weight (P_c) in gold, because of the difference between the specific gravity of gold (γ_o) and the one of silver (γ_a), there'd have been a difference between the volume of crown (V_c) and the volume of gold (V_o). If we made a comparison with the volume of an equal weight in gold (P_c), the crown's volume would be:

$$V_c = \frac{P_a}{\gamma_a} + \frac{P_c - P_a}{\gamma_o} = \frac{\gamma_o \cdot P_a + \gamma_a \cdot (P_c - P_a)}{\gamma_a \cdot \gamma_o} \tag{1}$$

where we see the unknown amount of silver (P_a), while the difference between the volume of the crown and the one of the equal weight (P_c) in gold would be:

$$V_c - V_o = \frac{\gamma_o \cdot P_c + (\gamma_a - \gamma_o) \cdot P_a}{\gamma_o \cdot \gamma_a} - \frac{P_c}{\gamma_o} = \frac{(\gamma_o - \gamma_a) \cdot P_c + (\gamma_a - \gamma_o) \cdot P_a}{\gamma_a \cdot \gamma_o} \qquad (2)$$

And if, instead, the crown should be compared in the water with an equal weight (P_c) in silver, its volume would be expressed by this formula:

$$V_c = \frac{P_o}{\gamma_o} + \frac{P_c - P_o}{\gamma_a} = \frac{\gamma_a \cdot P_o + \gamma_o \cdot (P_c - P_o)}{\gamma_o \cdot \gamma_a} \qquad (3)$$

and we'd find a difference between the volume of the crown and the one of the equal weight (P_c) in silver, which is

$$V_a - V_c = \frac{P_c}{\gamma_a} - \frac{\gamma_a \cdot P_c + (\gamma_o - \gamma_a) \cdot P_o}{\gamma_o \cdot \gamma_a} = \frac{(\gamma_o - \gamma_a) \cdot P_c + (\gamma_a - \gamma_o) \cdot P_o}{\gamma_o \cdot \gamma_a} \qquad (4)$$

where we find, instead, the unknown amount of gold (P_o). If we divide the expression (4) with the expression (2), and we simplify some terms, we find the same expression we had at the beginning (0).

$$\frac{P_o}{P_a} = \frac{V_a - V_c}{V_c - V_o} \qquad (0)$$

In the process above, we can notice that, arranging the terms of the former relationship, we obtain anyway:

$$\frac{V_a - V_c}{P_o} = \frac{V_c - V_o}{P_a} \approx \frac{1}{23{,}3} \qquad (5)$$

a constant value that's exactly equal to the difference in terms of weight in water between the unit weight in gold and silver, that's to say:

$$\frac{1}{23{,}3} \approx \frac{1}{\gamma_a} - \frac{1}{\gamma_o} \qquad (6)$$

And this difference in weight between silver and gold is due exactly to the different «hydrostatic thrust» that they get from water, as a consequence of their specific gravity. It's so possible to gather that, at least in principle, the reconstruction, «in the manner of Vitruvius» wasn't extraneous to the spirit of Archimedes. It's necessary, anyway, to warn that all the former reconstruction is a guess, because it has been run with actual logics and notes. For a Greek scientist, in fact, the relationship between different amounts as weight and volume wouldn't have had any meaning. Furthermore, a proof by Archimedes would be based on the «theory of proportions» among amounts of the same species and would have been developed through the «method of exhaustion». Making anyhow the hypothesis (taken from an exercise of an actual manual of physics) that the wreath would weight 5 kg and that it was made of gold (70%) and of silver (30%), the differences in terms of volume would have been expressed in deciliters, and so surely detectable by Archimedes who, among his many inventions, was also an improver of a water-clock. Sure, if we consider that the wreath for a «big head» had to be much larger than the little blocks of silver and gold of equal weight, the experiment could have been realized only in a vase of 20 cm diameter. In this case the differences in level would be surely narrow, but not paltry. It is suggestive to think, *"the great discoverer of volumes and areas determined with mathematical accuracy, is here forced to deal with this problem, using the practical measurement (necessarily imprecise)* [...] *of the amount of water displaced."* (4)

5. IN THE MANNER OF PRISCIAN

We resume the words of an authoritative Italian scientist to introduce another way to solve this crown problem, here improperly called «in the manner of Priscian». *"The general process, as Vitruvius writes it down, is not considered to be the one the great scientist from Syracuse used, and it's completely different the story we can read in a poem which was for a long time ascribed to Priscian; in this freely translated version we read that Archimedes took a pound of gold, and one of silver, and he put them on the plates of a balance, where they were of course in equilibrium; then he dipped them into the water, but as they lost their equilibrium for the overflow of gold, he decided to add some weight of silver, for example three drachms, to restore it, and from this he noticed that one pound and three dramms of silver equalized one pound of gold when they were in the water. After this, he weighted the crown, which had to be completely made of gold, and when he discovered that it weighted, for instance, six pounds, he took six more pounds of silver, and put them on the balance together with the first ones, dipping all of them into the water. If the crown had been* *really completely made of gold, eighteen drachms of silver, added to the former six, should have been enough to put the plates in equilibrium, but any drachm less than the eighteen proofed the existence in the crown of one third of pound of silver"* (15) The process mentioned above was related in the anonymous *Carmen de ponderibus* dated V century AC, which is present in several codes by the Latin grammarian Priscian. (13) In a section of the short poem we find two methods to solve the wreath problem, the former is essentially based on the principle by Archimedes. The technique is symbolically expressed by the following formula:

$$\frac{P_o}{P_a} = \frac{\sigma_a - \sigma_c}{\sigma_c - \sigma_o} \tag{7}$$

where can be noticed the losses of weight in water respectively of the crown (σ_c), of an equal weight in gold (σ_o) of the same weight in silver (σ_a), caused by the different push that different objects get from the water. The first «modern» presentation on this method, entirely based on the laws of lever and of floating, is related in the book *Magia naturalis* published in London by the Italian scholar Giambattista Della Porta. About in the same years also Galileo strongly criticized the reconstruction made by Vitruvius and, in his juvenile work called «La bilancetta», exposed *"a method came to my mind which very accurately solves our problem. I think it probable that this method is the same that Archimedes followed, since, besides being very accurate, it is based on demonstrations found by Archimedes himself."* (14) Also the author of a beautiful children's book dedicated to the genius from Syracuse (9) opts for the same version, explaining later the reasons which brought him to this choice: *"(1) I don't think the volumetric measure can be performed with great precision.* [...] *If the wreath has a complex structure (that's to say a high ratio surface/volume, how it seems to be likely), it needs to be dipped in a vase which has a much larger volume than hers, and consequently with a wide surface, which makes very small the height difference to be compared and consequently increases more and more the chance of a mistake.* [...] *In any case, in order to compare the overflowed waters, it would be better to weigh them. Why, then, don't we use a weight approach? (The mistake in measuring here, unlike volumetric measures, doesn't depend on the dimension of the vase where the objects are dipped; this seems to me the point!)* [...] *(3) Vitruvius presents the episode when he writes about Archimedes hydrostatics. The volume measures don't have, in this case, any connection with the object of discussion, while the measures done with the «hydrostatic balance» would be a perfect introduction to it. All that lets us think that Vitruvius source exposed a measure based on a hydrostatic push* *and that Vitruvius thought to «simplify» the argument (ignoring the precision of the measure itself)."* [16] If we wanted, anyhow, subject the same wreath of the former example (5 kg) to a control, using the above-mentioned method, we could have noticed a difference in weight of about 65 g. A clue definitely more important and much easier to grasp.

6. CONCLUSION

And finally, «pycnometer», «hydrostatic balance»? Currently this episode goes on circulating in both versions and with a certain confusion, as if they were antithetical one another. It's instead proved that both reconstructions are based on the same principles: the former measuring volume differences, the latter weight ones. In short, let's consider that between the two wreaths really found, similar to Hiero's and belonging to the same age, none is heavier than 1 kg. The golden wreath of Verginia, in Macedonia (IV century BC), for example, is just 700 g heavy. If we made a comparison between a 1 kg wreath, and a sample made of gold of equal weight, the «volumetric method» would show a difference of half a millimeter, while the «hydrostatic method» a difference of 13 g. This comparison allows us to say that the «hydrostatic method» is more careful and the easiest to be carried out. It seems therefore the most plausible but there's no chance to decide surely which was the process Archimedes really used, supposing that the episode called «The Golden Crown» is really reliable. This writer likes to think that this anecdote really happened and that, as many time before (for example the *Quadrature of the Parabola,* which was obtained and with a geometrical process, and with a mechanical one) Archimedes came to this result in a double way: the first time with a «volumetric method», the second one with a «hydrostatic method». There are no proofs of this, but it's nice to think that a careful scientist can find, one day, under some miniature, the traces of another work, which, in the Doric Greek of Archimedes, reveals us some other aspects of his wonderful genius.

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THE HERITAGE OF ARCHIMEDES IN SHIP HYDROSTATICS: 2000 YEARS FROM THEORIES TO APPLICATIONS

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ABSTRACT Archimedes left to posterity his famous treatise "On Floating Bodies", which establishes the physical foundations for the floatability and stability of ships and other maritime objects. Yet since this treatise was long lost and also simply ignored by practitioners, it took many centuries before Archimedes' brilliant insights were actually applied in ship design and ship safety assessment. This article traces the tedious acceptance of Archimedes' principles of hydrostatics and stability in practical applications. It will document important milestones and explain how this knowledge was passed down through the centuries and ultimately spread into ship design practice.

1. INTRODUCTION

Archimedes (ca. 287–212 B.C.) in his famous treatise "On Floating Bodies" [1, 2] laid the foundations of hydrostatics, especially for the equilibrium and stability of objects floating on the surface of a liquid or immersed in a liquid medium. Evidently his principles and brilliant theories are immediately applicable to ships and can thus form the basis of ship hydrostatics. These fundamental principles are apt to play a crucial role in ship design and ship safety assessment. Yet this knowledge from his treatise did not spread very far in Archimedes' lifetime and was lost or ignored by practitioners for more than a millennium until it was rediscovered many centuries later during the late Middle Ages. It then still took until the 18th c. before the theoretical principles established by Archimedes were actually applied in ship design and stability assessment. Why did this long delay occur?

This article will examine the long history of ship hydrostatics from Archimedes to the modern era and will document the most important milestones in this development. It will follow the route of knowledge

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transfer from its classical origins to current practical applications in ship design and operations. It will also discuss other prerequisites for making the physical principles of hydrostatics applicable to practical applications in ship design and stability. Overall a critical mass of knowledge has to be brought together in order to raise the theoretical knowledge to maturity for applications in practice. The following subjects are essential elements of knowledge for an adequate solution:

- Physical principles of hydrostatics for the equilibrium and stability of floating objects.
- Ship hull geometry definition and representation in some reliable medium, preferably at the design stage.
- Evaluation of ship geometry data by numerical calculation.
- Stability criteria and risk evaluation.

Archimedes firmly established the physical principles and took the opening moves in the other related topics. But it still took a long time before all other ingredients had reached sufficient maturity for actual application. We will describe this arduous road.

This article does not claim to present a complete history of ship hydrostatics. Rather it focusses on how the ingenious ideas of Archimedes were passed down to posterity, were lost and resurrected again, and then supplemented by other fundamental knowledge until they found their application in ship design, which Archimedes perhaps foresaw and which we take for granted today.

2. THEORIES

Archimedes

Precursors: Greek mathematics and Mediterranean shipbuilding, especially also in classical Greece, had reached an advanced level before Archimedes, on which he based his original achievements in the 3rd c. B.C. This background material which should be studied to appreciate the magnitude of his creative contributions can be found in the literature (e.g., Heath [3]), Nowacki [4]).

Force Equilibrium: Buoyancy and Displacement: Archimedes in his famous treatise “On Floating Bodies” (OFB) pronounced the fundamental laws of hydrostatics, i.e., the physical laws of equilibrium for bodies floating in a liquid at rest. Book I deals with the force equilibrium between buoyancy and displacement forces and contains the Principle of Archimedes, which holds for bodies of any shape. Book II treats the moment equilibrium and pertains to the stability of the floating condition, derived for the special case of a paraboloid of revolution. Indirectly hereby Archimedes also laid the foundations for ship hydrostatics since his approach is immediately applicable to ships, even if he did not mention ships anywhere in his treatise.

How did Archimedes arrive at his Principle of Hydrostatics? This is described in his own words in this treatise [1, 2]. He makes two essential axiomatic assumptions.

1. In Book I, preamble he states the properties of the liquid (Heath [1]):

"Let it be supposed that a liquid is of such character that its parts lying evenly and being continuous, that part which is thrust the less is driven along by that part which is thrust the more; and that each of its parts is thrust by the liquid which is perpendicularly above it…"

These lines infer a homogeneous, isotropic liquid whose parts are at rest when in equilibrium. Although the Greeks did not know the concept of pressure, the idea of a hydrostatic pressure distribution is implied here between the lines.

2. In Book I, §5 Archimedes postulates his Principle as follows (Heath [1]):

"Any solid lighter than the liquid will, if placed in the liquid, be so far immersed that the weight of the solid will be equal to the weight of the liquid displaced".

The proof is illustrated in Fig. 1 and described in more detail in Nowacki [4]. The surface of any liquid at rest is a spherical surface whose center point is at the center of the earth (section ALMND). The body EZTH be specifically lighter than the liquid. In two equal adjacent sectors of the liquid at rest the body EZTH floats in equilibrium on the surface such that its submerged volume BCTH is equal to volume RYCS in the neighboring sector. Since in equilibrium the total weight of the masses in each sector must be equal, the weight of the floating body EZTH must be the same as that of the volume of RYCS, hence must also be equal to the weight of the liquid volume it displaces.

Note that this elegant proof of the Principle of Archimedes (buoyancy force is equal and opposite to gravity force or displacement) is based entirely on an experiment of thought. The proof is entirely deductive from a few axioms regarding the liquid properties, no observations are required. It holds for floating bodies of arbitrary shape in an arbitrary type of liquid and was derived for liquids at rest without explicit knowledge of local pressure anywhere. Buoyancy and displacement are force resultants, which in equilibrium are equal in magnitude and opposite in direction. This proof is an outstanding example of Greek logical thought and of the brilliance of Archimedes.

Archimedes has also shown that this Principle holds for a fully immersed object of equal specific weight as the liquid (neutral force equilibrium), but does not apply when the solid is heavier than the liquid because the object then is grounded and loses as much weight as the displaced volume weighs, the rest of the weight is taken up by the grounding support force (Book I, §7).

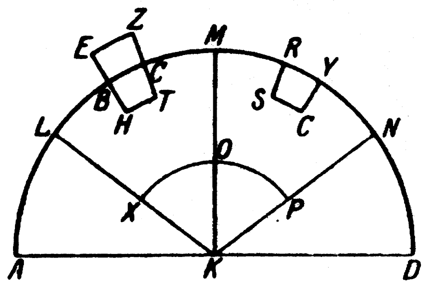

Fig. 1. Proof of Archimedes' Principle (from [2]).

<u>The Eureka Legend:</u> This evidence sheds some special light on the famous "Eureka" legend, as reported by Vitruvius [5], Book IX.3. According to this account Archimedes was challenged by king Hieron of Syracuse to determine whether a wreath, made for the king by a goldsmith for a sacrificial offering, was of pure gold or fraudulently made of gold mixed with silver. Archimedes is said to have sat in a brimful bathtub when he discovered a method to measure the volume his body displaced in the water: After leaving the tub he could fill up the water to the brim again with a measured volume of water. He was elated at this discovery and spontaneously ran through the streets of Syracuse nakedly shouting "Eureka" because he had found a method to prove the fraud. Archimedes went on to sink the wreath and two equally heavy pieces of pure silver and gold each in a bowl full of liquid to the brim, then after removing each object to refill the bowls with a measured volume of liquid. Then since the weights were known, the different volumes gave an indication of the different densities of the objects and the fraud was revealed.

Thus Archimedes thereby discovered a method for measuring the volumes of solid objects and, if their weights are known, their relative densities. But in this bathtub experiment he did not discover the law of equality of buoyancy and displacement, hence the principle of hydrostatics, as is sometimes falsely claimed. This law does not hold there because the human body in the tub will usually touch the ground and the ground force must be taken into account (Fig. 2).

Moment Equilibrium: Hydrostatic Stability: In Book II of OFB Archimedes deals with the moment equilibrium of a floating solid paraboloid of revolution when inclined from an initially upright position. Thereby he derives the righting moments of the inclined solid which he uses as a stability criterion: The equilibrium is stable, if - in the absence of any heeling moments – the inclined object restores itself to its upright position. How does Archimedes determine the righting moment in this case?

Fig. 2. Archimedes in the Bathtub.

First he makes the same assumptions regarding the properties of the liquid as in Book I. The liquid domain is again unbounded, the liquid is at rest. Then the floating object, initially at rest in an upright position, is inclined by a certain, finite angle, but so that the base of the paraboloid is not wetted (Fig. 3). The homogeneous paraboloid segment is cut off perpendicularly to its axis, the paraboloid segment axis length is not greater than 1.5 times its half-parameter. For this case Archimedes demonstrates that the righting moments are positive.

The actual proof applies several mechanical and geometrical principles, deduced in this treatise or derived by Archimedes in his earlier work (for details see Nowacki [4]): For the inclined paraboloid he disregards the underwater part under the water surface JS because its buoyancy and gravity forces are equal and opposite for the homogeneous solid and thus produce no moments. For the abovewater part he proves (by means of his centroid shift theorem [4]) that the vertical gravity force through its centroid C is equal to the incremental buoyancy force, due to the inclination, through B, but opposite in direction so that they form a couple or righting moment, tending to bring the body back to the upright position. Thus this shape for a solid of this specific weight is in stable equilibrium.

Although this derivation holds only for the homogeneous solid paraboloid and is limited to the incremental righting moments contributed by the immersed and abovewater parts, respectively, it can be shown that a similar reasoning can be developed for a solid of any shape and with nonhomogeneous mass distribution, hence also for ships. The lever arm between the buoyancy and displacement forces in the paraboloid is thus the ancestor of the "righting arm", which today is conventionally used for the same couple of forces in modern ship stability analysis. Positive righting arms are a necessary condition of upright stability.

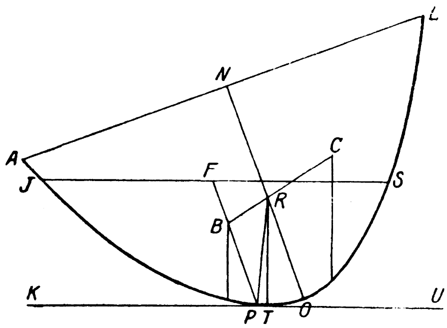

Fig. 3. Inclined Paraboloid (from [6]).

Achievements and Deficits: Archimedes thus laid the physical foundations for ship hydrostatics. He defined the resultants of buoyancy and displacement and pronounced the equilibrium principle of their equality in the same line of action and in opposite directions. From moment equilibrium he deduced a measure of hydrostatic stability by introducing the concept of righting moments based on the couple of buoyancy and displacement. This has remained the physical basis for judging the floating ability and stability of ships in design and operations. To evaluate ship properties at the design stage and during ship operations some further information is required:

- A reliable, complete *hull form definition*, in whatever medium (mould, model, drawing etc.).
- A method to calculate the volume and volume centroid of the underwater hull (center of buoyancy), for both the upright and inclined positions.
- A practical scheme to determine the centers of gravity of the ship's parts and therefrom the aggregate center of gravity of the entire ship.

– Criteria to assess the required safety margins in ship stability for different operating conditions and environments.

According to the available historical evidence Archimedes was not yet able to meet these additional requirements. Thus in the practice of ship design for many centuries the estimate of the ship's floating condition (draft and trim) and stability remained a matter of empirical judgment and approximation.

As for the calculation of volumes and their centroids Archimedes frequently applied the method of Eudoxus (410–356 B.C.), later known as the method of exhaustion (Boyer [7]), to simple shapes. In this scheme a polygonal approximant to the curved surface (or curve) is constructed and successively refined until the error drops below a given bound. The approximation becomes as close as desired, but falls short of calculus for lack of a limiting process to the infinitesimal [7]. Despite that, a numerical approximation for ship geometries might have been constructed on similar grounds, even in antiquity. But a continuous, arbitrarily refinable hull form definition was not available to Archimedes and his generation.

Some claim that Archimedes may have been involved in the conception of the famous *Syrakosia,* the giant ship ordered by Hieron, the ruler of Syracuse, or may have helped with engineering calculations, as e.g., suggested by Pomey and Tchernia in [8]. Bonino [9] has performed a thorough reconstruction or redesign of the vessel, based on the limited data, and arrives at a size of ca. 3000 tons of displacement and principal dimensions of length x beam x draft = 80x15.5x3.9 m. He has also built a realistic model replica illustrating the feasibility of this design. He concludes from the overall context of the shipbuilding methodologies of that period that Archimedes was not directly involved in any responsible design decisions although he may have acted as a consultant and advisor to Archias and Hieron. I share this cautious opinion.

History of Archimedes' Manuscripts

Today only 12 of Archimedes' treatises are preserved, several more existed in antiquity. They stem from Greek copies of his manuscripts and Latin translations. The adventurous history of these texts has been thoroughly researched by Heiberg [10] and updated by Dijksterhuis [11]. Clagett [12] has carefully examined the mediaeval reception of Archimedes. The story of a recent rediscovery of a palimpsest with Archimedes' texts is told by Netz and Noel [13]. This short survey will concentrate on events relevant to the history of "On Floating Bodies" (OFB). Much more detail is given by Nowacki [4].

The path of the preserved manuscript copies has followed a circuitous route. In essence only three master copies in Greek have existed, all stemming from mediaeval Byzantine sources in Constantinople, where Greek clerics collected the remainders of Archimedes' dispersed works in the 9th c. and later took some along into exile to Sicily under Norman and Hohenstaufen rule. They were numbered Codices A, B and C by Heiberg [10].

Codex A: In the papal libraries after 1266 with seven treatises by Archimedes, *but not OFB.* Copied several times, but the master irretrievably lost by 1564.

Codex B: A Latin translation of 1269 prepared by Willem van Moerbeke, a Flemish Dominican monk and papal translator. This translation was based in part on Codex A, in part on another Greek master *with OFB*, then existing in the papal archives, but soon lost. Thus Codex B contains OFB in Latin. This text formed the basis of several Renaissance humanist reprints after 1500, above all a brilliant reconstruction by Commandino [14] (1565) with both books of OFB. Commandino purged the text of apparent errors, removed some lacunae and completed missing arguments in proofs. This version became the most respected reference after the Renaissance (Clagett [12]). After 1600 many other editions followed in Greek, Latin and modern languages (Dijksterhuis [11]).

Codex C: Incredibly, a third Greek master copy was discovered in a Greek monastery in Constantinople in 1899 in a palimpsest, which contained the rinsed off and scraped off Archimedean text under a 13th c. prayer book, but still barely legible under a magnifying glass. This document was inspected, photographed, transcribed and immediately translated by Heiberg [15]. It did contain the only preserved Greek versions of both books of OFB. Codex C was lost during the Greek-Turkish wars in 1920–22, but resurfaced at an auction in New York in 1998, where the anonymous bidder who acquired it gave it to the Walters Art Museum in Baltimore for scientific reevaluation [13].

Thus OFB was accessible to scientists in Latin and increasingly in modern language translations since about 1600 and in the original Greek transcription by Heiberg [15] since 1907.

3. TOWARD APPLICATIONS

Late Antiquity and the Middle Ages

While the knowledge of Archimedes in hydrostatics lay dormant for at least a millennium in late antiquity and the early Middle Ages, shipbuilding technology did advance and underwent significant changes. Little specific is known about ship design methodology in antiquity, though it is evident in view of the complexity of some major shipbuilding projects that methods of advance planning and design must have existed (Pomey [16]). Archaeological sources from late antiquity show traces of prevailing practices, and later excavations of ship wrecks give indications of some basic reorientation in shipbuilding and ship design in the transition to the Middle Ages. This includes:

Marking: Bockius [18] excavated several Roman shipwrecks dating from the 4th c. A.D., which lay buried in the silt of the harbour and river-bed of the Rhine at Mainz, evidently river patrol boats of the Roman occupation period. He searched for traces of the shipbuildung process and found several transverse grooves on the inside of the keel planks, each sawed about 3 mm wide and arranged in uniform distances, as well as the remains of treenails or wooden pegs in the keel and side planking, carefully aligned in the same transverse planes. Since the hulls were built planks-first, he interpreted these findings as evidence of an assembly process, i.e., as layout markings and attachment points for template fixtures to hold the planks in place during assembly, but later to be removed to make room for transverse ribs as passive frame reinforcements in the same planes. This suggests that the idea of shape predefinition in transverse planes may have already existed in plank-first shipbuilding.

Skeleton-first assembly: Rieth [19] carefully describes the archaeological evidence for the important transition from plank-first to skeleton-first shipbuilding in the Mediterranean countries occurring during the 7th c. Here a skeleton of structurally active frames was erected in numerous transverse planes before the planking of the outside shell was attached to it. This necessitated a reorientation of the hull shape design process defining the desired shape in terms of planar transverse sections.

Moulding and lofting: While the use of templates or moulds for defining the shape of individual planar ship parts may be ancient, the use of unique master moulds, say, for the midship frame, from which all other transverse section shapes can be derived by a lofting process, was a new idea, apparently introduced in France just before 1300 (Rieth [20]). The individual section shapes at any longitudinal station of the ship can thus be deduced from the master mould (in French: Maître gabarit) by a transformation consisting of translation, rotation and clipping of shape elements (Nowacki [21]). Thus the shape of a single curve is sufficient to define the hull surface continuously at any desired point (except for the ship ends). This opens the door to the required volume and centroid calculations for ship hydrostatics.

Venice and the Italian Renaissance

During the Middle Ages and Renaissance Venice was a leading sea power in the Mediterranean and also a productive shipbuilding center, well-known for its Arsenal where many famous galleys were built. The contemporary written records on this shipbuilding activity are scarce. The earliest preserved documents stem from Michael of Rhodes (ca. 1435, McGee [22]) and Trombetta de Modon (ca. 1445). These are technical notebooks, written chiefly for specialists, with many illustrations, but little text. It was only much later by Drachio (1598, [23]) that explanations and commentaries were added that helped to understand this technology. In essence the Venetians had their own moulding and lofting techniques, based on a master mould (sesto) and rules for deducing section shapes at any desired longitudinal station (Alertz [24]).

The methods of lofting for ship parts were similar to those applied in other Mediterranean yards in Italy, France and Spain. They ensured a unique definition of hull shape and efficient fabrication of ship parts, allowing room for shape variation. The written sources deal chiefly with ship geometry, but do not make reference to design calculations, let alone to any thoughts from Archimedes in OFB.

This is disappointing since it was essentially during the same period that Italian humanists rediscovered OFB and made access to this classical knowledge feasible again. Van Moerbeke's translation (1269) and Commandino's brilliant revision [14] (Venice, 1565) were already mentioned. There are other indications that the ideas of buoyancy and displacement were at least intuitively known. Alberti, e.g., the famous Renaissance architect and writer, in one of his main works "De re aedificatoria" (ca. 1450), Book V, Ch. 12, alludes to his knowledge of the equality of buoyancy and displacement, at least for the cargo carrying capacity as an increment, though without giving any source. Leonardo likewise knew certain fragments of Archimedean thought. But in both cases they may also have run across some popularized pseudo-Archimedean text that was around since the 13th c. (Clagett [12]).

The Treatisers

Toward the end of the 16th and throughout the 17th c. a tradition developed in all major European seafaring nations to document the existing and evolving shipbuilding knowledge, whether practical or more theoretical, in more or less learned treatises for diverse purposes. The authors are often called treatisers. The treatises served as technical notebooks for insiders, as basic introductory texts for the general public or for the shipping community or even just as an opportunity to display scientific and technical excellence. The authors came from shipbuilding practice or from some scientific background or were knowledgeable in both aspects. In view of the rapid transitions in Europe to new technologies and methodologies during this very period the treatises are most valuable as contemporary sources on the intensive changes in practical and scientific knowhow. We will take a short survey of the major sources in our search for traces of Archimedean heritage. See Barker [25] for a more detailed overview. We will note the dates of treatise appearance in parentheses.

Portugal, riding on the wave of success of the age of exploration and possessing a strong position in shipbuilding, was also among the first to produce naval treatises. Pedro Nunes, a scientist, studied the theory of rowing ("O Comentario de Pedro Nunes à Navegação a Remos", 1566) and criticized the errors in Aristotle's approach in Problemata Mechanika (wrong conclusions from the law of the lever). Such publications challenged the scholastic dominance of Aristotle and paved the way for Archimedean thought.

On the practical side the shipbuilding treatises by Oliveira ("Ars nautica", 1570, "Livro da Fabrica das Naos", 1580), Lavanha ("Livro primeiro da architectura naval" [26], 1614–1616) and Fernandes ("Livro da Traças de Carpinteria", 1616) deserve to be noted. They deal essentially with ship geometry, moulding rules and ship construction. Lavanha cites Vitruvius and Alberti as precursors and raises the naval architect to comparable rank as the famous architects. He develops precise ship drawings and sketches. Fernandes already presents a rudimentary ship lines plan. These Portuguese sources contain no hydrostatic calculations or references to Archimedes.

In England William Bourne ("Treasure for Travaylers", 1578), one of the first treatisers there, already explains how to obtain a ship's volume estimate by taking its offsets when on dry ground by means of measuring rods relative to some suitable reference plane on the outside of the hull and up to the desired waterline. The offsets are then connected by linear approximants for estimating cross-sectional areas and likewise linearly volumes of ship segments between measured stations. In the end a reasonably rough volume estimate is obtained to which the Principle of Archimedes is applied to derive the ship's weight (or displacement) on that draft.

Other famous early treatisers (Mathew Baker/ John Wells: "Fragments of Early English Shipwrightry", 1570–1627, see Barker [27]; R. Dudley: "Arcano del Mare", 1646; E. Bushnell: "The Compleat Ship-Wright", 1664) deal chiefly with ship geometry, moulding methods and ship drawings up to first lines plans on paper. But they did not yet enter into Archimedean style calculations. However Anthony Deane ("Deane's Doctrine of Naval Architecture", 1670) resumed the subject of volume estimates by approximate planimetry of section areas, using circular arc or triangular approximants, and segment volume calculation. He thus obtained the ship's buoyancy force (= displacement) according to Archimedes for any desired draft. One motivation apparently was to provide enough freeboard to keep the gun-ports above water.

In Germany Joseph Furttenbach, a well-known architect and writer from Ulm, had traveled much in Italy as a young man and had picked up the basic naval architecture knowledge there. His treatise "Architectura navalis" (1629) concentrates on matters of ship geometry, ship construction and ship types, showing strong Italian influence, but not on hydrostatic calculations. His work remained rather solitary in Germany.

Early French treatisers (Fournier: "Hydrographie", 1643, Pardies: "La Statique ou la Science des Forces Mouvantes", 1673) were interested in nautical matters for textbooks in seamanship. It was actually the Jesuit Père Paul Hoste who first took on the challenge of calculating the displacement from lines plans (or offset measurement not unlike Bourne) and of defining a measure of ship stability on hydrostatic grounds, based on Aristotle and Archimedes. Unfortunately his stability analysis failed because he misinterpreted Archimedes' derivation and missed the effects of the shift of volume centroids by heeling inclination.

In the Netherlands, based on the pioneering work by Simon Stevin (1548–1620) to be discussed later, there existed an early understanding among practitioners for the principles of hydrostatics stemming from Archimedes. The Dutch mathematician Johannes Hudde (1628–1704) had proposed a method (1652), later called the difference-in-drafts method, for measuring the cargo payload (or tonnage) by taking the difference between the ship's displacement fully loaded minus empty. Offsets were taken in both floating conditions and the volume of the layer between the two waterlines was estimated numerically by means of trapezoids and triangles. The volume of this layer was converted to weight by Archimedes' Principle. In Britain Bushnell (1664) devised a similar technique.

Nicolaes Witsen (1641–1717) in his treatise [28] worked out a similar method (1671) in more detail, but also extended it to estimating the displacement for the whole hull. Certain details cast doubt on whether this method was ever practiced. Witsen also explicitly gives credit to Archimedes. For ship stability he follows Stevin, whose criterion was flawed (see below).

Cornelis van Yk in his treatise "De Nederlandsche Scheeps-Bouw-Konst Open Gestelt" (1697) cites Witsen , but as a practitioner has a more practical orientation. He pursues the method of difference–in-drafts for applications in tonnage measurement.

This short survey has been confined to traces of a growing understanding in Archimedean ship hydrostatics. Much more detail on the treatisers and their work is found in Barker [25] and Ferreiro [29].

In summary it is fair to state that by 1700 Archimedes' texts were known to scientists, but very little of his knowledge had found its way into ship applications. As for ship stability his criterion was not yet properly understood, let alone practiced in ship design or operations.

The Rebirth of Hydrostatics: Stevin, Galileo, Huygens

During the 17th c. the scientific discipline of hydrostatics was virtually reborn in a modern reincarnation. Although access to Archimedes' texts had much improved by 1600 so that scientists were able to study him literally, it took a number of very creative thinkers and physicists to reinvent hydrostatics and hydrostatic stability on new fundamental grounds and to apply it to their own new applications. Such prominent scientists as Stevin, Galileo, Huygens and Pascal made important contributions to this rebirth.

Simon Stevin (1548–1620), the famous Flemish/Dutch mechanician, astronomer and hydraulic engineer, worked on several fundamental problems of mechanics and also reestablished hydrostatics. He introduced the concept of hydrostatic pressure, which the Greeks had not known, and thus was able to determine hydraulic loads acting on submerged surfaces. He axiomatically developed a body of propositions embracing the whole of hydrostatics in his treatise "The Elements of Hydrostatics" [30] (1586 in Dutch, 1608 in Latin translation). His premises are tantamount to the Archimedean properties of the fluid. In a fluid at rest the hydrostatic pressure increases linearly with depth in proportion to the specific weight of the fluid. This was a brilliant breakthrough. Stevin also dealt with the stability of ships in his supplement "On the Floating Top-Heaviness", attached to [31], 1608. He had read Archimedes and praised him. But he had not fully understood the implications of the hydrostatic stability criterion so that he missed the influence upon stability of the volume shift from the emerging to the immersed side of the heeling ship, a stabilizing effect. Consequently he came to the erroneous conclusion that the ship's center of gravity *must always* lie *below* the center of buoyancy for a stable ship. Actually this is a sufficient, but not a necessary condition for ship stability.

Galileo Galilei (1564–1642), famous as an astronomer and physicist, especially in mechanics and strength of materials, also occupied himself with hydrostatics and its applications, which is less widely known. In fact, in 1612 he published a treatise "Discourse on bodies in water" [32], which is explicitly founded on Archimedean thought and deals with the floating of bodies on the surface of the water. This work originated from a dispute with Aristotelian opponents in Florence about the causes of buoyancy of floating objects [33]. Galileo accepted the Archimedean axiom that a body floats on the water surface if it is specifically lighter than water, but denied the Archimedean Principle for the equilibrium position. Rather he soon drifted off beyond Archimedean hydrostatics by recurring to kinematic principles from his theory of motion. His main contribution from this dispute therefore must be considered to lie in refuting the false Aristotelian theory of buoyancy.

Christiaan Huygens (1629–1695), the famous physicist, is little known for his excursion into hydrostatic stability. He never published his three volume treatise “De iis quae liquido supernatant” [34], which he wrote in 1650 at the youthful age of 21, because he regarded it as incomplete, and later (1679?) as “of small usefulness, if any, although Archimedes in Book II of ‘On Floating Bodies’ spent work on not dissimilar topics”. Incidentally it is reported that Huygens used Commandino’s version of the Latin translation of Archimedes, based on codex B. He wanted most of this work of his to be burnt. The manuscript was found in his legacy and was first published in 1908 [34]. The modern reader is bound to admire Huygens’ deep insights into Archimedes’ work as much as his own creative extensions. Huygens rederived Archimedes’ results for the stability of the sphere and the paraboloid using his own method and he provided original solutions for floating cones, parallelepipeds and cylinders. He studied some of these solids through a full cycle of rotation. He recognized that for homogeneous solids their specific weight and their aspect ratio are the essential parameters of hydrostatic stability. In conclusion Huygens was the first modern physicist who understood and was able to apply and extend Archimedes’ theory of hydrostatic stability. He did not proceed to apply his theory to ships or similar floating objects because he did not possess a suitable definition of ship geometry, a final obstacle.

Calculus

Archimedes skilfully and routinely used a method of geometrical proof in the derivation of areas, volumes and centroids of figures of simple given shapes, which later became known as the “method of exhaustion”, usually attributed to Eudoxus, a pupil of Plato (cf. Boyer [7]). Here a known curve is approximated by a regular polygon whose edges are subdivided successively, doubling the number of edges in each step, until the error between the curve and the polygon becomes as small as desired. After a finite number of steps the remaining error is estimated and the sum of the finite series with truncation error is taken. This proposed result is then confirmed by reductio ad absurdum of any differing assertions.

However the method of exhaustion is not equivalent to integration by infinitesimal calculus (cf. Boyer [7]). It is limited to a finite sequence of steps and relies on geometrical constructions for the proof. It cannot easily be extended to objects of arbitrary shape like ships. It does not comprise the limiting process of calculus. Calculus is based on the concept of an infinite series and derives its results analytically. Thus calculus can be applied to any analytically defined shape, hence also to ships of given arbitrary shape.

The invention of calculus had many precursors and contributors (cf. Boyer [7]). But it was the achievement of Newton and Leibniz to lay the foundations for consistent and procedurally well defined methods of calculus. These methods spread in Europe during the first few decades of the 18th c. Thus when the problems of ship hydrostatics after 1730 were revisited by two leading scientists, Bouguer and Euler, they had the mathematical tools at their disposal to reformulate the auxiliary quantities of areas, volumes and centroids in Archimedes' approach in terms of the elegant and definitive notation of calculus. Developments had now reached the stage where a reformulation of Archimedean hydrostatics as an application of continuum mechanics had become feasible and was at the threshhold of its application to ships.

Bouguer and Euler

After the advent of calculus and with the new concepts of analysis and of functions of one or several variables it became possible to review and restate many classical problems of mathematics and mechanics in new, original ways. For ship hydrostatics and stability the credit for a new, completely modernized approach, based for the first time on calculus, goes to two contemporary scientists, Pierre Bouguer (1698–1758) and Leonhard Euler (1707–1783), who worked on these problems separately, independently and without knowing of the other's work before their own large treatises were completed and ready to be published. Their original work can be well dated because they both participated in a prize contest held by the Parisian Academy of Sciences in 1727 on the optimum masting of sailing ships, where hydrostatics might have played a useful role to determine the equilibrium position of ships under sail. But they both failed to display any knowledge of Archimedean hydrostatics. However they continued to work on this issue during the 1730s, Bouguer essentially during a scientific expedition to the Andes in Peru from 1735 to 1744, Euler as a member of the Russian Academy of Sciences in St. Petersburg from 1737 to 1741.

Bouguer's famous "Traité du Navire" [35] appeared in 1746 soon after his return to France, Euler's fundamental "Scientia Navalis" [36] was published in 1749 after a major delay. But it is undisputed, also by both authors, that they had achieved their results independently and unaware of the other's parallel work (cf. Nowacki and Ferreiro [37]). Their results are essentially equivalent, though expressed in uniquely distinct ways, and have remained a valid basis for ship stability until today.

Bouguer in his pioneering treatise nowhere mentions Archimedes by name, but the spirit of his formulations leaves no doubt that he was familiar with Archimedes' work. e.g., he began his introduction of hydrostatics in Book II, section I, chapter I with this explanation of the buoyancy force:

"The principle of hydrostatics, which must serve as a rule in this whole matter and which one must always have in mind, is that a body that floats on top of a liquid is pushed upward by a force equal to the weight of the water or liquid whose space it occupies".

This is tantamount to the Principle of Archimedes, only slightly rephrased. In the following chapter the same result is also derived by integration of the hydrostatic pressure distribution over the submerged part of the hull surface. The pressure resultant or buoyancy force is then shown to be acting upward through the volume centroid of the submerged hull (or center of buoyancy), equal and opposite to the downward weight force (displacement) through the center of gravity of the hull.

For ship stability for infinitesimally small angles of heel (initial stability) Bouguer invented the metacenter as a stability criterion, i.e., the point of intersection of two infinitesimally adjacent buoyancy directions for a small angle of heel, the point g in Fig. 4. For a stable ship the center of gravity of the ship must not lie above the metacenter. This is a brilliant reinterpretation of Archimedes' stability measure for small angles of heel in terms of a geometric bound. Bouguer evaluated volumes and centroids for this measure by calculus and numerical approximation, also relying on Archimedes' centroid shift theorem.

Euler in the introduction to his "Scientia Navalis" pays full tribute to Archimedes. He begins his axiomatic foundation of hydrostatics with the statement:

"The pressure which the water exerts on a submerged body in specific points is normal to the body surface; and the force which any surface element sustains is equal to the weight of a vertical water column whose basis is equal to this element, whose height however equals the submergence of the element under the water surface".

All other results in ship hydrostatics can be derived from this axiom. e.g., the buoyancy force in the Principle of Archimedes is deduced by pressure integration by means of calculus over the hull of an arbitrary body shape. Euler also applies Archimedean criteria to the hydrostatic stability of ships for infinitesimal angles of heel when he says:

"The stability, which a body floating in water in an equilibrium position maintains, is measured by the moment of the restoring force if the body is inclined from its equilibrium position by a given infinitely small angle".

This stability criterion is formulated in terms of righting moments as by Archimedes, but unlike Bouguer. Physically the two formulations are equivalent. Euler calculates the righting moment taking into account the volume shift from the emerging to the immersed side and using Archimedes' centroid shift theorem. Figure 5 for an inclined cross section shows the stabilizing effect of this volume shift, caused by the couple of gravity force through G and buoyancy force through the new shifted center of buoyancy.

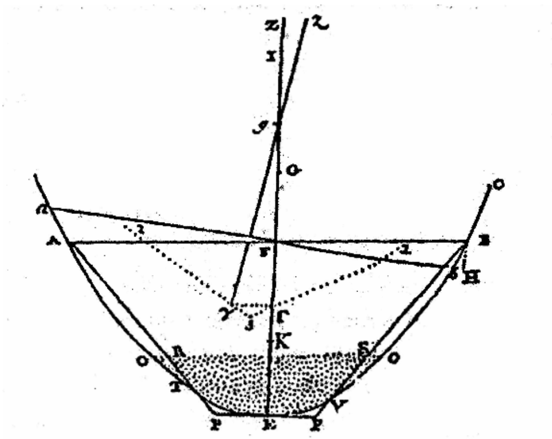

Fig. 4. Bouguer's Figure for the Derivation of the Metacenter (from [35]).

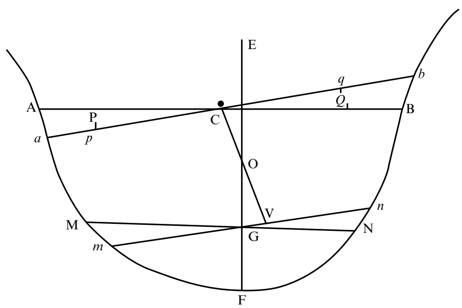

Fig. 5. Euler's Figure for Centroid Shift in An Inclined Cross Section (from [36]).

Note that both authors, Bouguer and Euler, have also addressed stability measures for finite angles of heel. Furthermore both also treated numerous other applications of ship stability in ship design and operations, e.g., ship loading and unloading, seaway motions, maneuvering under sail etc. which become amenable once the hydrostatic restoring reactions of the ship are known. Many more details about their achievements are presented by Ferreiro [29], especially on Bouguer, and by Nowacki [38], mainly on Euler.

Thus Bouguer and Euler have shown that the practical application of sability criteria to arbitrary ship shapes has been made possible by means of calculus formulations and their numerical evaluation.

After the publication of these fundamental treatises in ship theory Bouguer's results soon were widely distributed, his French text, augmented by numerical examples, was readily understood, textbooks with his methods were soon prepared for colleges in France, and the French Navy soon made stability assessments by the metacenter criterion an official requirement. Euler's Scientia was written in Latin, it was not widely circulated to practitioners, lacked numerical examples, and therefore remained relatively unknown in shipbuilding practice, though it made its mark on future scientific developments.

Chapman and Atwood

The Swedish naval constructor and scientist Frederik Henrik af Chapman (1721–1808) is the first and best witness for the actual application of the knowledge created by Archimedes, Bouguer and Euler being applied in actual ship design, construction and operation. Chapman, son of an English shipbuilder and immigrant to Sweden, grew up in an environment of practical shipbuilding and scientific openness. As a young man, practically trained and mathematically inclined, he spent a few years in England, Holland and France in a sort of "apprenticeship", picking up not only some practical trade skills, but also the scientific knowhow then available in those leading shipbuilding nations. He became familiar with the work of the Bernoullis, Bouguer and Euler, and hence with the Archimedean tradition. After his return to Sweden in 1757 he soon acquired much responsibility in Swedish naval and merchant ship design, rose to high rank and remained in a leading position throughout his lifetime. At the same time he not only practiced his scientific insights in his own actual designs, but also developed an ambition to publish his fundamental assumptions and conclusions in scientific treatises, foremost in his "Treatise on Shipbuilding" [39]. This gives us an intimate insider view of his use of scientific knowledge in practical design. Chapman made it a routine matter to calculate the displacement and a stability measure, the metacenter, at the design stage for every ship. For numerical integration he used the efficient quadrature rules acquired in private lessons from the mathematician Thomas Simpson in London. Chapman was also an excellent hull shape designer and lines plan draftsman. Thus he knew and discussed in his treatise how to influence hull shape design so as to achieve appropriate centroid locations and metacentric height. He also gave recommendations for the placement of ballast and cargo in ship operations in order to secure sufficient stability, but not too much metacentric height to avoid rough motions at sea. Further he had certain techniques for estimating the heeling moments by wind in sails, as already proposed by Bouguer and Euler, to derive the required safe margin for righting moments. Thus he rounded off the available physical knowledge, based on Archimedean thought, by further elements needed for safety assessment in practical design. This completed a cycle of 2000 years from the basic theoretical insights to practical applications.

Thomas Atwood (1745–1807), an English physicist and mathematician, assisted by the French naval constructor Vial du Clairbois, just before the end of the 18th c. added another missing piece to the puzzle of ship stability: They recognized that the initial stability for small angles of heel was not sufficient to ensure the ship's safety [40], as of course Bouguer and Euler had also already suggested, but they also proceeded to investigate the ship's righting moments at finite angles of heel, as Archimedes had done for the paraboloid. They used numerical quadrature rules again to calculate the "righting arm" of the vessel for a given draft, center of gravity and angle of inclination. Atwood also pointed out the nonlinear character of this function of heeling angle, which makes the rolling ship a nonlinear system. Thus by 1800 all prerequisites for hydrostatic displacement and stability calculations were available in practice when entering into the age of steam-driven steel ships.

4. CONCLUSIONS

It took about two millennia before the fundamental theories of Archimedes in hydrostatics were actually applied in the practice of ship design and operations. Archimedes laid the physical foundations for this technical purpose, but a number of other knowledge elements were still missing before this crucial assessment of ship safety could be performed on sound theoretical and practical grounds. Moreover access to Archimedes' manuscripts was interrupted for many centuries. The solution required further insights in hull geometry definition, mathematical analysis and data for physical criteria.

Geometry: While Archimedes still adhered to simple geometrical shape definitions, practical shipbuilding technology made many steps forward in hull geometry definition during subsequent centuries. After the skeleton-first construction principle was introduced (around A.D. 700), the use of moulds and in fact single master moulds for the whole ship became feasible (around 1300). Lofting ship parts only in the mould loft was then followed by drafting ship lines plans on paper (since about 1600) with more flexible construction rules [21]. In the 18th c. in addition analytical representations of hull geometry evolved, derived from offset data or form parameters. This much facilitated the numerical calculation of hull form features, especially areas, volumes and centroids. Thereby arbitrary hull shapes could be evaluated hydrostatically.

Analysis: Simple shapes in antiquity were often treated by the method of exhaustion to obtain area, volume and centroid information. But this approach had its limits when dealing with arbitrary practical hull shapes. Although numerical quadature rules were known and used to estimate tonnage since the Middle Ages and later applied to displacement calculations by some treatisers (by 1700), it is owed to the advent of calculus (by 1700) that first analytical and then numerical evaluations of all integral properties of ships could be performed for arbitrary hull shapes (following Bouguer and Euler after ca. 1750). Stability analysis for the metacenter also benefitted from the analytical concept of the center of curvature of a curve (Bouguer: Metacentric curve as an evolute).

Physics: Archimedes had elegantly derived the resultant hydrostatic force of buoyancy without resorting to effects in the liquid. An important alternative, the hydrostatic pressure, was introduced by Stevin (by 1600) so that hydrostatics could be newly developed from the viewpoint of continuum mechanics. This became the dominant basis of modern developments, also by Bouguer and Euler. This facilitated the expression of hydrostatic effects by calculus. Stability criteria for small angles of heel were thus stated in terms of infinitesimals (by 1750), while righting arms for finite angles of heel were formulated by means of calculus and evaluated numerically (by 1800).

Many further developments and insights were added to stability analysis during the 19th and 20th centuries before the current advanced level of risk-based ship design was reached [41]. But the foundations of our safety assessments of ships until today still rest on the principles and theories first pronounced by Archimedes.

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NOTES ON THE *SYRAKOSIA* AND ON ARCHIMEDES' APPROACH TO THE STABILITY OF FLOATING BODIES

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ABSTRACT A recent hypothesis on the giant ship *Syrakosia,* built around 235 B.C under the rule of Jeron II of Syracuse, is presented. In this enterprise Archimedes was a minister or a supervisor of the architect (Archias from Corinth). Comparison is made about opinions on the possibilities Archimedes could have had to plan the ship and to foresee her stability properties. The reconstruction of the actual procedures available to him to evaluate volumes and centres of buoyancy of different solids is studied also with models of the volumes of the ship and of the geometric solids mentioned in the "Floating bodies", to conclude that he could evaluate the stability properties on a quantitative basis only for the *orthoconoid*, the features of which were exhaustively studied in previous Archimedes' works. Therefore he must have left to the architect the task of both planning and evaluating empirically the stability of the ship. The last propositions of the first book of the "Floating bodies" suggest that Archimedes may have taken the experience of the *Syrakosia* as a guideline to approach the problem of stability of sectors of spheres, as the ship was launched when the hull reached the floating line and then she was completed afloat. Stability features of these phases appear to be comparable to those of different portions of sectors of the same sphere as presented by Archimedes.

1. INTRODUCTION

The *Syrakosia* is one of the chapters of the great naval architectural season which followed Alexander the Great's death and which interested in various examples Thracia, Greece and mainly the Ptolemaic reigns, which later became cultural references for all exceptional ships such as the Nemi ships or the obelisk carriers of Roman times. The analysis of the historical and cultural background to the *Syrakosia* and to Archimedes' works brought recently, starting from the Nemi ships from 1996 and from the

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exhibition "Eureka" held in Naples in 2005, to discuss and further study this aspect of the culture of the time. Different hypotheses have been proposed about Archimedes' involvement in the project and construction of the *Syrakosia* and about a possible connection between the principles of stability of floating bodies as discussed by Archimedes and the *Syrakosia* herself. Careful reading of sources and of the treatise on the "Floating bodies" and due consideration to the geometric tools ancient naval architects probably had to shape their hulls (as we can deduce from recent naval archaeology) contribute to fix more realistically the above items under discussion. The picture becomes clearer if we use models of the volumes and the procedures suggested by the "mechanical methods" available to Archimedes and if we compare his statements with the calculations of immersed volumes, centres of buoyancy and metacentric heigth of the same floating bodies and of the hull of the *Syrakosia* in her different phases. Archias from Corinth was the architect in charge of planning and building the ship, he applied the methods used and developed in Hellenistic times, which did not allow to calculate exactly the immersed volumes. The result of this part of the research is that probably Archimedes took inspiration from the *Syrakosia* for the study of the principles of floating of known solids, from the simplest (cylinder, prism, sphere), to the paraboloid, which allowed him to calculate exactly the centres of gravity and of buoyancy. This was not possible for the shape of hulls, because the exhaustion method cannot be applied to their shapes. The practice of trimming ships with ballast after launching (like for the second Nemi and for oared ships) show that, although empirical and legal rules were available for estimating cargo capacity, Archimedes' ideas could not be fully applied to ship's stability until the 18th–19th centuries.

2. THE SYRAKOSIA

Jeron II of Syracuse, in the years around 235 B.C., wanted to show the power of his reign and his technical capacities also with this extraordinary ship. This date is suggested according to the age of Archimedes and to the time in which he may have come back from his visit to Alexandria, hypothetically around 240 B.C.; we have to exclude war periods, the first Punic war in particular.

The ship kept her primacy until 220 B.C., when Ptolemy's IV *tesserakontere* was built.

The *Syrakosia* was described by Moschion in a long passage reported by Athenaeus from Naucratis in his "Deipnosophistai", written after Commodus' death (192 A.D.), in a chapter which includes the catalogue of Ptolemy's II fleet, the descriptions of Ptolemy's IV *thalamegos* and *tesserakontere,* as written by Callixenus from Rhodes. In the case of the *Syrakosia* we have the longest description of a ship in ancient literature and notwithstanding some lacunae (like the dimensions) it is complete enough to challenge the curiosity of scholars. From 16th cent. on many trials to reconstruct her were attempted (from Witsen in 1671 to Köster in 1934), but only between 1960 to 1970 precise and consistent deductions were drawn by Lionel Casson. After them the publication by Lucien Basch of a painting found in Crimea, showing a large ship comparable to the *Syrakosia,* a deeper knowledge of the naval culture in Hellenistic age, also after the further studies on the Nemi ships, other elements were added. These elements were basically the tonnage, the general aspect, the structure, the weight of the hull and the comparison with some ornamental parts, typical of Greek architecture in Sicily. A trial and error procedure allowed me to induce the displacement and the other main features of the ship, in a first hypothesis which I produced in the *Museo delle Navi Romane* in Nemi in 2001, ad then I published it more extensively in 2003 with a translation of Athenaeus' passage. Still it is an hypothesis, some adjustments are in progress and some details need still to be clarified, but the general reconstruction fits with all comparative documents and is accepted among the scholars (Fig. 1).

Fig. 1. Reconstruction of the Syrakosia, model by M. Tumbiolo in scale 1:200, based on drawings by the author, Trapani 2009.

The *Syrakosia* was planned by Archias from Corinth following Jeron's aim to have a sailing grain cargo ship. The name Archias from Corinth may be suspicious as it recollects that of the founder of Syracuse in 734 B.C. but the distinction of the roles of the architect *(architecton)*, the supervisor *(epoptes)* and the shipbuilders *(naupegoi),* as reported by Moschion, make think of a case of homonymy. Archias gave to the ship the shape of a supergalley with oaring teams made of 20, which are mentioned in Ptolemy's II catalogue (*eikoseres*) some 35 years before, but he did not put any oaring system on the *Syrakosia.* In the years 1960–1970 Lionel Casson proposed for the "supergalleys", having rowing teams of 16, 20, 30 and 40 men, that they could have been made of twin hulls, like catamarans. This theory comes from an interpretation of *double prow and double stern* mentioned by Callixenus, but in the end it did not solve the problem of the oaring systems; a careful reading of the literary passages, calculations of the needed displacements and the enormous technical problems involved in this solution make us exclude it. Carrying capacity of the *Syrakosia* was about 1940 tons, for a displacement of 3010 tons, as reported in the paragraph which follows. Athenaeus confirms the order of magnitude of her displacement when he states that the *Isthmia* (the ship with which Antigones Gonata won over the Ptolemies in 258 B.C) was about 1/3 or ¼ of the *Syrakosia.* Calculations of the reconstruction of the *Isthmia* give a displacement of about 730 tons, i.e. about ¼ of that of the *Syrakosia.* Timber was collected from the Etna, an exceptional pine was found in Calabria for the main mast, pitch came from Marseille and ropes from Spain.

The ship had three decks: hold deck, promenade deck and upper or combat deck. On the promenade deck there were the cabins in the middle, two open corridors aside, ending with rows of telamons, which were about 3 m high and they were similar to those of the temple of Zeus Olympic in Agrigento. The telamons were leaning on the protruding outrigger, or *apostis* (like that of the second Nemi ship), at the ends of which there were 20 stables for horses (probably in front), the galleys and relevant services astern. The inner part of the main level was divided into three corridors: a central passage and two rows of cabins for the passengers, which had mosaic floors, ending with a library, which had a planetarium or a sundial built by Archimedes, similar to that Marcellus brought to Rome from Syracuse in 212 B.C. The presence of this device suggests that the library had a sort of apse. The upper deck was open, dedicated to nautical manoeuvres and to fighting: three masts and eight towers (about 3 m high) rose over it. War machines, among which a type of *balista* expressively developed by Archimedes, were installed on them. On the upper deck there was probably the temple dedicated to Aphrodite (astern) and the cloak rooms of the *gymnasium*: the deck itself was the run track.

Building phases and details, like the lead sheathing, the mosaic floors, the fittings, the steering devices, wooden and iron anchors, water systems, etc. are comparable to those of the Nemi ships, while rigging was that typical of Hellenistic ships. Still we do not know how the bilge pumps could have worked, as Moschion refers of a screw pump applied by Archimedes, while the bilge pumps we know, including those of the Nemi ships, were scoop wheels.

Archimedes was a superintendent *(epoptes)*, or a sort of minister *(phylos)*, probably in charge of the solution of logistic problems and of special applications, leaving to Archias the actual project and realization. When problems were found to launch the lowest part of the hull (that sheathed with lead, under the main wale) he applied compound pulleys he developed on purpose for this operation.

The ship was a failure: she was too large for the harbour of Syracuse (clearly she was built outside that harbour): she was sent to Ptolemy III Euergetes as a present and during the trip to Egypt she brought a grain cargo to Athens. For this trip she was celebrated by Archimelocos with a pedantic piece of poetry. Once in Alexandria her name was changed into *Alexandrìda* and probably into *Isis,* as the painting discovered in Crimea shows: provided it is the same ship, the old *Syrakosia* could have been used for promoting the Ptolemy's image, but we can think that soon she was demolished.

3. TECHNICAL DATA OF THE *SYRAKOSIA*

Athenaeus passage was known since the Renaissance, then: B. Graser, 1864, calculated the load capacity of the *Syrakosia* in 4200 metric tons, considering the measure of grain in *medimni,* like his followers, who give:

- C. Torr, 1894, rist. 1964, p. 27, 3650 metric tons
- A. Köster, 1934, 3310 metric tons

L. Casson, in 1971 considers the measure of grain in *modii,* in sacks, giving the final load capacity as 1940 metric tons.

Other features, not mentioned in the previous paragraph:

- Main water tank: 75 tonn. (so far difficult to locate),
- 30 cabins 4 *triklinoi* (18 m^2) large on the promenade deck, on two rows and around a central corridor, plus the capitain's cabine (15 *triklinoi* large, 65 m^2), a bathroom 3 *triklinoi* large (13 m^2) and the library.
- Other cabins on the hold deck.
- The *atlantes* surrounding the promenade deck along the outrigger (*apostis)* were 6 cubits (2,7 – 3 m) tall.

– Men aboard: about 825; 20 horses, needing stables 1,5 x 3 m each.
– Combat deck with 8 towers 6 cubits high (2,7 – 3 m), with war machines.
– Plumbing system

CARGO:

60.000 *modii* of grain, 400 metric tons, in sacks, not in amphorae (17,500 would have been necessary, for an extra load of about 260 tons)

10.000 amphorae of fish	500
20.000 talents of wool	520
20.000 talents of mixed load	520
TOTAL:	1940 metric tons

No ballast is considered, as part of the cargo plays its role, as in Pliny, "Naturalis Historia". XVI, 76.

WEIGHTS:

Cargo + tank + men aboard + horses: 1940 + 75 + 57 + 8 = 2080 m. tons, plus the weight of the hull (structures similar to those of the Nemi ships) and of fittings 930 metric tons
TOTAL WEIGHT: 3010 metric tons

POSSIBLE DIMENSIONS:

Length at max floating (L):	80,0 m	Total length:	(tL)	87,0 m.
Breadth at max floating (b):	15,5 m	Total breadth:		17.5 m
Draught (d):				3,85 m
Height from keel to promenade deck:				5.6 m
Block coefficient (φ):				0,615
Height from promenade to combat deck:				3.5 m
Total height at main section (h), from keel to combat deck:				9.1 m

Displacement = L x b x d x φ = 80,0 x 15,5 x 3,85 x 0.615 =2936 m^3
i.e: 2936 x 1,026 = 3012 metric tons (1,026 is the density of sea water)

The comparatively low height (tL/h = 9.56) is suggested by the design of the hull, similar to that of a 20 supergalley, by the comparison with the Crimean encaustus painting of the *Isis* and with the Nemi ships.

4. ASPECTS OF ARCHIMEDES' APPROACH TO THE STABILITY OF FLOATING BODIES

L. Russo e F. Zevi extend Archimedes' work also to planning the ship and to the possibility to evaluate the features of her hull, as he did for the *orthoconoid*; Zevi proposes also a cultural connection with the *Isis* described by Lucian and a remind of his to Archimedes' principle. Similarly C. B. Boyer gave the opinion that: *he could have very well taught a theoretical course on naval architecture.* P. D. Napolitani suggests that a possible interpretation of the treatise on the "Floating bodies", in particular regarding the propositions on the equilibrium of segments of sphere and of the paraboloides, is that Archimedes was attempting to set out a mathematical modelling of floating hulls. H. Nowacki excludes that Archimedes could have applied the evaluations of the centres of gravity and of uplift to the shape of hulls, even though he could have had the possibility to perform them. P. Pomey, with Tchernia, suggests that Archimedes could have evaluated the stability of the *Syrakosia* on the basis of the volumes calculated from sections taken from the project, which, he proposes, was based on the moulds of a 20er galley. He proposes also that, being the ship built in two phases (before and after launching) the same phases could have been useful also to keep under better control floating and stability.

I wanted to review these positions on the basis of what we know and suggest about the *Syrakosia*, about project and building procedures used in Hellenistic times and trying to reproduce logical processes and calculations according to the methods used by Archimedes, with the help of models of the relevant volumes and then by comparing the results with those obtained with modern approach to centres of buoyancy and metacentres (the centres of oscillation of the hull), according to Normand's (1835–1906) formulas.

I think that P. D. Napolitani and H. Nowacki's positions are the most realistic: the first can also be recollected to the experience with the *Syrakosia* and his discussions with Archias from Corinth, but after that his interest for the principles of equilibrium and stability of floating bodies remained at a theoretical level of personal research. It appears that he could not extend practically the concepts found for selected solids to the actual cases of floating hulls, due to the difficulties to evaluate the volume of the immersed part, its centres of buoyancy and of gravity.

5. VOLUME OF THE IMMERSED PART OF THE HULL (Fig. 2)

To calculate the volume of the bottom of a hull, the method of exhaustion is not applicable due to the shape, which is not geometrically defined. Therefore a "mechanical" method must have been used, like the evaluation by comparison of weights of models of the interested volumes made in exact scale and with a uniform material. This procedure is not reported expressively by Archimedes, nor by Heron from Alexandria ("Definitions" N. 74), but such calculation of the volume by comparing the weights of a scale model and a reference volume might have been possible and is reported occasionally in Arabic treatises, like Al-Farisi and Al-Kasi (13^{th}–15^{th} cent.). It has the advantage of being simple, moreover the centre of buoyancy and the metacentre of the model are in the same scale, being geometric features. In the case of the model of the bottom of the *Syrakosia* I simplified the operation by weighing the parallelepipedal wooden block having the dimensions corresponding to the length, breadth, draught of the immersed part of the hull, then I carved the shape of that part of the hull from the same block according to the plan of the lines of the reconstruction. Then I compared the weight of the carved part of the hull to the original weight of the block, giving therefore the block coefficient ($\phi = 0.615$), or the ratio between the actual volume and that of the surrounding parallelepiped. This method corresponds to the comparison of the weight of the body with that of a cube having reference dimensions, but it is more precise and I am sure Archimedes could have used it in many occasions.

