Chapter 27 Exploring Coupled Housing and Land Market Interactions Through an Economic Agent-Based Model (CHALMS)

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 Abstract Land markets are characterized by spatially distributed exchanges of heterogeneous goods and decision-making by heterogeneous, adaptive participants. Land market dynamics influence and are influenced by spatially varying demands for residential housing through housing markets. This chapter describes a spatially disaggregated, economic agent-based model for exploring ex-urban growth patterns emerging from coupled interactions between housing and land markets (CHALMS). CHALMS simulates the conversion of farmland to housing development over time, through the actions of the agents in the land and housing markets. Three types of agents—consumers, farmers and a developer—make decisions based on microeconomic principles, and use stylized expectation formation models to adapt to dynamic market conditions. The location, price, and density of housing are represented explicitly, as are the location, price, and productivity of individual farms. The possibility of many possible system states, due to agent and landscape heterogeneity, stochastic processes, and path-dependence, requires multiple model runs, as does the expression of the spatial distribution of housing types, overall housing density, and land prices over time in terms of the most likely, or 'average', patterns. CHALMS captures stylized facts of diminishing population density and land prices at greater distances from the center city, increasing land prices over time, and dispersed leapfrog patterns of development evident in most suburban areas of the U.S.

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27.1 Introduction

 As urban sprawl and other undesirable development patterns become more prevalent, policy-makers and researchers alike are coming to grips with the complexity of forces that generate such patterns (Atkinson and Oleson [1996](#page-23-0); Brown et al. 2005; Brown and Robinson [2006](#page-23-0); Magliocca et al. [2011](#page-24-0)). Land-use patterns on the urban fringe emerge from many individual landowners' conversion decisions in response to changing economic opportunities and landscape features. With such complexity, land-use change simulation models have become valuable tools to understand processes of land conversion and development, and for analyzing the effects of land use policies. Various modeling methodologies have been applied in a wide range of disciplines, such as urban planning, landscape ecology, geography, and economics, to build the current understanding of land-use change (Irwin 2010; Parker et al. 2003; Veldkamp and Verburg [2004](#page-25-0)).

 However, most models represent either spatially detailed development patterns or individual-level decision-making—rarely integrating both elements explicitly. For example, models that provide spatially explicit representations of land-use patterns may lack an equally rigorous representation of agent decision-making processes (Fernandez et al. 2005; Parker and Filatova 2008; Parker et al. 2012). Models that take into account microeconomic agent decision-making, on the other hand, may fail to capture the full heterogeneity of those agents and inadequately describe spa-tial characteristics of model outcomes (Irwin [2010](#page-24-0)).

 This chapter describes the Coupled Housing and Land Markets model—CHALMS. It is an economic agent-based model (ABM) of housing and land markets that captures the conversion of farmland to residential housing of varying densities over time in a hypothetical, growing ex-urban area. The primary goal of this study is to develop some general theoretical insights into the individual-level processes that drive regional development patterns. CHALMS is unique among ABMs of land-use for its integration of: (1) microeconomic decision-making rules for consumer, farmer, and developer agents in a spatially explicit framework; (2) representation of heterogeneous agent characteristics and spatial goods (e.g. land productivity and housing sizes and densities); and (3) direct linkages between adaptive price expectations and demand and supply decisions of developer and farmer agents through housing and land markets. The model demonstrates how patterns in development density and land prices predicted by traditional urban economic theory can be reproduced in an ABM framework. In addition, it shows how disconnected, leapfrog development patterns emerge from the simulation of individual price expectations and market transactions.

 Section [27.2](#page-2-0) reviews the capabilities and limitations of current land-use modeling approaches, and describes how integrating the insights from recent economic and non-economic ABMs can provide a more complete representation of the processes driving urban growth. Section [27.3](#page-4-0) details the structure of CHALMS, agent representations, and market interactions. Section [27.4](#page-12-0) presents baseline results and outcomes of preliminary sensitivity analysis. Finally, Sect. [27.5](#page-18-0) concludes with a discussion of model capabilities and limitations and directions for future research.

 27.2 Some Previous Land-Use Modeling Approaches

27.2.1 Spatial Equilibrium Economic Models

 Economic models of urban land use are typically built on the assumption of spatial equilibrium. These models assume that over the long run housing rents will reach equilibrium and offset differences in spatially heterogeneous attributes such as transportation costs to the central business district (CBD), neighborhood amenities, and access to employment. Early models in the urban economics literature used a monocentric city framework in which location is defined purely by distance to a CBD where all jobs are located (Alonso [1964](#page-23-0); Muth 1969; Mills [1972](#page-24-0)). Decreasing housing rent and density gradients are a feature of these monocentric models—i.e., rents and housing density fall as distance to the CBD increases. The basic monocentric framework has been expanded to incorporate growth and uncertainty, include environmental and open space amenities, evaluate zoning and other regulations, and study a variety of other issues (Capozza and Helsley 1990; Mills [2005](#page-24-0); Wheaton 1974; Wu and Plantinga 2003). In recent years, economists have relaxed the mono-centricity assumption (e.g. Epple and Sieg [1999](#page-24-0); Walsh 2007).

 Although spatial equilibrium models have many desirable features—a rigorous representation of agent behavior and capitalization of spatial differences in amenities and other factors into land values (Irwin 2010)—several strong assumptions are made to ensure analytical tractability. First, spatial equilibrium is a particularly restrictive assumption, because out-of-equilibrium dynamics, such as path dependence of development location, are important drivers of urban systems (Arthur [2006](#page-25-0); Brown et al. 2005; Irwin 2010; Tesfatsion 2006). Second, in order to ensure analytical tractability, agent heterogeneity is typically quite limited.¹ More detailed discussions of the limitations of these assumptions are available elsewhere for a wide range of applications (Arthur et al. [1997](#page-23-0); Arthur [2006](#page-23-0); Axtell [2005](#page-23-0); Kirman 1992; Filatova et al. 2009; Irwin [2010](#page-24-0); Parker and Filatova 2008; Tesfatsion and Judd 2006). Since the intent here is to investigate the spatial and temporal dynamics of housing density patterns, a framework that can account for both agent preferences for spatially heterogeneous goods and idiosyncratic differences in decisionmaking processes is necessary. Path dependence of land-use patterns can then be explicitly linked to individual-level motivations of land conversion decisions.

