4
Requisites: the logic of intensions

In Section 2.4.1 we argued in favour of semantic anti-actualism: the actual of all the possible worlds should play no semantic role. In this and the following sections we outline an essentialism that likewise accords no privileged status to the actual world by making the notion of essence independent of world and time and a priori instead.[1] At the same time we are arguing in favour of ontological actualism: all the individuals at the actual world are all the individuals there are at all the other possible worlds as well (hence, there are no merely possible individuals, or *possibilia*).

Our essentialism is based on the idea that since no purely contingent intension can be essential of any individual, essences are borne by intensions rather than by individuals exemplifying intensions.[2] That an intension has an essence means that a relation-in-extension obtains a priori between an intension and other intensions such that, necessarily, whenever an individual (an ι-entity) exemplifies the intension at some $\langle w, t\rangle$ then the same individual also exemplifies certain other intensions at the same $\langle w, t\rangle$. This relation is called the *requisite* relation.[3] We base our essentialism on the requisite relation and call our position *intensional essentialism*, couching as it does essentialism in terms of interplay between intensions, regardless of who or what exemplifies a given intension. This is in line with our general top-down approach from construction to intension and from intension to extension.

Let the property of being a mammal be related by the requisite relation to the property of being a whale. Then, necessarily, *if* the individual *a* is a whale at $\langle w, t\rangle$ *then a* is also a mammal at $\langle w, t\rangle$. It is an open question (epistemologically and ontologically speaking) *whether a* is a whale at $\langle w, t\rangle$. Establishing whether it is requires investigation a posteriori. On the other hand, establishing whether *a* must be a mammal in case *a* happens to be a whale is a priori, the requisite relation being in-extension and as such independent of what is true at any $\langle w, t\rangle$. Thus, there is a sense in which intensional essentialism qualifies as anti-essentialism: Robert Stalnaker labels as 'bare particular anti-essentialism' any theory (such as ours) which includes bare particulars and which claims that no empirical property is essential of any individual (1979, p. 344).

Intensional essentialism is technically an algebra of individually necessary and jointly sufficient conditions for having a certain intension. This makes it possible to *define* a given intension by means of other intensions. The essence of an intension is

[1] This section and the next draw in part on material published as Jespersen and Materna (2002).

[2] By 'purely contingent intension' we mean an intension that is not constant and does not have an essential core. See Section 1.4.2.1 for the classification of empirical properties.

[3] Tichý first broached the notion of requisite in 1979, but abstained from further developing it in later works.

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identical to its set of requisites. The ⟨*w, t*⟩-relative extensions of a given intension are irrelevant, as we said; but so are the various equivalent constructions of the intension.

4.1 Requisites defined

Here we set out the logic of requisites. The requisite relations *Req* are a family of relations-in-extension between two intensions, hence of the polymorphous type $(\omicron\alpha_{\tau\omega}\beta_{\tau\omega})$, where possibly $\alpha = \beta$. Infinitely many combinations of *Req* are possible, but the following four are the philosophically relevant ones we wish to consider:

(1) $Req_1/(\omicron\ (\omicron\iota)_{\tau\omega}\ (\omicron\iota)_{\tau\omega})$: an individual *property* is a requisite of another such *property*.
(2) $Req_2/(\omicron\ \iota_{\tau\omega}\ \iota_{\tau\omega})$: an individual *office* is a requisite of another such *office*.
(3) $Req_3/(\omicron\ (\omicron\iota)_{\tau\omega}\ \iota_{\tau\omega})$: an individual *property* is a requisite of an individual *office*.
(4) $Req_4/(\omicron\ \iota_{\tau\omega}\ (\omicron\iota)_{\tau\omega})$: an individual *office* is a requisite of an individual *property*.

Partiality gives rise to the following complication both with respect to offices and properties. The requisite relation obtains for all worlds *w* and times *t*, and the values at ⟨*w, t*⟩ of particular intensions are irrelevant. Thus if an office *X* has the requisite intension *Y*, it is so no matter whether an office *X* is occupied or vacant at a given ⟨*w, t*⟩. For instance, even at those ⟨*w, t*⟩ where the office of King of France is vacant it is true that the property of being a king is a requisite of the office. Similarly, it is true at all ⟨*w, t*⟩ (including those where the office of President of USA is vacant) that the office of Commander-in-Chief is a requisite of the office of President of USA. Therefore, it does not suffice to add the antecedent condition that *X* be occupied. For, at a ⟨*w, t*⟩ where *X* is vacant, the antecedent condition is false, and so the intensional descent of *X* to ⟨*w, t*⟩ picks up *no* individual. In other words, the Compositions X_{wt}, $[{}^0Y_{wt} = {}^0X_{wt}]$ and $[{}^0Z_{wt}\ {}^0X_{wt}]$ will be *v*-improper $(Y/\iota_{\tau\omega};\ Z/(\omicron\iota)_{\tau\omega})$. The truth-functional connective of material implication $(\supset/(\omicron\omicron\omicron))$ is such that when applied to a missing argument (a truth-value gap), the result is *v*-improper as well, making the Composition $[[{}^0Occ_{wt}\ {}^0X] \supset [{}^0Y_{wt} = {}^0X_{wt}]]$ *v*-improper for ⟨*w, t*⟩.[4] The whole *definiens* $\forall w \forall t\ [[{}^0Occ_{wt}\ {}^0X] \supset [{}^0Y_{wt} = {}^0X_{wt}]]$ will, thus, construct **F**! $(Occ/(\omicron\iota_{\tau\omega})_{\tau\omega}$ is the property of an individual office of being occupied.)

A similar problem arises even in case of properties. The reason is because properties are isomorphic to characteristic functions, and these functions can also

[4] In a programming language, one would say that ⊃ is a *strict function* returning an error value for an error value: ⊥ → ⊥.

have truth-value gaps. For instance, the property of having stopped smoking comes with a bulk of requisites like, e.g., the property of being an ex-smoker. Thus, the predication of such a property Z of an individual a may also fail, causing $[{}^0Z_{wt}\ {}^0a]$ to be v-improper. The remedy is easy, fortunately—just use the propositional property of being true at $\langle w, t\rangle$: $True/(oo_{\tau\omega})_{\tau\omega}$. Given a proposition P, $[{}^0True_{wt}\ {}^0P]$ v-constructs **T** if P is true at $\langle w, t\rangle$; otherwise (i.e., if P is false or else undefined at $\langle w, t\rangle$) **F**.[5]

Now we are going to define the four above kinds of requisite relations.

Ad (1):

Definition 4.1 (*requisite relation between ι-properties*) Let X, Y be intensional constructions such that $X, Y/*_n \rightarrow (o\iota)_{\tau\omega}$; $x \rightarrow \iota$. Then

$$[{}^0Req_1\ Y\,X] = \forall w \forall t\ [\forall x\ [[{}^0True_{wt}\ \lambda w \lambda t\ [X_{wt}\,x]] \supset [{}^0True_{wt}\ \lambda w \lambda t\ [Y_{wt}\,x]]]]. \quad \square$$

Gloss *definiendum* as, 'Y is a requisite of X', and *definiens* as, 'Necessarily, at every $\langle w, t\rangle$, whatever x instantiates X at $\langle w, t\rangle$ also instantiates Y at $\langle w, t\rangle$.'

Example. *All whales are mammals*, provided the property of being a mammal is a requisite of the property of being a whale.[6]

Ad (2):

Definition 4.2 (*requisite relation between ι-offices*) Let X, Y be intensional constructions such that $X, Y/*_n \rightarrow \iota_{\tau\omega}$. Let $Occ/(o\iota_{\tau\omega})_{\tau\omega}$ be the property of an office of being occupied (or existing, as *existence* was defined in Section 2.3). Then

$$[{}^0Req_2\ Y\,X] = \forall w \forall t\ [[{}^0Occ_{wt}\ X] \supset [{}^0True_{wt}\ \lambda w \lambda t\ [X_{wt} = Y_{wt}]]]. \quad \square$$

Gloss *definiendum* as, 'Y is a requisite of X', and *definiens* as, 'Necessarily, if X is occupied at some $\langle w, t\rangle$ then whoever occupies X at $\langle w, t\rangle$ also occupies Y at this $\langle w, t\rangle$.'

Remark. Due to partiality, the relation between offices may not be symmetric. If the office X is occupied, then the office Y is occupied as well, and X and Y are occupied by the same individual. If Y is not occupied then X is not occupied either. However, Y can be occupied, and X vacant, at some $\langle w, t\rangle$. If being X is a *sufficient* condition for being Y, whereas being Y is a *necessary* condition for being X, it follows that the set of world/time pairs at which Y is occupied is a superset of the set of world/time pairs at which X is occupied. Suppose we rank individual offices in terms of the ordering defined by the subset relation between sets of worlds and times at which they are occupied according to the rule that a rarely occupied office

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[5] See Section 1.4.3.

[6] We also often say that the property of being a whale *implies* the property of being a mammal; or, in the vernacular of computer science, that the concept of whale *subsumes*, or *contains*, the concept of mammal.

is higher up the hierarchy than a frequently occupied one. Then X is higher up than Y. One could also say that X is, in a quite literal sense, more *exclusive* than Y.

Example. *The President of the USA is the Commander-in-Chief*. The latter office is a requisite of the former, such that whoever is the President is also the Commander-in-Chief. However, it may happen that the presidency goes vacant, while somebody occupies the office of Commander-in-Chief.

Ad (3):

Definition 4.3 (*requisite relation between a ι-property and a ι-office*) Let X, Y be intensional *constructions* such that $X/*_n \rightarrow \iota_{\tau\omega}$ and $Y/*_n \rightarrow (o\iota)_{\tau\omega}$. Then

$$[{}^0Req_3\ \ Y X] = \forall w \forall t\ [[{}^0Occ_{wt} X] \supset [{}^0True_{wt}\, \lambda w \lambda t\ [Y_{wt} X_{wt}]]]. \qquad \Box$$

Example. *The King of France is a king*.

Remark. 'The King of France is a king' is *ambiguous* between two readings—one necessarily true, the other contingently without a truth-value—as Tichý points out (1979, p. 408, 2004, p. 360).[7] One is the requisite (i.e., *de dicto*) reading:

$$[{}^0Req_3\ {}^0King\ \lambda w \lambda t\ [{}^0King\_of_{wt}\ {}^0France]].$$

Types: $King/(o\iota)_{\tau\omega}$; $King\_of/(\iota\iota)_{\tau\omega}$; $France/\iota$. If true, it is necessarily so, regardless of whether or not some $\langle w, t\rangle$ lacks an occupant of $\lambda w \lambda t\ [{}^0King\_of_{wt}\ {}^0France]$.

The other reading is the *de re* reading:

$$\lambda w \lambda t\ [{}^0King_{wt}\ \lambda w \lambda t\ [{}^0King\_of_{wt}\ {}^0France]_{wt}].$$

If true, it is so only because somebody occupies $\lambda w \lambda t\ [{}^0King\_of_{wt}\ {}^0France]$ at $\langle w, t\rangle$ and its occupant is in the extension of $King$ at $\langle w, t\rangle$.

Remark. When defining a requisite of an *office X*, the antecedent condition on X being occupied is required. Otherwise we shall have the following invalid argument on our hands (see Tichý, 1979, pp. 408ff, 2004, pp. 360ff).

P is a requisite of office *O*

The occupant of *O* instantiates *P*.

This inference pattern is fallacious,

> for the premise may be true even if *O* is vacant, in which case the conclusion, so far from being true, is vacuous (i.e., lacks a truth value). (Ibid., p. 408, p. 360, resp.)

[7] See Section 1.5.2.2 for the ambiguity between the two readings.

However, a valid inference rule can be obtained by adding an extra premise to the effect that the relevant office is occupied:

$$\frac{P \text{ is a requisite of office } O \qquad \text{Office } O \text{ is occupied}}{\text{The occupant of } O \text{ instantiates } P.}$$

Ad (4):

Definition 4.4 (*requisite relation between a ι-office and a ι-property*) Let X, Y be intensional constructions such that $X/*_n \rightarrow (o\iota)_{\tau\omega}$ and $Y/*_n \rightarrow \iota_{\tau\omega}$. Then

$$[{}^0Req_4\ \ Y\,X] = \forall w \forall t\ [\forall x\ [{}^0True_{wt}\, \lambda w \lambda t\ [X_{wt}\, x]] \supset [{}^0True_{wt}\, \lambda w \lambda t\ [Y_{wt} = x]]]]. \quad \square$$

Example. *God is omnipotent.* (That is, if somebody/something is omnipotent then he/she/it is God.)

Above we defined four types of requisite relation, namely Req_1, Req_2, Req_3, Req_4. While $Req_1/(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$ and $Req_2/(o\iota_{\tau\omega}\iota_{\tau\omega})$ are homogeneous, Req_3, Req_4 are heterogeneous. Since the latter two do not have a unique domain, it is not sensible to ask what sort of ordering they are. Not so with the former two. We define them as *quasi-orders* (a.k.a. *pre-orders*) over $(o(o\iota)_{\tau\omega})$, $(o\iota_{\tau\omega})$, respectively, that can be strengthened to *weak partial orderings*. However, they cannot be strengthened to strict orderings on pain of paradox, since they would then both be reflexive and irreflexive. We wish to retain reflexivity, such that any intension having requisites will count itself among its requisites. Otherwise there will be worlds and times at which an office X is occupied and $X_{wt} \neq X_{wt}$, and worlds and times at which a property Y is instantiated and $\neg[[Y_{wt}\, x] \supset [Y_{wt}\, x]]$.

Claim 4.1 Req_1 is a quasi-order on the set of ι-properties.

Proof. Let X, $Y \rightarrow (o\iota)_{\tau\omega}$. Then Req_1 belongs to the class $QO/(o(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega}))$ of quasi-orders over the set of individual properties:

Reflexivity. $[{}^0Req_1\, X\, X] =$
$\forall w \forall t\ [\forall x\ [[{}^0True_{wt}\, \lambda w \lambda t\ [X_{wt} x]] \supset [{}^0True_{wt}\, \lambda w \lambda t\ [X_{wt} x]]]]$

Transitivity. $[[[{}^0Req_1\ Y\, X] \wedge [{}^0Req_1\ Z\ Y]] \supset [{}^0Req_1\ Z\, X]] =$
$\Big[\forall w \forall t\ \Big[\forall x\ [[{}^0True_{wt}\, \lambda w \lambda t\ [X_{wt} x]] \supset [{}^0True_{wt}\, \lambda w \lambda t\ [Y_{wt} x]]] \wedge$
$[[{}^0True_{wt}\, \lambda w \lambda t\ [Y_{wt} x]] \supset [{}^0True_{wt}\, \lambda w \lambda t\ [Z_{wt} x]]]\Big] \supset$
$\forall w \forall t\ [\forall x\ [[{}^0True_{wt}\, \lambda w \lambda t\ [X_{wt} x]] \supset [{}^0True_{wt}\, \lambda w \lambda t\ [Z_{wt} x]]]]\Big]$

In order for a requisite relation to be a weak partial order, it will need to be also anti-symmetric. The Req_1 relation is, however, not anti-symmetric. If properties X, Y are mutually in the Req_1 relation, i.e., if

$$[[{}^0Req_1\ Y\ X] \wedge [{}^0Req_1\ X\ Y]]$$

then at each $\langle w, t\rangle$ the two properties are truly ascribed to exactly the same individuals. This does not entail, however, that X, Y are identical. It may be the case that there is an individual a such that $[X_{wt}\ a]$ v-constructs **F** whereas $[Y_{wt}\ a]$ is v-improper. For instance, the following properties X, Y differ only in truth-values for those individuals who never smoked (let $StopSmoke/(o\iota)_{\tau\omega}$ be the property of having stopped smoking[8]). Whereas X yields truth-value *gaps* on such individuals, Y is false of them:

$$X = \lambda w\lambda t\ \lambda x\ [{}^0StopSmoke_{wt}\, x]$$

$$Y = \lambda w\lambda t\ \lambda x\ [{}^0True_{wt}\, \lambda w\lambda t\ [{}^0StopSmoke_{wt}\ x]].$$

In order to abstract from such an insignificant difference, we introduce the equivalence relation $Eq/(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$ on the set of individual properties; $p, q \rightarrow (o\iota)_{\tau\omega}$; $=/(ooo)$:

$${}^0Eq = \lambda pq\ [\forall x\ [[{}^0True_{wt}\, \lambda w\lambda t\ [p_{wt}\ x]] = [{}^0True_{wt}\, \lambda w\lambda t\ [q_{wt}\ x]]]].$$

Now we define the Req_1' relation on the factor set of the set of ι-properties as follows. Let $[p]_{eq} = \lambda q\ [{}^0Eq\ p\ q]$ and $[Req_1'\ [p]_{eq}\ [q]_{eq}] = [Req_1\ p\ q]$. Then:

Claim 4.2 Req_1' is a weak partial order on the factor set of the set of ι-properties with respect to Eq.

Proof. It is sufficient to prove that Req_1' is well-defined. Let p', q' be ι-properties such that $[{}^0Eq\ p\ p']$ and $[{}^0Eq\ q\ q']$. Then

$[Req_1'\ [p]_{eq}\ [q]_{eq}] = [Req_1\ p\ q] =$

$\forall w \forall t\ [\forall x\ [[{}^0True_{wt}\, \lambda w\lambda t\ [p_{wt} x]] \supset [{}^0True_{wt}\, \lambda w\lambda t\ [q_{wt} x]]]] =$
$\forall w \forall t\ [\forall x\ [[{}^0True_{wt}\, \lambda w\lambda t\ [p'_{wt} x]] \supset [{}^0True_{wt}\, \lambda w\lambda t\ [q'_{wt} x]]]] =$

$[Req_1'\ [p']_{eq}\ [q']_{eq}]$.

Now obviously the relation Req_1' is antisymmetric:

$[[{}^0Req_1'\ [p]_{eq}\ [q]_{eq}] \wedge [{}^0Req_1'\ [q]_{eq}\ [p]_{eq}]] \supset [[p]_{eq} = [q]_{eq}]$.

Claim 4.3 Req_2 is a weak partial order defined on the set of ι-offices.

Proof. Let $X, Y \rightarrow \iota_{\tau\omega}$. Then the Req_2 relation belongs to the class $WO/(o(o\ \iota_{\tau\omega}\iota_{\tau\omega}))$ of weak partial orders over the set of individual offices.

[8] We take the property of having stopped smoking as presupposing that the individual previously smoked. For instance, that Charles stopped smoking can be true or false only if Charles was once a smoker. Similarly for the property of having stopped whacking one's wife. For more on presuppositions, see Section 1.5.2.1, Definition 1.14.

Reflexivity. $[{}^0Req_2\, X\, X] = [\forall w \forall t\, [[{}^0Occ_{wt}\, X] \supset [{}^0True_{wt}\, \lambda w \lambda t\, [X_{wt} = X_{wt}]]]]$.

Antisymmetry. $[[[{}^0Req_2\, Y\, X] \wedge [{}^0Req_2\, X\, Y]] \supset [X = Y]] =$
$\Big[\forall w \forall t\, \Big[[[{}^0Occ_{wt}\, X] \supset [{}^0True_{wt}\, \lambda w \lambda t\, [X_{wt} = Y_{wt}]]] \wedge$
$[[{}^0Occ_{wt}\, Y] \supset [{}^0True_{wt}\, \lambda w \lambda t\, [X_{wt} = Y_{wt}]]]\Big] \supset [X = Y]\Big]$

Transitivity. $[[[{}^0Req_2\, Y\, X] \wedge [{}^0Req_2\, Z\, Y]] \supset [{}^0Req_2\, Z\, X]] =$
$\Big[\forall w \forall t\, \Big[[[{}^0Occ_{wt} X] \supset [{}^0True_{wt}\, \lambda w \lambda t\, [X_{wt} = Y_{wt}]]] \wedge$
$[[{}^0Occ_{wt}\, Y] \supset [{}^0True_{wt}\, \lambda w \lambda t\, [Y_{wt} = Z_{wt}]]]\Big] \supset$
$\forall w \forall t\, [[{}^0Occ_{wt}\, X] \supset [{}^0True_{wt}\, \lambda w \lambda t\, [X_{wt} = Z_{wt}]]]\Big]$.