Fig. 2. Evaluation of the volume of the bottom part of the hull of the Syrakosia by weight, model by the author, in scale 1:200.

The methods probably used in Hellenistic age to design hulls and to control their shape during construction did not allow to evaluate the volumes directly from the project, as this was not based upon all orthogonal sections. The bases were the profile and the shape of the main transverse section, which had a rule for narrowing until the "active frames" towards the ends of the hull, together with the shape of the slices in which the shell was divided, corresponding to peculiar planks or wales, but not to orthogonal sections. These geometric operators were of practical nature, as shown by the use of traditional *garbi,* and not aimed at calculating the volume of the hull: only the method of drawing with orthogonal sections in connection to Bézoult or Simpson (17^{th} cent.) approximations allow to calculate the volume in a reasonable way. Empirical formulae were available to predict the cargo capacity in connection with the main measures, like those used in the Middle Ages, but they were fiscal formulae approximated to a standard shape. For this reason, the graphical tools available in Hellenistic times did not allow to evaluate the volumes of hulls, nor was Archias from Corinth able to tell Archimedes what was exactly the volume of the hull of the *Syrakosia,* unless he had a scale block model of the bottom part of the hull. I think this chance is unrealistic, or at least we need more documents to induce it.

It is clear that geometric solids, as those considered by Archimedes in his "Floating bodies" (cylinder, prisma, sphere, *orthoconoid*), did not put these problems of volume evaluation, in particular the *orthoconoid* was the most available for such evaluations.

6. CENTRE OF BUOYANCY

To find out the centre of buoyancy of our model of the bottom of the hull, again a "mechanical" method is available: hanging the model from one side in order to find the intersection of the vertical at the main transverse section with the centre line. The findings with the model of the bottom of the hull of the *Syrakosia* in scale 1:200 gave the position of the centre in good agreement with that calculated with Normand's formula (2.18 vs 2.11 m, see Fig. 3, 3–z, figure with the asterisk). With this experience we can induce that Archimedes could evaluate the position of the centre of buoyancy, or centre of gravity of the immersed part, as he calls it., provided he had a good model in scale of the bottom part of the hull.

Archimedes evaluated quantitatively the centre of buoyancy only for the *orthoconoid* in the second book of his "Floating bodies", in the first book he indicates the centres only qualitatively for the segments of sphere.

7. CENTRE OF GRAVITY

While for the geometric solids mentioned by Archimedes the evaluation of the centres of gravity follows the same method as that outlined for the centres of buoyancy, for a ship it becomes much more difficult a problem and, to my opinion, it could not be solved. The "mechanical" method would have implied two options: a model of the complete ship perfectly in scale and having the same buoyancy and trim as the original ship, or a model of the central section of the hull, with all structural elements in correct scale. Neither option can be suggested, as this approach appears far from Hellenistic culture. Moreover even the concept of centre of gravity of a ship was probably unknown: to improve stability ancient architects knew very well how to increase the breadth (raise the metacentre) or to ballast or generally to lower the weights.

8. THE BUILDING PHASES OF THE *SYRAKOSIA* AND A POSSIBLE CONNECTION WITH BOOK I, PROPOSITIONS VIII AND IX OF THE *FLOATING BODIES* (Fig. 3)

The *Syrakosia* was built in two phases: first the bottom (until the wale at the floating line) was built, sheathed with lead and launched, then the hull was completed afloat. The weight of this part of the hull was about 400 tons, according to the proposed reconstruction and still this weight was such as to create problems with launching. Archimedes solved these problems with a special capstan, but he did not manage the whole ship, as sometimes it is imagined. This part of the hull buoyed about 85 cms and the lead sheathing contributed very little to its stability, as it weighed about 15 tons (3.7% of the displacement of that section of the hull). When the hull was finished, but empty, it weighed a little less than 1000 tons and buoyed about 1.70 m. When the load was completed, the total weight could have reached about 3000 tons, with a correspondent buoyancy of 3.85 m. During these phases, the parts of the hull under completion had different reactions to rolling, because, said in modern words, the metacentre comparatively lowers and therefore the completed and loaded ship rolled more easily than the bottom part of the hull (see Fig. 3, 1–3).

Proposition VIII of the first book of the “Floating bodies” can suggest that Archimedes observed this behaviour of the ship when making the example of a segment of a sphere: if the floating body is a segment of a sphere smaller than a semisphere, metacentre is high over its upper surface, if it is a semisphere it lies a little over the centre and if it is higher it falls under its upper surface. Archimedes had not the concept of metacentre, but his constructions allow to locate it. Geometrically it does not displace dramatically with respect to the shape of the curve, but its effect on stability is evident. Archimedes does not quantify the position of the centre of buoyancy of a segment of sphere, but he indicates anyway it as responsible for the righting moment. Similarly, while the building phases of the *Syrakosia* proceeded, he observed the different reactions of the hull to rolling and I like to think that he discussed with Archias these reactions and the ways to minimize rolling in a ship. Ancient hulls tended to have a

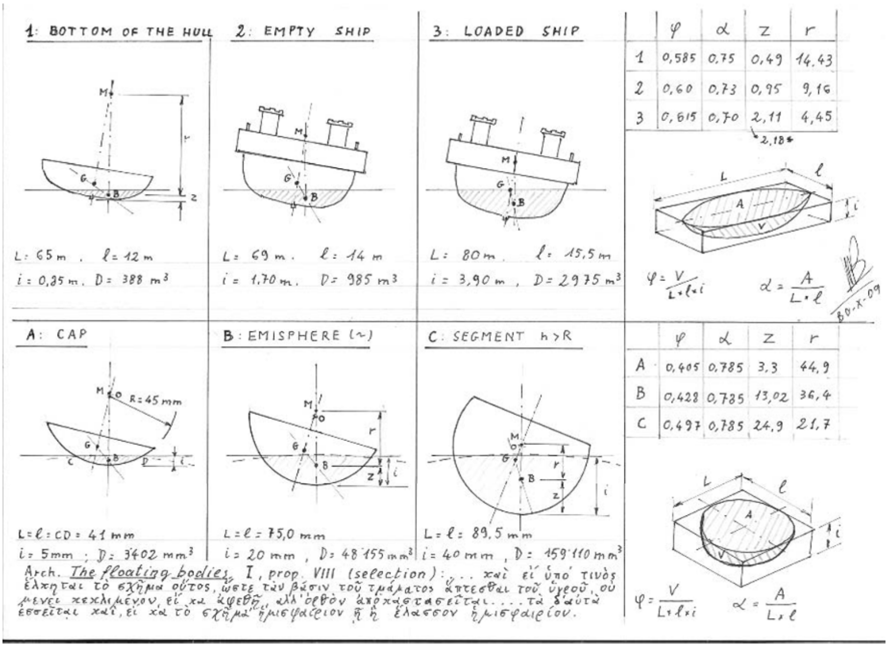

Fig. 3. The building phases of the Syrakosia (1, 2, 3) compared to segments of sphere (A, B, C) according to Propsition VIII of Book I of Archimdes' "Floating bodies": calculations of heights of centres of buoyancy (B, z) and of metacentres (M, r) with Normand's formulas (see Fig. 4).

high metacentre and a certain oversizing of the hull allowed to assure the foreseen cargo and trim. Adjustments with ballast were done when the ship was launched, and in the case of oared ships the tholepins were fixed in the final phase; in the case of the *Syrakosia* part of the cargo played the role of ballast, as hinted by Pliny ("N.H." XVI, 76). These thoughts, filtered through the principles soon discovered, may have induced Archimedes to compare his ideas how weights and centres of gravity and buoyancy could function and which limitations uprighting may have had.

In book I Archimedes considers a generic solid (a prisma or a cylinder?) and then a sector of sphere, on a spherical water surface, the main points of which are indicated qualitatively (centres of gravity and of buoyancy). In book II he deals only with the *orthotome,* this time floating on a flat surface; he could calculate all features (partial volumes, centres of gravity and of buoyancy) of that solid tanks to his previous researches.

Moreover the different cases and limitations to buoyancy of the *orthotome* of book II appear to hint to parts of the stability curve. The passage from spherical to flat water surface is indicative of a passage from general concepts to specific evaluations. To verify this aspect, I applied Normand's formulas for the height of the centre of buoyancy and of the metacentre to the mentioned sphere sectors (Fig. 3) and to an *othotome,* having a "parameter" equal to 3/2, considered as a floating hull (Fig. 4). Data evaluated Geometrically evaluated data fit exactly.

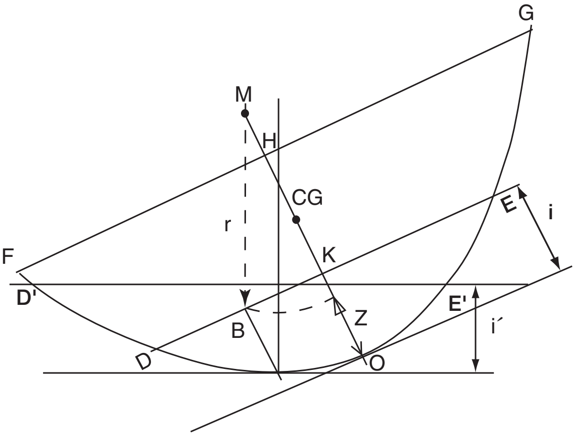

Fig. 4. Main stability features of an orthoconoid, Proposition 2, book II of the "Floating bodies", see following table.

There are examples which show that in antiquity the immersed volumes were not evaluated scientifically; among these the second Nemi ship provides with a good example: she was ballasted with 250–300 tons of concrete. The shape of the hull did not require ballast, as shown by the first Nemi ship, but the oaring system made it necessary to adjust the buoyancy, to have the tholepins at the correct height over the water surface. To avoid too high an angle of the oars, it was sufficient to sink the ship by about 30 cms, with respect to the finished ship. Calculations showed that 30 more cms of buoyancy were given by about 250 tons: just the amount of the otherwise useless ballast and this confirms the tendency of the architects to oversize, in order to be on the safe side, with the possibility to adjust later the trim according to needs.

Table 1. Stability features of the above orthocooid compared on a model by the author.

<table>
<tr><td colspan="2">Dotted line MB as r is extrapolated, as the concept of metacentre (M), or the centre of oscillation, is not present in Archimedes' work.
The Centre of Gravity (CG) is at 2/3 of the segment OH
The Centre of Buoyancy (B) is at 2/3 of OK</td></tr>
<tr><td>Diameters: FG = 121.5 mm, DE = 82 mm</td><td rowspan="6">Normand's formulas (1839–1909):

Height of the centre of buoyancy B :
$z = (0{,}833 – 0{,}333.\phi / \alpha) . i$

Metacentric radius, MB:
$r = \frac{(0{,}008 + 0{,}0745.\ \alpha^2).\ l^2}{\phi\ .\ i}$</td></tr>
<tr><td>Height HO = 50 mm</td></tr>
<tr><td>Draught KO (i) = 21 mm</td></tr>
<tr><td>$\phi = 0.39$, $\alpha = 0.785$, see Fig. 3</td></tr>
<tr><td>Geometric evaluation:
Z = 14.0 mm, r = MB = 43.5 mm</td></tr>
<tr><td>Normand's evaluation:
Z = 14.0 mm, r = MB = 43.9 mm</td></tr>
</table>

9. CONCLUSIONS

Hypotheses on the *Syrakosia* still need rechecks and discussions, but the general view provided by them appears to be consistent.

If the considerations developed here confirm Archimedes' ingenuity in observing the behaviour of floating bodies, probably including the *Syrakosia,* they show that his concepts could not be applied directly to her*,* nor have they been developed for the application to other ancient ships, like the Nemi or the oared ships, for which only empirically estimated formulas could have been available. A scientific calculation of the volumes and of the centres responsible for stability was available to Archimedes only for geometric solids, whose characteristics were known in advance. Application of stability concepts to shapes different from those of geometric solids was possible only after the 18th century, when modern naval architecture was developed, in connection with naval drawing based on orthogonal sections.

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WHAT DID ARCHIMEDES FIND AT "EUREKA" MOMENT?

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ABSTRACT Archimedes has been considered as he had found the Law of Buoyancy at "EUREKA" moment. Because, it seems to have been considered that the overflow water volume measurement that Vitruvius wrote, is impossible. After Archimedes, overflow volume measurements have not been done by using of a vessel having enough large opening to put in a practical size crown. Then, it is thought that Archimedes found the Law of Buoyancy at that moment and proved the theft of the goldsmith.

In this paper, the measurement of the overflow volume by golden crown etc. has been tried. At a result, it is proved that the measurement can be done with enough accuracy by using a vessel having enough large opening diameter.

Archimedes might have proved the theft of the goldsmith by almost the same method to this measurement.

From this result, at "EUREKA" moment, Archimedes did not find the Law of Buoyancy but found the solution of the king Hiero's problem and specific gravity of things.

Also this moment must have been when Archimedes got an inspiration of the idea of the law of Buoyancy.

1. INTRODUCTION

Archimedes ran through a street in Syracuse naked with much joy shouting "EUREKA! (I found it)" repeatedly.

After this moment, Archimedes made a silver lump and a gold lump of the same weight as the king Hiero's golden crown. ("Crown" is in Latin "Corona". Maybe its shape was a wreath.) Archimedes put them into a vessel filled with water one after another. And he proved the theft of a goldsmith of the golden crown by measuring the overflow water volume by these items.

This episode is the world famous story as the moment that Archimedes found the Law of Buoyancy. This episode was written by Vitruvius, an

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architect in BC 1st century, in the book "Ten books on Architecture".[1][2] However he did not write that Archimedes had found the Law of Buoyancy at this moment though he seemed to know Archimedes's achievements well. Of course, Archimedes did not write about this episode.

More than 1500 years later, Galileo wrote his first short treatise entitled "La Bilancetta" (The Little Balance) in 1586.[3] He concluded that Vitruvius made a mistake in reporting the episode of Archimedes. (He did not mention the name "Vitruvius".) Galileo might think that the measurement of the overflowed water volume by the golden crown etc. was difficult. (may be, impossible) Then, he reached the result that Archimedes had found the Law of Buoyancy at the moment and he made up "La Bilancetta".

Even in modern times, it may be believed that the measurement of overflow water volume is impossible. For example, the rationale is as follows. The golden crown is supposed to be 1000g. And 30% gold of the crown was replaced with silver. The difference of the volume between the golden crown and the same weight gold lump will be only 13cc. ("cc" or "cm^3" is used in this paper) If the opening diameter of the vessel is 20 cm, the difference of 13cc is only 0.4mm in height. Such a small difference cannot be measured.

In fact, any practical measurements along the Vitruvius's story seemed not to have been done by using a vessel having enough large opening diameter to put in a practical size crown.

If measurements of overflowed water volume by these items are possible, the mixed ratio of silver in the golden crown can be proved. There are not any contradictions in the story written by Vitruvius.

In this paper, measurements of the overflowed water volume by the golden crown are tried by using of the equipments that Archimedes would use in ancient Greece era.

The result of this trial is reported hereunder.

2. DIMENSIONS OF GOLDEN CROWN etc.

Archimedes Homepage by Prof. Chris Rorres of Drexel University was referred to.[4] The chosen conditions of the golden crown are as follows.

Weight of golden crown	: 1000g
Mixed ratio of silver in the crown	: 25%

From these conditions, dimensions of objects to be measured are shown in Table 1. And circular opening diameter of a vessel that Archimedes used, is decided to be 20cm. (Outside diameter of the crown is supposed to be less than 18cm).

Table 1. Dimensions of objects to be measured.

Object	Weight [g]	Density [g/cm^3]	VO [cc]
Silver lump	1000g	10.5	95.2
Gold lump	1000g	19.3	51.8
GCS	1000g	16.0	62.7

GCS Golden crown mixed with 25% Silver, VO Volume of Object

3. EXPERIMENTAL

3.1. Method of Overflowed Water Volume Measurement

3.2. Method According to the Vitruvius's Story

At first, a trial has been done according to Vitruvius's story.

1. A vessel having a round opening is prepared and its diameter is approximately 20cm(Vessel-a). It is filled with water up to nearly the highest point. At this point, any water does not overflow by water surface tension. And the surface level is higher than the brim. (Fig. 1, left) However, the real highest point cannot be known.
2. When an object that has approx.80cc volume is put in, much water overflowed from the circumference of the vessel. (Fig. 1, center)
3. After the overflow stopped, the object is taken out from the vessel. The water height of the vessel is slightly lower than the brim. (Fig. 1, right)
4. Then water is added up to the near height in item 1 by using of a pint measure.

Through these measurements, any water volumes could not be measured accurately. More than 50cc difference occurs at every measurement. From this result, through the method written by Vitruvius, it is understood that water volume will not be measured.

However, this result was as expected formerly.

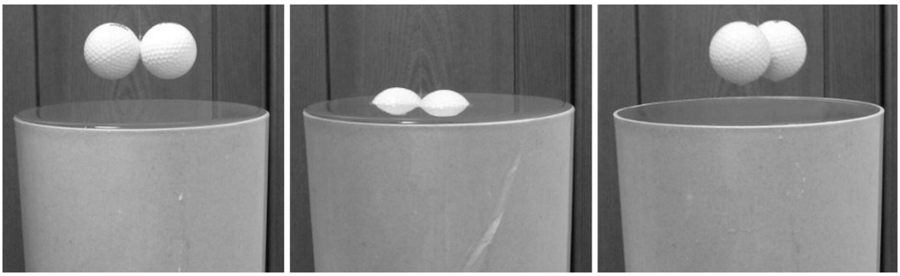

Fig. 1. Vitruvius's method is tested by using of vessel-a.

4. VESSEL HAVING A BEAK

A vessel having a beak is prepared as shown in Fig. 2. (Vessel-b) It is made by Polypropylene and its circular opening diameter is 21cm.

In ancient Greece, there were many kinds of ceramic wares, glass wares and metal wares. So, Archimedes would be able to use a vessel like this.

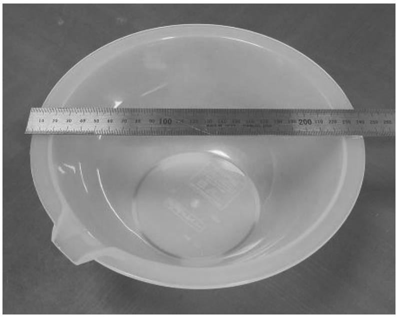

Fig. 2. Vessel-b having a beak.

Measurement of the overflowed water by the 80cc object is tried by using vessel-b through the Vitruvius method. At a result, two problems are found.

- Even using this vessel-b, the highest point of water cannot be determined. 15cc or more difference is occurred by the water surface tension.
- A part of overflowed water ran down along the vessel body. This also disturbs to achieve the accurate measurement.

5. EFFECT OF TONGUE

A triangle shape tongue is attached to the beak and tested. (It is made of Vinyl) Two effects were expected by attaching the tongue.

- The disturbance of the water surface tension will be decreased.
- The water flow is gathered to one stream along the tongue

Even by using this condition, an accurate measurement could not be obtained through Vitruvius's method. However, the overflow finish point seemed to make almost the same height repeatedly. Then this point is chosen as the basic point of measurements.

6. METHOD OF WATER VOLUME MEASUREMENT

The measuring method using the Overflow Finish Point (OFP) is as follows.

1. A container is put under the tongue to receive overflowed water.
2. Water is added gradually to the vessel until overflow begins. (Fig. 3 left)
3. At first, water flows out rapidly. The flow decreases slowly and turns into drops.
4. The water drop intervals become longer and longer. Then the flow stops. (Fig. 3 right) This is the basic measuring point (OFP). This point may be able to be considered as “the very brim” point that Vitruvius wrote in his book.
5. Next, the container is changed to an empty pint measure. (Vitruvius wrote as “sextarius measure”. It was about 540cc volume.)
6. An object that is to be measured such as the crown is put into the vessel. The object should be put in not so roughly but not so quietly.
7. At first, water flows out rapidly. The flow decreases slowly and stops at the OFP.
8. The sextarius measure is taken off and the gathered water is removed to a 250cc mess-cylinder. And the volume is read by the scale.
9. The measured object is taken out from the vessel.

This is the measuring procedure by using of OFP.

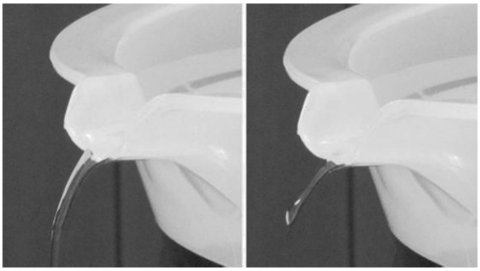

Fig. 3. Water overflow and stop at the OFP.

7. INVESTIGATION OF TONGUE SHAPE AND MEASURING CONDITION

By a combination of some large and small glass balls, a volume of 52.0cc is arranged.

Tongue’s shape, material and place have been investigated by using these balls through the procedure that is written in the upper clause.

The final chosen condition that enables an accurate measurement is as follows.

- The vessel is inclined 12 degrees so that the internal groove of the beak becomes level. (A-A’ line in Fig. 4)
- The tongue is attached outside of the beak. Better accuracy was obtained than it is attached inside.
- Cow skin is chosen as the tongue material. (Fig. 4)
- The tongue’s shape is thin triangle. At the end of the beak, the tongue width is 4mm. And the length is 20mm

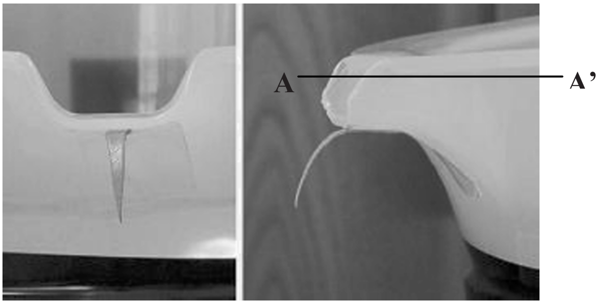

Fig. 4. Cow skin tongue.

By using of these conditions, it is found that very accurate measurement can be obtained. These conditions could be prepared by Archimedes.

8. MEASUREMENT OF THREE OBJECTS

The result of measurements of three objects is shown in Table 2. Their volumes are measured three times about each object by using of a mess-cylinder. At a result, only within 2.0cc difference against the input volume, was obtained.

Table 2. Result of overflowed water volume measurement.

Object	VO (cc)	IV (cc)			OV (cc)
		LGB	SGB	Total	
SL	95.2	1,3,4,7	1,2,3,5	95.0	95.5, 95.0, 96.0
GL	51.8	4,7	1,2,3,4	52.0	51.5, 52.0, 52.5
GCS	62.7	4,5,6	–	62.5	63.0, 64.5, 64.0

SL Silver lump, GL Gold lump, GCM Golden Crown mixed with 25% Silver,
VO Volume of Object, IV Input volume, LGB Large Glass Ball no.,
SGB Small Glass Ball no., OV Output Volume

However, there will be one more problem remaining.

The problem is that Archimedes seemed not to be able to express the volumes as numerical value.

9. LIQUID MEASURE IN ANCIENT GREECE (SYRACUSE)

In ancient Syracuse, they used "Attic measures". And the smallest unit of liquid measure would be "cyathos". Its volume was about 45.6cc. [5]

A sextarius measure that Archimedes used was volume of 540cc. A cup is used in this paper as a sextarius measure. (Fig. 5) Its volume is about 500cc. (A real sextarius measure might be approx. 10% larger than this.) This cup is graduated every 10cc.

Even by using of this cup, it was difficult to prove the theft of the goldsmith. Because the difference of the water height between the gold lump and the golden crown is so little, the goldsmith would never admit his theft. To measure a little volume of water, sextarius measure should have following features.

- Graduated at least every 1/10 cyathos.
- Made by transparent material such as glass.
- Much thinner and taller shape than the cup shown in fig. 5.

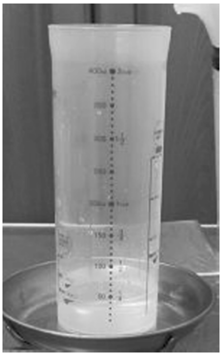

Fig. 5. Sextarius measure.

If a sextarius measure did not have these features, a small volume of water could not be measured by it. In ancient Syracuse, there would not be such kind of measure cup. They seemed to have no needs to measure less than a cathos (45.6cc) in their life.

10. MEASUREMENT OF OVERFLOWED WATER WEIGHT

In ancient Syracuse, they had very small unit about weight. The smallest weight unit was “chalcus” of 0.091g. Then Archimedes must have used a balance to measure the weight of water.

More than 1000 years before than Archimedes era, in ancient Babylonia, they measured time by weighing of dropped water weight from a water clock. Archimedes might have known this.

In this paper, a digital scale or a Roberval’s balance are used.

11. VOLUME MEASUREMENT OF GLASS BALLS

Glass balls are measured their volumes by the Law of Buoyancy. A digital scale is used. (A & D co.ltd. Type HJ-150, max150g, 0.1g digits)

A body in water will be lighter as the same weight as the water replaced by the body. The example of this measurement is shown in fig. 6.

1. A water cup is placed on the digital scale. And set the display clear to 0.0g. (Fig. 6, left)
2. A glass ball hung with a nylon thread is put into the cup.
3. Read the weight. (The right picture shows 21.3g)

Fig. 6. Glass ball volume measurement.

Where the density of water is 1.00g/cm^3. The result is shown in Table 3. From this result, some glass balls are combined to have the similar volume of objects to be measured.

Of course this measurement was not needed for Archimedes.

Table 3. Glass ball volume measurement (g=cc).

	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8
LGB	21.3	21.5	21.3	22.2	21.4	21.0	21.6	–
SGB	2.4	2.4	2.2	2.4	2.6	2.5	2.5	2.5

LGB Large glass ball, SGB Small glass ball,

12. METHOD OF OVERFLOWED WATER WEIGHT MEASUREMENT

A Roberval's balance is used. (Murakami Kouki Co.ltd type:MS-1, max.1kg)

At first, a sextarius measure is placed on the right dish. And a plastic cup is placed on the left dish and sand is put into the cup until make a balance with the sextarius measure. By using this sand cup as a counter weight, only weight of the water is measured at all measurement. The measurement procedure that was finally decided is as follows.

From item 1 to 7 are just the same as clause 3.1 "Method of Water Volume measurement".

8. The sextarius measure is taken off and is placed on the right dish of the balance.
9. The water is weighed by the balance. (see Fig. 7)
10. The measured object is taken out from the vessel.

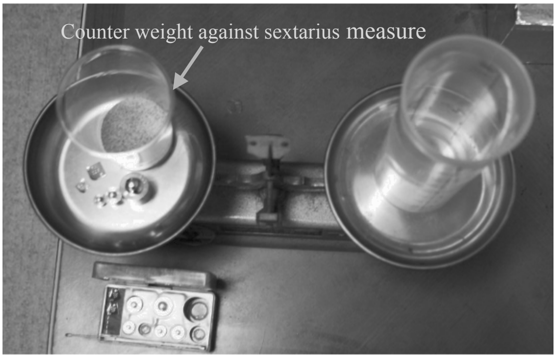

Fig. 7. Water weight measurement.

Now, the first measurement has completed. Before the next measurement, glass balls and the sextarius measure are wiped out to dry.

Three objects are measured through this procedure. (The first: a silver lump. The second: a gold lump. The third: the golden crown mixed with 25% silver)

13. RESULT OF WATER WEIGHT MEASUREMENT

The result is shown in Table 4. Each object is measured 5 times.

According to this result, the difference between the put-in volume (IV) and the overflowed volume (OV) was only –1.1g to +0.2g.

This result shows that this measurement has enough accuracy to prove the theft of the goldsmith.

Table 4. Result of overflowed water volume (by water weight).

Object	VO (cc)	IV (g=cc)			OV (g=cc)		DV (g=cc)
		LGB	SGB	Total		Ave.	
SL	95.2	1,3,4,6	1,2,3,4	95.2	94.1, 95.0, 95.2, 94.9, 94.9	94.8	-0.4
GL	51.8	1,6	1,2,3,4	51.7	50.6, 51.6, 51.7, 50.8, 50.7	51.1	-0.6
GCS	62.7	4,5,6	–	62.6	62.2, 62.6, 62.0, 61.6, 62.8	62.2	-0.4

SL Silver Lump, GL Gold Lump, GCS Golden Crown mixed with 25% Silver, VO Volume of Object, IV Input Volume, LGB Large Glass Ball No., SGB Small Glass Ball No., OV Output Volume, DV Difference of Volume (OV-IV)

These average values are changed from gram to chalcus (0.091g) as follows.

- The silver lump : 94.8g => 1042chalcus
- The gold lump : 51.1g => 562chalcus
- The golden crown mixed with 25% silver : 62.2g => 684chalcus

The differences become very clear by these values expressed in chalcus.

14. VERIFICATION BY ARCHIMEDES

This verification by Archimedes might have done in front of the king Hiero II.

After these measurements, Archimedes turned to the goldsmith and said.

"The difference between the silver lump and the gold lump is 480chalcus. And the difference between the gold lump and the crown is 122chalcus though they are the same weight. This is one fourth of the difference between the silver lump and the gold lump. You must have stolen the one fourth of the gold that the king handed to you and mixed the equivalent weight of silver in it!!"

15. CONCLUSION

Archimedes had been considered as he had found the Law of Buoyancy at the "EUREKA" moment. Because, it seems to have been considered that the overflow water volume measurement that Vitruvius wrote, is impossible.

However, in this paper, it is proved that the overflowed water volume measurement is possible with enough accuracy by using of a vessel having enough large opening to put in the golden crown. And by weighing the overflowed water weight, the volume differences of the objects become very clear. By using this method, Archimedes might have proved the goldsmith's theft.

The color of gold will change by mixing of 25% silver to greenish-yellow-gold. So the goldsmith might have also mixed copper (and other metal) to adjust the color of the crown. He could get these metals easily from the coins which were used usually. As the density of copper is 8.9, the volume of the crown will be larger than silver only. So, the theft of the goldsmith would be more easily found out.

In Archimedes era, the touch stone method had already been used. However, the crown is scraped off even a very little portion. Therefore Archimedes would never use this method.

As a result, the Vitruvius's story should be recognized as the fact. However, Vitruvius or some person might have made a mistake about the measuring procedure. He or someone could not record correctly with difficulties to understand Archimedes's measuring method in details.

As a conclusion of this paper, it is thought that Archimedes did not find the Law of Buoyancy at "EUREKA" moment.

At this moment, He must have found the solution of the king Hiero's problem and specific gravity of things.

Also this moment must have been when Archimedes got an inspiration of the beginning of the Law of Buoyancy.

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FLOATABILITY AND STABILITY OF SHIPS: 23 CENTURIES AFTER ARCHIMEDES

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ABSTRACT In this paper the main developments in ship buoyancy, stability and subdivision of ships since the milestone formulation of the basic laws of floatability and stability of floating bodies by Archimedes are reviewed. The continuous progress in the safety of ships as most effective transportation means and the links of the fundamental Archimedean studies to the modern naval architectural approaches to ship stability, design and safety are critically commented.

1. INTRODUCTION

Man has travelled since thousands of years throughout the oceans without first knowing how and why it was possible. The basic laws of hydrostatics of floating bodies were introduced by the Great Archimedes in 300 B.C. It is well established that he was the first to formulate the basic law of *buoyancy* and eventually *floatability*; namely, the ability of a solid body to float is trivially related to the equilibrium and balance of the gravitational (weight) and the hydrostatic pressure (buoyancy) force. Modern time naval architects were until recently less aware of the fact that Archimedes had also set the foundations of ship's *stability*; namely the ability of a floating (or fully submerged) body to regain its initial position after removing an applied disturbance, which is trivially related to the balance of a *couple of forces (or moments)*, namely the couple of weight and buoyancy forces. The basics of ship stability are laid down in Archimedes' most famous treatise *On Floating Bodies ('περί οχουμένων'*, literally translated from Greek '*on vehicles*'*)*, as has been recently elaborated by *Nowacki*, 2001.

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Many centuries after Archimedes, they were the French *P. Bouguer* (1746) with "*Traité du Navire*" and the Swiss *L. Euler* (1749) with "*Scientia Navalis*", who worked out (almost simultaneously) the principles of modern ship buoyancy and stability, of fluid resistance and a series of other specific problems of Ship Theory on the basis of *Newtonian* mechanics. The important notion of stability "metacenter" stems from *Bouguer* and was never used by *Euler*, who was not familiar with this terminology. But in fact both derived the magnitude of GM (the vertical distance between the ship's center of gravity G and the *metacenter* M, a property of ship's form) from the basic ship characteristics by the same expression in order to judge on the stability for *small* inclinations (*initial* stability). At the end of the 18th century and the years after, Atwood, Vial de Clairbois and Moseley set the foundations of ship stability at *large inclinations* and of *dynamic* ship stability (see *Nowacki & Ferreiro, 2003*).

During the industrial revolution in the 19th Century, the first ironclad steam powered and very large ships were introduced (*Great Eastern*, 1858), thus, the demand for even more thorough and practical approaches to ship's floatability and stability rapidly increased. Historical developments in ship's subdivision (and *damage* stability, which is ship's stability in case of loss of her watertight integrity, e.g., by collision, grounding etc.) in the 20th Century were marked by the most notable ship disaster, namely the dramatic loss of the *Titanic* on her maiden voyage (1912). This tragic accident mobilized the international maritime community and initiated the first international convention on the **S**afety **O**f **L**ife **A**t **S**ea (SOLAS) in 1914, noting, however, that the first international regulatory provisions regarding ship stability and subdivision were actually introduced only after WWII, namely in SOLAS 1948. This convention led also to the foundation of the **I**nternational **M**aritime **O**rganization (IMO,), which is today the United Nations specialized agency responsible for improving maritime safety and preventing pollution from ships worldwide.

More recent scientific and regulatory developments in the *intact* and *damage* ship stability concern the *dynamic stability of ships in waves* and considerations of ship's overall safety against capsize and other hazards for the various types of merchant (and naval) ships, with emphasis on the stability of passenger ships, for which the risk of loss of many lives on-board should be kept minimal. After the loss of *Estonia* in 1994, particular emphasis has been placed on improving the design of ferries (**R**oll-**O**n **R**oll–**O**ff or Ro-Ro ships, namely ships with large, unobstructed (car) deck areas) and to account for the possible flooding of the car deck which may have disastrous consequences on ship's survivability. Also, the safety of recently introduced ultra large passenger/cruise ships, carrying close to 9,000 passengers and crew (see the recently introduced *Genesis* class cruise ships) is of particular concern to the maritime industry, regulatory authorities and researchers, because of their innovative character and unprecedented size, requesting a thorough understanding of ship's stability behaviour in case of loss of watertight integrity (and of other main hazards, like fire etc.) and provisions of highest safety standards.

Developments in the methodology of ship's stability show the traditional *deterministic* assessment methods more and more being displaced by *probabilistic* and *first principles* approaches to ship's stability and safety, which are eventually integrated in *risk-based* design and operational procedures. For a recent review of developments in the *intact and damage stability* of ships, see reviews by *A. Francescutto (2007) and A. Papanikolaou (2007)*. Nevertheless, the fundamental laws of buoyancy and stability of Archimedes are carried on as most important ingredients of scientific and regulatory approaches to ship's floatability and stability over the centuries, as will be elaborated in the following sections.

2. GENERALIZATIONS OF ARCHIMEDES' PRINCIPLE

Archimedes' floatability principle may be derived from the simple, yet revolutionary, observation that a solid body *floats* in water (or at least has a *reduced weight*), although subject to its weight due to gravity because of an upward force, namely *buoyancy*, which is proportional to the displaced water mass. There is, however, an important difference in the characteristics of the two acting forces, i.e., the body's weight and buoyancy: the first is a force acting at the center of mass of the body, while the observed *buoyancy* is the resultant of the pressure forces exerted on the wetted surface of the body, which is acting through the imaginary center of the displaced water mass, or through the center of the immersed volume of the body, named the *center of buoyancy*. When interpreting the validity of Archimedes' principle in this form, complicated surface integrals of pressure forces are replaced by more convenient calculations of volumes and centers of volumes, which is a notable application example of Gauss' theorem in *Ship Geometry*.

An important departure from Archimedes' principle is related to the buoyancy in a *liquid in motion* under the effect of some external disturbance: typical is the case of a body floating in the presence of waves as more or less unavoidably happens to actual ships. This generalization does not allow us to use the simple calculations of volumes and centers of volumes of ship's hull, when floating in calm water, because nor the instantaneous pressures on the body (which include hydrodynamic effects) nor the actually wetted ship surface can be easily expressed mathematically, thus an exact pressure integration over the wetted surface is actually necessary. Several approximations are used in practice to address this complicated problem, except in modern nonlinear numerical simulation approaches: typically, the resultant action (force and moment) is calculated assuming a decomposition of the total pressure acting on the wetted surface in a principal part or the so-called *Froude-Krylov component* consisting of the pressure the actual wave would have exerted on the ship hull if the wave would not have been disturbed by the presence of the ship (*ghost ship*), and in other parts referring to the disturbance produced to the wave by the ship at rest (*diffraction* component) and by the motions of the ship (*radiation*). In typical modern approaches to ship motions in waves, these components are further decomposed and rearranged to give added (hydrodynamic) mass, hydrodynamic damping, restoring and forcing (wave excitation) terms in the equations of ship motion, which are obtained when applying Newtonian dynamics to the ship.

For the purpose of the following discussion, focusing on buoyancy and stability, we just note that besides modern numerical simulation methods, present practical approaches to ship's stability mostly focus on *static* stability characteristics in calm water, whereas those explicitly taking into account the effect of waves (ship *dynamics*) are based on the Froude-Krylov (Froude, 1861, Krylov, 1898) hypothesis and may eventually consider further simplifications of the wave effects based on the assumption of hydrostatic dependence of pressure under the actual free surface or the so-called Smith effect (Smith, 1883) averaging the effect of the orbital motion on the pressure with respect to distance from the free surface.

3. THE LINK OF SHIP BUOYANCY, STABILITY AND SUBDIVISION TO THE ARCHIMEDEAN WORK

Sinking because of insufficient buoyancy and capsizing due to insufficient stability are two of the most important threats to ship's survivability at sea. The safety from sinking and capsizing is thus an important part of the safety of navigation with the entailed safety of the life of people onboard, of carried cargo and with respect to the protection of environment in waterborne transportation.

The most characteristic discipline of Naval Architecture known as Buoyancy and Stability is directly founded on the roots of the Archimedean buoyancy principle, while it is less clear if his early findings about the *stability* of floating paraboloids (in Archimedes' treatise *On Floating Bodies*) remained unexploited for centuries (or were simply ignored or not referenced) and what was actually their impact on later developments in ship stability.

The development of Ship Stability as a science, came indeed much later, namely in the 18th century with approaches to ship stability on the basis of the concepts of metacenter and of uprighting moments. While ship buoyancy was and is a well defined notion, not the same can be said, in general, for ship stability. This is indeed a quite complex ship property, which is traditionally thought composed of the *initial stability* and the *stability at large inclinations*;

The *initial* stability is the stability *in the small* region around the upright position in the sense of theoretical mechanics, and the boundedness within the prescribed limitation of the transversal inclinations, either static or as a result of roll motion. When a ship is in upright hydrostatic equilibrium, weight and buoyancy constitute a couple of forces acting on the same vertical line through the center of gravity and the center of buoyancy respectively. Trivially, when the body is fully submerged (like a submarine) and the center of the weight force is below that of the buoyancy force, the resulting moment is uprighting and the body stable, this being the *only* stable position for the body. For a floating body, like a ship, a (small) inclination φ gives rise to two wedges of immersion/emersion, whose result is to displace the center of buoyancy so that weight and buoyancy no longer act along the same line and their moment can be uprighting or further overturning-heeling, so qualifying the ship as (initially) stable or unstable. The criterion of stability is here whether the center of the weight force G is below an *imaginary* center M (*metacenter*), or the distance GM is positive, noting that the position of the metacenter above keel-line KM is determined by ship's wetted hull form properties, namely as the sum of the vertical distance of the center of buoyancy above keel-line KB and the height of the metacenter above the center of buoyancy BM; the latter is determined by the ship's waterplane area properties (and is trivially zero for fully submerged bodies). For small inclinations φ, the uprighting moment is simply proportional to ship's displacement Δ times GM times φ.

For the stability at *large inclinations,* it is not sufficient to look only at the sufficiency of the position of the metacenter; rather more, we need to control the uprighting moment at large heeling angles and in particular the behaviour of ship's uprighting arm, which at small inclinations is equal to GM times φ, as a function of the heeling angle $GZ(\varphi)$; the characteristics of the $GZ(\varphi)$ curve (*static* righting lever) and of the underneath area (*dynamical* righting lever), which are unique for every ship hull form and loading condition, are then used in judging on ship's stability at large inclinations and consequently in formulating the ship's stability regulations.

In Archimedes' treatise *On Floating Bodies* the stability at *large inclinations* was addressed without use of the notion of metacenter, which is anyway an artificial (imaginary) center, but applying correctly the concept of balance of the couple of weight and buoyancy force. This was at that time possible for a paraboloidal solid, due to previous notable findings of Archimedes in the geometry of shapes. It is noted, that the parabolic *section*, which is not so far from slender shiplike sections, has the notable geometric peculiarity that, in upright position, the metacentric radius BM, which is an important part of form stability, is independent of the immersion of the body. It took some centuries for the naval architects to mathematically capture satisfactorily the geometrical properties of more complicated 3D shiplike hull forms and to better quantify ship stability. The revisiting of the Archimedean studies on the stability of the paraboloid is still exciting many mathematicians until today (*Rorres, 2004, Girstmair and Kirchner, 2008*).

The need of an internal *watertight subdivision* of the hull in compartments to resist the effect of damage (breach) of the hull and the subsequent flooding is in itself evident, but became more urgent when passing from wooden made ships to iron made ships and the ship size, as the number of people onboard, increased. Unfortunately, what we now consider *evident in itself* came into practice very slowly, too often as a late response to disastrous accidents.

It is indeed important to recall the interrelations between the different aspects of ship safety: buoyancy, stability and subdivision. A ship shall be disposing sufficient *reserve* buoyancy and stability in the intact ship condition to resist the environmental actions and the effect of possible hull damage. In particular, the *reserve* buoyancy, which is ensured by a minimum *freeboard* to ship's main deck constituting the upper limitation of the watertight ship body, has an important and positive effect not only on floatability, but also on the *reserve* (a margin) of stability at large inclinations. Two ships of similar dimensions, having the same initial stability (GM value) can have very different stability at large angles depending on their freeboard, as the tragic accident of the *HMS Captain* versus the comparable *HMS Monarch (two ships having comparable GM but very different freeboard),* demonstrated (*Reed*, 1870). It is fair to say that the problem of floatability and stability in damage condition was not addressed by Archimedes; however, the underlying basic concepts remain the same, namely the control of a balance of weight and buoyancy forces and moments, considering the effect of flooded water due to hull damage; this is commonly addressed as an *added weight* or *lost buoyancy* force, but else the Archimedean principles of floatability and stability fully apply.

Concluding as to the impact of Archimedes on current stability concepts, the various *static* stability quantities required for the assessment of compliance with present (large angles) stability criteria are still based on hydrostatic calculations in calm water, exactly as it would have been done by Archimedes if he would have generalized his calculations passing from simple mathematical forms to generic ship forms. The new notion included in the assessment, namely the *dynamic* stability or righting arm curve of areas, was at that time far from the scientific developments, as the concepts of work and energy had to wait for some later centuries to be introduced.

The present format of the assessment of initial stability is based on the concept of metacentric height GM and hence of metacenter. This is explained by the strong tradition in naval architecture and the fact that a single quantity/metric, like the metacenter, can effectively characterise the sufficiency of ship's stability at small heel angles. For the assessment of ship's stability at large inclinations, the characteristics of the uprighting moment arm lever $GZ(\varphi)$ are employed, noting that trivially the slope of the $GZ(\varphi)$ curve at $\varphi = 0$ is equal to GM.

4. THE DEVELOPMENT OF INTERNATIONAL REGULATIONS FOR ADEQUATE BUOYANCY, STABILITY AND SUBDIVISION

The first international regulations addressing in some way floatability and stability issues were the *Minimum Freeboard* and the *Subdivision* regulatory provisions. Relevant regulations concerning these parts of maritime safety have been previously developed in the frame of national bodies until the sinking of *Titanic* in 1912; this most remarkable accident in naval architectural history led to the first ever International Conference on the Safety of Life at Sea (SOLAS); that conference resulted to the first text of the SOLAS Convention, containing the basis of what was successively developed to become the so-called *factorial* subdivision; it was signed on January 20th, 1914, in London. The Convention met again in 1929 and 1948 and it was then taken onboard by the newborn IMO.

The *Minimum Freeboard* regulations have also a quite long history. They were first regulated in England in 1876 as a result of the efforts of *J. Hall* and *S. Plimsoll*, in an attempt to control ship's overloading by specific a minimum clearance of ship's open deck from her calm water draft. The first related international conference, however, was not convened until 1930 (*1st International Load Line Convention*).

Quite different is the history of developments of the stability regulations. Damage Stability came practically first with SOLAS'48, in relation to ship's subdivision and several decades after the loss of Titanic. Even later was the introduction of the first Intact Stability provisions, which were initiated by calls in the conclusions of SOLAS'60 and of SOLAS'74 International Conventions.

An accelerated change in international maritime law is noted after the United Nations family of organizations came into life after WWII, and here particularly the IMO. Developments at IMO until the end of the 80ties were still quite slow, with relatively small changes initially agreed regulations.

From the 90ties on, strong changes were triggered at IMO, first with respect to the Load Lines Convention through the findings about the sinking of the young age British capsize bulk-carrier *Derbyshire* in the early 80ties, which disappeared with all her crew in monster wave conditions. The Intact Stability Code was put in discussion in 2001 as a consequence of some inconsistency in the application of existing regulations to the design of large passenger vessels, continuously growing in size; also the spectacular accident of the containership APL-China in the Pacific revealed that *Parametric Rolling* of ships in waves may be a much more dangerous issue than expected.

The revision of the *Load Lines Convention* ended in 2000. The newly introduced features are related to a major role given in the development of the new standard to ship's seakeeping properties, to the requirements for the minimum bow height, the reserve of buoyancy, the height, strength and watertightness of the forecastle, of the cargo hold hatch covers and the strength of the forward compartment bulkheads.

The present situation and trends as far as *the intact and damage stability regulations* are concerned will be briefly addressed in the following two sections. The reader interested in greater details is referred to Francescutto (2007) and Papanikolaou (2007).

5. INTACT STABILITY OF SHIPS – RECENT REGULATORY DEVELOPMENTS AND TRENDS

The latest revision of the International Intact Stability Code, which started in 2001, led to the *2008 IS Code*. Further to this, the need for new criteria, based on more realistic physical approaches was stressed and a rational updated plan of action was decided, consisting in the development of:

- *vulnerability* criteria to identify the possible susceptibility of a ship to partial (excessive roll angles/accelerations) or total (capsizing) stability failures for each mode;
- procedures for direct assessment of: stability failures explicitly taking into account the *dead ship condition*, the stability variations in waves (*pure loss of stability and parametric resonance*) and the connections between stability and course-keeping qualities (manoeuvrability).

The idea of vulnerability criteria, to be developed in two levels (vulnerability and severity) is of paramount importance in the frame of criteria aimed at improving ship safety and making safety improvement more “cost-effective” against modes of failure not covered by present criteria. It could avoid the need for indiscriminate generalized application of heavy computational or experimental procedures (Bassler et al., 2009).

It is generally accepted that the new criteria will require calculations of stability in the presence of waves, probably based on the Froude-Krylov approach. It is not clear at this point, however if the third level criteria, or direct assessment, will be based on the best available computational techniques, including CFD, or if they will retain some level of the empiricism that negatively characterised the present generation of criteria.

6. DAMAGE STABILITY OF SHIPS – RECENT REGULATORY DEVELOPMENTS AND TRENDS

Since the loss of *Titanic* in 1912 and the first SOLAS Convention in 1914, ship damage stability regulations and relevant compliance criteria for passenger ships were slowly but steadily modified over the years, adapting to findings of new ship losses and the continuously improving state of art in the field, though to a lesser satisfactory degree from the scientific point of view. Notably, there were no specific damage stability criteria or sub-division requirements for *cargo ships* until the early 90ties, when SOLAS was amended to cater for *dry cargo ships' damage stability* by use of the so-called *probabilistic* concept.

The damage stability requirements for passenger ships, which were in force until very recently (namely, until the end of 2008), were *deterministic* or *rules-based assessment* concepts in nature; so, the so-called SOLAS 90 – *two compartment standard*, which was associated with stability criteria to ensure the survivability of the ship in case of flooding of up to *two adjacent compartments*; smaller passenger ships were in general of *one compartment* standard, whereas very large ships may have had *2+ and higher compartment standard*, depending on their size and number of people carried onboard; the standard was practically a half-empirical concept developed continuously over the years, namely by the analysis of damage cases and of stability data of ships that led to ships' capsize/sinking vs. the data of ships considered to be of “state of the art” in terms of stability/floatability properties.

Two very tragic accidents of *non SOLAS 90 European ships* in the last two decades (*Herald of Free Enterprise*, 1987 and particularly *Estonia*, 1994), in which both ships sunk because of the flooding of their main car deck, laid to an enhancement of the requirements of SOLAS 90 by the so-called *Stockholm Agreement* provisions, for all ferry ships operating regionally in NW Europe; these enhanced provisions were later on extended to all over Europe and to all developed countries, worldwide. *[IMO, Resolution 14, 1995], [Vassalos-Papanikolaou, 2002]*.

The modern *probabilistic* approach to the assessment of the damage stability of ships was introduced by the German *Kurt Wendel* in the late 50ties *[Wendel, 1960]* aiming at introducing a more rational method for the assessment of the probability of survival of a ship in case of damage (breach of ship's outer shell by collision or grounding). Additionally, the introduced assessment method allowed the definition of a global 'safety factor' (*Sicherheitsgrad*) through which the stability characteristics of ships of different size and type became directly comparable. The approach leads eventually to the determination of two characteristic safety factors for the ship under consideration. The first factor is the so-called *attained subdivision index* A, representing a measure for the probability of survival of the ship in case of a statistically probable damage. The second factor, namely to so-called *required subdivision index* R, is the minimum value for the attained index A and represents a generally accepted (imposed in regulations) survival level for the ship under consideration, corresponding to her size and the number of people onboard exposed to the collision hazard. The method allows conceptually through systematic application the optimization of the watertight subdivision of ships for the least number of watertight bulkheads at the greatest possible degree of safety against capsize and sinking. A fundamental property of the probabilistic damage stability method is the possibility to integrate a variety of general type of ship damages in a overall safety assessment risk-based concept (*risk-based design, operation and regulation, see, SAFEDOR, 2005–2009*, *Papanikolaou et al., 2009*).

Though the new probabilistic damage stability regulations for dry cargo and passenger ships (SOLAS 2009), which entered into force on January 1, 2009, represent a major step forward in achieving an improved safety standard through the rationalization and harmonization of damaged stability requirements, there are still serious concerns regarding the adopted formulation for the calculation of the survival probability of passenger ships, particularly for ROPAX and very large cruise vessels. Furthermore, the SOLAS 2009 damage stability regulations account only for *collision damages*, despite the fact that accidents statistics, particularly of passenger ships, indicate the profound importance of *grounding* accidents. A recently initiated EU project (GOALDS, 2009–2012), with strong partnership representing all stakeholders of the European maritime industry and relevant R&D organizations, aims to resolve the above critical issues in the next few years.

7. CONCLUSIONS

Looking into the scientific and regulatory developments in ship floatability and stability 23 centuries after Archimedes, it is trivial to say that developments, introduced slowly, have been significant, thus greatly improving the safety of ships and the people and cargo onboard even in very harsh environmental conditions. Transportation by ship, especially of bulk cargo, remains the most efficient and environmental friendly mode of transport. The Archimedean principles of buoyancy and stability of floating bodies are still governing ship's hull form development and design.

Time scales of most recent related developments (last two decades) were reduced drastically, owing to the fact that scientific approaches to ship safety came to maturity and expectations of society regarding maritime safety are extremely high.

An evident new development in maritime regulatory matters, including those related to ship's stability and subdivision, is the introduction of *proactive* rather than *reactive* methods. This is entirely in the frame of so-called Formal Safety Assessment (FSA) procedures, in which safety regulations and properties (like ship stability) are assessed in terms of *societal acceptance criteria*, eventually postulating an acceptable number of fatalities for people onboard of ships per year. Related to FSA procedures are innovative *holistic* approaches to ship design, namely *Risk-Based* Ship Design (RBD), thus design for acceptable risk levels, and design for *Goal-Based* Standards (GBS), currently in the focus of discussions at IMO.

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THE "SYRAKOUSIA" SHIP AND THE MECHANICAL KNOWLEDGE BETWEEN SYRACUSE AND ALEXANDRIA

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ABSTRACT In a very famous passage dealing with the life of Marcellus, Plutrarch says that Archimedes never wrote a text about mechanics and its practical applications (Plutarch, Life of Marcellus, 17): according to Plutarch, in fact, Archimedes would agreed with the classical Plato's attack against the knowledge originating from technology and the practice of science because of their vague and inaccurate nature. This paper, focusing on the building of the famous ship "Syrakousia" and its description according to the only existing reference in Atheneus of Naucratis (Deipnosophistae, V, 40–44), is an attempt to rethink Archimedes' position about mechanical knowledge and the cultural relationship between Syracuse and Alexandria.

1. INTRODUCTION

This paper focuses on the "Syrakousia" (fig. 1), the biggest ship ever built in Antiquity. After a brief description following the ancient literary sources, its goal is to understand the possible meaning of such impressive, oversized boat.

The "Syrakousia" story brings us to king Hieron II and Archimedes Syracuse (Midolo 1912; Favaro 1923; Dijksterhuis 1987; Geymonat 2006; Chondros 2007) and our knowledge of it relies upon literary sources. Even if the archaeological underwater discoveries have been adding new data to our ancient navy knowledge, the Syrakousia had to be an extraordinary, out of scale boat and the few existing illustrations of it are a product of fantasy.

In Archimedes' life ships seem to play an important role: enemies boats were the target of the "Manus ferrea" (fig. 2), the famous mechanical device, quoted by ancient authors, that Archimedes invented to lift and crash ships into the water during the second Punic war (Polybius, History, VIII, 6; Livy, History of Rome, XXIV, 34; Plutarch, Parallel lives. Marcellus,

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15; G. Tzetzes, Chiliades, II, 109–113; Zonara, Epitome ton istorion, IX, 4); moreover, Archimedes probably invented the burning mirrors, a scientific knowledge turned to become a new war technology used to burn the enemies ships (G. Tzetzes, Chiliades, II, 118–128; Di Pasquale 2004, pp. 31–76); finally, his studies about the equilibrium of floating bodies into the water were the necessary introduction to any future practical application to ship building (Nowacki 2002).

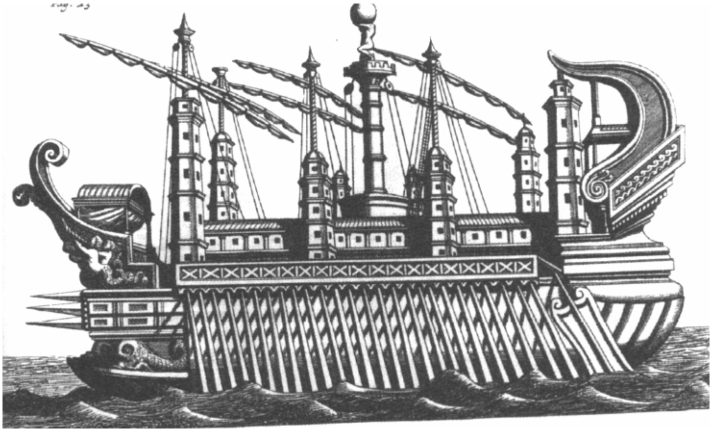


Fig. 1. The Hieron II Syrakousia ship (Witsen 1671).

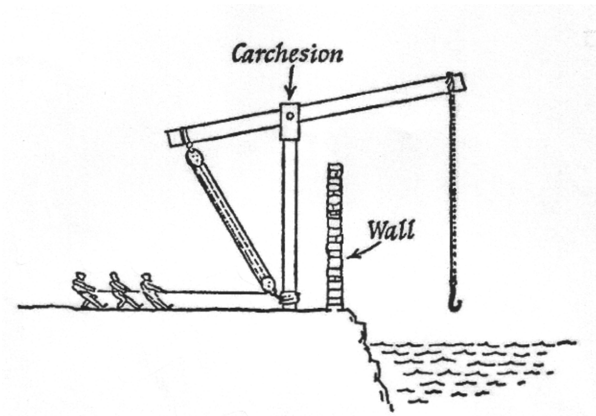


Fig. 2. The mechanical device of the manus ferrea, (J. G. Landels 1978).

This story deals with another ship and begins with a very famous passage from the life of Marcellus: according to Plutarch, Hieron II asked Archimedes to give a public demonstration about his mechanical knowledge (Plutarch, Parallel lives. Marcellus, 17; see also Loria 1928).

The demonstration Archimedes decided for concerned the possibility of moving, by himself, an enormous object. According to some ancient and middle ages literary sources the object chosen by Archimedes was the Syrakousia ship (Drachmann 1958, p. 279); our knowledge about this ship relies on Moschion, an author whose writings have gone lost.[1] The original Moschion's description of the Syrakousia survives thanks to Atheneus, a Greek philologist who lived in Rome at the end of the II century A.D. and wrote the Deipnosophistae: being interested in ancient navy and big ships of the passed centuries, Atheneus quoted the passage Moschion describes the Syrakousia (Atheneus, Deipnosophistae, V, 40–44).

The wood used to build the ship came from the forest of the Etna volcano, the same amount would have been sufficient to sixty normal size boats; moreover, the wood for the furniture came from Calabria, ropes from Spain, hemp and pitch from the river Rhone in France. Hundreds of artisans worked under the direction of the architect Archia from Corinth and they finished half the ship in six months: Archimedes superintended their work.

The fabrication of the ship originated a great inquiry about the best method of launching into the sea, so Archimedes decided that this would be the public demonstration that the king Hieron was waiting for: "[…] and then drew it along, smoothly and evenly as if it was floating in water, not with great labour, but sitting down at a distance, gently swinging with his hand the end of a compound tackle" (Plutarch, Parallel lives. Marcellus, 14; see also Proclus in Friedlein 1873, p. 63).

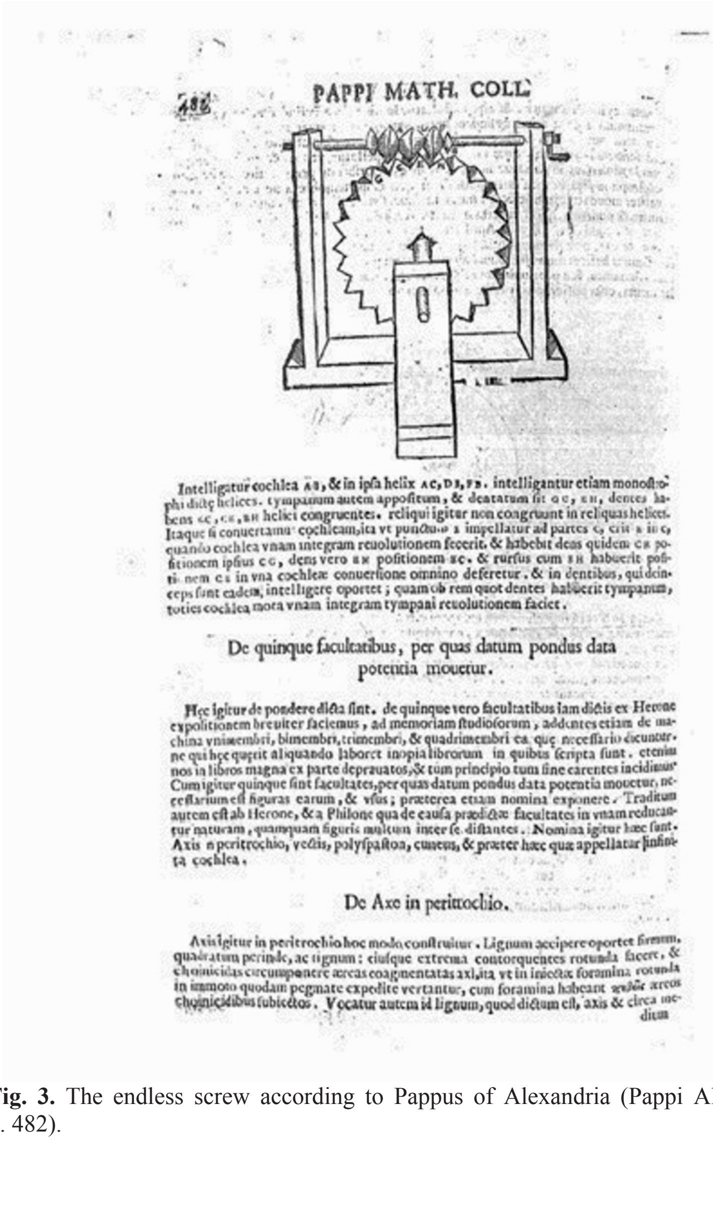
After this incredible demonstration, Archimedes probably pronounced the famous Δος μοι που στω, και κινω την γην, that is "give me a place to stand and I shall move the world" (G. Tzetzes, Chiliades, II, 29–130).[2] King Hieron suggested that there must be some limit, but Archimedes had demonstrated the contrary to be true, above all if the mechanical device he used was a windlass turned by means of an endless screw (fig. 3).

Now, why did Archimedes chose this experience? Why in Archimedes' mind, this had to be his first public demonstration? Archimedes wanted to focus on a problem that Aristotle had considered impossible. According to Aristotle, the lever and every mechanical combination based on the five

[1] According to Atheneus, Moschion wrote a book about Mechanics. In this book, Moschion demonstrated to be a keen observer and scholar in recording the history of mechanical inventions, when credited a certain Herakleids from Tarentum to be the inventor of a weapon called sambuké (Atheneus, *Deipnosophistae,* 14): "But Moschus, in the first book of his treatise on Mechanics, says that the sambuca is originally a Roman engine, and that Heraclides of Pontus was the original inventor of it".

[2] Even if without pronouncing exactly these words, the first record of this famous Archimedes claim is to be found in Plutarch, *Parallel lives. Marcellus,* 14.

basic machines (winch, pulley, lever, screw and wedge) had no power in complicated situations such as the moving of huge loads; the example Aristotle choose dealt with a boat: in fact, according to him, one man, by himself, cannot launch or move a boat with a lever or a combination of basic machines; on the contrary, he declared that this operation was possible only if a team of workers do it (Aristotle, Physica, VII, 5, 250a, 1–19; see also Mugler 1951 and 1970–1972).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Intelligatur cochlea AB, & in ipsa helix AC, DE, FB. intelligantur etiam monostrophi dictę helices. tympanum autem appositum, & dentatum sit GC, EH, dentes habens CG, CE, EH helici congruentes. reliqui igitur non congruunt in reliquas helices. Itaque si conuertamus cochleam, ita vt punctum A impellatur ad partes C, erit A in C, quando cochlea vnam integram reuolutionem fecerit. & habebit dens quidem CE positionem ipsius CG, dens vero EH positionem EC. & rursus cum EH habuerit positionem CE in vna cochleæ conuersione omnino deferetur. & in dentibus, qui deinceps sunt eadem, intelligere oportet; quam ob rem quot dentes habuerit tympanum, toties cochlea mota vnam integram tympani reuolutionem faciet.

