Chapter 21
Applied Pedestrian Modeling

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Abstract With an increasing world population and with more cost effective transportation, mass gatherings become ever more frequent. The total size of such gatherings is often as large as millions of people. Furthermore, everyday life in cities becomes increasingly crowded with people. This development has prompted better solutions to mitigate crowded places and make them safer as well as more efficient in terms of travel time. One way to approach this crowd problem is to use crowd modeling tools to assess and optimize locations where pedestrian crowds move around. Within the last decade, crowd modeling has become a mature science and there now exist well calibrated pedestrian models that can reproduce empirically observed crowd features. In this chapter, we will introduce the field of crowd modeling, explain how crowd models can be calibrated with empirical data, and expand a bit on how navigation works in these models.

21.1 Introduction and Motivation

In the past, pedestrian simulations have mainly been used to *qualitatively* reproduce and understand various aspects of crowds. Nowadays however, neither the computing performance nor the amount and detail of available empirical data restrict us from aiming at reproducing crowd dynamics *quantitatively* as well.

One of the reasons why the microscopic simulation of pedestrians as a field of research has taken off as late as about 1985, and has gained pace only during the last

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decade, is the availability of computing power. The simulation of pedestrians and especially real-world applications related to simulation of pedestrians has been at the edge of available standard computational power and will remain there for some more years to come. Compared to vehicle simulations that are effectively 1 dimensional, crowd simulations are 2D or even 3D, and this increase in dimensionality results in more degrees of freedom, which requires more sophisticated models with higher temporal and spatial resolution. Therefore, computing power (in terms of speed as well as RAM) plays a more decisive role in the simulation of pedestrians.

This has at least three implications: (1) Pedestrian modelers have often implicitly or explicitly restricted their creativity in using mathematical tools to make the model results more realistic with respect to computational costs. (2) While refined models successively replace more coarse-grained models for medium-scale applications, applications such as iterative approaches in large-scale projects only recently became possible to carry out with spatially continuous microscopic models. (3) Using pedestrian modeling for large-scale urban planning and transport projects often turns out to be a highly challenging task, since the right balance has to be found between computational time, model complexity and scale, and accuracy of results.

21.2 Modeling Approaches

There are various different ways to approach pedestrian modeling, and among the first ideas to simulate interacting agents in a swarm-like way, was proposed by Reynolds (1987) with his *Boids* model. Different approaches to pedestrian modeling can be classified in various different ways, for example according to their level of abstraction:

- *Microscopic* models describe each pedestrian as a unique entity with its own properties.
- *Macroscopic* models delineate the average or aggregate pedestrian dynamics by densities, flows, and velocities as functions of space and time.
- *Mesoscopic* (gas-kinetic) models are in between the two previously mentioned levels, taking into account the velocity distribution. Mesoscopic models often include individual entities but model interactions between them with common fields.

Alternatively, models can also be classified by their respective detail of description:

- *Discrete space* models sub-divide the environment into a lattice, and the spatial resolution of the model is limited by the cell size of the lattice.
- *Continuous space* models describe the spatial resolution down to an arbitrary level of detail.

Also time in the model can be either discrete or continuous. The latter can be achieved if there is no fixed time step in the model. If instead time is advanced until the next *event* occurs, then that requires non-trivial calculations.

21.2.1 Agent-Based Models

A class of models which is especially popular in the computer science community is agent-based models (ABMs) (O'Sullivan and Haklay 2000; Musse et al. 1998). These models are characterized by a high level of autonomy of the simulated pedestrians, where each pedestrian is controlled by a set of rules (see Crooks and Heppenstall 2012 for an overview). The advantages with these kinds of models are that the motion can look very realistic and that the agents can be adaptive and possess a high degree of artificial intelligence, with emergent phenomena arising from simulations. This also makes ABMs suitable for crowd animation (Treuille et al. 2007; Popovic et al. 2003).