27.2.2 Agent-Based Models

 Agent-based modeling (ABM) has emerged as an alternative method for modeling urban growth and land use change (see Crooks and Heppenstall [2012](#page-24-0) for an

¹ Some models include more heterogeneity than others. See, for example, Anas and Arnott (1991) and Epple and Sieg (1999) for models with heterogeneous consumers.

overview). Parker et al. (2003) provide a detailed review of the different types and applications of ABMs for modeling land use change. Although ABMs differ widely in their focus, assumptions, and formalizations of agent interactions (e.g. Benenson and Torrens 2004; Ettema [2010](#page-24-0); Filatova et al. 2007, 2009; Ligtenberg et al. 2004; McNamara and Werner [2008](#page-24-0); Otter et al. [2001](#page-24-0); Parker and Filatova 2008; Robinson and Brown [2009](#page-25-0)) they all rely on interactions between many distributed agents to form emergent larger-scale patterns (Manson [2001](#page-24-0)). Thus, microeconomic fundamentals can be incorporated into individual agents' decision-making rules to simulate emergent trends in a spatially explicit framework.

 However, examples of incorporating microeconomic decision-making rules into ABMs are few. Filatova et al. (2009) and earlier papers (Filatova et al. 2007; Parker and Filatova 2008) present the fullest, economically-based implementation of an agent-based land market to date. The authors relax the conventional spatial equilibrium assumption by explicitly modeling decentralized, bilateral transactions between land buyers and sellers. Transaction prices for land are determined by specifying a buyer's and seller's willingness to pay and willingness to accept, respectively, which are then adjusted to form bid and asking prices accounting for different market power scenarios (Filatova et al. 2009; Parker and Filatova 2008). The authors have provided valuable insights into methods for relaxing spatial equilibrium assumptions and incorporating microeconomic decision-making into the ABM framework. However, their model lacks a housing market and cannot capture the feedbacks between land and housing markets that influence spatial rent structures.

Ettema (2010) presents an economic ABM of a housing market, which explicitly simulates relocation and price setting processes. Housing prices are produced through bilateral transactions between a buyer and seller, and are constrained by the agents' perceptions of market conditions and by the buyer's budget constraint and housing preferences. The buyer's opportunity costs are explicitly considered by comparing utility derived from housing dwellings available in the current period to the maximum expected utility of potential housing in the future. Expectation formation, executed using Bayesian updating, is a key advance from this model design. However, the expectation formation process only accounts for price changes driven by changing consumer preferences attributed to life cycle effects. For the purposes of simulating spatially explicit development patterns—which the author acknowledges is beyond the scope of his current model—the model's design cannot accommodate spatial characteristics of housing goods or the formation of spatially heterogeneous price expectations.

Robinson and Brown (2009) present a detailed spatial representation of regional development patterns in a GIS-based ABM named dynamic ecological exurban development (DEED). Land and housing markets are integrated by the conversion of farm parcels to residential subdivisions of different densities by developers, and the acquisition of deeds to subdivision lots by residential household agents. In addition, township agents are able to specify zoning and land acquisition polices to alter development patterns. However, land conversion events are not based on microeconomic decisionmaking. Farm and residential parcel sales probabilistically occur on the basis of land or lot characteristics. No markets are represented in which competing land uses can be

valued, and the economic constraints or opportunity costs of the acting agents are not considered. Although the authors make a valuable contribution towards empirically grounding ABMs, this approach makes it difficult to gain general insights into the underlying economic forces that drive land conversion decisions.

 CHALMS builds upon the above ABMs by integrating many of their innovations into one framework capable of simulating development density patterns through coupled housing and land markets. Similar to Robinson and Brown (2009), housing and land markets are linked through the supply and demand functions of the developer and consumer households, respectively; however, our agents respond directly to and create market prices subject to economic constraints. Mechanisms of land and housing transactions in CHALMS are built upon the bilateral transaction framework developed by Parker and Filatova (2008), but are expanded to link the developer's rent expectations in the housing market to his bid prices in the land market. Price expectations play a similar role in CHALMS as they do in Ettema's model (2010). Adaptive expectations of future prices and market conditions are used to compare the utility of present and potential future transactions—directly influencing the timing of transactions. In addition, our agents' price expectation models are designed to capture spatially dependent price trends that directly affect the location of housing and land sales. These advances allow us to investigate both the supply- and demand-side forces driving spatial patterns of land conversion and development density over time.

27.3 Model Description

27.3.1 Model Structure

 A growing exurban area is represented in which land is converted from farming to residential housing of varying densities over time. Farmland differs randomly in its productive capacity across farms, and farmers differ in how they form expectations about future prices of their land. Farmers compare the returns from farming to expected profit from selling their land to a single representative developer and make the decision each period whether to continue farming or enter the land market. Inequality between farmers' total supply and the developer's demand for land establishes the bargaining power of farmers, which influences land transaction prices.

The developer determines the profitability of different types of housing that vary by both structure and lot size. He sells a housing good (i.e. a combination of a given house and lot size) to consumers who prefer to be close to the urban area to minimize transport costs, and are differentiated by both income and preferences over different housing types. CHALMS tracks development over time incorporating elements of path dependence and stochastic uncertainty that determine spatial development. A schematic of agent decision-making and market interactions, along with the sequence of events, is shown in Fig. [27.1](#page-5-0) . Price prediction models for farmers and the developer are used to form expectations of future land and housing prices, respectively, and are described in detail in the Appendix.