De quinque facultatibus, per quas datum pondus data potentia mouetur.

Hęc igitur de pondere dicta sint. de quinque vero facultatibus iam dictis ex Herone expositionem breuiter faciemus, ad memoriam studiosorum, addentes etiam de machina vnimembri, bimembri, trimembri, & quadrimembri ea quę necessario dicuntur. ne qui hęc quęrit aliquando laboret inopia librorum in quibus scripta sunt. etenim nos in libros magna ex parte deprauatos, & tum principio tum fine carentes incidimus. Cum igitur quinque sint facultates, per quas datum pondus data potentia mouetur, necessarium est figuras earum, & vsus; præterea etiam nomina exponere. Traditum autem est ab Herone, & a Philone qua de causa prædictæ facultates in vnam reducantur naturam, quamquam figuris multum inter se distantes. Nomina igitur hæc sunt. Axis in peritrochio, vectis, polyspaston, cuneus, & præter hæc quæ appellatur infinita cochlea.

De Axe in peritrochio.

Axis igitur in peritrochio hoc modo construitur. Lignum accipere oportet firmum, quadratum perinde, ac tignum: eiusque extrema contorquentes rotunda facere, & choinicidas circumponere æreas coagmentatas axi, ita vt in iniectæ foramina rotunda in immoto quodam pegmate expedite vertantur, cum foramina habeant æreos choinicidibus subiectos. Vocatur autem id lignum, quod dictum est, axis & circa medium

Fig. 3. The endless screw according to Pappus of Alexandria (Pappi Alexandrini 1660, p. 482).

Plutarch does not tell us how far Archimedes moved the ship, but since it was only a demonstration, few yards would have been enough making the story more credible, and a little movement was enough, as Drachmann already pointed out (Drachmann 1958).

The launching of the Syrakousia ship was the occasion Archimedes chose to demonstrate that Aristotle was wrong and, above all, to emphasize the wonderful and amazing power of the practice of science and the high educational value of the public demonstration.

2. A BRIEF DESCRIPTION OF THE SHIP

Summarizing directly from Atheneus' description the most important elements of the Syrakousia, we have to figure out that it consisted of three levels and giant sculptures representing Atlas were the dividing pillars (Zevi 2005; Pomey–Tchernia 2005). The cabins were decorated with paintings and wonderful mosaics representing the whole stories of the Iliad; there were eating rooms and baths with hot air circulation system and sweet water flowing from a 78 tons boiler through lead pipes; stables for horses were on each side of the boat, a salt water pool with fishes was on the prow; there were a gymnasium, promenades, gardens with all sorts of plants and a vineyard with an impressive water circulation system; next to the garden, there was a temple dedicated to Venus, “[...] containing three couches, with a floor of agate and other most beautiful stones of every sort which the island afforded” (Atheneus, Deipnosophistae, 41).

The ship had three masts: the problem of transporting them from the mountain down to the seaside was solved by Phileas of Tauromenion, another mechanic involved in the fabrication (Atheneus, Deipnosophistae, 43). The Syrakousia contained a library too, with a ceiling decorated with a painting representing the heavens: in the library there was another astronomical reference, a sundial “imitated from the dial at Akradina” (Atheneus, Deipnosophistae, 42); a sculpture representing Atlas holding on his shoulders a heavenly globe was situated at the top of the central mast of the ship; all these artefacts had to underline the astronomical knowledge necessary to navigators and the meeting between maritime experience and science.

The Syrakousia had a defensive apparatus too: on the upper level there were eight siege towers with catapults and boxes full of darts and stones; there was a deck too, with a new catapult capable of hurling a stone weighing thirty talents and an arrow twenty cubits long, invented by Archimedes himself. On each side of the boat there were mobile hooks used to catch enemies ships, a device forerunning the famous manus ferrea Archimedes will design and use during the second Punic war against the Roman fleet.

Technical apparatus of the ship consisted of twenty rows of oars, eight anchors and a set of screws to lift water. Finally, the ship transported sixty thousand measures of corn, ten thousand of Sicilian salt fish, twenty thousand talents weight of wool, for a total load amounting at 4000 tons (Casson 1971 and 1991; an expert in the field of ancient seafaring, L. Casson confirms the literary information and considers the Syrakousia the biggest ship of Antiquity).

3. THE POSSIBLE MEANING OF THE "SYRAKOUSIA" SHIP

Built in Syracuse after the middle half of the III century B.C., the Syrakousia not only was the symbol of the king Hieron II power, but also underlined the incredible fertility of the Sicilian land; above all, the Syrakousia let us understand the high level of both Archimedes' and Syracuse scientific knowledge and practice of science. Not by chance, its final destination is Alexandria, the place where, thanks to the famous cultural institutions, the library and the museum, a new high technical and scientific knowledge had been developing (Di Pasquale 2004, Russo 1996).

Archimedes had already spent some time in the Museum of Alexandria, being in touch with important scholars such as Eratosthenes, Conon and Dositheus; Dositheus was a friend or pupil of Conon, and on the latter's death, Archimedes, who had been in the habit of sending his mathematical works from Syracuse to Conon for discussion in the scientific circles of Alexandria, chose Dositheus as the recipient of several treatises. We can imagine Archimedes spending his time into the famous library, acquiring the basic mechanical knowledge by reading the old texts written by Poleidos and his pupils Diades and Carias, the engineers following Alexander's the great army, or by Biton and Architas of Tarentum (Thevenot 1693); above all, attending the library Archimedes learned the new development of mechanics in Alexandria. Ctesibius, the founder of the mechanical school, wrote at the beginning of the III century B.C. a Mechanikà, a mechanical encyclopaedia which clearly described the development of the practice of science at the moment (Ferrari 1984): theory of the lever, catapults fabrication, harbours and fortresses building, pneumatics, automata making and stratagems were the main topics of the new mechanics developing in Alexandria (Marsden 1999).

The Pseudo Aristotelic text on Mechanics, probably appeared at the beginning of the III century B.C., dealt with the hidden presence of the lever in a set of tools and devices, in order to explain their working thanks to the wonderful properties of the circumference (Micheli 1995). In Alexandria, it is now time for the transition to a new mechanics: as a consequence, the theory of the lever ceases to be the whole mechanical knowledge, becoming a single chapter inside a more complete set of knowledge dealing with the new practice of science.

In the middle of the III century B.C. Philon of Byzantium, another scholar in mechanics, writes a mechanical encyclopaedia dealing with the same Ctesibius' topics: this time too, the theory of the lever occupies a single chapter of the text.[3] We can imagine that once back to Syracuse Archimedes, even if he never wrote a text about ta mechanikà, started giving public demonstrations about his knowledge. From this point of view, the Syrakousia ship will be one of his most amazing public demonstrations, a mechanics summary sent from Syracuse to Alexandria by Hieron II and Archimedes.

In fact, the Atheneus description records several objects dealing with mechanical knowledge. The ship contains eight siege towers: the siege tower design, scale model, fabrication and final equilibrium were the hard work of a generation of skilful engineers who, for the first time during the V and IV century B.C., decided to write treatises containing technical drawings too (gone lost), in order to make clear their knowledge and to introduce the practice of science into the high level of the official culture. The fabrication of the perfect siege tower was a very hard task and ancient literature tell us many stories about their sudden collapse due to a wrong construction practice. Actually, when Atheneus talks about the siege towers of the Syrakousia, he underlines that "their size was proportioned to the burden" (Atheneus, Deipnosophistae, 43).

Even if the siege tower was overtaken by the invention of the catapult, its correct construction will be described, three centuries later, by Vitruvius (Vitruvius, De architectura, X, 19). The Syrakousia contained a set of catapults and among them was the one invented by Archimedes, which was capable of throwing very heavy stone projectiles. Catapults were a successful, significant invention: Hellenistic kingdoms and cities were in competition each other to obtain the most important engineers, whose services in terms of both money and prestige were great. Invented according to Diodorus (Diodorus of Sicily, XIV, 2, 2) in Syracuse during the IV century B.C., the catapult probably became a symbol of the innovative technological tradition developing in Syracuse. In any case, war industry was always supported by the Hellenistic kings, notably the Ptolemies of Egypt (Hacker 1968). Since

[3] Perhaps, Archimedes too collected some of his writings in a kind of mechanical collection: in fact, in the *Quadrature of the parabola* (VI, 10) he defines his writing *On planes equilibrium* a *Mechaniká* treatise. As a consequenze, he might have intended his writings *On planes equilibrium,* and the lost texts on *Sphere making* and *On beams* (Heron D'Alexandrie 1988, I, 25), the topics of a different mechanical encyclopaedia (Di Pasquale 2004, pp. 91–103).

the time of their invention, catapults had become the object of new theoretical and practical studies: their fabrication and working will be one of the chapters of the new mechanical encyclopaedias written by Ctesibius and Philon of Bizantium during Archimedes' life.

The Syrakousia presented water and hot air circulation systems for gardens and baths: water and air were the secret moving power of several pneumatics devices, and pneumatics was one of the most popular topics of the new mechanical science developed in Alexandria (Ferrari 1985; Brumbaugh 1966; Di Pasquale-Paolucci 2007, pp. 58–71). Both, air and water, had found a practical application in baths and gardens with decorative elements like fountains: the gardens and baths of the Syrakousia introduce, again, another comparison with the famous Alexandria Museum garden.

In fact, the philosophical schools of Plato and Aristotle had originated in green havens, the ideal spots for meditation, on the outskirts of Athens, and a garden was the meeting place and the title of the philosophical school of Epicurus (Repici 2007). From a place dedicated to otium and relaxing, the garden had later become a space devoted to study and experimentation mirroring the variety of nature suddenly discovered after the Alexander's the Great expeditions to East and now summarized in the new Hellenistic passion for luxurious gardens. A wider nature, a wider flora and fauna existed in respect to the one recorded into the books of ancient libraries.

The park of the Library and Museum of Alexandria, a real botanical garden abounding in plants and animals, was a true laboratory of nature, the ideal background for scholars involved in the study of botany, zoology and physical phenomena; microcosm mirroring the variety of the macrocosm, the park of the Museum of Alexandria was the most appropriate space for the innovative studies about plants carried out by Theophrastus and the ones by Aelian about the animals inhabits.

The Syrakousia had got a library too; even if we do not have any information about its content, its presence remind us the ancient authors' information relating the stories of the many ships entering the harbour of Alexandria to carry books from everywhere to fill the wonderful Ptolemies' library.

Some studies have recently demonstrated that it is a mistake to think of ancient libraries as places exclusively devoted to humanities and philosophy; scientific and high technological knowledge had its own place into the libraries, whose decorative elements often remind the reader both cultures: in the library of the Syrakousia there was a sundial and the ceiling was decorated with a painted representation of the sky introduced the passenger of the Syrakousia to the astronomical knowledge and its visual representation.

4. AFTER ALEXANDRIA

We assumed that Archimedes spent some time in Alexandria and that he used to send his writings to scientists working in the library and museum asking them for the approval. The Syrakousia goal is the same: the ship brings the knowledge from Syracuse to Alexandria, conveying to an impressive public, practical demonstration of the new mechanical science produced in the city of Syracuse and from there sent to the place where mechanics was developing (Traina 2000, p. 33).

A travelling encyclopaedia of mechanics, the Syrakousia seems to be the Syracuse and Archimedes answer to Alexandria and its developing mechanical knowledge. It is the equivalent of sending there a book, the mechanical treatise that Archimedes, according to Plutarch, never wrote. Archimedes and the future generations of mechanical scholars design the public scientific and technological demonstration as a crucial moment: if the growing mechanical knowledge could no longer be contained in a set of canonical texts, perhaps could be displayed thanks to impressive human artefacts (apart from the single case of the Syrakousia ship, it’s sufficient to consider the Hellenistic and Roman Imperial age impressive buildings, a meeting point between architecture and mechanics and a striking demonstration of the new public attitude of the practice of science).

As a paradigm of knowledge, public demonstration contained the parameters of the known culture and its expanding possibilities. The decision of displaying the results of years and years of mechanical research led to define knowledge as consensual, shaped in relation to the audience around kings and patrons.

It is a matter of fact that the Ancients had the greatest admiration for Archimedes and his inventions. The knowledge the Syrakousia ship embodied does not conclude its travel between the city of Syracuse and Alexandria. When Lucianus, being in Athens during the second half of the II century A.D., writes his dialogue “The ship, or the Wishes”, he tells the story of the giant ship Isis that survived a sea storm. Many scholars, today, read the description of the giant ship Isis to try to understand something more about the architecture of the Syrakousia. Nevertheless, there are other interesting information going to create a link between the two boats. In fact, Lucian tells us that the crowd that gathered at the Pireo Harbour to watch the arrival of the ship, stated that the mariners could survive because the boat was driven by the sure hand of an old man who moved the little rod of the enormous rudder (Lucian, The ship, 6).

Once again, four centuries after Archimedes, we find the story of the old little man capable of winning the giant dimension of the boat, the sea power and the storm thanks to a lever (for the diffusion of Archimedes myth in Rome see Jaeger 2008).

And perhaps it is not by chance that in this dialogue the name of the old man capable of driving the giant rudder with a little stick, that is the perfect image of the lever, is Heron, the famous scholar in mechanics at Alexandria and a contemporaneous of Lucian. The Archimedean science appears in another part of the same dialogue, when one of the protagonists declares to find more interesting “anchors, and the cabestan and the windlasses and the stern cabins” than the art ornaments (Lucian, The ship, 5); moreover, we find another important Archimedean echo when Adimantus, observing the ship, (Lucian, The ship, 18) Isis that was full of corn, starts dreaming to be the owner of it, but he wished it had been full of gold; as a consequence of this, his friend Lycinus claims (Lucian, The ship, 19): “Hey! The ship will sink. Corn and gold to the same bulk are not of the same weight”.

Finally, the Syrakousia and its wonderful mechanical content sailed once again between the XVI and XVII century, when ancient texts became the pillars of the new science and the Archimedes’ myth exerted its influence in art too. When Ferdinand I of the Medici decided to move the family scientific instruments collection from Palazzo Vecchio to the Uffizi Gallery, the Gran Duke wanted the room to be painted with an appropriate decoration (Camerota 2008). Thus, at the end of the XVI century (1599 – 1600), the artist Giulio Parigi painted a series of images whose purpose was to underline the cultural value of mathematics and geometry in the new Tuscan state. The main actor of this decorative program is Archimedes: we find not only the famous stories about “Eureka”, his interest in geometry even when spending some time at public baths, the manus ferrea and the burning mirrors, but also the new catapult he designed for the Syrakousia ship, the mechanical device to launch it (Fig. 4), and the lever with which, after that, he could have moved the hearth.

The image describing Archimedes presenting the armillary sphere to Hieron remind us his astronomical studies and the astronomical references into the Syrakousia ship library. Finally, the cupids facing the sphinx with mathematical instruments summarize the meaning of the whole decoration: the victory of reason embodied in Archimedean science against the sphinx, that is the ancient superstition, magic and false belief, a set of knowledge grown up in Egypt, the place where the ship was sailing to, carrying the mechanical science of Archimedes and the city of Syracuse.

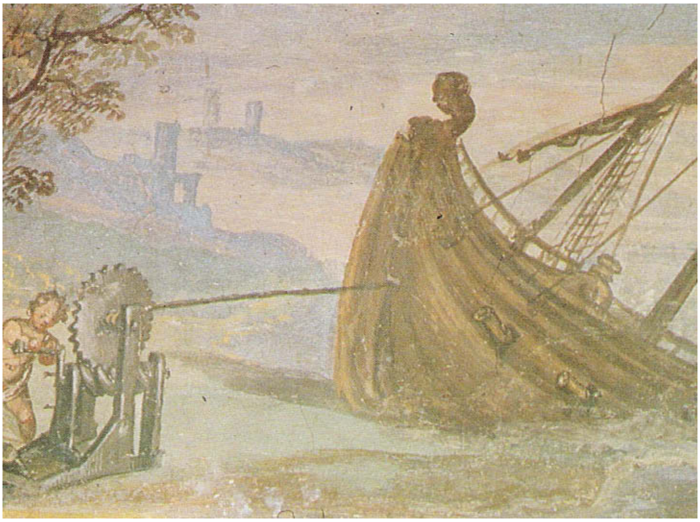

Fig. 4. Giulio Parig (1599–1600), Florence, Uffizi Gallery, “Stanzino delle Matematiche”, Archimedes and the mechanical device to move the “Syrakousia” ship.

5. CONCLUSION

According to the few existing sources dealing with Archimedes’ life, he spent his time between Syracuse and Alexandria, where he attended the prestigious cultural institutions of the Library and Museum. Archimedes was in Alexandria during the new development of the typical mechanical science and treatises were written down by Ctesibius and Philon of Byzantium. Even if Archimedes never wrote a book on “Mechanics”, he superintended the fabrication of the “Syrakousia”, the biggest ship ever built in antiquity, full of scientific, technological and mechanical items: like a travelling mechanical encyclopaedia, the final destination of the ship was the harbour of Alexandria, to show the high level of Archimedes’ mechanics.

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4. LEGACY AND INFLUENCE IN PHILOSOPHY

BROWSING IN A RENAISSANCE PHILOLOGIST'S TOOLBOX: ARCHIMEDES' RULE

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ABSTRACT A letter preserved in a manuscript of the 16th century belonged to Gian Vincenzo Pinelli, shows the high esteem bestowed on Archimedes at that time. An anonymous philologist was working on the critical edition of Pliny the Elder's *Naturalis Historia* and asked a mathematician for help in amending a doubtful *lectio* of a passage in the 2nd book. The philologist's interpretation is discussed and rejected by the mathematician thanks to the use of Archimedes' rule for the approximation of *pi*.

1. INTRODUCTION

The Biblioteca Ambrosiana in Milan houses a miscellaneous manuscript (R 94 Sup.), dating back to the 16th century; it is composed by 44 texts, written by different hands in Greek, Latin or Italian vernacular, about various subjects (geography, politics, mathematics, mechanics, ancient comedy, metallurgy, numismatics and alchemy) by well-known authors [1–3].

The manuscript belonged to Gian Vincenzo Pinelli (1535–1601) from Padua, the famous humanist and polymath owner of a huge library consisting of thousands of books and hundreds of manuscripts [4–7]. On some folios of R 94 Sup., he annotated his own remarks about the passages in the margin.

2. THE MANUSCRIPT CONTENT

F. 178 contains a text that could be either a letter, unfortunately undated and unsigned, without any indication of the recipient, apart from the fact that the sender addresses him in a very formal way, as My Lord («*Vostra Signoria*») or part of a dialogue, a very common Renaissance genre, modelled upon Plato's works. The text subject is the emendation of a

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passage taken from Pliny the Elder's *Naturalis Historia* (2, 21) and the discussion about the computation contained in two sentences.

In the 2^{nd} book of his work *Historia naturalis* [8], Pliny describes the world and in §21 he tries to estimate its circumference length including atmosphere, grounding on Ptolemaic cosmology and on geometry. Pliny is aware that not all scholars agree about the extension of this space, full of mist, clouds, and winds between the Earth and the Moon, and he admits that claiming to calculate it precisely would be an almost foolish pastime («*id enim velle paene dementis otii est*»). The Latin author only wants to apply a simple geometrical rule, in order to roughly estimate that value; while doing this, he implicitly refers to Archimedes' rule about the ratio between diameter and circumference, by saying that the diameter is always the third part and a little less than the seventh of the third of the circumference («*semperque dimetiens tertiam partem ambitus et tertiae paulo minus septimam colligat*»). Some lines after, Pliny also gives a numerical (approximated) example: it is evident that, if a circumference is divided into 22 parts, its diameter length will be seven times the length of these parts («*quantas enim dimetiens habeat septimas, tantas habere circulum duoetvicesimas ... constet*») and concludes that by this means we could measure the heaven as we could do with a plumb («*tamquam plane a perpendiculo mensura caeli*»).

The two sentences above are quoted and analysed in the 16^{th} century letter from both a linguistic and a mathematical point of view. The anonymous sender (in the following lines we will call him the mathematician, in order to simplify our expression), as we can easily argue from the text, must have been solicited by the recipient (the philologist) to discuss about his proposal of philological emendation about a passage of Pliny's text. He proposes to correct the *lectio* «*semperque dimetiens tertiam partem ambitus et tertiae paulo minus septimam colligate*», by deleting the conjunction *et* and the final *m* in the word *septimam*, in order to obtain «*semperque dimetiens tertiam partem ambitus tertiae paulo minus septima colligat*» and to define an algorithm for the calculation of the diameter, once the circumference is known, and vice versa. The procedure he suggests is the following: if we divide the circumference length (let it be C) by 3 (let the result be C/3), C/3 again by 3 (C/9), and C/9 by 7 (C/63), and we subtract C/63 from C/3, we will have the diameter length (d).

Symbolically and in modern words:

$$d = \frac{C}{3} - \frac{C}{3 \cdot 3 \cdot 7} = \frac{C}{3} - \frac{C}{63} = \frac{20}{63} C, \qquad (1)$$

where 20/63 = 0.317… = $(3.15)^{-1}$: an approximation of pi^{-1}.

The mathematician agrees with the philologist about the grammatical correctness of the proposed *lectio*, but he doesn't agree about its mathematical content, mainly referring to Archimedes' rule, implicitly quoted by Pliny some lines after. Theoretically, he writes, the procedure appears to be correct, but it properly works only for the example quoted by Pliny; if we want to verify it with different values (i.e., 21 instead of 22) and we do the calculation according to Archimedes' rule, we obtain 6 and 15/22. Assuming that the rounded down circumference/diameter ratio is 3 + 1/7, as the mathematician does, we have:

$$d = \frac{21}{3 + \frac{1}{7}} = \frac{21}{\frac{22}{7}} = 6.6\overline{81} = 6 + \frac{15}{22}. \qquad (2)$$

On the contrary, the rule proposed by the philologist would yield 6 and 2/3:

$$d = \frac{20}{63} \cdot 21 = \frac{20}{3} = 6.\overline{6} = 6 + \frac{2}{3}. \qquad (3)$$

In order to give further evidence of the correctness of his demonstration, the mathematician shows that calculating *d* starting from 22 will give the same result, regardless of the procedure: both the Archimedes' rule and the other algorithm will yield 7 as diameter length.

As a matter of fact, he shows that:

$$d = \frac{20}{63} \cdot 22 = 6.98... = 6 + \frac{62}{63} \approx 7\,. \qquad (4)$$

So the mathematician concludes that «*Though what you are saying, My Lord, is a subtle remark, nonetheless the numbers don't agree with Archimedes' rule*» («*Quantunque però quello che dice V.S. sia intelletto sottile, non dimeno non corrispondono i numeri con la regola d'Archimede*»).

3. ABOUT THE CONTENT

The text content arises some comments; the first one is that the mathematician doesn't want to obtain the best approximation of the result value: he wants exactly the same result as given by Archimedes' rule, probably because he knows that it has already been rounded down. As a matter of fact, according to Archimedes, the circumference/diameter ratio value, which nowadays is called *pi*, is the following:

$$3+\frac{10}{71}<pi<3+\frac{10}{70}. \qquad (5)$$

The second remark is that, if we do the same calculation with the currently used value of *pi*, we obtain a result (6.6845..) that is nearer to the one calculated by means of Archimedes' rule than to the other one.

From the point of view of historiography, it would be very interesting to identify the two persons involved in this dispute, although no explicit clues are present in the document. Not even the fact that the philologist is working on Pliny's text can help in such an inquiry: the *Naturalis Historia* has been copied and abridged a lot during the Dark and Middle Ages, because it was the only ancient source of scientific topics, so, starting from the 15th century, some 50 Italian and European scholars passionately tried to amend the text, to correct content mistakes of the manuscript tradition, and to comment on the most interesting passages [9–12]. Even the marquis Lionello d'Este (1407–1450), who was taught in letters by the humanist Guarino Veronese and owned copies of Pliny's book, took part in this work. As mentioned above, the sender addresses the recipient as «My Lord», an expression that may involve a social difference; while showing the incorrectness of the procedure suggested by his interlocutor, the mathematician is also very careful to emphasize its smartness («*auenga che l'interpretatione sia* [soti]~~a~~*ltiss.*[a]*, non dimeno non corrisponde col calcolo*»*, «*Quantunque però quello che dice V.S. sia intelletto sottile, non dimeno non corrispondono i numeri con la regola d'Archimede*»), to avoid offending the sensibilities of the recipient. These remarks could lead to consider the possibility of identifying the recipient in the marquis of Este, but unfortunately at the moment no firm evidence is available.

However, it is not surprising that some of the most distinguished philologists were humanists who lived at the Este's court in Ferrara: in 1492 Niccolò da Lonigo (1428–1524), better known as Leonicenus, wrote *De Plinii et aliorum plurium erroribus* about medicine [13], and in 1493 Ermolao Barbaro il giovane (1454–1493) composed his *Castigationes Plinianae*, where he stated he had corrected more than 5,000 mistakes; Pandolfo Collenuccio (1444–1504) replied with his *Pliniana defensio* in 1493.

At the end of his volume, Barbaro advised his audience not to deal with ancient scientific works, as if they were untouchable real *auctoritates*, even if they were edited by an eminent humanist as he was; he urged his readers to look for the truth through a careful evaluation of the sources: precisely what the mathematician of the Ambrosiana manuscript does.

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* *«although the interpretation is very acute, it cannot find consistency in the calculation»*

He shows without a doubt his great confidence in Archimedes' results that lets the mathematician find a definitive answer to the philologist's question. Such an attitude is not surprising. Those were the years when the ancient Greek scientific literature (unknown, little-known or misunderstood during the Middle Ages) was translated and published, first in Latin, and later in vernacular. That was the case of Ptolemy, but also of the Greek mathematicians, whose celebrated texts were translated and printed in the 16th century: Archimedes, first, then Apollonius, Pappus, Heron. As underlined by relevant historians of science, such as Koyré and Ghidini [14, 15], the revival and the assimilation of Archimedes' works during the Renaissance would be one of the roots of the 17th century's scientific revolution.

4. CONCLUSIONS

This study aims to contribute to a fuller understanding of Renaissance culture, especially regarding its relationship with classical authors dealing with science, since it is an understudied issue.

The shortage of useful clues to an explicit identification of the two scholars is a major problem not only for a more precise historical contextualization of the episode, but also for a possible thorough analysis of the study about the works of the two scholars (here arbitrarily named 'philologist' and 'mathematician' solely on the basis of their apparent role), about their links with other scholars, about their training and their influence.

It is rather difficult to speculate on what future discoveries may lead to their identification, particularly if we take into account that the *lectio* proposed by the philologist is accepted neither in contemporary nor in subsequent critical editions of Pliny's work, and therefore the studied manuscript offers only a documentation of a preliminary phase of work, later allegedly discarded.

However, what undoubtedly emerges from this study, is a proof that the gap between humanistic and scientific culture during the Renaissance, is thinner than it could be expected, if, browsing in a Renaissance philologist's toolbox, we find the Archimedes' rule and, implicitly, an admission of the great esteem for this Greek mathematician and his valuable works.

5. APPENDIX: THE TRANSCRIPTION OF THE TEXT

<178r>Presupposta la regola d'Archimede et detta poco di sotto da Plinio con queste parole: "*Quantas enim dimetiens habeat septimas tantas habere circulum duo et uicesimas, etc*." siegue ch'essendo la circonferenza uentidue il diametro sara sette et pero uolendo uerificare la correttione dirò se la

circonferenza essendo 22 da il diametro 7. Se la circonferenza fosse 21, che darebbe il diametro? Operando secondo la regola, uengono sei et 15/22 di modo che il diametro abbraccia meno della terza parte della circonferenza <ossia tolto> poco meno della settima della terza parte [della terza parte della circonferenza]. Come uolesse inferire il diametro essere meno della terza parte della circonferenza in alcuni rotti. Come se la circonferenza fosse 22 il suo terzo sarebbe 7 et 1/3 ~~et quel~~ [per il] terzo e meno il diametro del te~~sto~~[rzo] della circonferenza, ma a questo modo non corrisponde con le parole di Plinio poi che egli non parla per settime et terze. Corregendo adunque al modo che l'gentilhuomo dice cioe leuando la [\et/] et la <u>m</u> per modo che dica "*semperque dimetiens tertiam partem ambitus tertiae paulominus septima colligat*". Il senso uiene a corrispondere co numeri 21 et 6 con 15/22 ne in altri numeri si può facilmente uerificare si come appare in 22 et 7. Et quantunque con lo scemamento della congiontione *et*, et della lettera <u>*m*</u> pare che 'l luogo corra, a me pero da non poco da dubitare se Plinio hauesse scelto tra numeri che si posson dare alla circonferenza, il 21. Et però io stimarei ch'egli forse s'auuiluppasse nell'esplicare tale proportione et che l'hauerla detta à quel modo che sta senza essaminarla altrimente con numeri et senza uenirne alla prattica dell'operazione hauesse giudicato che stesse bene. Et po auuenire facilmente a chi non e essercitato molto nelle cose dell'arte, si come stimo che fosse Plinio hauendo hauuto il capo pieno di tante <178v> et tantae cose et diuerse et simili. Non nieghero la correttione non essere dotta et erudita, et corrispondere alla regola d'Archimede contratta pero à quei numeri ciò è del 21 et del 6 et 15/22. Al modo poi che uorrebbe V.S. che s'intendesse la correttione ciò è pigliare il terzo della circonferenza et di tal terzo la terza parte et di tal terza parte il settimo et sottrar poi questo settimo dal terzo della circonferenza, et il resto secondo lei uerrebbe il diametro: auenga che l'interpretatione sia [soti]~~a~~ltiss.[a], non dimeno non corrisponde col calcolo. Perche presuposto la circonferenza 21 il terzo di quello e 7 o 21/3 operando con rotti et il terzo di 21/3 e 21/9 et il settimo di 21/9 e 21/63 che tolto da sette cioè dal terzo della circonferenza resta 6 et 42/63 che sono 2/3 et operando con numeri ridotti a minor dinominatione faremo cosi 7/1 et il terzo della circonferenza, et il suo terzo e 7/3 et il suo settimo e 1/3 che tolto da sette resta 6 et 2/3 cioe come prima et tanto uerrebbe ad essere il diametro, ma cosi non è il uero. Poiche 6 et 2/3 a 21 non ha quella proportione c'ha 7 a 22, ma si bene 6 et 15/22 a 21 ha quella proportione che 7 a 22. Ma uariando i numeri et pigliando il terzo di 22 uiene ad essere 22/3 et il terzo di questo e 22/9 et il settimo di questo e 22/63 et questo scemato da 22/3 resta 6 et 62/63 et douerebbe restare 7 apunto. Quantunque però quello che dice V.S. sia intelletto sottile, non dimeno non corrispondono i numeri con la regola d'Archimede.

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THE MYSTERY OF ARCHIMEDES. ARCHIMEDES, PHYSICIST AND MATHEMATICIAN, ANTI-PLATONIC AND ANTI-ARISTOTELIAN PHILOSOPHER

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ABSTRACT Irony of fate! Democritus is the only philosopher mentioned in one of his works by Archimedes, and wrote about mathematical things, while neither Plato nor Aristotle are mentioned by him nor have they written about mathematics, but only witnesses scattered here and there in their writings and very often confused, yet they're considered Archimedes' inspirers! (Boyer 1939, Delsedine 1970, Frajese 1974, Gambiano 1992, Reymond 1979). But Archimedes violates the prohibitions of Plato and Aristotle and is inspired by the philosophy of Democritus. It is argued about Archimedes' sections-weights (toma…-b£rea) and Aristotle's and Democritus' indivisible magnitudes (¥toma megšqh). (Luria 1970, Mugler 1970, Ver Ecke 1959, Furley 1967).

1. INTRODUCTION

The discovery of the mechanical method of Archimedes of Syracuse unravels the mystery about his admirable geometrical demonstrations.

In 212 B.C. during the sack of Syracuse from the Roman armed forces led by the consul Marcellus, Archimedes from Syracuse was dying at the hands of a Roman soldier; so narrates Plutarch, who adds that the consul Marcellus, faced with the devastation of Syracuse, couldn't hide the pain and compassion felt in his heart when having seen in a flash of an eye that kingdom of happiness and splendour completely erased (Plutarch).

Syracuse was not only "kingdom of happiness and splendour". It is there that with Archimedes ancient science reaches its highest peaks.

Archimedes' postulate, that denies the existence of indivisibles and infinitesimals, is still the basis of modern calculus.

Archimedes enunciates his postulate in geometric terms: *Further, of unequal lines, unequal surfaces, and unequal solids, the greater exceeds the less by such a magnitude as, when added to itself, can be made to*

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exceed any assigned magnitude among those which are comparable with (it and with) one another". (T.L. Heath 1912).

Today it is considered as an assumption of the real numbers system, after the arithmetization of the analysis during the second half of the 19th with Cantor, Dedekind, and Weierstrass: "**If a and b are two real numbers of the system and a<b, a real number n such as n a>b always exists**."

In the sixteenth and seventeenth centuries, with the discovery and the study of Archimedes' works, a certain aura of mystery surrounds his image. Studied by Galileo "with infinite stupor" (Galilei 1964) and admiration, venerated by all scientists of the time for his bold inventions and adopted as a strict model, Archimedes makes the conviction arise on his work, which also becomes a legend, that he purposely removed the tracks of his investigation, as if he had buried for posterity the secret of his method of research. (Heath S.T. 1981).

It is only in 1906 that the aura of mystery is reduced, thanks to the J.L. Heiberg, a Danish philologist and editor of Euclid's and Archimedes' works. In an ancient palimpsest found in Constantinople at the St. Sepulchre's Monastery, the scholar comes across Archimedes' writings on top of which an euchology had been placed. An important work by Archimedes comes to light, directed to Eratosthenes from Alex*a*ndria, in which several mechanical theorems are shown and through which Archimedes reveals how he managed to calculate the area of the parabolic segment, the volume of the sphere, its surface area, the volumes of the right-angled conoid and of the spheroid, as well as their gravity centres, etc, **through the use of mechanical principles and of the indivisibles, which were denied by his postulate geometry.**

This is what Archimedes writes to Eratosthenes:

Seeing moreover in you, as I say, an earnest student, a man of considerable eminence in philosophy, and an admirer [of mathematical inquiry], I thought fit to write out for you and explain in detail in the same book the peculiarity of a certain method, by which it will be possible for you to get a start to enable you to investigate some of the problems in mathematics by means of mechanics. This procedure is, I am persuaded, no less useful even for the proof of the theorems themselves; for certain things first became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards because their investigation by the said method did not furnish an actual demonstration. But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge. This is a reason why, in the case of the theorems the proof of which Eudoxus was the first to discover, namely, that the cone is a *third part of the cylinder, and the pyramid of the prism, having the same base and equal height, we should give no small share of the credit to Democritus, who was the first to make the assertion with regard to the said figure though he did not prove it* (T.L. Heath 1912).

With Archimedes' direct testimony and from the examination of the "mechanical theorems" Archimedes' the mystery is revealed, having found the secret way of his discovering admirable geometrical results.

2. THE METHOD OF EXHAUSTION AND CRITICISM OF THE INDIVISIBLES OF ARISTOTLE

Mathematicians of all times have always admired the rigor of Archimedes' demonstrations, who in an admirable way succeeded in utilizing the so called "**method of exhaustion**"(**Appendix**); a method which had most probably been introduced by Eudoxus of Cnidus in order to avoid the use of the so called "indivisibles", which, still, in Archimedes' days were said to be useless in mathematics, perhaps due to the "Zeno's paradoxes". Nothing is known about Eudoxus regarding his conception of the indivisibles; but the method of exhaustion avoids their use.

Instead we preserve a firm logical-philosophical closing speech against the indivisibles on the part of Aristotle. He denies that the line could be composed of points, the surface of lines and the solid of surfaces. The continuity of geometrical magnitudes, according to Aristotle, is constructed by more and more divisible magnitudes.

In a passage from "*On the Generation and Corruption*" Aristotle asks himself:

The primary <reals> are indivisible magnitudes? Or is no magnitude indivisible? For the answer we give to this question makes the greatest difference. And again, if the primary <reals> are indivisible magnitudes, are these bodies, as Democritus and Leucippus maintain? Or are they planes, as is asserted in the Timaeus? (Aristotle 1a).

The absurd conclusion, according to Aristotle, whoever admits atomic or indivisible magnitudes (clearly referring this to Democritus) is that any magnitude composed from their disappears, being the components atomic necessarily nothing.

If its constituents are nothings, then it might both come-to-be out of nothings and exist as a composite of nothings: and thus presumably the whole body will be nothing but an appearance (Aristotle 1b).

Aristotle denies that infinitesimal magnitudes of forces could even exist, **dF**, as it is written today, such that summed up could surpass any resistance data, as big as one wishes, but still always finite.

In fact, he writes in his work "*Physics*":

If a given motive power causes a certain amount of motion, half that power will cause motion either of any particular amount or in any length of time: otherwise one man might move a ship, since both the motive power of the ship-haulers and the distance that they all cause the ship to traverse are divisible into as many parts as there are men. (Aristotle 2a).

Aristotle denies that a point can describe, by its motion, a straight segment or a curve.

He continues writing about this in his quoted work:

It may be shown in the following way that there can be no motion of a point or of any other indivisible. That which is in motion can never traverse a space greater than itself without first traversing a space equal to or less than itself. That being so, it is evident that the point also must first traverse a space equal to or less than itself. But since it is indivisible, there can be no space less than itself for it to traverse first: so it will have to traverse a distance equal to itself. Thus the line will be composed of points, for the point, as it continually traverses a distance equal to itself, will be a measure of the whole line. But since this is impossible, it is likewise impossible for the indivisible to be in motion. (Aristotle 2b)

The indivisibles expelled, thanks to Aristotle, from physics and from geometry, have taken on with time a considerable heuristic value in the research of mathematics and physics, as it has been demonstrated in centuries to come.

3. THE MECHANICAL METHOD AND THE USE OF THE INDIVISIBLES. DEMOCRITUS AND ARCHIMEDES.

According to the testimony of Archimedes himself Democritus had already made use of them exactly, and even Archimedes relies even on these, with his "**mechanical method**" and theorems on the balance of the bodies, in order to reach his so brilliant geometrical results.

Dijksterhuis leaves no doubt of the fact that Archimedes, stating that the results achieved with the "mechanical method" didn't constitute an actual demonstration, he didn't really refer to the mechanical theorems, which he had already used in the *Quadrature of the Parabola (an official publication satisfying all requirements of exactness),* but rather to the use of the indivisibles *(the mathematical deficiency is exclusively a consequence of the use of indivisibles*). (Dijksterhuis 1956).

The historian of mathematics Boyer is of the same opinion, who, besides, writes:

"*Archimedes of Syracuse displayed two natures, for he tempered the strong transcendental imagination of Plato* (Peano 1958) *with the meticulously correct procedure of Euclid* " (Boyer 1939).

It was also written that by the discovery of the method, Archimedes ***allows us to look into his*** **mechanical** ***workshop*** (The phrase is of the historical mathematician H.G. Zeuthen). In our opinion, with the discovery of the mechanical method, Archimedes lets us look into his **philosophic-epistemological workshop**, if it is true that the letter is addressed to "*a man of considerable eminence in philosophy*", Eratosthenes, as stated in the quoted text of Archimedes.

Thus, in such a workshop we find Plato or Aristotle no longer, but we only find Democritus with his physical and geometrical one.

In Archimedes' view there's a relation between physical atomism and geometrical one; the same ratio that is valid for geometrical lines is also valid for physical ones, imagined homogeneously balancing themselves onto a lever. On the contrary, we can say that his physics and his geometry are built in contrast to Plato stated in the following passage by Plutarch:

Eudoxus and Archytas had been the first originators of this far-famed and highly-prized art of mechanics, which they employed as an elegant illustration of geometrical truths, and as means of sustaining experimentally, to the satisfaction of the senses, conclusions too intricate for proof by words and diagrams. As, for example, to solve the problem, so often required in constructing geometrical figures, given the two extremes, to find the two mean lines of a proportion, both these mathematicians had recourse to the aid of instruments, adapting to their purpose certain curves and sections of lines. But what with Plato's indignation at it, and his invectives against it as the mere corruption and annihilation of the one good of geometry, which was thus shamefully turning its back upon the unembodied objects of pure intelligence to recur to sensation, and to ask help (not to be obtained without base supervisions and depravation) from matter; so it was that mechanics came to be separated from geometry, and, repudiated and neglected by philosophers, took its place as a military art. (Plutarch)

We can find the two principles of Democritus' philosophy of *the full* (*tÕ plÁrej*) and *the void* (tÕ kenÕn) in his philosophic-epistemological workshop.

Geometrical figures as a cone, a cylinder, a sphere, a spheroid, or conoid etc, idealized and imagined empty, are filled completely from time to time through circles-sections-weights, deemed full physical elements, putting them together *(sumplhrwqšntoj,* Archimedean term). Figures in their physical fullness, with unitary density like their elements, carried onto a lever, are in equilibrium, thanks to their weights according to a determined ratio.

If these are the two physical principles, which have nothing to do with neither Plato's nor Aristotle's philosophy (and with which Archimedes built his mechanical demonstrations), then it is to avoid to read Archimedes through Aristotle's eyes and with the criticism of his school of the so-called indivisibles magnitudes or indivisible bodies (¥toma megšqh or ¥toma sèmata), which we believe Dijsterhuis refers implicitly to in his quoted passage.

In fact, these indivisible magnitudes are not mentioned in the Archimedean text, and the concept of atom (or better of **full element**, *tÕ plÁrej*) in Archimedes **had a very different epistemological meaning.**

(In reality, writes S.T. Heath, *they* -parabolic segment and triangle - *are made up of indefinitely narrow strips, but the width –* ***dx****, we might say - being the same for the elements of the triangle and segment respectively, divides out*. (S.T. Heath 1981). And still Boyer writes: *a collection of thin laminae or material strips* (Boyer 1939).

For Parmenides and Democritus' rationalist tradition the real element is the thought thing, not the sensible thing, and it is thanks to the former we can **observe** (this is the meaning of qewre‹n of the *mechanical trÒpoj* of Archimedes) the sensible thing in a real way.

Aristotle, an empiricist, confuses the physical thing with the thought thing, therefore it is clear that an indivisible magnitude, as a physical thing, is a contradictory thing, but if it is considered as **theoretical element** (as I think that was the **atom-idea** in Democritus, according to some testimony on him, and this seems to be in Archimedes, or **the idea of full**), then it escapes the contradiction.

("Democritus calls the atoms the full" and *"Democritus believed that indivisible bodies were principles of the things, but as theoretical elements, lÒgoi qewrht£.")* (Luria S, 1970).

Atoms or the full elements are indivisible because they are conceived in such a way, but, as they are full and fill a physical space, they have parts and are magnitudes.

For Simplicius this is the meaning of atom in Democritus. (Luria S. 1970).

This explains why Democritus, according to the testimony, argued that atoms could be both very small and big like a world.

Only within this onto-epistemological framework the modern concepts of "material point", of "rigid body", incompatible with a type of Aristotelian philosophy, could develop.

4. ARCHIMEDES VIOLATES THE PROHIBITIONS OF PLATO AND ARISTOTLE

Archimedes violates the platonic prohibition of linking mechanics and geometry. It arises, on the other hand, from the reading of the quoted passage, the fundamental suspicion that Eudoxus himself had initiated the use of the mechanical method for solving geometrical problems, but was stopped by the platonic prohibition.

Archimedes also challenges and violates Aristotle's prohibition that infinitesimal quantities of forces can sum themselves up to overcome any finite resistance, as big as one wishes, introducing the concept of moment of a force with its law on the lever.

In anti-Aristotelian function the famous episode of the ship launching told by Plutarch can be interpreted:

Archimedes, however, in writing to King Hiero, whose friend and near relation he was, had stated that given the force, any given weight might be moved, and even boasted, we are told, relying on the strength of demonstration, that if there were another earth, by going into it he could remove this. Hiero being struck with amazement at this, and entreating him to make good this problem by actual experiment, and show some great weight moved by a small engine, he fixed accordingly upon a ship of burden out of the king's arsenal, which could not be drawn out of the dock without great labour and many men; and, loading her with many passengers and a full freight, sitting himself the while far off, with no great endeavour, but only holding the head of the pulley in his hand and drawing the cords by degrees, he drew the ship in a straight line, as smoothly and evenly as if she had been in the sea (Plutarch).

Describing how his spiral originated from the movement of a point on a straight line segment moving in a circular motion around one of its fixed extremities continues violating the anti-Aristotelian prohibition mentioned above.

5. CONCLUSION FROM THE MYSTERY OF ARCHIMEDES TO THE MYSTERY OF DEMOCRITUS

Then we have not understood yet from where Dijksterhuis had received from the conviction that the reading to Eratosthenes was "*private*" (Boyer 1939) if in it we read: *I am myself in the position of having first made the discovery of the theorem now to be published [by the method indicated], and I deem it necessary to expound the method partly because I have already spoken of it and I do not want to be thought to have uttered vain words, but equally because I am persuaded that it will be of no little service to mathematics; for I apprehend that some, either of my contemporaries or of my successors, will, by means of the method when once established, be able to discover other theorems in addition, which have not yet occurred to me*. (T.L. Heath, 1912).

On the other hand, it isn't it likely any more to maintain that Archimedes had matured the conviction that the philosophy of the indivisibles has nothing to be ashamed of, nor to hide from, it being perfectly in harmony with the demonstrations of Eudoxus' method of exhaustion, and does all that brings us to fecund results in the field of science?

Let us consent a bitter consideration here, does not the mystery of the disappearance of Archimedes' method during the centuries recall the "mystery" of the disappearance of the very vast production of Democritus, who, as we know, is never mentioned by Plato, differing from all the other so called "pre-Socratics"?

To conclude, Archimedes is not just a great mathematician or a great physicist, but is also a great philosopher, who has an organic conception of the universe, in which neither the concept of indivisible magnitudes is that of the Aristotle's criticism nor the concept of m£qhma is that of Plato, since for Archimedes physics, geometry and philosophy are together, when we want to build the knowledge of real.

In any case, in particular in my paper I have tried to show the **t**hesis that "indivisible magnitudes", or better Archimedes' "sections-weights", have not had a secret life of only in a workshop, as Dijksterhuis maintains, but have had a long existence inside **a tradition of philosophical-episthemological though**t, which I call "**Italic**", according to the doxographer D. Laertius. (D. Laertius 1925). That goes under the names of Pythagoras, Parmenides, Zeno, Archytas, Eudoxus, Democritus.

On the basis of his own testimony Archimedes continues such tradition, but it is suffocated in the Alexandrian culture under the weight of Plato and Aristotle's **Ionic tradition** (Boscarino 1999).

By taking the "Italic" tradition again, Galilei could start physics as modern science.

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APPENDIX

The method of exhaustion

By the method of exhaustion we prove that the measures of two magnitudes A and B are equal, m(A) = m(B), if the following proposition is satisfied. Proposition: Let A and B be two homogeneous magnitudes. Consider two classes H and H' of homogeneous magnitudes to A and B with elements $\mathbf{H}_n$ and $\mathbf{H'}_n$ respectively, such that: 1) There exists an n such that $A–\mathbf{H}_n < E$ for all E (positive real numbers); 2) $\mathbf{H}_n \leq \mathbf{B}$ for all n; 1') There exists a n such that $\mathbf{H'}_n–A < E$ for all E (positive real numbers); 2') $B \leq \mathbf{H'}_n$ for all n. Then, we have m(A) = m(B). Proof. a) We first consider A and B such that A>B. Putting A–B = E, by 1) we get $A- \mathbf{H}_n < E = A–B$, that implies $B < \mathbf{H}_n$ in contradiction with hypothesis 2). Thus A cannot be greater than B. b) Analogously, we prove that A cannot be less than B. In fact, considering A<B and putting B–A = E by 1') we get $\mathbf{H'}_n – A < E = B–A$ that implies $\mathbf{H'}_n<B$ in contradiction with 2'). Then, the statement A = B is now an immediate consequence of a) and b).

In order to avoid confusing the modern methods of calculus, by its concept of limit, with the method of exhaustion, see ***C.B. Boyer,*** *The concepts of the Calculus, 1939,* New York, pp. 33-37, pp. 50–51.

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ARCHIMEDES TO ERATOSTHENES: "METHOD FOR MECHANICAL THEOREMS"

Ἀρχιμήδους Περί τῶν μηχανικῶν θεωρημάτων πρὸς Ἐρατοσθένην ἔφοδος.
Ἀρχιμήδης Ἐρατοσθένει εὖ πράττειν.

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ABSTRACT This essay is meant to be a **contribution to a philosophical interpretation** of a letter from Archimedes to Eratosthenes.

1. INTRODUCTION

This essay implies the following parts: preliminary remarks, a synoptic table, philosophical comments about the letter, informations about the names mentioned and references.

2. PRELIMINARY REMARKS

The writer is a mechanical engineer (machinator by Vitruvius) with a degree in philosophy who is devoted to scientifics and technical issues. The opportunity to be present at the world conference on "the genius of Archimedes" grew out of Archimedes' letter where he wrote "think mathematical questions by means of mechanics". This did me idea to write an essay. "*de Archimedis epistula sub specie philosophiae*".

I quoted Vitruvius because "*machinator*" (latin term) is a word with metaphorical ideas and images. It is sufficient to say "*deus ex machine*" where "ex machine" denotes the machine that carries the god on the stage and the skillful device too on the part of the god, to find the solutions to puzzling situations (used by Aristophanes). I believe that those ideas show a link with statements contained in the letter written by Archimedes to Eratosthenes.

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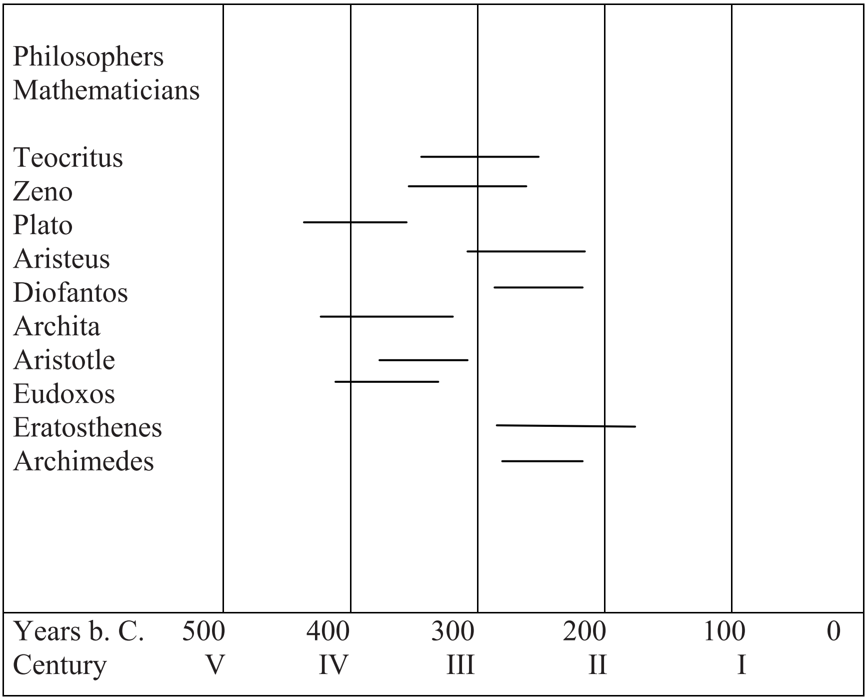
The letter to Eratosthenes can be found in the book by E. Rufini [8] “the method of Archimedes” from wich I took only the most interesting passages.

I would like to suggest the possibility for dialectic philosophical opportunities where the research of the truth is like an asymptotic curve to the straight line of knowledge. The philosophical angle is only an attempt wich may be called wishful thinking or, better, “thoughts in freedom”.

3. THE SYNOPTIC TABLE SHOWS THE LIFES OF THE BELOW MENTIONED NAMES RELATED TO ARCHIMEDES



4. PASSAGES FROM THE LETTER:

The selected passages of Archimedes’ letter are: “I know Your reputation as a scholar and excellent teacher of philosophy” - “You can appreciate the research in mathematics” - “I thought to show You the peculiarity of a certain way for mathematical problems […] by means of mechanical solutions” - “some things appeared to me first by mechanics and afterwards I proved them geometrically" - "the research by this system is not a true demonstration […] so I decided to give You the demonstration by writing a letter […]".

5. ABOUT WORD: "METHOD"

First I am searching for a definition of the word "method" to do a comparison with the meaning of normal term: "a logical continuous structure to give good results in the solutions of problems with regular and required means. A whole unity of regulations to take methodical proceedings to avoid errors".

I read seven translation of the letter (the first one, of course, in greek, the second in latin, see below). The most important passages are those pieces where Archimedes explains his concepts but (except in the title) Archimedes does not use the word "΄έφοδος" translate in "method", Why?

"τρόπου τινός ιδιότητα" = the peculiarity of a certain way
"*methodi cuiusdam proprietatem perscribere tibi*" = I will write to You the property of a certain method
According to Napoletani [6]: " such a particular way of …."
" " Google : " the peculiarity of a certain system for …"
" " Rufini [8] : " the particularity of a method for …"
" " Frajese [3]: " the characteristics of a method for …"
" " Dijksterhuis [2] : " a certain particular method to …."

It is noteworthy that only in the title appears the word greek method: The others sentences are circumlocutions. May be one can ask if the word which from the year 1906 has been translated with "method" (in quotes to avoid a logic tautology) can be correct and adequate.

Plato wrote for people who are admitted to Academos garden "nobody can enter if he is unlearned in geometry"; so to pay a homage to Plato, I researched the word: it is found in Sophista, in Phaedro, in Teethetos, but the word is μέθοδον, ΄οδῶ, καθοδόν, with μετά, μεθ', κατά, or only ΄οδῶ.

The word ΄οδός (walk, road, way) appears with several proposition but Archimedes uses it only with the prefix ΄επί, ΄εφ'.

The word ΄έφοδος with ΄επί, ΄εφ' (latin = de) has been used by Thucydides, Xenophon, Euripides, Aristotle, with the meaning of expedient rather than method.

During the search for a "**something philosophical**" I read the excellent work by P.D. Napoletani [6] where many of expressions have raised doubts about the meaning of the word "method".

"the use of the term method has a particular peculiarity—can be used as a general word?—certain methods are used according to the different situations—Archimedes did not have a method for all problems".

After which for avoid in order to fall into nominalism (see medieval philosophy) but altought impossible to modify the accepted use of "method" it seems appropriated to attribute to Archimedes a certain "metis" greek word that is shrewdness or cleverness (not the cunning!). I would suggest to the readers of the letter to keep in mind the word "stratagem".

I wish do a mention found in the book: by G. Granger[4] "the science and the sciences": a certain Hansen says "the contest to justify a problem (may be like Archimedes?) is like a recipe".—Napolitani also states "the method of Archimedes is like a recipe for…."

6. SOMETHING PHILOSOPHICAL?

The genius of Archimedes shines in the examples listed in the letter when he makes use of scale lever, barycentre and static moment to mechanize geometry when, in his time, the greek geometry used only rule and compasses. It is a process with several applications; Archimedes works with deductive coherence of a construction by an engineer and he gives, by the accumulation of demonstrations, a great capacity that he can transmit to others after him.

Archimedes does not say how he has arrived to use mechanical means but his works can be likened to a proposition by E. Mach: "the invention can not be born suddenly and the sensorial knowledge follows a logic course that shall meet in the deed of the human reason".

The technological tales about Archimedes have been magnified as cause and effect or viceversa for his unbelievable reputation: it is enough to think of the intuition he exhibited in the problem of Gerone's golden garland, in any case it must be noted how much Archimedes did in developing and exploiting natural forces: it must be also point out what Pasini said: "there are in the world two powerful sources of forces: the first one is Nature which hides them jealously and the second one is human Genius which seeks them, reveals and applies them". As Archimedes did.

It is right to recall the idea that sense and reason contribute to the solution in two ways: from deduction through sensorial experiences to hypothesis and viceversa from hypothesis to facts. Can I deduce (may be not totally correct) that this is the same way as when Archimedes says: "first by mechanics, then by geometry" ?. That is practice and theory, or experience and reason. Archimedes uses scale and level (it is well known that those was already used by workers but non geometrically) like a know how. Perhaps Archimedes thought of a mechanical improvement of common means starting from a idea and afterwards developing by technical way? It is sure that Archimedes preferred theory (also for intellectual and personal thinking typical of his time) but he never refused to make use of technical and empiric knowledges.

From Basalla we know: “technology is nothing more than another name for applied science. Technology is as old as human kind. “and from Bachelard (if possible) we can point out the “phaenomenotechnics” (phenomenetechniques) that are the sensible applications of the empirical knowledge.

G. Loria thinks that Archimedes uses the most clever reasoning as if they were “disguised integrations” (mathematically speaking) and notes that he distinguishes between “ingenuity method” from “method of demonstration” exempting his countrymen not from the “smallest” but from “the infinitely small”. Leonardo da Vinci writes that mathematic is like a tool for mathematic but instead Archimedes writes that the mechanic serves the purpose for the solution of mathematical problems.. The historical contest justifies both assertions.

The few points mentioned above can be considered at the edge of the philosophical science but the try a place Archimedes inside comprehensive philosophical view, gives rise to some objective problems. For the subjective ones there is no difficulty because all that has been written about:

from the exhaustion (so called in 1660 a.C. by Gregorio di San Vincenzo and F. Picquet)

to the theory of the infinitesimal

from the approximation

to the limits of the integrals

is all to be classified in the field of the “POST-CURSORI” as they have been called.

All this we are dealing with a document of 2300 years with erasures and smodges, lost and after found (casually) and restored only in 1906 when it was mathematically out of date.

I too adopt a …. method: apofatic and catafatic (words well known in the middle age – patristica) that means: positive and negative opinions. May I write : on-off?

Historical gradient would have been exceptional for the history of the sciencephilosophy if the letter have been found 500 years earlier. Altough several mathematicians, one for all Bonaventura Cavalieri drew from Archimedes’ works but many questions remained without answer and, for this reason, many things were hypotetical in merit and in substance.

Was he a nature philosopher? No doubt but “*artis philosophandi magister*” is non applicable. In the western world philosophy as a special activity of the human thought can not definited once for all because only unitary meaning can not be equal to all philosophers we have known from 600 years b.C.; at the most we can consider philosophy as a whole (please don’t confuse with the logic formalism!). Archimedes did not deal with philosophical arguments but we can suppose (*quae pro alio suppositio dicitur*) and think (as the modern mathematicians) “a hidden variable” with research for a philosophical seam (mineral term is properly used).

It can see in the geometric demonstration XVI and XVII of the letter to Eratosthenes. Archimedes gives to “stratagem” (= method) the value of research: the invention of a system of comparison between wellknown geometric figures (using scale and level) with a condition of equality between two terms.

Problem: (I cut off all that is mathematic): A = B: yes or not.

Archimedes supposes A different in comparison of B that is A unequal B. We put first A < B; we can prove it impossible (ex absurdo). Therefore we put A>B; as before it impossible. So if we know that A can not be > or < than B it means that A = B.

Is it a law of not contradiction or viceversa? One can not say: if A > B is false, the A < B is true, but A < B too is false, then both A and B are both false. May be that A is not “non A”?.

Is it possible by syllogism to insert this?: if A implies B and B is true then A is true; and if A implies B and B is false, A too is false and also even worse if is not true for A and B but “equal” there is a “reduction ad absurdum” if A can not be> or < B. Contradiction of “*tertium non datur*”? Can the syllogism be applied? I do not know.

But Archimedes’point of view is correct.

I cite Poincarè: “geometry is not more true than another geometry, only it is easier” but Poincarè is the father of conventionalism!.

Archimedes used technical and mechanical systems that are barely suitable to greek geometry. For instance: the grouping and the condensation of the section of the figures using the stratagem of compression (Napolitani writes that it is better to translate the compression in “indirect pass to the limit”) without forgetting that the successive approximations of stratagem is not exhaustion.

Plutarch writes (about the superimposed sections): “if one cuts a cone with a plane parallel to the cone’s base and infinitely near to the base, the obtained circle can be equal or unequal to the base. If equal the cone becomes like a cylinder, in unequal the cone becomes like a staircase.”

The greek general concepts are: to find a solution to a problem, it is better to consider it already solved—anlysis is the invention of demonstration—multiplication verifies what results from division: these three concepts come to fore reading the Archimedes’ letter.

Ghoete wrote: “what is ingenuity? the result which one is looking for”.

7. CLOSING

I can not give remarks: this is up to the readers: the arguments of the abstract are concluded **but it is my personal believe that the genius of Archimedes** (and even without considering the letter to Eratosthenes) **was worth an inquiry in the field of philosophy** (the queen of the human disciplines) only the sake of showing his manifold genius and fame.

In any case, after what has been said about Archimedes I wish that the homo sapiens sapiens to day also called “technologicus” could find the way, the **΄έφοδος** to reach the poetry; not that of any poet but the “POETRY” itself because it is there if one wishes find it.

INFORMATION ABOUT THE NAMES MANTIONED

Eratosthenes: director of the Alexandria library
Vitruvius Pollio: roman architect and engineer – first century a.C.
Plato: philosopher
E. Mach: scientist and philosopher – (1838–1916)
L. Pasini: naturalist, geologist – (1804–1870)
G. Bachelard: philosopher – (1884–1962)
Gregorio di San Vincenzo and C. Picquet: mathematician – XVII century
G. Basalla: scientist and inventor
Leonardo da Vinci: scientist, engineer, inventor, painter – (1452–1519)
B. Cavalieri: mathematician – (1598–1647)
G. Loria: mathematician
Plutarch: writer – (45–125)

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ARCHIMEDES IN SEVENTEENTH CENTURY PHILOSOPHY

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ABSTRACT In this paper our aim is to examine the special interest that some major seventeenth century philosophical figures have shown in the achievements of Archimedean geometry. Given the excellence of the work of Archimedes, it is to his work that philosophers like Descartes, Spinoza or Pascal have been referring in order either to clarify some points of their own philosophies, or to find a sound basis for the modern mathematical conception of nature.

1. INTRODUCTION

The legacy of Archimedes' work does not run short of brilliant pages. A lot of mathematicians as well as philosophers and scientists have acknowledged their debt to the work of the great Syracusan. In this respect the seventeenth century was no different. The work of Archimedes was well-known to every well-educated individual of that century and was praised as an exceptional and outstanding example not only of what mathematical science can achieve but also of what human intellect is capable of. What is of special interest for us here is the ways that leading figures of the new philosophical trend which is usually described in textbooks as "early modern philosophy" have been using Archimedes' work for their own purposes. We should bear in mind that these philosophers were struggling to relieve seventeenth century philosophy of the heavy burden of the scholastic medieval philosophical tradition. To their eyes, the work of Archimedes stood as an example of sound thought and as such had a major impact on their new view of man as well as of the world that surrounds him.

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2. ARCHIMEDES AND THE SCIENTIFIC REVOLUTION

Let us be more precise and have a look at the impact that Archimedes' work had on the great revolution that was taking place during the seventeenth century, namely the scientific revolution. To start with, let us consider the fact that Galileo's work in physics was inspired to a great extend by the work of his Syracusan predecessor. In his attempt to modify what at the time was the well-established model of a closed world within which bodies are characterized by their qualitative features and to build a new geometrical science of motion, Galileo drew mainly from the Archimedean tradition of bringing together geometry with mechanics and finding solutions that satisfy both sciences (Koyré, 1978).