A disadvantage is that these kinds of models tend to be very complicated, which makes it hard to approach them analytically, and they typically also need a lot of computational effort. However, the separating line between ABMs and other types of microscopic models is not that clear, and in a sense, most models could be referred to or reformulated as ABMs.

21.2.2 Social-Force Model

The social-force model (Helbing and Molnar 1995; Helbing and Johansson 2009) is a microscopic model, which is continuous both in space and time. It is influenced by Newtonian mechanics, generalized to the motion of pedestrians. The forces consist of repulsive forces with respect to other pedestrians and boundaries, friction forces, attractive forces among group members, and driving forces related to desired velocities. A superposition of all these forces gives a resultant force which determines the acceleration of the pedestrians. Finally, by integrating over time, velocities and positions are obtained from the accelerations.

21.2.3 Cellular Automata Models

Another popular approach to pedestrian modeling is based on cellular automata (CA) (Bolay 1998; Blue and Adler 2000; Meyer-König et al. 2002; Batty et al. 2003; Nishinari et al. 2004; Kretz 2007; Iltanen 2012), which is a microscopic model, discrete both in time and space.

The exact specification of these models differs, but the common idea is to divide the walkable space into a lattice, where each cell has an area corresponding to the size of a human body projected onto the floor, approximately 40 × 40 cm. Each cell can either be occupied by *nobody* or by *one pedestrian*. The movements of pedestrians are carried out by iterating the time in steps in intervals of about 0.3–1.0 s. In each time step the pedestrians can move to unoccupied neighboring cells. However, even though the basic idea of CA models is simple, it often becomes complex with many rules for how the movement should be performed.

Since CA models are discrete in both time and space, and due to the fact that they use only local interactions, they are often used for simulating large crowds. One drawback of CA models, however, is the central role of the underlying lattice, which introduces artificial symmetries and tends to cause problems. An example is the tendency for deadlocks in counterflow situations at relatively low demand. The reason is that the grid structure promotes exact head-on movement. It is possible to solve this problem at the cost of giving up a part of the advantage of CA models, namely their computational efficiency. The grid structure itself poses a limit to the spatial precision. A bottleneck with a width of three cells can represent a real width just above 80 cm to just below 160 cm. Conversely, a real width of 100 cm can end up as a bottleneck with two or three cells in the model.

21.2.4 Fluid-Dynamic Models

When the crowd density is high, flows of pedestrians resembles fluid flows. Therefore, a macroscopic approach to crowd modeling is to use fluid-dynamic models (Helbing 1992; Hughes 2003) adapted to the simulation of pedestrian crowds.

An advantage of fluid-dynamic modeling of pedestrians is that it becomes possible to make analytical evaluations of changes in the infrastructure or changes in the boundary conditions.

21.2.5 Queuing Models

Queuing models (Watts 1987; Lovas 1994) make further simplifications to crowds. They are used to analyze how pedestrians are moving around in a network of modules, where the nodes and links can, for example, be doors and rooms, or intersections and roads. It is important to stress that the dynamics inside each node is not explicitly taken into consideration.

The idea is rather to grasp how the different modules are interacting with each other, by analyzing queues in the system. Each node has a certain 'service rate' and pedestrians move to the next queue as soon as they have been 'served'.

21.3 Calibration

No matter on which principles a pedestrian model is built, there is probably no model in existence without parameters. This opens the possibility and imposes the necessity to calibrate the models by comparison with empirical data. Calibration can be approached in at least three different ways: one is to measure pair-wise interactions of pedestrians in different situations, calibrate the model such that it