 Fig. 27.1 Conceptual map of agent and market interactions in CHALMS. The *numbers* indicate the (counter-clockwise) sequence of events within one simulated time period (t). Agents (*italics)* are labeled with the underlying conceptual model that governs their behavior. Inter-temporal processes $(t+1)$ shown include updating developer's rent prediction models, updating the farmers' land price prediction models, and exogenous growth of the consumer population (Taken from Magliocca et al. 2011)

27.3.2 Formation of Agent Price Expectations

27.3.2.1 Consumer Utility, Willingness to Pay (WTP), and Willingness to Bid (WTB)

 A consumer *c* calculates standard Cobb-Douglas utility derived from the consumption of a general consumption good and a housing good. Each housing good can be considered a 'bundle' of 1 of 18 different housing types, which are distinguished by different combinations of three different house sizes (h) —1,500, 2,000, and 2,500 square feet—and six different lot sizes (l) — $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 5, and 10 acre; these lot and house sizes are meant to represent a typical ex-urban area. Consumer *c*'s utility function is assumed to have a Cobb-Douglas form:

$$
U(c,n) = \left(I_c - P_{ask|n} - \psi_n\right)^{\alpha_c} h_n^{\beta_c} l_n^{\gamma_c}
$$
 (27.1)

where I_c is income, ψ_n is the travel cost from the location of house *n* to the CBD, and β_c and γ_c are the consumer's idiosyncratic preferences for house and lot sizes, respectively. $P_{ask/n}$ is the developer's asking price for house *n*, which is determined by Eqs. [27.15](#page-11-0) or [27.16](#page-11-0) below, depending on whether the house is being re-sold or is newly constructed, respectively (see Sect. [27.3.4.1](#page-10-0)).

 The WTP of consumer *c* for any given house *n* is then equal to the portion of the consumer's income that he/see is willing to pay for housing as given by the Cobb-Douglas structure:

$$
WTP(c,n) = (I_c - \psi_n)(\beta_c + \gamma_c)
$$
 (27.2)

 Although this functional form for the utility function implies that consumers would pay the same amount for all housing net of transportation costs, consumers identify the housing option with the greatest utility and adjust their bids on other

houses relative to this most preferred option. First, the maximum utility possible across all houses, *U** , is found. Holding *U** constant for all housing options, the rent, R^* , that would produce the same utility to the consumer as the most preferred choice (i.e. an optimal rent such that the consumer would be indifferent among housing options) is calculated for each housing option.

$$
R^*(c,n) = I_c - \Psi_n - \left(\frac{U^*}{h_n^{\beta_c} l_n^{\gamma_c}}\right)^{\frac{1}{\alpha_c}}
$$
 (27.3)

Second, the difference between the rent being asked by the developer, P_{genb} , and the optimal rent, R^* , is used to form a willingness to bid (WTB) from WTP for each house.

$$
WTB(c,n) = WTP(c,n) - \left(P_{ask|n} - R^*(c,n)\right) \tag{27.4}
$$

 Consumers therefore bid more or less than the constant share of income for housing depending on their income and idiosyncratic preferences for house and lot size, and on the seller's asking prices for the houses actually available at a point in time. It is important to note that the full heterogeneity of consumer preferences is captured, and bids reflect the relative utility of each housing option offered.

27.3.2.2 Developer's Rent and Return Projections and Willingness to Pay (WTP) for Land

 The developer is assumed to use housing information, such as incomes and utilities of residents and records of past housing prices, to form rent expectations, which in reality would be available from a 'real estate agent' or similar source. Housing information is recorded in discrete 'zones' of five by five blocks of cells, which segment the entire simulated landscape. This information includes the average expected rent, lot size, house size, number of bidders before sale, percent that sale price was above/below the original asking price, the number of houses of each type in the zone, and residents' income and utility levels for all houses in each zone. For any given house, the developer uses financial prediction models (see [Appendix](#page-19-0), Eqs. A.1–A.6) to form a rent expectation (R_{env}) for that house in $t+1$ given past price information from the neighboring zones. Based on rent expectations for existing housing, the developer makes spatially explicit rent projections for all housing types for all undeveloped cells.

 Rent projections are made by one of the three different methods described below. Projected rents are a combination of weighted local and regional (city-wide) rent information. For a given housing type to be built in a given location, a similar housing type within a local geographic area provides rent information from which a direct extrapolation can be made based on distance and local price trends. However when a similar housing type does not occur locally, the developer must rely on rent prediction methods that draw from similar housing types in a larger geographic

region. In this 'regional case', rent predictions are less direct than in the 'local case'. Thus, the appropriate rent projection method is adopted based on the amount of rent information available in a given area. For each undeveloped cell, a rent for each housing type is projected taking into account the distance of the given cell from the CBD and associated travel costs.

 For a given undeveloped cell, the distance to every other grid cell is calculated and mapped. The specified parameter, n_{close} , sets the number of closest cells to be considered as a local search area for rent information. Using n_{close} developed cells, a distance-from-the-CBD-weighted average rent is calculated for each housing type present. This subset of local houses, n_{close} , is the basis of rent projections so that high demand in particular areas (e.g. due to desirable housing types and/or a relative shortage of housing in close proximity to the CBD) can be capitalized into rents that may exceed what is predicted based on only the travel cost gradient. Depending on whether the housing type for which a rent projection is being made is present in n_{class} search cells, one of the following methods for projecting rent is used:

 1. If the housing type for which a projection is made *is present* in the *n* closest cell:

$$
R_{proj}^{loc}(i,lt) = R_{lt}^{loc} - mcD^{loc}(i,lt)
$$
 (27.5)

where R_h^{loc} is the local distance-weighted average rent for housing of type *lt* within he closest developed cells, *mc* is the travel cost per cell (converted from \$/ mile), and $D^{loc}(i, l_t)$ is the distance from the cell *i* to the closest developed cell of the same housing type *lt* .

$$
R_{proj}^{reg}(i,lt) = R_{lt}^{reg} - mc\left(D_i - D_{lt}^{reg}\right)
$$
\n(27.6)

where R_t^{reg} is the regional average rent for housing type ltt , D_i is the distance from the CBD of cell *i*, and D_l^{reg} is the average distance from the CBD of all housing of type *lt* in the region. The resulting rent projection is given by:

$$
R_{proj}(i,lt) = w_{loc} R_{proj}^{loc}(i,lt) + w_{reg} R_{proj}^{reg}(i,lt)
$$
\n(27.7)

where w_{loc} and w_{rec} are local and regional weights of 0.3 and 0.7, respectively.