It might be objected here that Galileo's work is mainly focused on physical and astronomical problems and that the mathematical work of Archimedes never really treats problems concerning the reality of material things: at best his work treats idealized and abstract cases that may be projected upon the existing world (consider, for example, his treatment of a balance without any friction in his *On Plane Equilibriums*), so that all things considered, at least the part of Archimedes' work that has survived through the ages, concerns purely geometrical theory. But this is exactly the point when someone is dealing with the problem of the relation between Archimedes' and Galileo's work. Galileo's new physics and astronomy (that is, if one decides to leave aside the part of astronomy that has to do solely with observation) is in fact a kind of geometry that is based on the geometry of Archimedes. Galileo's new science treats nature in a geometrical way because, according to him, nature is a text written in mathematical and geometrical terms and must be read in these terms (Galileo, 1957). Thus, Galilean science consists of a brand new way of conceiving the world that surrounds us and forms an essentially mathematized view of nature. Archimedes and Galileo share in common their geometrical way of thinking about the world.

3. THE SEARCH FOR A PREDECESSOR IN DESCARTES' WRITINGS

The new science that emerged during the seventeenth century was escorted by a whole new philosophical treatment of the questions concerning both the world and the human being. This new philosophy that emerged in ways that can be described as parallel with those followed by the new revolutionary science, was itself deliberately revolutionary in its effort to take the place of the scholastic conception of things. Such an endeavor led several leading philosophers of that century towards a search for distinguished figures of the past who could be counted as models of sound argumentation.

This search for a predecessor who could count as a reliable reference can be found in a crucial text of one of the greatest philosophers of the modern era, a philosopher who tried to build his philosophy solely by doubting about all that he had learned and by asking himself if there was anything that could be counted as capable of resisting such a doubt. It was Descartes, of course, who raised his deliberate and methodological doubt to the status of the first move towards the recovery of indubitable truth.

As it is widely known, Descartes is one of the leading figures not only of early modern philosophy, but also of early modern science. He was the inventor of analytical geometry and he contributed largely to the construction of a worldview that is based on mathematical physics, despite the fact that Cartesian physics is not *stricto sensu* mathematical as long as it only provides a few mathematical laws of nature and is mainly occupied with the invention of mechanisms that explain natural phenomena. But Descartes' major achievement is his metaphysical theory, a theory that sets the traditional problems of the theory of knowledge on new ground. This theory consists of a new way of defining and conceiving the essence of man and his ability to grasp certain knowledge. The completion of the enterprise is described in a text emblematic for seventeenth century metaphysics, the *Meditations on first philosophy*, published in latin in 1641 under the title *Meditationes de prima philosophia*. This text, divided into six meditations that each mark a step towards the conception of indubitable knowledge, has as its starting point a systematic and well-organized leap into total uncertainty. This leap is based on some arguments that in Descartes' philosophy are supposed to show that we cannot be sure even of the truth of simple mathematical propositions such as the sum of two and three.

By contrast with what takes place during the first meditation of the *Meditations on first philosophy*, the second meditation introduces a new issue. Descartes, or the unidentified narrator in Descartes' text, starts to ask himself if the universal doubt leads to some kind of knowledge, if he knows anything at all. This questioning will lead him to the famous "*ego sum, ego existo*", but before that he defines in theory the kind of indubitable knowledge which he is seeking. In this context he does name Archimedes: "Archimedes used to demand just one firm and immovable point in order to shift the entire earth; so I too can hope for great things if I manage to find just one thing, however slight, that is certain and unshakeable." (Descartes, 1984, p. 16).

This allusion to the well-known story that relates to Archimedes' famous ability with machines does not have to do only with Archimedes' ability as an engineer, but with Archimedes' mathematical science, a science aiming at certainty at any cost. In order to grasp the full meaning of this allusion to Archimedean science we should not forget that for a seventeenth century scientist who belongs to the scientific revolution movement, the knowledge of nature and of her laws is a question posed in mathematical terms. Thus, the way in which man interferes with nature does not exclude mathematical practice and mathematical certainty. And this certainty is to be found above all in the rigid way of Greek geometrical thinking. In Descartes' eyes such a scientist as Archimedes can, on a given stable basis, work towards a real revolution of what is taken to be true, of what is thought to be certainly known. It is in this sense that Descartes claims in this particular step of his metaphysical itinerary that by way of analogy with the Archimedean demand for a "firm and immovable point" he is seeking for an axis of certainty. It is to be noted that in Descartes' correspondence (Letter to Mersenne, 27 May 1638 and Letter to Hyperaspistes, August 1641) Archimedes' name figures in a textual context that implies the demand for absolute certainty.

According to Archimedes, even the earth can be put in motion if an immobile point is given. In the same manner, all that Descartes' metaphysics wants is to show that if I know certainly that I am (and, to be sure, such a knowledge according to Descartes' meditational enquiry entails the knowledge of *what* I am, i.e. the knowledge of the property my essence consists of), I can start building on firm foundations a new philosophy. When the Cartesian allusions to Archimedes are read from this point of view, Archimedean science can be seen as the inspirer not only of modern science, but also of modern philosophy, at least the kind of philosophy that Descartes thinks is the best to lay the foundations of a mechanical worldview.

4. SPINOZA AND THE INFINITE

It is time for us now to turn to an examination of the ways the legacy of Archimedes is treated in the work of another major philosophical figure of the seventeenth century: Spinoza. Spinoza's work from many points of view can be seen as a turning point in the history of early modern philosophy. Whilst reading Spinoza one should keep in mind the fact that he, although generally counted amongst those philosophers who form the group of intellectuals usually covered under the umbrella of the so-called "post-cartesianism", opposed many facets of Descartes' work, both philosophical and scientific. It should be enough for the scope of our study to keep in mind that Spinoza's thought moves towards a philosophical system that can be characterized as "pantheism" (although the word never appears in his texts). According to this philosophical system neither God nor man are uniquely thinking substances. God must be considered as the only genuine existing substance, a substance that consists of infinitely many attributes (from this definition it follows that according to Spinoza, God is amongst other things an extended being given that God's essence consists of the infinitely many attributes). As to man, his mind and body must be understood as finite modes of two of the divine attributes, namely the attributes of Extension and Thought.

By treating human nature in such a way, Spinoza is opposing the Cartesian definition of man as essentially thinking substance. But he is also setting new standards as to what may count as of significant value for a philosophical theory, because considerations concerning the material extended world are, for such a philosophy, as important as considerations concerning the human soul. If it be so, it is of little wonder that Spinoza dedicated a lot of pages in his first publication to an exposition of Cartesian physics. This early work, whose full title is *Descartes' Principles of Philosophy demonstrated in the geometric manner*, was published in latin in 1663 under the title *Renati Des Cartes Principiorum Philosophiae pars I et II more geometrico demonstratae*. In it Spinoza reiterates the basic dogmas of Cartesian natural philosophy as well as those of Cartesian metaphysics, while transforming their order into a deductive one and changing their demonstrative apparatus. Spinoza follows here a rigid geometrical method of proceeding. He, thus, sets forward the main steps of Cartesian metaphysics and physics in the form of theorems, themselves being demonstrated with the auxiliary invocation of a set of definitions and axioms. It should be clear by now that Spinoza feels at ease with ancient Greek geometry and its rigid method.

Amongst other things in this work Spinoza presents the fundamental principles on which Cartesian physics build a consistent theory of inertial motion. In Proposition 16 of the second part of this work, Spinoza claims that "Every body which moves in a circle, as for example, a stone in a sling, is continuously determined to go on moving along a tangent" (Spinoza, 1988, p. 278). This theorem is followed by two demonstrations, each of them containing an allusion to a different geometrical theory of the past. The first one refers the reader to two propositions of the third Book of Euclid's *Elements*, whilst the second one proceeds in a way supposedly borrowed from Archimedes who is mentioned incidentally. These two demonstrations have no equivalent in any of Descartes' texts, so one might say that they constitute Spinoza's contribution to the theory of inertial motion.

The second demonstration of the theorem passes from the study of a special case to a generalization while using Archimedean techniques. The special case consists of the study of the properties of a hexagon inscribed in a circle and makes use of the example of a body lying at rest on one of the hexagon's sides. This body says Spinoza, when put into motion from an external cause, will be determined to continue its motion along the extension of the side of the hexagon. The same, according to Spinoza's demonstration, may be attested concerning a figure of infinitely many sides, or, as the philosopher adds in a bracketed phrase, "a circle, according to Archimedes' definition" (Spinoza, 1988, p. 280).

It might of course be objected that Archimedes never states such a definition of the circle. The great Syracusan inscribes and circumscribes polygons and then makes use of the axiom of Eudoxus in order to show that with the use of polygons with very many sides one can get as close to the exact dimensions of the circle as he wishes to. Why, then, is Spinoza arguing here that a figure of infinitely many sides is what Archimedes defines as a circle? Why is he taking such liberties *vis à vis* the Archimedean way of thinking?

As we have already seen, Spinoza knew very well the method of ancient Greek geometry and used it for his own purposes within his philosophical system (he wrote several philosophical texts using the geometrical deductive method). He thus knew for certain that a Greek mathematician would never accept such a definition of the circle, a definition much closer to the use of infinitesimals. It is early modern mathematics that introduced concepts such as these, leading to the infinitesimal calculus. Kepler, for instance, introduced geometrical elements that might allow the passage from a polygon to the curved line of the perimeter of a circle (Mancosu, 1996). But it is obvious that Spinoza prefers to show the way Archimedean geometry – a geometry that, as we have already seen, served as a model for the newly introduced mathematical physics – entails considerations concerning the infinite. Given that Spinoza already granted God the attribute of Extension, one can easily comprehend that, according to the philosophy of Spinoza, Extension should not be counted as a purely passive substratum. Considered as a divine attribute, Extension involves in Spinoza's philosophy the divine productive power, i.e. the infinite itself. This is why Spinoza provides a demonstration based on the possibility of a passage from the finite to the infinite.

It is in his *Method of Mechanical Problems, for Eratosthenes* that Archimedes takes the surface of a finite geometrical figure (for instance, a triangle) as composed of the segments contained in it, and he does it while making it clear that this way of proceeding is not a proper way of demonstrating. This Archimedean text was not, of course, available to seventeenth century readers, as it was published in the twentieth century by Heiberg. But the fact is that the use of Archimedean techniques allows, under certain conditions, considerations concerning the infinite and its relation to the finite. In Spinoza's philosophy this relation is a very close one, and according to him this relation is based on metaphysical foundations and has a lot to do with the construction of an adequate theory of mathematical physics.

5. PASCAL'S THREE ORDERS

Pascal was one of the greatest mathematicians of his times. The solutions he brought to the problems of the conic sections and of projective geometry (a work inspired by the previous work on the same subject by Desargues) is enough to grant him the title of a mathematical genius – not to mention his pioneering probability theory. But it is the philosophical aspects of some leading figures of seventeenth century thought that is of some interest for the present study.

Pascal has produced along with his scientific production a tremendous body of work on philosophy which was not published during his lifetime. This huge amount of notes was published posthumously under the general title "Pensées". These notes were written as a preliminary study for what was meant to be an apology for Christian religion. Archimedes, nevertheless, is not absent from these notes; he is mentioned by Pascal in a passage dealing with the distinction of the three orders of things that he himself introduces. This tripartite scheme can also be found, for example, in the fragment 933 of the *Pensées* (the numbers of the fragments are given according to the Lafuma edition cited in the Bibliography) and must be related to the twofold scheme already introduced into the letter Pascal addressed to Queen Christine of Sweden.

In this letter (Pascal, 1963, pp. 279–280) Pascal distinguishes two different empires, two different orders, the order of power or order of bodies, and the order of knowledge or order of minds. These two orders have nothing in common and each one is, considered in its autonomy, great. What we should keep in mind is exactly the fact that Pascal stretches out: between kings and geniuses there is no common measure. They may both be sovereigns, each one in their own domain, but science and authority form two distinct areas of excellence.

This two-level division of human excellence is transformed into a three-level comparison of human activities in the *Pensées* in a quite different context. In this scheme, as was the case with the scheme proposed in the above mentioned letter, each one of the three orders is of a different and unique kind, that is, each order is incommensurable with the other two. Thus, the three orders now proposed are marked by the existence of a gap between them: there is no common measure between body and mind, at least no more than between mind and heart (Pascal speaks in the *Pensées* indifferently of the order of the heart or the order of charity: see, for example, the fragment 298). Each one of these orders has his own representatives. According to Pascal kings and captains appertain to the first order, intellectual people to the second, Jesus Christ as well as saints to the third.

Each order exercises no attraction to people whose interests belong to another order. Whatever is part of a certain order cannot produce any effect on what is of another order nor can it even be perceived by it: "All the brilliance of greatness has no attraction for people who are involved in pursuits of the mind. The greatness of intellectual people is invisible to kings, the rich, captains, to all those great in a material sense." (Pascal, 1995, p. 86).

This whole scheme, a scheme regarding the existence of a number of incommensurable levels, is already to be found in Pascal's mathematics. It is exactly this scheme that Pascal had in mind when he was working on his *Arithmetical Triangle*. In this scientific essay he points out that an entity of a lesser dimension – a point, for instance – does not at all augment the quantity of a higher level, a quantity with more dimensions than the aforesaid, when it is added to it. Therefore points add nothing at all to lines, lines add nothing at all to surfaces, etc. (Pascal, 1963, p. 94).

It is obvious that it is by way of comparison to this sound geometrical argument that Pascal proceeds when he has to deal with the classification of the orders of human activities in the *Pensées*. In other words, he draws his inspiration from a commonplace geometrical problem, the problem of the constitution of the continuum and the relation of the latter to the existence of indivisible magnitudes (Cavalieri's famous but also highly problematic *indivisibilia*; see Koyré, 1973). Pascal tries to establish an analogy between geometry and metaphysics and, of course, in order to illustrate his thought he chooses the work of Archimedes whom he obviously considers as the greater mathematician (or even scientist) of all times. Or, perhaps, things may be the other way around: he wants to establish an analogy between the highest achievements of human intellect and the achievements of divine charity – Archimedean mathematics being for him at the highest point of human capacities – and that is why he chooses a scheme inspired by the geometrical problem of the indivisibles.

So, Pascal picks one single intellectual in order to show the gap that exists between greatness taken in a material sense (i.e. the greatness of kings and captains) and intellectual greatness. There is one man, who occupies, *mutatis mutandis*, in the intellectual order the same position as kings in the material order: Archimedes. The great Syracusan, according to Pascal, has given to those who work within the limits of the order of intellectual greatness, victories that are not visible by kings and captains. Archimedes did not have to be a king or a prince in order to excel in his mathematics, he did not have to act the prince in his mathematical work. Pascal, following Plutarch's testimony, points out that Archimedes was indeed a prince, but this was a fact totally irrelevant to the significance of his scientific production. "It would have been pointless for Archimedes to act the prince in his mathematical books, even though he was one." (Pascal, 1995, p. 86).

The true value of any achievement is to be appraised according to its belonging to a certain context that authorizes the comparison between what is of the same kind. The real difference between Archimedes' production and what kings achieve in the order which is theirs is infinite, thus might not be counted as a difference taken in the same way as when we compare different things that are susceptible of less and more. Things pertaining to different orders have between them the same *ratio* as finite and infinite quantities. What is finite does not count in front of an infinite quantity and when added or subtracted to the latter does not add to its greatness or deduct from it something of any importance whatsoever. This is a persisting pattern in Pascal's writings who insists on the fact that different kinds of quantities have nothing in common and cannot be added. Thus, in the famous fragment of the *Pensées* entitled "Infinity nothingness" he writes: "A unit added to infinity does not increase it at all, any more than a foot added to an infinite length. The finite dissolves in the presence of the infinite and becomes pure nothingness. So it is with our mind before God, with our justice before divine justice. There is not so great a disproportion between our justice and God's justice as there is between unity and infinity." (Pascal, 1995, p. 152).

Unity-infinity-God. This tripartite scheme can guide us to a better understanding of the exact place of Archimedes' figure in Pascal's philosophy. Archimedes is for Pascal a scientist of outstanding greatness. His merit can be and must be distinguished from any other because his merits are infinitely superior to those of any king, given that they are not of the kind of merits that one can appreciate by looking at them with his eyes. "Archimedes in obscurity would still be venerated. He did not fight battles for the eyes to see, but he furnished every mind with his discoveries. How brilliantly he shone in those minds!" (Pascal, 1995, p. 86).

Let us not forget that Archimedes has in his mathematical work made great use of the principle attributed to Eudoxus. According to this basic principle (stated in Euclid's *Elements*, Book V, Definition 4 and Book X, Proposition 1) we can compare quantities of the same kind, but not of different kinds. Thus, Archimedes does not make use of planes in order to calculate, for instance, the volume of a "paraboloid of revolution" in his

On Conoids and Spheroids; he cuts the solid into volumes, instead. Pascal uses this principle not only in his mathematical work (we have already made an allusion to the way he refers to this principle in his mathematical treatise on the Arithmetical Triangle) but also in his philosophy. By distinguishing in the *Pensées* three different orders of human activities he sets the scene for an exact comparison of what can really bear an adequate comparison. In other words, Pascal promotes an idea according to which the three different orders must be treated just like different kinds of quantities; thus, Archimedes himself represents in this scheme a quantity, the greatest quantity that one can find in the order of intellectual power. This quantity, according to Pascal, should be compared only to what is of its own kind and not of a different one: "Out of all bodies together we could not succeed in making one little thought. It is impossible, and of another order. Out of all bodies and minds we could not draw one impulse of true charity. It is impossible, and of another, supernatural, order." (Pascal, 1995, p. 87). By treating intellectual people as a special kind of order and by comparing them only with what is relevant to their special qualities, Pascal attests his deep conviction that the principle of continuity can and must be applied to all levels of reality. In other words, Pascal treats the magnitude that Archimedes represents in the mathematical domain in an Archimedean manner!

It is not an accident that Pascal selected Archimedes as the highest point human intellect can reach by using its own powers. Geometry is the only human science capable of producing flawless demonstrations, as it is, according to Pascal, the only one to follow the true method. It possesses a method that one should apply to the entirety of human intellectual activity because, as Pascal points out, "we can see that in contests between minds that are equally strong in all other respects, the mathematical one wins." (Pascal, 1963, p. 349).

Archimedes, as one of the greatest mathematicians ever, has left such an impulse on the minds of mathematicians of all times that it is no wonder that Pascal chosen him as a figure of excellent brilliance in order to illustrate what he calls "the order of mind". In a certain way Archimedes was well beyond his time, and his achievements bear the trace of a mind that has reached the limits of the capacities of inventiveness within the realm of rational deduction.

If things are as clear as that, then on the level that is the level of purely human activity Archimedes has in Pascal's eyes no other rival; that is not to say, of course, that this level of activity is the only one that Pascal cares about. Quite the opposite: there exists another level where human ability is powerless and unable to achieve any goals at all. This level is the level of charity, accessible only by supernatural intervention. The whole argument about the existence of radically different orders makes it clear that what is of the domain of a certain order cannot be compared with things that are of the domain of another order: “It would have been pointless for Our Lord Jesus Christ, in order to shine in his reign of holiness, to come as a king. But he truly came in brilliance in his order. It is quite absurd to be shocked at the lowliness of Jesus Christ, as if that lowliness was of the same order as that of the greatness which he came to reveal.” (Pascal, 1995, p. 87). In the same manner, Archimedes stands out in the domain of intellectual capacities when he is compared with what is of the same kind as his. The work that he has produced is, in Pascal’s eyes, not just of infinite value for the science of mathematics, but of great importance when considered with purely philosophical criteria. It represents the highest point of human intellectual capacity, given that it consists of a collection of works forming the peak of mathematical craftsmanship, mathematical science being itself the summit of rational knowledge.

6. CONCLUSION

We have tried in our study to detect the main features that characterize the way some of the leading seventeenth century philosophical, but also scientific, figures have been referring to the work of Archimedes in their writings. Archimedes’ name is, according to them, connected to the search for truth not only in the domain of geometry but also in the domain of intellectual honesty and devotion to the ideal of truth. Seen from the perspective of Cartesian metaphysics, Archimedes is granted the status of a man in search of the absolute, of what is even impossible to find. The demand for a point in absolute rest in space is an example of such a request for what is, absolutely speaking, out of the reach of any kind of doubt. It is on a claim as absolute as this that the metaphysics of Descartes is based upon: what shall be considered as truth, says Descartes, must bear the trial of doubt and show that it can resist it.

Considered from this point of view, Descartes’ claim as well as his reference to Archimedes is not totally irrelevant to the way Spinoza’s reference to the work of Archimedes is intended to be understood. As we have seen, the main point of this reference is the one that has to do with the Archimedean use of the “method of exhaustion” and its complications. It is mainly the problem of the infinite and of its nature that matters for Spinoza, and his quite free, it is true, interpretation of the Archimedean geometry allows him to introduce the notion of infinite in a theory concerning the nature of material beings. Spinoza appeals to the Greek mathematician in order to show that a physical theory is capable of integrating infinitesimal techniques and thus can be considered as compatible with his philosophical theory. Spinozism, one should not forget, is a theory that builds God's relation to nature on the model of an immanent causal relation.

Pascal, on the contrary, pleads for the cause of Christian religion, but in a way that is a peculiar one. He distinguishes three different orders (the order of bodies, the order of the mind, and the order of charity *alias* the order of the heart). The nature of each of them does not bear any comparison with the other two, just like a finite magnitude cannot be compared to the infinite. Archimedes is for Pascal the man who has reached the highest point in the order of the mind. From this point of view the great Syracusan is somehow related to Pascal's considerations concerning the two infinites (the infinitely small and the infinitely big) that surround man.

What is impressing as an outcome of our analysis of these seventeenth century texts is that each one of them relates in one way or another the name and the mathematical work of Archimedes with a question regarding the infinite (the absolutely, thus infinitely, certain principle of knowledge, the infinite in nature, the relation between the infinite and the finite). Maybe this must be taken as indicative of the significance of Archimedes' work, a work that contains just two mentions (both of them found in the opening lines of the *Sand-Reckoner*) of the word *apeiron*. In the eyes of the philosophers we have been studying, Archimedes' geometry opens new perspectives for the science of nature as well as for the science of man.

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5. LEGACY AND INFLUENCE IN SCIENCE AND TECHNOLOGY

CROSS-FERTILISATION OF SCIENCE AND TECHNOLOGY IN THE TIME OF ARCHIMĒDĒS

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ABSTRACT The mutually positive interaction between Science and Technology is first reminded, and the early traces of such a crossfertilisation are sought in Ancient Greece. Subsequently, this phenomenon is examined during the Hellenistic period. Several technical achievements are found to be inspired by scientific knowledge, whereas Technology did offer to Science some practical ideas and, above all, lots of measuring devices. Within this Alexandrian spirit, Archimēdēs was educated and has produced his mathematical and engineering works. Some of his inventions, probably inspired by his own mathematical findings, are mentioned. A more detailed analysis is presented on the scientific bases of the archimēdean planetarium, admired by Cicero. Further on, the innovative views of Archimēdēs are presented on the hybrid demonstration of some geometrical theorems, via both mechanical and theoretical means. Besides, the strange view of Plutarch is critically examined, according to which Archimēdēs considered as "unworthy and vile" any activity related to machines. In conclusion, this assertion is found to be completely unsupported and arbitrary. Finally, the scientific rationality of the design of machines during the Italian Renaissance is mentioned as a confirmation of the validity of the crossfertilisation process.

1. INTRODUCTION

Empirical Technology appears very early – almost simultaneously with humans (animals too possess their own technology). The subject of this lecture is to examine the ways such a Technology may later on be fertilised by Science, and how Science itself may profit of available Technology; and, more specifically, how such a crossfertilisation has taken place during the 3rd cent. BCE, the period when Archimēdēs lived and published his scientific work. For this kind of historical investigation, one needs first to understand if such a crossfertilisation is feasible in principle. Subsequently, we need to follow the steps of such a process in Ancient Greece, before the examination of the related phenomena in the time of Archimēdēs.

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2. HOW SUCH A PROCESS MAY WORK

Let us consider this question first, independently of historical periods. In other words, we will try to identify the conditions under which Technology may enhance scientific thinking, and vice versa. To this end, it would be profitable to recall first the processes that generate Technology: Self-preservation does impose some Needs[1] that humans occasionally are unable to satisfy by means of their own natural forces; and they "invent" tools to intensify or lengthen their hands. Such inventions may be produced with the following procedure: Empirical trials (occasionally guided by imitation of the Nature) may lead by chance to the solution of the problem to "artificially" satisfy the given Need. This empirical process may be profitably enhanced by available previous knowledge or by additional knowledge purposely sought under the pressure of circumstances. Let us now describe the conditions under which Technology may influence Science, and vice versa.

i) This "technological process" may favourably affect future scientific endeavours, along the following lines:
   - Trials may evolve in purposeful "experiments" – the basis of Science.
   - Empirically accumulated knowledge, may be subjected to systematic classifications[2] – the basis for subsequent rational thinking.
   - Besides, available Technology may be used in order to produce measuring devices – the basis of scientifically proved observations.

ii) On the other hand, as soon as scientific thinking is able to predict a physical phenomenon (qualitatively or quantitavely), technological innovation in the field may be drastically assisted.

These seem in principle to be the possible interactions between Technology and Science. It remains to see if such was the case in Ancient Greece.

3. EARLY INTERACTIONS BETWEEN SCIENCE AND TECHNOLOGY IN ANCIENT GREECE

It seems relevant to start with the "mother science" (Geometry, that is) founded in Ionia during the 6th cent. BCE by Thalēs of Milētos: Thanks to

[1] It is remarkable that early enough greek scholars have recognised that the "Need" was the cause for Technology to be generated (Moschion, 4th cent. BCE, fragm. 6 N^2 sn.)

[2] This was also the case of Francis Bacon and Tycho Brahe...

the capacity of tracing on the ground very long parallel lines and of measuring distances between inaccessible points, Thalēs[3] himself was able to divert the Alys river behind the army of the Lydian King, so that the army would continue its march forwards through a dry river-bed (Herodotus, I-70). Similarly, about the same time, Eupalinos the engineer of the 1km long tunnel of Samos, was able to design geometrically a lot of details of precisely driving the tunnel through two fronts.

Another technological achievement was enhanced by the mathematical conceptualization of Music by the Pythagoreans: A distinguished Pythagorean, Archytas (around 400 BCE), offered the possibility to construct stringed musical instruments (tetrachordon) by means of simple geometrical measurements.[4] Similarly, Vitruvius (I, 1.9) reminds us that the knowledge of **music theory** is necessary in order to construct organs and other musical instruments. Besides, Plato himself clearly underlined the favourable consequences of Science on Technology: “In any Technique if you take apart Arithmetics, Maesurement and Statics, it would remain a superfluous residual” (Philēvos, 55 E). Such favourable influences of scientific developments on the Technology of the time, were to be made more clear during the Hellenistic period – the culmination of ancient greek Technology.

4. CROSSFERTILISATION DURING THE HELLENISTIC PERIOD

In fact, in the cosmopolitan world of the expanded greek civilisation, after Alexander the great, many more “philosophers” were now taking an interest in observation, measurement and indeed construction – being free of mythological thinking. That is why Alexandrian Technology flourished, together with Science – especially under the productive cover of the Mouseion of Alexandria.

4.1. Enhancement of Technology thanks to Science

The best way to introduce this subject would be to quote Vitruvius, the first Roman technical writer (and perhaps the last “Greek” technical writer): “Aristarchos, Plilolaos, Apollonios, Eratosthenes, Archimedes and Scopinas, have bequeathed to future generations several **machines**, invented and

[3] That is why Plato admired Thalēs as a “*σοφός*“, precisely because of this engineering achievement (Republica, 600 a)
[4] His lost book “Mēchanika” was based on mathematical principles.

constructed on the basis of **calculus** and **physical laws**", (I, 1.17-26)[5]. The enhancement of Technology thanks to Science, was recognised two thousand years ago…

In fact, after the classical period, Greek technology seems to flourish in a remarkable way. Let us only recall those developments that apparently were favourably influenced by scientific contributions.

a) In Lavrion, counterweights were facilitating the lifting of excavated ore through double wells, whereas the invention of helicoidal washing-channels substantially improved separation of mineral fragments. It is believed that such specific technological developments were inspired by scientific thinking.

b) Ktēsibius (285–222 BCE, Alexandria), the inventor of Hydraulis and the compressed-air-catapult, most probably was aware of the theory of Straton of Lampsakos (ca. 300 BCE) on the compressibility of gazes. In fact, his disciple Philon of Byzantium mentions "Ktēsibius' experiments on the nature of air, its power and its velocity of motion"; in other words, in the new Alexandrian spirit, the physicist Ktēsibius was in the same time an Engineer – a happy wedding of Science and Technology.

c) Philon of Byzantium (260–180 BCE, Alexandria), describes the "capacity" of a catapult, for the first time by means of a mathematical formula

$$d=1{,}1\ \sqrt[3]{w}$$

where
d = the diameter of the twisted sinew (the cord of the catapult), [in attic fingers, *δάκτυλοι*]
w = the weight of the spear to be thrown [in drachmas]
whereas the numerical coefficient includes a 10% increase "against uncertainties", according to Philon.

d) Almost every writer of engineering subjects includes in his technical treatises chapters (or "books") presenting Mathematics and Mechanics – indicating that it would not be possible for them to describe "machines" without a scientific background.

[5] More precisely, here is the text of Vitruvius: "…qui multas res <mechanicas> organicas, gnomonicas, **numero naturalibusque rationibus** inventas atque explicatas, posteris reliquerunt..."

4.2. Enhancement of Science thanks to Technology

On the other hand, Hellenistic Technology has offered to Science several counter services, briefly enumerated here-after.

i) "The parallels between Erasistratos[6] model of the h e a r t, and central features of the new Alexandrian mechanical Technology are striking". This is the opinion of H. von Staden (Yale university[7]), referring to the two chambers and valves of Ktēsibius's double-piston water pump!

ii) But the most important contribution of Hellenistic Technology to Science was the large number of measuring devices and instruments that allowed for further developments in Geography, Astronomy, Chemistry and Medicine of that time. It is worth to mention some examples of this category.

   – Thanks to his "spygmometer" Herophilos[8] was able to quantify basic medical data. The instrument was a small scale special clepsydra, containing appropriate wedges in order to modulate it for various ages of the patient. Rhythm (vv, -v, –, v-), speed, size and vehemence of the pulse were measured.
   – Dioptras and chorobats were improved thanks to progresses in manufacturing more solid and malleable alloys. Similarly, fine and more precise balances were manufactured.
   – Thanks to water clocks (Ktēsibius, Archimēdēs), long time intervals were objectively measured, as opposed to the traditional sundials.
   – Similarly, long distances may now be accurately measured by means of hodometers or sea-dromometers (gearwheeled contrivances, Archimēdēs).
   – The construction of astrolabs was improved, and mechanically complicated planetaria were technically feasible (see §5).

   These technological devices were, so to say, a **repayment** of Technology to Science, for the fertilisation of the first, thanks to the contributions of the second - the way we followed them in §4.1.

iii) Last but not least, I will submit that another (epistemological in nature) contribution of Engineers to Science is the consciousness of u n c e r t a i n t i e s of "scientific" results: Philon of Byzantium, in

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6 Alexandrian physician, 305–240 BCE

7 "Body and Machine. Interaction between Medicine, Mechanics and Philosophy in early Alexandria" in "Alexandria and Alexandrianism", J. Paul Getty Museum 1996, (p. 93)

8 Alexandrian physician, 331–250 BCE

his Poliorkētika, 3.50.20, explains why it is not always possible to solve a problem only by means of the logic and the principles of Mechanics.

5. ARCHIMĒDĒS: SCIENTIST OR ENGINEER?

a) I maintain we have proved that crossfertilisation between Science and Technology was established in greek regions as a normal procedure well before Archimēdēs (287–212 BCE). It is within this spirit that Archimēdēs studied in Alexandria, where both scientists and engineers were called sages "σοφοί". (Besides, this was also the term Plato used for Thalēs when he was citing Thalēs's **technical** achievements). In his book "Greek science after Aristotle" (1973), G. Lloyd explains how the technical writers did not merely describe complicated machines, but they also examined the principles of Mechanics intervening in the construction of these machines. This was the prevailing philosophy in Alexandria. We therefore maintain that, most probably, Archimēdēs had not faced the dilemma of the title of this chapter. In what follows, we will seek further evidence on this continuing crossfertilisation between Science and Technology in the work of Archimēdēs himself.

b) I will use the neologism "Endho-fertilisation" for those technical inventions that are based on scientific findings of the inventor **himself.** Such is the case with the archimēdean helicoidal pump – an achievement of the 3rd cent. BCE in Egypt: This invention is entirely based on the knowledge of the mathematical curve "helix" – an archimēdean finding. The helicoidal "worm gear" is also based on the archimēdean helix. In this respect, Athenaeus (2nd cent. CE) in his book "Philosophers in dinner", 5. 2006e, writes: "Hieron ordered [the hull] to be dragged down into the sea, […].Archimedes the Mechanicos alone has been able to drag it down with a few persons to help. He was able to launch such an enormous ship by means of a device involving a screw, for he was the first to invent devices employing the force of the screw". Here we should note that a simple screw would be unable to move such a big ship, along a considerable distance. A "worm gear" should be used instead, i.e. a threated shaft (a "worm screw"), together with a toothed wheel meshing into it. We may therefore reasonably suppose that Archimēdēs, with his fundamental knowledge of helix and with his devotion to Mechanics, has in fact used an helicoidally threaded shaft and, most probably, in combination with gearwheels. However, in the same context we must also consider the view of Plutarch (50–125 CE). In his book "Marcellus" (XIV), referring to the same ship and the same Engineer, he wrote: “A royal freighter (with three sails set) was dragged in the navy-yard with much effort […]. [Archimedes] being seated at a distance and without any effort, with his calm hand pulling the end of a compound pulley, brought the ship smoothly and precisely as if it were sailing to the sea”. I maintain however that there might be a contradiction here. The last and decisive phase of launching cannot be effectuated by t r a c t i o n applied in the front of a ship – and by whom? By a person seated in the sea? On the contrary, such a final stage of launching necessitates a p r e s s u r e to be exercised to the back of the ship. Perhaps Plutarch had confused this final stage of launching with the initial stage of dragging the ship by means of pulley-blocks, to bring it closer to the coast. I am therefore inclined to conclude that, most probably, Athenaeus was right in saying that Archimēdēs used a screw (a worm gear actually), and I subscribe to the opinion of A. I. Wilson[9] that the worm drive was invented by Archimēdēs around 250 BCE - as another case of “endho-fertilisation”.

c) Thanks to his engineering way of thinking, Archimēdēs made a rational revolution against scientific purism: He succeeded to be more productive in his pure mathematical endeavours, legitimising a fruitful introductory “mechanical” way of thinking. He clearly explains to Eratosthenēs (Mēchanika Theorēmata, H 428, 15 to 28) how “some Mathematics could be examined by means of Mechanics. And I am sure that this is not less useful for the demonstration of these theorems as well. Because some of these [theorems] that were disclosed to me mechanically, were later on demonstrated geometrically […]. Since it is easier to find a (geometrical) demonstration, having already some (mechanical) knowledge of these questions” – a thought experiment, that is.

I maintain that this is a high moment in History of Science: The ultimate goal of **objective** understanding of the World is better served by such hybrid intellectual procedures, than it does by means of others. Such procedures are therefore preferable than any others – without platonic[10] discriminations. That is why I subscribe to the following view of H.C. Horst Nowacki[11]: “Archimēdēs, feeling as a physicist

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9 A. I. Wilson: “Machines in Greek and Roman Technology”, in the Oxford Handbook of Engineering and Technology in the classical world”, Ed. J. P. Oleson, Oxford U.P., 2008, (p. 340).

10 I am referring here to the disagreements of Plato regarding the intellectual tools used by Eudoxos, Archytas and Menaichmos to solve the dēlian problem.

11 “Archimedes and Ship Stability”, in “Int. Conf. on maritime research and technology”, Crete, Oct. 2001.

and engineer, was able to bring his creative imagination to bear on scientific problems by allowing at least tentative conclusions from careful observation of nature, and from heuristic inductive reasoning, prior to rigorous proof. He was able to reconcile two methodologies and to combine observations with logic, inductive with deductive reasoning (in application to mathematics, mechanics and engineering)".

Nobody had before Archimēdēs dared that physical consideration of theoretical mathematics[12] – and I wish to enlist this achievement too into the "endho-fertilisation" events we were describing in the above §b.

d) The culmination of this mutual enhancement between Science and Technology is perhaps the archimēdean Planetarium described by Cicero (de republica, XIV, 22). Cicero is referring to two versions of Planetaria of Archimēdēs, brought to Rome by Marcellus, after the conquest of Syracuse (212 BCE). The first (placed in the temple of Virtue) was a solid and compact ("plenae") sphere and it was a very early invention ("vetus inventum") like those previously "constructed by Thales of Miletus and later worked by Eudoxus […], with the constellations and stars **fixed** in the sky". We may assume that this earlier Planetarium was used for demonstrations of instant and static positions of celestial bodies in the sky. On the second "sphere" Cicero is alluding to, "were delineated the motions of the sun, the moon and the five planets (more than could be shown on the solid globe). The invention of Archimēdēs deserved admiration because he found out, by means of one rotation, how it would be possible (in dissimilar motions) to maintain various orbits of **unequal** speed". It is rather clear that such a complete differentiation of motions can be ensured only by means of gearwheels. The scenario of "pulleys and ropes" to achieve the same remarkable result in a portable device, is rather hard to be feasible. On the other hand, the latin expression "una conversio" (one rotation) as a cause of various unequal motions, may refer to an external knob. One may therefore maintain that this second version of the archimēdean Planetarium was most probably functioning by means of sets of **gearwheels**. Cicero is clear about the material this device was made of: bronze. And this was also the view of Lactanius[13](3rd cent. CE), as opposed to Claudius Claudianus (4th cent. CE) maintaining that the contrivance was made of glass. But all of these later latin

[12] R. Netz, W. Noel: "The Archimedes Codex", Weidenfeld and Nicolson, 2007, (§6.c).
[13] In E. Stamatis: "Archimedes", greek edition, Tech. Chamber of Greece, Athens 1970 (four volumes).

writers, including Sextus Empiricus (2^{nd} cent. CE), do admire Archimēdēs for the way he produced the motions of the celestial bodies. In conclusion, it is difficult to disagree with Oe. Wikander[14], maintaining that "even though there is no explicit mention of gears in Archimede's "globe", we must presume that cogwheels transmitted the motion to its various different parts".

At the end of § XIV, Cicero describes the way one was able, by rotating the globe, to reproduce eclipses of the sun on the planetarium. If Cicero is right about the chronology of the device he describes[15], the Antikythera Mechanism cannot be considered as unicum.

The attribution of such an early achievement to Archimēdēs is also supported by the fact that he had devoted to the subject an entire book "On sphere making" (Σφαιροποιία)- one of the lost books that he has undoubtedly written[16](and sphere-making was about Planetaria).

For the purposes of this lecture, the main point of interest is that several Sciences could have contributed to the production of such a Planetarium:

– Happily enough, astronomical knowledge of the time was sufficient for the purpose: Autolykos (ca. 300 BCE) has well established spherical astronomy, whereas the metonic cycle was known both to the Greeks and Persians since the 5^{th} cent. BCE.

– Besides, theoretical Geometry (at least since Euclid, ca. 300 BCE) was well advanced for epicyclic motions of the Moon to be precisely described.

– On the other hand, pseudo-Aristoteles[17](ca. 270 BCE) had studied the cinematics of tangent cycles as a motion transmission contrivance, so that gearwheels to be appropriately designed.

Such scientific inputs were probably amalgamated with the available Technology of the time[18], for such an archimēdean Planetarium to be

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[14] In J. P. Oleson (Ed.): "The Oxford Handbook of Engineering and Technology in classical world", op. cit., (p. 792).

[15] Unfortunately, from this point on (of § XIV), two and a half pages of the manuscript are lost: more technical details were probably offered in these pages.

[16] See i.a. G. Lloyd: "Greek Science after Aristotle", 1973,(§4).

[17] Recent research however shows that "Mechanika Problemata" was a genuine work of Aristotle himself, according to M.E. Bottechia Deho ("Problemi meccanici", Rubbetino, Univ. di Calabria), 2000.

[18] See i.a. Tassios T.P.: "Prerequisities for the Antikythera Mechanism to be produced in the 2nd cent. BCE", 24th Int. Congress of the History of Science and Technology, Budapest, 2009.

produced, around the end of the 3rd cent. BCE. Historical data are not sufficient to offer further evidence on this happy wedding, but the **double inclination** of Archimēdēs towards science and technology is in favour of such an achievement.

e) This view however was apparently objected by Plutarch (Marcellus, §§ XIV and XVII)- and by some modern scholars taking for granted Plutarch's opinion. It is our duty to critically re-examine this opinion.

- First, I will invoke the broader spirit of the Hellenistic times in favour of an amalgamation of science and technology, as I think we proved it in the previous §4. Archimēdēs, with his continuous connections with Alexandria, could hardly deviate from such a broad norm.
- Second, Archimēdēs himself had repeatedly shown his deep knowledge and appreciation of Mechanics and its applications – as we have indicated in §5 b, c, d.
- Third, Plutarch along his book on Marcellus describes in details lots of "marvellous" and "terrible" machines invented and personally used by Archimēdēs. The frequency of such references seems to be in serious contradiction with Plutarch's assertion that "Archimēdēs considered as unworthy and vile any mechanical occupation".
- Four, almost all ancient writers express their admiration both for Archimēdēs mathematical **and** mechanical works. And Plutarch does not offer any foundation of his remarkable disagreement.
- On the other hand, it does seem true that "Archimēdēs did not leave behind any book on his [works] for which he got divine fame". If this passage of Plutarch referred to Archimēdēs's technical writings, we should recall that several of his lost books belonged to this category: "On balances" (quoted by Pappus), "On mirrors" (quoted by Olympiodoros), "Sphere-making" (quoted by Pappus and Proclus), "Whinch, Water clocks, Pneumatics" (mentioned by Tzetzis), "On sea-dromometers"(mentioned by Tzetzis). Allthough there is no evidence on the fate of all these books, the testimonia about them are not in harmony with Plutarch's contention.

In view of this rather extensive argumentation, we must conclude that "Plutarch's, assertion […] is not only unsupported, but it is contradicted by earlier and later commentators"[19]. In fact, such a hypothetical, deeply philosophical, attitude of Archimēdēs, should have been personally declared by Archimēdēs himself – in as much as such an attitude would constitute a fundamental revolution against the great Ktēsibius and the entire Alexandrian School. But **nowhere** in his entire written work has Archimēdēs ever expressed such a dramatic statement. Nor in his very many personal letters to Eratosthenēs, Dositheos etc.

I am afraid we have taken too seriously this arbitrary opinion of Plutarch under his "platonic predisposition"[20]. Yet, it seems to me that Plutarch had rather **misunderstood** Plato in this respect: Plutarch, in ch. XIV of "Marcellus", introduces the (fake) archimēdean view "the sage did not consider seriously [his] machines", and directly connects it with the case of the platonian disciples Eudoxos and Archytas, accusing them that "they initiated this beloved and famous endeavour with instrumental geometry". It is precisely here that Plutarch's misunderstanding lies. The instrument Plutarch is referring to, was foreordained to construct conic sections serving to solve the dēlian problem – a **completely** different scope than Archimēdēs's machines: Machines serve Society – Archytas's instrument served Mathematics. And it was precisely this second specific scope that Plato has condemned because of the risk "to destroy and corrupt the good of Geometry". This understandable platonic purism has **nothing** to do with the ideal to defend the City or to feed the poor – noble and entirely platonic scopes!

I therefore subscribe to the conclusion of Pappus on the same matter, saying that "Archimēdēs was considered a superman and enjoyed a general admiration **precisely** because of his achievements in Mechanics"[21].

6. INSTEAD OF AN EPILOGUE

As a confirmation of the evident correctness of the idea of crossfertilisation we dealt with in this lecture, I will follow the thread of the Science / Technology dialogue after the period of the Late Antiquity, very briefly though.

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[19] D.C. Simms: "Archimedes: Nailing Plutarch's lie", in Cultural crossfertilisation of South Italy and Western Greece through history, Patras, Sept. 2007.

[20] G.L. Lloyd: "Greek Science after Aristotle", 1973 (ch. 7).

[21] S. Cuomo: "Pappus of Alexandria and the Mathematics of Late Antiquity", ch. 3.

I will not comment here the much less documented Byzantine Period, and I will go directly to the Italian Renaissance. My first indication will be the fact that the last of the repetitive editions of the works of Heron of Alexandria appears in Venice, in the year of 1589 ("Gli Automati", Trad. B. Bladi, Appr. G. Porro) - a possible indication of the continuous interests of the Italian Scholars on the hybrid (science/technology) orientation of the Greeks. Now, one should systematically observe the relevant developments in 14th up to 15th century. But I will select only one emblematic case, related to the crossfertilisation issues: "Leonardo da Vinci developed several new machines. […]. Like many others, Leonardo concluded a [previous] process to give rigorousness and scientific dignity to mechanism design […] within an early scientific perspective", (M. Ceccarelli 2008)[22].

Several data insinuate that L.d.V. was indeed aware of the Alexandrian spirit:

- Interest for the work of Archimēdēs (manuscript B, Institut de France)
- Investigation for a manuscript of Archimēdēs in Padova (L.25/1502)
- Possession of the Philon's book on waters (2nd manuscript of Madrid)
- Interest for the "Centers of Gravity" of Archimēdēs (Manuscript F/o, Institut de France, 1508)
- Inspiration from Euclid (Ms B42b, CA 148 v-b, CA 283 v-e)
- Reference to the honours the Romans offered to Archimēdēs after his death (BM 279 v)
- Wish to meet J. Argyropoulos, professor of greek language (G. At. 11b, 37b).
- A possible crossfertilisation of centuries that is…
- Sic transit Technē mundis!

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ARCHIMEDES IN ANCIENT ROMAN WORLD

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ABSTRACT Cicero's rediscovery of Archimedes' tomb shows the interest for the Sicilian scientist in Rome, even if in Italy Archimedes' geometry was put into practice only by architects and by *Gromatici*, a sort of practical technicians who worked primarily in military and agricultural fields (we have some clear information about their work in a wonderful manuscript of the sixth century now in Wolfenbüttel). Some poets of the classical period were interested in the combination of numbers (like Catullus' 5 or 7 and Virgil's *Georgics* 2), but they never did open references to Archimedes, for metrical difficulties and embarrassed by his astonishing killing during the Roman occupation of Syracuse. Archimedes' life and death had an important part on the confluence of eastern and western culture in the third and second centuries B.C., but a good image of the scientist received serious obstacles by the difficulties of his theoretical works (Cicero also didn't read and understand the mathematical and physical ones) and by his strong and open struggle against the Romans.

1. INTRODUCTION

Archimedes was the greatest mathematician of classical antiquity and among the greatest scientists of all time. Gifted with a prodigious and audacious intuition, he brought to completion his discoveries by subjecting them to a rigorous logic. He lived intensely bound to his people and to his time, so much that his involvement in civil life favored the growth and conservation of innumerable anecdotes about him. At more than two thousand years distance the stories are still enjoyable and full of fascination. The ancient Romans were upset for at least four or five hundred years by Archimedes' extraordinary and seemingly contradictory personality. But it was difficult for them to accept that Archimedes - the scientist who disclosed unsuspected correspondences in the geometric figures (e.g. spirals, curved planes, characterized by infinite rotations growing in an arithmetic progression around a point), the investigator of some deep secrets of the nature, able to build powerful civilian and

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military machines – was so much harsh against the Roman supremacy in Sicily and the south Mediterranean sea. Bound by kinship and friendship with the tyrant Hiero II and his young nephew Hieronymus, who succeeded him at the age of fifteen in 215 BC, shortly after the terrible defeat of Rome at Cannes, Archimedes was convinced by the elders of Syracuse to turn his art somewhat from abstract notions to material things, and by applying his philosophy somehow to the needs which make themselves felt, to render it more evident to the common mind, to prepare offensive and defensive engines to be used in every kind of siege warfare.

In the most part of western Europe, two millennium later, similar conflicting attitudes were reserved to Galileo, by contemporaries and also by the general society of 18th and 19th century: in fact, the scientist born in Pisa was admired for the revolutionary significance he gave to the new astronomical and physical researches, but was feared for his conviction by the religious Catholic power and the isolation he lived during the last years in Arcetri (only nowadays Roman Church recognised that his process was for herself a terrible mistake). It can be said that the contradictory and difficult relationship between Galileo and his contemporaries was the core reason that drew a veil of silence over the admiration mixed with fear that the Roman world held on Archimedes, a really controversial chapter too in the world history of science.

Archimedes moved to Alexandria, then the intellectual capital of the world, around 243 BC, a little less than thirty years after Theocritus, Syracuse's greatest poet and founder of the bucolic genre. For his own part Archimedes refused to settle down in Egypt, but in Alexandria he became a friend of the scientists of the generation that immediately followed Euclid. In particular he befriended the geographer Eratosthenes of Cyrene, to whom he dedicated the *Method*, the astronomer Conon of Samos, for whom he showed great esteem, and Dositheus, to whom he dedicated the treatise *On the Sphere and the Cylinder* and even *Spirals* and *Conoids and Spheroids*. With these colleagues Archimedes exchanged letters from Sicily, subjecting his own works to their judgment before producing the final draft, so that they could discuss them and suggest to him further modifications and improvements. But in contrast to Euclid's works, those of Archimedes did not have a specific didactic intent: he omits the minutia and often trusts his reader to understand some passages of his reasoning that are anything but easy. It is not possible to find in geometry more profound and difficult questions treated in simpler and purer forms. Certainly the scientist from Syracuse was not content to give the ultimate refinement to subjects already known in whole or in part, rather he dedicated himself with passion to innovative discoveries and inventions. In the life and works of Archimedes science and technology are melded for the first time, with surprising advantages for both. He never founded a school, but he cultivated science with the spirit of an engineer rather than of a professor.

2. CICERO'S DISCOVERY OF ARCHIMEDES' TOMB

In ancient times (the period which Mary Jaeger [4] take into her investigation), Cicero's enthusiasm for his discovery of Archimedes' tomb (*Tusculanae Disputationes* 5.64–66) represented a major event: "When I was quaestor [in 75 BC] I tracked out his grave, which was unknown to the Syracusans (as they totally denied its existence), and found it enclosed all round and covered with brambles and thickets… So you see, one of the most famous cities of Greece, once indeed a great school of learning as well, would have been ignorant of the tomb of its one most ingenious citizen, had not a man of Arpinum point it out". Figure 1, But it must be noted that about this finding there isn't any other contemporary evidence. Besides, under the long Roman rule of Sicily there wasn't any follow-up to Cicero's discovery and all the monument's trails got lost.

Fig. 1. Cicero discovering Archimedes' tomb in a painting by Pierre-Henry de Valenciennes, now in Toulouse (Musée des Augustins). (Courtesy of the Musée des Augustins)

In the Latin works the most consistency stories about Archimedes are about the tricky of the crown which we read in the 9th book of *De architectura* by Vitruvius – an important text, but not normally included in the studies of Roman intellectuals. It must be pointed out that in that story the witty aspects prevail on scientific observations, as we get from the treaty *On Floating Bodies*, a work that poses the scientific bases of all hydrostatics.

3. PRESENCE IN ROME OF ARCHIMEDES' MATHEMATICAL WORKS

As I said, the reading and comprehension of Archimedes' mathematical works were in fact relegated to the technical works of the *Gromatici*, as it appears, for example, in *The Sand-reckoner* (Ψαμμίτης) quoted by Hyginus at the end of the first century: *Nam et Archimedem, virum preclari ingenii et magnarum rerum inventorem, ferunt scripsisse, quantum arenarum capere posset mundus, si repleretur* (p. 148 Thulin: "It is said that Archimedes, man of the finest brain and inventor of imposing machines, calculated the grains of sand that the entire world could contain"). In my opinion, we can find echoes of Archimedes' numbers also in some lines of well known Latin poets, like Catullus, Virgil and later Silius and Ausonius, but for metrical reasons the name of the scientist doesn't appear openly, and at the beginning of *Carmina* 1.28 Horace carefully preferred to replace his name with the one of Pythagorean Architas, not so fearful for the Romans, but who never did researches in geometry (Robin G.M. Nisbet - Margaret Hubbard [8], p. 321).

The drawing in Fig. 2 explains some of the most important discoveries of Archimedes in the difficult field of Mathematics (e.g. how to resolve the quadrature of the circle, the area and the volume of the sphere). They have been very much appreciated by Galilei and the best scientists of 17th, 18th and 19th century.

4. ARCHIMEDES' MYSTERIOUS DEATH

Lorenzo Braccesi [1], a classical very expert in ancient Sicilian history, recently sharply puts the accent on the Roman tradition about the death of Archimedes. He believes that the consolatory reconstruction of his death was done for political goals, following the stereotype of Roman *clementia*, as felt by Marcellus when he received the news of the tragic event: *quem cum audisset interfectum permoleste tulisse* (Cicero, *Verr.* 2.131: "when he

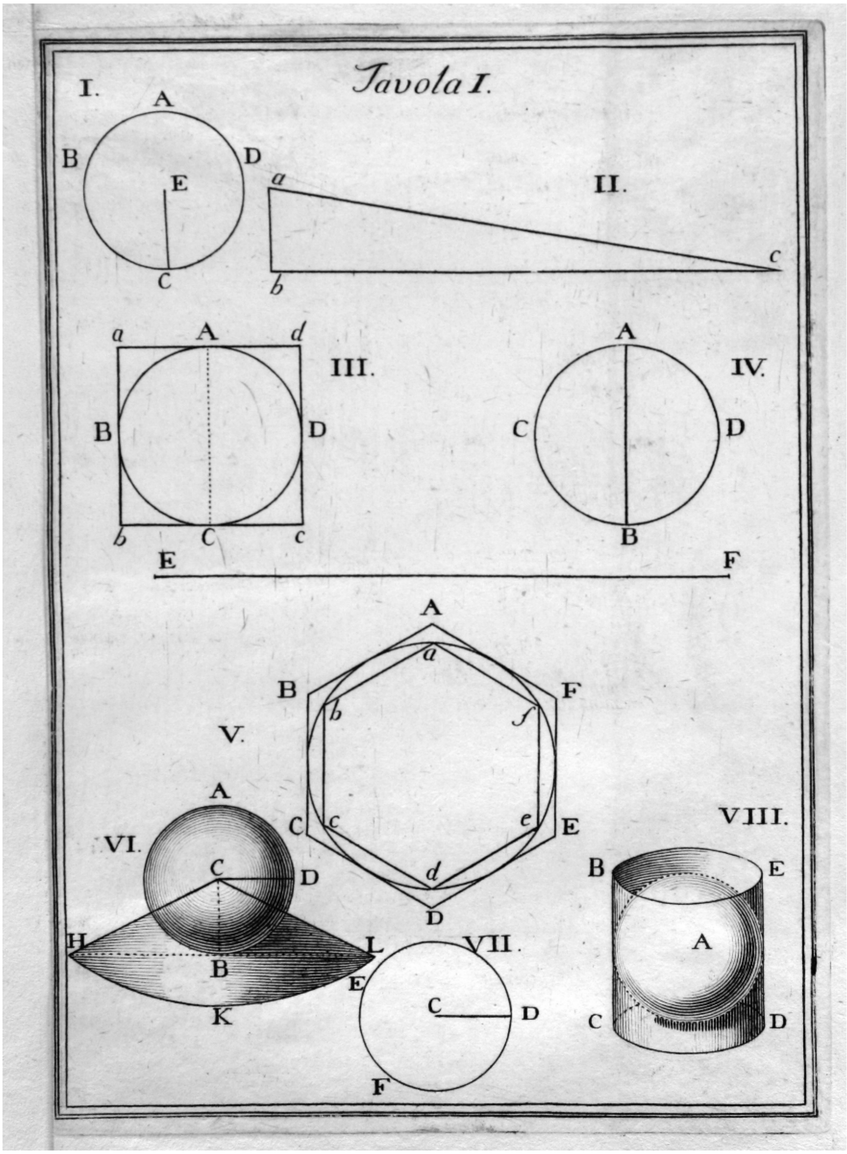

Fig. 2. Table I from 'Notizie istoriche e critiche intorno alla vita, alle invenzioni ed agli scritti di Archimede Siracusano' by Gian Maria Mazucchelli, Rizzardi, Brescia, 1737.

heard of his death, he was hardly moved"). Undoubtedly, that was not a glorious page for the history of Rome, also if Archimedes had been the most dangerous opponent to Marcellus' army!

Studying with particular care the story told by Valerius Maximus (8.7.7), Braccesi reaches the conclusion that Archimedes' death was really an execution of a task (with the head to produce as a proof to the Roman commander), a planned assassination to prevent him, once left alive, from placing himself at Hannibal's service, at the time the most dangerous opponent to the Romans.

On the other hand, for Marcellus, who desired a solemn triumph at home, it would be ridiculous to have a famous scientist 75 years old as a prisoner in irons after his own chariot of commander (*imperator*). Besides, at that time, the secret service of a conquering country was not so sophisticated as in our days and didn't activate any action for protecting the technicians of the opponent side with the hope of a future collaboration (as happened in 1945 with the Germans scientists and technicians who went to American or Russian laboratories).

5. TWO WONDERFUL PLANETARIUMS FROM SYRACUSE TO ROME

An important presence, and testimony too, of Archimedes in Italy were the two great and complex planetariums made by himself (Cicero again left us an account of them in *De republica* 1.21), brought from Syracuse to Rome as spoils of war: one was set in Marcellus' private house and the other in the temple of *Virtus*. Instead of offering an image of the Sicilian scientist, they were vivid images of Romans discovering, displaying, and manipulating artefacts associated with him. The images of the two spheres is emblematic of Cicero's way of casting the Roman appropriation of Greek cultural capital as both inheritance and rediscovery. In the planetariums the movements of the Sun, the Moon, and the five planets then known (Mercury, Venus, Mars, Jupiter, and Saturn, visible to the naked eye) were imitated with precision and the formation of the eclipses was represented: "the invention of Archimedes deserved special attention because he had thought out a way to represent accurately by a single device for turning the globe those various and divergent movements with their different rates of speed" (in the same passage from Cicero). Arab sources cite a work of Archimedes, *On the Construction of the Sphere*, in which the scientist would have given specific instructions for building a planetarium.

6. CONCLUSIONS

The relocation of the two planetaries in Rome will be a significant token "of the transfer of cultural capital from one people to another and from one generation to the next", something similar to the transfer of a large amount of memories kept for centuries in Europe and now in American museums and universities, as subtly observed by Mary Jaeger: "Cicero' story of the spheres anticipated the story of the later adventures of the Archimedes palimpsest, a scholarly work transferred from the Greek world to a new 'imperial' center (the United States) and given new status as a prestige object and an object of scholarship" ([4], 67–68 and 152, 154).

Archimedes' fame holds even in late antiquity and the Middle Ages. Cassiodorus in the 6th century tells us that Boethius had translated some of his treatises, but not even fragments of them have survived. In particular it was the Arabs who produced translations of Archimedes already in the 9th century, and the Syracusan's fame was so great in that culture that even works that did not belong to him were published in his name (and a range of fantastic inventions, such as the *burning mirror,* was attributed to him). During the Renaissance, the rediscovery, translation, and in-depth study of Archimedes' works gave an extraordinary impulse to the foundation and development of all aspects of modern science, both in methods of research and in geometric concepts and has constituted an important basis for future developments in mathematics and geometry. Among the admirers of Archimedes in that time were the Medicis (rulers of Florence), Piero della Francesca, Leonardo da Vinci, Niccolò Tartaglia, Federico Commandino and lastly Galileo Galilei, who claims to have read and studied his works "with infinite stupor" and cites him explicitly more than one hundred times, defining him, depending on the occasion, as *superhumanus, inimitabilis, divinissimus*. I am sure that Archimedes will be a vivid example also in the future of science, everyday and everywhere.

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ARCHIMEDES: RUSSIAN RESEARCHES

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ABSTRACT The short review of the most considerable Russian researches of engineering and scientific activity of Archimedes is given. Special attention is focused on Ivan Nikolayevich Veselovsky's original research - one of the largest Russian experts in this area.

1. INTRODUCTION

The review and the analysis of works of Archimedes in areas of mathematics and mechanics are included into all corresponding courses of history of all mechanic-mathematical and physical faculties of Russia. At all times the biography made by certain Heracleides of Oxyrhynchus, who lived in II century BC, had served for the ancient as the main source of biographic data on Archimedes and it had not reached us. Further the works of Titus Livy, Cicero, Diodorus, Silius Italicus, Valerius Maximus serve us as sources for Archimedes's biography. In the list of the literature we imparted the enumeration of some popular scientific books, school literature, university textbooks and scientific editions in Russian. Considerably the attitude to Archimedes's works at various stages of development of Russia represents more interest. There is short review of the most significant Russian researches of engineering and scientific activity of Archimedes in the article. The special attention is given to the review of original research of I.N. Veselovsky – one of the largest Russian researchers of a life and Archimedes activity.

2. THE REVIEW OF THE RUSSIAN RESEARCHES OF ENGINEERING AND SCIENTIFIC ACTIVITY OF ARCHIMEDES

In the given review the data on the most interesting, in the authors' opinion, works, devoted to Archimedes's activity and simultaneously not being prefaces and comments to the translations of his works, are resulted.

S.A. Paipetis and M. Ceccarelli (eds.), *The Genius of Archimedes – 23 Centuries of Influence on Mathematics, Science and Engineering*, History of Mechanism and Machine Science 11,
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Lourier S.J., **"Archimedes"** [1] In the book the sketch of a life and Archimedes's activity is given in a traditional key. At that time, at least in Russian and Soviet literature, the history of discovering of ***«Εφοδιχον»*** (Euphodic) was extracted in prime.

In this case we see that good luck accompanies knowledge, and «has found» and «has discovered» are absolutely different concepts in a science. The private-senior lecturer of the St. Petersburg university P. Papadopulo-Keramevs had found a parchment of a late origin not quite washed off Ancient Greek text (Palimpsest) of mathematical maintenance under the text of his interest in the library of a monastery of St. Sabas near Jerusalem. Not being the mathematician, Papadopulo-Keramevs has not given great value to the opening. In 1906 the Danish philologist and the mathematician of prof. Geiberg found small endurance from this manuscript in the catalogue of the Jerusalem library. In this short endurance prof. Gejberg [2] learnt ***ex ungue leonem*** («on claws of a lion») Archimedes product. He managed to find this manuscript and he found out in it the Greek texts of some Archimedes's compositions. The manuscript was compiled in X century. Between XII and XIV centuries, as it often happened, the same parchment has been used again for the theological text. The old text, thus tried to wash off, but, fortunately, not with full success. Archimedes's book contains a statement of the method connected with mechanical theorems and is devoted to Eratosthenes - the Alexandria mathematician and astronomer. It was familiar to Hero of Alexandria (II century BC), who named it "Euphodic" (*Εφοδιχον*) - method, guidance. Besides, there was the Archimedes's work on hydrostatics "About floating bodies" stated in it.

Kagan V.F., **"Archimedes"** [3] Besides known treatments of Archimedes's works the question on plausibility of legends on Archimedes engineering activity is discussed in the book, and the assumption that the stories about lifting of the Roman ships from water and moving a ship on sand are no more than a legend which was born owing to natural desire of Archimedes's descendants to give more shine to a figure of the greatest person.