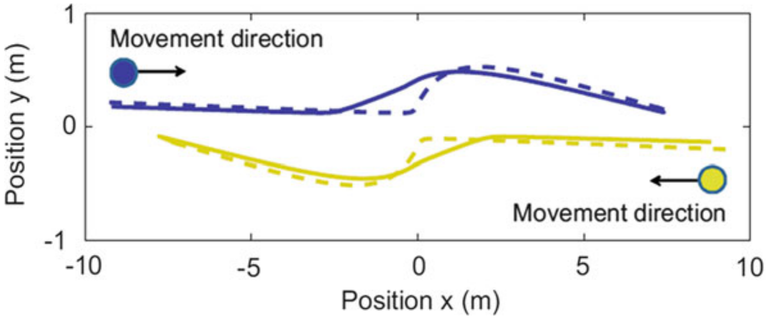

Fig. 21.1 Resulting trajectories from a simulation of two pedestrians who are approaching each other at a 180-degree angle. The simulation is carried out with the social-force model, with two different model specifications. The *dashed lines* are the resulting trajectories for an isotropic model and the *solid lines* are the resulting trajectories for an 'elliptical' anisotropic model, which gives smoother evading maneuvers and also a better fit to empirical data

reproduces these interactions and assume that the model with these parameters yields realistic results when pedestrians move in crowds (Johansson 2009). See Fig. 21.1.

The second approach is to measure aggregated macroscopic properties of moving crowds and calibrate the parameters according to these (Fischer 1933; Hankin and Wright 1958; Older 1968; Navin and Wheeler 1969; Fruin and Strakosch 1971; Predtechenskii and Milinskii 1978; Weidmann 1993; Virkler and Elayadath 1994; Muir et al. 1996; Hoogendoorn and Daamen 2005; Kretz et al. 2006a, b; Seyfried et al. 2009; Chattaraj et al. 2009; Seyfried et al. 2010a, b). The third approach is to calibrate the parameters for minimal deviation of individual trajectories of pedestrians moving in a crowd (Johansson et al. 2007; Hoogendoorn and Daamen 2009; Bauer and Kitazawa 2010), where the borderline between the second and third approach is fuzzy (Portz and Seyfried 2011).

These approaches are different methods of calibration, but they can also be combined, e.g. using method 3 for calibration and method 1 for validation to make sure that the model reproduces empirically obtained patterns on a macroscopic scale.

21.3.1 Shortest Path vs. Quickest Path

One aspect of pedestrian motion that has received very little attention is in terms of calibration work. It is this aspect which distinguishes most pedestrians from vehicles: pedestrians often choose between slowing down to walk a shorter path within a dense crowd or take some detour to keep the walking speed higher in a less dense region of the crowd (see Fig. 21.2). There is some theoretical and modeling work available on this issue (Kretz 2009a, b; Kirik et al. 2009; Steffen and Seyfried 2009; Dressler et al. 2010; Venel 2010; Rogsch and Klingsch 2010; PTV Planung Transport Verkehr AG 2010), but the empirical data are much sparser than for corridor movement.

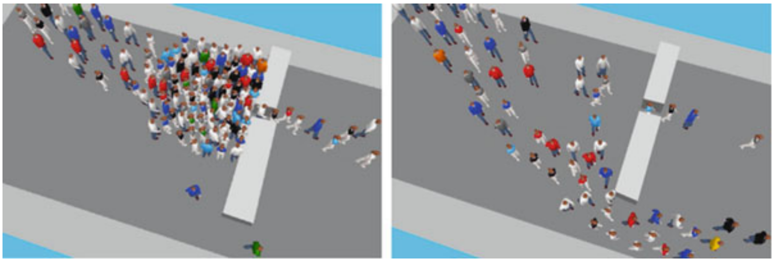

Fig. 21.2 A snapshot from a simulation with the social-force model. If the pedestrians take the shortest path (*left*), they gather in front of the bottleneck, and do not use the second path. When the pedestrians use the fastest path instead (*right*), they balance over the two possible routes

The current state of empirical crowd research is that it has even sometimes taken one step back and explicitly excluded this degree of freedom (evading vs. decelerating) in pedestrian motion by measuring the speed density relation for single file movement. This has yielded fruitful results (Chattaraj et al. 2009; Portz and Seyfried 2011; Seyfried et al. 2005, 2010b).