- 2. If the lot type for which a projection is being made *is not present* in the *n* closest cells, but exists somewhere in the city, the rent projection is solely based on regional rental information and is given by Eq. 27.6 for $R_{\text{proj}}^{\text{reg}}(i, lt)$.
- 3. If the lot type for which a projection is being made *is not present* in the *n* closest cells, and it *does not exist* anywhere else in the city, then rent projections are made based on average utilities:

$$
R_{proj}^{loc}(i,lt) = I_n^{loc} - \psi_i - \left(\frac{U_n^{loc}}{h^{\beta_n} l^{\gamma_n}}\right)^{\frac{1}{\alpha_n}}
$$
(27.8)

Mean (Std. dev) farm size, in acres	128 (70.67)	
Mean (Std. dev) agricultural return, in \$/acre	\$2,486 (\$249)	
Building cost per square foot	$$85 - 165	
Infrastructure costs per housing unit ^a		
One acre lots or smaller	\$6,000-\$17,000	
2 acre lots	\$11,000-\$20,000	
$5+$ acre lots	\$13,000-\$25,000	
Share of income on housing expenditure, $\beta + \gamma$		
Low income	$.35 - .42$	
Middle income	$.27 - .34$	
High income	$.18 - .26$	
Proportion of housing expenditure on land, $\gamma/(B+\gamma)$	$.10 - .90$	
Transportation costs (costs/mile)		
Time ^b	\$1.30	
Out of pocket (BTS 2007)	\$0.54	
Exogenous rate of population growth	10%	
\mathbf{B} and \mathbf{B} and \mathbf{A} and \mathbf{B} and \mathbf{B} and \mathbf{B} and \mathbf{B} and \mathbf{B}		

 Table 27.1 Selection of model parameters

distance)

^aBased on Frank (1989) and Fodor (1997)
^bWe assumed time costs to be a function of ^bWe assumed time costs to be a function of average road speed (30 mph), average number of workers per house (2), average wage per person (\$30/h), value of time as a percent of wage (50%), and the road network indirectness coefficient (0.3) (this is the ratio of network distance to the Euclidean

where I_n^{loc} is the average income (available from zonal housing information, see above in Sect. [27.3.2.2](#page-6-0)) households located in the *n* closest cells, and U_n^{loc} is the average utility of households located in the *n* closest cells.

$$
R_{proj}^{reg}(i,lt) = I^{reg} - \psi_i - \left(\frac{U^{reg}}{h^{\beta}l^{\gamma}}\right)^{\frac{1}{\alpha}}
$$
(27.9)

where I^{reg} and U^{reg} are the average household income and utility, respectively, over the entire region. The rent projection for housing type *lt* in cell *i* is then given by Eq. [27.7](#page-7-0) .

 Based on projected rents, potential returns are calculated for every housing type in every undeveloped cell by subtracting the costs of construction and infrastructure (Table 27.1), which vary by housing type, and the price of land for the given cell. The maximum return for each cell is calculated as the housing type with the maximum return over all possible housing types (subject to zoning constraints) for the given cell. Maximum returns are then projected onto the gridded landscape to be used by the developer to determine the type and location of housing construction that maximizes profit across all vacant holdings.

 Given the rent projections for every undeveloped cell, the rent associated with the housing type that produces the maximum return in each cell i of farm F is specified as R_{marti} . The developer's WTP for a given farm *F* is the average R_{marti} over the extent of the farm:

$$
WTP(F,t) = \frac{\sum_{j=F_i} R_{\text{max}|j}}{A_F} \tag{27.10}
$$

where A_r is the total acreage of farm *F*.

27.3.2.3 Formation of Farmer's Willingness to Accept (WTA)

 Farmer expectations of land prices are formed using a randomly allocated set of 20 prediction models. Each prediction model uses one of six different methods for forming predictions based on up to 10 years of past land prices from which to extrapolate the next period's price expectation (Eqs. [A.1](#page-20-0) [– A.6](#page-20-0) in Appendix). A farmer's decision to sell to a developer or continue farming is based on the expected return from selling his farm relative to the value of the farm's agricultural return per acre in perpetuity, V_{gas} . The projected land price for cell *i* on farm *F*, $P_{\text{f,proj}}$, which consists of spatially discounted (Eq. [A.10](#page-22-0) in Appendix) and predicted (Eqs. [A.1 – A.6](#page-20-0) in Appendix) price components, is compared to the farmer's baseline WTA.

$$
WTA(F_i, t) = \max \left\{ P_{Lproj|F_i}, V_{agr|F_i} \right\} \tag{27.11}
$$

 The farmer's WTA is dynamically set to the greater of the two values. This enables the farmer to capture speculative gains from sale of his/her land when development pressure is high, while enforcing a rational threshold below which the farmer would be better-off farming.

27.3.3 Land Market Interactions

27.3.3.1 Bargaining Power

 If the developer's WTP for a given farm is greater than the farmer's WTA for his land, then the two enter into bilateral negotiation to determine the final transaction price of each parcel. Bargaining power in the land market, ε , is adapted from Parker and Filatova (2008) and captures differences in the developer's demand for and the farmers' supply of land at the initial WTP of the developer.

$$
\varepsilon = \frac{\left(d_{Land} - A_{F^*}\right)}{\left(d_{Land} + A_{F^*}\right)}\tag{27.12}
$$

where d_{Land} is the acreage demanded by the developer and A_{F*} is the acreage supplied by participating farmers. *F** is the subset of all farmers for which the condition

 $WTP > WTA$ is true. If the developer demands more land than farmers supply, ε is positive and farmers bid above their WTA (see Sect. [27.3.2.3 \)](#page-9-0). If farmers supply more land than is demanded by the developer, ε is negative and the developer will bid below his initial WTP (see Sect. [27.3.2.1 \)](#page-5-0). Bargaining power is dynamic because the amount of land supplied by farmers depends on the initial WTP of the developer. Also, the developer's WTP for a given farm depends on the level of rents in the housing market. Thus, housing and land markets are explicitly linked.