Ivanovsky M.P., **Golden rule** [4] This small book is written for schoolchildren. The review of historical background from the 1st Punic war till Archimedes's activity is given and the condition of science and engineering of that epoch is shown in it. Activity of Ctesibius, Heron and Galileo, who were Archimedes' followers, was examined. Special attention is given to the examples of Archimedes's practical activities of: «Archimedes screw» and its application in the modern techniques, subtractive and differential collar and other examples. The analysis of technical possibilities of "artillery" of that time (ballistas, "scorpions", catapults), ravens and beaks is given. Mechanisms from legends about Archimedes (overturning of the Roman ships etc.) are considered from the point of view of realisation. It is thought that it was the best book about Archimedes for schoolchildren.

Veselovsky I.N.: **Archimedes** [5, 6] In some measure Veselovsky's sights were naturally influenced by the fact that the graduate of Lomonosov Moscow State University, who worked at the largest technical university of the country – Bauman Moscow Higher Technical School, today Bauman Moscow State Technical University, and conducted a course of Theoretical Mechanics. Working at engineering university and constant acquaintance to practical problems has led him to the understanding of the role of engineering in formation of the exact sciences.

On Fig. 1 the kind of a cover and the title page of the book [16] published by the State Publishing House of the Technical-Theoretical Literature in a series «Classics of physics» is resulted.

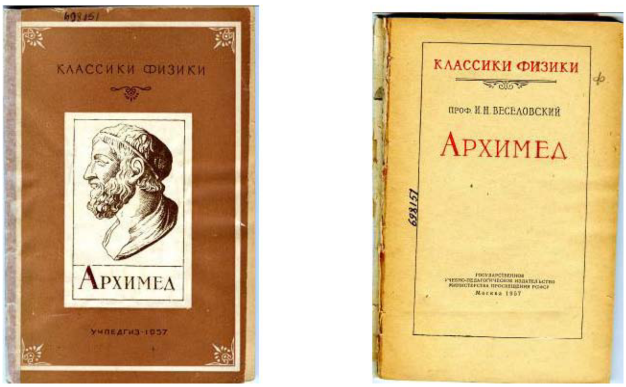

Fig. 1. The cover and title list of I.N. Veselovsky's book "Archimedes".

Authors represent in compressed way I.N. Veselovsky's position, hardly known to mechanicians and mathematicians, and furthermore to the engineers who do not do researches in history of science and technics, even within Russia. Difficulties of the analysis of Archimedes's life and activity consist in «having to scoop the data on his life from the writers mentioning Archimedes, or in the general historical works, as for example, at Polybius and Titus Livy, the historians, or writers who came across Archimedes as, for example, Plutarch». Actually, the information contains in descriptions of history of three Punic wars, Archimedes took part in two of which, that is events 264–146 BC. Two main purposes of this work are to give Archimedes characteristic which could add new lines to its figure;

to offer experts in the history of physical and mathematical sciences field a new view on the facts of Archimedes's biography of, exactly:

1. The characteristics of development of antic mechanics in the pre-Archimedes epoch;
2. Establishment of the fact that all mathematical Archimedes's products which have reached us completely were written by him at mature age (about 50 years);
3. Evolution of Archimedes's method of exhaustion;
4. Establishment of rather late occurrence of books «About balance of flat figures»;
5. Analysis of books «About swimming».

2.1. State of Engineering, Mechanics and Mathematics Before Archimedes

In the review of condition of engineering, mechanics and mathematics before Archimedes Veselovsky notices that the Greek siege artillery – catapults and ballistas – were invented at that time. The distance which a stone could cover at identical degree of elastic element tension was considered proportional to its volume, ***what is the size of potential energy in modern understanding.*** Establishment of this fact promoted stereometry development: a series of solutions of the volumes measurement main task – volume contraction, exceeding available in the set number of times. In connection with finding of the solution of a cube doubling problem the theory of new curves, opened by Eutocius and his brother Menehemus, – so-called conic sections: an ellipse, a hyperbole, a parabola; began to develop. In engineering the theory of mechanical similarity began to develop: it was found out that the simple increase in volume does not always give corresponding increase in useful results. War becomes the expensive enterprise, feasible only to the large-scale state. At the end of IV century the siege machine started to play such role under the prosecution of war that one Spartan commander named them «a tomb of human valour». In the struggle of Syracuse against the Carthagin invasion industrial mobilization, first in the world history, was carried out.

2.2. Archimedes's Engineering Works

The first Punic War had lasted from 264 till 241 BC. Syracuse took part in it firstly on the Carthaginians' party, and then, having had been dropped out of the war, began to adhere to neutrality politics. That is why it is possible to assume that Archimedes worked on the "first" speciality – the military engineer – and his researches basically had special character, which reveals in Archimedes's works in mechanics area, at this time. He had already used the concepts of the gravity centre and conditions of balance of a lever in «A parabola quadrature», and refers to the former works, which titles have strongly pronounced mechanical character. Many historians regarded as a merit to Archimedes that ***he had found time to help his native city in a trouble***. However, attentive reading of the Polybius' story gives one an absolutely other picture. First of all, the Roman risked to attack only one time and, besides, soon after the arrival so there was no time for preparation of protection. On the contrary, city defense had been prepared beforehand. During the process of the Roman approach to the city the long-range missile machines, went into action, then – the machines of closer action. For this purpose the action, called "square zeroing in" by modern artillerists, preliminary had to be done. It is impossible to make this "zeroing in" at the enemy's sight; it should be necessarily made beforehand. Polybius wrote absolutely definitely about the fact that the city was ready to the siege in advance: «… Hieron has given means on them (missile machines), and Archimedes has invented and masterful constructed machines». So, Archimedes's participation in defense was not casual. An image of Archimedes - the main military engineer of Hieron, the Syracuse governor, appears instead of the absent-minded scientist-mathematician in front of us. Of course, it should not be understood in the sense that Archimedes was the only military engineer. Anyway, the base, on which Archimedes mathematical and mechanical achievements were constructed, is opened by now.

2.3. The Mechanic and Mathematician Archimedes

On the basis of Archimedes's epistle Veselovskiy offers the following possible chronological order of Archimedes's prodution. Five epistles to Dosifeus (the student of Alexandrian astronomer and mathematician Conon of Samos, who Archimedes was in the habit of sending his compositions for criticism to): the mention in the message about Conon death allows us to establish the date before which all specified compositions could not be written, that is 246 year when Archimedes was 41 year old.

- A parabola Quadrature;
- Two books «About a sphere and a cylinder»;
- About spirals;
- About conoids and spheroids.

The sequence of these compositions is established absolutely precisely on the basis of introductions to these books, and the book «About spirals» had to stand last and only the difficulties, which were met during working on "conoids", had forced the delay of that book. Further:

- “About balance of flat figures”;
- “Euphodic”. Two books «About floating bodies», the second of which assumes “Conoids” and “Euphodic” to be written. Independently stand:
- “Circle Measurement” (the problem, which geometrical solution is given in the book «About spirals», is sorted out. Some features of the proof method applied in it allow to think that, at least, it is simultaneous to two last epistle to Dosifeus);
- “Psammite”. It is written after “Circle Measurement” and before 216 BC (the year of Helon’s death, Hieron’s son and joint ruler).

After 241 BC Archimedes had the chance to visit Alexandria – the centre of Hellenistic science of that time – and to strike up acquaintances local scientists, including Konon and Eratosthenes. Archimedes saw Konon if not as the teacher, than as the instructor which opinion he valued; he behaved as equal with equal with Eratosthenes, and even a little bit as with the higher. Two letters to Eratosthenes are known. These are “Euphodic” and the «Problem about bulls», opened in a XVIII century by German playwright and poet Lessing (the accessory to its Archimedes without the sufficient bases is challenged by some), where Archimedes does not ask Eratosthenes’s opinion any more. At the same time, as in accordance of Diodorus of Sicilia’s works (the historian, 2nd half of II after BC), Archimedes invented the “Archimedes’s screw”, which has been used to lift water.

Archimedes, as we know it, was solving such problem of antique technics as those, which could be mathematically processed. At present time we have to economize materials, which a special field of mechanics – the resistance of materials – is looking after. Constructions of the antique world did not demand the economy of materials. Receipt of big force by means of small, which the ancient solved by means of machines, was the primary goal of antique technics. Of the two laws for two principal kinds of simple machines – the lever and an inclined plane – the ancient knew only the laws of balance of the lever.

It is possible to make some representation about Archimedes’s early products on the fragments which have reached us. One of such products is a composition on mechanics which has not reached us, whether it was “about levers”, whether simply “mechanics”. Rod concept of all Archimedes’s statics is the concept about the center of gravity which, apparently, was established by Archimedes. He possibly got his understanding of the concept about the center of gravity on the basis of purely practical researches on load distribution between supports bearing it. There are some fragments from

Archimedes composition «Book of supports» from the Arabian translation of Heron's the Alexandrian (1st century AD) "Mechanics". He received the correct solution for a two-basic beam if the supports were on the ends of the beams. For a console beam he was not so lucky. For a console and three-basic beam his reasoning led to an incorrect conclusion. However, as the sizes of Greek constructions were rather small, there was no need to achieve exact values for pressure upon support. The correct decision for a multi-support beam was only received in XVIII after Euler's works.

In ancient Greece as well as in Europe the physics had been making a part of philosophy and stated without big mathematical wisdom till XVII. Therefore, Archimedes's attempt to state the balance theory strictly logically and to present it as mathematical science is possible to be considered as the beginning of mathematical physics.

In the Polibius's story it is said that Archimedes grasped the Roman swimming up ships by means of paws and tightened them upwards by machines. Archimedes could find the soil for establishment and working out of his law (the balance of floating bodies), in experiments with such cars, at least in very approximate calculations, in particular, if the fact that all preparations for defense of Syracuse it have been made beforehand is considered.

"Euphodic - *Eφοδιχον*" - "a way, means". In "Euphodic" Archimedes presents the mechanical proofs with which help he had come to his theorems and also puts new problems before Eratosthenes. Besides, he gives new theorems (about the center of gravity of a semi sphere, any segment of a sphere, segments of a paraboloid, a hyperboloid, ellipsoid rotations). Geiberg had found about half of Greek text of the treatise «About floating bodies» on the same parchment. Before Geiberg found it that text was only known in the Latin translation made in XIII century by Belgian monk Wilhelm von Moerbecke for Foma Aquinatus, known Catholic theologian monk. It is needed to look for real Archimedes's successors only in XVII when the works of Galileo, Huygens, Newton, marked the beginning of modern physical and mathematical sciences.

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APPENDIX

I. Some Researches of Archimedes's Activity in Russian Editions

- Bogolyubov V.I.: The note "On the Matter of Archimedes's law"; (The defence of sense certificate's infallibility), Tobolsk, Governmental typography, 1902 (in Russian)
- Bogolyubov V.I.: About the purposes of my two booklets: "On the Matter of Archimedes's law" and "The Earth stands but the Sun goes" – Tobolsk, Governmental typography, 1904 (in Russian)
- Krivenko I.S.: The design of transmissions with Archimedes's and involute worms, S. Petersburg, 1999 (in Russian)

II. Some Popular-Scientific Books about Archimedes in Russian

- The cosmic distances of Archimedes/ Zhitomirsky S.V., Kurtik G.E. – Moscow, 1988 (in Russian)
- Lishevsky V.P.: Verityhunters: Stories about science maker, Moscow, 1990 (in Russian)
- Goryaev M. A.: The History of Physics from Archimedes till Einstein, S.-Petersburg, 2002 (in Russian)
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ARCHIMEDEAN SCIENCE AND THE SCIENTIFIC REVOLUTION

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ABSTRACT According to Richard Westfall (Westfall, 1977) the Scientific Revolution of the seventeenth century was dominated by two themes: the Platonic-pythagorean tradition "which looked on nature in geometric terms" and mechanical philosophy "which conceived of nature as a huge machine". This paper is an attempt to study the appropriation of Archimedean science in the Scientific Revolution in Western Europe.

KEYWORDS: Archimedean tradition, scientific revolution, history of mechanics, Galileo's scientific method.

1. INTRODUCTION

The seventeenth century Scientific Revolution is a theme that continues to attract the attention of many historians of science. These scientists portray this process as occurring roughly in the following sequence: Copernicus' reformulation (1473–1543) of Ptolemy's solution (100–170) of the problem of planets with the need to restore their lost harmony; the acceptance by Kepler (1571–1630) and Galileo (1564–1642) of its realistic proposition; based on this perspective the development of mathematical tools to study the heavens; the mathematization of free fall and projectile motion to confirm the realistic basis of Copernicanism; and the development of a new inertial conception of motion, associating an abstract idealized concept of nature, linked to empirical and artificial means of experiment (Cohen, 1994).

The main objective of this paper is to present and discuss how the Archimedean 'legacy' was received and transformed by the long process that occurred during the revolution in science which culminated with Galileo and Newton's mechanicism (1642–1727) (Dugas, 1988). The paper pays special attention to the relationship between Archimedean techniques and the scientific method presented mainly by Galileo (Cohen, 1985). The

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origins of integral calculus are also referred as an important part of Archimedes' 'legacy' (Urbaneja, 2008).

2. A BIOGRAPHICAL NOTE ON ARCHIMEDES

Archimedes (see Fig. 1) was born in Syracuse, Sicily around the year of 287 B.C. His father was Phidias, an astronomer who investigated the size of the sun and moon and their distance from the earth. In his youth Archimedes seems to have spent some time in Egypt, where he invented the water-screw as a means of pumping water out of the Nile to irrigate fields (Hutchins, 1952).

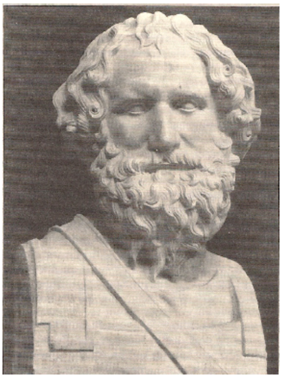

Fig. 1. Marble Bust of Archimedes in Naple Museum.

He seems to have studied with Euclid's pupils in Alexandria. It was probably there that he became friends with Conon of Samos (280B.C.–220B.C.) and Eratosthenes (285B.C.–194B.C.). He communicated his discoveries to these two before they were made public and it was for Eratosthenes that he wrote the 'Method'. After the death of Conon, Archimedes sent his discoveries to Conon's friend and pupil, Dositheus of Pelusium, to whom he dedicated other treatises.

Archimedes won great fame because of his mechanical inventions. At the request of King Hiero he made catapults, battering rams, cranes and many other engines and devices of war, which were later used with enormous success in the defense of Syracuse against the Romans. Another military story told by Lucian was that he used mirrors to set Roman ships on fire.

As described by Cicero (106B.C.–43B.C.) in his 'Republic' Archimedes constructed an astronomical machine sufficiently accurate to show eclipses of the sun and the moon. This apparatus consisted of concentric glass spheres moved by water power and represents the Eudoxian system of the world.

Archimedes only wrote about mathematical subjects, except in the lost work 'On Sphere-making'. His work dealt with arithmetic, geometry, mechanics and hydrostatics (Heath, 2002). He wrote no text books, unlike Euclid and Apollonius (262B.C.–200B.C.). Although some of his writings have been lost the most important have survived.

Archimedes' concern with mathematics is considered to be the cause of his death in the invasion that followed the capture of Syracuse by Marcellus in 212 B.C. According to some historians Archimedes was so intent on a mathematical diagram that when ordered by a soldier to attend the victorious general he refused. He was then slain by the enraged soldier. In accordance with Archimedes' wishes, his family and friends inscribed on his tomb the figure of his favorite theorem, a sphere and a circumscribed cylinder and the ratio of the containing solid to the contained (Arquimedes, 2006).

3. ARCHIMEDES' NEW SCIENCE

Statics and Hydrostatics – Aristotle's mechanics is integrated in a theory of physics which is part of a system of the world. Archimedes made statics an autonomous theoretical science, based on postulates of an experimental origin and supported by mathematical demonstrations (Serres, 1997).

In Book One of the treatise 'On the Equilibrium of Planes' Archimedes developed the principle of the lever. He enunciated eight postulates which provided the foundation for fifteenth propositions. In Book One of his treatise 'On Floating Bodies' Archimedes developed his famous Principle. It originally appears in Proposition 3 which states: "Of solids those which, size for size, are of equal weight with a fluid will, if let down into the fluid, be immersed so that they do not project above the surface but do not sink lower". In Book Two of the same treatise, Archimedes modified the principle which is the subject of Proposition 5, Book one, to the following form: "Any solid lighter than a fluid will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced" (Archimedes, 1952).

Both principles became fundamental principles of mechanics. The principle of the lever gave rise to the principle of virtual velocities which is equivalent to the equilibrium conditions for a static problem. Also, Galileo uses the principle of the lever and quotes Archimedes in the Second Day of the Discorsi. Archimedes' Principle is the origin of the Galilean theory of motion, as we will see below (Archimedes, 1952).

Historically it was perhaps Leonardo da Vinci (1452–1519) who recognized the importance of the general concept of moments in a static sense (Dugas, 1954). Galileo Galilei asserts that the moment was the inclination of the same body considered in the situation which it occupied on the arm of a lever or a balance (Drake, 1995).

The principle of virtual works was formalized by Jean Baptiste Fourier (1768–1830), but the letter sent by Johann Bernoulli (1667–1748) to Pierre Varignon (1654–1772), written in January 26, 1717, needs to be acknowledged as a historical fact of great importance in the history of the principle (Fourier, 1798).

The Geometrization of Mechanics and the Mechanization of Geometry – 'The Method Treating of Mechanical Problems', is a treatise addressed to Eratosthenes in which Archimedes reveals the method used in his mathematical discoveries. In fact, the same method was applied in other treatises, such as 'On the Equilibrium of Planes', 'Quadrature of the Parabola' (Fig. 2) and 'On Floating Bodies'.

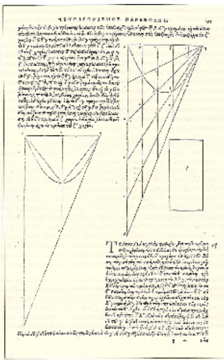

Fig. 2. Excerpt of propositions 16 and 17 of "On the Parabola Quadrature".

The essence of 'The Method' can be deduced by analyzing any problem solved by Archimedes. It is possible to identify three steps in its development. The first step is a pure geometrical approach, where the objects are selected in order to compare unknown quantities with known ones using the properties of the lever. In the second step, a mechanical approach is applied. The equilibrium laws of the lever regarding the fulcrum are used to make comparisons between geometrical quantities. In the final step the operations made in the second step are repeated but now in relation to the entire figures under consideration. This means, if the positions of the center of gravity of figures and the volume of one of them are known, the equilibrium conditions permit the volume of the other to be found (Archimedes, 1952).

'The Method' only survived on the Palimpsest discovered in 1906, as shown in Fig. 3. The most significant contribution made by the Palimpsest appears in proposition 14 of 'The Method' where Archimedes is measuring the volume of a cylindrical segment. In that proposition Archimedes pushes the discussion of the mathematical use of actual infinity approximately about 2000 years back in time.

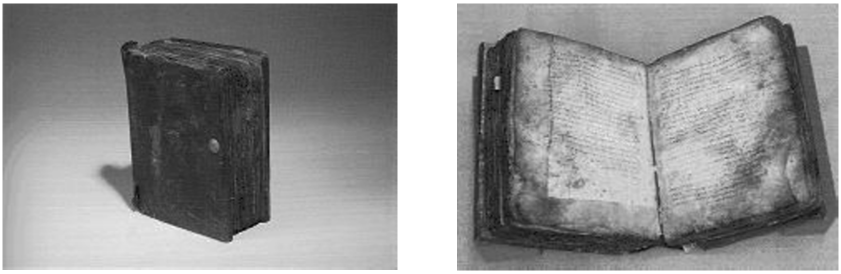

Fig. 3. Two Views of The Palimpsest.

The Origin of Integral Calculus – It is possible to see in Eudoxus-Archimedes' method of exhaustion the origin of integral calculus. This method is based on the theory of proportions presented by Eudoxus (408–355A.C.) of Cnidus. It consists of finding the area of a shape by inscribing inside it a sequence of polygons whose areas converge to the area of the containing shape. If the sequence is correctly constructed, the difference in area between the nth polygon and the containing shape will become arbitrarily small as n becomes large. As this difference becomes arbitrarily small, the possible values for the area of the shape are systematically 'exhausted' by the lower bound areas successively established by the sequence members (Eves, 1997).

The idea behind 'the Method' is attributed to Antiphon, but the theory was given greater rigor by Eudoxus. The first use of the term exhaustion appeared in 1647 by George de Saint-Vincent in his '*Opus geometricum quadraturae circuli et sectionum coni'*. Archimedes used this method for several geometric purposes, such as the length, area, volume and center of gravity of many geometric figures. However, the method of exhaustion is unable to efficiently solve the problems which it suggests. Only after the development of analytic geometry by Fermat (1601–1665) and Descartes (1596–1650), as well as the appearance of the concept of limits, was it possible to construct automatic algebraic operations to solve them (Descartes, 1991).

4. THE RECEPTION OF ARCHIMEDES' WORK IN WESTERN EUROPE

Archimedes' works were translated directly from Greek by William of Moerbeke (1215–1286), a Flemish Dominican. He was made Latin bishop of Corinth in Greece about 1286. At the request of Thomas Aquinas (1225–1274) he undertook a complete translation of the works of Aristotle. The reason for the request was the concern that by the thirteenth century Arabic versions had distorted the original meaning of Aristotle (384B.C.–322B.C.). Another concern was that the influence of the rationalist Averroes (1126–1198) could be a source of philosophical and theological error. Thus, Moerbeke was the first to translate the 'Politics' into Latin, which unlike other parts of Aristotelian corpus had not been translated into Arabic. The translations of Moerbeke (Fig. 4) were already standard classics by the 14th century. He also translated mathematical treatises by Hero of Alexandria and Archimedes. Then, he translated almost all of Archimedes works except 'The Method' and 'Stomachion'. In the early 1450s Pope Nicholas V (1397–1455) commissioned Jacobus de Sancto Cassiano Cremonensis to make a new translation of Archimedes with commentaries of Eutocius. This translation became the standard version and was printed in 1544 (Arquimedes, 2007).

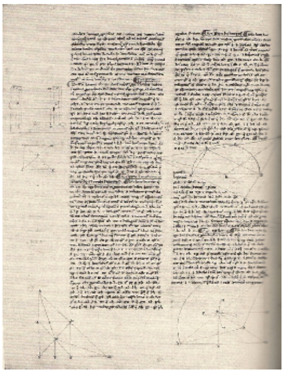

Fig. 4. Moerbeke's Translation to Latin of Archimedes Works.

Another translation of Archimedes was made by Niccolo Tartaglia (1500–1557) in 1543 in Venice. Tartaglia belongs to an important Italian school of mechanics. He published 'Nova Scientia' in 1537 and 'Quesiti et

Inventioni Diversi' in 1546, where dynamical problems were solved. In 1544 the *Editio Princeps* of Greek and Latin versions of Archimedes appeared in Basel.

Two new translations of Archimedes appeared in order to reconstruct his work. The authors of these translations were Frederico Commandino (1506–1575) and Francesco Maurolico (1494–1575). Because of the fruitful consequences of the translations by the former we will look at some of its details and implications.

Commandino studied philosophy and medicine at Padua from 1534–1544. He dedicated himself to studying the mathematical classics and he had the mathematical knowledge, as well as the language skills, to edit and translate these classics. He worked on the classic texts of Archimedes, Ptolemy, Euclid, Aristarchus, Pappus, Apollonius, Eutocius, Hero and Serenus.

The first translation of his published was an edition of Archimedes in 1558. In 1565 Commandino published his original work 'De Centro Gravitatis', described by Stillman Drake (Drake, 1981), Galileo's biographer, as a "pioneer treatise on centers of gravity in the Archimedean tradition". In the preface of this work Commandino refers to his edition of Archimedes' 'On Floating Bodies'.

Commandino also has influenced his pupil Guidobaldo del Monte (1545–1607), an important mathematician who was part of Galileo's circle. Guidobaldo studied at the University of Padua in 1564 and helped Galileo in his academic career. Under his patronage Galileo was appointed to a professorship of mathematics at the University of Pisa in 1589. Guidobaldo helped Galileo again in 1592 when he had to apply for the chair of mathematics at the University of Padua. He was a critic of Galileo's principle of isochronisms of the pendulum, one of Galileo's major discoveries which Guidobaldo believed was impossible.

5. THE MATHEMATIZATION OF NATURE IN THE SCIENTIFIC REVOLUTION

The supreme instance of abstraction in the scientific method is the use of mathematics, especially geometry, for the study of physical problems. Nature as mathematized by Galileo finds itself represented at a different level of abstraction than nature realized in daily experience. This difference appears to Galileo in a form of a problem of how to ensure that the mathematically expressed laws he found were valid in some way at the level of experience too. The necessary means to establish this were discovered by Galileo in experiments. He had already used 'mental experiments' to heuristically explain mathematical regularities that are behind free fall and projectile motion. Experiments also appeared to Galileo to provide a means by which to bridge the gap between phenomena represented at an idealized and at an empirical level (Galileu, 1988).

Kepler, with the fundamental help of Ticho Brahe's accurate (1546–1601) observations, mathematizes nature in the sense of creating a mathematical physics of the heavens, completely distinct from the model on which many astronomers' best efforts had been expended since the time of Ptolemy, including Copernicus himself. Galileo realized the impossibility of 'Archimedenizing' the Aristotelian concept of motion and saw also that free fall and projectile motion were key phenomena and could be the cornerstones of a new science of motion. In addition, Galileo was convinced that this new science could support the heliocentric doctrine and to provide its realistic basis. Bringing Archimedean techniques to study these phenomena, he mathematized nature in a modest range of terrestrial phenomena (Hall, 1981). Both Galileo and Kepler established the universe of precision. The Scientific Revolution had begun.

To study how Archimedean science was firstly used by Galileo, we have to focus our analysis on the return of Galileo to Pisa in 1589. In Pisa at that time philosophical discussions were greatly concern with motion. Francesco Buonamici (1533–1575), while he was one of Galileo's professors completed an important work, 'De Motu', published in Florence in 1591. The text is directly influenced by Archimedean themes. He discussed the problem of Hiero's crown and used Archimedes' treatise 'On Floating Bodies'.

Despite this influence Buonamici states that the Archimedean analysis on the decrease of the weight of bodies immersed in water did not have a universal character because it was restricted as a mathematical explanation. The progress Galileo made by developing a theory of motion supported by natural causes is remarkable. Here he found in Archimedes the foundation on which to construct a realistic theory of motion. It seems that it happens in the period spent in Pisa, where he wrote the work known as the manuscripts of 'De Motu Antiquora', with the objective of describing the natural motion of bodies. Galileo never published this work. However, its subject appears later in his 'Discorsi', published in 1638.

The development of Galileo's ideas about motion considers gravity as the unique cause of motion (free fall and projectile motion) and the explanation of the upright motion of light bodies immersed in water is due to Archimedes' principle. The fact that fluid is heavier than the bodies is the cause of upright motion. While Buonamici saw only a mathematical explanation to this phenomenon, Galileo inverted the approach completely, renewing the conceptual basis of the theory of motion and thus creating a new science (Geymonat, 1997).

Looking at the history of mechanics during the above mentioned period, it is important to note the contribution of Giovanni Benedetti (1530–1590) because he anticipates some of Galileo's ideas (Koyré, 1973). He stated that the speed of a falling body would depend on its surface because of friction with the air, and only in a vacuum would bodies of different sizes fall at the same speed. He also stated that bodies composed of the same material fall at the same speed regardless of their weight. He justified his claim with an argument using Archimedes' results on bodies in a fluid.

Benedetti's 'Diversarum Speculationum', which appeared in 1585, contains a section on mechanics, and in it circular motion is studied. He wrote that if a body is released from circular motion it will travel in a straight line which is tangent to the original circle of motion. This result also anticipates some of the achievements of Huygens (1629–1695) and Newton.

6. CONCLUSION

Archimedes was the first Greek mathematician to put into evidence the addition of terms of an infinite series. This result appears when he calculated the area under a parabola segment using the method of exhaustion. A good approximation to the $\prod$ number is obtained with this method (Heath, 2003). Archimedes also anticipates by several centuries the modern concepts of power series developed by Taylor (1685–1731) and MacLaurin (1698–1746).

The questions that arise out of Archimedean problems show the impossibility of Greek mathematics being able to solve these problems with the classical theory of proportions. Neither the tools nor the general methods were available for this. Only the Scientific Revolution would provide the solution.

After the appearance of a new mathematical language of analytic geometry with the pioneering work of François Viète (1540–1603) and Descartes the necessary innovation was introduced into the mathematical corpus creating new conditions and other possibilities for science.

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ARCHIMEDES' BURNING MIRRORS: MYTH OR REALITY?

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ABSTRACT This paper is devoted to the questions connected with «Archimedes' burning mirrors» myth. The scientific disputes, no and con arguments concerning these death mirrors or helio-concentrators are considered. Possibilities of current usage of helio-energetics on the basis of geometrical optics are presented.

KEYWORDS: Burning mirror, history of science, helio-energetics, geometrical optic.

1. INTRODUCTION

To stimulate economical growth and to develop the country without environmental damage, the modern world needs less expensive and ecologically safe energy. To provide an access to cheap, large, non-polluting and stable energetic sources is the most important global problem. In the middle of the 20th century, thermal power stations were replaced by hydraulic and nuclear ones. The latters seemed to be more profitable and environmentally safe. However, after Chernobyl accident many countries refused to develop nuclear energy. Hydroelectric power stations have roused many censure, because water reservoirs take a lot of space and disturb the regular water drain, they also change the environment. The accident in Sajano-Shushensky hydroelectric power station has revealed another disadvantage of giant structures.

Among recoverable energy sources solar radiation is the most perspective, taking the size of resources, ecological safety and prevalence into consideration. Every second the Sun radiates energy, which is thousand billions times greater, than the output radiation at nuclear explosion of 1 kg U235. The best way of strengthening energetic safety is the introduction of non-polluting and cheap technologies in energetics. Solar technology is one of the ways of solution the given problem; helio

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devices are the most rational in terms of using solar energy. Different designs of heliostations have been offered. Some of them use semi-conductor panels with photo elements; others due to the difference between temperature expenses create an air stream which rotates the generator turbine, the third ones work under the thermal power station scheme, using a solar energy for transformation water into steam.

Fig. 1. The machine room of heliostation, built in 1985.

In the USSR, the first industrial power heliostation was built in 1985, in the Crimea [1] (Fig. 1). 40 hectare area was covered with more than 2 thousand flat mirrors, controlled by two coordinates and directed to the top of the tower, where the steam generator was placed. The resulting steam was transmitted to the machine room, towards the steam turbine. However, that energy was too expensive, so using the station was acknowledged financially invalid. After the disintegration of the USSR, the station became the property of the Ukraine. At present the wind power station is planned to be build in that country.

All Earth energy sources are known to come from the Sun. It is the solar energy that the people indebt for all their technological achievements. The water circulation is caused by the Sun. Unequally heating different parts of the world the Sun causes the circulation of the air. All fossil fuel, which is used in modern energetic, is formed from the solar rays. But is it actually possible to use the solar energy directly? It seems like not a difficult problem. In his childhood everyone tried to burn out pictures in small wooden planks, using magnifying glass. It takes less than a minute until we can see black dot and light smoke. Using exactly this method, in J. Verne's novel "The Mysterious Island", engineer Cyrus Smith saved his friends, when the fire turned off. Using two glasses from watches he created a lens, and got the fire. This easy method of getting high temperature was in common use even in Ancient Ages. By using mirrors ancient engineers introduced another way of concentrating solar rays.

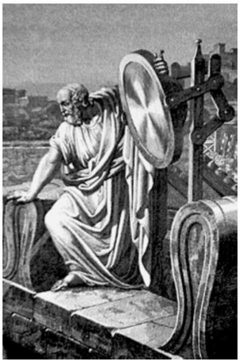

Fig. 2. Archimedes ruling a burning mirror.

2. ARCHIMEDES' BURNING MIRRORS

The topic of this paper is the first examples of using solar energy by Archimedes, the greatest ancient scientist. Among his fundamental works, which have not been brought to us, there was a work in geometrical optics, called "Catoptrics". We do not know what exactly was described there, and we do not know how the legend about burning Roman ships originally appeared (Fig. 2). Researchers have different opinions on Archimedes' burning mirrors. Some of them think that it is technically impossible; others think that if this is true, then igniting Roman ships at a large distance is a rumor; and just a few of them think that the actions, described in the legend, did exist in reality. Many explorers had been trying to investigate burning mirrors for a long time [2–8]. For the first time that legend was mentioned in ancient manuscripts, written in 902 A.D., and today they are kept in the Museum of Tareq Rajab [2]. Scientists divide researching burning mirrors into different stages. The first stage corresponds to the time, when people trusted any ancient sources and considered them as the absolute truth; also they tried to learn the theoretical reasoning of burning mirrors and how to reconstruct them. The second stage was based on the new achievements in optics of Johannes Kepler and Rene Descartes. They came up with the idea, which had doubted the opportunity of burning ships using mirrors. After their discoveries, this legend was acknowledged to be impossible. The third stage, started with Buffon's experiments and refuted Descartes' conclusion and recovered the possibility of existence of Archimedes' invention. But Danish philologist Geiberg doubted the existence of burning mirrors. In 2005, the scientists of The Massachusetts Institute of

Technology carried out another experiment of “Archimedes’ death rays”. This experiment proved the technical possibility of burning wooden samples by using mirrors. However, it did not answer the main question whether Archimedes had realized that idea.

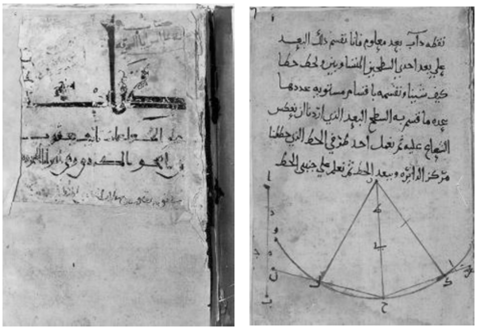

Fig. 3. Pages from the Arabian manuscript from A.D. 902 (translated from Greek).

Professor Roshdi Rashid found the manuscript of 902 A.D., and now it is the most ancient source of Archimedes’ mirrors [2] (Fig. 3). It is considered to be the translation from more ancient Greek manuscript. Another copy written in 14th century in Cairo, is located in India at present. The manuscript provides a deep historical research of earlier works about geometrical optics. It also admits that the research of burning mirrors was one of the most important subjects of researching in Alexandria in the 2nd-3rd centuries B.C. Conon of Alexandria, Archimedes, Dosithcus and Apollonitis were the main researchers. The manuscript refers to the earliest Greek sources and makes clear the fact, that Archimedes elaborated the theory of canonical sections and catoptrics. In the 2nd century he elaborated the fundamental work in catoptrics. Archimedes discovered two important fields of optics: investigation of burning mirrors and usage of parabolas and hyperboles in optical systems. After his death, Apollonitis continued working on canonical sections.

At the beginning of the 9th century scientists were not satisfied with the results of Greek and Byzantine predecessors, and they continued researching the rays, converging them to the dot at different distances from the mirror (Fig. 4). A century later this activity continued with geometrical research of the focal points with far-located sources using different types of mirrors, which resulted in developing double-convex and flat-convex lenses. That led to a new field of science – the dioptric. By the end of the 10th century Arabian and Greek scientists working on oval and parabolic mirrors had developed basic conceptions of the dioptric. But even today,

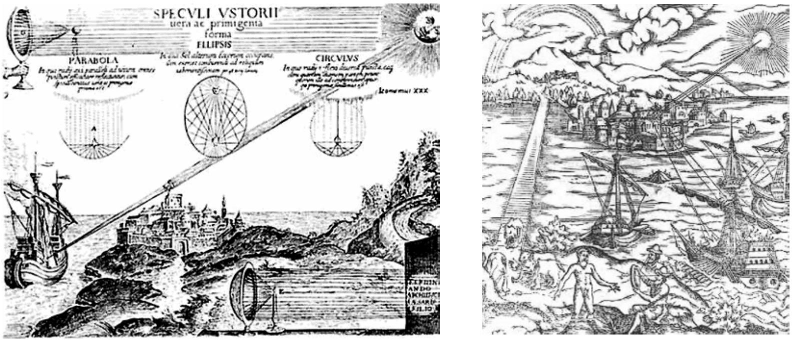

Fig. 4. Engravings depicting set fire to a Roman fleet by mirrors.

there exist many arguments on possibility of Archimedes' mirrors realization. In that case the biggest doubt is the absence of any mention about the mirrors in the history of Syracuse siege from the works by Polybius, Tius Livius and Plutarch. None of them mentioned any form of fire being used as a weapon, let alone burning mirrors. The fact that Polybius, the most important authority, had never said or written anything about Syracuse even 50 years later its downfall seems to be an important reason against of the existence of Archimedes' mirrors. Polybius usually described which military devices and technique had been used in the wars with all details. But he did not mention anything about using mirrors in his works about defense of Syracuse. However we should keep in mind another his peculiarity – his distrust [3]. The authority and popularity of Polybius were very great. His opinion meant a lot, so it is not surprising, that Tius Livius and Plutarch also did not mention anything about mirrors [3, 4, 5].

Later in the 2nd century A.D. Lucian, Greek satirist, stated that Archimedes had set the enemy's triremes on fire by artificial means [6, 7]. This fact might imply that the latter had spouted burning substances into them. Lucian did not say, as some commentators suggest, that Archimedes had used burning mirrors, only that "he had burnt the enemy's ships (triremes) by means of his science" [8]. Galen, a Roman doctor, clarified in his notes that burning Roman ships in the battle of Syracuse was well-known fact (Fig. 4). This may also imply that Archimedes spouted burning substances into them. Galen wrote: "In some way, I think, Archimedes is said to have set the enemy's triremes on fire by means of pyreia." [9] There are three points to be noted about this sentence. First, it implies that Galen only heard of Archimedes' feat as a story, a tale; second, he, like Lucian, wrote "triremes"; third, the Greek word pyreia means "flammable materials" and not "burning mirror." However, the third point has given rise to a considerable controversy, or even to a different legend.

The first statement about the burning mirrors Archimedes had used in Syracuse belongs to Anthemius, the Byzantine mathematician and the architect from Tralles [10]. In his work "Mechanical Paradoxes" he studied in details the creation of burning mirrors. He mentioned that there might have been at least 24 mirrors managed by a special mechanical device, which provided navigation to the object. The Byzantine authors such as Eustathius, Zonaras, and Tzetzes of the 12th century A.D. wrote about Archimedes' burning mirrors. The earliest unequivocal statement available to us that Archimedes used a burning mirror to set the Roman fleet on fire was made by Anthemius living 700 years after the event, and he only referred to it according to the tradition. The earliest circumstantial available accounts of the use of a burning mirror are those by Tzetzes and Zonaras, nearly 1,400 years after the event. But these historical evidences aren't convincing now. In the 17th century, Rene Descartes, a philosopher and mathematician, in his "The dioptrics" work gave a detailed theoretical analysis of burning mirrors and justified, that burning ships at a long distance was impossible. He wrote, that "only people, who do not know much in dioptrics are sure the fable to be true; as these mirrors had to be so huge, so they probably had never existed" [11].

Did Archimedes have enough knowledge for designing these mirrors? Paraboloid of revolution is the most efficient form for the mirror (or a section of it, which includes the center and the axes of the mirror). He might have known the main property of parabola: a parabola has the focus; that is, parallel rays of light are concentrated at one point. It is extremely difficult to decide whether Archimedes had all this knowledge, although we know he did have some. It is well-known that he used the method of exhaustion [12] and he realized that a set of lines could give as close an approximation to a parabola as required. He had sufficient knowledge for inventing, designing and constructing burning mirrors.

In 1747 Buffon (Fig. 5) published his 6th unknown memoirs called "The invention of mirrors for burning objects from long distances" [13]. In this work he described series of his experiments with burning mirrors. Firstly he made some calculations, and they showed that the size of mirrors might be very big – the diameter should be about 10 meters. Of course he could not make it by himself, that's why for the next experiment he used the mirror 13 times smaller than the calculated one. The mirror, constructed contained 168 flat mirrors, with the total area of 5.85 m^2. Using that mirror he could burn the tree, which was 50 meters away from him.

In 1741 M.V. Lomonosov wrote the work and called it "The reasoning about catoptrical and dioptrical burning devices" [14]. He described the

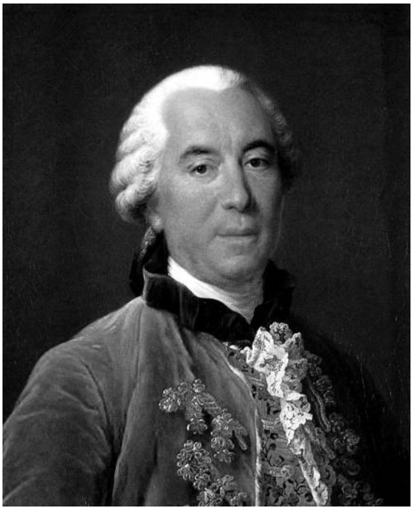

Fig. 5. Georges-Louis Leclerc, Comte de Buffon (1707–1788) and his burning mirror.

device, made of several mirrors, which directed solar rays towards the lenses and concentrated them in the focal point. Lomonosov constructed his device during chemical experiments. The members of The Russian Academy also were interested in Buffon's experiments. In 1747, before going abroad Tauberg I.I., the adviser, was told to learn about "the mirror invented in Paris". Today the problem of researching Archimedes' mirrors still exists. In 1973 Ioannis Sakkas, Greek scientist, made another experiment on burning ships using mirrors [15]. He lined 60 soldiers in Skaramangas harbor (near to Athens), each soldier holding a huge flat mirror 90x50 cm of size (Fig. 6). There was a resinous wooden boat, 50 meters away from the coast. By Sakkas' signal the soldiers were navigating reflected solar rays from the mirrors at the boat. After a couple of minutes the boat was full of smoke, and then got burnt. That was another justification of burning ships using mirrors. In 2005 a group of scientists from The Massachusetts Institute of Technology, sponsored by the Discovery channel repeated the experiment of "Archimedes' death ray" [16]. The experiment showed, that technically that was possible, but did not answer the question whether Archimedes had used the mirrors to burn enemy's ships. David Wallace, the professor from MIT wrote: "Who can say whether Archimedes did it or not? He's one of the great mathematical minds in history. I wouldn't want to underestimate his intelligence or ability."


Fig. 6. The experiment of Sakkas and the experiment of The Massachusetts Institute of Technology on burning mirrors.


Fig. 7. The solar tower (Seville, Spain) and V.S. Severyanin's project on helioconcentrator.

3. CONCLUSION

Today probably nobody is able to justify that Archimedes really made the arson of Roman ships during the defense of Syracuse. However, it is obvious, that the possibility of creating this kind of a weapon was described by Archimedes and many other scientists. Practically, we do not know much about Archimedes' life and works. The historical evidence of Archimedes' burning mirrors is feeble, contradictory in itself. Modern experiments suggest a burning mirror is highly unlikely to produce ignition on a moving ship, let alone a continuing fire [17]. We definitely can say that burning mirrors were not the basic weapon of defense of Syracuse, because there were more effective and reliable weapons. It is quiet possible that mirrors were used for blinding and putting burns to Romans or as a device for pointing the target. The most information has the idea of the legends or myths. But those legends gave birth to many theoretical and experimental scientific researches. Fields of geometrical optics were based on the legend of burning mirrors, and today developing helio-energetics and new designs of heliostations are based on geometrical optics. Figure 7 shows a new design of a helioconcentrator, assembled in Brest State Technical University, managed by Professor V.S. Severyanin [18] and a model of a modern electrical heliostation.

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THE INFLUENCE OF ARCHIMEDES IN THE MACHINE BOOKS FROM THE RENAISSANCE TO THE 19TH CENTURY

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ABSTRACT The influence of Archimedes on the so-called *theatre of machines* books is reviewed using original manuscripts from the 15th to the 19th centuries. The evidence shows continuity of knowledge of ancient Greek theory of machines as well as Archimedes principles of statics, hydrostatics and concepts of centers of gravity on the development of machine science.

1. INTRODUCTION

We know very little of the life of Archimedes (c. 287–212 BCE) except through the writings of later historians such as Polybius and Plutarch. There is also a lack of direct evidence for his technical contributions and inventions except through indirect commentaries and apocryphal stories of later chroniclers. There have been many scholarly interpretations of Archimedes works including those by Heath (1887), Dijksterhuis (1987), as well as Simms (1995). Our focus in this short paper is on the influence of Archimedes on the development of machine and mechanisms science and engineering. Many machine inventions have been attributed to Archimedes, such as the endless screw, the water screw pump, military hardware etc. [See for example, Chondros, 2007, or Koetsier and Blauwendraat, 2004 for a discussion of the screw pump.] We shall not attempt to revisit these interpretations. Instead we aim to examine the extent to which Archimedes' ideas or those attributed to him, were acknowledged by later historians, engineers, architects and scientists in writings related to the history of machines. Our sources will be the class of so-called 'theatre of machines' books, especially those from the 15th to the 18th centuries as well as the textbooks and monographs appearing in the 19th century during the great age of machines related to the industrial revolution.

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When we refer to the ideas and inventions of Archimedes we cannot hope to determine whether in fact these inventions were created by the Syracusan. Throughout the history of science in western civilization, scholars and historians have often used the names of historical figures as a tag to identify scientific concepts and inventions such as 'Archimedes' screw pump' or the 'Watt steam engine' or 'Newton's equations of motion' even when we have evidence that the screw pump may have originated in Egypt before the time of Archimedes, or that James Watt had *improved*, not invented the steam engine of Newcomen. Thus we shall look at the influence of *Archimedes-attributed* technical ideas such as the screw-pump, pulleys, cranes, hydrostatics, buoyancy and others on later inventions and technical ideas in engineering.

We must also acknowledge the fact that in technology, and especially in the theory and practice of machine construction, most advances have come about through evolution and through the accumulation of many centuries of experience as well as the development of new materials and the need to satisfy new demands of society, commerce and warfare. The development of a guild class of skilled craftspeople was another contributor to machine technology. Thus there is probably as much credit for new inventions due to workshop artisans. This evolution of machines also belies the trend in the past half century to attribute the development of technology solely to the development of science or the proof of mathematical theorems. [See e.g. Moon, 2007, Part II.]

Archimedes is often cited in the machine books as classical Greek name-dropping, homage to the ancient past, in reference to his notable accomplishments such as lifting the great ship of Syracuse, or to his inventions such as the screw pump. Archimedes is also mentioned for his work in hydrostatics and his mathematical studies.

In mathematics, Archimedes is especially noted for his "method" of formulating propositions in geometric quadrature, and center of gravity based on the principle of equilibrium and the lever. In both his studies of the lever and in hydrostatics his approach starts from statics: i.e. he uses no dynamical ideas. In hydrostatics, Archimedes introduced the idea of stability or the notion of immanent dynamics, but his work essentially avoids the notion of change. This is compatible with the early work in kinematics of mechanisms and machines that treated machines as constrained geometry of motion.

Among the several references on Archimedes and his inventions and scientific work is Vitruvius (80–15 BCE). In his *Books on Architecture*, is the Archimedes story of moving the world with a lever. Dijksterhuis discusses Archimedes' use of the equilibrium concept to motivate theorems in geometry and quadrature (See also the edition of Heath, 1887). More recently Simms (1995) gives a detailed critique of claims by Plutarch, Livy and Polybius on the engineering achievements of Archimedes in the battles and siege of Syracuse by the Romans. The awareness of Archimedes work in western science began in the Renaissance as discussed by Sarton [A History of Science: Chapter V] The first printed edition of Archimedes in appeared in 1503 in Venice. The first important edition printed in Latin was in 1543 by Niccolo Tartaglia. Archimedes *Statics* was translated into Latin by Guido Ubaldi del Monte 1588. According to Sarton, Archimedes *Method* was discovered in 1906 in a palimpsest i.e. as an erased work underneath writing of another manuscript. Our review is focused mostly on the European machine books from the Renaissance to the 19th century spanning the centuries over which Archimedes' work were translated into Western languages.

Fig. 1. Cover of Theatre of Machines book by Jabob de Strada, 1617–1618. Archimedes is shown on the left and Vitruvius on the right.

2. ARCHIMEDES IN THE THEATRE OF MACHINE BOOKS 1400–1800

There are many reviews of the so-called ‘theatre of machines’ books that catalog collections of machines, Fig. 1. [See e.g. Keller (1964), Moon (2007), Bautista Paz et al. (2007)] A table of many of these works may be found in Moon (2007) Table II-4, p. 147. Many of these works have been scanned by Cornell University as part of a project to document the model collection of Franz Reuleaux. These books may be found in the website http://KMODDL.library.cornell.edu. [See Moon (2004) for a description of KMODDL.] They may also be found directly at the URL http://digital.library.cornell.edu/k/kmoddl/index.html. These books were not only scanned but were optical-character-scanned for search capabilities. For many of these works we were able to scan for the name “Archimedes”. This was not always successful in the case of old German script. Where possible, we have used original manuscripts and books related to the history of machines in the Rare Books Library at Cornell University. In all over 20 books were surveyed. Almost all the books were in their original languages. We have tried to read and understand how the author was influenced by Archimedes and to cite specific pages, folios and plates were the reader can find references to Archimedes.

We begin our discussion with Vitruvius Pollio (c 27 BCE) because the architect-engineer Francesco di Giorgio Martini [1439–1501] was involved in one of the early attempts to translate Vitruvius into Latin in the 15th century and the published work of Francesco di Giorgio influenced many of the later machine book writers including Leonardo di Vinci. We end with the 19th century German engineer Franz Reuleaux and the American steam engine engineer, Robert Thurston.

Vitruvius: *De architectura (c. 15 BCE)*
In the English translation of 1929, based on the 8th century Latin translation, Archimedes is mentioned in several so-called books of Vitruvius. Referring to the Harvard University Press edition of 1931–1970, Ctesibius and Archimedes are mentioned in Book I (p. 13) in the context of ‘knowing the principles of nature’ in order to solve practical problems involving water. Also in Book I (p. 23) Archimedes is cited with others as having left “many treatises on machinery and clocks, in which mathematics and natural laws are used to discover and explain.” In the preface of Book VII (p. 75) Archimedes, Ctesibius and Philo of Byzantium are described as having written books on machinery. In Book VIII (p. 181) there is reference to Archimedes work on hydrostatics. In Book IX (p. 203) Vitruvius describes how Archimedes discovered a method to assess the

amount of gold in the King's crown by using the concept of buoyancy. Oddly in the section on machines, Book X, there is no explicit mention of Archimedes except indirectly in the discussion of the water screw pump (pp. 307–311).

Francesco di Giorgio Martini: *Trattato di architecttura (c. 1470–1480)*
There are several printings and editions of this work, each different perhaps because the copies were done in different scriptoria. In the facsimile edition of the *Codice Torinese Saluzziano*, there is a screw pump shown in Tav 85 f.46v along with three other water pumps. Archimedes name is not written here. However Francesco di Giorgio Martini made an attempt at translating Vitruvius and therefore was certainly aware of Archimedes and some of his accomplishments in the realm of machines. A similar screw pump may be found in the manuscripts of Leonardo da Vinci. (See below). It is known that Leonardo had a copy of one of Francesco di Giorgio Martini's codices and may have been influenced by this more senior engineer.

Roberto Valturio: *De Re Militari (c. 1455–1460)*
Valturio was not an engineer and this book was written for his patron as a summary of military techniques. There are 12 so-called books with many of the machines in Book X.

There are three references to Archimedes in *De Re Militari,* two in Book 2, Chapter 5 (p. 31 in the Cornell scanned copy) and one citation in Book 10 Chapter 4 (p. 254). The citation in Book X likely refers to machines.

Leonardo da Vinci: *Codex Madrid, Codex Atlantico*, *(c. 1490–1515)*
The citation of Archimedes in the Notebooks of Leonardo has been documented by MacCurdy (1906). Leonardo made several lists of the books in his library. In Codex Madrid II [Folio 2 verso] he mentions "Euclid on geometry" and "Problems of Aristotle" and "Quadrature of the circle" which may refer to Archimedes. In the *Notebooks*, MacCurdy translates a passage from Leonardo noting that there are works of Archimedes in the library of the Bishop of Padua [Manuscript L Cover 1 verso] implying that he had access to them. There is also a reference to the "complete Archimedes in the possession of the brother of Monsignor of Sant' Agnosto in Rome". Archimedes is also noted as the inventor of the steam cannon or 'architronito' in Notebook B 33 recto. In another Notebook, Leonardo refers to "Archimedes De Ponderibus". There are other citations in the manuscripts of Leonardo to Archimedes' work in mathematics. See also Hart (1961, 1925) for a discussion of Leonardo and

Archimedes. There is also a drawing of a screw pump similar to that of Francesco di Giorgio (Figure 2) suggesting that Leonardo may have been influenced by the latter. The influence of Archimedes on Leonardo may have been more on the subject of mathematics however.

Fig. 2. Screw pump of Leonardo da Vinci.

Jacques Besson: *Theatre des instruments mathematique et mechanique (1569–1578)*
In the 16th century, when Latin editions of Archimedes work just began to appear, Jacques Besson's "Theatre of Machines" has a plate [#54] with a sketch of a method to upright a capsized ship attributed to Archimedes. Clearly Besson must have read Plutarch's description of Archimedes launching a ship for King Hieron. In addition, the cover page has an engraving with two bearded, robed, ancient men one of whom might represent Archimedes with a ship at his foot. (See Strada below)

Agostino Ramelli: *Le diverse et artificiose machine del capitano Agostino Ramelli (1588)*
The original was written in both Italian and French. This classic work has been translated into English in a 1976 Dover Edition. In the Preface (page 50 of this edition), Ramelli pays homage to the Greek mathematicians including the 'divine Archimedes'. Most of the book is devoted to engravings and descriptions of machines. In Ramelli we see the widespread use of the Archimedes screw pump without attribution (see plates 45–48) suggesting that it was common knowledge in contemporary engineering design, independent of the publication of Archimedes mathematical works.

Vittorio Zonca: *Novo teatro di machine et edificii (1607)*
Zonca is listed in the title as an architect of Padua. In his sources he mentions the "maestii' Archimedes, Aristotle, and Vitruvius. The illustrations

of many machines have the quality of Besson and Ramelli, a generation earlier.

Jacobus Strada: *Künstlicher Abriss allerhand Wasser Wind Ross, und Handt Mühlen (1617–18)*
It is not clear who was the real author of this book (See Moon 2007, or Keller 1964). However, this handsome book of machines has a front cover with an engraving of Archimedes on the left and Vitruvius on the right. Archimedes carries a balance with two small spheres in his right hand and a glass jar of water in which to demonstrate the force of buoyancy. Clearly this was recognition of the debt inventors of machines has to Greek engineers. There is a section called "Wasser schrauben" or water screw pumps with a text reference to Vitruvius. Plates 30, 39 show the use of the water screw pump in two different machines.

Giovanni Branca: *Le Machine, 1629*
This is another 17th century theatre of machines book with 77 full page engraving plates. The engraving on the cover has Vitruvius on the left and Archimedes on the right, similar to that of Strada.

Otto von Guericke: *Experimenta Nova, 1672*
In this classic work on the use on the use of the force of vacuum, there is a reference to Archimedes' work on Geometry on page 66 of the Cornell scanned edition.

Jacob Leupold: *Theatri Machinarum Generale 1724*
On page 48 there is a reference in the text to Archimedes' work in geometry. In Leupold's work on water machines, *Theatri Machinarum Hydraulicarum*, there is another reference to Archimedes. It should be noted that James Watt studied German in order to read the work of Leupold at the time he was making improvements to Newcomen's steam engine. Thus the trail of influence of the Greek engineers and mathematicians on the inventors of the industrial revolution of the 19th century remained unbroken.

3. NINETEENTH CENTURY MACHINE BOOKS

Written machine design theory in the 16th–18th centuries, is dominated by the *'theatre of machines'* books written in Latin, Italian, French and German. In the 18th and 19th centuries the mode of transmission of written knowledge of machines evolves into popular lectures on practical science and finally into the technical textbook. This was especially true in the

English speaking countries. The dominance of English hegemony in steam engine development was helped in part by the education of the ordinary public on the practical principles of mathematics and science. An example are the lectures of James Ferguson (1710–1776) that were published in many editions in the UK and North America into the early 18th century. Ferguson was a self educated Scotsman who excelled in astronomy and instrument making and was elected to the Royal Society.

James Ferguson: *Lectures on Select Subjects in Mechanics, Hydrostatics, Hydraulics and Optics,* 1760 (7th edition in 1790.)
Ferguson has references to Archimedes as well as Isaac Newton. One reference on page 138 (7th ed) cites Archimedes' work in geometry of the circle and cylinder. In the section on "of Hydraulic Engines" (p. 153) he describes Archimedes use of the principle of buoyancy to check the purity of the King Hiero's gold crown in Syracuse. Ferguson's lectures had an influence on American inventors such as Oliver Evans who invented a high pressure steam engine as well as an automatic grain mill using an Archimedian Screw.

Oliver Evans: *The Young Mill-Wright and Miller Guide, 1795–1834*
Evans was a Philadelphian inventor whose initial fame was the invention of an automatic grain mill using a conveyor system. Part of this conveyor device used an Archimedes' screw pump for dry grain. This may be the first use of this pump for granular material. His book is full of practical advice on mill construction, but is prefaced by popular lectures on mechanics and hydraulics in the style of James Ferguson of Scotland. He also describes the so-called *mechanical powers,* or simple machines first enumerated by the school of Aristotle [Wheel and axel, lever, screw, inclined plane and pulley]. This again follows the lectures of Ferguson. A detailed engraving of the Archimedes' Screw conveyor is shown in Plate VIII, Article 89. Evans work on the automatic grain mill was known in Europe as well as North America. Evans also invented a high pressure steam engine circa 1790.

Jose Maria de Lanz and Augustin de Betancourt: *Analytical Essay on the Construction of Machines* 1808, 1817:
This is a classic book on the French classification of mechanisms that originated in the Ecole Polytechnique in the late 18th century. In the English translation, there is a reference to the "theory of the screw of Archimedes", noting that a discussion may be found in the work of Daniel Bernoulli on hydrodynamics.

J-.A. Borgnis: *Traite Complete De Mecanique: Composition des Machines (1818)*

This work is clearly motivated by the classification scheme of Lanz and Betancourt (1808). In Borgnis (1818) Archimedes is mentioned in the introduction/preface in reference to his work in geometry and his inventions. He is mentioned later on page 71 with respect to Plate VI Figure 19 for the spiral screw pump, Fig. 3. [See also Plate VI, Fig. 20, ‘Spirales a axe oblique’]

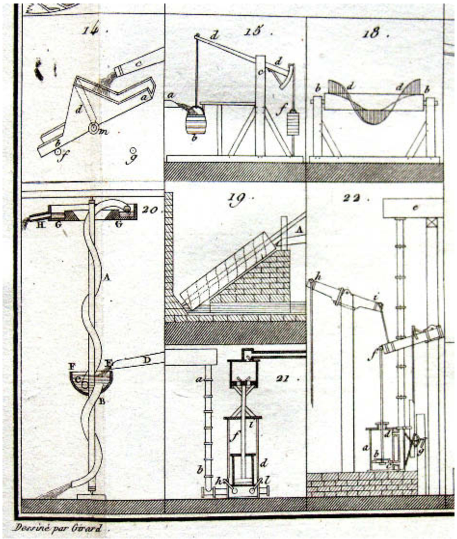

Fig. 3. Spiral of Archimedes in Borgnis (1818) Plate VI with its figures 18, 19, 20.

John Nicholson: *The Operative Mechanic (1826)*

This book is subtitled ‘a practical display of the manufactories and mechanical arts of the United Kingdom’. Under the section on hydraulic engines there is a reference to the Archimedes Screw (p. 246, 228). There is also a description of “Ctesibes” Pump” [sic] on page 259. There are figures of the Archimedes screw or spiral pump on Plate 27, Fig. 218–220. It is interesting that this practical book has references to the Greek engineers, showing the author’s desire to establish the continuity of machine invention from the ancients to the 19th century.

Robert Willis: *Principles of Mechanisms (1841)*

Willis’s work is one of the first extensive textbooks on the kinematics of mechanisms and influenced the work of Franz Reuleaux. His only reference to Archimedes is on page 75 on the “Spiral of Archimedes”.

Julius Weisbach: *Lehrbuch der Ingeniuer und Maschinen Mechanik (1848–49)*
This was a popular textbook in both the German states and North America. In the English translation, there is a reference on page 335, section 5 (Vol. 1) to Archimedes and the problem of finding the specific gravity of a mixture.

Henry T. Brown: *Five Hundred and Seven Mechanical Movements (1868)*
Brown was a patent attorney who published a magazine on new technology. He gathered articles from this journal into a popular 19th century theatre of mechanisms with small figures and short descriptions. On page 107 he describes "an application of Archimedes screw to raise water,–." The small sketch on page 106 (#443) is not very clear however.

Robert H. Thurston: *A History of the Growth of the Steam Engine (1878)*
Thurston was a famous mechanical engine with expertise in the steam engine. He taught at Steven's Tech and Cornell University in the late 19th century. In his history of the steam engine he has a reference to both Archimedes and Hero in the introduction (p. 4) and another on page 12 with reference to the ancient steam gun or Architonnerre. Towards the end of the book he describes a ship called the *Archimedes,* (125 feet long, 232 tons) built by the American firm Ship Propeller Co.

Franz Reuleaux: *The Constructor (1893)*
Oddly there is no reference to Archimedes in volume 1 of Reuleaux's famous Lehrbuch der Kinematik published in 1876, though he has a long list of references to other works in kinematics including Leupold. However in the English edition of his machine design book *The Constructor,* he has a reference to the Archimedian screw and the Archimedian Tympanon.

Franz Reuleaux: *Lehrbuch der Kinematik, Band 2 (1900)*
At the turn of the century, Reuleaux published the second edition of his earlier well-known work on kinematics. Here he has over 20 citations relating to Archimedes. (Pages XVII, 196, 197, 200, 206, 448, 449, 778). The citation on page 196, refers to Archimedes use of the principle of buoyancy to test the quality of the gold in King Hieron's gold crown.

4. CONCLUSION

We have by no means compiled a complete list of all the influential books in the history of machines leading up to the industrial revolution of the 19th century. But our survey provides detailed evidence of the influence of Greek science, mathematics and machine engineering through the work of Archimedes and others on the succeeding generations of machine theoreticians and engineering educators. We cannot know whether the artisans who built machines from the Renaissance to the early machine age of the 19th century, were aware of or were influenced by Archimedes in any direct way. There were also 'theatre of machines' books as well as 19th century machine design textbooks such as Salomon de Caus (published in 1624) where the author did not find any mention of Archimedes.

The author as well as others, has made a case for an evolutionary theory of the development of technology, especially machine invention and design. (See, Moon 2007) In this model of human technical achievements, great mathematicians, scientists and engineers such as Archimedes, Leonardo, Galileo, Watt and Reuleaux were not singular but were the best of their peers and that the contributions of artisans and guilds were also important contributors to human advances in technology.

In this short survey we found evidence that machine book authors were sometimes influenced as much by Archimedes writings in mathematics as in specific designs for machines. We can conclude however that the later codification of engineering and machine knowledge through the use of mathematics and scientific principles clearly owed a debt to the early knowledge of Archimedes and other ancient writers.

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ARCHIMEDES INFLUENCE IN SCIENCE AND ENGINEERING

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ABSTRACT Archimedes (ca. 287–212 BC) was born in Syracuse, in the Greek colony of Sicily. He studied mathematics probably at the Museum in Alexandria. Archimedes made important contributions to the field of mathematics. Archimedes discovered fundamental theorems concerning the center of gravity of plane geometric shapes and solids. He is the founder of statics and of hydrostatics. Archimedes was both a great engineer and a great inventor, his machines fascinated subsequent writers, and he earned the honorary title "father of experimental science". Archimedes systematized the design of simple machines and the study of their functions and developed a rigorous theory of levers and the kinematics of the screw. His works contain a set of concrete principles upon which mechanics could be developed as a science using mathematics and reason. His contribution separates engineering science from technology and crafts, often confused for matters arrived at empirically through a process of long evolution. His works have influenced science and engineering from the Byzantine period to the Industrial Revolution and the New Era.