Remark. *Antisymmetry* requires the consistent *identity* of the offices constructed by X, Y: $[X = Y]$. The two offices are identical iff at all worlds/times they are either co-occupied by the same individual or are both vacant: $\forall w \forall t\, [[{}^0True_{wt}\, \lambda w \lambda t\, [X_{wt} = Y_{wt}]] \vee [{}^0Undef_{wt}\, \lambda w \lambda t\, [X_{wt} = Y_{wt}]]] = \forall w \forall t\, \neg[{}^0False_{wt}\, \lambda w \lambda t\, [X_{wt} = Y_{wt}]]$.

This concludes our definition of the logic of the requisite relations. We turn now to the definition of essence as a set of requisites. Tichý considered only requisites of offices (though of offices of any degree):

> [T]he *requisite* of an office is any property such that, for any world *w* and time *t*, if *x* occupies the office in *w* at *t* then *x* instantiates the property in *w* at *t* (1979, p. 408, 2004, p. 360).

But the underlying idea readily generalises, in that the requisite of an intension Int_i is any Int_j, such that, for any $\langle w, t\rangle$, if x either instantiates or occupies Int_i then x instantiates or occupies Int_j at $\langle w, t\rangle$. Obviously, Tichý considered only essences of offices:

> [T]he conjunction of all [the requisites of an office] is fittingly called its *essence*. The essence of an office is thus a property such that the having of it by *x* in world *w* at time *t* is not only necessary but also sufficient for *x* to occupy the office in *w* at *t*. Whereas a requisite of an office is *part* of what it takes for something to occupy it, the *essence* is *all* it takes. An office can thus be defined by specifying its essence (Ibid).

Again, the underlying idea readily generalises, such that any Int_j can be defined by specifying its essence. However, unlike Tichý, we are not restricting requisites to properties suitable for the occupants of offices of degree n, $n \geq 1$: witness (1), (2), (4). Now, intensional essentialism simply says: specify the intensions that are the requisites of a given intension, pool those requisites, this will give you the essence of your intension. However, this drags type-theoretic complications along with it, since the requisites of Int_i may well be of different types. For instance, let $X/(o\iota)_{\tau\omega}$; $x \rightarrow \iota_{\tau\omega}$. Then we can formally specify all those of X's requisites that are of type $\iota_{\tau\omega}$:

$$\lambda x\, [{}^0Req_4\, x\, {}^0X].$$

But if *X*'s requisites also count intensions of type $(o\iota)_{\tau\omega}$ then they need to be specified separately, $y \rightarrow (o\iota)_{\tau\omega}$:

$$\lambda y\ [{}^0Req_1\ y\ {}^0X].$$

The former is a construction of a set of ι-offices; the latter, a construction of a set of ι-properties. Thus, *contra* Tichý, we are banned from holding that the essence of an intension is 'the conjunction of all its requisites' (ibid.), if this would mean the *set* of all its requisites. There can be no such set, as soon as more than one characteristic function is involved, as with $\lambda x\ [\ldots x\ldots]$ and $\lambda y\ [\ldots y\ldots]$ above. We are *not* permitted to do what would ostensibly be the obvious thing to do; namely, forming their union:

$$\lambda x\ [{}^0Req_4\ x\ {}^0X] \cup \lambda y\ [{}^0Req_1\ y\ {}^0X].$$

This is off-limits, as the union would contain elements of more than one type. This predicament becomes evident if we attempt to type $\cup$. It must be a function from pairs of sets to sets, and the types of its arguments in this case are known: $(o\iota_{\tau\omega})$ and $(o(o\iota)_{\tau\omega})$, respectively. But the question of what the type of its value would be affords no answer.

However, there are two solutions possible, one arguably superior to the other. One solution makes the essence of an intension a pair whose first element is a set of $\iota_{\tau\omega}$-entities and whose second element is a set of $(o\iota)_{\tau\omega}$-entities. (It is obvious how to generalise this solution to cover any two homogeneous or heterogeneous combinations of requisites.) The other solution makes the essence of an intension a set of $(o\iota)_{\tau\omega}$-entities only, without thereby restricting the requisites to such entities.

Here is the definition of the essence of the intension $Y \rightarrow (o\iota)_{\tau\omega}$, according to which its essence is a heterogeneous pair of sets of intensions (of a set of ι-properties and a set of ι-offices, respectively), where $x_1 \rightarrow (o(o\iota)_{\tau\omega})$; $x_2 \rightarrow (o\iota_{\tau\omega})$; $c \rightarrow (o\iota)_{\tau\omega}$; $d \rightarrow \iota_{\tau\omega}$; $Essence_1'/((o(o(o\iota)_{\tau\omega})(o\iota_{\tau\omega}))(o\iota)_{\tau\omega})$.

$$[{}^0Essence_1'\ Y] = [\lambda x_1 x_2\ [x_1 = \lambda c\ [{}^0Req_1\ c\ Y] \wedge [x_2 = \lambda d\ [{}^0Req_4\ d\ Y]]]].$$

A *pair* is here a relation-in-extension between two sets of arbitrary intensions; therefore, the polymorphous type of $Essence_1$ is $((o\ (o\gamma_{\tau\omega})\ (o\beta_{\tau\omega}))\ \alpha_{\tau\omega})$.

Here is the definition of the essence of $Y \rightarrow (o\iota)_{\tau\omega}$, according to which its essence is a set of ι-properties. That is, $Essence_2'/((o(o\iota)_{\tau\omega})(o\iota)_{\tau\omega})$ is a function from a ι-property to a set of ι-properties. If $p \rightarrow (o\iota)_{\tau\omega}$ then

$$[{}^0Essence_2'\ Y] = \lambda p\ [{}^0Req_1\ p\ Y].$$

The polymorphous type of *Essence*$_2$ is $((o(o\alpha)_{\tau\omega})\ \beta_{\tau\omega})$: given an arbitrary intension of type $\beta_{\tau\omega}$, *Essence*$_2$ returns the set of α-properties that are the requisites of this arbitrary intension. In general, if $Z \rightarrow \beta_{\tau\omega}$; $q \rightarrow (o\alpha)_{\tau\omega}$; $Req_n/((o\ (o\alpha)_{\tau\omega})\ \beta_{\tau\omega})$ then

$$[{}^0Essence_2\ Z] = \lambda q\ [{}^0Req_n\ q\ Z].$$

The reason why the definition of essence can be made homogeneous is because, given an arbitrary intension, there will always be a corresponding property. For instance, the ι-office *the tallest woman* will correspond to the property *being an x such that x is identical to the tallest woman*. If this office is A then the corresponding property is $(x \rightarrow \iota)$

$$\lambda w \lambda t\ [\lambda x\ [x = {}^0A_{wt}]].$$

Which of *Essence*$_1$, *Essence*$_2$ is preferable? *Essence*$_2$, in our view. It is more elegant, first of all, in that it makes it possible to define the essence of a given intension by means of one type of intension only. But there is also a substantial reason for preferring *Essence*$_2$. If we were to consider more requisite relations than just Req_1 through Req_4 we would need to specify an n-tuple of intensions, n>2, each element being of a different intensional type. It would remain indeterminate which particular intensional types to insert into the n-tuple and what the value of n would be. In particular, even if n were just countably infinite, it would be impossible to identify any appropriate construction of the tuple.

4.2 Intensional essentialism

Here we motivate intensional essentialism philosophically in opposition to extensional essentialism and its adjacent notion of metaphysical modality.

Intensional essentialism is opposed to standard contemporary essentialism, which is set within an extensional framework, according to which essential properties are borne by extensional entities such as individuals.[9] Extensional essentialism

[9] Nortmann distinguishes between what he calls *property essentialism* and *individual essentialism* (2002, pp. 8ff). The property essentialist inquires about the essence of those properties that he considers accidental in whatever bearers they may have. This inquiry contributes nothing to the question of what the nature of any bearer of the relevant property is, if the property is contingently borne by the bearer. If an individual a has the property F at time T then this only means that the following may be known a priori: *If* something has F at T *then* it has F throughout its existence. But whether a actually has F is something that can, in general, not be known a priori. (Ibid., pp. 26–27.) TIL comes close to qualifying as property essentialism in Nortmann's sense; though not entirely – we do not require that if a is an F then a must be an F from beginning to end of its cycle. First, we do not wish to exclude nomologically deviant worlds in which an F-object may shed F at some point without ending its cycle. This is to say that in such a world a

has received extensive attention in the vast literature on 'Aristotelian essentialism' following in the wake of the development of quantified modal logic, Kripke (1980) arguably being *the* modern classic.

Typical questions would be whether Socrates is essentially a human being or essentially Plato's teacher. We ask a different kind of question, such as whether being human is an essential property of any occupant of the property of being Plato's teacher (i.e., whether it is a requisite of this property).[10] The leading idea is that modality *de dicto* is based on a priori relations between intensions, while modality *de re* is based on bare particulars. (For modality *de dicto* and *de re*, see Section 4.6).

Our extensive reliance on intensional entities at the expense of extensional ones is 'pre-revolutionary' in the general sense that TIL has not joined the current orthodoxy ushered in by Kripke, Kaplan, etc., that began as a 'revolution' against Carnap, Church, etc. Simchen (2004, esp. pp. 528–40) provides a precise description of the change in perspective and priorities that the 'revolution' (as he terms it) brought about. Pre-revolutionary possibility was analytical possibility, which was simply a matter of consistency of the co-instantiation of intensions, with little concern for 'what *things* would have been like had they been different from the ways they are' (2006, p. 24; emphasis ours.) In keeping with this, 'the conditions [i.e., the 'purely qualitative manners of presenting portions of our surroundings', 2004, p. 543]…should be just as they are in the complete absence of any world to satisfy them' and 'the world [supplies] mere satisfiers for independently constituted conditions' (ibid., p. 530, p. 531, resp.). We agree wholeheartedly, TIL being ('Platonic') realism *ante rem*.[11] Kripke, by contrast, holds that

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may end its cycle as an elephant without thereby dying (but, e.g., becoming a different sort of mammal or something much more exotic). Second, our ι-objects are incapable of going out of existence, so *coming into and going out of existence* will often have to be recast as *being born and dying*, *being created and destroyed*, etc., and then only in an individual's capacity as an *F*-thing, a *G*-thing, etc.

[10] Bordering on morbidity, Kim Il Sung was made Eternal President of the Democratic People's Republic of Korea in 1998, 4 years after his death. If a dead human being is not a human being then it is not a requisite of this office to be a human being. Since the ontological status of deceased people is far from obvious (just as it is uncertain whether *deceased* is a privative modifier), it is far from obvious what it takes to occupy the office. This suggests that the office of Eternal President of the DPRK is ill-defined; so, strictly speaking, there may be no such office (but only a vacuous title with no office to back it up). Nor is it entirely clear what it actually means to say that Kim was made Eternal President after his death; for, assuming that a dead person is not a person, *who* acquired, in 1998, the property of being the occupant of the office of Eternal President? Colloquially, one would say that Kim did (as we just did a few lines up); but he died in 1994, so in what (non-ghoulish) sense was *he* around in 1998 to acquire any new properties at all? Our concern is with the exact requisites of an (alleged) office and the possibility of a deceased person (hence, probably non-person) occupying it.

[11] On a similar note, Sartre says, '[Essence] precedes existence for Leibniz, and the chronological order depends on the eternal order of logic' (1943, p. 469). The priority of essence over existence holds for complete individual offices; i.e., entire life-stories. The only dash of contingency is choosing one such office at the expense of all the rest. Once that choice is made, all the rest

> [W]e begin with the objects, which we *have*, and can identify, in the actual world. We can then ask whether certain things might have been true of the objects (1980, p. 53).

Transposed into the key of intensional essentialism, the conceptual order would be the other way around; we begin with the conditions (intensions) that we have and can identify regardless of any particular possible world. We can then ask whether certain conditions might have been satisfied by something (extensions) at this or that world.[12]

We make two negative claims. The first is that the predication

Necessarily, *a* is an *F*

is false, if *a* is an extensional entity and *F* a purely contingent property. That is, we reject individual essentialism. The second is that no purely contingent relation-in-intension between any two different individuals ever obtains of necessity. So

Necessarily, *a* has origin *o*

where, e.g., *a* is a wooden table and *o* a chunk of wood, is false. That is, we reject the thesis of the necessity of origin. This is not to say that it could not be made a requisite of some particular individual office that its bearer must have its material origin in either a specific individual or whatever occupies a specific office. It could; but then *necessity of origin* is no longer a relation-in-intension between two individuals, but a relation-in-extension between either a property and an office or between two offices. Whether *Necessarily, a is an F* or *Necessarily, a has origin o*, we find ourselves rejecting the category of so-called metaphysical modality due

follows as a matter of necessity. We side with Leibniz against Sartre in saying that essence does precede existence. But we also deny that what an individual is and does throughout its life-span is a matter of unfolding an entire, pre-programmed individual office. One could, in principle, individuate any two individuals strictly in terms of what is true of either of them with respect to worlds and times. But such a principle of individuation would be of no use to us humans, since the respective sets of truths applying to two individuals are infinite and as such cannot be grasped in full by humans. If individuals would enter into our ontology only as values of intensions, especially of individual offices, we would never be entitled to Trivialize an individual *a*: ^0a. We would never get 'closer' to individuals than in terms of A_{wt}, $A/\iota_{\tau\omega}$.

[12] Whether *a* is an *F* or the *G*, for instance, is, logically speaking, irrelevant. What is relevant is only whether some individual or other is an *F* or the *G* at some $\langle w, t\rangle$ of evaluation, whether the same individual is both the *G* and the *H* at $\langle w, t\rangle$, etc., and not whether it is *a*, *b*, *c*, etc. This will come across as exceedingly cynical if *a* is a human being; for it may well be extremely relevant to *a* whether he or she is an *F* or the *G* or both the *G* and the *H* (etc.). We wish to emphasise, therefore, that our top-down approach from condition to satisfier combined with an exterior, or outside-in, perspective on individuals is no theory of the good (human) life, but adopted for strictly logical and semantic purposes, having 'little to do with how men (or men and animals) fare in [the actual world]', as Rescher says about Leibniz's struggle with squaring the well-being of rational creatures with God's choice of *the* possible world that will combine the fewest and simplest laws with the greatest multitude of phenomena (1986, p. 157).

to Kripke (1980), which is supposed to be the sort of modality a posteriori underlying the modifier *Necessarily* in both cases.

Consider this example from contemporary analytic metaphysics, which has taken on a life of its own, spawning a literature dedicated particularly to it. The example will prove helpful in discussing both, 'Necessarily, *a* is an *F'* and, 'Necessarily, *a* has origin *o*'.

Pointing at his wooden lectern in the auditorium, Kripke says:

> In the case of this table, we may not know what block of wood the table came from. Now could *this table* have been made from a completely *different* block of wood, or even of water cleverly hardened into ice [?] (1980, p. 113).

Kripke argues that, necessarily, that wooden table is wooden, and that, necessarily, its material origin is the block of wood it was actually hewn from. Our objection to individual-essentialist predication is its *circularity*. Our objection to necessity of origin is its *infinite regress*.

First, individual essentialism. Consider two individuals, *a* and *b*, of which we already know that they are both tables but only one is wooden while the other merely appears to be so. Our task is to decide whether it is *a* or it is *b* that is the wooden table of the two. Pick one of the tables and apply a (low-key) scientific procedure to check whether it is wooden. Let the outcome be that it is, indeed, wooden. If we know this, our knowledge is a posteriori, because we applied an empirical procedure, and of a *contingent* truth, because if we had checked the other table it would have been false that the inspected table was wooden. Our knowledge is insufficient to establish whether it is true that *a* is the wooden table:

> But if [it is not knowable *a priori* that *a* is wooden] it is hard to see how, on Kripke's theory, it can be knowable at all. For…if we do not know [that *a* is wooden] to start with, no amount of inspecting or testing a table will tell us that it is *a* rather than *b* that we are dealing with. Accordingly, no amount of inspecting and testing will tell us that *a* is wooden (Tichý, 1983, pp. 239–40, 2004, pp. 521–22.)

Semi-formally:

(1) the inspected table = the wooden table
(2) *a* = the inspected table
(3) *a* = the wooden table
(4) if *x* is the wooden table then *x* is wooden
(5) *b* = the inspected table

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(6) *a* is wooden.

The argument is valid, of course, but we cannot know it to be sound, as long as we do not know whether it is *a* or if it is *b* that is the inspected table. As long as we do not know whether (2) is true, we cannot know whether (3) is true; but then we cannot know whether it is (5) or its rival (*b* = the inspected table) that is true. The truth-value of (2) can be ascertained only if it is already known that being wooden is an essential property of every wooden table (understood *de re*), such that woodenness can be used to tell *a* from *b*, no matter whatever other properties *a*, *b* may have at a given ⟨*w, t*⟩. Remember that, according to Kripkean essentialism, if *b* fails to be wooden at one world *W* then *b* fails to be wooden at all other worlds accessible from *W*.

Tichý's verdict is that

> Kripke's individual essentialism … involves an epistemological circle. In order to establish that an object has an essential property, we have to inspect that object. But we cannot be sure that we are inspecting the right object unless we know that the object has that essential property. The Kripkean essentialist is thus saddled with the absurd conclusion that no particular table can be known to be wooden [.]
> (1983, p. 240, 2004, p. 522.)

The circularity objection readily extends from individuals to natural kinds. In order to establish whether all individuals belonging to a particular species share some particular essential property, we have to inspect such individuals (say, cats). But we cannot be sure that the individuals we are inspecting are cats, unless we know that the inspected individuals possess that essential property.[13] Remember that, according to Kripkean essentialism, if *x* is a cat at one world *W* then *x* is a cat at all worlds accessible from *W* at which *x* exists.

Then, necessity of origin. In its crudest form the thesis is that the binary relation *Origin* holding between two individuals *a*, *b*, such that *a* is the material origin of some artefact or organism *b*, obtains as a matter of 'metaphysical' necessity:

$$Origin\,\langle a, b\rangle \supset \Box\; Origin\,\langle a, b\rangle.$$

An object owes its origin to other objects, the way a child owes its origin to its parents or a statue owes its origin to a lump of bronze (say).[14] But those objects are also anchored to other objects, and so on backwards into the bottomless past.[15] A full description or comprehension of an individual's origin would include an amount of other things so vast, it could not possibly be surveyed. The notion of origin will be epistemologically and conceptually inoperative unless made manageable

[13] See Kripke (1980, pp. 125–27) regarding 'the actual cats that we *have*' *versus* demons masquerading as cats, where it is assumed that we would be able to know of something that it is a cat prior to knowing what the species-specific essence of cats is.

[14] Though a statue owes its origin to much more than just some lump of matter. A statue is an artistic artefact that also embodies an artistic idea which is materialized by means of a lump of matter. From the point of view of artistic idea, it matters little which lump of matter happens to embody Michelangelo's ideal male youth. Yes, the statue at *Accademia* in Florence is the original and the one in front of *Palazzo Vecchio* is a copy; but they manifest the same idea(l) of male youth. The bottom-line is that a statue is at the intersection of matter and idea, and is not reducible to a chunk of clay, marble, or stone.