27.3.3.2 Formation of Farmer's Asking Price (P_{askII})

 After bargaining power is observed (Sect. [27.3.3.1](#page-9-0)), farmers participating in the market $(F^*$, i.e. WTP>WTA for their farm) form an asking price in response to market conditions to maximize their gains from trade (Parker and Filatova 2008).

$$
P_{ask|L}(F_i^*, t) = \max \left\{ WTA(F_i^*, t)^* (1 + \varepsilon), V_{ags|F_i^*} \right\}
$$
 (27.13)

The asking price of the market-participating farmer, F_i^* , is equal to or greater than the value of his land in agriculture. If the developer demands more land than farmers supply, each farmer will mark up his asking price to potentially maximize gains from trade.

27.3.3.3 Formation of the Developer's Bid Price (P_{bidII})

 After bargaining power is observed (Sect. [27.3.3.1](#page-9-0)), the developer forms a bid price for each farm for which the condition, WTP > WTA, is true.

$$
P_{bid|L}(F_i^*, t) = \min \{WTP(F_i^*, t)^*(1+\varepsilon), WTP(F_i^*, t)\}
$$
 (27.14)

The developer's bid price for the farm of a market-participating farmer (F_i^*) is equal to or less than his initial WTP for the farm. If farmers supply more land than the developer demands, the developer will mark down his bid price for each farm to maximize both gains from trade and profit from sales of houses in that location.

27.3.4 Housing Market Interactions

27.3.4.1 Formation of Asking Prices for Houses (P_{ackHH})

 Houses enter the housing market as either new construction or as pre-existing, recently vacated houses. For existing housing, the asking price equals the developer's expected rent, which is formed using the price expectation models described in Sect. [27.3.2.2](#page-6-0) and the Appendix (Eqs. [A.1](#page-20-0)–A.6). For newly constructed houses, the asking price equals the developer's projected rent subject to varying levels of rent information, as described in Sect. [27.3.2.2](#page-6-0) and specified by Eqs. [27.5](#page-7-0)–27.9.

 27.3.4.2 Housing Market Competition

The set of houses on which consumer c bids, H_j are identified by the criteria:

$$
\left\{ H_j \in H_n : WTB(c, j) \ge P_{ask|j} \Omega_{lt} \right\} \tag{27.15}
$$

 Consumer *c* will bid on houses for which his *WTB* is greater than or equal to the developer's asking prices, P_{set} , multiplied by the bid level, Ω_{μ} for housing type *lt*. The bid level is the running average percentage that sale prices have been above/ below the original asking prices for houses of type *lt* in the past.

The housing market competition factor, *HMC*, describes the competition for housing that each consumer faces in the housing market. It is calculated by comparing the number of houses consumer *c* will bid on to the number of other consumers bidding on the same houses:

$$
HMC_c = \frac{(NC - NH)}{(NC + NH)}
$$
\n(27.16)

where *NH* is the number of houses in H_j and *NC* is the number of other consumers bidding on H_j .

27.3.4.3 Formation of Consumer Bidding for Housing

 After *HMC* is observed (Eq. 27.16), consumer *c* sets his bid price for each particular house *j* in the set H_j in relation to his optimal rent for that house, $R^*(c, j)$, in response to market conditions:

$$
P_{bid}(c,j) = R^*(c,j) + HMC_c \left[WTP(c) - P_{ask}(j) \right]
$$
 (27.17)

If HMC_c is positive, competition for housing for consumer c is high and his bids will be set above his optimal rents. If HMC_c is negative, competition for housing for consumer c is low and his bids will be set below the asking prices. If HMC_c is zero, the number of consumers bidding on consumer *c* 's set of houses is the same as the number of houses *c* is bidding on, and his bids will equal his optimal rents. The adjustment of the consumers' bid prices in response to market conditions allows consumers to try to simultaneously maximize their gains from trade and the likelihood that they will be the highest bidder.

27.3.4.4 Rules for Matching Consumers with Houses

 After the bidding process is completed, the highest bidder on each house is identified. Consumers possessing at least one 'winning bid' are put into a subset of 'winning bidders'. For each consumer in the set of winning bidders, the set of houses for which the consumer owns the highest bid is identified. The consumer's utility is recalculated (using Eq. [27.1](#page-5-0)) for each of these houses using his winning bid instead of the initial asking price. Given these new levels of utility, the consumer is matched with the house for which he is the highest bidder and derives the highest utility. Once a consumer is matched with a house, both the consumer and house are removed from the market. The matching process is repeated with the remaining bids (which are kept constant) until all consumers are matched, all houses are occupied, or all positive bids are exhausted. This process ensures consumers are matched to houses that generate their maximum possible utility levels given competitive bids from other consumers and discrete housing options provided by the developer.

27.4 Model Experiments

CHALMS was run on an 80×80 gridded landscape with each cell representing an acre for a total region of 6,400 acres, or 10 square miles. The CBD was set in the middle of the top row at coordinates (1,40) with an established ex-urban developed area shown as the dark blue half-moon at the top of Fig. 27.2 . Although CHALMS was able to replicate 18 different housing types, where type is defined by lot and housing size, initial development only consisted of randomly placed housing types

Fig. 27.2 Initial landscape configuration. Each polygon represents the location of one of 50 farms. The semi-circle (top center) represents the initial 'city' location

			Mean			
Housing	Lot size	Housing type	number		Mean annual	
type	(acres)	description	of lots	Std. dev.	rents $(2007 \text{ } $)$	Std. dev.
1	$\frac{1}{4}$ ac lots	Small house	87	58	7,737.35	797.10
$\overline{2}$		Medium house	51	41	12,156.92	883.71
3		Large house	104	77	14,502.24	788.30
$\overline{4}$	$\frac{1}{2}$ ac lots	Small house	144	110	9,252.51	1,426.78
5		Medium house	173	124	12,382.07	1,430.72
6		Large house	155	76	15,946.08	981.74
7	1 ac lots	Small house	429	185	12,218.53	689.40
8		Medium house	231	110	14,786.07	605.53
9		Large house	141	76	18,559.56	856.78
10	2 ac lots	Small house	475	88	19,653.39	629.50
11		Medium house	358	77	21,342.20	653.40
12		Large house	183	40	24,739.68	716.58
13	5 ac lots	Small house	Ω	Ω		
14		Medium house	Ω	Ω		
15		Large house	Ω	Ω		
16	10 ac lots	Small house	30	32	30,461.25	4,374.33
17		Medium house	12	26	32,581.86	3,424.89
18		Large house	1	3	33,047.47	2,959.10