1. INTRODUCTION

Archimedes (ca. 287–212 BC) was born in Syracuse, in the Greek colony of Sicily (Fig. 1). His father was the astronomer and mathematician Phidias, and he was related to King Hieron II (308–216 BC). The name of his father – Pheidias – suggests an origin, at least some generations back, in an artistic background (Stamatis 1973). At the time of Archimedes, Syracuse was an independent Greek city-state with a 500-year history. The colony of Syracuse was established by Corinthians, led by Archias in 734 BC. The city grew and prospered, and in the course of the 5th century BC the wealth, cultural development, political power and victorious wars

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against Athenians and Carthaginians ensured for a long time the dominance of Syracuse as the most powerful Greek city over the entire south-western Mediterranean basin.

Fig. 1. Archimedes portrait Courtesy of the MacTutor History of Mathematics Archive run by the School of Mathematics and Statistics at the University of St Andrews, Fife, Scotland).

The decline of Greek civilization coincides with the rise of Alexandria, founded in honour of Alexander the Great (356–323 BC) in the Nile Delta in Egypt. Alexandria was the greatest city of the ancient world, the capital of Egypt from its founding in 332 BC to AD 642, and became the most important scientific centre in the world at that time and a centre of Hellenic scholarship and science. In this University, the Museum (meaning, the house of Muses, the protectresses of the Arts and Sciences) flourished a number of great mathematicians and engineers (Dimarogonas 2001). Euclid was one of the most well known scholars who lived in Alexandria and his Elements in Geometry with an elegant logical structure based on a small number of self-evident axioms undoubtedly influenced the work of Archimedes (Sacheri 1986).

What we know of Archimedes' life comes from two radically different lines of tradition (Bell 1965). One is his extant writings and the other is the ancient biographical and historical tradition, usually combining the factual with the legendary. The earliest source is Polybius a competent historian writing a couple of generations after Archimedes' death and from the histories authored by Plutarch, Cicero, and other historians several centuries after his death. Due to the length of time between Archimedes' death and his biographers' inconsistencies among their writings may arise. Plutarch and Polybius describe giant mechanisms for lifting ships from the sea, ship-burning mirrors, and a steam gun designed and built by Archimedes.

According to some sources Archimedes went to Alexandria about 250–240 BC to study in the Museum under Conon of Samos, a mathematician and astronomer (the custodian of the Alexandrian library after Euclid's death), Eratosthenes and other mathematicians who had been students of Euclid (Dijksterhuis, E.J. 1987, Landels 2000, Lazos 1995, Netz 2004, Simms 1995). According to Schneider (1979), we are not really sure that Archimedes actually was and studied in Alexandria. It is a supposition; he knew the Alexandrian mathematicians but there is no direct evidence that he ever was in Egypt. He knew Conon but theoretically it is possible that Conon visited Syracuse and Archimedes never was in Egypt. Also, the fact that Archimedes published his works in the form of correspondence with the principal mathematicians of his time is a serious reason to expect that he was able to study through this web network. The Method discovered in 2004 with the Palimpsest is a correspondence with Eratosthenes.

The attribution of works to Archimedes is a difficult historical question. His works were preserved mainly through Latin and Greek-Latin versions handwritten and then printed from the thirteenth to the seventeenth centuries. Translations into modern European languages came later. The *Works* of Archimedes as well as other extant manuscripts had a difficult path to follow through the ages. (Stamatis 1973, Heath 2002, Netz 2004, Chondros 2007). The standard edition in Greek and Latin of the works of Archimedes with the ancient commentaries is that of Johan Ludwig Heiberg and Evangelos S. Stamates (eds.), *Opera omnia, cum commentariis Eutocii*, 3 vol. (1910–15, reprinted in 1972). According to Netz (Netz 2004) in the corpus surviving in Greek – where Eutocius' commentaries are considered as well 29 works may be ascribed to Archimedes.

Archimedes' contributions to mathematical knowledge were diverse. Archimedes was the first mathematician to introduce mechanical curves as legitimate objects of study (Fig. 2 left). In *Spiral Lines* Archimedes defines what is now called Archimedes' spiral. This is the first mechanical curve (i.e., traced by a moving point) ever considered by a Greek mathematician.

In *Measurement of a Circle*, he described his method for calculating the ratio between the circumference of a circle and its diameter. By a method that involved measuring the perimeter of inscribed and circumscribed polygons Archimedes correctly determined that the value of π was somewhere between 3.1408 and 3.1428.

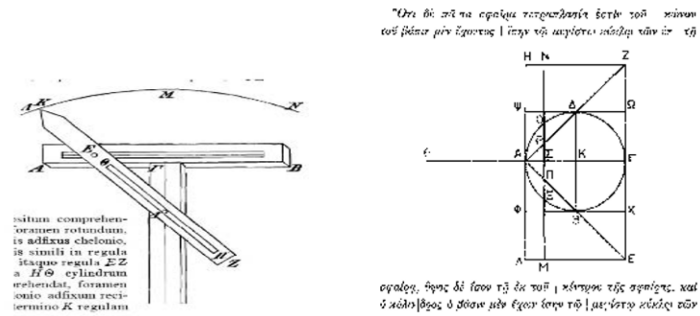

Fig. 2. Left: Mechanical curve. From Heiberg, 1972, p. 99. Right: From the book On Conoids and Spheroids (Stamatis 1973 B p. 396).

Archimedes settled in his native city, Syracuse, where he devoted the rest of his life to the study of mathematics and building machines and mechanisms. In addition to his mathematical studies, Archimedes was both a great engineer and a great inventor. He invented the field of statics, enunciated the law of the lever, the law of equilibrium of fluids, and the law of buoyancy, and he contributed to knowledge concerning at least three of the five simple machines – winch, pulley, lever, wedge, and screw – known to antiquity. He discovered the concept of specific gravity and conducted experiments on buoyancy. He is credited with inventing the compound pulley, the catapult, and the Archimedes Screw, an auger-like device for raising water. He conducted important studies on gravity, balance, and equilibrium that grew out of his work with levers and demonstrated the power of mechanical advantage (Drachman 1963, Heath 2001, Archimedes–Apanta (The Works) Vols. 1–3 2002).

Archimedes systematized the design of simple machines and the study of their functions and developed a rigorous theory of levers and the kinematics of the screw. (Dimarogonas 2001). He designed and built Syracusia (The Lady of Syracuse), the largest ship of his times, 80 m long, 4,000 ton displacement, with three decks. The ship made only its maiden trip to Alexandria because it was too slow and there were no harbor facilities anywhere to handle her (Dimarogonas 2001, Archimedes–Apanta Vol. 6 2002). Archimedes was also known as an outstanding astronomer; his observations of solstices were used by other astronomers of the era.

During Archimedes' lifetime the first two of the three Punic Wars between the Romans and the Carthaginians were fought. The series of wars between Rome and Carthage were known to the Romans as the "Punic Wars" because of the Latin name for the Carthaginians: Punici, derived from Phoenici, referring to the Carthaginians' Phoenician ancestry. During the Second Punic War (218–201 BC) – the great World War of the classical Mediterranean, Syracuse allied itself with Carthage, and when the Roman general Marcellus began a siege on the city in 214 BC, Archimedes was called upon by King Hieron to aid in its defence and later worked as a military engineer for Syracuse (Plutarch ca. 45–120 AD).

The historical accounts of Archimedes' war-faring inventions are vivid and possibly exaggerated. It is claimed that he devised catapult launchers that threw heavy beams and stones at the Roman ships, burning-glasses that reflected the sun's rays and set ships on fire, and either invented or improved upon a device that would remain one of the most important forms of warfare technology for almost two millennia: the catapult. Plutarchos and Polybios (201–120 BC) describe giant mechanisms for lifting ships from the sea, ship-burning mirrors and a steam gun designed and built by Archimedes. The latter fascinated Leonardo da Vinci, however the validity of these stories is questionable. The death of Archimedes by the hands of a Roman soldier is symbolical of a world-change of the first magnitude: the Greeks, with their love of abstract science, were superseded in the leadership of the European world by the practical Romans (Whitehead 1958).

2. ARCHIMEDES' MACHINES AND MECHANISMS

The first known written record of the word *machine* appears in Homer and Herodotus to describe political manipulation (Dimarogonas 1999, Chondros 2004). The word was not used with its modern meaning until Aeschylus used it to describe the theatrical device used to bring the gods or the heroes of the drama on stage; whence the Latin term *deus ex machina*. Mechanema (mechanism), in turn, as used by Aristophanes, means "an assemblage of machines." None of these theatrical machines, made of perishable materials, is extant. However, there are numerous references to such machines in extant Greek plays and also in vase paintings, from which they can be reconstructed. They were large mechanisms consisting of booms, wheels, and ropes that could raise weights perhaps as great as one ton and, in some cases move them back and forth violently to depict traveling through space, when the play demanded it. The designers and builders of these mechanisms were called by Aristophanes *mechanopoioi* (machine-makers), meaning machine designers in modern terminology.

Archimedes' mechanical skill, together with his theoretical knowledge, enabled him to design and construct many ingenious machines. Archimedes contributed greatly to the theory of the lever, screw, and pulley, although he did not invent any of these machines. Of these three, the lever is perhaps the oldest. The lever and the wedge had been used in various forms for centuries prior to Archimedes. Levers appeared as early as 5000 BC in the form of a simple balance scale (steelyard), and within a few thousand years workers in the Near East and India were using a crane-like lever, called the shaduf, to lift containers of water (Fig. 3).

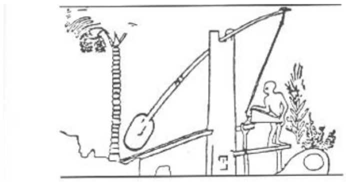

Fig. 3. The shaduf, first used in Mesopotamia in about 3000 BC.

Where the lever was concerned, the law of the lever already appears in *Mechanical Problems* written by Aristotle or a pupil of his, and possibly based on work by Archytas, but definitely older than Archimedes' work. Archimedes' contribution lay in his theoretical explanation considering that a pulley operates according to much the same principle as a lever and the principle of the mechanical advantage was introduced. A single pulley provides little mechanical advantage, but by about 400 B.C. the Greeks had put to use compound pulleys, or ones that contained several wheels. This mechanism was crucial for the development of large cranes and artillery machines. Archimedes perfected the existing technology, creating the first fully realized block-and-tackle system using compound pulleys and cranes (Lazos C. 1995). This he demonstrated, according to one story, by moving a fully loaded ship single-handedly while remaining seated some distance away. In the late modern era, compound pulley systems would find application in such everyday devices as elevators and escalators (Zrnić 2007).

Archimedes name is associated with the invention of a hand-cranked manual pump, known as "Archimedes' screw" that is still used in many parts of the world. Archimedes provided the theory for the screw geometry and construction, in this case with a formula for a simple spiral. The invention consists of a metal pipe in a corkscrew shape that draws water upward as it revolves. Vitruvius in his book *De Architectura* (Book X, Chapter VI, The Water Screw) provides details for the construction and the operation of the water screw (Fig. 4).

This idea of enclosing a screw inside a cylinder is in essence the first water pump. Its open structure is capable of lifting fluids even if they

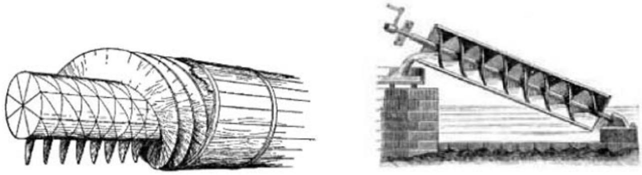

Fig. 4. The Water Screw, from Vitruvius "De Architectura" (Book X, Chapter VI) and reconstruction from Lazos (Lazos C. 1995).

contain large amounts of debris. This device soon gained application throughout the ancient world. A screw-driven olive press was found in the ruins of Pompeii, destroyed by the eruption of Mount Vesuvius in 79 AD. Hero later mentioned the use of a screw-type machine in his *Mechanica*. The Archimedean screw has been the basis for the creation of many other tools, such as the combine and auger drills. Following Drachmann and others Koetsier and Blauwendraat (2004) argue that it is reasonable to assume that Archimedes invented both the infinite screw and the screw-pump. They argue that these inventions can be related to Archimedes' interest in the problem of the quadrature of the circle. Moreover, they discuss aspects of the development of the theory of the screw-pump.

The Greeks from Syracuse developed the first catapults, a result of engineering research financed by the tyrant Dionysius the Elder in the early 4th century BC. Early catapults probably fired arrows from a bow not much stronger than one a man could draw. By mechanizing the drawing and releasing of the arrow, however, the catapult inventors made possible the construction of much more powerful bows. To mechanize the archer's motions the catapult engineers incorporated a number of appropriate design features (Soedel and Foley 1979, Dimarogonas 1993, Dimarogonas 1995, Rossi and Russo 2009). One of the crucial steps in designing the torsion springs was establishing a ratio between the diameter and the length of the cylindrical bundle of elastic cords. All the surviving catapult specifications imply that an optimum cylindrical configuration was indeed reached, and it could not be departed from except in special circumstances, such as the exclusively short range machines that Archimedes built at Syracuse. This optimization of the cord bundle was completed by roughly 270 BC, perhaps by the group of Greek engineers working for the Ptolemaic dynasty in Egypt, Thera and at Rhodes. The investigations and the experiments of the catapult researchers were, according to Philo, "heavily subsidized because they had ambitious kings who fostered craftsmanship." This phase of the investigations culminated in quantified results of a distinctly modern kind.

Archytas of Tarentum and Eudoxus of Cnidus had devised elegant theoretical solutions for the stone-thrower formula, but they were three-dimensional, very awkward physically and of no use in performing calculations. There the matter stood until the advent of the torsion bow. Most of the next group of solvers of the cube-root problem had either a direct or an indirect connection with catapults. The next solver of the cube-root problem was Eratosthenes, a friend of Archimedes and a native of Alexandria, which was then a centre of catapult research. Eratosthenes stated explicitly that the catapult was the chief practical reason for working on cube-root problems. Archimedes dedicated his book *On Method* to Eratosthenes, and thus we can assume that Eratosthenes was interested in engineering problems (Soedel and Foley 1979, Dimarogonas 1993).

The catapult engineers having arrived at an optimal volume and configuration for the torsion-spring bundle continued their experiments until they had optimized the dimensions for the remaining pieces of the machine. Eventually the catapult engineers wrote their texts in such a way that the dimensions of the major parts were given as multiples of the diameter of the spring. Once this diameter had been calculated for the size of the projectile desired, the rest of the machine was automatically brought to the proper scale. The surviving texts that contain this information testify to a level of engineering rationality that was not achieved again until the time of the Industrial Revolution. The last major improvement in catapult design came in later Roman times, when the basic material of the frame was changed from wood to iron. This innovation made possible a reduction in size, an increase in stress levels and a greater freedom of travel for the bow arms.

Gears were discussed in Aristotle and were well-known to Archimedes and the Alexandrian engineers. Almost concurrently with the decline of Alexandria, the differential gear was known to the Chinese (Dimarogonas 2001). As an astronomer, he developed an incredibly accurate self-moving model of the Sun, Moon, and constellations, which even showed eclipses in a time-lapse manner. The model used a system of screws and pulleys to move the globes at various speeds and on different courses (Archimedes–Apanta Vol. 6 2002). Cicero (106–43 BC) writes that the Roman consul Marcellus brought two devices back to Rome from the sacked city of Syracuse. One device mapped the sky on a sphere and the other predicted the motions of the sun and the moon and the planets. He credits Thales and Eudoxus for constructing these devices. For some time this was assumed to be a legend of doubtful nature, but the discovery of the *Antikythera mechanism* (De Solla Price 1975, Dimarogonas 2001) has changed the view of this issue, and it is indeed probable that Archimedes possessed and constructed such devices. Also, Pappus of Alexandria and Sextus Empiricus (*Archimedes– Apanta* Vol. 6 2002) write that Archimedes had written a practical book on the construction of such spheres entitled *On Sphere-Making*.

Archimedes' discoveries in catoptrics are reported (Lazos 1995, Archimedes–Apanta 2002). It is said that Archimedes prevented one Roman attack on Syracuse by using a large array of mirrors (speculated to have been highly polished shields) to reflect sunlight onto the attacking ships causing them to catch fire. Tests were performed in Greece by engineer Sakas in 1974 (Lazos 1995) and by another group at MIT in USA (MIT experiments web-site) in 2004 and both concluded that the mirror weapon was a possibility.

Archimedes discovered fundamental theorems concerning the center of gravity of plane geometric shapes and solids. His most famous theorem, which traditionally became known as Archimedes' Principle (Fig. 5), was used to determine the weight of a body immersed in a liquid. Based on this Archimedes principle, shipbuilders understood that a boat should have a large enough volume to displace enough water to balance its weight. Around 1586 Galileo Galilei invented a hydrostatic balance for weighing metals in air and water after apparently being inspired by the work of Archimedes.

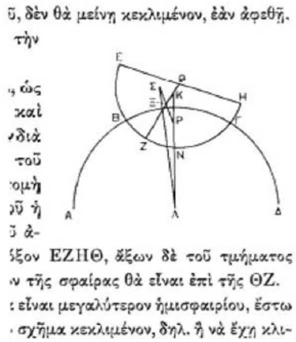

Fig. 5. From The first book On Floating Bodies ΟΧΟΥΜΕΝΩΝ Α. Proof of the vertical equilibrium of a spherical body lighter than the liquid immersed in (Stamatis 1973 B p. 287).

3. ARCHIMEDES INFLUENCE IN SCIENCE AND ENGINEERING

Archimedes' work was not as widely recognized in classical antiquity as that of Euclid, and some of his treatises are believed to have been lost when the Library of Alexandria was damaged at various periods in its history. Some of his writings survived through Latin and Arabic translations made during the Middle Ages, and these documents provided Renaissance scholars with an influential source of ideas. The translation of many of Archimedes' works in the sixteenth century contributed greatly to the spread of knowledge of them, and influenced the work of the foremost mathematicians and physicists of the next century, including Johannes Kepler, Galileo Galilei, Descartes and Pierre de Fermat (O'Connor and Robertson 2006). Archimedes together with Isaac Newton (1643–1727) and Carl Friedrich Gauss (1777–1855) is regarded as one of the three greatest mathematicians of all times (Bell 1965). The contemporary development of calculus and continuum mechanics led to the rapid development of mechanics by the mid 19th century. The fundamental contribution of Galileo and Newton is the revival and redefinition of classical physics and mechanics just as greater progress was being demanded from natural science.

Archimedes established the principles of plane and solid geometry. Some of Archimedes' accomplishments were founded with mathematical principles, such as his calculation of the first reliable value for π to calculate the areas and volumes of curved surfaces and circular forms. He also created a system of exponential notation to allow him to prove that nothing exists that is too large to be measured. Archimedes invented the field of statics, enunciated the law of the lever, the law of equilibrium of fluids, and the law of buoyancy. He discovered the concept of specific gravity and conducted experiments on buoyancy. He invented the entire field of hydrostatics with the discovery of the Archimedes' Principle. Archimedes studied fluids at rest, hydrostatics, and it was nearly 2000 years before Daniel Bernoulli took the next step when he combined Archimedes' idea of pressure with Newton's laws of motion to develop the subject of fluid dynamics (Dijksterhuis 1987, Stamatis 1973).

Archimedes systematized the simple machines and the study of their functions. The lever and the wedge are our technology heritage from the paleolithic era. Archimedes first designed in a systematic way those machines and mechanisms and developed a rigorous theory of lever and the kinematics of the screw. His contribution separates engineering science from technology and crafts, often confused for matters arrived at empirically through a process of long evolution. "Give me a place to stand, and I shall move the Earth," Archimedes is said to have promised (Dijksterhuis 1987). Archimedes was referring to the law of the lever, which (in the variant form of the law of the balance) he had proved in his treatise, *Planes in Equilibrium*. One can say that Archimedes moved the Earth – in principle – without standing anywhere. Also, Archimedes figured out that the Earth and a pebble are the same kind of thing, differing only in size. This revolutionary idea yields to imagine a vantage point from which the earth and the pebble can both be seen for what they are. Archimedes went one better, and offered to move the Earth, if someone would supply him with this vantage point, and a suitable lever.

A book collecting several treatises by Archimedes was prepared in the sixth century AD by Isidore of Miletus and Anthemios the Tralleus, the architects of Aghia-Sofia in Constantinopole. It is believed that this collection of works was a “State-of-the-Art” review for the construction of this huge building. This book was copied by Leo the geometer or his associates, once again in Constantinople, in the ninth century AD (Lazos 1995, Archimedes–Apanta 2002, Netz 2004).

The early modern era is highlighted by the works of Galileo (1564–1642) and Newton (1642–1727) and includes the early stages of mechanization and the Industrial Revolution. The subjects of Mechanical Engineering have attracted more and more interest since early Renaissance both for practical applications and from a theoretical viewpoint in response to an increase of societal needs (O’Connor and Robertson 2006). The contribution of Leonardo Da Vinci, Galilei and Newton, the redefinition of classical physics and mechanics, the separation of the study of kinematics and the study of machinery in the 18^{th} century, the early mechanization and the progress during the Industrial Revolution yielded the development of engineering design as a systematic process in modern era.

Archimedes earned the honorary title “father of experimental science” because he not only discussed and explained many basic scientific principles, but he also tested them in a process of trial and experimentation (Bendick 1997). His works contain a set of concrete principles upon which design can be developed as a science using mathematics and reason (Dimarogonas 2001). The aforementioned design principles can be traced to Filippo Brunelleschi, a Renaissance architect famed for designing the cupola for Santa Maria del Fiore in Florence in the 1420s (Salustri, Mechanical Engineering, 2004). He introduced a method of design based on a six-step design process, identical in essence to the design principles of Archimedes, consisting of 1. analyzing the design requirements, 2. making a concept design, 3. making a detailed design, 4. planning the manufacturing process, 5. manufacturing the parts and 6. assembling the parts. Brunelleschi’s six-step design process is considered the first systematic design process in engineering history and was carried out for 500 years.

In 1964 Sandor (Dimarogonas 2001) proposed a seven steps strategy for machine design that is similar to that of Brunelleschi. The seven steps proposed by Sandor are: formulation of the problem, design concepts, synthesis, analyzable model, analysis-experiment-optimization, presentation.

Similar sequential design procedures were in use until the 1970s, when the notion of engineers working on product design in teams manufacturing and mechanical engineers took hold, and by the 1980s many engineering firms adopted this concept, called concurrent engineering (Dimarogonas 2001). This switch to concurrent engineering has changed the way engineers do their work, and around that time the advent of computer aided design has revolutionized engineering design.

4. CONCLUSIONS

It was among the Eleatic philosophers that important beginnings of logic were developed by Platon and Aristoteles into a science and served as an instrument for the parallel development of the natural sciences, especially mathematics and physics, by such pioneers as Pythagoras, Aristoteles, Euclid and Archimedes (Dimarogonas 2001). The search for Reason led to the development of a generalized science as distinct from a set of unrelated empirical rules. Pythagoreans, for example, sought the principles of geometry, originally practiced by Egyptians, in ultimate ideas and investigated its theorems abstractly and in a purely rigorous way (Proclos Diadochos A.D. 410–485). The rigorous proof was introduced, based in deductive logic and mathematical symbolism. Experimentation was established as a method for scientific reasoning.

Archimedes made important contributions to the field of mathematics. Plutarch wrote: "He placed his whole affection and ambition in those purer speculations where there can be no reference to the vulgar needs of life." Some of his mathematical proofs involve the use of infinitesimals. His contribution to the calculation of an approximate value for π was a remarkable achievement, since the ancient Greek number system was awkward and used letters rather than the positional notation system used today.

The Law of the Lever and the Law of Buoyancy are two of the most fundamental laws of nature and two of the first laws of nature articulated and quantified (Rorres 2001). Archimedes studies greatly enhanced mathematics, mechanics and engineering. His practical applications remain vital today. Archimedes earned the honorary title "father of experimental science" because he not only discussed and explained many basic scientific principles, but he also tested them in a process of trial and experimentation (Bendick 1997). His works contain a set of concrete principles upon which engineering was developed as a science using mathematics and reason.

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6. LEGACY AND INFLUENCE IN TEACHING AND HISTORY ASPECTS

THE FOUNDER-CULT OF HIERON II AT AKRAI: THE ROCK-RELIEF FROM INTAGLIATELLA'S LATOMY

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ABSTRACT In Akrai, a Syracusan archaic sub-colony, there is a rock-relief in *Intagliatella*'s urban latomy. In this paper, after a trip around the history of studies related to the monument, we propose a new interpretation on the ground of signs, drawn near its position to the town's entrance, following relevant historical considerations about the figure of Hieron II (306–215 B.C.), the most important king of Hellenistic Syracuse, as well as a friend and protector of Archimedes (287–212 B.C.) who lived and worked in his court.

1. INTRODUCTION

During the 3rd century B.C., Sicily found itself again at the centre of the Mediterranean political scene. Two new contenders, the Carthaginians and the Romans, fought for supremacy in the West. Italiotes and Siceliotes became second-rate actors and, entering into an alliance with either, lived their last years of independence.

In Sicily, the long reign of Hieron II (269–215 B.C.) represented the bright swan-song for Western Greeks [De Sensi Sestito 1977]. Archimedes, the greatest scientist of the Antiquity, lived and operated in his court [Voza C. 2002][1].

His good relationship with the king Hieron, perhaps also due to the distant consanguinity[2], provided him with the ideal tools for his restless researcher's spirit.

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1 On the location of his grave to Syracuse, see at last Scirpo 2007.

2 So Plutarch believes him (*Vita Marcelli*, XII) but not Cicero (*Tusculanes disputationes*, V, 64) who called him *humilem homunculum.*

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At the end of the First Punic war (264–241 B.C.), Rome pledged herself for the ally Hieron II, who had faithfully helped her with troops and supplies in the hard fight with the Punics. Among the so many testimonies found in the sources on the King's friendship for the Romans, suffice is to mention the support afforded after the battles of the Trebbia [*Polybius*, III, 75], of the Trasimenus [*Livius*, XXII, 37] and of Cannes [*Livius*, XXXII, 38]. A Senate decree established which Sicilians cities were assigned to Hieron II [*Diodorus Siculus*, XXIII, 4.1]. Akrai was among them (*fig. 1*), a Syracusan archaic sub-colony [Scirpo 2004; 2005], near today's Palazzolo Acreide [Scirpo 1996–2004; 2005–2009 *in press*].

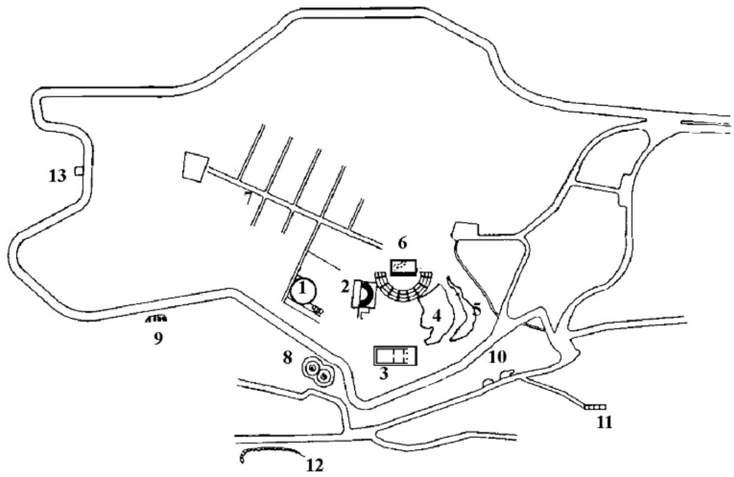

Fig. 1. Archaeological area of Akrai. The Intagliatella's quarry is no. 5 (from Scirpo 2004, fig. 6).

2. A DIFFICULT QUEST

Akrai entered soon into 17th century's antiquarian literature, as testified by many quotations that Europeans travellers, searching for a part of Magna Graecia, devoted her, since Greece, being under the Ottoman yoke, was still inaccessible. One for all was Jean Houel who, financed by the French Government, went to Sicily in 1776 and performed sketches that to Paris, where he returned over three years later, providing the necessary information to have the 264 tables of the his *Voyage pittoresque des isles de Sicile, de Malta et de Lipari*, engraved and published in four volumes between 1782 and 1787 [Houel, III, 1785, pp. 111–112, 119, tavv. CXCVI-CXCIX].

Recently was open to Palazzolo Acreide a *Travellers' Museum in Sicily* [Gringeri Pantano 2008].

In the *Intagliatella*'s latomy[3] (*fig. 2*), a huge rock relief of great dimensions is found (2,13 x 0,83 m) that, since the beginning of local archaeological research, attracted scholars' attention.

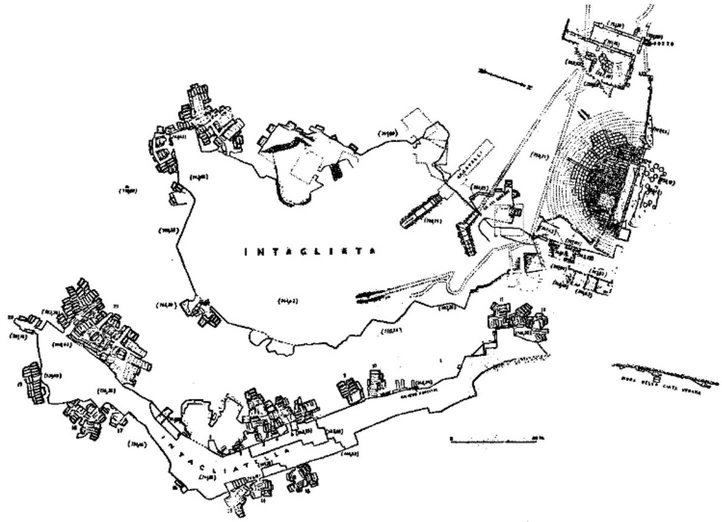

Fig. 2. Urban Latomias of Akrai. (from Bernabò Brea 1956, tav. B).

In fact the first to quote and present a sketch of it was Gabriele Judica [1819, pp. 91–92, tav. VII], in his monograph dedicated to the ancient Akrai. However, the Baron couldn't read the figures, complaining about loss of heads (*fig. 3*).

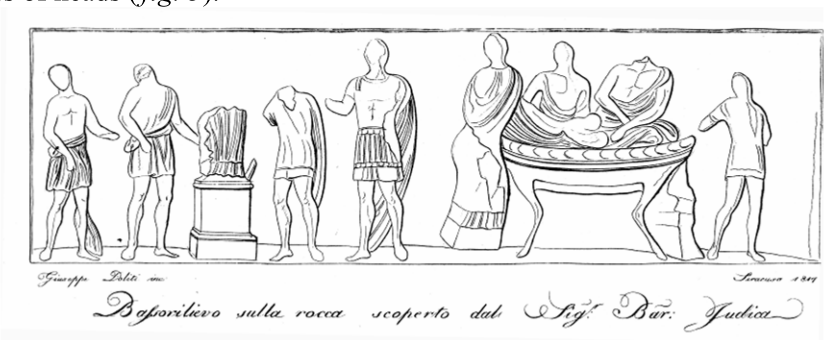

Fig. 3. Rock relief. Engraving of G. Politi (from Judica 1819, tav. VII).

3 With this name, from the Greek word (*Λατομεῖον*), we call the ancient stone quarries, diffused in all the Ancient world, also used by the Greeks as jails.

A few years later, the Duke of Serradifalco [1840, p. 155, tav. XXXV. 1] mentioned it among the memorable antiquities of Sicily and illustrated it (*fig. 4*), as Julius Schubring [1867, p. 665] also made it.

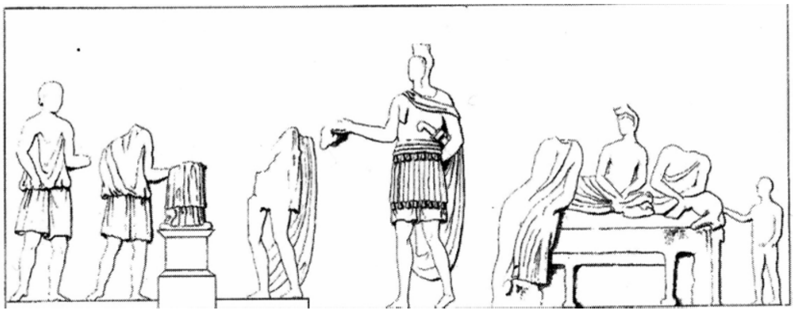

Fig. 4. Rock relief. Engraving of F. Cavallari (from Serradifalco 1840, tav. XXXV, 1).

Biagio Pace [1945, p. 513] was the first to analyzed its nature, trying to set it against his chronological and stylistic background.

The stylistic and thematic analysis by Luigi Bernabò Brea [1956, pp. 61, 63-65, tav. XII. 1], framed it artistically as a modest attempt of *"adaptation of an eminently Roman motive to the service of an essentially Greek religious concept."*

He then proposed to be dated it in the first half of the 1st century B.C., certainly before the establishment of the August's *principatum*. Filippo Coarelli attended to the relief in two occasions. Firstly [1980, p. 168], he dated it after 300 B.C., and classified it certainty as a work of religious character as the *pinakes* of the zone, with parallel from Asia Minor's rock-sculpture, similar but chronologically anterior. In a second time [1997[4], p. 298], he postponed its creation to the 3rd century, considering it a function of celebration of the heroic cult of dead warriors.

Moreover, Nicola Bonacasa mentions it twice: the first one [Bonacasa & Joly 1985, p. 309, fig. 351], in his long *excursus* on the Greek plastics of Sicily, reserving to it only a mediocre judgment. In a second time [1996, p. 429], he seems that to have attributed a heroic function to the lying dead.

Recently Salvatore Distefano [2006, pp. 24–26; 2009, p. 233, note 105] gave a new interpretation to the relief: in a scene of libation, there is a Roman judge in toga, the ones lying down are goddesses Aphrodite, Artemis and finally standing is perhaps Apollo. The cult seems to be related to the Akrai's unknown *oikistes*. In the Thucy-dides's passage (VI, 5.2) where the foundations of Akrai and Kasmenai are mentioned, seventy and ninety years after the Syrakousai's foundation respectively (734 B.C.), the name of the founder is not preserved by the Athenian historian (and probably even from his source, Antiochus of Syracuse). On the contrary, Daskon and Menekolos are quoted, *oikistai* of the rebellious Kamarina (598 B.C.). In this colony of population, the most heterogeneous elements of the Metropolis are welcome to conclude the Syracusan infiltration in the south-eastern area of Sicily [Di Vita 1956].

3. A NEW INTERPRETATION

Departing from this last suggestion, in my opinion, a new interpretation can be formulated, however keeping in mind some landmarks like:

- The relief position
- The style
- The representation and its interpretation
- The presence of divinities of the Hellenistic Syracuse and Akrai Pantheon.

The relief (*figs. 5–6*) is first of all situated at the entrance of the so called "hieronian" city area: in fact the surrounding public monuments (theatre and *bouleuterion*), already brought to light by Judica, are all together unanimously datable in the 3rd century b.C. [Bernabò Brea 1956, p. 39 - Distefano 2009, pp. 226–231]. The town urban plan shows a Hellenistic phase that traces a precedent archaic phase, going back to the foundation of the colony [Voza 1971; 1973; 1999, p. 139]. The eastern entrance, the «Syracusan Gate», had clearly to be placed to east of the theatre at a lower quota. The cave ground was probably «extra-moenia» and the western rocky face perhaps had to hold up the urban walls. The urban wall's chronology, based on the comparison with the Syracusan one, goes to a period from the end of the 4rth to the first half of the 3rd century B.C. [Bernabò Brea 1956, p. 23]. So the relief's position assumes a precise meaning, of welcome to the foreigner, as Akrai was thought to be the city of Hieron II.

The style is the element that brings closer to the Hellenistic period. The stylistic comparisons proposed by Coarelli are pertinent for that it concerns the typology. The realization, expression of a handicraft rather than artistic sensibility, doesn't certainly diminish however its importance in any way, made clear by non indifferent dimensions and above all by its position.

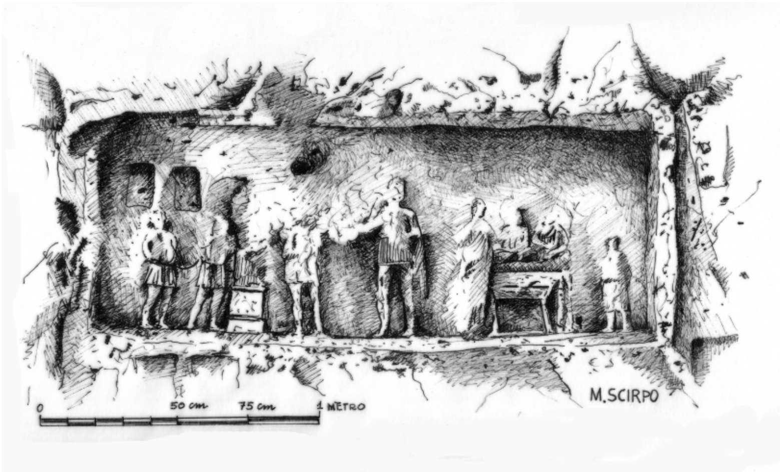

Fig. 5. Rock relief. Sketch of M. Scirpo.

It is then possible to read the relief as the representation of a libation for Hieron II, honoured as *οικιστής* in Akrai with heroic cult. The hypothesis of his birth in Akrai from Hierokles, an aristocratic Syracusan exile, is pursued on the grounds of the so many signs that tie the sovereign to the small Hyblean polis [Distefano 2009, pp. 217–219].

This relief could bring the Syracusan royal family (the son and successor, Gelon II in the foreground, his wife, Philistis, standing, the two daughters, Damareta and Heraklea, lying down and on the extreme right, Hieronymus, his grandson and future last king of the Pentapolis[4]) that assists to the libation on the altar, surrounded by three young people that could be the *triakadarcheis*. The institution of the *triàkadis* is testified in Akrai by three epigraphs (*IG* 209, 211, 212) [Sartori 1980, p. 279].

Consequently, the relief could be datable at the 3rd century B.C. and perhaps, more precisely, at the end of the long Hieron's *basileia*. If we reduce the period of time when all the members of the Syracusan royal family were in life, then the relief could be dated to the triennium 219–216 B.C.

To continue the tradition of town founders inaugurated by the Dinomenides, who he already wanted to attach it to with his royal title [De Sensi Sestito 1977, p. 183], Hieron II didn't hesitate, therefore, to exhibit his benevolence toward all the cities of his kingdom, with the construction of great public buildings [Bell 1999] but also with by restructurings of some great sanctuaries, among them the one of Cybele in Akrai [Pedrucci

[4] On the brief (only 1 year) kingdom of Hieronymus, see Ciancio 1972.

2009][5]. Moreover, the choice of gods from the Pantheon of Syracuse [Reichert Südbeck 2000] and Akrai would make the hieronian politics of sovereignty's issue from the Gods. True political tool, the dynastic cult was born from the demand of the Hellenistic sovereigns to found a family pantheon that could admit the various social classes inside their multi-ethnic kingdoms together with the Greek traditional cults [Consolo Langher 2000]. Hieron II exploited the concept of the sovereignty's origin from Zeus, indissolubly tying himself with the father of the Gods. This bond with the cult of Zeus is present in the hieronian coinage too [Germanà Bozza & Scirpo 2007 *in press*]. In his difficult politics of equilibrium, without making show of the diadem in private, but giving show of munificence and patronage, the Syracusan sovereign wanted to cut out for himself a place among the others Hellenistic monarchs.

4. CONCLUSION

The small Syracusan sub-colony that received honours from the *basileus* (perhaps also its fellow-citizen), wanted to thank him with the founder-cult's institution in the heart of the so-called *Via Sacra*, so that the futures generations of hard-working Acrenses could worthily honour the memory of the more wise and illuminated sovereign that Greek Sicily remembers.

Fig. 6. Rock relief. Actual view.

[5] For the existence of the sanctuary already in late-archaic period, see Scirpo 2007 (in press) and Scirpo 2009 (in press).

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ARCHIMEDES: RUSSIAN EDITIONS OF WORKS

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ABSTRACT The data on all known editions of Archimedes's works in Russian from the middle of XVIII till the middle of XX centuries and the most interesting comments to Archimedes's works and translations of them are presented.

PREFACE

Since the 7th May of 1960 the physical faculty of Moscow State University has been annually spending the «Day of the physicist», named «Archimedes Festival» in the beginning of academic year. Presence of two great modern physicists was important event of the 2nd Archimedes Festival: Niles Bor and Lev Landau, as they were the Nobel Prize laureates. Andrey Slavnov, the known Russian physicist, academician of the Russian Academy of Sciences, who also was in the managing chair of theoretical physics of the Moscow State University, the chief of theoretical physics department in the Institute of Mathematics of the Russian Academy of Sciences (n.a. Stekloff), and student of the physical faculty in 1956–62 years, shot an amateur film about this event. Two frames from this film are shown in fig. 1

Fig. 1. Autumn, 1956. The 1st Archimedes's Festival. Left frame: the opening (the main entrance of the University); right photo – Niles Bor and Lev Landau.

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1. INTRODUCTION

It is hardly needed to enumerate Archimedes's services to modern technique, sciences, education in schools and universities. Review and analysis of Archimedes's works in the field of mathematics and mechanics is included in all the corresponding courses of history of all mechanic-mathematical and physical faculties in Russia. In the list of literature we gave the references to a row of popular scientific books, school literature, university text-books and scientific publications in Russian. The relation to Archimedes's works at various stages of development of Russia represents considerably more interest. According to I.N. Veselovskiy [11] the seven publications of Archimedes's works in Russian were known near 1962. For the unknown reasons that list does not include publications [1] and [10], which will be mentioned here. It is essential that the attitude to Archimedes's works was the bigger interest of researchers. Thus, the presented proceeding had two main targets. The first one is to introduce Russian editions of Archimedes's works. And the second one is to emphasize the main reasons for publishing the Archimedes's proceedings. In the paper some extracts from the prefaces to translations are quoted, which let us know the reasons for publishing of the works and the attitude of the publishers to them, and the meaning of Archimedes's proceedings in school and higher education as well. Besides we gave some material which is hardly known or not known at all not only in Russia.

In the first chapter of this paper the data on first publications of Archimedes's works from the middle of XVIII till the first third of XIX are presented; in the second one – the data on the publications from the first third of XIX till the Great October Revolution in 1917; in the third one – the data on publications after the Revolution in 1917 and till the middle of XX. All editions consist of preface, description of Archimedes biography, extensive commentary for all books or for each fragment of work. Most impressive fragments of comments are considered here.

2. THE FIRST WORKS OF ARCHIMEDES

The requirement of Russian scientific and technical community for studying of works on mathematics, physics, mechanics and engineering, including Archimedes's works, arose at the beginning of XVIII in connection with Peter's the Great reforms and was connected with development of the industry and, as consequence, with formation of university and technical education. The first translations of Archimedes's works to Russian were completed near 1745 [1] 1823 [2] and 1824 [3]. They were already considered as a bibliographical rarity at the end of XIX. We managed to find them in the Russian State Library (Lenin Library in the past), Museum of Book. At all times the biography made by a certain Heraclitus of Oksoronikh, who lived in II BC, which has not reached us served as the main source of biographic data on Archimedes. Further the works of Titus Livy, Cicero, Diodorus, Valerius Maximus serve as the source of Archimedes's biography. The same sources appear also in the Russian editions of Archimedes's works. The translation of 8 Euclid's geometry books into Russian was published in 1739:

EUCLID'S ELEMENTS IN CHOSEN OF TWELVE NEWTON'S BOOKS

The Academy of Navy was established in St. Petersburg at 1715. It was the first special marine high school in Russia. Under decree of Peter I, the Academy of Navy Publishing House was found at 2nd of January, 1721. Near 1722, the first book was published by it – "The Static Science or Mechanica". Marine maps, guidelines and navigation aids, decrees, shipping orders, manuals, patents and ship passports were also published. Before XIX century the prints were usually made from copper (bronze?) engravers by hand stamps with lower quality.

The selection of Newton's book was made by Andrew Farhvarsson, who was professor of mathematics. The translation from Latin into Russian was made by Ivan Satarov, the surgeon. Book was printed in Printing House of the Nautical Academy, found in 1721, in St.-Petersburg in 1739 and contained 284 pages and 10 tables with 309 figures. Andrew Farhvarsson the professor of the Mathematics in the Aberdine University was invited by Peter I to Russia for teaching in the mathematical-navigation school.

In 1745 in the same printing house the translation of Archimedes's theorems from Latin was published for the first time.

ARCHIMEDES'S THEOREMS CHOSEN BY ANDREA TACQUET THE JESUIT AND ABRIDGED BY GEORGE PETER DOMICQIO [1].

Translation of this work was also made by Ivan Satarov. That book contained 172 pages and 1 table with 32 figures. It should be noted that the data on the publication of 1745 is most likely to be included only in the publication of the year 1823 [2]. The statement anticipated G.P. Domicqio's preface, in which he explained the reasons for choosing the themes for translation:

- Concrete acquaintance to Archimedes works as “many praise them more, rather than read, admire more than understand”;
- Archimedes theorems serve as continuation of studying of Euclid’s geometry, published in 1739;
- The choice of translation was connected with practical advantage; therefore the material about the cylinder and sphere were added to the book.

In the Fig. 2a–2b the title, fragment of the text and drawings to the theorems are represented. Translation language of the first editions is hard and archaic for modern readers, especially in terminology section. But the didactics was good for that text and graphs were made excellently.

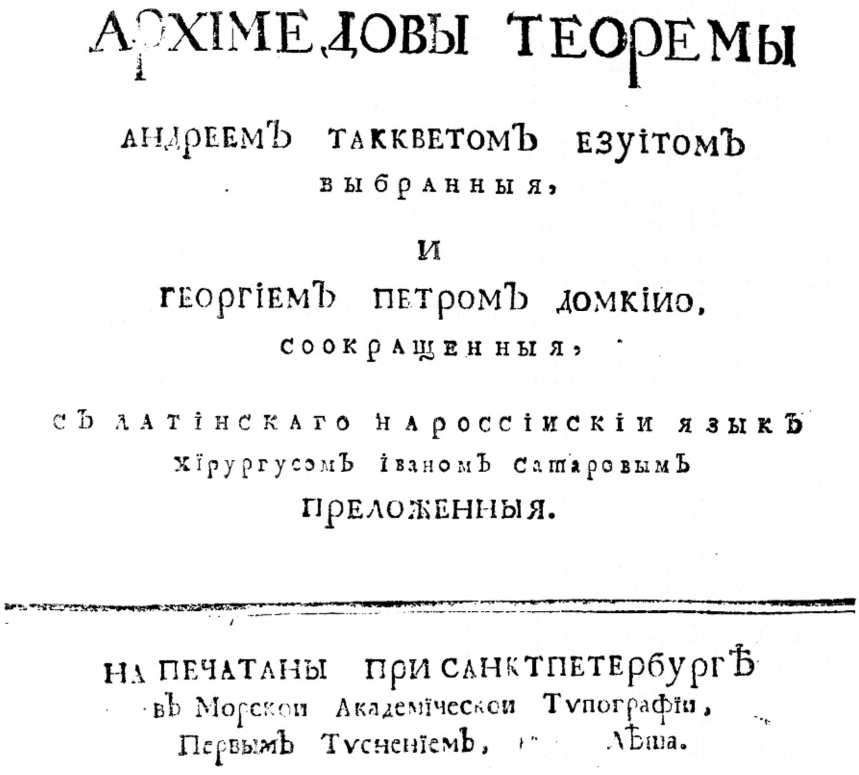
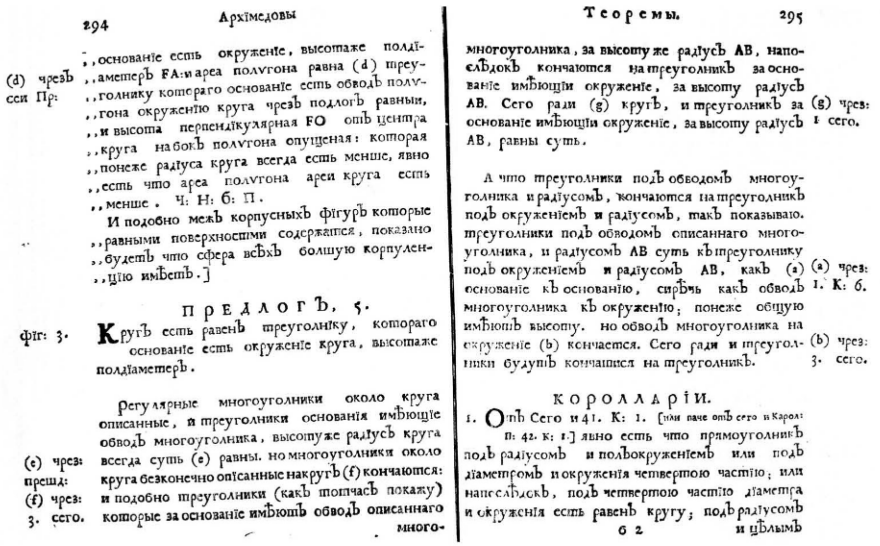

Архімедовы теоремы
Андреемъ Такк ветомъ Езуітомъ
выбранныя,
и
Георгіемъ Петромъ Домкіио,
сокращенныя,
съ латінскаго на россіискіи языкъ
хїрургусомъ Іваномъ Сатаровымъ
преложенныя.
Напечатаны при Санктпетербургѣ
въ Морскои Академіческои Тvпографіи,
Первымъ Тvсненіемъ, [illegible] лѣта.

Fig. 2a. Title, of the “Archimedes’ Theorems”.


294 Архімедовы

,,основаніе есть окруженіе, высотаже полді-
(d) чрезъ ,,аметеръ FA: и ареа полvгона равна (d) треу-
сси Пр: ,,голнику котораго основаніе есть обводъ полv-
,,гона окруженію круга чрезъ подлогъ равным,
,,и высота перпендікулярная FO отъ центра
,,круга на бокъ полvгона опущеная: которая
,,понеже радіуса круга всегда есть менше, явно
,,есть что ареа полvгона ареи круга есть
,,менше. Ч: Н: б: П.

И подобно межъ корпусныхъ фігуръ которые
,,равными поверхностми содержатся, показано
,,будетъ что сфера всѣхъ болшую корпулен-
,,цію имѣетъ.]

ПРЕДЛОГЪ, 5.

фіг: 3. Кругъ есть равенъ треуголніку, котораго основаніе есть окруженіе круга, высотаже полдіаметеръ.

Регулярные многоуголники около круга
описанные, и треуголники основанія имѣющіе
обводъ многоуголника, высотуже радіусъ круга
(e) чрез: всегда суть (e) равны. но многоуголники около
прешд: круга безконечно опісанные на кругъ (f) кончаются:
(f) чрез: и подобно треуголники (какъ тотчасъ покажу)
3. сего. которые за основаніе имѣютъ обводъ описаннаго
много-

Теоремы. 295

многоуголника, за высотуже радіусъ AB, напо-
слѣдокъ кончаются на треуголникъ за осно-
ваніе имѣющіи окруженіе, за высоту радіусъ
AB. Сего ради (g) кругъ, и треуголникъ за (g) чрез:
основаніе имѣющіи окруженіе, за высоту радіусъ 1 сего.
AB, равны суть.

А что треуголники подъ обводомъ многоу-
голника и радіусомъ, кончаются на треуголникъ
подъ окруженіемъ и радіусомъ, такъ показываю.
треуголники подъ обводомъ описаннаго много-
уголника, и радіусомъ AB суть къ треуголнику
подъ окруженіемъ и радіусомъ AB, какъ (a) (a) чрез:
основаніе къ основанію, сирѣчь какъ обводъ 1. К: 6.
многоуголника къ окруженію; понеже общую
имѣютъ высоту. но обводъ многоуголника на
окруженіе (b) кончается. Сего ради и треугол- (b) чрез:
ники будутъ кончатися на треуголникъ. 3. сего.

КОРОЛЛАРІИ.

1. Отъ Сего и 41. К: 1. [или паче отъ сего и Карол: п: 42. к: 1.] явно есть что прямоуголникъ
подъ радіусомъ и полъокруженіемъ или подъ
діаметромъ и окруженія четвертою частію; или
напослѣдокъ, подъ четвертою частію діаметра
и окруженія есть равенъ кругу; подъ радіусомъ
6 2 и цѣлымъ

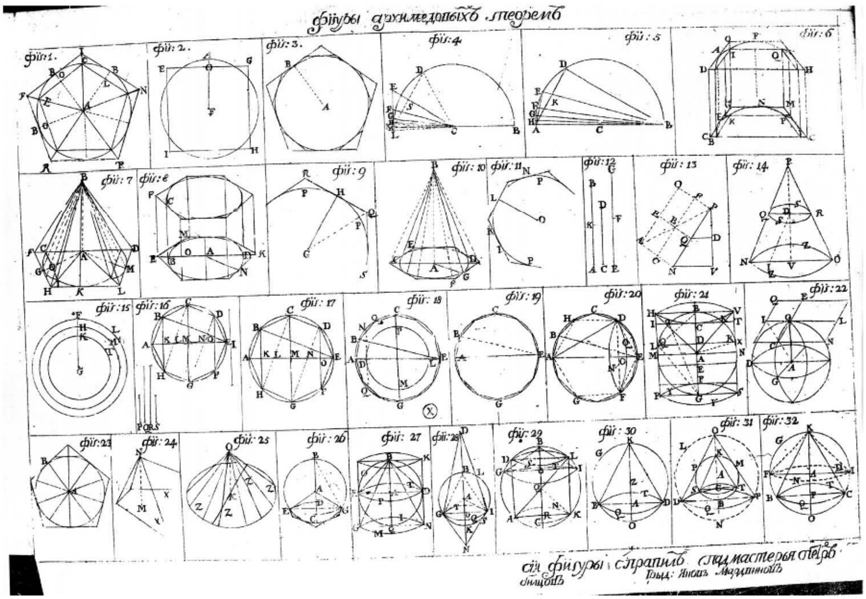


Fig. 2b. Fragment of the text and drawings for the "Theorems".

It should be noted that since 1726 till 1740 the quantity of books printed in Russia in comparison with the past reduced. At that time about 12 names were printed per year, while at the 1st quarter of XVIII century about 19 of them, with the exception for church service books, were printed within the same time.

The next 2 publications of Archimedes' works [2], [3] were fulfilled in 1823 and 1824. They were the translations that is requested by Public Education Deputy, and they were publishes in Deputy Publishing House. All of these books, as well as book of 1745, were the bibliographic rarities even at beginnings of XX century. Language of their translation (although of its precision and quality) was archaic and very heavy for reading, and especially, for studying.

ARCHIMEDES'S TWO BOOKS: ABOUT A SPHERE AND A CYLINDER, THE MEASUREMENT OF A CIRCLE AND A LEMMA/TRANSLATION FROM GREEK (LEMMAS - FROM LATIN) MADE BY F. PETRUSHEVSKY, WITH NOTES AND ADDINGS, THE DEPARTMENT OF PUBLIC EDUCATION, 1823.
The book contains 240 pages of text, 7 tables with general number of figures of 83, 17 pages of preface, written by F. Petrushevsky[1], texts of two books about a sphere and a cylinder (141 pages), the book about the circle measurement (7 pages) and a lemma (17 pages). Notes to those books were mostly taken by the translator from Eutocius' interpretations of Archimedes's compositions. In the Fig. 3a–3b the title page, tabs to the first book and lemmas are represented. It is noticed in the preface that in 1823 it was considered that "almost all Archimedes's works are saved and has reached us". This works are "About a sphere and a cylinder", "Measurement of a circle", "About conoids and spheroids", "About snails and curls (about spirals)"; "About the balance of planes"; "About a parabola quadrature"; "Psammite"; "About bodies shipped in water"; "Lemmas". The translator notices that last two books "are found only in translation in Arabian, others are kept and published in the original, written in pure and decent Dorian syllable. However, in "Books about a cylinder and a sphere" and in "Measurement of a circle" Attic expressions are quite often met that allows to assume that they were entered by copyists".

1 **Petrushevsky Foma Ivanovich**, (1785–1848) – the Russian metrology scientist, former student of St. Petersburg Pedagogical University. He translated "Eight books of Euclid's beginnings, containing the bases of geometry" (St.-Petersburg, 1819), "Eight books of Euclid's beginnings, containing the general theory of numbers of ancient geometers" (St.-Petersburg, 1835), "Archimedes's two books about a sphere and a cylinder, measurement of a circle and lemma" (St.-Petersburg, 1823), "Archimedes's Psammit, or Calculation of sand in the space, equal to a sphere of stationary stars" (St.-Petersburg, 1824), having supplied them with additions and notes. Petrushevsky was awarded with Demidov prize in 1835 for translation of these compositions.

АРХИМЕДА

ДВѢ КНИГИ
О ШАРѢ И ЦИЛИНДРѢ,
ИЗМѢРЕНІЕ КРУГА
И ЛЕММЫ.

ПЕРЕВОДЪ СЪ ГРЕЧЕСКАГО

(Леммы съ Латинскаго)

Ѳ. ПЕТРУШЕВСКАГО.

Съ примѣчаніями и пополненіями.

САНКТПЕТЕРБУРГЪ,

ВЪ ТИПОГРАФІИ ДЕПАРТАМЕНТА НАРОДНАГО ПРОСВѢЩЕНІЯ.

1823.

Fig. 3a. Title page, of “Lemmas”.

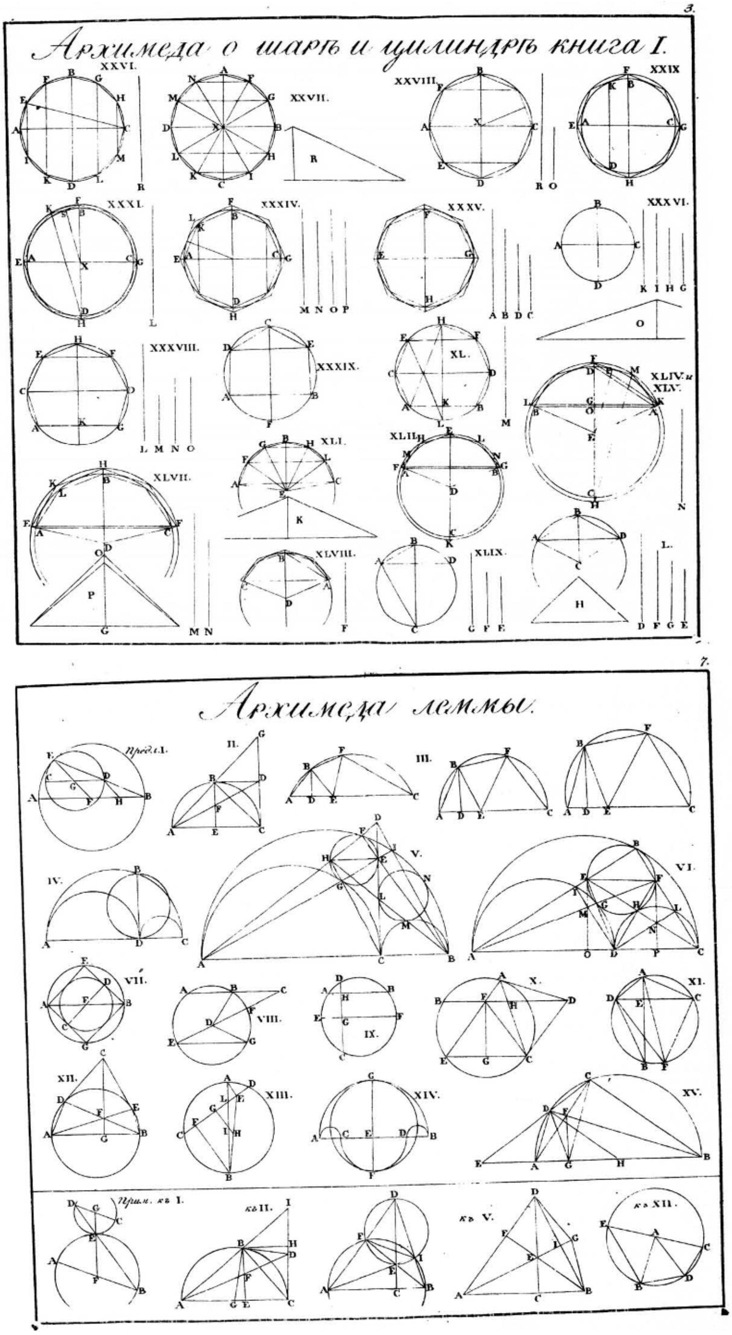
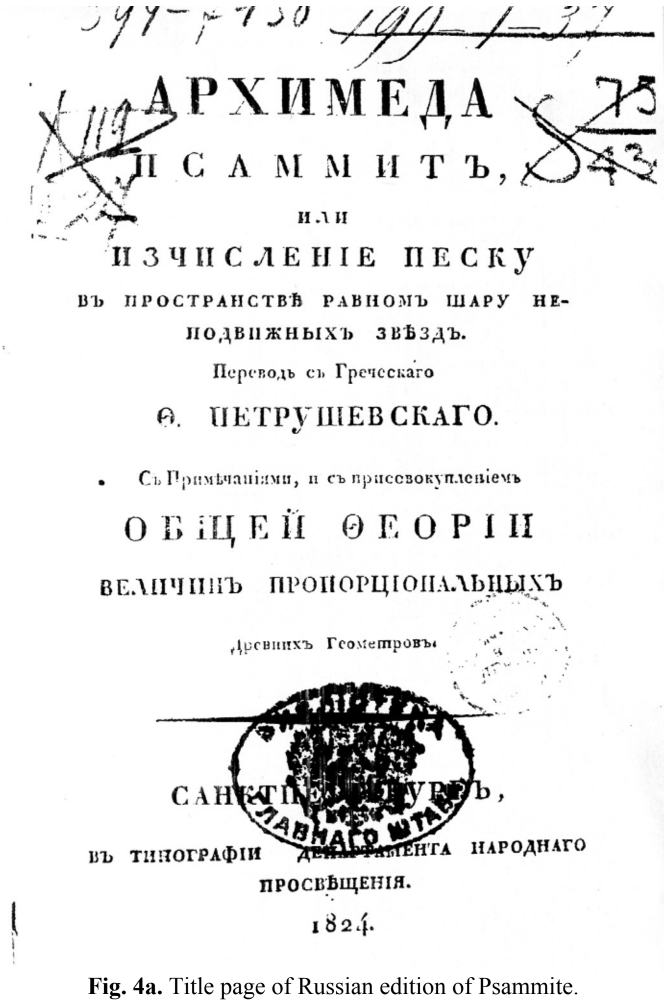


Fig. 3b. Tabs to the first book and tabs of "Lemmas".