[15] Berkovski notes that, 'The full specification of Napoleon's origin will be recursive. If the question is how we identify Letizia Bonaparte [Napoleon's mother], the same proof of origin is to be repeated for her, her own parent, and so forth.' (2005, p. 17.) Berkovski, however, fails to point out that the recursion is going to be infinite, unless terminated by *fiat*. (We thank Berkovski for permitting us to quote from his unpublished manuscript.)

by arbitrarily stipulating a point at which the backtracking were to end. This is not to say that the notion of origin might not underpin some form of essentialism. In fact, we can express the thesis by means of Req_2, since the relation between origin and destination (i.e., the resulting artefact or organism) is not symmetric, as in effect argued by Rohrbaugh and deRosset (2004, pp. 718ff).[16,17]

Both $\Box\ Fa$ and $\Box\ Origin\ \langle a, b\rangle$ are supposed to be conclusions of the argument schema that Kripke introduces (1971, p. 153):

(1) $P \supset \Box P$
(2) P

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(3) $\Box P$.

Thus, let P be $(a = b)$, 'a', 'b' Kripkean proper names:

(1.1) $(a = b) \supset \Box (a = b)$
(2.1) $a = b$

(3.1) $\Box (a = b)$

Kripke argues that the necessity in the consequent of (1.1) and in (3.1) is metaphysical necessity a posteriori. If $a = b$, then necessarily so, since everything is necessarily self-identical. Or phrased in the idiom of rigid designation: if two rigid designators co-designate at one world they do so at all worlds (with provisos for inexistence); i.e., '$a = b$' will express a necessary truth.

But the necessitation of $a = b$ can be argued for on strictly logical grounds. If true, (3.1) is just the logical triviality that a is self-identical. Only the triviality of the argument is masked by the notation. When unmasked, the argument is

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[16] If a table, T_1, has its origin in a hunk of wood, H_1, at one world then T_1 must have its origin in H_1 at all other worlds as well, except that there are worlds where H_1 fails to exist and T_1, therefore, also fails to exist. However, there are still other worlds at which H_1 exists without T_1 existing; the existence of H_1 is a necessary but not sufficient condition for T_1 to exist. (Hence, the set of worlds at which T_1 exists is a proper subset of the set of worlds at which H_1 exists.) Rohrbaugh and deRosset allow that at worlds lacking T_1, H_1 may be the origin of wooden objects different from T_1 or of no artefacts at all. However, their principle of *origin uniqueness* (ibid., p. 715) is not immune to the infinite-regress objection. The principle grounds the necessary distinctness of T_1, T_2 in the distinctness of their origins H_1, H_2. But the necessary distinctness of H_1, H_2 must in turn be grounded in the distinctness of *their* origins; and so on, with no end in sight.

[17] Cameron (2005, p. 264) says, 'Given a block of wood I could make a table that was four-legged or three-legged, tall or short, round or square, thin or wide. Am I to believe that it would be the same table I was making in each case?' Cameron thinks not, citing a lack of essentialist intuitions. But the obvious answer is Yes—for being four-legged and all the rest are all *accidental* properties of one and the same table (entailing that the table might be many different *kinds* of table).

(1.1.1)	$(a = a) \supset \Box (a = a)$
(2.1.1)	$a = a$
(3.1.1)	$\Box (a = a)$.

Further, there is nothing a posteriori in the premises or the conclusion of either of the notational variants of the argument. It only requires linguistic competence to know whether '*a*', '*b*' co-denote, and linguistic competence is acquired a priori.[18]

The necessitation of *Fa* and *Origin*⟨*a*, *b*⟩, by contrast, must be another, since neither is a logical truth. Once we start casting about for an alternative sort of necessity, the only reasonable candidate would be nomological necessity, since the necessity of *Fa* and *Origin*⟨*a*, *b*⟩ is supposed to be a posteriori. If 'metaphysical' necessity reduces to nomological necessity, then the former is redundant and can be done away with. It is not obvious to us what the added value of the category of 'metaphysical' modality might be. In TIL, at least, there is neither need nor room for it. What we suggest instead are analytic and nomological necessity, but the former seems to be too strong, and the latter too weak, to match the intended modal profile of metaphysical necessity.[19]

Alleged cases of necessity a posteriori are what Kripke terms 'theoretical identifications' (1980, pp. 99ff).[20] One famous example is that water is H_2O.[21] TIL makes available two ways of construing this 'identification'. The first is

$$[{}^0Req_1\ {}^0F\ {}^0G]$$

$F, G/(o\iota)_{\tau\omega}$. If F, G are co-intensional, then $F = G$, which is trivial; so *being water* and *having the molecular structure* H_2O would need to be two different properties. But if they are different, which should be a requisite of the other? The choice is

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[18] See Sections 3.3, 4.3.

[19] Though there is no way to find out, since no advocate of metaphysical modality that we are aware of has ever bothered to actually define the notion. This is not a satisfactory situation, considering the frequency and abandon with which the notion is being bandied about.

[20] Kripke famously claims that, 'One might very well discover essence empirically.' (1980, p. 110.) We agree with Nortmann's qualification of this claim. A chemist, he says, may very well discover the essence (e.g., the molecular structure) of some liquid; but he can hardly be said to have discovered *that* this molecular structure (or whatever) is the essence of the liquid in question. Discovering what the essence of some stuff is, is not a purely empirical matter ('keine allein in der Natur vorfindbare Tatsache'), as it also contains conventional components, (ibid., p. 10.) *Perhaps* Kripke makes a similar qualification in 1971 (p. 153) when claiming that one knows by philosophical analysis a priori that if some table is made of wood then it is necessarily not made of ice, while knowing a posteriori whether some particular table is wooden.

[21] The discussion appears to fall within a larger discussion of identity sentences, yet two examples of theoretical identifications Kripke gives are 'light is *a* stream of photons' and 'lightning is *an* electrical discharge' (ibid., p. 116, emphasis ours); so theoretical identifications need not be phrased as identity sentences; so it is not certain that the identification of water as H_2O should be, either.

obvious: *Having the molecular structure* H_2O should be the requisite in order to define liquids as water. The resulting modality is a priori, *necessary* and *analytic*. It leaves open the possibilities that G (i.e., *being water*) should have more requisites than just F (i.e., *having the molecular structure* H_2O) and, if so, that something should have the molecular structure H_2O without being water. The second construal is

$$\lambda w\,[\forall t\,[[{}^0Exist_{wt}\,{}^0G] \supset [\forall x\,[{}^0True_{wt}\,\lambda w\lambda t\,[{}^0G_{wt}\,x] \supset [{}^0F_{wt}\,x]]]]].$$

The resulting modality is a posteriori, logically *contingent* and at least *quasi-nomological*. It leaves open the possibility that there be possible worlds outside the set of worlds so constructed at which it does not follow that if x has G then x also has F. It falls to chemistry to ascertain whether the actual world is an element of the set of worlds just constructed. Neither construal, however, can be the full story about whether water is H_2O. For H_2O will in turn have to be 'identified' (we would prefer: defined), which can happen only relative to a body of chemical propositions; i.e., a chemical theory. A venture into philosophy of science would take us too far afield; here we intended merely to point out the possibilities of (partially) defining G in terms of F and of offering a construal on which Kripke's theoretical identification comes out both a posteriori and necessary, albeit not 'metaphysically' but physically so.

The thesis of the necessity of origin claims that it is 'metaphysically' necessary for a given individual a to have its material origin in some other particular individual b. If a is a wooden table then if a is a wooden table and b is a chunk of wood then if b is a's origin then this is so as a matter of 'metaphysical' necessity. Or if a is a person (better: a human body) and b another person (better: another human body) then if b is a's origin then this is so as a matter of 'metaphysical' necessity. TIL offers two ways of construing

Necessarily, a is an offspring of b.

The first makes the property of being an offspring of b a requisite of the office A:

$$[{}^0Req_3\,\lambda w\lambda t\,[\lambda x\,[{}^0Offspring_{wt}\,x\,{}^0b]]\,{}^0A].$$

Types: $x \rightarrow \iota$; $Offspring/(o\iota\iota)_{\tau\omega}$; b/ι; $A/\iota_{\tau\omega}$.

Alternatively, *Offspring* may take an office B as its argument.[22] Whether b or B, this construal is not in Kripke's spirit, since *Offspring* is a relation-in-*intension* between a particular individual b and whatever individual (if any) is the value of A at $\langle w, t\rangle$ or between two such values (i.e., B_{wt}, A_{wt}). Kripke seems to envision a as 'growing out of' b and being related in-extension to b. Lacking a notion akin to

[22] Some sort of 'pedigree essentialism' construed in terms of requisites may be relevant to inheritance in monarchies, clan-based Stalinist regimes and suchlike. By the way, Kripke's origin essentialism comes with a tacit physicalist premise pertaining to personal identity that we see no cogent reason for adopting.

requisite, Kripke is in no position to define a relation-in-extension between two intensions, but only relations-in-intension or relations- in-extension between two individuals. The former will make it contingent that *a* is related to *b* as its offspring; only the latter will make their relation necessary. But care must be taken not to turn the Kripkean offspring relation into a logical or mathematical relation. The means Kripke has available to him are his *accessibility* relations between worlds. The worlds at which the offspring relation between two individuals obtains need to be restricted to a proper subset of logical space. Some worlds will need to be inaccessible from the actual world, such that there will be worlds at which it is not true that *a* is the offspring of *b*.[23] The accessibility relations can be modelled as functions from a world *w* of evaluation to a set of worlds accessible from *w*, making their type ((οω)ω). As is seen, the accessibility relation is an intension. In any sub-S5 system the resulting set of worlds will be a proper subset of logical space. Hence, the accessibility relation characterizing the given system will be a non-trivial intension. Hence, it will be a posteriori whether *b* is the origin of *a*. In S5 the equivalence relation will still be an intension, but non-triviality will instead have to be obtained by means of varying domains, making it a posteriori whether *a* and *b* exist.

The recourse to accessibility relations is a viable alternative to our requisite proposal. But there is a philosophical problem. Which is that the accessibility proposal fails to indicate which particular modal system (equipped with a particular accessibility relation defined over its frames) models metaphysical necessity best. This failure is symptomatic of what we see as a lacuna in Kripke's oeuvre. There is the mathematical logic of the accessibility relations. And there is the intuitively argued philosophy of metaphysical modality. But there is no philosophical logic to bridge between the two by privileging one particular modal system. (On the other hand, it is widely agreed that S5 best models logical modality, that S4 best models intuitionistic logic and epistemic modality, that S4.3 best models temporal modality, etc.)

Our second construal is

$$\lambda w\ [\forall t\ [{}^0Occ_{wt}\ {}^0A] \supset [{}^0Offspring_{wt}\ {}^0A_{wt}\ \ {}^0b]].$$

(Again, *b* may be replaced by B_{wt}.) This makes it at least quasi-nomologically necessary that whenever *A* is occupied its occupant originates from b/B_{wt}. The modal profile of this proposition is necessity a posteriori. The construal fails to exclude that there be a possible world at which *A* is occupied and A_{wt} is not an offspring of b/B_{wt}. This is fine, since any such world will be inaccessible from the world of evaluation. But again, it is not clear what the formal properties of the accessibility

[23] Thus, Cocchiarella says, '[N]ot only need not all the worlds in a given logical space be in the model structure..., even the worlds in the model structure need not all be possible alternatives to one another... Clearly, such a restriction...only deepens the sense in which the necessity in question is no longer a logical but a material or metaphysical modality.' (1984, p. 323).

relation would have to be. Hence, it is not clear, either, which modal system is best suited to model such a possibility.

We wish to push the point that metaphysical necessity is best identified with (or 'reduced to') physical/nomological necessity.[24] The connection between physical and metaphysical modality is inspired by a remark made by Graeme Forbes:

> We need a theory according to which our conception of the thisness of an individual is formed in the temporal case and then projected to transworld identity, to fix the boundaries of significance on de re hypotheses about the individual (1985, p. 147, n. 11).

It seems fair enough that once individual *a* is a wooden table, *a* could not, 'metaphysically' speaking, have been an elephant nor ever become one. The physical building-blocks making up a wooden table are not the right stuff for making an elephant(!), or *vice versa*. As for the traffic up and down the temporal axis we have no quarrel with metaphysical modality thus construed. The construction following below constructs a set of worlds V such that for each individual x which, at any $w \in V$, is a table there is no moment t at which x is an elephant. In more natural English, if something is a table in V then it is never an elephant in V.

$$\lambda w\ [\forall t\ [\forall x\ [[{}^0Table_{wt}\ x] \supset \neg[\exists t'\ [{}^0Elephant_{wt'}\ x]]]]].$$

The laws of physics, biology, chemistry, etc., that rule within V rule out the physical possibility that a table turn into an elephant. Even cutting-edge physical, biochemical, etc., engineering, no matter its stage of development, will bump up against the laws of nature that hold sway in V. However, there must be other classes of worlds where a table can indeed turn into an elephant. The laws of nature obtaining at those worlds are well likely to defy human comprehension. What is more, it is even conceivable, and logically possible, that there should be worlds devoid of laws of nature.[25] Such mind-boggling worlds must be capable of existing,

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[24] This is argued by, e.g., Cocchiarella (1984, p. 325), Berkovski (2005) , Farrell (1981), whereas Kripke appears to be suggesting that metaphysical modality is identical, or else very close, to logical modality; see, e.g., (1980, pp. 99, 125). Rohrbaugh and DeRosset (2004) assume an independent category of metaphysical necessity, yet fail in our view to differentiate it from either nomological or logical necessity. On the one hand, when talking about the production processes from hunks of wood to wooden tables, the possibilities and impossibilities they consider are in effect nomological (ibid., pp. 711ff). On the other hand, all four formulae in the formal argument in ibid. (p. 715, fn. 18) contain strings like '…□…⇒…'. But ⇒ is entailment, which is a *logical* relation between (hyper-) propositions and too strong for metaphysical necessity, provided it is to be something other than logical necessity. Furthermore, '…□…⇒…' looks like overkill. Entailment is defined as the necessitation of implication, $\Box(p \supset q)$; so what is the point and sense of necessitating entailment?

[25] We do not consider the—admittedly interesting—question of whether possible worlds devoid of laws of nature could possibly have elephants and tables in them. A reasoned answer to this question would presuppose a discussion of what the nomological prerequisites are for a given intension to be instantiated.

since otherwise the laws of logic and mathematics would coincide extensionally with the laws of the natural sciences.[26] For instance, Hanson says,

> No one has ever succeeded in building [a *perpetuum mobile*]. And, given *our* physical world, no one ever will. … But it need not be self-contradictory to suppose [this circumstance] to obtain; it would just be false. [Both "A *perpetuum mobile* is impossible" and "Nothing travels father than light" are] not conceivably false and yet not tautologically true (1967, p. 88).

We propose the term 'temporal essentialism' to stand for the doctrine above in terms of which we interpret metaphysical modality. The temporal essentialist now makes the further claim that no individual that *exists* within V exists without V. If, *per impossibile*, this were the case then we might indeed have an example of an individual that was a table in one world and an elephant in another. But the notion of metaphysical modality was launched exactly to narrow the modal span of an object down to what is physically, or temporally, possible within some subset of *all* the logically possible worlds. A thought-provoking passage in Forbes reads:

> It is presumably true that more or less anything can develop into more or less anything, given sufficiently sophisticated engineering, so taking the acorn c which grows into a certain oak tree in the actual world, we can consider a world where c is treated in such a way that it develops into a small vegetable. Then (PI) entails that that oak tree could have been, e.g., a cabbage, and therefore that there are entities which can be oak trees in some world and cabbages in others (Ibid., p. 146).

(PI) says: if x at world u has the same propagules as y at world v then $x = y$. Forbes rejects (PI) on the ground that the principle points toward bare particulars by allowing what he calls 'ungrounded identity'. However, our bare particular anti-essentialism is not predicated on applying engineering, whether sophisticated or pedestrian, to acorns, zygotes, or whatnot. Introducing cunning engineering into the story gives the wrong idea about what counterfactual scenarios involving essences are all about.

Metaphysical modality depends on fixing some set of worlds within which one member plays the role of the 'home world' from which all the other worlds are targeted as 'merely possible or non-actual'. But such a set of worlds would, *ex hypothesī*, not exhaust all of *logical* space. We suspect that the notion of metaphysical modality is fuelled by the illusion that philosophical investigations can somehow fix the modal span of at least some kinds of object. For instance, an acorn, genetically or otherwise tampered with, may turn into a cabbage rather than an oak, but surely not into an elephant or a wooden table. Or so the intuition goes.

But why not? It is hardly acceptable that the laws of nature of some particular set of worlds, for instance, those of the set of worlds containing the actual world as a member, should play any role in analytic philosophising, which is concerned with conceptual analysis. Yet this is exactly what happens when the empirical

[26] Worlds whose laws of nature deviate from the actual ones are what G. Priest calls 'nomologically impossible worlds' (1992, p. 292).

laws defining *V* are allowed to determine which properties *b* might possibly have had and which not. The kind of engineering that could possibly be applied to Forbes' *c* in *V* will be hedged in by the laws of *V*. It follows then that *c* may at most exhibit its full *physical*, or 'metaphysical', potential within *V*, but not its full *logical* potential. We are, therefore, in flat opposition to the second half of the quote by Forbes:

> In the time of a single world, the same individual can undergo a change of sex, but it is less clear that an individual of one sex could have been, from the outset, an individual of another [.] (Ibid., p. 148).

If 'from the outset' means from the beginning of time within *V* then the truth of the claim presupposes metaphysical necessity. If 'from the outset' means from the beginning of time within logical space *in totō* then bare particular anti-essentialism is only happy to embrace that possibility. The way we look at it, the question should not be whether anything can become anything else thanks to engineering, which is something drawing upon the notion of natural laws. Instead the question ought to be whether anything could turn into anything else thanks to logic. In the case of intensions, the answer is a resounding No. In the case of individuals, the answer is a no less resounding Yes. In logical space the sky is the limit (Which is not to say that TIL spills over into the space of logical impossibilities).

Our quarrel with temporal essentialism is not only to do with its stealing empirical laws into questions of essence. A narrower objection concerns *existence*. Consider this Closure:

$$\lambda w\ [\forall t\ [[{}^0 Wooden_{wt}\ {}^0 a] \equiv [{}^0 Exist'_{wt}\ {}^0 a]]].$$

Types: *Wooden*, $Exist'/(o\iota)_{\tau\omega}$. The point is this: individual *a* exists′ wherever and whenever *a* is wooden and is wooden whenever and wherever it exists; so *a* is essentially wooden.

Our objection concerns existence as an $(o\iota)_{\tau\omega}$-entity.[27] Within an intensional system the tendency would be to conceive of existence as something along the lines of an $(o(\alpha_{\tau\omega}))_{\tau\omega}$-entity: an empirical property of intensions. By contrast, *Exist′* above would come out a trivial intension, returning as it would for every $\langle w, t\rangle$ the set of those objects that are the elements of the universe of discourse. Existence, on our theory, is the property an intension *Int* exemplifies at those $\langle w, t\rangle$ pairs at which *Int* is occupied/instantiated. What is fundamentally at play is probably that when we speak of *individuals*, intending ι-entities (i.e., bare particulars), those who construe existence as a non-trivial property of what they call 'individuals' intend what we would take to be something like *persons*, typing personhood as $(o\iota)_{\tau\omega}$. For now it will suffice to observe that conceptualising *Person* as an intension turns it into the right sort of thing to come into and go out of existence non-trivially. Thus, rather than operating with varying domains we operate with

[27] See also Section 2.3.

modally and temporally varying extensions of *Person*. It is along these lines we would make sense of the claims that there might have been more or fewer persons, or that there might have been other persons than those who actually exist.