Table 27.2 Number of lots by type of house/lot combination, at $t = 20$

1 through 12 (see Table 27.2 for a description of the housing types). Fifty farms surrounded the initial development and are shown as different colored patches in Fig. 27.2.² Initially, 334 consumers participated in the housing market, and an exogenous growth rate of 10% a year was assumed. Incomes of incoming households are assumed to vary from \$20,000 for the lowest quintile to \$200,000 for the highest quintile.³ Travel costs for households were assumed to depend both on time and monetary costs (Table [27.1 \)](#page-8-0). As new households moved to the region, they demanded housing; a single developer for the region responded by buying land from farmers and building houses. Thus, farmland was gradually converted to developed uses over time.

CHALMS was run 30 times⁴ and each run tracks growth over a 20-year period. Farmers' locations and agricultural returns were held constant across all runs, as were the distribution and location of housing types in the initial city. Draws from income and consumer preference distributions and the initial assignment of all prediction models (i.e. for farmers' price predictions and distance discounting, and

² Colors are used in Fig. [27.2](#page-12-0) to delineate the farms but have no other meaning.

³ These data were based on median household incomes for suburban counties in the Mid-Atlantic region (Delaware, Maryland, Pennsylvania, and Virginia) from the 2000 Census. In general, the model is meant to represent a hypothetical community on the urban fringe in one of these states; we parameterize the model using data from this region.

⁴ Thirty runs were determined to be a sufficient sample size as given by $n = z_\alpha^2 \left(\frac{\sigma^2}{\delta^2} \right)$ for estimates of mean rents and number of lots (Table 27.2) at the 95% confidence level of mean rents and number of lots (Table 27.2) at the 95% confidence level.

Fig. 27.3 'Average' development pattern maps for time steps (a) 5 , (b) 10, (c) 15, and (d) 20. Housing types are color-coded from 1 (*dark*) to 18 (*light*)

developer's price predictions) were allowed to vary randomly across each of the 30 runs. Holding landscape features constant across runs eliminates sources of geographic variability, while exploring the effects of path-dependence and stochastic processes on development patterns that result from agent heterogeneity.

 Stochastic elements in CHALMS (i.e. random draws from consumer income and preference distributions and assignment of prediction models) limit the insight of any single model realization. Instead, maps of the most likely, or 'average', development patterns were constructed (Fig. 27.3a–d). For each time step displayed, the development pattern consists only of cells that were developed above a threshold frequency, which was calibrated to produce an 'average' development pattern that closely approximated the calculated average percent-developed area and dispersion across 30 runs. Within each of those cells, the housing type with the highest probability of occurrence is mapped. In addition, Fig. [27.4](#page-15-0) shows the probability of development at any density occurring across 30 runs.

27.4.1 Results

 Table [27.2](#page-13-0) provides a description of housing and lot sizes associated with each housing type, and summary statistics of final outcomes across 30 model runs. Even though the initial landscape configuration was held constant across runs, the housing

Fig. 27.4 Probability of final development patterns of any density occurring at $t = 20$ across 30 runs. Land that was always developed is color-coded as *white* , while land that had a low probability of developing is color-coded as *black*

types built across runs showed a good deal of variation. This variation reflected the importance of heterogeneity in consumer demand. The most frequently developed housing types were those with small or medium sized houses on 1- and 2-acre lots, which were affordable for most consumers. No 5-acre lots were built over the entire period, but 10-acre lots show up. The absence of 5-acre lots was due to the combined effects of high construction costs relative to expected rents, and the wealthiest consumers demanding houses on 10-acre lots.

 The results exhibit a general development pattern that is consistent with urban economic theory: as shown in Fig. [27.5](#page-16-0) , housing density tends to decrease, and average lot sizes increase, as distance from the CBD increases (Mills 1972; Brueckner and Fansler [1983](#page-23-0)). Also consistent with urban economic theory, land prices tend to decrease with distance from the CBD (Fig. [27.6](#page-16-0)) and increase over time as population grows and demand for land increases (Fig. [27.7 \)](#page-17-0). The results also show a pattern that is typical of urban "sprawl" (Fig. [27.8](#page-17-0)): a divergent relationship over time between the number of lots and acreage developed (Heimlich and Anderson [2001](#page-24-0)).

In the first five time steps, development density and location were primarily driven by consumer demand and relative farm productivity. From the initial housing stock, consumers generally derived higher utility from 1- to 2-acre lots than from other lot sizes. This resulted in strong competition for those housing types and a subsequent bidding up of their rents. Relatively high rent levels prompted the developer to purchase land and capitalize on the strong demand for 1- and 2-acre lots.

Fig. 27.5 Mean density by zone after 20 time steps. Zones form concentric circles at equal intervals away from the CBD. Rounded interval values are shown in miles

 Fig. 27.6 Average price of farmland sold for development over 30 runs, at any time step, as a function of distance from the CBD

The development pressure filtered through to the land market, where farmers adjusted their WTA levels upward attempting to capture gains from sale above their return from agriculture. This price signal was strongest close to the initial development, resulting in high land prices that decreased with distance. Thus, the first farms sold were those with relatively low asking prices, distant from initial development (weak price signal) and comparatively low productivity (low initial asking price). As a result, early development progressed in a 'leapfrog' pattern (Fig. [27.3a, b](#page-14-0)) with

 Fig. 27.7 Average price of farmland sold for development over 30 runs in each time period

 Fig. 27.8 Comparison of number of lots versus acreage developed over time

farms far from initial development sold first. Furthermore, because of strong demand and high returns net of land prices and construction costs, 1- and 2-acre lots were built on the first farms sold.