21.3.2 Principle of the Weakest Elements in Real-World Projects

Neither a model that perfectly describes single file movement nor a model that is perfectly calibrated for straight and wide corridor movement offers sufficient help to a project manager who is faced with a project that includes one or more corners and the movement around corners is corrupted in all models available to him, as the precision level of the entire project will normally be set by the worst precision of all elements of the simulation. Movement of a large crowd around a corner is the simplest situation one can think of where a pedestrian has to choose between trying to walk the quickest or the shortest path or something in between. Empirical efforts in the science of pedestrians need to and will turn to this aspect soon in the future.

21.3.3 Other Influences and Effects

The option set of travel time vs. travel distance can be generalized to a concept of generalized costs, as it has been done in a number of the utility-based models. Then not only travel time and travel distance can be combined to a utility for an individual pedestrian, but also, for example, the discomfort of walking on a bicycle lane or over muddy terrain or the comfort of walking shaded from rain or sunshine can be integrated in just the same manner. It is well known that the free speed (or desired speed) of pedestrians depends on their demographics, e.g. age, sex, air temperature, trip purpose (e.g. commuting or leisure time), time of day, trip circumstances (e.g. early for a train or late), culture and probably some more factors (Weidmann 1993; Chattaraj et al. 2009; Buchmüller and Weidmann 2006). Some of these factors are correlated (e.g. certain trip purposes occur not distributed equally over a day, air temperature has a typical course during a day, etc.). As the desired speed is relevant in any other movement situation, we may infer that these parameters also influence all other movement situations.

21.4 Navigation

A crowd simulation project is set up by defining the relevant boundaries: the spatial boundaries of the walkable area (i.e. the geometry of the model) and the boundary between external knowledge about the dynamics of the pedestrians and where the model needs to take over – in other words the localized demand, which is the inflow into the model, and how it varies over time. The last elementary definition a modeler needs to do is to set the destinations for the agents, which are set in the model.

The first and most basic element that a dynamics model then needs to include is a navigation or wayfinding method from the positions of the inflow (‘sources’) of the pedestrians to the given destinations (‘sinks’). The remainder of the section will deal with that task.

One way to achieve shortest route navigation to the destination is to make use of a visibility graph (de Berg et al. 1997). Simply speaking, a visibility graph is the graph of mutually visible corners of obstacles of a pedestrian movement geometry. With Dijkstra’s algorithm (Dijkstra 1959), the shortest path from the agent’s current position to the closest corner point of the destination polygon can be found. When using this method, one is faced with the difficulty of where to place the navigational points exactly: if only individual agents are moving through the geometry in low density, the navigation points can be very close to the corners of the obstacles. If the agents are moving in large groups, then the navigation points need to be placed further away. Moreover, there has to be some minimum distance that allows agents to come close to these points such that an agent can proceed toward the next navigation point.

As an example in the social-force model (Helbing and Johansson 2009), the beeline direction from the current position of an agent toward the next navigation point would then typically use the direction of the desired velocity (the absolute value of the desired velocity is an external parameter).

A method that avoids this difficulty, but which requires more computational effort, is that of a floor field (also called “static potential”), which is a grid placed over the geometry, where the distance towards the destination (under consideration of the obstacles) is written to each grid point. Plainly spoken, it is a localized look-up table of distances. There are numerous methods to calculate this static potential. Typically the calculation time rises when the deviation from the Euclidean distance