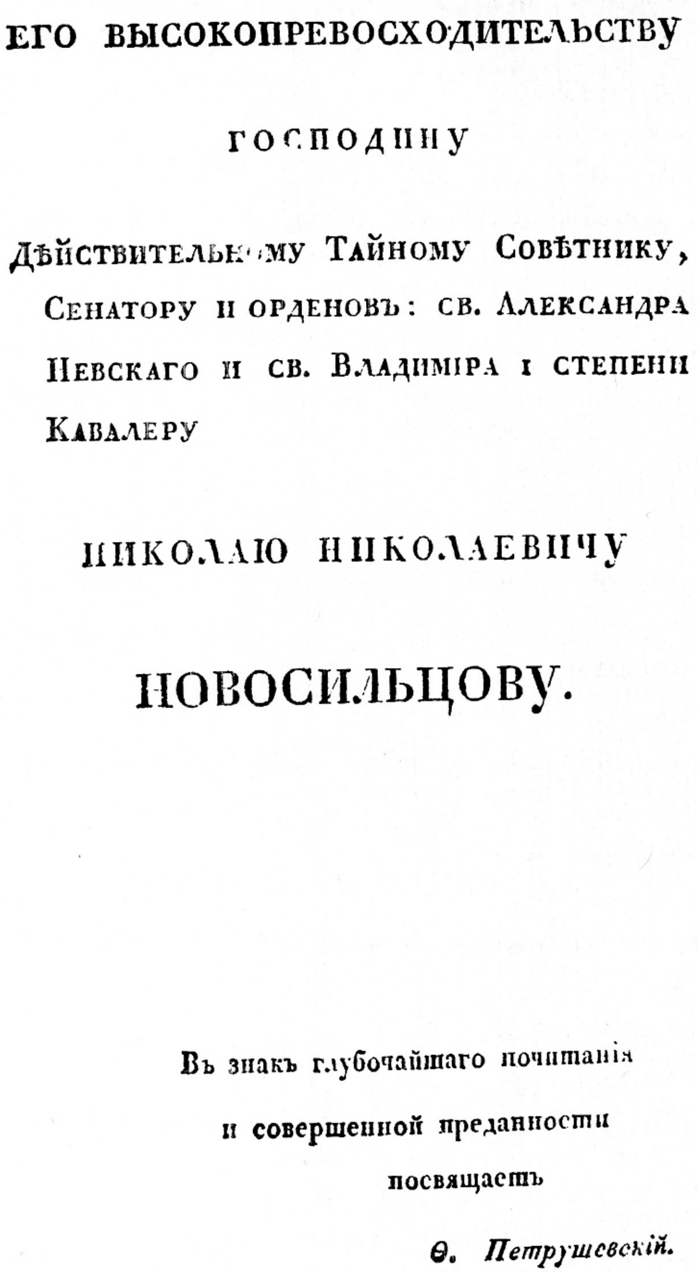
PSAMMITE, OR THE "MEASUREMENT OF SAND IN THE SPACE, EQUAL TO A SPHERE OF STATIONARY STARS". TRANS. FROM GREEK BY F. PETRUSHEVSKY, WITH NOTES AND ADDITION OF THE COMMON THEORY OF THE EXTENTS AND PROPORTIONAL OF IMMEMORIAL GEOMETRICS. - S. PETERSBURG, DEPARTMENT PUBLIC EDUCATION TYPOGRAPHY, 1824 [3].

The book contains "Psammite" (32 pages), notes, and general theory of sizes of proportional (54 page), preface of Petrushevsky as well. In Fig. 4a–4c the title page, dedication to count Novosiltsev and an insert are shown.



АРХИМЕДА
ПСАММИТЪ,
или
ИЗЧИСЛЕНІЕ ПЕСКУ
ВЪ ПРОСТРАНСТВѢ РАВНОМЪ ШАРУ НЕ-ПОДВИЖНЫХЪ ЗВѢЗДЪ.
Переводъ съ Греческаго
Ѳ. ПЕТРУШЕВСКАГО.
Съ Примѣчаніями, и съ присовокупленіемъ
ОБЩЕЙ ѲЕОРІИ
ВЕЛИЧИНЪ ПРОПОРЦІОНАЛЬНЫХЪ
Древнихъ Геометровъ.
САНКТПЕТЕРБУРГЪ,
ВЪ ТИПОГРАФІИ ДЕПАРТАМЕНТА НАРОДНАГО ПРОСВѢЩЕНІЯ.
1824.

Fig. 4a. Title page of Russian edition of Psammite.



ЕГО ВЫСОКОПРЕВОСХОДИТЕЛЬСТВУ

ГОСПОДИНУ

Дѣйствительному Тайному Совѣтнику, Сенатору и орденовъ: св. Александра Невскаго и св. Владиміра I степени Кавалеру

НИКОЛАЮ НИКОЛАЕВИЧУ

НОВОСИЛЬЦОВУ.

Въ знакъ глубочайшаго почитанія
и совершенной преданности
посвящаетъ

Ѳ. Петрушевскій.

Fig. 4b. Dedication to count Novosiltsev on the first page of Psammite.

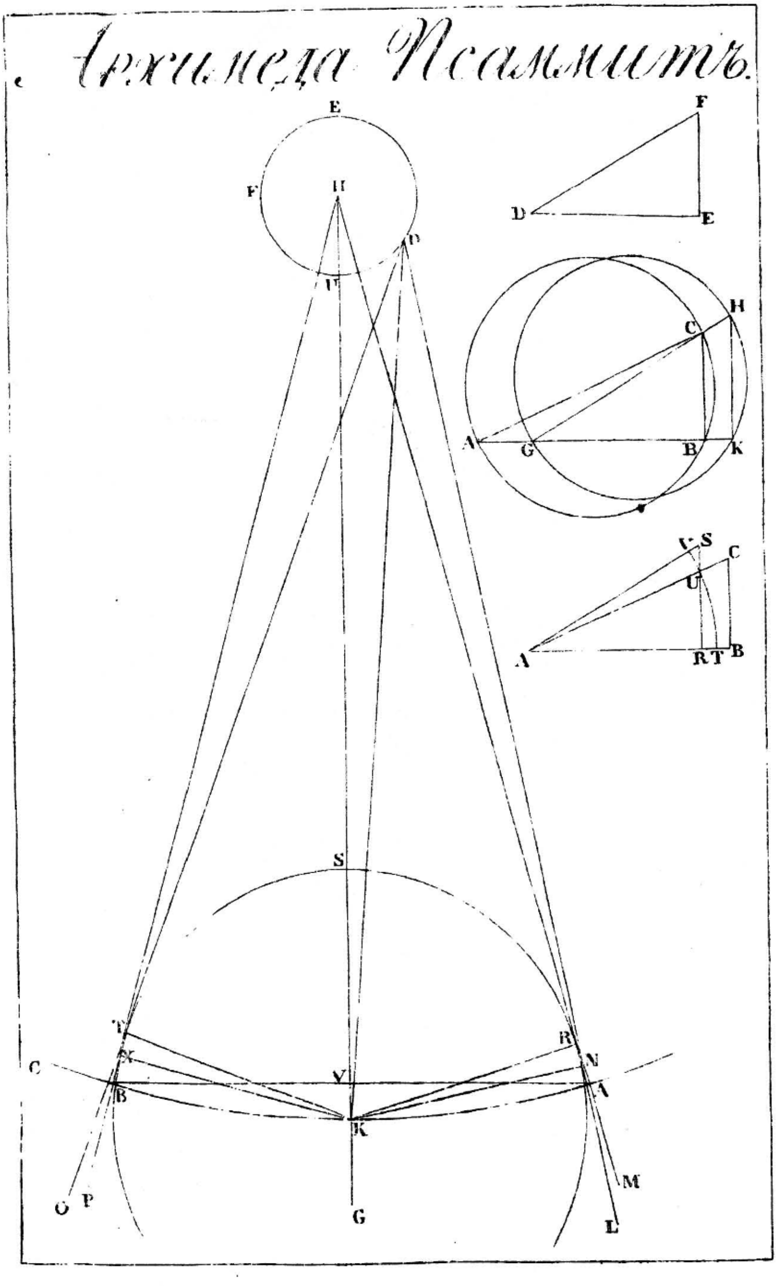


Fig. 4c. Tab page of Psammite.

3. PUBLICATIONS SINCE FIRST THIRD 19TH CENTURY BEFORE REVOLUTION OF 1917

The Treatise **«About Measurement of a Circle»** [4], the appendix to the translation of Euclid's elements; translation and comments to the treatise are made by prof. Vashchenko-Zaharchenko of M.E.[2], News of the Kiev University, 1880. New composition of Archimedes: **The Epistole of Archimedes to Eratosthenes** about some theorems of mechanic/ [Publ.] Prof. J.L. Heiberg: With the preface of the private-senior lecturer I.O. Timchenko; Trans. from German, in the edition of "The reporter of experimental physics and basic mathematic" – Mathesis, Odessa, 1909 [5].

In 1906 prof. J.L. Heiberg, the Danish philologist and mathematician, found a small endurance from one ancient manuscript of mathematical maintenance in the library catalogue of a monastery of St. Savvy near Jerusalem. This endurance was resulted by P. Papadopulo-Keramevs, the private-senior lecturer of the Petersburg University, from not quite washed off Ancient Greek text, which he had found out on a parchment of his interest under the text of later origin. Not being the mathematician, Papadopulo-Keramevs did not give great value to the opening, but prof. J.L. Heiberg recognized **ex ungue leonem** («on claws of a lion») Archimedes's product in that short endurance. He managed to find that manuscript, and he found Greek texts of some Archimedes's compositions in it. The manuscript was made in X century. Between XII and XIV centuries, as it often happened, the same parchment was used again for a theological text thus tried to wash off the old text but, fortunately, not very successfully [12]. Archimedes's book contains a statement of the method connected with mechanical theorems and is devoted to the Alexandrian mathematician and astronomer - Eratosthenes. It was known to Heron of Alexandria (II century BC), who named it "Euphodic" – method, guidance (*Εφοδιχον*). Besides, the work "About floating bodies" was stated in it.

In Geiberg's preface it is stated that eight mathematic compositions, rewritten by Archimedes, except newly opened "Euphodic", have reached us to the present moment: two books about the balance of flat figures together with the book about the quadrature of a parabola; two books about a sphere and a cylinder; about the measurement of a circle; about spiral lines or spirals; about conoids and spheroids; Psammit, or the calculation of sand grains; two books about floating bodies, lemmas.

---

[2] **Vashenko-Zakharchenko, Mikhail Egorovich**, the mathematician (1825–1912). He published "Euclid's elements" with explanatory introduction and interpretation in "News of the Kiev University" (1878, 79, 80, and the separate edition).

Some books have reached us in the original, which means in the Dorian dialect of Greek, who ones spoke in Syracuse: books about the balance of flat figures; about spirals; about conoids and spheroids, Psammit. The books about a sphere and a cylinder have reached us in the latest editing, in which the Dorian dialect was replaced with general Greek. The book about floating bodies had been famous until the last time as Wilh.v. Moerbecke (East Flandrian monk) said. All Archimedes's compositions, except Psammit, contain more or less of the latest insertions. "Lemmas" were translated from Arabian into Latin and in the editing, in which it has reached us, could not have been written by Archimedes; however, many of them are likely to belong to Archimedes.

About quadrature of a circle – Archimedes, Huygens, Legandre, Lambert, with appendix of questions' history, made by F. Rudio/Trans. from German under the red. and with notes of S.N. Bronstein, the private-senior lecturer of the Kharkov University–Odessa, Mathesis, 1911 [6].

4. PUBLICATIONS AFTER REVOLUTION IN 1917

Industrialization of the country demanded considerable number of scientific and engineering personnel (I. Stalin: "The personnel mean everything"). The series "classicists of natural science" (Aristotle, Descartes, Archimedes etc., including literature about sciences and techniques at antique and middle centuries) was published. Most of these editions were published by State technical-theoretical publishing house of USSR Scientific Academy. The quantity of scientific and technical editions grew great; there were 5000–10000 printed copies. They were (usually) the books of quadric (near A5) format, very cheap for good accessibility from students, and printed on middle-lower quality paper.

ARCHIMEDES, CALCULATION OF SAND GRAINS (PSAMMITE). TRANSLATION, COMMENT AND SHORT SKETCH OF ARCHIMEDES'S SCIENTIFIC ACTIVITY WERE WRITTEN BY PROF. G.N. POPOV [7].

The first edition of Archimedes's works after the Revolution, "Psammite" was published in E.V. Visotskiy's "Sower" Publishing House in 1922. In 1932 and 1933 Prof. G.N. Popov published processed edition of that work in the State Technical-Theoretical Publishing House [8]. He used the best in his opinion: the edition of Archimedes's compositions for translation, which contained the Greek text with translation made by Prof. Geiberg.

The precondition for a choice of this work was that its maintenance did not demand much knowledge in mathematics, and, anyway, it was easier, than the main Archimedes's treatises devoted to geometry. In fig. 5 the title page of the edition of "Psammite" (1932) [5] is presented.

Fig. 5. Title page of the edition of "Psammite" (1932).

In 1932 the State Technical-Theoretical Publishing House (Moscow-Leningrad) published the book of the Beginning of a Hydrostatics: Archimedes, S. Stevin, Galileo, B. Pascal [9]/Translation and notes were written by prof. A.N. Dolgov. The book was republished in 1933. The translation was made using the text of the treatise, placed in volume of II full three-volume collection of Archimedes's works, repeatedly published by Prof. Geiberg in the Greek and Latin languages ("**Archimedes opera omnia cum commentariis Eutocius itrum editit J.L. Heiberg**", Leipzig, 1910–1915).

Prof. A.N. Dolgov gives a magnificent substantiation of the choice of authors: "… works of the ancient were based exclusively on empirical data and were not based on any revealed principles of hydromechanics. … the first absolutely exact and clear formulation of one of hydrostatics organic laws, with which the history of its development as a science in effect begins, we find only at Archimedes's … hydrostatics general laws were found and strictly enough proved by elementary geometrical and mechanical argumentation, accessible to those, who are not familiar with higher mathematics. These reasoning have not have lost their value till present as they are extremely natural and do not differ much from modern methods of stating … Thereof they can serve as fine addition to both a little dogmatic formulations of hydrostatic principles in physics textbooks and formally exact and strict constructions, met in modern courses of hydraulics".

CZWALINA A., ARCHIMEDES, LEIPZIG-TEUNBER, 1925 [10]/THE TRANSLATION FROM GERMAN OF PROF. V.I. KONTOVT. THE STATE TECHNICAL-THEORETICAL PUBLISHING HOUSE, MOSCOW-LENINGRAD, 1934.

There was the list of the major Archimedes's compositions: his life description, sketches of his works on algebra, geometry, mechanics, calculation of sand grains included into the book. Especially it is necessary to note three cardinal services to modern science of Archimedes the author marked.

1. Universe limits in the ancient's conception were limited to ices, heat and ocean. Behind these limits was "divine". So, in the ancient's mathematics there were no concepts of continuity and infinity. The concept of continuity lies across the side of the Euclid's mathematics. To come to continuity, it was necessary to take a step towards infinity. Archimedes took this step and operated with variable and continuous: at Euklid's we do not meet a straight line or a circle being able to be received with a point movement.
2. "Archimedes met spiral quadrature problems in a problem about a cone quadrature. Thus, huge synthesis grows from the analysis of separate problems. He does not have predecessors in it".
3. Archimedes had begun the mathematical physics, had found mechanics basic laws: he was the first mathematician to connect mathematics with other sciences meaningly.

In the book there is a poetic essay about last hour of Archimedes: "… Syracuse are taken. Everyone is running to save their life. But the grey-haired 75 year old man is sitting by himself in a garden. He is drawing geometrical figures. Of what he was thinking – it is not known, but we will not be mistaken, if we say that he was thinking of what nobody had not thought of for another 1000 years".

"ARCHIMEDES. COMPOSITIONS". MOSCOW, 1962. TRANSLATION, INTRODUCTORY ARTICLE AND COMMENTS MADE BY I.N. VESELOVSKY [11].

It was the expensive book of large format (70x108 with $^{1}/_{16}$), written for mathematical, mechanical scientists, physicists and historicians of science. The "Compositions" was printed at quantity of 4000 items.

In this edition I.N. Veselovsky "tried to collect every Archimedes's work that had been left whole. The translation was made with use of 2nd edition of the text of Archimedes's compositions, published by Geiberg [13]. Besides, the translator added all texts concerning Archimedes, which Pappus and Heron had, in comments. At last, the offered book included Archimedes's Arabian texts, in particular the translation of the "Book about a heptagon", which was published (1962) for the first time.

I. Veselovsky wrote that while translating ancient classics it is possible, for example, to keep strictly to the character of the original statement (Ver Eecke) or to give it in a modern statement (T.H. Heath). He chose the third way: «having kept Archimedes's statement as far as its reading will not complicate the reader, he had added modern algebraic formulations». Into the structure of Archimedes's compositions were entered:

- Mechanical fragments (made using citations of Archimedes's compositions by Heron, Pappus, Simplicus, Eutocius);
- A quadrature of a parabola;
- About a sphere and a cylinder;
- About conoids and spheroids;
- About spirals;
- Measurement of a circle;
- About the balance of flat figures, or about the gravity centers of flat figures;
- The Letter about mechanical theorems to Eratosthenes;
- About floating bodies;
- Psammite;
- Katoptrika;
- About the device of heavenly sphere;
- The Problem, which Archimedes found in epigrams and sent to solve to Alexandrian scientists, working with similar problems in the letter to Eratosthenes;
- Stomahius;
- About polyhedrons;
- Abu-El-Hasan Sabit ibn Hens' Treatise about construction of a corporal figure circumscribed about a sphere with fourteen bases;
- The Archimedes's Book of lemmas in translation of Tebit Ben Kora and with explanations of scientist Al-Mohtasso Gali Ben Ahmad the Nasuene;

- Archimedes's book about plotting of the circle divided into seven equal parts. Translated by Abu-El-Hasan Sabit ibn Kuri Al-Harani;
- Archimedes's theorems remained in Al-Biruni's statement;
- The book about touching circles of Archimedes, who was killed in 212 BC.

In Fig. 6a–6c the title page and two pages of the book [11] are presented.

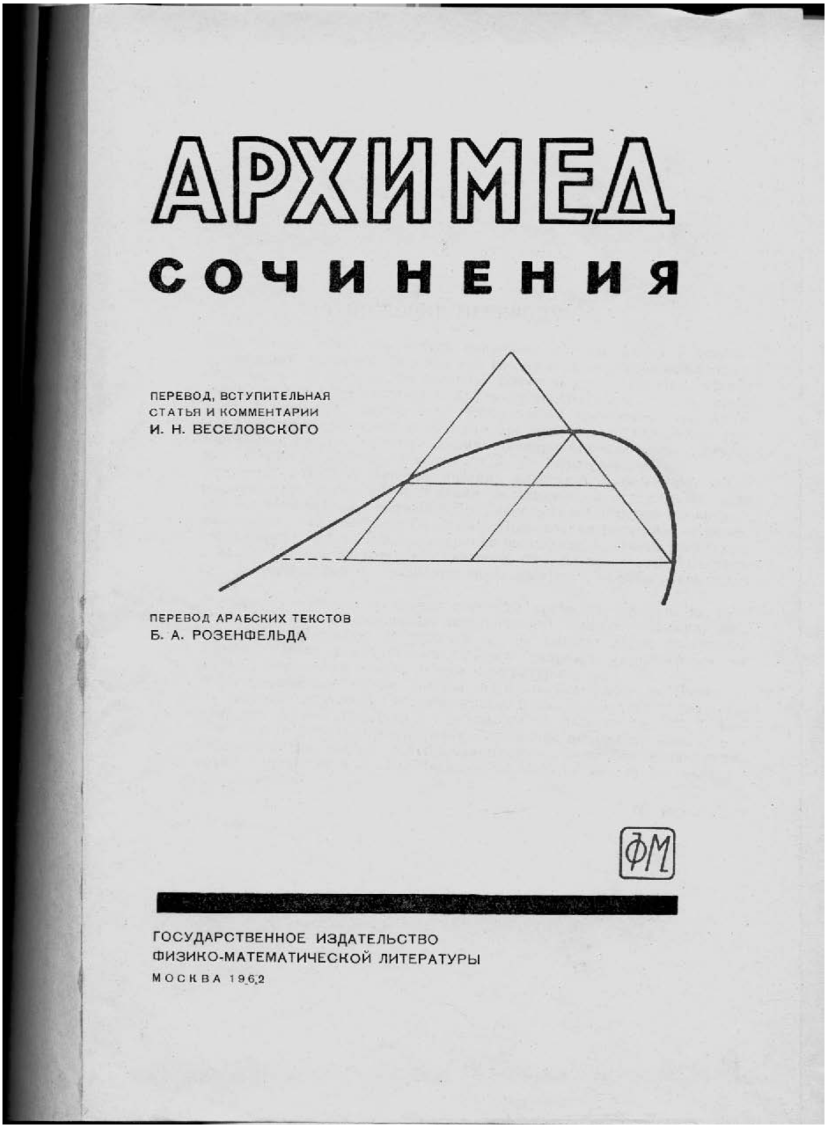
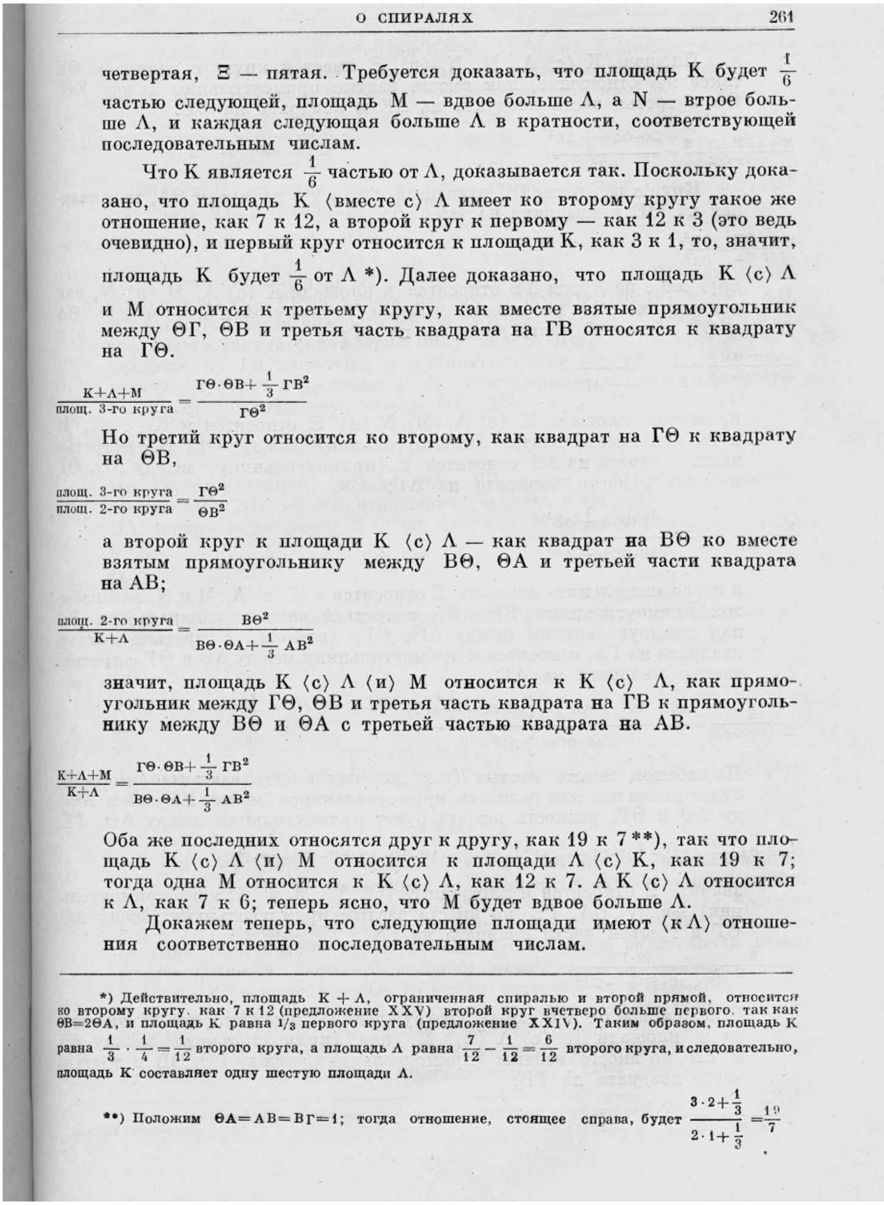


Fig. 6a. The title page of the translation of "Book about spirals".



четвертая, Ξ — пятая. Требуется доказать, что площадь K будет $\frac{1}{6}$ частью следующей, площадь M — вдвое больше Λ, а N — втрое больше Λ, и каждая следующая больше Λ в кратности, соответствующей последовательным числам.

Что K является $\frac{1}{6}$ частью от Λ, доказывается так. Поскольку доказано, что площадь K ⟨вместе с⟩ Λ имеет ко второму кругу такое же отношение, как 7 к 12, а второй круг к первому — как 12 к 3 (это ведь очевидно), и первый круг относится к площади K, как 3 к 1, то, значит, площадь K будет $\frac{1}{6}$ от Λ *). Далее доказано, что площадь K ⟨с⟩ Λ и M относится к третьему кругу, как вместе взятые прямоугольник между ΘΓ, ΘB и третья часть квадрата на ΓB относятся к квадрату на ΓΘ.

$$\frac{K+\Lambda+M}{\text{площ. 3-го круга}} = \frac{\Gamma\Theta\cdot\Theta B+\frac{1}{3}\Gamma B^2}{\Gamma\Theta^2}$$

Но третий круг относится ко второму, как квадрат на ΓΘ к квадрату на ΘB,

$$\frac{\text{площ. 3-го круга}}{\text{площ. 2-го круга}} = \frac{\Gamma\Theta^2}{\Theta B^2}$$

а второй круг к площади K ⟨с⟩ Λ — как квадрат на BΘ ко вместе взятым прямоугольнику между BΘ, ΘA и третьей части квадрата на AB;

$$\frac{\text{площ. 2-го круга}}{K+\Lambda} = \frac{B\Theta^2}{B\Theta\cdot\Theta A+\frac{1}{3}AB^2}$$

значит, площадь K ⟨с⟩ Λ ⟨и⟩ M относится к K ⟨с⟩ Λ, как прямоугольник между ΓΘ, ΘB и третья часть квадрата на ΓB к прямоугольнику между BΘ и ΘA с третьей частью квадрата на AB.

$$\frac{K+\Lambda+M}{K+\Lambda} = \frac{\Gamma\Theta\cdot\Theta B+\frac{1}{3}\Gamma B^2}{B\Theta\cdot\Theta A+\frac{1}{3}AB^2}$$

Оба же последних относятся друг к другу, как 19 к 7 **), так что площадь K ⟨с⟩ Λ ⟨и⟩ M относится к площади Λ ⟨с⟩ K, как 19 к 7; тогда одна M относится к K ⟨с⟩ Λ, как 12 к 7. А K ⟨с⟩ Λ относится к Λ, как 7 к 6; теперь ясно, что M будет вдвое больше Λ.

Докажем теперь, что следующие площади имеют ⟨к Λ⟩ отношения соответственно последовательным числам.

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*) Действительно, площадь K + Λ, ограниченная спиралью и второй прямой, относится ко второму кругу, как 7 к 12 (предложение XXV), второй круг вчетверо больше первого, так как ΘB=2ΘA, и площадь K равна 1/3 первого круга (предложение XXIV). Таким образом, площадь K равна $\frac{1}{3}\cdot\frac{1}{4}=\frac{1}{12}$ второго круга, а площадь Λ равна $\frac{7}{12}-\frac{1}{12}=\frac{6}{12}$ второго круга, и следовательно, площадь K составляет одну шестую площади Λ.

**) Положим ΘA=AB=BΓ=1; тогда отношение, стоящее справа, будет $\frac{3\cdot 2+\frac{1}{3}}{2\cdot 1+\frac{1}{3}}=\frac{19}{7}$.

Fig. 6b. Text fragment of the translation of “Book about spirals”.

меньше описанной фигуры. Но он не меньше, а больше; значит, сектор Xϙ будет не больше площади, заключающейся между спиралью АВГΔЕ и прямыми АΘ, ΘЕ.

Но он не будет также и меньше. Действительно, пусть он будет меньше, и пусть все остальное будет сделано совершенно так же. Тогда опять можно вписать в эту площадь плоскую фигуру, состоящую из подобных секторов, так, чтобы упомянутая площадь была больше вписанной фигуры на ⟨величину⟩ меньшую той, на которую эта же самая площадь превосходит сектор Xϙ. Впишем ее, и пусть из секторов, составляющих вписанную фигуру, наибольшим будет ΘВГ {рис. 28}, наименьшим же ОΘЕ; ясно, что вписанная фигура будет больше сектора Xϙ. Теперь опять имеются некоторые линии, одинаково возвышающиеся одна над другой,— именно, доходящие от Θ до спирали, из которых наибольшей будет ΘА, а наименьшей ΘЕ; имеются также и другие линии,— именно доходящие из Θ до окружности сектора ΘАZ, кроме лишь ΘА, по количеству на одну меньше одинаково возвышающихся одна над другой, по величине же равные друг другу и наибольшей ⟨из них⟩; на каждой из них построены подобные секторы, лишь на наибольшей из одинаково возвышающихся одна над другой сектор не построен; тогда секторы на прямых, равных друг другу и наибольшей, к секторам на одинаково возвышающихся одна над другой, кроме лишь сектора на наибольшей, будут иметь большее отношение, чем квадрат на ΘА к ⟨прямоугольнику⟩ между ΘА, ΘЕ, и третьей части квадрата на части ЕZ, так что и сектор ΘАZ имеет ко вписанной фигуре большее отношение, чем к сектору Xϙ; таким образом, сектор Xϙ будет больше вписанной фигуры. Но он не больше, а меньше; значит, сектор Xϙ не будет и меньше площади, заключенной между спиралью АВГΔЕ и прямыми АΘ, ΘЕ; значит, он будет ей равен.

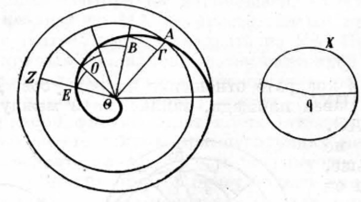

Рис. 28.

XXVII

Из площадей, заключенных между спиралями и прямыми на начале вращения, третья будет вдвое больше второй, четвертая втрое больше ее, пятая вчетверо, и всегда каждая следующая будет больше второй площади в кратности, соответствующей последовательным числам, а первая площадь будет шестой частью второй.

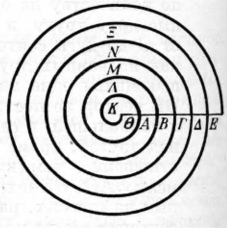

Рис. 29.

Пусть предложенная спираль будет описана в течение первого оборота, второго и взятых в любом числе следующих, пусть началом спирали будет точка Θ {рис. 29}, началом же вращения прямая ΘЕ, а из площадей пусть К будет первая, Λ — вторая, М — третья, N —

Fig. 6c. Second text fragment of the translation of “Book about spirals”.

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ARCHIMEDES IN PROGRAM ON HISTORY OF MECHANICS IN LOMONOSOV MOSCOW ST. UNIVERSITY

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"Archimedes had a greater genius than
can be compatible with human nature"
Cicero, De republica, v.1

ABSTRACT Steps in elaboration of teaching courses on the history of mechanics and mathematics are illustrated. The courses which are taught in Moscow University and in other universities in Russia, together with their respective textbooks, have always paid a great attention to Archimedes and his original manuscripts. The review of Archimedes' works that are considered at the lectures on history of mechanics and mathematics in MSU, Department of Mechanics and Mathematics, is given and methodological problems connected with evaluation of Archimedes' creative ability are discussed.

1. INTRODUCTION

The history of mathematics and mechanics is a discipline that, on one hand, can be considered as a part of the history of science tightly connected with philosophy and, from the other hand, as a discipline studying the subject itself (mathematics or mechanics) in its historical observation.

Archimedes' role in science is highly evaluated both in pure mathematics and mechanics, the theme "Archimedes" covering significant positions in the corresponding courses.

This paper consists of three sections. The **first section** deals with teaching the history of mechanics and mathematics in Moscow University as well as in other universities in Russia; the **second one** provides a summary of Archimedes' works that are considered in the "History of Mechanics" course at the MSU Department of Mechanics and Mathematics.

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2. TEACHING THE HISTORY OF MECHANICS AND MATHEMATICS IN MOSCOW UNIVERSITY AND OTHER UNIVERSITIES IN RUSSIA

Moscow St. University (MSU) is one of the largest centres on studying and teaching the history of mathematics and mechanics which is, first, an *obligatory subject* on mathematics and mechanics in the educational program for students, and, second, one of specialization courses at the Department, students being involved in respective researches when preparing their term papers and graduation dissertations and carrying out thesis investigations. The history of mathematics and mechanics acquired, as a discipline, such an important position in the system of teaching and researching in higher educational institutions in the Soviet times[1].

In 1945, at the MSU natural science departments, systematic courses on the history of mathematics and certain yearly courses on the history of mechanics were included in the educational process (Mechanics and Mathematics Department), and so were some courses on the history of astronomy and physics (Physics Department)[2]. The theme "Archimedes" became of considerable importance in the courses, as well as in respective textbooks which appeared later.

The evolution of the lecturing process will be shown in more detail by the example of the course on the history of mechanics in MSU where it has been taught as an obligatory course since 1945.

Nikolai Dmitrievich Moiseev (1902–1955), a famous specialist in celestial mechanics and theory of stability, being in charge of the Celestial Mechanics Chair at the Astronomy Department in 1938–1955 [1], was the first lecturer of the subject. He was a scientist having encyclopedic scope of knowledge, revealing competence in many European and classic languages. In his course on the history of mechanics, Moiseev considered a lot of tractates and separate works, included epistolary fragments from the letters between prominent scientists where the analysis of Archimedes' discoveries was on essential positions. As a rule, N.D. Moiseev used his

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[1] Before 1917, however, many mathematicians revealed vivid interest to the history of their science. Historical data were conveyed in lectures and monographs. In the period from 1882 to 1919, privat-docent V.V. Bobynin lectured in Moscow University a facultative course on the history of mathematical sciences.

[2] The history of chemistry at the Chemistry Department, the history of geology at the Geology Department, and the history of geography at the Geography Department were lectured. Lecturing the history of biology at the Biology Department was entirely prohibited for 5–6 years because of 'lysenkovshchina" (the term introduced after the name of Academician Lysenkov). In the years 1970–80, Prof. N.N. Polyakhov, Dean of the Mathematics and Mechanics Department in the University, was teaching a term course on the history of mechanics in former Leningrad (now city of Saint-Petersburg).

own translations from original languages. An idea on Moiseev's course can be obtained from his well-known "Essays on Mechanics Elaboration" [2]. It is worth noticing that Archimedes' works, in translation by Prof. I.N. Veselovsky who added his own valuable comments to them [5], were not published in Russian until 1962. For that moment, N.D. Moiseev was already late, and his textbook [2] could only appear owing to the activity of his apprentices I.A. Tyulina and E.N. Rakcheev. After N.D. Moiseev' death, the course on the history of mechanics at the MSU Mechanics and Mathematics Department has been irremovably lectured by I.A.Tyulina. In recent years, V.N. Chinenova[3] has joined to the lecturing process. Traditionally, the key role in the course has belonged to Archimedes and his works. While telling on Archimedes's biography, we do not concentrate on numerous mythological and legendary facts from the life of this unordinary scientist; those who are interested in them are referred to popular literature, for example, to the headliner by N. Vitkovsky and S. Ortoly "Archimedes' Bath". The most attention in lectures is paid to his inventions and discoveries, his methods of scientific analysis. It is emphasized that the main property of Archimedes's creations is keeping close connection between mathematical methods and problems in mechanics and physics.

In 1960s and 70s "The History of Mechanics" course (covering Archimedes's activity) was lectured in the universities of Rostov, Voronezh, Dnepropetrovsk, Saransk, and in 1980s, in the universities of Tbilisy and Alma-Ata. Since 1960s, the specialized course on the history of mechanics was lectured in Bauman Moscow Higher Technical School by the prominent specialist in this field, Prof. I.N. Veselovsky, who translated Archimedes's works into Russian [5] and published the monograph "Essays on the History of Theoretical Mechanics" [7][4]. In Vilnius Pedagogical Institute there was a course on the history of mechanics which was lectured by the assistant professor L.L. Kulvetsas since 1970.

In the courses on the history of physics (lectured at MSU Physics Department as well as in other universities, for instance, in Tambov) Archimedes' life and creative activity were also illustrated. Prof. P.S. Kudryavtsev (Tambov) developed a periodic system for researching the history of physics that was linked with personalities; see, for example, the chapter "Ancient Mechanics and Archimedes" in his monograph [8].

In MSU, Prof. K.A. Rybnikov prepared the course on the history of mathematics and produced a textbook [9] which provided Archimedes's

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3 In 1979 the monograph by I.A. Tyulina, "The History and Metodology of Mechanics" [3], and in 2002 the textbook by I.A. Tyulina and V.N. Chinenova, "The History of Mechanics"[4], were published.

4 Later this course was lectured by assistant professor G.I. Gataulina.

detailed scientific biography and considered his infinitesimal methods. Lecturers involved in teaching this course were repeatedly giving reports and publishing articles with analyses of Archimedes' mathematical works [10].

3. ARCHIMEDES' WORKS IN THE MSU COURSE "HISTORY OF MECHANICS"

Archimedes's works on statics and hydrostatics are considered to be mathematical physics of the ancient world. These works based on geometry are an example of geometrical deductive method. In each separate case, for instance, for the theory of lever or theory of body flotation, the needed physical postulates (axioms) are introduced and a corresponding theoretical proposition is developed with the help of geometry.

Before providing the main aspects of the theory concerning equilibrium of weights, developed by Archimedes, which are lectured in the course on the history of mechanics at the Mechanics and Mathematics Department, specific properties of the 3rd century B.C. are given, technical requirements of the time and cognitive preconditions of Archimedes' barycentric teaching are considered. Since another approach was also applied at the same time to the problems of statistics (considering possible velocities of weights) it is made comparison of particularities in technical problems which generated these two directions in developing the statics (geometrical and kinematical). Such a comparison helps students to deepen their understanding the subject. Definition of "centre of gravity" was known to be introduced by Archimedes in his works "Book of Supports" and "On Balance" which did not survive till our time and of which little trace remained[5]. The concept of "Centre of Gravity" appears in a natural manner when resolving the mechanical problems connected with the equilibrium of heavy bars and plates which are supported in one or more points, centre of gravity being such a point that supports a body and keeps its equilibrium even if other supporting points are deleted. A general criterion for equilibrium of a suspended heavy body is formulated as follows: "If the centre of gravity of a heavy suspended body is situated on the vertical line passing through the fulcrum, then the body is in the equilibrium" [2, p. 47]. Archimedes defined a centre of gravity of a body as a point where various vertical lines crossing the body in corresponding equilibrium positions and passing through different fulcrums intersected each other [4, p. 37].

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[5] We know about these works only from references that are cited in other Archimedes' compositions and in works belonging to such Alexandria school representatives as Heron and Pappus who were well familiar with Archimedes' teaching.

A strict aspect in teaching on equilibrium of loads (in Euclid's style) considered in Archimedes' works is more easily conceived by students than a pragmatic approach from which a principle of possible displacements later originated.

Archimedes' work "On the Equilibrium of Planes, or on the Centre of Gravity of Plane Figures" contains seven postulates and is cited according to the work [2, pp. 48–53]. When providing the gist of this work, we follow main Archimedes' considerations.

Since N.D. Moiseev's book is a bibliographic rarity and his fragmentary translation of Archimedes' works is seldom mentioned in educational and scientific literature, some of these postulates are worthwhile to be cited:

1. "Equal weights suspended at equal distances (from a fulcrum) are in equilibrium.
2. Two equal weights suspended at different distances are not in equilibrium; the weight suspended at a longer distance will fall down.
3. If two weights suspended at certain distances are in equilibrium, and something is added to one of them, they will no longer be in equilibrium; the greater weight will fall down.
4. Similarly, if something is taken off from one of those weights, they will no longer be in equilibrium; the lighter weight will fall down.
6. If certain weights suspended at certain distances are in equilibrium, equivalent weights suspended at the same distances will also be in equilibrium" [2, p. 48].

Postulates 5 and 7 mean that equal and coinciding and homogeneous figures, when they are superpositioned, have the centres of gravity which also coincide under superposition of figures, the commensurable figures have the centres of gravity located in a similar manner and, finally, a centre of gravity of a convex figure is inside the figure.

The following three theorems, in fact, serve to explain the substance and sense of the postulates.

The subsequent group of theorems, 4 and 5, describes the possibility for loads, suspended to a lever, to be concentrated and deconcentrated, their common centre of gravity being unchanged, so that the equilibrium occurred before the operation is not upset. The main condition for keeping the equilibrium is considered to be the fact that the centre of gravity of the suspended body is located on a vertical line passing through a fulcrum.

Thus, theorem 4 affirms that "the centre of gravity of the magnitude made up of two magnitudes is the point situated at the middle of the line which joins the centres of gravity of these two equal magnitudes". Theorem 5 considers "the magnitude made up of three equal magnitudes" located in such a manner that the centre of gravity of the central magnitude is situated at the middle of the line joining the centres of gravity of the two extreme magnitudes. The theorem affirms that the centre of gravity of the combined magnitude will coincide with the centre of gravity of the middle magnitude.

On the assumption of these postulates, Archimedes proves the law of lever according to which two magnitudes, commensurable or incommensurable, keep the equilibrium at distances which are inversely proportional to the magnitudes themselves. Theorem 6 deals with the equilibrium of a direct lever with unequal arms: "the commensurable magnitudes situated at distances which are reciprocally proportional to their weights are in equilibrium"[6].

Let us produce the proof of this theorem in the form that is proposed to the students (it is very close to Archimedes' proof) [4, p. 38].

Given: the magnitudes of loads are reciprocally proportional to their distances from the fulcrum O (Fig. 1) along the lever straight line:

$$\frac{\alpha}{\beta} = \frac{OB}{OA} \tag{1}$$

It is necessary to prove that the system is in equilibrium.

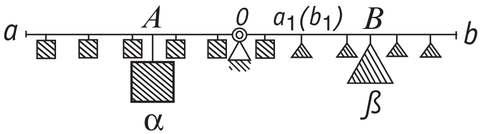

Fig. 1.

On the extension of the horizontal lever line AB. The intervals Aa_1 and Aa, which are equal to the arm OB, are plotted to the right and to the left from the point A. Similarly, two intervals, Bb_1 and Bb, each being equal to the arm OA, are plotted from the point B. Thus, a new lever with double extension $ab = 2\,AB$ is produced, the fulcrum point O being at the middle of the lever. Archimedes deconcentrated the given loads α and β according to proportion (1): the load α is substituted by the loads equal to the half of the unit of weight each over the unit of length

[6] Archimedes. Works, p. 274.

aa_1; the load β is analogously distributed along the segment bb_1. A homogeneous heavy bar, *ab*, is produced which is supported at the middle, *O*, or at the centre of gravity, and is not capable to break its equilibrium state by itself. The system of weights distributed along the bar ab is equivalent to the system of two original weights located at the direct lever AB with unequal arms because distribution of two given weights, α and β, could not upset the equilibrium of the original lever. If the equilibrium is maintained (the centres of gravity of the weights α and β keep their original positions when distributed along the length of the bar), then the original lever AB, should necessarily be in equilibrium[7].

The following, seventh theorem extends this proposition to the case when weights and arms are incommensurable, the proof being given with the help of "the method of exhaustion" (the passage to the limit).

The other theorems of Archimedes' treatise "On the equilibrium of Planes" are devoted to the problem concerning the location of the centres of gravity of particular plane figures with homogeneous distribution of surface specific weight. It was there that Archimedes set the location of the centres of gravity for a parallelogram, a triangle, a trapezium, and a parabolic segment.

Barycentric concept was also utilized by Archimedes in his work "Message to Eratosthenes on the Mechanical Theorems" which appeared after the treatise "On the Equilibrium of Planes". In this work, Archimedes abided by the heliocentric hypothesis of the Universe, according to the views of Aristarchus from Samos. This work developed the mechanical method of solving geometrical problems.

Archimedes proved some theorems concerning correlations between certain parts of volumes for homogeneous bodies: a sphere, a cylinder, and a cone. This correlation is interpreted mechanically, in terms of the lever rule, that is, two-arm balance.

The same method of mechanical analogy was utilized by Archimedes in his work "On the Quadrature of Parabola".

These are the main principles contributed by Archimedes to the foundation of the scientific theory concerning the equilibrium of supported and suspended bodies.

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7 Sometimes we provide the proof in that well-known manner which was used by Galileo in his second Day of Discources [11, pp. 220–222].

As to the theory of the equilibrium of heavy fluid and floating bodies, Archimedes' treatise "On Floating Bodies" is of fundamental significance. The second lecture in our course on the history of mechanics is devoted to this work. It is worth noticing that in doing so we use both the translation by I.N. Veselovsky [5] and the translation by A.N. Dolgov which is given in the compilation "The Foundations of Hydrostatics" [12].

The following hypothesis is fundamental for Archimedes' conclusions: "The nature of a fluid is such that if its parts are equivalently placed and continuous with each other, that which is the least compressed is driven along by that which is the more compressed. Each part of the fluid is compressed by the fluid which is above it in a vertical direction, provided the fluid is not contained in a vessel and not compressed by something else" [5, p. 328].

The first two propositions of Archimedes' treatise state that the surface of any fluid at rest surrounding the Earth is spherical and that the centre of this circumferential sphere coincides with the centre of the Earth. On the base of these experimental premises, Archimedes proves propositions III – VI. Propositions V, VI, VII state the so-called basic Archimedes' law.

Proposition VII reads as follows: "If a body is placed in a fluid which is lighter than itself, it will fall to the bottom. In the fluid the body will be lighter by an amount which is the weight of the fluid which has the same volume as the body itself" [5, p. 332].

After that, we concentrate our students on the problems of equilibrium and stability of floating bodies which were considered by Archimedes. The basic method of the investigation is perturbation of an equilibrium state.

All the propositions of the treatise are proved by means of the unique method in which the centre of gravity of the entire body and that of the immersed part of the body are determined. The body is in equilibrium if these points are situated on one vertical line when the gravity of the body and the force of hydrostatic pressure act in opposite directions along the same line. The equilibrium is stable if the body, when being declined from the equilibrium state, tends to return into this state [2, p. 52].

The second part of the treatise considers particular cases of equilibrium and stability of spherical segments and paraboloids of revolution floating in the fluid.

We also fix our students' attention on the idea of hydrostatic balance and later, in the section of mechanics, on medieval East conceptions where this idea was developed and used by Arabic scientists (see, for example, composition by Abd al-Rahman al-Khazini, "The Book of the Balance of Wisdom").

Subsequently, the works of Archimedes' followers who lived in 16th and 17th centuries are described, for instance, those of Guidobaldo del

Monte, Simon Stevin, Galileo (in his young years), and G. Personne de Roberval.

Many students carry out their essays and graduation papers particularly on the basis of Archimedes' works. Specialization occurs beginning from the 3rd year, however, certain term papers which analyze the works by Archimedes and his followers are performed by students in their earlier years.

When considering infinitesimal methods applied in the mathematics of ancient world, the course on the history of mathematics lectured in MSU analyzes Archimedes' works. Connections between mathematics and mechanics derived by Archimedes were revealed especially well in his work "Message to Eratosthenes (Ephod)". A detailed review of this work is provided [9, pp. 56–57].

The same barycentric method was applied by Archimedes in his work "On the Quadrature of Parabola".

History of mathematics considers the following opinion on the ancient science, which seems to us to be unjustified, according to which the whole science was dramatically divisible into two parts, theoretical "pure" science and applied one, any adherent points between them being hardly traced. Many philosophers and historians of the époque pronounced that mechanics and other applications of mathematics belonged to a number of dishonorable occupations. It is sufficient to recollect Plutarch who tended to assure his readers that Archimedes himself had believed his researches in mechanics and theory of mechanisms to be something peripheral, that "Archimedes did not attach much importance to all machines constructed by him, he considered them to be simple mechanical toys for enjoyment in free time mostly because tyrant Hiero insisted who permanently persuaded Archimedes to occupy not only with pure intellectual subjects but also with certain material things" [7, p. 16]. I.N. Veselovsky cites this argument with the reference to "Vitae parallelae" by Plutarch.

In the history of mathematics, however, few examples could be found when problems of theory and applications were coupled in such a close and inextricable connection as they were in Archimedes' creative work. I.G. Bashmakova, in her course of the history of mathematics at our Department, has analyzed Archimedes' work "On Floating Bodies" [10]. She demonstratively showed that for investigation of the equilibrium stability of floating bodies, Archimedes brightly elaborated and used both integration methods and the geometry of conics. "In doing so, Archimedes exposed and treated the problem of stability in a perfectly strict manner, in the spirit of his purely mathematical compositions… In his work, "On the Method", Archimedes alternatively used the lever principle for new mathematical factors to be established" [10, p. 787]. We absolutely agree with I.G. Bashmakova who comes to the conclusion that the work "On Floating Bodies" is the most profound among other Archimedes' compositions.

Not without reason, Lagrange in his "Analytical Mechanics" wrote: "This book is one of the most excellent memorials of Archimedes' genius, it contains the theory of stability of floating bodies to which very little was added by contemporary scientists" [13, v.1, p. 135]. The scientists did not put forward the theory till the 19th century when Archimedes' methods themselves were elaborated as well.

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ARCHIMEDES DISCOVERS AND INVENTIONS IN THE RUSSIAN EDUCATION

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ABSTRACT This article is focused on the way Archimedes' works and inventions are covered in the academic curricula in Russia. The authors consider different education levels: from the elementary school to the higher technical education. Archimedes' biography, his inventions and the laws discovered by him are described in a great number of educational literature, both for schoolchildren and for the students of technical universities. Archimedes' works on hydrostatics, geometry, center of gravity, mechanics of simple machines (levers, pulleys) and the structure of the machines he created are described there.

KEYWORDS: Archimedes works and inventions, primary school, higher education.

1. INTRODUCTION

Descriptions of Archimedes' inventions and discoveries and calculation methods are included in school curriculum of all levels, from elementary to high school. His name and his works are mentioned both in children's books and cartoons and in the university textbooks. Students can learn about Archimedes and his works at the lessons of physics, mathematics, history, and astronomy. His inventions are widely used in today's world and students in all countries study his works until now.

In the time of Archimedes scholars developed science solving both theoretical and practical problems, which required knowledge in various scientific fields: mechanics, hydraulics, mathematics. Science was not yet divided into branches and scientists did not have a narrow specialization. This is the explanation of the diversity of spheres where Archimedes used to work and where he developed new methods or made discoveries.

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2. ARCHIMEDES IN ELEMENTARY SCHOOL

In really young age Russian children get to know about Archimedes, his inventions and life in ancient Greece.

All Russian children love the story told in the book “Kolya, Olya, and Archimedes” (Fig. 1) by L. Zavalnyuk where two schoolchildren, a boy and a girl, find themselves in Ancient Greece.

Fig. 1. Book “Kolya, Olya, and Archimedes”.

There they meet Archimedes who tells them about his inventions. There is a cartoon based on the book where one can see a large number of Archimedes’ mechanisms and his other inventions. The cartoon is oriented towards younger audience, it arouses children’s interest and develops their creative thinking.

3. ARCHIMEDES IN SCHOOL MATHEMATICS

The majority of Archimeses’s inventions are fundamential for many branches of science. That’s why the name of Archimedes is so often mentioned in school education because its aims are to teach children basic knowledge.

The most famous Archimedes' discovery in mathematics is the calculation of the π number. Some math textbooks describe a way of π computation using regular polygons inscribed in a circle, according to Archimedes' method [1]. Planimetry books use problems named after the great scientist; thus, in [2] two problems are given: 1) "Archimedes' problem: The polyline AMB consisting of two segments (AM >MB) in inscribed in the arc AB of a circumference. Prove that the foot of the perpendicular KH dropped from the middle point K of the arc AB onto the segment AM divides the polyline in half, i.e. AH = HM + MB."

And 2) "The problem of Archimedes' arbelos: Prove that the radius of a circumference tangent to S1, S2 and segment BD is equal to the radius of a circumference tangent to S2, S3 and segment BD."

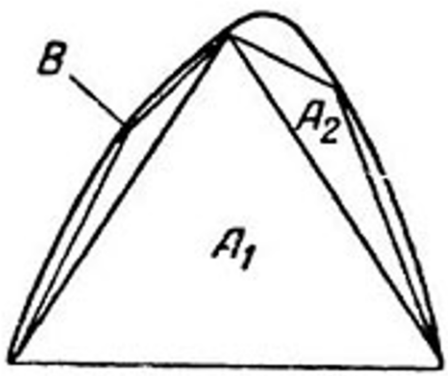

Fig. 2. Scheme for problem of Archimedes of polyline.

In the 80ies of the last century a series of school books called "Encyclopedia of young ……" was published, where omission points stand for "mathematician", "physicist", "historian", etc. These books featured issues beyond the school curriculum. In one of the books from the math line quite a number of chapters deal with Archimedes: geometry axioms, integral calculus (exhaustion method), progressions, calculation of the π number. The authors mention that Archimedes considered his main discovery to be the relationship between the volume of a cylinder and the volume of the inscribed sphere (their ratio is 3 : 2). According to his will, a cylinder with the inscribed sphere was placed on his tomb (Fig. 3).

Fig. 3. Sphere inscribed in cylinder.

4. ARCHIMEDES IN SCHOOL PHYSICS

Modern physics textbooks tell a legend of the golden crown and king Hiero whom Archimedes helped to find out whether the crown was really made of solid gold. This anecdote is often given as an illustration of Archimedes' law and as a practical means to find the volume of bodies having intricate shape [5, 6].

"Give me a place to stand on, and I will move the Earth!" One can find this quotation almost in every textbook chapter devoted to levers (Fig. 4). In one of physics textbooks there is an amusing calculation of a possibility to move the Earth: "A legend says that that Archimedes, excited by the discovery of the law of the lever, exclaimed, 'Give me a place to stand on, and I will move the Earth!' Archimedes certainly could not have fulfilled this task even if he had been given a pivot (which would have to be outside the Earth) and a lever of the required length. To raise the Earth even by 1 cm the long arm of the lever would have to describe an arc of

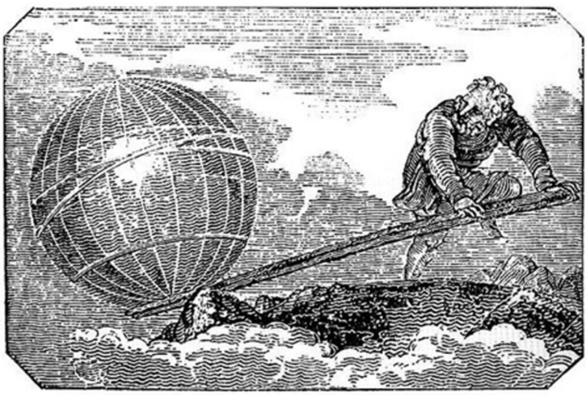

Fig. 4. Archimedes moves the Earth (antique print).

enormous length. It would take millions of years to move the long arm of the lever along this trajectory at a speed of 1 m/s.” [6]

Nowadays there are a lot of Internet resources devoted to school physics. Many of them provide additional materials for “the curious” and non-typical problems focused on developing creative thinking and keenness of wit. Teachers and students often share their experience. The website called “Cool Physics for the Curious” has an interesting article on the two mechanisms created by Archimedes: the movable pulley and the fixed pulley. From the article by the 7th grade students [7] one can learn how these devices are used today.

Every Russian school student associates Archimedes’s name primarily with the force named after him, i.e. with Archimedes’ expulsive force. On physics websites one can often come across lessons devoted to this law.

The website called “Physics for Teachers and Students” has a great number of articles on all parts of physics. Take, for example, the article on hydrostatics [8] describing interesting physical experiments. There the students get acquainted with Archimedes’s principle and investigate how the expulsive force acts in their experiments. In the first experiment a glass of water is placed on one of the pans of the balanced scales, with a load hanging above the glass from the support. The students have to guess what will happen to the scales if the thread holding the load is made as long as to submerse the load into the water. Another experiment: there is a glass with a small ball in it floating in a vessel with water. The students have to say how the water level will change if the ball - a) made of wood and b) made of steel – is removed from the glass into the water. (Fig. 5b).

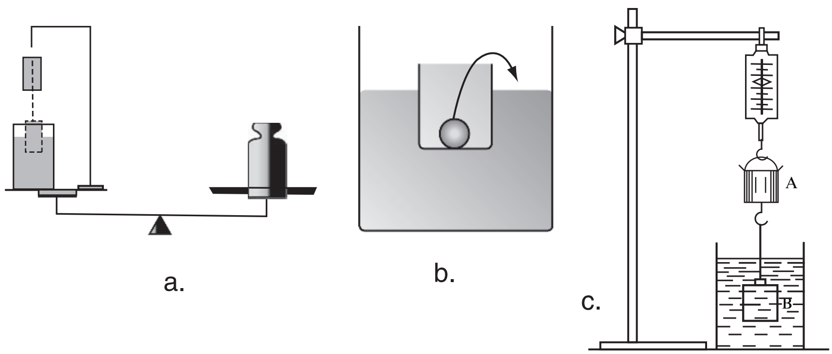

Fig. 5. Scheme of school experiment.

Another website, in an article on hydrostatics [9], there is an interesting description of Archimedes’ principle in the following wording:

"The buoyant force acting upon a body immersed in a fluid is equal in modulus to the gravity acting upon the fluid in the volume of the body (the displaced volume), is directed upwards and is applied in the center of gravity of the volume." The experiment with 'Archimedes' bucket proves the correctness of this law: "Let us suspend to the dynamometer an empty bucket A (Archimedes' bucket) and a solid cylinder B having a volume equal to the capacity of the bucket. Then, let us put a vessel with water under the cylinder and immerse the cylinder in the water; the balance will be upset and the tension of the dynamometer will decrease. If now we fill the bucket with water the dynamometer spring will stretch up to its former length. The weight loss of the cylinder is equal to the weight of the water in the volume of the cylinder."

5. ARCHIMEDES IN THE MECHANICS OF HIGHER SCHOOL

The aims of higher school education is to teach special knnowlege in branches of science chosen by a student. Education in higher school is a profound study of the required academic disciplines.

Simple mechanisms like a movable and fixed pulley, a crab and a lever were used in construction - for load lifting, in the military – for stone throwing. They considerably increase human resources, that's why in modern textbooks we can find their description. There is a separate chapter called "Archimedes the Engineer" in the book [10], which is devoted to mechanisms invented by him (Fig. 6). In this book, there is a detailed description of the use of stone-throwing and other siege machines. Pictures of these machines can be found on the pages of school textbooks [5]. The book also describes other inventions, for example, Archimedes screw. It was used for water pumping from mines and holds of ships, it for irrigation systems and for grain loading.

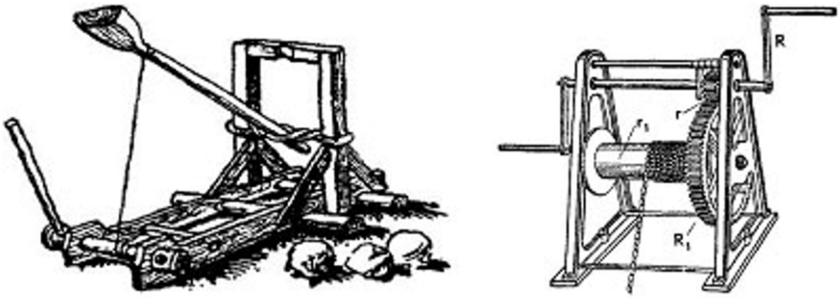

Fig. 6. Pictures from textbooks on Mechanics with schemes of mechanisms of Archimedes.

The textbook on Theoretical Mechanics characterizes Archimedes in the following way, “Archimedes became the founder of mechanics as a science. Archimedes produced an accurate solution of the problem on the balance of forces applied to the lever and created a teaching of the center of gravity of bodies. Besides, Archimedes discovered and formulated the law on hydrostatic pressure on the fluid on the immersed body, the law that is named after him.” In the course of lectures on the Theory of machines and Mechanisms [12] it is mentioned that Archimedes invented the screw, improved the cog-wheel, built water-lifting structures and a number of military machines.

The kinematic approach to the solution of problems used by Archimedes [13] became the prototype of the principle of possible displacements in theoretical mechanics. Later it was put by Galileo Galilei as follows: “the ratio of the distances covered by the bodies within the same periods of time is inverse to the ratio of their weights”.

6. A COURSE ON MATHEMATICS

In math textbooks [4] Archimedes is often mentioned in connection with a number of methods he developed for calculating the volume and area of figures:

– exhaustion method used to calculate figures’ area (Fig. 7), volume of solid bodies and length of curves. (Though his invention is ascribed to Eudoxus this method is described in some of Archimedes’ works);
– method of mechanical analogies described in the Letter to Eratosthenes (Ephod) and using mechanical analogies to calculate the volume of the sphere; the same method of mechanical analogy is used by Archimedes in his work On the Quadrature of the Parabola:
– method of integral sums described in his works On the Sphere and the Cylinder, On Spirals, On Conoids and Spheroids. This method was used to calculate the volume of the ellipsoid.

Archimedes’ spiral is mentioned in connection with the polar frame of reference [13]. Archimedes did not only described this curve but also calculated the area of its coil and learned how to plot a tangent to it.

In the course of Theoretical Mechanics Archimedes’ methods are used to determine the center of gravity of a geometrical figure and of a body [12]. This method is mentioned in the section on balancing rotors and static balancing of lever mechanisms [11].

In the courses of History of Science and Technology studied in many universities Archimedes is shown as a new type of a scientist who was not just involved with the "high theory" but also widely used it in the practical work of a military engineer [15]. Archimedes' works are translated into Russia and have been published repeatedly, and his scientific and engineering activity has been studied by Russian historians [16].

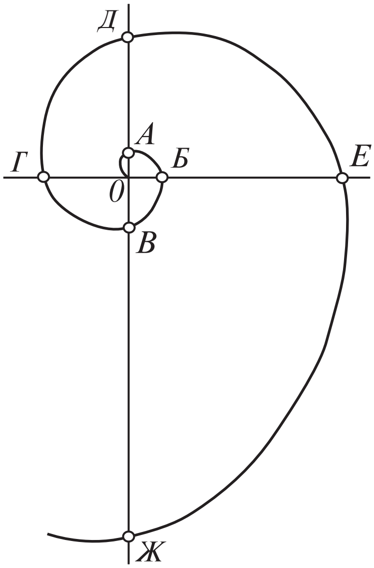

Fig. 7. Picture from textbooks on Mathematics (exhaustion method).

7. CONCLUSION

As is shown in the article, Russian school and university students get thoroughly acquainted with actually all Archimedes' discoveries and inventions in the course of their study. In the animated film "Kolya, Olya, and Archimedes" Archimedes says, "The descendants' memory is a great award for a scholar". This world conference is another proof of this idea.

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ARCHIMEDES IN SECONDARY SCHOOLS: A TEACHING PROPOSAL FOR THE MATH CURRICULUM

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ABSTRACT The aim is to propose, at various levels in secondary schools, Archimedes' idea for calculating π using the computer as programming tool. In this way, it will be possible to remember the work of one of the greatest geniuses in history and, at the same time, carry out an interdisciplinary project, particularly relevant to the current debate on the Math curriculum.