 As time progressed, increased land prices coupled with consumer demand prompted the construction of houses on comparatively small (1-acre or less) or large (10- acre) lot sizes. Rents for these relatively scarce housing types rose faster than those of other housing types in the existing housing stock and prompted a shift in construction. Concurrently, development pressure and land scarcity drove land prices upward as population growth spurred competition for housing and farmers

reacted to an upward trend in past land prices. Faced with higher asking prices from farmers and consumer demand for scarce housing options, the developer shifted lot sizes and location. Generally, smaller lots—i.e., higher density housing—occurred on expensive land closer to the initial 'city', while lower density housing was built on remaining land far from the initial 'city'. Spatial and temporal variability around this general pattern were due to heterogeneity in farmers' expectations of selling prices and consumer housing demands and the resulting profitability in each particular location.

 Figure [27.4](#page-15-0) provides a sense of the probability distribution of spatial outcomes across model runs. Comparisons between Figs. [27.3a](#page-14-0) , b and [27.4](#page-15-0) demonstrate that several farms have a greater than 85% probability of being developed early in any given run. Those farms have relatively poor land and sufficiently low expectations of land prices to prompt early development consistently across runs. After time step 10, however, the remaining farms shown as developed in Fig. [27.3c ,](#page-14-0) d generally have much lower probabilities of being developed in any given model run. At this point in the simulations, land prices are determined more by the agents' price expectation models than by differences in agricultural land productivity. Thus, development patterns are less dependent on landscape features and become more directly influenced by stochasticity inherent in agents' price expectation models (Brown et al. [2005](#page-23-0)) . This leads to increased stochasticity in development patterns in the last half of the simulations.

27.5 Discussion and Conclusions

 CHALMS is an ABM of urban growth and land-use that integrates microeconomic fundamentals into a framework capable of capturing full heterogeneity and spatially explicit development patterns. Optimizing behavior of heterogeneous consumers, farmers, and a developer, a spatially differentiated landscape, population growth, and a variety of housing and lot types are included as part of the development process. At the same time, bounded rationality, or the lack of perfect foresight, is assumed on the part of all agents. CHALMS describes the dynamics and spatial outcomes of the development process in a hypothetical ex-urban locale.

 CHALMS as it currently exists has some limitations. The current version is simulated on a simplified landscape that lacks natural features such as water bodies, topography, or soil quality, which would influence a particular location's attractiveness for development and/or suitability for agriculture. In the real world, many of the features that constitute good agricultural land are often favorable for development, which compounds their influence on development patterns. In addition, proximitybased valuation of natural amenities or publicly provided goods by consumers is not represented, which has been shown to significantly influence development patterns (Filatova et al. [2007](#page-24-0); Irwin and Bockstael [2002](#page-24-0); Wu and Plantinga [2003](#page-25-0)). Future model iterations will incorporate more detailed natural landscape features and associated proximity-based valuation to explore their effects on development patterns.

 Another limitation is the representation of only one developer. This was a simplification made to ease interpretation of simulated interactions and outcomes in both the land and housing markets. The introduction of competition between developers may change current development patterns. Although incorporating the above elements into the model's structure would likely improve its realism, such elements would also add further complexity into an already complex model. Moreover, the existing framework allows us to establish baseline development patterns subject to heterogeneous consumer preferences and incomes and farm productivity without the added complexity of a more detailed landscape. Thus, further testing of model sensitivities and outcomes will take priority before additional landscape features are introduced.

 Our results demonstrate qualitative behaviors consistent with urban economic theory that emerge from explicitly coupling housing and land markets in the ABM framework. The interplay between markets and agents' heterogeneous preferences and perceptions reproduces many trends predicted by conventional urban economic models but also shows a dispersed, "leapfrog" development pattern that is common in ex-urban areas. This has three important implications. First, CHALMS demonstrates that housing and land markets influence and are influenced by one another. Thus, simulating feedbacks that emerge between markets is critical for understanding the forces that drive urban growth patterns. Second, our formalization of economic agents shows that microeconomic decision-making can be incorporated into an ABM framework to reproduce regional patterns consistent with those produced by conventional spatial equilibrium approaches. Finally, by simulating urban growth from the 'bottom-up', ABMs allow the researcher to represent full agent and environment heterogeneity and build an individual-level understanding of the dynamics of growing urban systems—a combination of advantages unique to the agent-based modeling approach.

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27.6 Appendix

27.6.1 Prediction Models

27.6.1.1 Financial Prediction Models

 Developers and farmers make pricing decisions informed by expectations of future housing and land prices, respectively. Adapted from price expectation models used in agent-based financial literature (e.g. Arthur 1994, 2006; Axtell 2005), agents try to predict the next period's price based on current and past price information. An agent is given a set of 20 prediction models. Each prediction model may use one of six different prediction methods, and there may be more than one model applying the same prediction method in the agent's set of 20 models. Some of these prediction methods map past and present prices (P) into the next period using various extrapolation methods.

1. *Mean model*: predicts that $P(t+1)$ will be the mean price of the last *x* periods.

$$
P(t+1) = \frac{\sum_{i=t-1} P(t_i)}{x}
$$
 (A.1)

2. *Cycle model*: predicts that $P(t+1)$ will be the same as x periods ago (cycle detector).

$$
P(t+1) = P(t-x) \tag{A.2}
$$

3. *Projection model*: predicts that $P(t+1)$ will be the least-squares, non-linear trend over the last *x* periods.

$$
P(t+1) = aP(t_s)^2 + bP(t_s) + c;
$$
 (A.3)

where *t_s* is the time span of *t*−*x* to *t*, and *a*, *b*, and *c* are coefficients of fit. Other methods translate changes from only the last period's price to next period's price.

4. *Mirror model*: predicts that $P(t+1)$ will be a given fraction ξ of the difference in this period's price, P(t), from last period's price, P(t−1), from the mirror image around half of P(t).

$$
P(t+1) = 0.5P(t) + [0.5P(t) - (1 - \xi)(P(t) - P(t-1))]
$$
(A.4)

5. *Re-scale model*: predicts that $P(t+1)$ will be a given factor ζ of this period's price bounded by [0,2].

$$
P(t+1) = \zeta P(t) \tag{A.5}
$$

6. *Regional model*: predicts that $P(t+1)$ is influenced by regional price information coming from neighboring agents.