Tichý's strongest argument against varying domains is this:

> Suppose that an unactualized world *W* featuring a unique winged horse has been successfully specified. Will the winged horse of *W* constitute an example of an individual absent from the actual world? Not necessarily. Having wings is surely a contingent matter. Hence the horse which is winged in *W* will presumably be wingless in some other worlds. The actual world, where wingless horses are legion, may well be one of these worlds. Should this turn out to be the case, the individual in question would not be missing from the actual world after all. Thus in order to furnish an example of an individual which is actually missing, *W* would have to be specified as a world in which the [office] of the winged horse is filled by an individual numerically distinct from all individuals existing in the actual world. But how can this be done? If there are non-existent individuals, there will presumably be more than one. Clearly any world in which one of them is the winged horse is distinct from any world in which another one is. *W* won't be specified until it is specified *which* non-existent individual is its winged horse. The task of giving an example of a non-existent individual is thus hardly facilitated by appeal to the [office] *the winged horse*. To be able to exploit the [office] in pinpointing such an individual, one has to have an epistemic handle on the individual's numerical identity in the first place (1988, p. 181).

The argument, in a nutshell, is the following. A non-actual individual cannot be identified by ostension but only by description. So one might attempt to identify some numerically specific individual as the unique *F* at $\langle w, t\rangle$. But the individual office of the unique *F* will not be powerful enough to identify, or pinpoint, some numerically specific individual, for the occupant of the *F*-office at $\langle w, t\rangle$ will just be whoever or whatever is the unique *F* at $\langle w, t\rangle$. (Worse, the *F*-office may even fail to take a value at $\langle w, t\rangle$.) The specification of *which* (non-actual) individual is the unique *F* at $\langle w, t\rangle$ will thus be circular. This incapacity to pinpoint a numerically specific individual is shared by all offices. What is required is identification of an individual independently of its satisfying some condition at some $\langle w, t\rangle$. This brings us back to ostension; but again, ostension is inapplicable to non-actuals.[28]

If existence is no longer a property non-trivially applicable to individuals, but is instead a property of intensions, the construction $\lambda w\ [\forall t\ [[{}^0Wooden_{wt}\ {}^0a] \equiv [{}^0Exist_{wt}\ {}^0a]]]$, $Exist/(o(\iota_{\tau\omega}))_{\tau\omega}$, will simply involve a type-theoretic category mistake. It would be impossible, for this reason, to define non-trivial essential properties in terms of the (non-) existence of individuals. For instance, one among countless ways of defining equivalence classes of worlds is in terms of the existence of some particular individual *a*. The essential properties of *a* will be just those that *a*

[28] A recent discussion of a cluster of arguments whose conclusion is that Aristotle exists necessarily is a good example of what we have in mind (see Stephanou, 2000). Stephanou assumes that people would find the conclusion counter-intuitive because they would find it unacceptable that Aristotle should exist of necessity. But nobody is in a position to know whom Stephanou is talking about for want of a description of the intended individual. An individual office would have come in handy.

exemplifies in all worlds within that class. However, since existence applies only trivially to individuals, none of *a*'s properties exemplified anywhere will be both essential and non-trivial.

By adhering to a fixed domain of discourse, TIL adheres to ontological actualism. The only individuals we acknowledge are actual, eschewing merely possible individuals. Simchen also espouses ontological actualism in 2004, 2006,[29] but proposes a rival solution to how actualism can maintain

> [T]hat there are no merely possible things in the face of properties that are both actually uninstantiated and cannot be had contingently (2006, p. 9).

Simchen's solution centres around a notion of innate *potentiality* suggesting some brand of ('Aristotelian') realism *in rē*:

> An oak seed is no possible oak and a fertilized human egg is no possible human. But an oak seed and a fertilized human egg are *potentially* an oak and a human, respectively. Potentiality is a matter pertaining to what the seed and the egg might become. Potentiality is possible becoming (2006, p. 21).

The intuitive idea seems to be that an individual with a given property (e.g., the property of being a fertilized human egg) is a thing with a potential that circumscribes the modal variability of this thing and anything like it. For instance, though the properties of being a donkey and being a talker may be co-instantiated consistently, no donkey is potentially a talking donkey (nor is any talker potentially a talking donkey): 'So it is impossible that there be a talking donkey.' (Ibid., p. 8.)

But Simchen pays little attention to the fact that anchoring modal variability to potentiality goes via *nomological* modality. We may grant that, given the actual laws of nature, no donkey will have the potential, or make-up, to be able to talk. But, if a donkey lacking the actual potentiality to talk is transplanted to a world obeying relevantly different laws of nature (or perhaps none at all?), its potentiality thus embedded may (logically speaking) well manifest itself differently in such a way as to enable it to talk. Or the other way around with a talker being a donkey at such a world.[30] So the properties of being a donkey and a talker will have intersecting extensions at at least one $\langle w, t\rangle$. Whether the actual world and the present moment is such a pair can be established only empirically.

Our solution to the problem Simchen wishes ontological actualism to take on is this. If *G* (e.g., *being a mammal*) is actually uninstantiated,

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[29] Simchen claims, 'To be an [ontological] actualist requires dealing with the metaphysics directly and letting the logic track the metaphysics rather than the other way around.' (2006, p. 18.) We beg to differ, since this conceptual order of priority is tantamount to rejecting analytic philosophy as we know and love it. Analytic philosophy starts out with a logical analysis of expressions and concepts pertaining to a particular discourse on (say) metaphysics and only then enters the sphere of (say) metaphysics proper.

[30] Similarly, though less interestingly, it is logically possible that there be nomologically more restrictive worlds at which the donkeys' (the talkers') potential is a fraction of what it is at the actual world.

$$\lambda w \lambda t \neg [{}^0Exist_{wt} \; {}^0G]$$

and if G is a requisite of F (e.g., *being a whale*),

$$[{}^0Req \; {}^0G \; {}^0F]$$

then necessarily, if there had been F's then there would also have been G's. In case G is instantiated at any $\langle w, t \rangle$ only in its capacity as requisite, then G cannot be had contingently. That is, no individual is ever a mammal, pure and simple. Something is a mammal only in virtue of being some particular kind of animal (be it a zebra, a human being, a wombat, or whatnot).

Now, having rejected individual essentialism, our particulars are 'bare', not in the sense of lacking properties at any world and time, but in the sense of possessing no purely contingent properties of necessity.[31] The introduction of bare particulars is the only way to preserve the non-triviality of the predication that a is an F. Imagine now that there is an object before you that you wish to take a closer look at. After turning it inside out and upside down, you make the observations that it is a table, is wooden, is two metres long, and dark-brown. Could these four pieces of knowledge have been obtained a priori? Surely not. Only empirical inquiry can decide what is actually and presently true of the individual you are taking apart. At the beginning of the inquiry the individual can rationally be checked for any property whatsoever: is it a planet, a table, an elephant, a speck of dust, etc.? At this initial stage logic is no guide to any of its actual properties. As the results start coming in, logic will become useful, however. For instance, if the object before you is a Roman Catholic cardinal, you may infer, thanks to the requisites of cardinalhood, that the individual is also a human being, a man of faith, fluent in Latin, and a host of others. Also an infinite string of properties can be ruled out. Since no cardinal is inanimate, it follows that he is not inanimate, and since only inanimate objects can be planets, he cannot be a planet. The point is that the empirical investigation must begin from absolute scratch. If some purely contingent properties were true of the individual a priori, the empirical tests would already have something to begin from. But then it would not be informative to get to know that the object before you was a table, say, rather than a cardinal; it would be just as exciting as getting to know that the individual was self-identical. Yet it seems incontrovertible that by correctly ranking a among the tables and not among the cardinals you have made a discovery about the actual world: you have established that the actual world belongs to that set of worlds where it is true that a is a table. Had a radically different world been actual instead, a would not have been a table, but a cardinal, a banknote, a drop of water, or whatever, and your ranking a among the tables would have been a miss instead of a hit.

David Lewis (crediting Tichý with making him think less unfavourably of bare particulars) would call bare particular anti-essentialism *extreme haecceitism* (1986, pp. 293ff). A haecceitist is someone who thinks that above and beyond its

[31] For purely contingent properties, see Section 1.4.2.1. See Bergmann (1967, pp. 24ff) for the term 'bare particular'.

qualities an individual has a non-qualitative core. A haecceitist is extreme if no qualities are privileged in the sense of forming a protective belt around the core. Lewis, needless to say, has little time for haecceitism, but basically argues that if somebody wants to be a haecceitist then they would be much better off being an extreme haecceitist. The reason is that the latter discharges themselves of a burden that the former will have to lift. The burden is how to lay down the qualitative constraints which would constitute the protective belt of some individual (or species or natural kind as well, presumably). Certain choices of qualities might intuitively have something going for them, but justifying those intuitions is hard. We would add that it is hard also to formulate such a protective belt of qualities if those qualities are to be drawn from among purely contingent intensions without infringing their non-triviality. The situation is somewhat simpler for the extreme haecceitist. In Lewis' words,

> A moderate haecceitist says that there are qualitative constraints on haecceitistic difference; there is no world at all, however inaccessible, where you are a poached egg. Why not? He owes us some sort of answer, and it may be no easy thing to find a good one. Once you start it's hard to stop—those theories that allow haecceitistic differences at all do not provide any very good way to limit them. The extreme haecceitist needn't explain the limits—because he says there aren't any (Ibid., p. 241).

We draw from this the morale that since we are trafficking in bare particulars, we ought to make sure that they really are bare and not clad, however scantily, in a few select intrinsic non-trivial qualities. Otherwise we end up with individual essentialism. In Tichý's words,

> [T]he notion of object and that of an [intension] of an object are conflated and the result is presented as the doctrine of individual essentialism. According to this doctrine, the properties instantiated by an individual divide into two kinds: accidental and essential. Accidental properties are those that the individual might conceivably lack. Essential properties are those which the individual could not possibly lack. It is beyond dispute that every individual instantiates properties which are essential in this sense. Self-identity, and membership of any class to which the individual belongs, are examples of such. Elizabeth II, for example, could not possibly fail to be identical with herself, or fail to be a member of a class consisting of herself and Prince Philip, and so on. But the thesis of individual essentialism is to the effect that not all essential properties are of this trivial sort; some of them, it maintains, are substantive and their possession by an individual can be established only empirically (1988, p. 185).

That is, also TIL admits of a kind of individual essentialism, but of a hollow kind, since the necessity of $a = a$ or $a \in \{\ldots, a, \ldots\}$ is logical, not 'metaphysical'. There is nothing about those two necessities that could furnish a with a qualitative core.

It might seem as if we had flung the door open to anarchy at this point. As Stalnaker rightly observes, any individual might have had the properties of any other:

> [I]f [Babe Ruth] does have the logical potential to be a billiard ball, it is of no interest that he does since on the bare particular theory this does not distinguish him from anything else (Ibid., p. 349).

True, individuals are indistinguishable as far as their logical potential goes. What is possibly true of one individual is also possibly true of any other individual. Still, no two individuals can be (in) the extension of the same intensions at all the same possible worlds at the same time. For instance, no two individuals can be the extension of the intension *the King of France* at the same $\langle w, t\rangle$. Since individuals are ground types in TIL, they are logical atoms and, therefore, pairwise disjoint, their identity and difference being a matter of bare, or numerical, identity and difference. On the other hand, each and every individual gets to be King of France at some $\langle w, t\rangle$ or other. Individuals, which are the same for all worlds and times, are in and by themselves nothing but numerical individuators that exemplify any empirical property only contingently. As Ruth Barcan Marcus says in so many words, what we want is the 'description-neutral peg on which to hang descriptions across possible worlds' (1993, p. 61). Individuals as such are of little logical importance, since they are not themselves functions, but only functional arguments or values.

So while anarchy, if you like, does rule in the extensional basement, order reigns on the intensional ground floor in virtue of the requisites hosted there. The state space of any individual is bounded only by type-theoretic constraints. For instance, it is impossible for any individual to be a prime number, since functions of type ($o\tau$) do not apply to ι-objects. But intensions are bounded not only by type theory but also by other intensions.[32] What we are interested in is studying the conceptual interplay between intensions, and not the interplay between intensions and extensions, which is roughly the question of which individuals have which properties at which worlds and times.[33]

For a final illustration of the distinction between intensional and extensional, or individual, essentialism, consider the difference between two construals *de dicto* and a construal *de re* of the sentence

'Wooden tables are necessarily wooden'.

The intensional, or *de dicto*, construals make it a necessary truth that wooden tables are wooden. The first construal expresses the Composition

[32] Along similar lines, Jaakko Hintikka says, '[I]n the question, Who administers the oath to a new President?, the relevant alternatives might be the different officers (offices) (Secretary of State, Chief Justice, Speaker of the House, etc.) rather than persons holding them. Then my criterion of answerhood will require that the questioner knows what office it is that an answer refers to, not that he knows who the person is who holds it' (Hintikka, 1962, p. 45). Similarly, Fred Dretske says, 'Once an object occupies such an office, its activities are constrained by the set of relations connecting that office to other offices…; it must do some things, and it cannot do other things' (1977, pp. 264ff). To be sure, Dretske is concerned to make an analogy between legal and nomological modalities, but his discussion of what he himself dubs 'offices' is kindred to ours, particularly 'by talking about the relevant properties rather than the sets of things that have these properties' (ibid., p. 266).

[33] For examples of such interplay, see Sections 3.3.1, 4.3.

$$[{}^0Req_1\ {}^0Wooden\ \lambda w \lambda t\ \lambda x\ [[{}^0Wooden_{wt}\ x] \wedge [{}^0Table_{wt}\ x]]]$$

Type: *Wooden*, *Table*/$(o\iota)_{\tau\omega}$.

The second construal *de dicto* mixes *Wooden*/$(o\iota)_{\tau\omega}$ with the property modifier *Wooden′*/$((o\iota)_{\tau\omega}\,(o\iota)_{\tau\omega})$:[34]

$$[{}^0Req_1\ {}^0Wooden\ [{}^0Wooden'\ {}^0Table]].$$

The extensional, or *de re*, construal makes the sentence denote a falsehood, for now it expresses the Closure

$$\lambda w \lambda t\ \forall x\ [[[{}^0Wooden_{wt}\ x] \wedge [{}^0Table_{wt}\ x]] \supset \forall w'\ \forall t'\ [{}^0Wooden_{w't'}\ x]].$$

The so constructed proposition returns **T** at those $\langle w, t\rangle$ at which it holds that, for all x, if x exemplifies *Table* and *Wooden* then it is necessary that x exemplifies *Wooden*. Does any $\langle w, t\rangle$ satisfy this truth-condition? Yes; thanks to the truth-table for $\supset$, the condition is satisfied by all and only $\langle w, t\rangle$ pairs that falsify the antecedent. The proposition returns **F** for all the remaining $\langle w, t\rangle$-pairs.

By way of summary, it is rigid what the requisites of an intension are, and it is flexible who or what instantiates or occupies a given intension. This general point can be rephrased thus. Any instance of the instantiation or occupation relation between an individual *a* and a purely contingent intension *Int* is accidental; and: some instances of the co-instantiation or co-occupation relation between any two intensions Int_i, Int_j are analytically necessary, such that every instance of necessary co-instantiation or co-occupation of intensions is an instance of a *requisite* relation.

4.2.1 Quine's mathematical cyclist

Quine put forward his by now famous biking-mathematician example, in 1960, to create the paradox that it is both necessary and not necessary of the same individual that it be rational and bipedal. However, the argument rides on flat tyres, as pointed out in Plantinga (1974, Chapter 2), Marcus (1993, Chapter 1). At the same time, we agree with the purpose for which Quine put forward his argument; he wished to show that individual essentialism is incoherent. In the previous Section 4.2 we also argued against individual essentialism—but in favour of an essentialism of a different ilk, which we called *intensional essentialism*.

We can make explicit the fallacy of Quine's argument using the notion of requisite (see Section 4.1). Quine's argument goes as follows.

[34] See Section 4.4 on property modification.

1. *Mathematicians are necessarily rational but not necessarily bipedal.*
2. *Cyclists are necessarily bipedal but not necessarily rational.*
3. *Charles is both a cyclist and a mathematician.*
4. ∴*Charles is necessarily rational but not necessarily bipedal.*
5. ∴*Charles is necessarily bipedal but not necessarily rational.*

Contradiction, for conclusion (4) contradicts conclusion (5).

Let $M, R, C, B/(o\iota)_{\tau\omega}$); Ch/ι. Then

1′. $[^0Req\ ^0R\ ^0M], [^0\neg[^0Req\ ^0B\ ^0M]$

2′. $[^0Req\ ^0B\ ^0C],\ [^0\neg[^0Req\ ^0R\ ^0C]$

3′. $\lambda w\lambda t\ [^0C_{wt}\ ^0Ch],\ \lambda w\lambda t\ [^0M_{wt}\ ^0Ch]$.

The definition of the requisite relation between individual properties yields:

1″. $\forall w\forall t\ [\forall x\ [[^0M_{wt}x] \supset [^0R_{wt}x]]], \exists w\exists t\ [\exists x\ [[^0M_{wt}x] \wedge \neg[^0B_{wt}x]]]$

2″. $\forall w\forall t\ [\forall x\ [[^0C_{wt}x] \supset [^0B_{wt}x]]],\ \exists w\exists t\ [\exists x\ [[^0C_{wt}x] \wedge \neg[^0R_{wt}x]]]$.

From 1″ and 3′ we get

$$\lambda w\lambda t\ [^0R_{wt}\ ^0Ch], \text{ but not } \forall w\forall t\ [^0R_{wt}\ ^0Ch],$$

and from 2″ and 3′

$$\lambda w\lambda t\ [^0B_{wt}\ ^0Ch], \text{ but not } \forall w\forall t\ [^0B_{wt}\ ^0Ch].$$

It is not possible to derive $\lambda w\lambda t\ [\neg[^0B_{wt}\ ^0Ch]]$ or $\lambda w\lambda t\ [\neg[^0R_{wt}\ ^0Ch]]$. The point is that (3) is not *Charles is necessarily both a cyclist and a mathematician.*

Here explicit intensionalization has shown what also Marcus showed when pointing out that from $\Box(A \supset B)$ it follows that $(\Box A \supset \Box B)$, while $(A \supset \Box B)$ does not. The fallacy thrives on confusing the necessitation of the consequence with the necessitation of the consequent.

4.3 Requisites and substitution in simple sentences

The discussion of the semantics, pragmatics and logic of so-called simple sentences like ‘It is raining’ has received renewed attention over the last 10 years in the form of a substitution puzzle involving ‘Superman’ and ‘Clark Kent’.[35] The discussion is due to Saul (1997). According to Saul’s (negative) characterisation, simple sentences are ‘sentences which contain no attitude, modal, or quotational constructions’ (1997, p. 102, n. 1).

[35] See, for instance, Barber (2000), Forbes (1997, 1999), Moore (1999), Pitt (2001), Predelli (2004), and Spencer (2006). For further critique of Saul’s puzzles, see Jespersen (2008b).