1. INTRODUCTION AND PRELIMINARY

In the current teaching practice in Italian secondary education, as is apparent from available textbooks, there is little evidence in the math curriculum, bar a few exceptions, of reference to the genius of Archimedes (Syracuse 287–212BC). This is a serious gap not merely on account of the fact that he was one of the greatest mathematicians of all time, but also, and above all, because such omission deprives students of an opportunity to investigate vital areas of debate in an interdisciplinary context. One argument linked to the name of Archimedes, of great interest to mathematicians ever since, is the calculation of π [Beckmann, 1971], in other words the constant ratio between the length of the circumference and its diameter. Archimedes' contribution [Frajese, 1974] to this constitutes a milestone that lends itself for its simplicity to explanation and experimentation at various levels within secondary schools. For its infinite mathematical depth, Archimedes' idea has a didactic relevance that goes beyond its practical utility; indeed, it can be proposed as a problem and approached from various angles with diverse instruments, also including the formulation of an algorithm to be tested in a programming environment tailored to the capabilities of the students. Thus a link can be formed not only between the different branches of Mathematics such as algebra, geometry, trigonometry and calculus, but also with information science, in particular with

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the computer as programming tool. In his book "*Regarding the Measurement of the Circle*" [Frajese, 1974] Archimedes establishes that the ratio between the surface of a circle and the square of its radius is equal to the ratio between its circumference and diameter; then, considering the polygons inscribed and circumscribed with 6, 12, 24, 48, and 96 sides he calculates the successive approximations of π that lead him to the following values:

$$3+\frac{10}{71}<\pi<3+\frac{1}{7} \qquad \text{o} \qquad \frac{223}{71}<\pi<\frac{22}{7}$$

In other terms, he obtains: $3.1408<\pi<3.1429$. This is an extraordinary achievement because at the time there were no algebraic notations available; moreover, Archimedes did not use our positional system to elaborate his calculations nor any calculating instrument. The geometrical method used by Archimedes involves pure abstract calculations (not measurements!). He considers a circle of radius 1 circumscribed and inscribed with polygons of 3×2^n sides. Let us indicate a_n the semi-perimeter of the circumscribed polygon and b_n that of the inscribed polygon. Geometrically, it can be demonstrated [Delahaye, 1997] that for n=1 (hexagon)

$$a_1 = 2\sqrt{3} \qquad b_1 = 3$$

and for every n:

$$\frac{1}{a_n}+\frac{1}{b_n}=\frac{2}{a_{n+1}} \qquad b_n \times a_{n+1} = \left(b_{n+1}\right)^2$$

Thus:

$$\begin{cases} a_{n+1}=\dfrac{2a_nb_n}{a_n+b_n} \\ b_{n+1}=\sqrt{b_na_{n+1}} \end{cases} \qquad n=1,2,\ldots\ldots \tag{1.1}$$

By utilizing this recurring formula, it is possible to approximate π with the desired precision, provided that we are capable of calculating the square roots (that which, at that time, was realizable only by 'trial and error'). Archimedes' calculations stop at the values a_5 and b_5.

On the basis of these considerations, the rest of the work is organized as follows: in the second section an exemplification is proposed for lower secondary school (age of student 10–13) and in sections 4 and 5 for higher secondary school - for the first two year and second three year period, respectively. Finally, the article is concluded with a number of didactic reflections.

2. EXEMPLIFICATION FOR LOWER SENCODARY SCHOOLS

After the pupils have understood (for instance through basic experiments with simple materials) that the ratio between the length of the circumference and its diameter is constant, that is, indicated with c the length of the circumference and with r its radius, the ratio does not vary with the

$$\frac{c}{2r} \tag{2.1}$$

varying circumference. There remains the problem of numeric calculus of such constant. The said constant value has been traditionally indicated with the Greek letter π (pi), so we have

$$\frac{c}{2r} = \pi \tag{2.2}$$

Archimedes' idea, as we have mentioned, consists of inscribing and circumscribing regular polygons to the circumference and of approximating the length of the circumference with the perimeters of these polygons. Bearing in mind the actual level of learning, as a first step we propose to circumscribe a square to a circumference. This can be easily done using the programming environment MatCos [Costabile and Serpe, 2003 and 2009; The MatCos software can be in demand to the first author], Fig. 1. Indeed, the following programming reaches the goal:

Code MC1: square circumscribed to a circumference

```
axe=straightlineNum; A=Point_su(axe); B=Point_su(axe);
r=straightline(A,B); cancel(axe); PenStyle(5);
m=distance(A,B);p=Perpendicular(r,A);
p1=Perpendicular(r,B);
PenColour(128,0,0); PenThickness(2); PenStyle(1);
l=segment(A,B); l1=segment(A,m,p); l2=segment(B,m,p1);
C=l2.extreme(2); D=l1.exstreme(2); l3=segment(D,C);
PenStyle (5); PenThickness(1); cancel(P,P1,r);
s=straightline(A,C); s1=straightline(B,D);
Q=intersection(s,s1);
PenStyle(1); PenColour(0,0,255); PenThickness(2);
g=circ(Q,M/2); f=diameter(g); cancel(s,s1);
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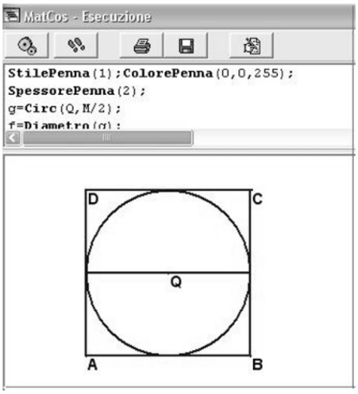

Fig. 1. Output of a square circumscribed to a circumference.

It can be easily ascertained, both with geometrical considerations and with practical verifications using the software, that the side of the square is equal to the diameter. Moreover, observing that the length of the circumference is less than the perimeter of the square we can write the following inequality:

$$c < 8r \quad \text{and thus} \qquad \frac{c}{2r} < \frac{8r}{2r} = 4$$

that is in (2.2) we have the first result:

$$\pi < 4 \tag{2.3}$$

This result can be improved considering the regular hexagon inscribed and circumscribed to the circumference, Fig. 2. This can be done with the following program:

Code MC2: hexagon inscribed and circumscribed to a circumference

```
P=point; r=readnumber("radius"); c=circ(P,r);
A=list; B=list; D=list; A(1)=point_on(c);
   for(i from 2 to 6) execute;
   A(i)=rotation(A(i-1),P,60,anticlockwise);
   end;
PenColour (128,0,64); polygon(A); segment(A(1),P);
segment(A(2),P); PenStyle(5);
r=straightline(a(1),a(2));
s=perpendicular(r,p); h=intersection(r,s);
B(1)=intersection(c,s); t1=tangent(C,B(1));
   for(i from 2 to 6) execute;
   b(i)=rotation(b(i-1),P,60,anticlockwise);
```

```
    t2=tangent(c,b(i)); t3=tangent(c,b(i-1));
    d(i-1)=intersection(t2,t3);
    cancel(t2,t3);
    end;
n=distance(d(1),d(2));
print("the side of the circumscribed hexagon is" , "
",n);
t4=tangent(c,b(6)); d(6)=intersection(t1,t4);
cancel(r,s,t1,t4); PenColour(128,0,64); polygon(d);
```

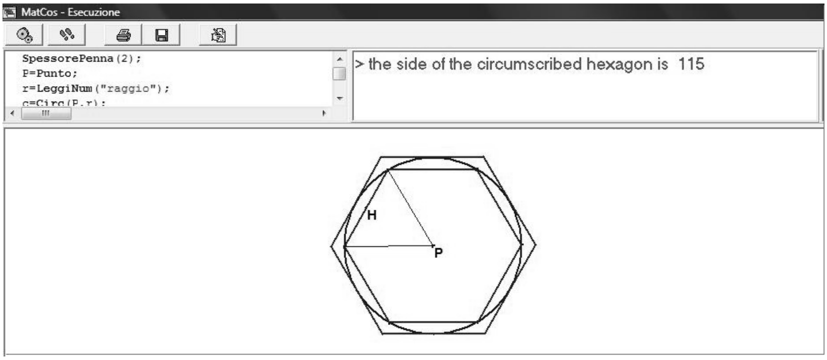

Fig. 2. Output of the hexagon inscribed and circumscribed to a circumference whit radius r=100.

By executing this program many times it becomes apparent that the length of the side of the hexagon inscribed is the same as the radius, thus the perimeter is *6r*, from which the inequality:

$$p < c \;\Rightarrow\; \frac{p}{2r} < \frac{c}{2r} = \pi \;\Rightarrow\; 3 < \pi \tag{2.4}$$

As far as the side of the circumscribed hexagon is concerned, we can obtain the measurement directly with the software. For example, for a circumference with radius *100* we find *l=115* and so the perimeter equals:

$$p = 6\times 115 \tag{2.5}$$

Keeping in mind that in this case the inequality is $c < p$, one easily finds

$$\pi = \frac{6\times 115}{2\times 100} \approx 3,4$$

Thus, the regular hexagon inscribed and circumscribed produces the inequality

$$3 < \pi < 3,4 \tag{2.6}$$

The theoretical justification that the radius of the circumference equals the side of the regular hexagon inscribed can be obtained also for lower secondary schools. The measurement of the side of the circumscribed hexagon is more difficult. A better result can be obtained inscribing and circumscribing a regular dodecagon. The code of the program follows the previous idea, though it should be noted that in this case the angle of rotation should be 30°, Fig.3.

Thus we have the following code:

Code MC3: dodecagon inscribed and circumscribed to a circumference

```
P=point; r=readnumber("radius"); c=circ(P,r);
A=list; B=list; D=list; A(1)=pointatrandom_on(c);
for(i from 2 to 12) execute;
  A(i)=rotation(A(i-1),P,30,anticlockwise);
end;
m=distance(a(2),a(3)); print(m*6/r);
PenColour(128,0,64);polygon(A); PenColour (0,0,255);
segment(A(1),P); segment(A(2),P);
PenStyle(5); r1=straightline(a(1),a(2));
s=perpendicular(r1,p); h=intersection(r1,s);
B(1)=intersection(c,s); t1=tangent(C,B(1));
for(i from 2 to 12) esecute;
  b(i)=wheel(b(i-1),P,30,anticlockwise);
  t2=tangent(c,b(i)); t3=tangent(c,b(i-1));
  d(i-1)=intersection(t2,t3);
  cancel(t2,t3);
end;
m1=distance(d(2),d(3)); print(m1*6/r);
t4=tangent(c,b(12)); d(12)=intersection(t1,t4);
cancel(r,s,t1,t4); PenColour(128,0,64); PenStyle(1);
polygon(d);
```

By analogy with what has been said so far, we find the inequality

$$3,1 < \pi < 3,2 \tag{2.7}$$

which is narrower than (2.6).

Didactic activity in the lower secondary school can thus be concluded stating that π is an irrational number, that is, it has unlimited non periodic decimal representation, whose approximation to the cents is *3,14*.

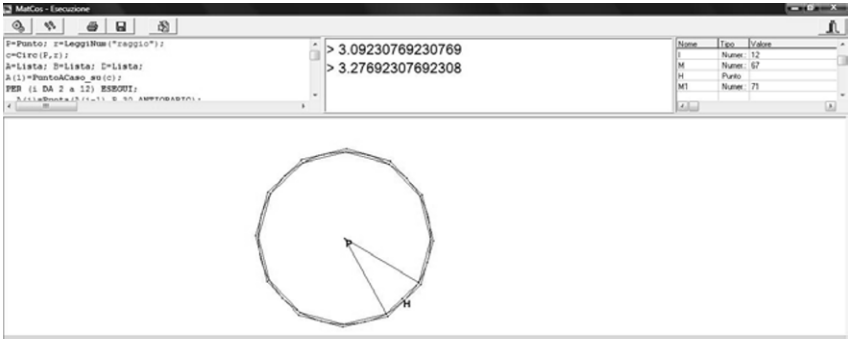

Fig. 3. Output of a dodecagon inscribed and circumscribed to a circumference with radius r=130.

3. EXEMPLIFICATION FOR HIGHER SECONDARY SCHOOL: FIRST TWO YEAR PERIOD

In the first two year period of higher secondary school the topic of calculus of π should be dealt with, and can be thoroughly investigated using Archimedes' idea with the aid of the computer as a programming tool. Indeed, especially in the 'Liceo Scientifico' (Scientific Lyceum), you can first of all demonstrate with the Method of Exhaustion that the ratio

$$\frac{c}{2r} \quad \text{is constant} \tag{3.1}$$

Thus, inscribing and circumscribing to the circumference regular polygons with an increasing number of sides, indicating with p_n e P_n the perimeters of the polygons inscribed and circumscribed respectively, with n sides, we have the inequality

$$\frac{p_n}{2} < \pi < \frac{P_n}{2} \tag{3.2}$$

where, to simplify, the radius of the circumference has been supposed equal to 1.

The inequality (3.2) enables the calculation, at least in theory, of increasingly accurate approximations of π. In order to calculate p_n e P_n you obviously need to know the measure of the side of the regular polygon with n sides inscribed and circumscribed. In broad terms, it is not easy to approach a similar problem in the first two year period of secondary school. Nevertheless, we can still reach our aim with a useful simplification which takes into consideration polygons of 3×2^n $n=1,2,\ldots$ sides, as originally suggested by Archimedes.

In fact, with simple applications of Pythagoras and Euclid's theorems one can demonstrate recurring formulae which are equivalent to (1.1) and easy to program. In more precise terms, with the use of Pythagoras' theorem alone we have:

Theorem 1 [see e.g. Costabile, 1992] - If l_n *(n=3,4,5,...)* indicates the side of the regular polygon of *n* sides inscribed in the circumference with radius 1 (for convenience), then the side l_{2n} of the regular polygon inscribed of *2n* sides is given by:

$$l_{2n} = \sqrt{2 - \sqrt{4 - l_n^2}} \tag{3.3}$$

This recurring formula, keeping in mind that the side of the hexagon inscribed in the circumference is equal to the radius, enables us to calculate the side of the polygon of *12, 24, 48, 96,....* sides. For example we have:

$$l_6 = 1$$

$$l_{12} = \sqrt{2 - \sqrt{3}}$$

$$l_{24} = \sqrt{2 - \sqrt{2 - \sqrt{3}}}$$

....

As regards the side of the regular polygon of *n* circumscribed to the circumference, using Euclid and Pythagoras' theorems we can demonstrate the following:

Theorem 2 [see e.g. Costabile, 1992] - Let L_n be the side of the regular polygon of *n* sides circumscribed to the circumference with radius 1 and l_n the side of the regular polygon with the same number of sides inscribed, then we have the following relation

$$L_n = \frac{2l_n}{\sqrt{4 - l_n^2}} \tag{3.4}$$

Combining (3.3) and (3.4) we derive the inequality

$$\frac{nl_n}{2} < \pi < \frac{nl_n}{\sqrt{4 - l_n^2}} \tag{3.5}$$

which enables us to calculate easily approximations -rounding up and rounding down- of π, starting from *n=6* and doubling the number of sides at each step. The formulae (3.3) and (3.4) give the same results as formula (1.1), that is with the Archimedes method. We can obtain an alternative demonstration of (1.1) with trigonometric identities [see e.g. Weisstein, 2010 – From MathWorld – A Wolfram Web]. On the chain of inequalities (3.5) we can construct an algorithm which calculates π with a previously established precision; we can assume as estimate of the error:

$$\frac{1}{2}(P_n - p_n)$$

that is
$$nl_n\left[\frac{2-\sqrt{4-l_n^2}}{2\sqrt{4-l_n^2}}\right] \tag{3.6}$$

The codification in MatCos of the algorithm in question can be the following:

Code MC4: Archimedes' algorithm

```
n=6; L=1; eps=readnumber("desidered precision");
d=10;
Execute Until (d>eps);
   Pinf=n*L/2;  Psup=n*L/SquareRoot(4-L^2);
   d=(Psup-Pinf);
   n=n*2;
   L= SquareRoot(2-SquareRoot(4-L^2));
   end;
Print("for n= ",n, " the rounding down value is ",pinf);
Print("per n= ",n, " the rounding up value is ",psup);
Print("Pi is comprised between ", Pinf, " and ", Psup);
```

Assuming $\varepsilon = 10^{-3}$ we obtain the following values in output (Fig. 4):

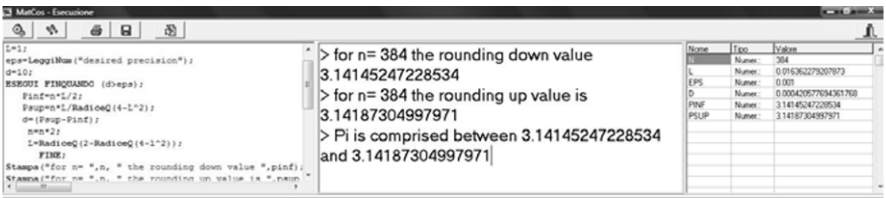

Fig. 4. Output of π with (2.6) and precision ε=10^{-3}.

The precision can be increased up to $\varepsilon = 10^{-13}$, thus the following values in output can be obtained (Fig. 5):

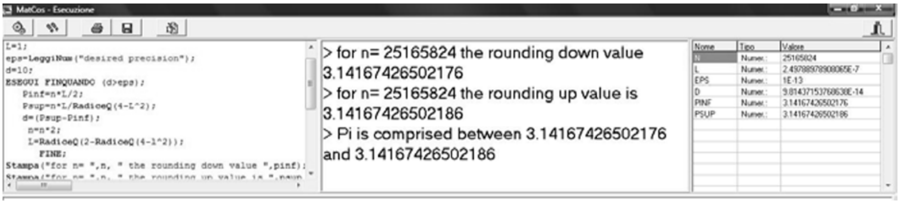

Fig. 5. Output of π with (2.6) and precision ε=10^{-13}.

For $\varepsilon = 10^{-14}$ the result we obtain is wrong, Fig. 6. This is not due to errors in the algorithm or in the MatCos code, but for cause of the error of rounding and his propagation. This gives us the chance to introduce and investigate the relevant topic of the error of rounding which is due to the nature of numbers and the use of Finite Arithmetic. In fact, substituting the formula (3.3) with the other equivalent

$$l_{2n} = \frac{l_n}{\sqrt{2 + \sqrt{4 - l_n^2}}} \tag{3.7}$$

the previous program (Code MC4) produces positive results also for $\varepsilon = 10^{-14}$, that is it gives 15 exact digits.

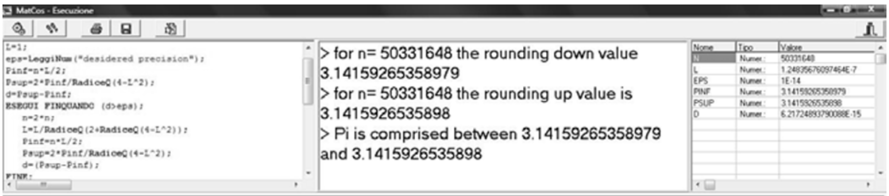

Fig. 6. Output of π with (2.7) and precision ε=10^{-14}.

4. EXEMPLIFICATION FOR HIGHER SECONDARY SCHOOL: THREE YEAR PERIOD

During the three year period it is advisable to go back to the calculus of π after the fundamental notions of trigonometry have been acquired, as well as the Taylor series, if a more in depth study of error is required [see e.g. Costabile, 2003]. Applying the Law of Sines and well-known trigonometric identities, the side of the regular polygon inscribed in the circumference with radius 1 is:

$$l_n = 2\sin\frac{\pi}{n}$$

and thus: $$\frac{p_n}{2} = \frac{nl_n}{2} = n\sin\frac{\pi}{n} \tag{4.1}$$

Taking into account Taylor – McLaurin's development of the function $\sin x$ we have:

$$\frac{p_n}{2} = n\left[\frac{\pi}{n} - \frac{1}{3!}\left(\frac{\pi}{n}\right)^3 + \frac{1}{5!}\left(\frac{\pi}{n}\right)^5 + ...\right] \tag{4.2}$$

from which $$\pi - \frac{p_n}{2} = O\left(n^{-2}\right) \tag{4.3}$$

which provides the asymptotic development of the error. Finally, keeping in mind that for $x > 0$ $\sin x > x - \frac{x^3}{6}$, from (4.1) we can obtain the increase of the error:

$$\pi - \frac{p_n}{2} < \frac{\pi^3}{6n^2} < \frac{\left(\frac{22}{7}\right)^3}{6n^2} \approx \frac{5.17}{n^2} \tag{4.4}$$

Combining (4.1) and (4.4) we can obtain an algorithm with a-priori estimate of the error, which is easy to implement in MatCos since the command $\sin(\alpha)$ is available with α allocated in degrees, Fig. 7. A possible code is the following:

Code MC5: Archimedes' algorithm with a-priori estimate of the error

```
n=readnumber("number of sides of initial polygon");
eps=readnumber("required error");
d=10; p=n*sin(180/n);
   Execute Unitile (d>eps);
   n=n*2;
   p=n*sin(180/n); d=5.17/(n^2);
   end;
print(" the approximated value is ", p);
print(" number of sides is ", n);
print(" effective error is ", pi-p);
print(" estimated error is ", d);
```

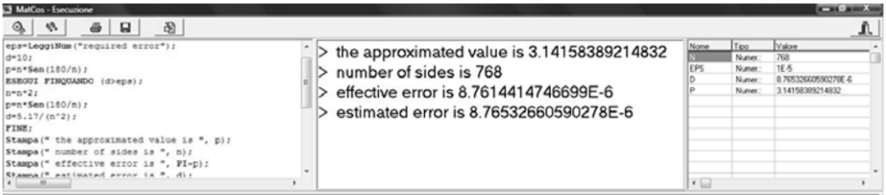

Fig. 7. Output with n=24 assigned and precision $\varepsilon = 10^{-5}$.

5. CONCLUSION

The outlined process appears to reach the aims set out in the introduction. The suggested didactic activity is varied, with ideas ranging across different branches of Mathematics. Furthermore, the use of the computer within a programming environment on the one hand explores an interdisciplinary approach between Mathematics and Information Technology, while on the other it offers a modern approach which connects the past to the present and is in line with recent scientific-technological innovations. Also, the high educational value of programming, albeit at a simple level, in terms of the development of logical-rational skills, creativity, deduction and intuition are well-known. Finally, the themes proposed here represent the opportunity for an historical overview of mathematical thought, and point out how Archimedes can still be a master in the field of Mathematics, and how his genius still inspires modern science.

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MECHANICAL ADVANTAGE: THE ARCHIMEDEAN TRADITION OF ACQUIRING GEOMETRIC INSIGHT FROM MECHANICAL METAPHOR

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ABSTRACT Archimedes' genius was derived in no small part from his ability to effortlessly interpret problems in both geometric and mechanical ways. We explore, in a modern context, the application of mechanical reasoning to geometric problem solving. The general form of this inherently Archimedean approach is described and it's specific use is demonstrated with regard to the problem of finding the geodesics of a surface. Archimedes' approach to thinking about problems may be his greatest contribution, and in that spirit we present some work related to teaching Archimedes' ideas at an elementary level. The aim is to cultivate the same sort of creative problem solving employed by Archimedes, in young students with nascent mechanical reasoning skills.

1. INTRODUCTION

Perhaps the most unique and significant aspect of Archimedes' genius was his ability to interchangeably interpret problems in both geometric and mechanistic contexts. This is most evident in his *Method of Mechanical Theorems* [Heath][Netz], where geometric problems were viewed through the lens of mechanical principles. Specifically, the law of the lever and the concept of center of gravity were used as conceptual tools, in conjunction with the idea of infinitesimals, to evaluate areas and volumes of specific

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geometric forms. While he did not consider this approach mathematically rigorous Archimedes, nevertheless, viewed it as a vital step toward gaining insight into a geometric problem. Further, it provided an intuitive way of acquiring an answer to a problem, which he could later formally prove by another method, such as the method of exhaustion. While this mechanistic reasoning may not have been considered by Archimedes to be sufficiently rigorous, as Alan Hirshfeld suggests in *Eureka Man: The Life and Legacy of Archimedes*, it would "pass muster today as an acceptable form of proof in mathematics" [Hirshfeld].

Archimedes approach of applying mechanical reasoning to mathematical problem solving, pioneered in the *Method*, continues to have relevance over two thousand years after his death. While the science of mechanics has advanced significantly since Archimedes' study of statics, his approach of using mechanical metaphors to address mathematical problems has been highly applicable in the intervening centuries and up to the present day. Its philosophical legacy can be seen in, among other places, the 17$^{\text{th}}$ century mechanical philosophy of René Descartes and Pierre Gassendi [Westfall]. Whereas Archimedes interpreted abstract mathematical problems in terms of mechanical metaphors, the mechanical philosophers interpreted all natural phenomena in terms of hidden mechanisms. That same thematic thread of mechanical interpretation persists today in modern science.

In this paper we will explore, in a modern context, the application of mechanical analogy to geometric problem solving. A concrete problem in the field of differential geometry will be addressed. While the mathematics and mechanics to be presented post-dates Archimedes by over two thousand years, the *Mechanic of Syracuse* is still very present in the method to be pursued. It is in fact this method (and *his Method*), spanning two millennia, that attest to Archimedes' genius more so than any particular problem he solved. Archimedes' approach to thinking about problems may in fact be his greatest contribution.

Regarding Archimedes' approach to solving problems, we will present some work related to teaching Archimedes' ideas at an elementary level. The aim is to imbue an understanding of Archimedes' ideas in a young audience, with the ultimate aim of cultivating the same sort of creative problem solving skills employed by Archimedes. Imparting an Archimedean approach to problem solving is particularly important at an elementary stage since children are already in possession of nascent mechanical reasoning skills (what better example of a lever is there than a playground see-saw?). If honed, these skills can form a template for solving more advanced problems throughout their educational life.

2. MECHANICAL ANALOGS TO GEOMETRIC PROBLEMS

Archimedes' approach of applying mechanical thought devices to geometric problems can be viewed in a more general way than he demonstrated in the *Method*. In this view, a mechanical analog is constructed that can be reasoned about more intuitively than the abstract reasoning required to solve the original problem [Levi]. In constructing this mechanical analog a mechanical surrogate must be chosen that relates to the original problem through a binding principle. The binding principle asserts a set of truths about the behavior of the mechanical surrogate, that connect the surrogate to the original problem. As a consequence we can solve the mechanical analog as way of arriving at the solution to our original problem (see *Fig. 1*). This mechanical reasoning does not need to replace geometric reasoning to be effective but, rather, can complement it and offer new conceptual insights.

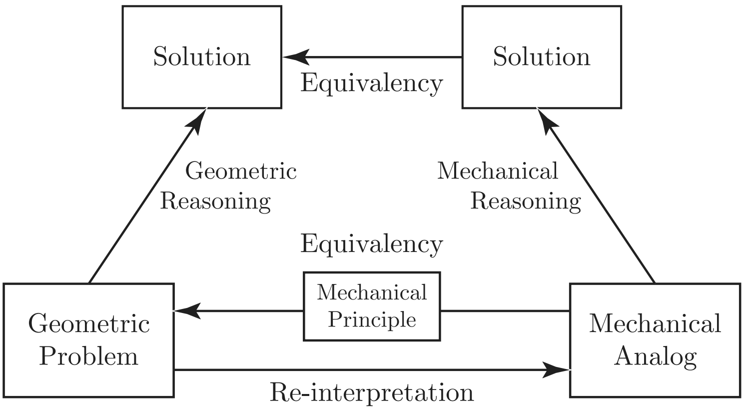

Fig. 1. The original geometric problem is re-interpreted using a mechanical analog. The analog is related to the original problem through a binding mechanical principle. Using mechanical reasoning we can solve the analog to arrive at the solution to our original problem.

2.1. A Case Study: Finding Geodesics

We turn our attention to the problem of computing the shortest line (curve) between two points on a surface. Such a line is referred to as a geodesic. For a given surface there is a family of geodesics that represent the shortest lines between any two points on the surface. This is a problem that can be addressed using the tools of differential geometry and, as we shall see, mechanics. While this problem can be formulated in any dimension we will focus on the case of a surface embedded in $\mathbb{R}^3$.

2.2. An Approach Using Differential Geometry

In *3* dimensions geodesics can be computed by solving a set of second order nonlinear differential equations. If we parameterize our surface as $\boldsymbol{r}(u,v) \in \mathbb{R}^3$, the differential equations are given by [Do Carmo],

$$u'' + \Gamma^1_{11}u'^2 + 2\Gamma^1_{12}u'v' + \Gamma^1_{22}v'^2 = 0 \tag{1}$$

$$v'' + \Gamma^2_{11}u'^2 + 2\Gamma^2_{12}u'v' + \Gamma^2_{22}v'^2 = 0 \tag{2}$$

where the Christoffel symbols, Γ, are found by solving the linear system,

$$\Gamma^1_{11}\boldsymbol{r}_u\boldsymbol{r}_u + \Gamma^2_{11}\boldsymbol{r}_u\boldsymbol{r}_v = \boldsymbol{r}_{uu}\boldsymbol{r}_u \tag{3}$$

$$\Gamma^1_{11}\boldsymbol{r}_u\boldsymbol{r}_v + \Gamma^2_{11}\boldsymbol{r}_v\boldsymbol{r}_v = \boldsymbol{r}_{uu}\boldsymbol{r}_v \tag{4}$$

$$\Gamma^1_{12}\boldsymbol{r}_u\boldsymbol{r}_u + \Gamma^2_{12}\boldsymbol{r}_u\boldsymbol{r}_v = \boldsymbol{r}_{uv}\boldsymbol{r}_u \tag{5}$$

$$\Gamma^1_{12}\boldsymbol{r}_u\boldsymbol{r}_v + \Gamma^2_{12}\boldsymbol{r}_v\boldsymbol{r}_v = \boldsymbol{r}_{uv}\boldsymbol{r}_v \tag{6}$$

$$\Gamma^1_{22}\boldsymbol{r}_u\boldsymbol{r}_u + \Gamma^2_{22}\boldsymbol{r}_u\boldsymbol{r}_v = \boldsymbol{r}_{vv}\boldsymbol{r}_u \tag{7}$$

$$\Gamma^1_{22}\boldsymbol{r}_u\boldsymbol{r}_v + \Gamma^2_{22}\boldsymbol{r}_v\boldsymbol{r}_v = \boldsymbol{r}_{vv}\boldsymbol{r}_v \tag{8}$$

Equations (1) and (2) can be expressed in compact form as,

$$\boldsymbol{u}'' = -\begin{pmatrix} \boldsymbol{u}'^T\boldsymbol{\Gamma}^1\boldsymbol{u}' \\ \boldsymbol{u}'^T\boldsymbol{\Gamma}^2\boldsymbol{u}' \end{pmatrix} \tag{9}$$

2.3. A Mechanical Analog: *Hertz' Principle*

We will know construct a mechanical analog to the problem of finding geodesics on a curved surface. This approach was described in [De Sapio] and is considered here in the context of mechanical re-interpretation. Consider the case of a particle moving in *3* dimensions, $\boldsymbol{r}(\mathrm{t}) \in \mathbb{R}^3$, under holonomic constraints but no applied force. That is,

$$M\ddot{\boldsymbol{r}} = \boldsymbol{\Phi}^T\lambda \qquad \text{subject to,} \quad \phi(\boldsymbol{r}) = 0 \tag{10}$$

The holonomic constraint, $\phi(\boldsymbol{r}) = 0$, restricts motion to a surface $Q^p \subset \mathbb{R}^3$, where $\phi(\boldsymbol{r}) = 0$ is an implicit representation of the surface which is represented parametrically by $\boldsymbol{r}(u,v)$. The gradient of $\phi(\boldsymbol{r})$ yields the constraint Jacobian matrix, $\boldsymbol{\Phi} = \nabla_r\phi = \partial\phi/\partial\boldsymbol{r}$.

To connect this mechanical system to the problem of finding geodesics on a surface we begin by restating a little known basic axiom of classical mechanics, *Hertz' Principle of Least Curvature* [Hertz][Lutzen]. This principle is equivalent to other formulations of classical mechanics (Newtonian, Lagrangian, Hamilitonian, etc.) and can be derived as a special case of *Gauss' Principle of Least Constraint* [De Sapio]. It states that under force-free constrained motion a system will follow the line of least extrinsic curvature, k, on the constrained motion surface, Q^p. Further, this constrained minimization implies $\nabla k^2 \perp T_r(Q^p)$, where $T_r(Q^p)$ denotes the tangent space of Q^p at the point $\boldsymbol{r}$. We note,

$$\nabla k^2 = \frac{\partial}{\partial \boldsymbol{r}''} k^2 = \frac{\partial}{\partial \boldsymbol{r}''} \left\| \boldsymbol{r}'' \right\|^2 = 2\boldsymbol{r}'' = 2\frac{d^2\boldsymbol{r}}{ds^2} \tag{11}$$

So,

$$\frac{d^2\boldsymbol{r}}{ds^2} \perp T_r(Q^p) \tag{12}$$

This implies that the covariant derivative, D/ds, of the tangent vanishes,

$$\frac{D}{ds}\frac{d\boldsymbol{r}}{ds} = \mathrm{proj}_T\left(\frac{d}{ds}\frac{d\boldsymbol{r}}{ds}\right) = \mathrm{proj}_T\left(\frac{d^2\boldsymbol{r}}{ds^2}\right) = \boldsymbol{0} \tag{13}$$

where $\mathrm{proj}_T()$ denotes the projection of a vector onto the tangent space. The intrinsic geodesic curvature,

$$k_g = \left\| \frac{D}{ds}\frac{d\boldsymbol{r}}{ds} \right\| \tag{14}$$

is thus *zero*. This implies that under force-free constrained motion a system will follow geodesics (lines for which $k_g = 0$) on the constrained motion surface. This is illustrated in *Fig. 2*.

Since a geodesic minimizes arc length the condition of *zero* geodesic curvature is equivalent to finding a line that minimizes the *action* defined in terms of arc length. That is,

$$\delta I = \delta \int ds = 0 \tag{15}$$

subject to the constraints. Equivalently, for a system with no external forces this fact can be concluded from Jacobi's form of least action [Goldstein] which states that,

$$\delta I = \delta \int_{t_o}^{t_f} T\, dt = 0 \tag{16}$$

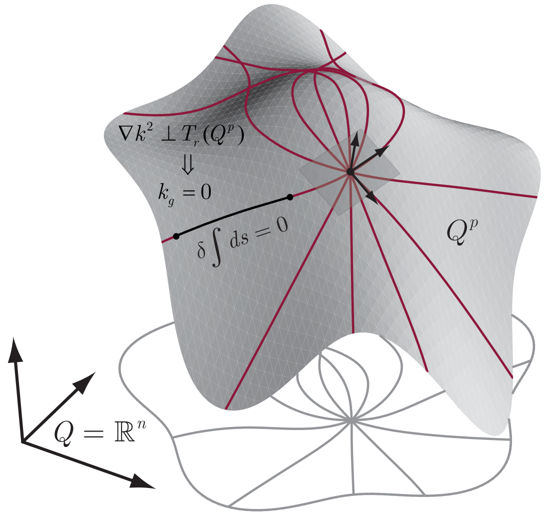

Fig. 2. Force-free geodesics on a constrained motion surface. Force-free motion minimizes the extrinsic curvature, k, subject to the constraints, yielding zero geodesic (intrinsic) curvature, k_g.

subject to the constraints. In this case the Lagrangian has been replaced by the kinetic energy alone (no potential energy). Since,

$$dt = \sqrt{M/(2T)}\, ds \tag{17}$$

we have,

$$\delta I = \delta \int \sqrt{MT/2}\, ds = 0 \tag{18}$$

Because T is constant for this system (18) implies that arc length is minimized on the constrained motion surface.

Returning to our mechanical system, the *Principle of Least Curvature* states that the particle will trace out geodesics on the constrained motion surface, $Q^p \subset \mathbb{R}^3$. Thus, to find the geodesics of a surface we can solve the analogous mechanical system of (10). The *Principle of Least Curvature* acts as the binding mechanical principle used to re-interpret the geometric problem of finding geodesics (*Fig. 3*).

We can recast the differential-algebraic equations of (10) as a set of differential equations [De Sapio]. Using the acceleration form of the constraint equations, $\ddot{\phi} = \mathbf{\Phi}\ddot{\boldsymbol{r}} + \dot{\mathbf{\Phi}}\dot{\boldsymbol{r}} = 0$, the system of (9) can be solved to yield the differential equations,

$$\ddot{\boldsymbol{r}} = -\mathbf{\Phi}^T(\mathbf{\Phi}\mathbf{\Phi}^T)^{-1}\dot{\mathbf{\Phi}}\dot{\boldsymbol{r}} \tag{19}$$

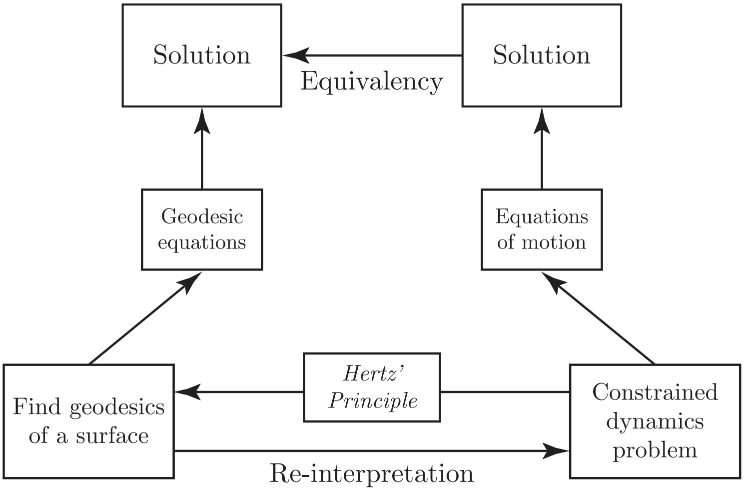

Fig. 3. The original problem of finding geodesics is re-interpreted using the mechanical analog of a particle constrained to move on a surface. Hertz' Principle of Least Curvature acts as the binding principle.

or, incorporating constraint stabilization [De Sapio],

$$\ddot{\boldsymbol{r}} = -\boldsymbol{\Phi}^T(\boldsymbol{\Phi}\boldsymbol{\Phi}^T)^{-1}(\dot{\boldsymbol{\Phi}}\dot{\boldsymbol{r}} + \beta\boldsymbol{\Phi}\dot{\boldsymbol{r}} + \alpha\boldsymbol{\phi}) \tag{20}$$

Thus, (20) represents a set of mechanically derived equations that can be solved, in the same manner that the geodesic equations of (9) can be solved, to compute geodesics for a surface. Equation (20) is not limited to $\mathbb{R}^3$ and can be used to compute geodesics in $\mathbb{R}^n$.

An Example

We can apply (20) to the problem of computing geodesics for the surface,

$$\phi(x, y, z) = 4 + x^2 + y^2 - (z - \frac{1}{3}\cos 3x \cos 3y)^2 = 0 \tag{21}$$

The constraint Jacobian is computed directly from ϕ as,

$$\boldsymbol{\Phi} = \begin{pmatrix} \partial\phi/\partial x & \partial\phi/\partial y & \partial\phi/\partial z \end{pmatrix} \tag{22}$$

and (20) yields a system of *3* second order nonlinear differential equations in *x*, *y*, and *z*,

$$\begin{pmatrix} \ddot{x} & \ddot{y} & \ddot{z} \end{pmatrix}^T = \ddot{\boldsymbol{r}} = -\boldsymbol{\Phi}^T(\boldsymbol{\Phi}\boldsymbol{\Phi}^T)^{-1}(\dot{\boldsymbol{\Phi}}\dot{\boldsymbol{r}} + \beta\boldsymbol{\Phi}\dot{\boldsymbol{r}} + \alpha\boldsymbol{\phi}) \tag{23}$$

Specifying the point $(x_o, y_o, z_o) = (.25, -.25, 1.85)$ as one initial condition we can solve (23) using different departure directions, $(\dot{x}_o, \dot{y}_o, \dot{z}_o)$. The resulting geodesics are shown in *Fig. 4*. It is noted that the specific time parameterization used does not affect the shape of the curves, only the speed at which they are traversed. Therefore, only the direction (not the magnitude) of the initial velocity dictates the curve.

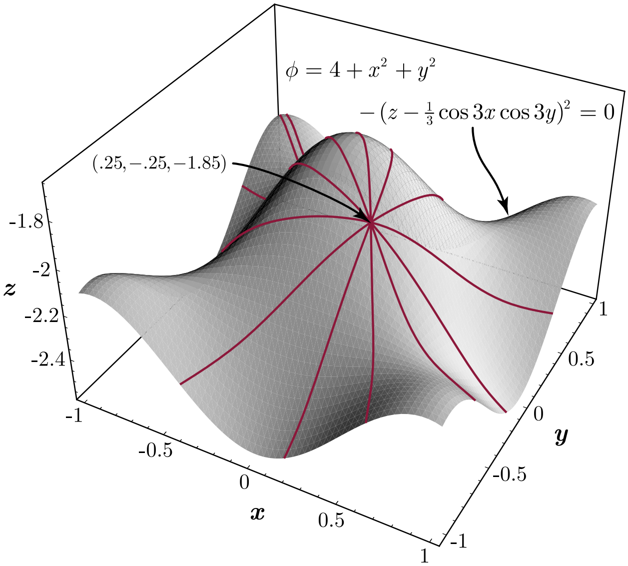

Fig. 4. Solving the system of (20), force-free geodesics were computed for a surface (21). All geodesics were chosen to emanate from a single point.

3. TEACHING ARCHIMEDES AT AN ELEMENTARY LEVEL

Having exercised, at a fairly advanced level, the Archimedean approach of applying mechanical reasoning to geometric problems, it is worthwhile to take a step back and approach Archimedes' ideas at an elementary level. Addressing a young audience affords the opportunity to begin cultivating problem solving through mechanical reasoning. This methodology, when introduced at a young age, can form a framework for effective problem solving in mathematics and science through later more advanced stages of educational development.

3.1. Children and Movement

Children have an intuitive understanding of movement. The toys of today push sand out of sandboxes, lift stuffed animals out of glass cages at fairs, and fancy see-saws grace the playgrounds of schools worldwide. Children are familiar with catapults, slingshots, and the way in which a bow and arrow projects items into the air with force and speed. On a daily basis students witness how a swing set can transfer movement depending on the push it receives from a helping hand. How better to study the basic principles of Archimedes then to begin by teaching children his rudimentary ideas in their mechanical context?

3.2. The Archimedes Study at Sleepy Hollow School

Located in Orinda, California, Sleepy Hollow School, is one of the highest performing elementary schools in the state of California. According to the school's accountability report card in 2007, 96% of the students scored advanced on the mathematics state standardized test (STAR) and 94% of the students scored advanced on the science STAR test. Sleepy Hollow participates in a diversified teaching program where the students' own level is assessed and their needs are addressed weekly in a targeted teaching math group.

In a project open to fourth grade students, teachers encouraged specific students to participate in a hands-on project focussed on learning about Archimedes. Three candidate students who achieved advanced scores on the state math and science tests and demonstrated an eagerness to learn were chosen. The three girls each demonstrated critical thinking skills during interviews. According to the faculty coordinator, "All of these young ladies spoke with confidence, but also with a desire to find out more. Each wanted to know the *whys* and the *hows* behind Archimedes' ideas and inventions".

The three students participated in athletics and possessed an understanding of the basic concepts of movement, force, resistance, and balance. Additionally, each student scored above 85% on a short number sense test, demonstrating that they had sufficient skills and schema necessary to make logical reflections. They also scored at an advanced level on their 2008 third grade mathematics state exam. Each student participates in a weekly extension math group for accelerated students. Moreover, given a chance to challenge themselves each of them eagerly volunteered for the study, were willing to do additional work, and expressed an enthusiasm for collaborating with each other.

Using the mentor text *Archimedes and the Door of Science* by Jeanne Bendick the children first learned who Archimedes was [Bendick]. They were fascinated that Archimedes grew up in a world with no concept of *zero*! His world was one of constant discovery and discussion; so much had not yet been established. The children began to relate to his creativity and desire to discover things. Using Bendick's book the children reviewed some of his most notable discoveries. He pioneered the science of mechanics and hydrostatics, introduced the laws of levers and pulleys, the principal of buoyancy, and the principal of specific gravity. Most notably, they learned that Archimedes gave the world a logical way to think about mathematics. Keeping this in mind the students and faculty coordinator decided to model their investigations after Archimedes' exploratory style. They would read, learn, and then try to logically relate the information to their day-to-day life.

Using Bendick's text as a guide the students reviewed Archimedes' contributions to the study of motion. In a chapter dedicated to the Archimedean screw, the text and diagrams provided ample explanation of how Archimedes moved water from the ground to arid places. In the discussions the students were asked, "Why was this invention necessary?" and "How does this invention demonstrate movement?". The students' answers demonstrated a practical understanding: "This is useful because everyone, whether living on desert lands or on good soil, get to have access to the water".

The investigation of the Archimedean screw related directly to an independent classroom study on the Cahuila tribe of California Indians who used similar innovative techniques to irrigate their land. This provoked thoughtful conversation on the development of technological ideas in geographically and culturally diverse civilizations. One student observed that, "This is like the Cahuila Tribe, they also had to move water to grow food". Another student made an observation relating to the modern world, "It's almost like sprinklers". It was also noted that, "The screw is useful because it gets the right amount of water you want to your land … you can spin it fast to get a lot, or spin it slow for a little".

The students then focused on Archimedes' investigation of the lever. They learned that Archimedes began to experiment with force and machinery, and that he used a pulley to reign in a ship for King Hiero of Syracuse. The concept of a machine as any device that helps one do work more easily was emphasized, as was the notion that a lever is a machine that allows a person to multiply their force. This was intriguing to the students. Bendick's illustrations of first, second, and third class levers helped communicate the function of levers in daily use. One student wrote in her journal that levers are important because "you can shift weight, do work, and understand movement". Each student learned the term "fulcrum" and realized that balance, resistance, force, and work affect the way in which things move.

More significant than appreciating individual inventions the students reflected on Archimedes' ability as a critical thinker and role-model for his peers. One student wrote, "He took an idea and put it into a machine. By understanding how one machine worked he was able to learn about others". Another wrote, "He took one idea and made others. By understanding how motion worked he could make other machines". These observations echo the thoughtful analysis that Archimedes encouraged during his life.

The final part of the Archimedes project involved reasoning about geometry. The students at this level understand the concepts of perimeter and area for simple shapes like rectangles, however, they have not yet learned about the perimeter and area of circles. Using the method of exhaustion, applied at a simple level, they could reason about the area of a circle and approximate it with no a priori knowledge of the formula or of π. They were shown how a circle of radius r fits neatly between an inscribed square and a circumscribed square (see *Fig. 5*).

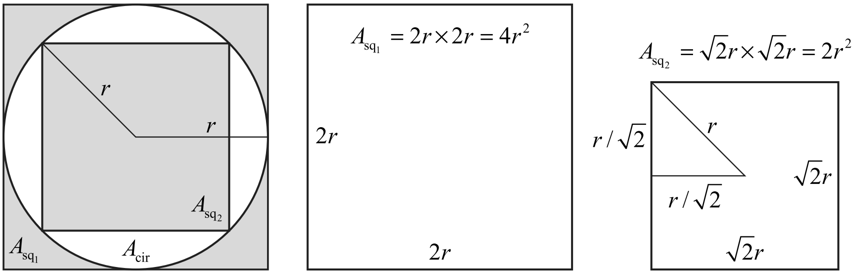

Fig. 5. Approximating the area of a circle by examining an inscribed square and a circumscribed square.

They easily knew how to compute the area of the circumscribed square in this example. Familiar with the Pythagorean theorem they were able, with a little more effort, to compute the area of the inscribed square. They understood that the area of the circle had to be somewhere in between the areas of the two squares, or that, $A_{sq_1} > A_{cir} > A_{sq_2}$. With the area of the small square being $2r^2$ and the area of the large square being $4r^2$, they guessed that the area of the circle was $3r^2$. They were impressed with how close their approximation was when they were told that the actual answer was πr^2, and that π is a special number that is approximately 3.14. With a little help they could see how even better approximations could be made by using polygons with more sides.

The students were then introduced to thinking about geometric problems using mechanical devices. This connected Archimedes' mechanical ideas with his geometric ideas. When considering the volume of a sphere they immediately suggested using the idea of exhaustion, but with inscribed and circumscribed boxes. They were then presented with the problem whose solution Archimedes was very proud of, the sphere inside a cylinder. They were asked, "How much bigger is the cylinder than the sphere?".

To answer this question they were instructed to think about Archimedes' lever. They knew that another name for it was a balance and that it could compare the weight of two objects; "if one object was twice as heavy as another it would balance at half the distance to the fulcrum as the other object". Using a balance with adjustable lever arms they experimented with wooden blocks of different shapes and sizes and adjusted the lever arms to get them to balance. They know that objects with shorter lever arms were "heavier by that same amount". By making carefully measured clay models of a sphere and cylinder and by adjusting the lever arms they guessed that a cylinder is 1½ times as big as the sphere that sits inside it (see *Fig. 6*). They were reminded that the materials that the objects were made of needed to be the same, since a "small object made of something heavy could weigh more than a big object made of something light".

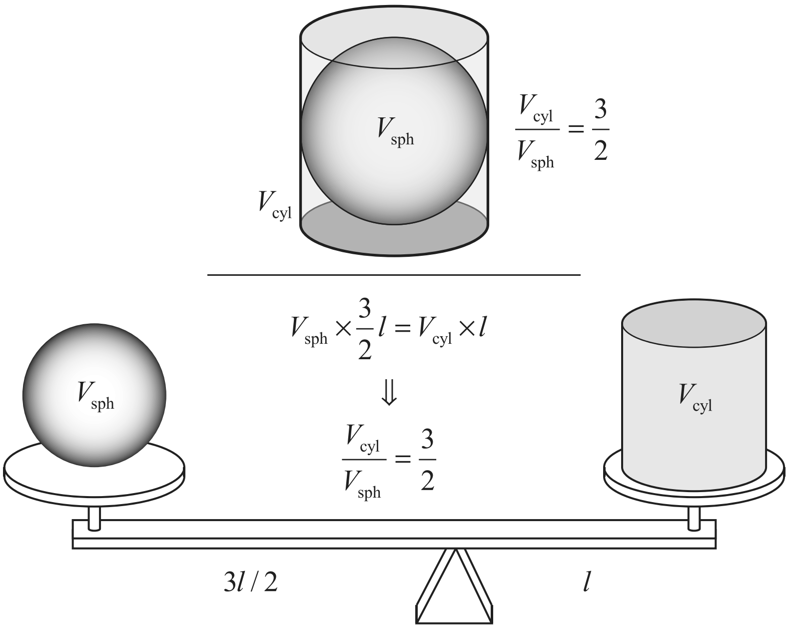

Fig. 6. Comparing the volumes of a cylinder and an inscribed sphere. The concept of the lever provided an intuitive way of reasoning about volumes.

4. CONCLUSION

The Archimedean approach of mechanical reasoning, as an aid to problem solving, has been our central theme in this paper. Archimedes' use of this technique in his *Method of Mechanical Theorems* is one of his most fundamental contributions. It is of vast utility in addressing modern problems over two millennia after Archimedes' time. We have described its general form and demonstrated its specific use with regard to the problem of finding the geodesics of a surface. The *Principle of Least Curvature* was used to re-interpret the geometric problem as a mechanical problem.

Mechanical reasoning in mathematical problem solving, with its ancestry in Archimedes' *Method*, complements mathematical reasoning and offers new intuitions and insights into abstract problems. It is a technique that, as Mark Levi [Levi] observes, "was responsible for some fundamental mathematical discoveries from Archimedes, to Riemann, to Poincare, up to the present day". However, as is also noted this initial intuitive reasoning tends to be forgotten and "students are often unaware of the intuitive foundations of subjects they study".

While this is unfortunate there is constantly the opportunity to introduce an Archimedean approach to problem solving to students at an early age. In this spirit, we have presented work with elementary school students that was aimed at teaching some of Archimedes' fundamental mechanical ideas. It is hoped that with an early introduction to Archimedes' ideas into the educational system, students can embrace and practice an intuitive approach to problem solving that will be preserved well into the future.

ACKNOWLEDGEMENTS

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THE DEATH OF ARCHIMEDES: A REASSESSMENT

(*Marcellus illacrimasse dicitur*: Marcellus is said to have wept)

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ABSTRACT The conquest of Syracuse, which to begin with the Romans expected to lead to a speedy victory, soon turned into a long hard war thanks to Archimede's extraordinary military defence machines. During the war the scientist was killed - probably by mistake - by a Roman soldier who infringed Marcello's orders. The author analyses historical sources and outlines some discrepancies that give a totally different reading of the events, a reading more in agreement with Roman politics.

1. INTRODUCTION

Was it a tragic mistake or a state-sponsored assassination? The flow of historical events seems to lead inevitably towards this fatal outcome: the death of Archimedes and the conquest of Syracuse.

This single tragic event actually heralded what was to be a change in direction in Roman politics, one which involved the legacy of the Greek world of which Archimedes appeared to be a "pivotal point" or a "nerve centre" for this fundamental transition.

How did Rome contend with this legacy? How did it assess the figure of Archimedes who seems to have been the quintessence of Hellenistic civilisation?

With hindsight, a reconstruction of the events seems to generate causes and effects which certainly did not seem to be so carefully thought out and planned when they occurred, even if they responded to an ideology of conquest and power, directed and predetermined.

If a reconstruction of the events is carried out on the basis of all the documents at our disposal, primarily the account of the events given by historians, the truth should emerge. However, there are some crucial points in the narration about which these historians seem to disagree and of which they give different versions, though they appear to agree on everything else.

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And this, we know, is a sign of falsehood, designed to hide feelings of guilt.

1.1. The Bibliographic Sources

But why the falsehood and what guilt?

The accounts of the death of Archimedes given by Greek and Roman historians are those in Latin by Cicero, Livy, Silius Italicus, Valerius Maximus and Pliny the elder and in Greek by Plutarch, while the relative passage, again in Greek, by Polybius is full of gaps.

Cicero tells us in *De finibus* that Archimedes was so passionately dedicated to his studies that, concentrating on tracing signs in the dust, he did not notice that his country had been conquered. Then in *In Verrem* he says that when the consul Marcellus was informed that Archimedes had been killed, he was filled with sadness and arranged for his burial, seeking out his relatives, honouring them and giving them assistance.

Livy too presents him as being intent on tracing geometrical figures in the sand, saying that he was killed by a Roman soldier who did not know who he was. He even mentions how troubled and upset Marcellus was, "*supermoleste tulisset*" (he had borne it with the greatest difficulty) when he learnt what had happened and also how the consul arranged the funeral and helped his relatives (all details that reinforce and add plausibility to the idea that he had meant to allow the scientist to be spared).

Silius Italicus also underlines the fact that Archimedes was killed by a soldier who was unaware of his identity "while he was intent on studying geometrical figures traced in the sand, not at all disturbed by the terrible ruin of the city". Valerius Maximus illustrates his account with important details, pointing out that Marcellus had ordered Archimedes life to be spared, fascinated as he was by his genius, even if he was aware that the victory had been delayed by his "machinations". However, the crazed greed of a soldier who violently broke into the house of the scientist while he was intent on tracing figures on the ground meant that instead of obeying the order to give his name "*quisnam esset interrogabat*", Archimedes expressed his desire to protect his drawing, at which point, "contrary to the orders of the victor" he was killed.

Pliny the elder also notes that when Syracuse was taken, Marcellus "had ordered that only one should be spared", Archimedes, "and that the mean '*imprudentia'* of a Roman soldier meant that the order was given in vain".

Plutarch goes even further in three versions of the death of the scientist. In the first of these, Archimedes seems so immersed in solving a problem that he does not notice the conquest of the city. When the Roman solder appears and demands that he follow him, Archimedes asks him to be "patient" and wait because he does not want to leave what he is studying incomplete without proof: so the soldier, overcome by anger, ran him through. In the second version everything happens more rapidly because the soldier appears with his sword unsheathed and following the same sequence of questions and answers, becomes angry and stabs him. In the third, however, there are a number of soldiers who meet him "while he was carrying in a chest to Marcellus" one of his scientific instruments made up of dials, spheres and quadrants with which the size of the sun could be measured. Believing that he was carrying gold, they killed him for plunder.

Plutarch concludes his versions, however, by saying that "all the historians agree in saying that Marcellus was greatly aggrieved by the death of Archimedes and refused to look upon his killer, as if it were sacrilege; having found his relatives, he honoured them".

1.2. The Absent-minded Scientist

The accounts given so far by Roman sources and supporters of Rome need to be examined together as a whole in order to underline a few essential facts: firstly, that concerning the consul Marcellus who shows integrity and prudence with his order to save Archimedes and his subsequent desire to honour his memory. Another fact concerns the conventional image of the scientist who goes down in history with a very powerful iconography as a person shut up in an inner world which totally cuts him off from reality to the extent that he is so preoccupied that he fails to notice the conquest of the city. The third element highlights the figure of the Roman soldier: the descriptions first of his foolishness and subsequently of his coarseness and brutal desire for conquest become increasingly negative. All these accounts end with the sorrow of Marcellus at the destruction of the Greek metropolis, sorrow made even more intense and grievous by the unexpected killing of the scientist.

Was this suffering real? There are those who, ever since, have rightly questioned the truthfulness of the accounts, above all the order to spare Archimedes, which was so lightly disobeyed by the Roman soldier, however uncouth and carried away by events he may have been. (In one version – that of Tzetzes – it is said that Marcellus immediately grabbed an axe and killed the offender.)

How were the orders given? What description of Archimedes was given to the troops? Surely they should have been able to recognise him even at a some distance, when his appearance alone made people quiver and shake!

What soldier, however stupid and uncouth, would have vented his anger on an old man meditating over drawings and diagrams, without hesitation and contrary to precise orders from the consul? And shouldn't these orders have been to spare precisely an old man meditating over drawings and diagrams? And if he had been captured would Archimedes have been forced to follow the victor's triumphant procession as a magnificent prize from Syracuse and would his genius have been put at the service of the Roman senate and people? Is all this plausible?

We obviously incline towards the idea that it was highly improbable, since we can imagine the peremptory nature of military orders and the fear of disobeying them.

And even if, despite this, the soldier were a complete imbecile, how is it possible that Archimedes was also oblivious to reality?

And what about what was said elsewhere? Has it been forgotten that he and he alone was the heart of the defence, a Briareus with a hundred arms, one capable of the impossible, one who had transformed what should have been a quick victory into a dreadful defeat and exhausting siege? When all is considered with everything that came into play, the violence, cunning and betrayal, wasn't Archimedes himself the real objective that had to be struck down?

If there was only one verdict of the Roman senate on Carthage, one without appeal, "*Delenda Carthago*" (Destroy Carthage), what must have been decided for Syracuse, an enemy for too short a time to be defined an arch-enemy, but whose unexpected resistance and above all its unexpected capacity for resistance demonstrated by Archimedes was in reality the only real obstacle to Rome's political plans in Sicily, solidly based on the conquest of territory and therefore of land to be exploited for the good of the senate and the Roman people according to Roman law? And let us remember that with the fall of Syracuse, once Archimedes was dead, Sicily was to become the first province of Rome.

We have seen much of an anecdotal nature wrapped around little nuggets of truth. The various layers of the original accounts overlap and are channelled into the conventions of the narrative style of contemporary writers or writers that came immediately after them, and so the account is added to and embroidered with those details that were introduced and codified in the form of a plausible and decorous narration.

Livy, Valerius Maximus, Silius Italicus and Plutarch all underline the same characteristic in Archimedes which divorces him from reality. Biographies will also have been written of Archimedes at the time and we know from Eutocius of Ascalon, who wrote a commentary on Archimede's *Quadrature of the Circle*, that his contemporary Heraclides did write his biography.

In reality what is noticed is the traditional and cultural attitude of the Romans to the killing of Archimedes, for which Rome was responsible, however it happened. The obfuscation of the facts and their “correction” is only found on some points that thus become a clear gauge of the mechanism.

What is the justification for that stereotype pushed to the point that it becomes a bizarre caricature, that of the scientist totally divorced from reality, while at the same time he is seen as being “*concretely*” and “*fully*” involved in the defence of the city, and therefore appears as a person to be feared and certainly not harmless? How then could the story of Archimedes' death be told, if not by taking a step back from it and inventing a vile person on whom to lay the blame and on whom the loathing and regret of right-minded people might be poured?

One has the impression that the pretence of respect for Archimedes' greatness arises more from his fame, already established and unexpectedly experienced by the Romans themselves, than from any appreciation of the scientific contribution of this Syracusan.

In this perspective, it can also be understood why virtually no mention is made of his most sensational feat, that of burning the ships with mirrors, because it was indisputable proof that science could actually have practical applications.

1.3. A State Murder

What we have here is a grievous crime which pro-Roman sources recount in a misleading manner to hide the real facts with the aim of justifying the crime or embroidering on it to cover up the true, decidedly political, responsibilities. They invented and exploited the convenient image of a genius divorced from reality, immersed in the lofty thoughts of an investigator and thinker. This perspective pushes into the background his almost legendary role as the heart of the defence of Syracuse with his fearful inventions against which no effective counter-action could be found. It was inevitable that if victory were to be won, such an enemy had to be eliminated.

Then as now, “state-sponsored assassination” went hand-in-hand with a state funeral: great emphasis is given to the description of Marcellus' grief along with the decision, reported by the sources, to fully honour his death in an official way. This detail undoubtedly reflects the reality of what happened, a hypocritical touch to the carefully contrived falsehood of a tragic death at the hand of a brutal soldier.

The state-sponsored assassination was cleverly camouflaged as an accident and the way it happened was reported with shrewd realistic details supported by the account of the only thing which we believe really occurred, the state funeral. This was reported truthfully, as it really was grandiose and monumental, but it is emphasised in such a way as to cover up the tragic truth which they wanted to hide.

We are therefore looking at a “version” of the death of Archimedes, knowingly given the official seal where political strategy is concerned but unknowingly accepted by the scientific world. For the latter, however, it creates and in fact establishes for the future expectations of what science should be: pure speculation and theory, divorced from any practical application.

1.4. An Alternative Story

In order to gain a better grasp of the facts and to reinforce and give credit to what is not just a suspicion aroused by the lack of perfect agreement between the accounts compared, it is worth examining a source which presents a version that is again different and which we might define as sympathetic to Carthage.

This source is Cassius Dio, who provides the basis for Zonaras’ and Tzetzes’ extracts. Cassius Dio, like Diodorus Siculus, according to Tzetzes, told the story of Archimedes’ death. Now Dio’s source was Coelius Antipater who lived close to the time in which the events he narrated occurred and he in turn based his account on those of writers who supported the Carthaginians, writers such as Silenus of Calacte, the author of a history of Hannibal or other biographers of the time such as Sosilus who was the Carthaginian general’s tutor.

In this version, which we know about thanks to the summaries of Tzetzes and Zonaras, we are given a few details, which initially hardly seem important, but stand out against what is basically the same background. They concern the attitude of Archimedes at the moment in which the city is conquered and therefore at the moment of his death. In these desperate moments, surprised by a soldier in his house, Archimedes is said to have exclaimed in anger, “My head, but not my drawing!” and the advancing enemy soldier then told him to move away from the drawing, after which he was killed.

The account in Tzetzes follows the same format up to the point where the soldier tells him to move away from the drawing, but then adds that having realised that this was a Roman, Archimedes started to shout, “Somebody give me one of my machines”, clearly to defend himself by striking the enemy who had dared to confront him. Then he was killed.

Tzetzes adds that Marcellus’ wept and perhaps killed the assassin himself with an axe and then arranged for an illustrious funeral to be held for the scientist, who was buried among the tombs of his countrymen with full funeral rites.

This offers us a different vision of Archimedes, this time angry, still immersed in defence projects, and ready to take material action against the enemy.

It is in this version that we see a plausible account of the man who was solely responsible for the strenuous defence of the city, whose action, effective to the end, and hardly absent minded, could only have been reported by “non aligned” sources, who had acquired their material from elsewhere.

2. CONCLUSION

A reanalysis of the literary sources forces one to reflect deeply on the truthfulness of the traditional version of the events concerning the death of Archimedes. That which appears to have been a state-sponsored assassination resulted not only in the death of an indomitable enemy of Rome and the end of the power of Syracuse itself, it also represented a serious blow to the progress of science which, with the work of the great Syracusan, had reached one of the highest peaks in its development.

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