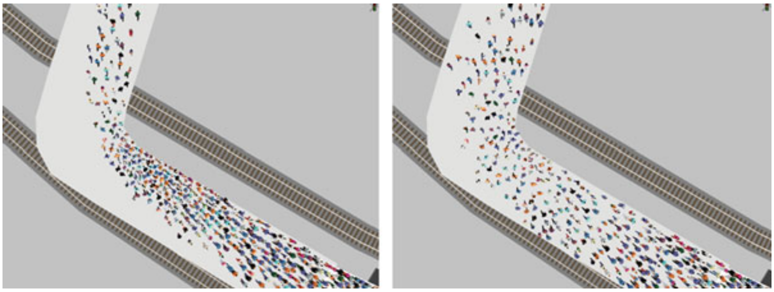

Fig. 21.3 A static potential field (*left*) compared to a dynamic potential field (*right*). The snapshots are taken at the same time instant in two identical simulation scenarios. Notice how pedestrians get stuck and delayed at the corner when a static potential field is used

is reduced (Kimmel and Sethian 1998; Jeong and Whitaker 2008; Kretz et al. 2010). The negative gradient of the static potential at the position of an agent gives the direction of the shortest path from that agent to the destination. Thus, the negative gradient of the static potential is used as the direction of the desired velocity. Recently a method has been proposed that directly and efficiently calculates the gradients without the need to calculate the static potential (Schultz et al. 2010).

The Fast Marching Method (Kimmel and Sethian 1998) and Fast Iterative Method (Jeong and Whitaker 2008) are well suited to also calculate a floor field, which contains the estimated remaining travel time from a grid cell to the destination (PTV Planung Transport Verkehr AG 2010). Contrast this with the method employed in Sect. 21.3.1. As the distribution of agents has a major impact on the estimated remaining travel time, and as the distribution of agents naturally changes in each simulation time step, such a floor field needs to be recalculated frequently. Therefore, it is called the ‘dynamic potential’. In this way it is possible to make agents in the social-force model evade groups of other agents dwelling around or being jammed at a bottleneck or the inner side of a corner early on, already by the direction of their desired velocity (see Fig. 21.3). For the dynamics of the whole system this means that jams do not grow endlessly and that agents distribute better. Therefore this method can be seen as a kind of non-iterative assignment in two continuous spatial dimensions.

Let us assume that an agent wants to reach its destination as quickly as possible. In principle the ‘bee line’ would be the quickest way. An agent walking the shortest path under consideration of obstacles is modeled as someone, who accepts that inevitably obstacles prevent one from walking along the bee line. In principle the shortest path under consideration of obstacles would also be the quickest path under consideration of obstacles. An agent walking into the direction of the estimated quickest path under consideration of all other agents is modeled as someone who accepts that jams will inevitably cause delays and therefore might prevent the shortest path from being the quickest.

While real pedestrians can be assumed to have a very good comprehension of the situation around them, and while there are situations where it is safe to assume that early arrival is the single-most important movement criterion (passengers in a station who are late for a train), modeling pedestrians to walk on the shortest path under consideration of obstacles can nevertheless be justified in many situations. First there are situations in which the quickest path is not much different from the shortest and where inter-pedestrian forces can reproduce these differences.

Second, there can be situations, where the shortest path is valued more than the quickest. As has been stated above the quickest path/shortest path trade-off can be seen as a special case of a generalized cost. Instead of calculating a field of estimated remaining travel time to the destination, it is also possible to calculate a field of generalized cost to reach the destination associated with the field. This shows that by using the gradient of such a field as the direction of the desired velocity in the social-force model, it is possible to connect the force-based approach with the utility-based approach. This can be interpreted such that the information entering the direction of the desired velocity models the free planning process of an agent, while the forces act according to their name, and they force the agent to evade other agents at rather small distances to avoid collisions.

21.5 Conclusions

Pedestrian crowd modeling has emerged as a mature and active field of research, where models are challenged on their ability to reproduce empirically observed features. This has resulted in crowd simulation tools, both commercial and freely availably ones, that are routinely used in the planning of major events, and also for optimizing transport systems, assessing building evacuations, optimizing the organization of airports and train stations, etc. Some of the challenges ahead are to reach consensus as to which modeling approaches yield the most realistic results. Another ongoing challenge is to make crowd modeling tools more autonomous. Earlier crowd modeling tools relied heavily on the user to specify every single detail in the model scenario, whereas in more recent models, pedestrians find their way around complex spaces, they queue, and they even interact with and use public transport.

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