In Section 2.7 we discuss the principle of substitution, claiming that the substitution of co-referential expressions is valid when the expressions occur in *de re* supposition. The 'Superman'/'Clark Kent' puzzle appears to throw doubts on the principle. In this section we show that the substitution principle *is* valid in the *de re* case. The 'puzzle' can be easily explained away by showing (a) that there are two readings (one *de dicto*, the other *de re*) of the sentence 'Superman is Clark Kent', such that on its *de re* reading 'Superman' and 'Clark Kent' are not necessarily co-referential, (b) that substitution is invalid due to a shift in time, and (c) that the *de dicto* reading of 'Superman is Clark Kent' is analysed as expressing that an antisymmetric requisite relation obtains between the two individual offices of Superman and Clark Kent.[36]

First, we examine whether a certain argument whose validity Saul has drawn into doubt is valid. We conclude, uncontroversially, that the argument is obviously valid, provided Leibniz's Law applies. Then we offer an alternative analysis of the premises and the conclusion based on the notion of requisite (see Section 4.1) and along the lines of the analysis of 'Hesperus is Phosphorus' (see Section 3.3.1). The TIL analysis is intended to bring out the rational core of the anti-substitution sentiments that the 'Millian' analysis is unable to bring out. On the 'Millian' analysis of 'Superman is Clark Kent' the sentence just means that an individual bearing two different names is self-identical. Our aim is to demonstrate that a purely semantic explanation of the anti-substitution intuitions rivalling the prevalent pragmatics-based ones is available.[37]

The puzzle we investigate here substitutes 'Clark Kent' for 'Superman' in 'Clark Kent enters the phone booth and Superman emerges' (see Saul, ibid., p. 102). Call it (*).[38] If the anti-substitution intuitions are correct, then (*) will at at least one world/time have this distribution of truth-values:

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[36] If being Superman is a *sufficient* condition for being Clark Kent, whereas being Clark Kent is a *necessary* condition for being Superman, it follows that the Superman office is *more exclusive* than the Clark Kent office, in the sense specified in Section 4.1, Definition 4.2, and the following Remark.

[37] Another attempt at a purely semantic approach is Forbes (1999), which introduces (so-called!) logophors that receive no mention in the sentences under analysis. What speaks against Forbes' proposal is, as Predelli observes (2004, p. 112), that Forbes' allegedly simple sentences are not simple, logophors being a quotational device.

[38] Saul'z puzzle bears some resemblance to the Partee puzzle from around 1970 (see Section 2.6):

The temperature is 90°F
The temperature is rising
———————————
90°F is rising.

Saul also wishes to come up with a flawed argument in order to make a point, but none of Saul's arguments in 1997 is invalid, provided Saul's semantic stipulations are accepted.

(*)

(1)	Clark Kent enters and Superman emerges	**T**
(2)	Superman = Clark Kent	**T**
	———————————————	
(3)	Clark Kent enters and Clark Kent emerges	**F**

But then (*) would have to be an *invalid* argument. Yet, if (as Saul assumes) 'Superman' and 'Clark Kent' are 'Millian' names of individuals and if Leibniz's Law is valid, then the substitution of 'Clark Kent' for 'Superman' in (3) does go through. Hence, Saul's puzzle thrives on the collision between a valid argument and an intuition to the effect that the argument is, or ought to be, invalid.

Below we consider whether one-way and two-way substitution are valid. These are the general forms of one-way and two-way substitution, respectively:

$$\frac{\ldots a \ldots b \ldots}{\ldots a \ldots a \ldots}\,(\textit{one-way}_1) \qquad \frac{\ldots a \ldots b \ldots}{\ldots b \ldots b \ldots}\,(\textit{one-way}_2)$$

$$\frac{\ldots a \ldots b \ldots}{\ldots b \ldots a \ldots}\,(\textit{two-way})$$

Two-way substitution is trivially valid due to self-implication, if the expressions are co-denoting semantic proper names, the respective conclusions being but a rephrasing of a premise in the respective arguments. But substitution can be rendered non-trivial. The solution we offer below is an extensive elaboration of one of the several candidate solutions that Saul herself considers and rejects. The solution goes a long way toward accommodating her anti-substitution intuitions by validating only one-way substitution. At the same time, it also contains the extra means to validate two-way substitution in those cases when this ought to be validated, and to block it when it should not be validated. Not so with the 'Millian' approach to 'Superman', 'Clark Kent', which validates two-way substitution *tout court*.

The semantic solution we are proposing is capable of proving that (*) is valid, whereas another argument is invalid:

(**)

(1)	Clark Kent enters and Superman emerges
(2)	Superman = Clark Kent
	———————————————
(3)	Superman enters and Superman emerges

But then the semantics of 'Superman' and 'Clark Kent' must be different from the naïve one that 'Millianism' embodies.

Our solution to the phone booth puzzle is pivoted on, first, making both ‘Superman’ and ‘Clark Kent’ denote individual offices rather than individuals and, second, conjoining these two offices by an antisymmetric requisite relation (see Section 4.1). While anything remotely like requisites and the requisite relation seem to play no role in the extensive literature on the Superman puzzle, Saul gives individual offices (what she calls ‘ordinary senses’ of singular terms) short shrift by claiming that they cannot have any of the properties that apply to individuals, such as entering and emerging from phone booths. Of course, they cannot. But Saul overlooks the fact that if an individual office is extensionalized then an individual fully capable of entering and exiting from phone booths will emerge.

More specifically, we argue that a non-trivial semantic analysis of the example should take account of the *diachronicity* of Clark Kent’s entrance and Superman’s exit while preserving the internal link between being Superman and being Clark Kent. We suggest the following. If ‘Superman’ and ‘Clark Kent’ denote two different individual offices, then ‘Superman is Clark Kent’ no longer expresses the self-identity of an individual bearing two names, but the fact that two named offices are held together by the requisite relation: wherever and whenever someone occupies the office of Superman the same individual also occupies the Clark Kent office, whereas there are exceptions to the converse. This link is preserved by arranging the two offices in a requisite relation, such that the occupant of the Superman office co-occupies the Clark Kent office, while the converse is not always true, since the Clark Kent office may be occupied without the Superman office being occupied. The semantic analysis always validates the substitution of ‘Clark Kent’ for ‘Superman’, but validates the substitution of ‘Superman’ for ‘Clark Kent’ only if the additional condition is met that somebody should occupy the Superman office.

The *rule of substitution* that Saul tacitly assumes is Leibniz’s Law of substitution of identicals for identicals. The general formulation of the rule is as follows, ‘Φ’ an n-ary predicate and ‘μ’, ‘ν’ singular terms:

(*Leibniz’s Law*)

$$\frac{\Phi\langle\mu_1, \ldots, \mu_n\rangle \qquad \mu_i = \nu_i}{\Phi\langle\mu_1, \ldots, \nu_i, \ldots, \mu_n\rangle, \text{ for any } i \in (1, \ldots, n).}$$

First-order predicate logic with identity suffices throughout to spell out the relevant measure of logical structure in (*). It is obvious what the logical structure of the respective arguments is, once we assume this logical framework and accept Saul’s assumption that the terms involved denote individuals.

(1) $Fa \wedge Gb$

(2) $a = b$

---

(3) $Fa \wedge Ga$

Leibniz's Law assumes the following form in the case of (*):

$$\frac{\begin{array}{c}\Phi(\mu_1)\\ \mu_1 = \nu_1\end{array}}{\Phi(\nu_1).}$$

Conclusion (3) follows uncontroversially from {(1), (2)} via Leibniz's Law. So does (3′), as well as (3″):

(3′) $Fb \wedge Gb$
(3″) $Fb \wedge Ga$

But those who harbour anti-substitution sentiments may object that, although we *may* validly substitute both one-way and two-way, we *ought* not to do so at least in certain simple sentences. There are pragmatic constraints on the uses of '*a*', '*b*' that are not reflected in their semantics. Pragmatically speaking, '*a*', '*b*' are not interchangeable. Consonantly with this, Saul says, when outlining a similar response,

> We accompany our favourite standard semantic account with the explanatory claim that such truth-preserving substitutions may well yield sentences which are quite misleading, due to false pragmatic implicatures (1997, p. 106).

This suggests a two-tiered policy combining valid substitution with false implicatures.[39] The perhaps most important consequence of this policy is that it locates the origin of Saul's puzzle in pragmatics and not in semantics or logic. This sort of cohabitation between pragmatics and semantics has something to be said for it, *if* 'Superman' and 'Clark Kent' are names of individuals. It pretty much allows us to have our cake and eat it. One may admit that the conclusion is contrived or baffling while at the same time leaving the validity of Leibniz's Law unscathed by (*). Analogously, the 'paradoxes of material implication' are both almost universally deemed unnatural and are at the same time classically valid.

If 'Superman' and 'Clark Kent' denote individuals then, from a logical point of view, (*) is no puzzle at all.[40] Yet both one-way and two-way substitution do leave one with a sense of dodgy reasoning. In our view this uneasy feeling can be put down to the fact that the diachronicity of Clark Kent's entrance and Superman's

[39] However, see Spencer (2006) for a convincing case that 'Russellian' philosophers of language in effect overstretch Grice's concept of implicature.

[40] Note that if *Clark Kent*, *Superman* are of type ι then both ^{0}*Kent* and ^{0}*Superman* occur with ι-intensional supposition in the premises. Thus two-way substitution is valid in intensional contexts; see Section 2.7 for the intensional rule of substitution.

exit is wholly absent from the (shallow) logical analysis considered so far.[41] It does seem to matter that it is *Clark Kent* who enters and, later, *Superman* who exits. An easy fix would be to construe Clark Kent and Superman as two different individuals, such that one guy enters and another guy later exits. But this 'fix' would falsify the second premise, that Superman is Clark Kent. So we need to preserve some internal link between being Superman and being Clark Kent. Only that link should not be self-identity.

The sentence 'Superman is Clark Kent' lends itself to two readings. On its *de dicto* reading it denotes the necessarily true proposition *TRUE* that the Clark Kent office is a requisite of the Superman office:

$$\lambda w \lambda t\ [{}^0 Req_2\ {}^0 Kent\ {}^0 Superman].$$

That is,

$$\lambda w \lambda t\ [\forall w \forall t\ [[{}^0 Occ_{wt}\ {}^0 Superman] \supset [{}^0 True_{wt}\ \lambda w \lambda t\ [{}^0 Superman_{wt} = {}^0 Kent_{wt}]]]].$$

Types: $Req_2/(o\iota_{\tau\omega}\iota_{\tau\omega})$; $Occ/(o\iota_{\tau\omega})_{\tau\omega}$; $Superman$, $Kent/\iota_{\tau\omega}$; $True/((oo_{\tau\omega})_{\tau\omega})$.

On its *de re* reading the sentence denotes the properly partial proposition *P* constructed by the Closure

$$\lambda w \lambda t\ [{}^0 Superman_{wt} = {}^0 Kent_{wt}].$$

Though *P* comes close to being necessarily true, it is not equal to *TRUE*. There are worlds/times at which *P* lacks a truth-value; namely, those worlds/times at which either Superman or Clark Kent (or both) fails to exist.

We need to operate with two distinct instants of time, for Clark Kent's entering the phone booth cannot be simultaneous with Superman's exiting it without rendering 'Superman is Clark Kent' false—nobody, including superhuman aliens, can enter and exit in one go. So Clark Kent's entrance must precede Superman's exit. To bring out the temporal profile of 'Clark Kent went into the phone booth, and Superman came out' in the logical syntax, the truth-conditions spelt out below come with an explicit time indication to capture temporal variability. Let T_0, T_1 be moments of time, such that T_0 precedes T_1. These two times are those of Clark's entrance and Superman's exit, respectively. Further, let *W* be some possible world the scenario is set at.

Our semantic analysis validates two-way substitution only if the additional condition that somebody occupy the Superman office when Clark Kent enters is met, while the substitution of 'Clark Kent' for 'Superman' in '…exits…' follows unconditionally. So we always have one-way substitution, but two-way substitution only conditionally. The lack of symmetry is due to two factors. One is the

[41] The only other commentator that we know of to point out the *non-symmetry* between being Superman and being Clark Kent caused by the *diachronicity* between Clark Kent's entrance and Superman's exit is Zimmermann (2005, p. 55, pp. 68ff).

diachronicity between Clark Kent's entrance and Superman's exit. The other is the construal both of Superman and Clark Kent as individual offices, arranged in the following (antisymmetric) relation: necessarily, whoever occupies the Superman office co-occupies the Clark Kent office, though not always *vice versa*. In plain English, if you are Superman then you are also Clark Kent, while if you are Clark Kent then you may, or may not, be Superman. Consequently, in virtue of Leibniz's Law, whatever is true of the occupant of the Superman office is true of the occupant of the Clark Kent office, while again the converse is not always true.[42] The Clark Kent office being a requisite of the Superman office, if the occupant of the Superman office exits at WT_1 then so does the occupant of the Clark Kent office at WT_1, hence 'Clark Kent' may be substituted for 'Superman' in the second conjunct. But the occupant of the Clark Kent office enters at WT_0 without the occupant of the Superman office entering at WT_0, in case the Superman office is vacant at WT_0. As it stands, the argument does not allow us to infer that the Superman office is occupied at WT_0. Hence, 'Superman' may not be substituted for 'Clark Kent' in the first conjunct. The key to two-way substitution, then, consists in adding the premise that the Superman office is occupied at WT_0.

The intuition that (*) is invalid might be fuelled by the fact that {(1), (2), (3)} also lends itself to the following interpretation:

(1*) Clark Kent (who is not Superman yet) enters and Superman (hence also Clark) emerges
(2*) Superman = Clark Kent (at the moment of emerging)

(3*) Clark Kent enters and Clark Kent (but *not* Superman) emerges

This argument is obviously *invalid*. The situation depicted in the argument is a possible one; the occupant of the office of Clark Kent, distinct from the occupant of the office of Superman (as this office is vacant when Clark Kent enters) enters, and while in the booth he obtains the additional properties required to make him occupy the Superman office. Hence, the occupant of the Clark Kent office, when emerging, co-occupies the Superman office; from this it can be inferred that Superman exists. Therefore, it is impossible that anyone who would emerge and be Clark Kent would not also be Superman. Such a reading is borne out by pragmatic considerations: in order to pick out an individual, it makes good sense to denote the more exclusive office, if possible. When using a less exclusive office, we want to express the fact that some individual lacks some properties that are requisites of the more exclusive office. For instance, when referring to Johannes Ratzinger, we would typically use the term 'the Pope' and not 'the Head of State of the Vatican', the office of Head of State being a requisite of the office of Pope. If somebody

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[42] The Superman office might just as well have been a requisite of the Clark Kent office, but since the sentence to be analysed is 'Superman is Clark Kent' and not 'Clark Kent is Superman' (cf. Saul, ibid., p. 104, display [11]), the antisymmetry is in this particular direction.

would use the latter term without mentioning Ratzinger's papacy, the hearer would typically suppose that Ratzinger had resigned as Pope.[43]

The interplay between occupancy and vacancy can be phrased in terms of rules. Let X, Y be variables ranging over individual offices and let *Occ* be the property of being occupied. Then:

$$\frac{[{}^0Req_2\ Y\ X] \quad [{}^0Occ_{wt}\ X]}{[{}^0Occ_{wt}\ Y]} \qquad \frac{[{}^0Req_2\ Y\ X] \quad \neg[{}^0Occ_{wt}\ Y]}{\neg[{}^0Occ_{wt}\ X]}.$$

On the other hand, if $\neg[{}^0Occ_{wt}\ X]$ then neither $\neg[{}^0Occ_{wt}\ Y]$ nor $[{}^0Occ_{wt}\ Y]$ follows. And if $[{}^0Occ_{wt}\ Y]$ then neither $[{}^0Occ_{wt}\ X]$ nor $\neg[{}^0Occ_{wt}\ X]$ follows.

Let us generalise how the anti-symmetry between X and Y is decisive for which predications *de re* are true. If at $\langle w, t\rangle$ X_{wt} (i.e., the occupant of X) has the property H then at $\langle w, t\rangle$ Y_{wt} is also an H. But if at $\langle w, t\rangle$ Y_{wt} is an H then either X is occupied and its occupant is an H or X is vacant and it is not true that X_{wt} is an H. In terms of rules:[44]

First Rule of Predication de re (*P1*)

$$\frac{[{}^0Req_2\ Y\ X] \quad [{}^0H_{wt}\ X_{wt}]}{[{}^0H_{wt}\ Y_{wt}]}.$$

Second Rule of Predication de re (*P2*)

$$\frac{[{}^0Req_2\ Y\ X] \quad [{}^0Occ_{wt}\ X] \quad [{}^0H_{wt}\ Y_{wt}]}{[{}^0H_{wt}\ X_{wt}]}.$$

Since the requisite relation between any two offices holds for all worlds and times, 'Superman is Clark Kent' expresses (on the *de dicto* reading we are championing) a necessary truth. But it is not a requisite either of the Superman or the

[43] To the best of our knowledge, the offices of Pope and Head of State of the Vatican are distinct, and can be ordered in the requisite relation, such that the latter is a requisite of the former. This relation obtains on condition that it be conceptually possible that the office of Head of State is occupied while the papacy goes vacant. This scenario might obtain if, for instance, somebody is the political leader of the Vatican while nobody is its religious leader.

[44] What underlies both rules is the principle of predication *de re* explained in Section 2.6.

Clark office that whoever is its occupant at some given instant must be identical to whoever is the occupant either of the Superman or Clark office at some earlier or later moment. In other words, it is not necessary that there be diachronic co-occupation of both offices by the same individual 'throughout' the conjunction 'Clark Kent enters and Superman exits'. Thus, for instance, it is possible that whoever occupies the Clark office at WT_0 not be identical to whoever occupies the Superman office at WT_1. Consequently, if this possibility is realised, the one who is Clark at WT_0 is not the one who is Clark at WT_1.[45] Odd it may be; impossible not. There is no logically compelling reason why, for instance, the following scenario should not obtain: the occupant of the Clark office enters the phone booth at WT_0 and ceases occupying the office upon entering, whereas someone else already waiting inside exits at WT_1 either as the occupant of the Superman office (hence, also of the Clark office) or as the occupant of the Clark office (though not necessarily as the occupant of the Superman office).

Such a scenario cannot be articulated in a language that construes 'Superman' and 'Clark Kent' as 'Millian' names. Any such language obliterates the differences between (being) Superman and (being) Clark Kent and also renders the diachronicity between Clark's entrance and Superman's exit irrelevant, since the one who enters must be identical to the one who exits.

We are now able to specify our take on two-way substitution in Saul's phone booth argument when interpreted in terms of offices and requisites. Here is the argument in (slightly stilted) prose first.

(i) The Clark Kent office is a requisite of the Superman office
(ii) At WT_0, the Superman office is occupied
(iii) At WT_0, the occupant of the Clark Kent office enters, and at WT_1, the occupant of the Superman office exits

---

(iv) At WT_0, the occupant of the Superman office enters, and at WT_1, the occupant of the Clark Kent office exits.

Let $t_0, t_1 \rightarrow \tau$ such that $t_0 < t_1$ (t_0 preceding t_1). Let X, Y be variables ranging over individual offices ($X, Y \rightarrow \iota_{\tau\omega}$) and F, G variables ranging over properties ($F, G \rightarrow (o\iota)_{\tau\omega}$). Then the logical form (see Section 1.5.1, Definition 1.11) of the argument underlying this three-premise analysis of the inference of 'Superman enters the phone booth and Clark Kent exits' is

(1)	$[F_{wt0}\ Y_{wt0}] \wedge [G_{wt1}\ X_{wt1}]$	Assumption
(2)	$[{}^0Req\ Y\ X]$	Assumption
(3)	$[{}^0Occ_{wt0}\ X]$	Assumption
(4)	$[F_{wt0}\ Y_{wt0}]$	1, $\wedge$E

45 Similarly, when in a monarchy the previous king is dead and the new king is proclaimed—'The king is dead. Long live the king!'—the old king and the new king are two different individuals.

(5)	$[F_{wt0}\ X_{wt0}]$	2, 3, 4, *P2*
(6)	$[G_{wt1}\ X_{wt1}]$	1, $\wedge$E
(7)	$[G_{wt1}\ Y_{wt1}]$	2, 6, *P1*
(8)	$[F_{wt0}\ X_{wt0}] \wedge [G_{wt1}\ Y_{wt1}]$	5, 7, $\wedge$I.

The two intermediate conclusions are (5) and (7). The main conclusion, (8), then follows by adjoining them by means of conjunction introduction. One-way substitution (invalidating two-way substitution) is obtained by leaving out (3), so that (5) cannot be inferred.