 For farmers, land prices are a function of land scarcity as measured by the number of remaining farmers, N_f in the region at time *t*.

$$
P(t+1) = P(t) \left(1 + \frac{1}{N_f} \right) \tag{A.6}
$$

For developers, the expected price of house types with size, *h*, on lot size, *l*, in a given neighborhood, N_b , is the mean of the prices of the houses and lots of the same sizes in adjacent neighborhoods, N_{net} . N_{net} are neighbors in the cardinal directions.

$$
P\left(N_{b|hl}, t+1\right) = mean\left\{P\left(N_{nei|hl}, t\right)\right\} \tag{A.7}
$$

 All models in the agent's set of prediction models are used to predict the price in the next time period $(P(t + 1))$. In time $t + 1$, the actual price is known and an error squared is calculated for each model by squaring the difference between the predicted price and the actual price. The prediction model with the least error is used to make the agent's pricing decisions in the current period. This same process of prediction and evaluation is used every period so that the most successful prediction model is used every time.

27.6.1.2 Developer's 'New Consumers' Prediction Models and Demand for Land

Adapted from Arthur's (1994) "El Farol Problem", the developer attempts to predict the population at time *t* using past population information from the last 10 years. Population information for time *t* is not known until new consumers bid for houses on the housing market (Sect. $A.2$). Just as agents are allocated 20 financial prediction models, developers are allocated 20 population prediction models. However, instead of receiving six different predictions methods, developers receive only the first five prediction methods listed above in Sect. A.1.1. For trends in population from time *t−x* to *t* −1 (where *x* ranges from 2 to 10 years in the past), developers attempt to predict how many new consumers will enter the market in time *t* .

The developer uses this prediction as the number of new consumers in time *t*, which corresponds to the number of new houses that need to be supplied in time *t* for new consumers, N_{max} . In addition, the developer observes the number of consumers who bid on houses but were not the highest bidder on any house in *t*−1 and therefore did not locate in the region, N_{old} . By combining the number of houses needed for new consumers (N_{new}) and consumers from the last period that did not locate (N_{old}) , the number of new houses that need to be constructed in the current period (H_{new}) is calculated.

$$
H_{new}(t) = N_{new}(t) + N_{old}(t-1)
$$
\n(A.8)

Based on the developer's rent projections (Sect. A.1.2), the H_{new} most profitable houses are chosen for construction later in the period. Given this housing set and the associated land required to build each, the developer calculates how much land will be needed in the current period. The developer's demand for land is then the difference between the amount of land needed for new construction and the amount of vacant land already owned by the developer from previous land purchases (if any). For example, if the developer calculates ten new houses are needed in time *t* and the ten most profitable houses require 2 acres each, but the developer already owns 5 acres that are vacant, then the developer's demand for land in the current period will be 15 acres.

 27.6.1.3 Farmer's Spatial Discounting Models

 Land is an immobile good with spatially heterogeneous attributes, thus land prices vary in space and time. Farmers observe the price and location of one or more land transactions through time. A farmer then attempts to discount the observed transaction price(s) based on the distance from his location. The spatially discounted price(s) accounts for spatially variable land values and enables an adjustment of land prices based solely on trends in the market land price.

A coefficient of spatial discounting is predicted using a genetic algorithm that enables the farmer to 'learn' the best coefficient over time. Initially, each farmer is allocated a 'population' of 100 random coefficients bounded by [−200, 200]. After the transaction price(s) is observed, it is discounted using each coefficient in the farmer's 'population' of the coefficients and compared to the farmer's current asking price to evaluate the 'fitness' of each coefficient.

$$
\chi_i(t) = \chi_i(t-1) + \left| \left(\frac{P_{ask|F}(t) - P_{L|F}(t)}{\overline{D}_F} \right) - \beta_i \right|; \tag{A.9}
$$

where the fitness, χ _{*i*}, of coefficient β _{*i*} is the absolute value of the difference between the current asking price of farmer *F*, $P_{ask|F}$ and the average of the transaction price(s), P_{L} , divided by the average distance, \overline{D}_{F} , of the observed transaction price(s) from farmer *F*. 'Fitness' is measured as such so that the 'most fit' coefficient will be the one with the least error. The 'most fit' coefficient is designated as 'active' and is used as β_L in Eq. A.10 to spatially discount observed transaction prices.

 The farmer spatially discounts the observed transaction price(s) by predicting the coefficient of spatial discounting in a linear extrapolation to give the spatially discounted price, $P_{I|F}$, faced by farmer *F*.

$$
P_{L|F}(t) = \beta_L \overline{D}_F + \overline{P}_L(t); \tag{A.10}
$$

The coefficient of spatial discounting, β_L , represents the marginal discount of the observed transaction price(s) per cell away from farmer *F* . The spatially discounted price, P_{LIF} , is then given as an input into the farmer's financial prediction models $(Sect. A.1.1).$

27.6.2 Housing Market Competition Factor

 The housing market competition factor has several characteristics that demand further explanation. *HMC* can change over time based on the income distribution of new consumers and the type and price of new houses that come onto the market. Holding incomes of existing and new consumers constant, if relatively more expensive homes are introduced to the market, the number of consumers that can afford to

bid on the most expensive housing is reduced. This affects the amount of competition faced by consumers of varying income levels. For example, the wealthiest consumers would face reduced competition (i.e. *HMC* is only slightly positive or even negative), because fewer consumers can afford to bid on the most expensive houses. Conversely, lower income consumers would face increased competition for the remaining houses (i.e. *HMC* becomes increasingly more positive), because there are comparatively fewer houses per lower income consumer to bid on.

 This interaction between housing prices and incomes can occur with a change in the distribution of incomes too. Holding housing prices constant, if the income distribution skews towards lower incomes, competition for housing would increase for lower income consumers, but it would not change for higher income consumers. If the income distribution skews towards higher incomes, competition for housing would increase for all consumers. Wealthier consumers would experience comparatively more competition for the most expensive houses. Lower income consumers would also experience increased competition for housing, because higher income consumers that were not the highest bidders on more expensive houses would likely outbid most lower income consumers for the remaining housing.

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