We finish by briefly addressing two other puzzles.[46]

Superman is more successful with women than Clark Kent	*Rab*
Superman is Clark Kent	$a = b$
———	
Superman is more successful with women than Superman	*Raa*.

Superman leaps tall buildings, and Clark Kent does not	$Fb \wedge \neg Fa$
Superman is Clark Kent	$a = b$
———	
Superman leaps tall buildings, and Superman does not	$Fa \wedge \neg Fa$.

As long as ‘Superman’, ‘Clark Kent’ are names of individuals, these two arguments are valid (though unsound). Consequently, Superman cannot outdo Clark Kent in anything. But since the plotline of the *Superman* comics drives us to accept that Superman *does* outdo Clark Kent (in courtship, in leaping tall buildings, etc.), the arguments seem to be puzzles. However, if ‘Superman’ and ‘Clark Kent’ are names of offices, then both sets of premises are *true* on their *de dicto* reading. The arguments then come out *invalid*, because the conclusions are false. The conclusion of the first argument is false, because the relation of being more successful is irreflexive; the conclusion of the second argument is a contradiction.

If Superman and Clark Kent are re-construed as offices, our take on the first puzzle is this. An individual occupying the office of Superman (thereby co-occupying the office of Clark Kent) is more successful with women than an individual occupying only the office of Clark Kent, because the office of Superman is (in some unspecified way) greater than the office of Clark Kent.

The puzzle of Superman, but not Clark Kent, leaping tall buildings can be solved in the same manner. The first premise cannot be true on its *de re* reading, for the first conjunct entails that Clark Kent leaps tall buildings, which contradicts the second conjunct. However, on its *de dicto* reading the first premise is true. It expresses again a relation between the offices of Superman and Clark Kent that

[46] The first is culled from Saul (1997, p. 103), while the second is adapted from ‘Superman leaps tall buildings more frequently than Clark Kent’, originally occurring in Joseph G. Moore (1999, p. 92, n. 1).

makes the former greater than the latter (assuming that greatness is exemplified by, for instance, leaping tall buildings). No occupant of the Clark Kent office ever leaps tall buildings, when the Superman office is vacant. When the Superman office is occupied, some of its occupants leap tall buildings. This is to say, due to the requisite relation between the two offices, that whenever Superman leaps tall buildings then Clark Kent also does so.

Formally, both puzzles are unravelled thus:

$$\frac{[{}^0Greater_{wt}\ {}^0Superman\ {}^0Kent]}{\begin{array}{c}\forall w\forall t\ [[{}^0True_{wt}\ \lambda w\lambda t\ [{}^0Suc_{wt}\ {}^0Kent_{wt}]] \supset [{}^0True_{wt}\ \lambda w\lambda t\ [{}^0Suc_{wt}\ {}^0Superman_{wt}]]] \wedge \\ \exists w^*\exists t^*\ [\neg[{}^0Suc_{w^*t^*}\ {}^0Kent_{w^*t^*}] \wedge [{}^0Occ_{w^*t^*}\ {}^0Kent] \wedge \neg[{}^0Occ_{w^*t^*}\ {}^0Superman]]\end{array}}$$

Gloss: 'Being successful as Superman is a necessary condition for Clark Kent to be successful (at whatever). When the Superman office is vacant, Clark Kent is not successful.'

4.4 Property modification and pseudo-detachment

Gamut (the Dutch equivalent of Bourbaki) claims that if Jumbo is a small elephant, then it does not follow that Jumbo is small (1991, §6.3.11). We are going to show that the conclusion does follow. To this end we define *the rule of pseudo-detachment* (PD). The rule validates a certain inference schema, which on first approximation is formalized as follows:

$$\frac{a \text{ is an } AB}{a \text{ is an } A}$$

where 'a' names an appropriate subject of predication (e.g., an individual or a property), while 'A' is an adjective and 'B' a noun phrase compatible with a.

The reason why we need the rule of pseudo-detachment is that A as it occurs in AB is a *modifier* and, therefore, cannot be transferred to the conclusion to figure as a *property*. If a is an individual and B a function of type $(o\iota)_{\tau\omega}$, whereas A is a function of type $((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$ then no actual detachment of A from AB is possible, and Gamut is insofar right. But PD makes it possible to replace the modifier A by the property A^* compatible with a to obtain the conclusion that a is an A^*. PD introduces a new property A^* 'from the outside' rather than by obtaining A 'from the inside', by extracting a part from a compound already introduced. The applicability of PD presupposes the validity of existential generalisation over properties and of substituting identical properties, something we are not going to doubt.

It might be objected, however, that the rule of pseudo-detachment is far too liberal. Apparently it is nonsensical for *A* to stand on its own, except when *A* is an intersective modifier. It seems indisputable that somebody can be happy, full stop, whence follows we may factor out ‘happy’ from ‘happy B’. But the objection applies to two particular kinds of modifier. One kind are the non-subsective ones, which divide into *privative* like *forged* and *former*, and *modal* such as *alleged* and *apparent*. The other kind of modifier are *scalar* and other relative ones, e.g., *small* as in *small elephant* and *good* as in *good flutist*. Our claim that PD is logically valid entails that forged banknotes are forged and small elephants are small (though definitely not that forged banknotes are banknotes). For instance, if you factor out *small* from *small elephant*, say, the conclusion says that Jumbo is small, period. Yet this would seem a strange thing to say, for something appears to be missing: Jumbo is a small *what*? Nothing or nobody can be said to be small—or forged, temporary, larger than, the best, good, notorious, or whatnot, without any sort of qualification. A complement providing some sort of qualification to provide an answer to the question, ‘a … *what*?’ is required. Or so it appears. We are going to show why we, nonetheless, find the conclusion reasonable whatever the property and how to dismantle the objection that PD is an invalid rule.

First case. We consider the following argument valid:

a is a forged banknote and *b* is a forged passport

---

a and *b* are forged.

For instance, if the customs officers seize a forged banknote and a forged passport, they may want to lump together all the forged things they have seized that day, abstracting from the particular nature of the forged objects. This lumping together is feasible only if it is logically possible to, as it were, abstract *forged* from *a being a forged B* and *b being a forged C* to form the new predications that *a is forged* and that *b is forged*, which are subsequently telescoped into a conjunction.

Second case. We consider this argument valid, too:

c is a small elephant

---

c is small.

For instance, somebody may insist that *c* is a large mammal, a large land-living animal, and certainly much larger than any mouse around. But it ought to be possible to counter that *c* is also small, most other elephants dwarfing *c*.

Properties are scalar when requiring a scale for their application. Without a scale to differentiate small elephants from average-sized and large elephants, it becomes nonsensical to predicate smallness of an elephant. The reason is because nothing is absolutely small (or average-sized or large), but only small relative to something else. This ‘something else’ is other elephants, when we say about some elephant that it is small. Exactly how many, or which, elephants it takes to constitute a norm is another matter. What is important, from a logical and linguistic point of view, is that some scale or other be already in place. So it would seem a non-starter to argue that we may, nonetheless, predicate smallness of *c* without specifying a scale. As indeed it would be, as the rule stands. The claim that *c* is small invites the standard rejoinder, 'A small *what*?' But one thing is to indicate a specific scale; another thing is to indicate an unspecified scale. The essence of our solution is to claim that *c* is small with respect to some scale without stating which one. The obvious way to introduce an unspecified scale is to existentially quantify over scales. We model scales as properties: if Jumbo is a small elephant, it is with respect to the property of being an elephant that Jumbo is small. But now, if we introduce quantification over properties, does it not, then, become trivial to say that there is some property or other relative to which *c*, or any other individual, is small (or large, or whatnot)? Yes, it does. It is to say very little, virtually nothing, that there is some property with respect to which *c* is small. But this very triviality explains why we do not hesitate to embrace the rule of pseudo-detachment.

The temporary rule above is incomplete as it stands, for two related reasons. First, on our interpretation the two occurrences of *A* denote two different functions that are type-theoretically distinct. The first occurrence is as a modifier; the second, as a property: a distinction the rule above glosses over.[47] Secondly, therefore, the full pseudo-detachment rule must contain more premises to bridge between the original premise and the conclusion.

Here is the full pseudo-detachment rule, SI being substitution of identicals.[48]

(1)	*a* is an *AB*	Assumption
(2)	*a* is an (*A* something)	1, EG
(3)	*A** is the property (*A* something)	Definition
(4)	*a* is an *A**	2, 3, SI

[47] In *some* natural languages the two types seem to be flagged grammatically. For instance, 'Jumbo ist ein *kleiner* Elefant', but 'Jumbo ist *klein'*. Strictly speaking, however, we are imposing a particular interpretation on German grammar by claiming that the form '*kleiner*' as it occurs in 'ein *kleiner* Elefant' signals that the adjective denotes a modifier here. It could be objected that '*glückliches*' as it occurs in 'Karl ist ein *glückliches* Kind' is an intersective adjective and that the sentence has been generated by telescoping the conjunction 'Karl ist *glücklich*, und Karl ist ein Kind'. Though 'ein kleiner Elefant' and 'ein glückliches Kind' are grammatically on a par, 'klein' denotes a modifier and 'glücklich' a property. But even if we grant this point, we are still able to claim that the morphology of German grammar displays a grammatical link between 'kleiner' and 'Elefant' (which is absent in the corresponding English phrase 'small elephant') that shows that 'kleiner' calls for complementation, as is indeed characteristic of modifying predicates. ('Jumbo ist kleiner' is actually well-formed, but means that Jumbo is *smaller*, not small, and demands complementation.)

[48] More precisely, substitution of identical properties according to the intensional rule of substitution; see Section 2.7.

Let $[AB]$ be the property resulting from applying A to B, and let $[AB]_{wt}$ be the result of applying the property $[AB]$ to the world and time variables w, t to obtain a set, in the form of a characteristic function, applicable to a. Further, let = be the identity relation between properties, and let p range over properties, x over individuals. Then the *proof* of the rule is this:

1. $[[AB]_{wt}\, a]$	assumption
2. $\exists p\, [[Ap]_{wt}\, a]$	1, EG
3. $[\lambda x\, \exists p\, [[Ap]_{wt} x]\, a]$	2, β-expansion
4. $[\lambda w' \lambda t'\, [\lambda x\, \exists p\, [[Ap]_{w't'} x]]_{wt}\, a]$	3, β-expansion
5. $A^* = \lambda w' \lambda t'\, [\lambda x\, \exists p\, [[Ap]_{w't'} x]]$	definition
6. $[A^*_{wt}\, a]$	4, 5, Leibniz's Law

Any valuation of the free occurrences of the variables w, t that makes the first premise true will also make the second, third and fourth steps true. The fifth premise is introduced as valid by definition. Hence, any valuation of w, t that makes the first premise true will, together with step five, make the conclusion true. Therefore, the following argument is valid:

$$\frac{\lambda w \lambda t\, [[AB]_{wt}\, a];\;\; A^* = \lambda w' \lambda t'\, [\lambda x\, \exists p\, [[Ap]_{w't'} x]]}{\lambda w \lambda t\, [A^*_{wt}\, a]}$$

Here is an instance of the rule.

(1′) *a* is a forged banknote
(2′) *a* is a forged something
(3′) Forged* is the property of being a forged something
(4′) *a* is forged*.

If it is to be a logically valid rule, PD must apply indiscriminately to intersective, subsective, modal/intensional and privative modifiers (We do not consider the so-called modal modifiers, which appear to be well-nigh logically lawless[49]).

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[49] 'Modal modifier' is the term used by, e.g., Partee; see (2001, p. 7). Such modifiers are also known as 'intensional'. E.g., Cresswell (1978, p. 17) suggests that 'Arabella walked across the park for fifteen minutes' fails to entail, 'Arabella walked across the park', making *for* an intensional modifier. Nor does 'Arabella walked across the park for fifteen minutes' exclude that Arabella did walk across the park. Rotstein and Winter (2004, p. 276, n. 14) point out that, 'Many modifiers, especially intensional ones, are neither restrictive nor co-restrictive. For instance, the sentence *John is hopefully a good student* does not entail that John is a good student, and it does not entail that John is not a good student. Hence, the (sentential or predicational) modifier *hopefully* is neither restrictive nor co-restrictive.' Similarly, the intensional property modifier *alleged* allows that some alleged assassins are assassins while others are not. It is interesting to note a strong similarity between modal modifiers and *non-factive attitudes*. If, for instance, *a believes* that *b* is an assassin then it does not follow that *b* is an assassin, but nor does its negation. We suppose that a deeper study of modal/intensional modifiers (a hitherto marginalized kind of modifiers in formal semantics and linguistics) will reveal that many of them are attitudinal in nature, as exemplified by *a hoped-for result* or *being presumed innocent*.

Here is a taxonomy of the three kinds of modifier, $\|\,$ forming sets from properties.[50] (In Section 4.1 *requisite* was defined as a relation-in-extension of type $(o\alpha_{\tau\omega}\beta_{\tau\omega})$ that inputs an ordered pair of intensions and yields **T** iff the first element is a *requisite* of the second element.)

Intersective. 'If *a* is a *happy* child, then *a* is happy and *a* is a child'.

$AB(a) \therefore A^*(a) \wedge B(a)$.
Necessarily, $\|AB\| = \|A^*\| \cap \|B\|$.
$[{}^0Req\ \lambda w\lambda t\ [[A^*_{wt}\ x] \wedge [B_{wt}\ x]]\ [AB]]$.

Types: $A \rightarrow ((o\alpha)_{\tau\omega}(o\alpha)_{\tau\omega})$; $A^*, B \rightarrow (o\alpha)_{\tau\omega}$; $x \rightarrow \alpha$; $Req/(o(o\alpha)_{\tau\omega}(o\alpha)_{\tau\omega})$.

Thus the class of modifiers which are intersective with respect to a property *F* is defined as

$$\lambda g\ [{}^0Req\ [\lambda w\lambda t\ \lambda x\ [[g^*_{wt}\ x] \wedge [{}^0F_{wt} x]]]\ [g\ {}^0F]].$$

Types: $g \rightarrow ((o\alpha)_{\tau\omega}(o\alpha)_{\tau\omega})$; $F/(o\alpha)_{\tau\omega}$; $g^* \rightarrow (o\alpha)_{\tau\omega}$; $x \rightarrow \alpha$.

Intersectivity is the least interesting form of modification, since antecedent and consequent, or premise and conclusion, are equivalent. Still, even in the case of the apparently logically trivial intersectives we cannot transfer *A* from the premise to the conclusion. The reason, again, is that a modifier cannot also occur as a property. Hence *A** instead of just *A*.

Subsective. 'If *a* is a *skilful* surgeon, then *a* is a surgeon.'

$AB(a) \therefore B(a)$.
Necessarily, $\|AB\| \subseteq \|B\|$.
$[{}^0Req\ B\ [AB]]$.

Thus the class of modifiers which are subsective with respect to a property *F* is defined as

$$\lambda g\ [{}^0Req\ {}^0F\ [g\ {}^0F]].$$

The major difference between subsective and intersective modification is that subsectivity bans this sort of argument: $AB(a)$, $C(a) \therefore AC(a)$. Charles may be a skilful surgeon, and he may be a drummer too, but this does not make him a skilful drummer. Scalar properties are subsective modifiers. Again, Jumbo may be a small elephant, as well as a mammal, but this does not make Jumbo a small mammal.

[50] Hence $\|$ is an operation of extensionalising properties, which corresponds in TIL to B_{wt}.

Privative. 'If a is a *forged* banknote, then a is not a banknote', 'If b is an *ex*-Stalinist, then b is not a Stalinist.'[51,52]

$$AB(a) \therefore \neg B(a).$$
Necessarily, $\|AB\| \cap \|B\| = \varnothing$

or equivalently,

Necessarily, $\|AB\| \subset \|\text{non-}B\|$.
$[^0Req\ \lambda w \lambda t\ [\neg[B_{wt}\ x]]\ [AB]]$.

Thus the class of modifiers which are privative with respect to a property F is defined as

$$\lambda g\ [^0Req\ \lambda w \lambda t\ \lambda x\ \neg[^0F_{wt}\ x]\ [g\ ^0F]].$$

The pseudo-detachment schema is immune to whether A in AB is an intersective, subsective or privative modifier. Happy children are happy, skilful surgeons are skilful, fake Ming vases are fake. The reason is the same for all three. The existential generalisation in the pseudo-detachment schema quantifies B away, replacing AB by $\exists p(\text{A}p)$. Since we impose no restrictions on which of the three kinds of modifier A in AB may be, it follows that anything that is capable of being an AB-object is *ipso facto* capable of being an Ap-object. Via the identification of Ap with A^*, the indifference to the particular nature of the modifier A is transferred to A^*.

Up until now we have been arguing intuitively why the pseudo-detachment schema ought to be valid and shown in prose how to assign a semantics to it that is capable of validating it. Now we commit ourselves to a full-fledged semantics in accordance with TIL. The functional application of A to B is the logical operation underlying the formation of a compound predicate 'AB' containing a modifying

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[51] In the second case, however, it can be inferred that b once was a Stalinist.

[52] Partee (2001) attempts to reduce privative modifiers to subsective modifiers so that 'the [linguistic] data become much more orderly' (ibid.). In her case guns would divide into fake guns and real guns, and fur into fake fur and real fur. Her argument is that only this reduction can do justice to the meaningfulness of asking the following sort of question: 'Is this gun real or fake?' At first blush, however, it would seem the question pre-empts the answer: if some individual is correctly identified as a gun, then surely it is a real gun, something being a gun if, and only if, it is a real gun. However, if we go along with the example, we think the argument is easily rebutted by putting scare quotes around 'gun' so that the question becomes, 'Is this 'gun' fake or real?' The scare quotes indicate that 'gun' is something like 'gun-like', including toy guns, which are not guns. If the answer is that the gun-like object is a fake gun (hence not a gun), the scare quotes stay on. If the answer is that it is a real gun (i.e., a gun), the scare quotes are lifted. Similarly with 'Is this 'fur' fake or real?' A more direct way of phrasing the question would be, 'Is this fur?', which does not pre-empt the answer and which does not presuppose that there be two kinds of fur, fake and real. For an intuitive test, ask yourself what the sum is of a fake 10-Euro bill and a 10-Euro bill. For a comparison between the respective kinds of procedural semantics of TIL and Martin-Löf's constructive type theory, see Jespersen and Primiero (forthcoming).

adjective '*A*' and a modified noun '*B*' and denoting the property *AB*. This is why *A* needs to be of type $((\text{o}\iota)_{\tau\omega}(\text{o}\iota)_{\tau\omega})$.[53] Let *A*, *B*, *A** be constructions, typed as above. Then the Composition [*AB*] of *A* and *B* *v*-constructs the property of being an *AB*. The predication of this property of *a* proceeds as explained in Section 2.4.2:

$$\lambda w \lambda t\ [[AB]_{wt}\ {}^0a].$$

$[AB]_{wt}$ *v*-constructs the set of *AB*-things at $\langle w, t\rangle$. $\lambda w\lambda t\ [[AB]_{wt}\ {}^0a]$ constructs a proposition that is true at all and only those worlds and times at which *a* is in the extension of the property constructed by [*AB*]. Notice that the *w* and *t* parameters, for intensional descent, must be appended to [*AB*] and not to either of *A*, *B* in isolation. Wrong typing aside, the very point of employing 'modified' properties would be lost if $[AB]_{wt}$ were replaced by either $[A_{wt} B_{wt}]$ or $[AB_{wt}]$.

PD, dressed up in full TIL notation, is this:

$$\frac{\begin{array}{c}[[AB]_{wt}\ {}^0a]\\ [A^* = \lambda w\lambda t\ \lambda x\ {}^0\exists p\ [[Ap]_{wt} x]]\end{array}}{[A^*_{wt}\ {}^0a].}$$

Types: $\pi = (\text{o}\iota)_{\tau\omega}$; $\exists/(\text{o}(\text{o}\pi))$; $p/{*}1 \rightarrow_v \pi$; $A \rightarrow (\pi\pi)$; $A^*, B \rightarrow \pi$; $=/(\text{o}\pi\pi)$.

The whole argument stipulates that there is a logically necessary connection between two properties, *AB* and *A**. The stipulation is to the effect that whenever something is an *AB*-thing it is an *A**-thing. But the connection is established only via worlds, times and individuals, which accounts for the use of intensional descent. However, since we are already operating within an intensional system, why not link the two intensions directly? This can be done using the requisite relation. Here is how. The specific type of the relation we need here is $Req_1/(\text{o}\pi\pi)$: $[{}^0Req_1\ A^*\ AB]$.[54] When employing the schema of pseudo-detachment below, we shall condense it into this one-premise rule:

$$\frac{[[AB]_{wt}\ {}^0a]}{[A^*_{wt}\ {}^0a].}$$

The schema extends to all (appropriately typed) simple-type objects. For instance, let the inference be, 'Spelunking is an exciting hobby; therefore, spelunking is exciting'. Then *a* is of type π, $B \rightarrow (\text{o}\pi)_{\tau\omega}$, $A \rightarrow ((\text{o}\pi)_{\tau\omega}\ (\text{o}\pi)_{\tau\omega})$, and $A^* \rightarrow (\text{o}\pi)_{\tau\omega}$.

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[53] For the sake of simplicity we now consider only individual properties. Generalization to any type of property is straightforward.

[54] See Section 4.1, Definition 4.1.

Let us return to the first of the two examples set out above. We can now easily show why this argument must be valid:

Charles has a forged banknote and a forged passport

Charles has (at least) two forged things.

$$\lambda w \lambda t\ \exists xy\ [[{}^0Have_{wt}\ {}^0Charles\ x] \wedge [{}^0Have_{wt}\ {}^0Charles\ y] \wedge [[{}^0Forged\ {}^0Bank]_{wt}\ x] \wedge [[{}^0Forged\ {}^0Pass]_{wt}\ y] \wedge [{}^0{\neq}\ x\ y]]$$

$$\lambda w \lambda t\ \exists xy\ [[{}^0Have_{wt}\ {}^0Charles\ x] \wedge [{}^0Have_{wt}\ {}^0Charles\ y] \wedge [{}^0Forged^*_{wt}\ x] \wedge [{}^0Forged^*_{wt}\ y] \wedge [{}^0{\neq}\ x\ y]]$$

$$\lambda w \lambda t\ [{}^0Card\ \lambda x\ [[{}^0Have_{wt}\ {}^0Charles\ x] \wedge [{}^0Forged^*_{wt}\ x]] >= {}^0 2].$$

Types: *Card*(*inality of a set of individuals*)/$(\tau(o\iota))$; *Bank*(*note*), *Pass*(*port*), *Forged**/$(o\iota)_{\tau\omega}$; *Have*/$(o\iota\iota)_{\tau\omega}$; *Forged*/$((o\iota)_{\tau\omega}\,(o\iota)_{\tau\omega})$.

Since *Forged* is privative, a forged banknote is *not* a banknote that is forged, such that there would be two kinds of banknotes: those that are genuine and those that are forged. The sum of four genuine banknotes and one forged banknote is four banknotes and not five (though five pieces of paper).[55] This is also to say that *Genuine* is an idle modifier: anything is a genuine *F* iff it is an *F*.[56] This is not to say that the same material object may not be genuine in one respect and fail to be genuine in another. For instance, an artefact being passed off as a paper banknote may fail to be a banknote (being a forged banknote), while being indeed made of paper (rather than polymer, say), thereby being a paper artefact. ('The "banknote" is fake, the paper is real').

Now we are going to tackle four conceivable objections to the validity of PD.

First objection. If Jumbo is a small elephant and if Jumbo is a big mammal, then Jumbo is not a small mammal; hence Jumbo is small and Jumbo is not small. Contradiction!

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[55] In colloquial speech we may ask, 'Is this a genuine banknote or a *Monopoly* banknote?', where it would be sufficient to ask, 'Is this a banknote or a *Monopoly* banknote?', *Monopoly* having the effect of a privative modifier not unlike *toy* in *being a toy gun*.

[56] There is a viable alternative to construing *Genuine*, *True* and suchlike as trivial modifiers, though not along the lines suggested by Partee (2001). Consider the sentence, 'True men of the desert know no fear'. It would be tempting to construe it as having only rhetoric import, as 'true' does in, 'True beer lovers prefer Czech Budweiser to American Budweiser'. But the property *True man of the desert* may also partition a set of men of the desert into those who are male desert dwellers and those who are male desert dwellers plus something more. The latter would be the natural-born male desert dwellers. They know no fear; the former may, and some of them no doubt will. One might, though need not, go one step further and construe *Knowing_no_fear* as a requisite of *True_man_of_the_desert*: $[{}^0Req\ Know\_no\_fear\ [{}^0True\ {}^0Man\_of\_the\_desert]]$.

The contradiction is only apparent, however. To show that there is no contradiction, we apply PD:

$$\frac{\lambda w \lambda t\ [[{}^0Small\ {}^0Elephant]_{wt}\ {}^0Jumbo]}{\lambda w \lambda t\ \exists p\ [[{}^0Small\ p]_{wt}\ {}^0Jumbo]}$$

$$\frac{\lambda w \lambda t\ [[{}^0Big\ {}^0Mammal]_{wt}\ {}^0Jumbo]}{\lambda w \lambda t\ \exists q\ [[{}^0Big\ q]_{wt}\ {}^0Jumbo].}$$

Additional types: *Small*, *Big*/(ππ); *Mammal*, *Elephant*/π; *Jumbo*/ι; $p, q/*_1 \rightarrow \pi$.

Now the only conclusion we can draw is that Jumbo is small (i.e., a small something) and big (i.e., a big something else). To obtain a contradiction, we would need an additional premise; namely, that, necessarily, any individual that is large (i.e., a large something) is not small (the same ‘something’). Symbolically,

$$\forall w \forall t\ \forall x\ \forall p\ [[[{}^0Big\, p]_{wt}\ x] \supset \neg[[{}^0Small\ p]_{wt}\ x]].$$

Applying this fact to Jumbo, we have:

$$\forall w \forall t\ \forall p\ [[[{}^0Big\, p]_{wt}\ {}^0Jumbo] \supset \neg[[{}^0Small\ p]_{wt}\ {}^0Jumbo]].$$

This construction is equivalent to

$$\forall w \forall t\ \neg\exists p\ [[[{}^0Big\, p]_{wt}\ {}^0Jumbo] \wedge [[{}^0Small\ p]_{wt}\ {}^0Jumbo]].$$

But the only conclusion we obtained by applying PD expresses the construction:

$$\lambda w \lambda t\ [\exists p\ [[{}^0Small\ p]_{wt}\ {}^0Jumbo] \wedge \exists q\ [[{}^0Big\ q]_{wt}\ {}^0Jumbo]],$$

which obviously does not entail that

$$\lambda w \lambda t\ \exists p\ [[[{}^0Small\ p]_{wt}\ {}^0Jumbo] \wedge [[{}^0Big\ p]_{wt}\ {}^0Jumbo]].$$

Hence, no contradiction.

The conclusion ought to strike us as being trivial. If we grant, as we should, that nobody is absolutely good or absolutely bad, then everybody has something they do well and something they do poorly. And if we grant, as we should, that nobody and nothing is absolutely small or absolutely large, then everybody is made small by something and made large by something else. That is, everybody is both good and bad, which here just means being good at something and being bad at something else, without generating paradox:

$$\lambda w \lambda t\ \forall x\ [\exists p\ [[{}^0Good\ p]_{wt}\ x] \wedge \exists q\ [[{}^0Bad\ q]_{wt}\ x]].$$

But nobody can be good at something and bad *at the same thing* simultaneously:

$$\forall w \forall t\ \forall x\ \neg\exists p\ [[[{}^0Good p]_{wt}\ x] \wedge [[{}^0Bad\ p]_{wt}\ x]].$$

Additional type: *Good*, *Bad*/(ππ).

Second objection. It would appear that too liberal a use of pseudo-detachment, together with an innocuous-sounding premise, enables the following argument:

$$\frac{\text{Jumbo is a small elephant} \wedge \text{Mickey is a big mouse}}{\text{Jumbo is small} \wedge \text{Mickey is big}}$$

$$\frac{\text{If } x \text{ is big and } y \text{ is small, then } x \text{ is bigger than } y}{\text{Mickey is bigger than Jumbo.}}$$

Similarly, if Jumbo is a small elephant and Mickey a big mouse, we cannot deduce that Mickey is bigger than Jumbo. We can only infer the necessary truth that *if* x is a small something and y is a big object of the same kind, *then* y is a bigger object of that kind than x:

$$\forall w \forall t\ \forall x\ \forall y\ \forall p\ [[[[{}^0Small\ p]_{wt} x] \wedge [[{}^0Big\ p]_{wt}\ y]] \supset [{}^0Bigger_{wt}\ y\ x]].$$

Type: *Bigger*/$(o\iota\iota)_{\tau\omega}$. This cannot be used to generate a contradiction from these constructions as premises:

$$\exists p\ [[{}^0Small\ p]_{wt}\ {}^0a];\ \exists q\ [[{}^0Big\ q]_{wt}\ {}^0b]].$$

Geach (1956) launches an argument similar to the one we just dismantled to argue against a rule of inference has the same effect of PD of pseudo-detaching a property. He claims that that rule would license an invalid argument. And indeed, the following argument *is* invalid:

> *a* is a big flea, so *a* is a flea and *a* is big; *b* is a small elephant, so *b* is an elephant and *b* is small; so *a* is a big animal and *b* is a small animal (Ibid., p. 33).

But pseudo-detachment licenses no such argument. Geach's illegitimate move is to steal the property *being an animal* into the conclusion, thereby making *a* and *b* commensurate. Indeed, both fleas and elephants are animals, but *a*'s being big and *b*'s being small follow from *a*'s being a *flea* and *b*'s being an *elephant*, so pseudo-detachment only licenses the following two inferences, $p \neq q$:

$$\exists p\ [[{}^0Big\ p]_{wt}\ {}^0a];\ \exists q\ [[{}^0Small\ q]_{wt}\ {}^0b].$$

And a big p may well be smaller than a small q, depending on the values assigned to p, q.

Third objection. If we do not hesitate to use ‘small’ not only as a modifier expression but also as a predicate, then it would seem we could not possibly block the following fallacy:

Jumbo is small
Jumbo is an elephant

Jumbo is a small elephant.

$$\lambda w \lambda t\ \exists p\ [[{}^0Small\ p]_{wt}\ a]$$
$$\lambda w \lambda t\ [{}^0Elephant_{wt}\ {}^0a]$$

$$\lambda w \lambda t\ [[{}^0Small\ {}^0Elephant]_{wt}\ {}^0a].$$

But we can block this obviously invalid argument. The premises do not guarantee that the property p with respect to which Jumbo is small is identical to the property *Elephant*. As was already pointed out, one cannot start out with a *premise* that says that Jumbo is small (is a small something) and conclude that Jumbo is a small *B*.

Fourth objection. If it is valid to infer that Charles is happy from Charles’ being a happy child, then it would seem that if the premise is that *a* and *b* are French fries then *a* and *b* are French; which should not follow, of course. And if some piece of paper is a forged banknote then it appears to follow that the piece of paper is forged; which should not follow, of course.

The morale is that we must be careful not to mechanically apply the rule of pseudo-detachment without conducting a prior semantic analysis of the terms and the grammar of the premises. The first example concerns illegitimate substituends for ‘*AB*’; the second, illegitimate substituends for ‘*a*’.

First example.

a, *b* are French fries

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a, *b* are French.

Valid? No. ‘French fries’ is not a compound descriptive name of a property, describing things that are both French and fries, though the surface grammar of English would make it appear to be such a name. From a logical and semantic point of view, ‘French fries’ is a non-composite expression (an idiom) denoting the property of being some particular kind of sliced and fried potatoes. Not surprisingly, some other languages use grammatically simple expressions for this property, such as ‘patat’ in Dutch and ‘hranolky’ in Czech, with no ‘Franse’ or ‘francouzské’ appended.

Second example.

$$\frac{\text{This piece of paper is a forged banknote}}{\text{This piece of paper is forged}}$$

The conclusion might mean that a certain piece of paper is a forged piece of paper. But then we generate the contradiction that something is a piece of paper and also a forged piece of paper (i.e., a piece of non-paper). The contradiction comes about because nothing seems to block ‘forged’, which qualifies ‘banknote’ in the premise, from qualifying ‘This piece of paper’ in the conclusion. Such a reading of the argument is fallacious, however, because ‘This piece of paper is forged’ can be read either as above or as ‘This piece of paper is a forged *something’*. Obviously the latter is the correct reading that does follow from the premise and that does not generate paradox.

4.4.1 Malfunction: subsective vs. privative modification

Property modification is also indispensable when analysing properties like *being a malfunctioning F*, since something malfunctions only with respect to a property and not absolutely. The modifier *Malfunctioning* is also susceptible to pseudo-detachment. If a is a malfunctioning F, it follows that a is malfunctioning*; namely, with respect to the ∃-bound property p. Let *Malf* (*Malfunctioning*)/$((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$; = $(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$; $p \rightarrow (o\iota)_{\tau\omega}$; $x \rightarrow \iota$. Then this argument is valid:

$$\frac{\begin{array}{c}[[{}^0Malf\ {}^0F]_{wt}\ {}^0a] \\ \exists p\ [[{}^0Malf\ p]_{wt}\ {}^0a] \\ {}^0Malf^* = \lambda w\lambda t\ \lambda x\ \exists p\ [[{}^0Malf\ p]_{wt}\ x]\end{array}}{[{}^0Malf^*_{wt}\ {}^0a].}$$

A logically interesting question is whether the converse holds; i.e., whether a malfunctioning F (be it an organism or a device) is no less of an F for that.[57] For instance, is a malfunctioning heart a heart? Is a malfunctioning piston a piston?

[57] Proper-function theory holds that a malfunctions as an F iff a falls short of fulfilling its proper function, and systemic-function theory holds that a malfunctions as an F iff a lacks the current capacity to function as an F. See Kroes and Meijers (2006) and Jespersen and Carrara (ms).

Logically, this is the question whether *Malf* is subsective or privative. Here we do not take a stand either way, because this would require a thorough philosophical discussion that would stray too much from modification proper. Instead we show how the notion of requisite (see Section 4.1) may be useful in defining *Malf*, whether subsective or privative. Therefore, we distinguish between $Malf_s$ and $Malf_p$, where $Malf_s$ is subsective:

$$(Malf_s\ F)a \therefore Fa,$$

and $Malf_p$, privative:

$$(Malf_p\ F)a \therefore \neg Fa.$$

Once the definitions of $Malf_s$, $Malf_p$ in terms of requisites are in place, it is straightforward how to infer that a malfunctioning$_s$ F is an F and a malfunctioning$_p$ F is not an F.

The tricky bit consists rather in the bit of homework needed to be done before setting out the definitions of $Malf_s$, $Malf_p$. Technically, we need the modifier *Func_as* (for '*Functioning_as*'), as well as two additional mappings. *Func_as* forms the property *functioning_as_an_F* from the property *F*. One of the elements of the essence of *F* is a property specifying what *F*-objects are for. Thus, if *F* is *being a gun*, then let its *what-for* property be *firing_bullets*. So to function as a gun is to be used to fire bullets (without necessarily being *designed* as a gun, to leave room for improper, or unintended, use of an artefact).

As a notational convention, let 'π' abbreviate '$(o\iota)_{\tau\omega}$'. The first mapping we need is of type (π(oπ)): given the essence of *F* as argument, the mapping extracts the property that specifies what *F*'s are for; so this mapping is the '*what-for*' mapping. We dub it '*Extract*'. The second mapping we need is one we encountered in Section 4.1; namely, $Essence_2$, here specifically of type ((oπ)π). Given *F* as argument, $Essence_2$ returns the essence of *F*; so *Essence* is the 'essence-extracting' mapping. Finally, let F' be the *what-for* property of some property *F*. Given F' as argument, *Func_as*, of type (ππ), yields as value the property of functioning as an *F*. The property *functioning_as_an_F* is then formed thus:

$$[{}^0Func\_as\ [{}^0Extract\ [{}^0Essence_2\ {}^0F]]].$$

If *F* is *being a gun*, as above, then $[{}^0Essence_2\ {}^0F]$ is the essence of *being a gun*, $[{}^0Extract\ [{}^0Essence_2\ {}^0F]]$ is the property *firing_bullets*, and $[{}^0Func\_as\ [{}^0Extract\ [{}^0Essence_2\ {}^0F]]]$ is the property *functioning as a gun*. The predication of that property of *a* then looks like this:

$$\lambda w \lambda t\ [[{}^0Func\_as\ [{}^0Extract\ [{}^0Essence_2\ {}^0F]]]_{wt}\ {}^0a].$$

What we have done here is merely spell out the bare logical bones of how to form the property *functioning_as_an_F*. This is not an ad hoc solution, however, but more of a schema of what any logical analysis of that property would have to look like, provided properties have essences and a what-for property. With $[^0Func\_as\ [^0Extract\ [^0Essence_2\ ^0F]]]$ in place, we now show how to define the modifiers $Malf_s$, $Malf_p$ of type (ππ). We must define two mappings from π to π. The property figuring as functional argument is a property *F*. But what is going to be the property figuring as functional value? The property *being an F and not functioning as an F* in case of $Malf_s$ and the property *not being an F and not functioning as an F* in case of $Malf_p$. The definitions are as follows.

Definition 4.5 (***subsective malfunctioning: Malf$_s$***). Let *p, q* → π; *Req*/(οππ). Then the *subsective modifier Malf$_s$*/(ππ) is the mapping

$$\lambda p\ \iota q\ [[^0Req\ p\ q] \wedge [^0Req\ \lambda w\lambda t\ \lambda x\ \neg[[^0Func\_as\ [^0Extract\ [^0Essence_2\ p]]]_{wt}\ x]\ q]]. \qquad \square$$

Corollary 1. The set $\{[^0Malf_s\ ^0F], ^0F\}$ is a subset of the *essence* of the property $[^0Malf_s\ ^0F]$.

Definition 4.6 (***privative malfunctioning: Malf$_p$***). Let *p, q* → π; *Req*/(οππ). Then the *privative modifier Malf$_p$*/(ππ) is the mapping

$$\lambda p\ \iota q\ [[^0Req\ \lambda w\lambda t\ \lambda x\ \neg[p_{wt}\ x]\ q] \wedge [^0Req\ \lambda w\lambda t\ \lambda x\ \neg[[^0Func\_as\ [^0Extract\ [^0Essence_2\ p]]]_{wt}\ x]]]\ q]]. \qquad \square$$

Corollary 2. The set $\{[^0Malf_p\ ^0F], \lambda w\lambda t\ \lambda x\ \neg[^0F_{wt}\ x]\}$ is a subset of the *essence* of the property $[^0Malf_P\ ^0F]$.

With these two definitions in place, the following two derivations are straightforward. Both derivations invoke the propositional property $True/(οο_{τω})_{τω}$ introduced in Section 1.5.2.1. Where $P \to ο_{τω}$, the definition of *True* is:

$[^0True_{wt}\,P]$ *v*-constructs **T** iff P_{wt} *v*-constructs **T**,
otherwise $[^0True_{wt}\,P]$ *v*-constructs **F**.

Thus, the rule of *True* introduction is

$$P_{wt} \vdash [^0True_{wt}\,P],$$

and the rule of *True* elimination,

$$[^0True_{wt}\,P] \vdash P_{wt}.$$

First, the derivation that a malfunctioning$_s$ F is still an F. We are to prove that the argument

$$\frac{\lambda w \lambda t\ [[Malf_s\ F]_{wt}\ a]}{\lambda w \lambda t\ [F_{wt}\ a]}$$

is valid. From Definition 1.13 of *valid argument*, it follows that we are to prove that for any $\langle w, t \rangle$ at which $[[Malf_s\ F]_{wt}\ a]$ v-constructs **T**, $[F_{wt}\ a]$ v-constructs **T** as well.

(1) $[[Malf_s\ F]_{wt}\ a]$ Assumption

(2) $[True_{wt}\ \lambda w \lambda t\ [[Malf_s\ F]_{wt}\ a]]$ 1′, *True* I

(3) $[Req\ F\ [Malf_s\ F]]$ Corollary 1

(4) $\forall w' \forall t'\ \forall x\ [[True_{w't'}\ \lambda w \lambda t\ [[Malf_s\ F]_{wt} x]] \supset [True_{w't'}\ \lambda w \lambda t\ [F_{wt}\ x]]]$
Definition 4.1

(5) $[[True_{wt}\ \lambda w \lambda t\ [[Malf_s\ F]_{wt}\ a]] \supset [True_{wt}\ \lambda w \lambda t\ [F_{wt}\ a]]]$ 4, ∀E, *a*/*x*, *w*/*w*′

(6) $[True_{wt}\ \lambda w \lambda t\ [F_{wt}\ a]]$ 2, 5, MPP

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(7) $[F_{wt}\ a]$ 6, *True* E.

Second, the derivation that a malfunctioning$_p$ F is not an F:

(1′) $[[Malf_p\ F]_{wt}\ a]$ Assumption

(2′) $[True_{wt}\ \lambda w \lambda t\ [[Malf_s\ F]_{wt}\ a]]$ 1′, *True* I

(3′) $[Req\ [\lambda w \lambda t\ [\lambda x\ \neg[F_{wt}\ x]]]\ [Malf_p\ F]]$ Corollary 2

(4′) $\forall w' \forall t'\ \forall x\ [[True_{w't'}\ \lambda w \lambda t\ [[Malf_p\ F]_{wt} x]] \supset [True_{w't'}\ \lambda w \lambda t\ \neg[F_{wt}\ x]]]$
Definition 4.1

(5′) $[[True_{wt}\ \lambda w \lambda t\ [[Malf_s\ F]_{wt}\ a]] \supset [True_{wt}\ \lambda w \lambda t\ \neg[F_{wt}\ a]]]$ 4′, ∀E, *a*/*x*, *w*/*w*′

(6′) $[True_{wt}\ \lambda w \lambda t\ \neg[F_{wt}\ a]]]$ 2′, 5′, MPP
(7′) $\neg[F_{wt}\ a]$ 6′, *True* E.

These two definitions and derivations apply across the board to any modifier that comes in both a subsective and a privative variant, so there is insofar no particular reason to exemplify the distinction between subsective and privative modification by means of *Malf* rather than any other modifier also susceptible to this bifurcation. But it is philosophically interesting that both variants apply to *Malf*, since the respective associated notions of (biological or technical) *function* are going to be somewhat different.[58]

Having presented both variants of *Malf*, we will put them to good philosophical use by showing how to solve the following puzzle. Consider multiple-function artefacts. Jesse Hughes (2009, n. 12, §4) brings up the example of a claw hammer, saying that a claw hammer with a broken claw functions properly with respect to pounding nails, but not with respect to prying nails. Hughes is right about this. But what general lessons can be extracted? Assume that to function as a claw is to pry nails and to function as a hammer is to pound nails. Then there are two options. The first option is this: something is a functioning claw hammer if, and only if, it functions as a claw *and* it functions as a hammer. The second option is this: something is a functioning claw hammer if, and only if, it functions as a claw *or* it functions as a hammer (*or*, inclusive disjunction). The first option entails this: something is a malfunctioning claw hammer if, and only if, it fails to function as a claw or it fails to function as a hammer. That is, it is possible that something be a malfunctioning claw hammer and still function as a claw or as a hammer, depending on whether its clawing capacity or its hammering capacity is compromised. The second option entails this: something is a malfunctioning claw hammer if, and only if, it fails to function as a claw and it fails to function as a hammer. That is, a malfunctioning claw hammer functions neither as a claw nor as a hammer. As is seen, the two respective entailments are instances of De Morgan's laws: the negation of a conjunction is equivalent to the disjunction of the negations of its conjuncts; the negation of a disjunction is equivalent to the conjunction of the negations of its disjuncts.

The difference between the subsective and the privative view of malfunction can be schematised in the following manner. Let *Claw* be a property modifier and *Claw** a property. Then the subsective view validates the inference

$$\frac{\dfrac{[[{}^{0}Malf_{s}\ [{}^{0}Claw\ {}^{0}Hammer]]_{wt}\ {}^{0}a]}{[[{}^{0}Claw\ {}^{0}Hammer]_{wt}\ {}^{0}a]}}{[{}^{0}Hammer_{wt}\ {}^{0}a]}$$

That a claw hammer is a claw is in turn inferable via the rule of pseudo-detachment. The privative view validates the inference

[58] For details, see Jespersen and Carrara (ms.).

$$\frac{\dfrac{[[{}^0Malf_p\ [{}^0Claw\ {}^0Hammer]]_{wt}\ {}^0a]}{\neg[[{}^0Claw\ {}^0Hammer]_{wt}\ {}^0a]}}{[\neg[{}^0Claw*_{wt}\ {}^0a] \vee \neg[{}^0Hammer_{wt}\ {}^0a]]}$$

That is, a malfunctioning claw hammer is a non-(claw hammer) that is either not a claw or not a hammer, or neither a claw nor a hammer.

So what is the essence of a multiple-function tool like a claw hammer? It is the union of the sets of essential properties defining each of its functions. Where *being a claw* and *being a hammer* each comes with a set of essential properties, *being a claw hammer* has as its essence the union of these two sets.

4.5 Nomological necessity

By 'nomological or nomic necessity' we understand the sort of necessity that pertains to laws of nature. We are not attempting to analyze causality, which we expect to possess a somewhat more elaborate modal profile than the one we suggest for nomic necessity.

Nomic necessity is logically contingent, so the source of the universality that any kind of necessity requires is another. We obtain universality by suspending temporal variability, which means that it is true (false) at all instants of time that if so-and-so then so-and-so, or that a certain equality between magnitudes obtains. For instance, for all times t, for all individuals x, if x is hot air at t, then x rises at t. This seems to correspond to the sound intuition that the laws of nature are always the same, yet might logically speaking have been different.[59] Nomic necessity is a law-bound correlation between two empirical properties or two propositions (states-of-affairs, events), such that if one is exemplified or obtains then so must the other, or between two magnitudes such that they are equal at all t.

Our position on laws of nature is kindred to the *universalism* of Armstrong, Tooley and Dretske, especially because we are also after contingent ('soft') necessitation and apply a top-down approach starting out with universals. But ours is importantly different in at least one regard. The universalist contention that physical necessity is a relation between universals—$N(F, G)$, in universalist notation—can be expressed by means of *Req*. Thus, if F, G are properties of type $(o\iota)_{\tau\omega}$ then we have $[{}^0Req\ {}^0G\ {}^0F]$. Similarly, if P, Q are propositions then we have $[{}^0Req'\ {}^0Q\ {}^0P]$, meaning that, for all $\langle w, t\rangle$, if P is true at $\langle w, t\rangle$ then so is Q. But the fact that this can be done goes to show that universalism is too strong, as soon as the physical necessities that apply to some logically possible worlds are supposed not to extend

[59] Cf. Mitchell (2000, p. 247): 'Laws are about our world for all time.' However, we bracket the question of whether theoretical physics will eventually bear out this assumption.

to all other logically possible worlds as well.[60] Thus, $[^0Req\ ^0Q\ ^0P]$ is definable as *P entails Q*; yet this is surely too strong a relation to capture nomic necessity.

On the other hand, this is too weak:

$$\lambda w \lambda t\ [\forall x\ [^0F_{wt}\ x] \supset [^0G_{wt}\ x]].$$

This amounts to an empirical generalisation holding for a set of times relative to a set of worlds. It is incapable of guaranteeing that the implication from being an F to being a G may not previously have failed to hold or may not later fail to hold. If true, it just reports the fact that at the given time of evaluation it is true that all F's are G's.

This would seem to steer the right course, though:

$$\lambda w\ [\forall t\ \forall x\ [^0F_{wt}\ x] \supset [^0G_{wt}\ x]].$$

What is constructed is a set of worlds; namely, the set of worlds at which it holds for all times and all individuals that F's are G's. This does not exclude the logical possibility of counterlegals, only not within this set of worlds. So the Closure arguably succeeds in pinning down at least one essential feature of nomic necessity.

The type $((o o_{\tau\omega})\omega)$ is the type of propositional properties—given a world, we are given a set of propositions; to wit, those eternally true at the given world. One such empirical property is the property of being a nomically necessary proposition. Thus, the Closure

$$\lambda w\ [\lambda p\ \forall t\ p_{wt}]$$

constructs a function from worlds to sets of propositions. The set is the set of propositions p that are eternal at a given world. Nomically necessary propositions constitute a subset of this set.[61]

Some laws are phrased as *generalizations*: 'As a matter of nomic necessity, all F's are G's'. Others are phrased as *equations*: 'As a matter of nomic necessity, the magnitude M is proportional to the magnitude N'. The best-known example of the latter is probably Einstein's 1905 equation of mass with energy,

$$E = mc^2.$$

It bears witness to the thoroughgoing mathematization of physics that the syntactic form of the formula does not reveal that the proportion between energy and mass times the speed of light squared is empirical. Assuming that the special theory of

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[60] As pointed out in Materna (2005, n. 1, p. 62), the source of the problem is that N is a relation-in-*extension*, according to Dretske (1977, p. 263), aligning N with mathematical and logical relations.

[61] See Materna (2005).

relativity is true, Einstein's discovery of this equivalence was an empirical one. What he discovered was the physical law that two particular magnitudes coincide or are proportional to one another. A unit of energy will be equal to the result of multiplying a unit of mass by the square of the constant of the speed of light. So his equation will be an instance of the following logical form:

$$\lambda w\, [\forall t\, [M_{wt} = N_{wt}]].$$

Types: $M, N \to \tau_{\tau\omega}$ (i.e., constructions of magnitudes); $=/(o\tau\tau)$.

When making explicit the empirical character of $E = mc^2$, it is obvious that E, m must be modally and temporally parameterized. But so must c. Though a constant value, the value is constant only relative to a proper subset of the space of all logically possible worlds. It is a logical possibility that in at least some nomologically deviant universe light will have a different velocity.[62] Einstein's equation is constructible thus:

$$\lambda w\, [\forall t\, [[{}^0Mult\ m_{wt}\ [{}^0Square\ {}^0c_{wt}]] = E_{wt}]].$$

Types: $Mult(iplication)/(\tau\tau\tau)$; $Square/(\tau\tau)$; $=/(o\tau\tau)$; $E, m \to \tau_{\tau\omega}$; $c/\tau_{\tau\omega}$.

What is constructed is the set of worlds at which it is eternally true that E_{wt} is identical to the result of multiplying m_{wt} with the square of c_{wt}.

The crux of our conception of nomic necessity is as modally flexible and temporally rigid. But 'freezing' the temporal parameter is not uncontroversial. For would it not be analytically possible for something both to be a law of nature and either change or be replaced over time? And if there are laws of nature now, were they always in place? In particular, to put it naïvely, if there was a Big Bang at the dawn of time, were the (presumed) laws and the values of the (presumed) constants settled simultaneously with the inception of the physical universe or did a fraction of whatever time unit pass before they were? It would be tempting to answer Yes and No, respectively, backing the answer up by a definition of law of nature to the effect that such laws apply to *all* times within a set of worlds. However, we are open to the possibility that something more deserving of the predicate 'law of nature' may have a somewhat more complicated modal profile. Thus, it is probably a non-negotiable constraint on any viable notion of nomic necessity that it be logically and analytically contingent.[63] But this constraint will most probably

[62] Though we acknowledge that essentialists about the velocity of light will claim that c is the same value for all logically possible physical universes. This is not to say that light will travel at the speed of c in all logically possible universes; for at some of them light will not travel at all or light will be missing altogether. So it still constitutes a non-trivial, empirical discovery that the speed of light is c and not any other numerical value.

[63] So-called *necessitarians* flatly deny, of course, that logical contingency is a constraint at all. Instead (strong) necessitarianism holds that the laws of the actual world are identical to the laws of all other logically possible worlds. For a clear statement of (strong) necessitarianism, see Bird (2004). Bird's theory is based on the highly problematic premise of dispositional essentialism,

turn out not to be sufficient. It thus remains a partly open issue how to exactly capture nomic necessity in TIL.

4.6 Counterfactuals

It is a well-established constraint on laws of nature that they must sustain counterfactuals in contrast to cosmic coincidences, which do not. The idea is this: *x* is not hot air; but if it had been, then *x* would have risen, as a matter of nomic necessity. By contrast, as a matter of cosmic coincidence, each *x* that is hot air also rises. But this generalization is not guaranteed to extend to any or every new *x* that is hot air, so it may happen, as a matter of fact, that *x* is hot air and *x* fails to rise.

We are sympathetic to this constraint, for it ought to be a defining feature of laws of nature that they are capable of issuing just this sort of guarantee for any universe in which they hold sway. It seems obvious, however, that the relationship between laws and counterfactuals needs to be another than so-called scientific essentialism claims, as convincingly argued in Lange (2004). The root of the problem is that scientific essentialism (e.g., Ellis' dispositional essentialism) does not keep natural kinds and laws sufficiently separate. According to essentialism, a natural kind such as electrons comes with an essence, e.g., in the form of an electric charge, that determines which lawful proportions electrons enter into. Therefore,

> [T]he laws in which […] a natural kind figures must be laws in any world in which that […] natural kind exists. In a world with different […] natural kinds, the law is again true—vacuously—but it is not a law in that world (Lange, 2004, p. 227).

That is, if you have electrons then you thereby also have certain laws; so without those laws you do not have electrons. But this imposes excessively narrow constraints on counterfactuals:

> If a counterlegal automatically posits an entirely new population of natural kinds, then essentialism cannot readily account for the preservation of certain laws […] under that counterlegal supposition (Ibid., p. 233).

Essentialism appears in fact to make counterlegals impossible, for a counterfactual involving electrons cannot also be a counterlegal on pain of either not admitting electrons into that counterfactual scenario or redefining the notion of electron by means of that contrary-to-fact law. Yet

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which is a variant of extensional essentialism as applied to natural kinds. Given that properties are defined (and individuated) in terms of their dispositional essences, it is little wonder that if (as Bird argues) all logically possible worlds share the same properties then all worlds must share the same laws. But it does leave one wondering how the strong necessitarians avoid making it cognizable a priori what the laws of nature are.

> Essentialism was supposed to *explain why* the laws and natural kinds would have been no different under various counterfactual perturbations. (Ibid., p. 234.)

In our view the logical root of the predicament of scientific essentialism is that it is a version of *extensional* essentialism (as discussed in Section 4.2). One obvious way out of the problem that Lange has raised would be to define natural kinds and laws of nature independently of one another, so that electrons (and not just near-identical replicas) may exist in a universe where some or all of those laws deviate from the actual ones. Natural kinds could be defined in terms of requisites (see Section 4.1) and laws of nature at least partially as sketched in Section 4.5.

For want of a fully-fledged notion of nomic necessity, we are not yet in a position to demonstrate how laws of nature exactly sustain counterfactuals. We do have, however, a full theory of counterfactuals. The problem of counterfactuals can be illustrated by the following example.

(Cond) '*If* Charles had owned something, then *he* would have taken care of *it*.'

The sentence indicates what would be the case if its antecedent *were* true. This is to be contrasted with an indicative conditional, which indicates what the case is if its antecedent *is* true. The latter would be expressed by the sentence

'If Charles owns something, then he takes care of it'

and analysed simply by means of material implication.

The problem of how to analyse counterfactual sentences like (Cond) is a wide-ranging one. Here we just outline the gist of the solution based on Tichý's tacit-premise theory.[64] Tichý proposes an amendment of the Mill-Ramsey-Chisholm theory, which is the following. A counterfactual sentence expresses a construction of the form

$$\lambda w \lambda t\ [A \angle B]$$

where w, t can occur free in the construction A and/or in the construction B (A, $B/*_1 \rightarrow_v o_{\tau\omega}$) and the implication function $\angle$ is of type $(o\ o_{\tau\omega}\ o_{\tau\omega})$, taking a pair of propositions to a truth-value. This function takes the value **T** if in all the world-time couples in which the proposition *v*-constructed by A is true it holds that also the consequent proposition *v*-constructed by B is true. Moreover, parts of the construction A are often tacitly understood rather than explicitly spelled out in the antecedent of the conditional sentence. The reason for using the implication function $\angle$ (instead of the common material implication $\supset$) is the fact that arguments of this function can often be *v*-constructed by open propositional constructions or picked out by a propositional office of type $(o_{\tau\omega})_{\tau\omega}$, as for instance in the sentence,

[64] For details, see Tichý (1984, 2004, pp. 543–75).

'If John's most favourite proposition had been true
then he would have weighed a ton.'

Hence, a conditional sentence is not an analytical sentence, as it does not express a construction of the proposition *TRUE*. Rather it is an *empirical* sentence. Informally, the explicitly stated antecedent proposition is itself too weak to logically imply the consequent proposition. However, the proposition denoted by the conditional sentence is nevertheless true, if in all those worlds and times $\langle w^*, t^*\rangle$ that differ from the actual $\langle w, t\rangle$ only in some obvious aspects, the antecedent proposition implies the consequent proposition. The antecedent proposition is then *v*-constructed in such a way that in $\langle w^*, t^*\rangle$ that tacit assumption is true (hence 'counterfactual').

In (Cond) the tacit premise is the proposition that Charles does not own anything. Hence, the sentence can be explained as expressing the proposition that Charles does not own anything, but in all the worlds and times that are the same as the actual, except that Charles does own something, it is true that Charles takes care of his belongings.

Let us analyse first a simpler case of the conditional statement without taking into account the anaphoric reference of 'he':

(Cond1) 'Charles does not own anything,
but if he had owned something then he would have taken care of it.'

Types: *Charles*/ι; *Own* (something)/$(o\iota\iota)_{\tau\omega}$; (take) *Care* (of something)/$(o\iota\iota)_{\tau\omega}$; x, $y \rightarrow_v \iota$.

(Cond1′) $\lambda w\lambda t\, [\lambda w^*\lambda t^*\, [\neg\exists x\, [{}^0Own_{wt}\, {}^0Charles\ x] \wedge \exists x\, [{}^0Own_{w^*t^*}\, {}^0Charles\ x]] \angle \lambda w^*\lambda t^*\, \forall y\, [[{}^0Own_{w^*t^*}\, {}^0Charles\ y] \supset [{}^0Care_{w^*t^*}\, {}^0Charles\ y]]]$.

Now we have to take into account the fact that the meaning of the consequent clause 'if *he* had owned something then *he* would have taken care of it' is the open construction (the variable $he \rightarrow_v \iota$)

$$\lambda w^*\lambda t^*\, \forall y\, [[{}^0Own_{w^*t^*}\ he\ y] \supset [{}^0Care_{w^*t^*}\ he\ y]]$$

that is to be completed by substituting the Trivialization of *Charles* for the variable *he*. To this end we use the substitution function Sub_1:[65]

$$^2[{}^0Sub_1\ {}^{00}Charles\ {}^0he\ {}^0[\lambda w^*\lambda t^*\, \forall y\, [[{}^0Own_{w^*t^*}\ he\ y] \supset [{}^0Care_{w^*t^*}\ he\ y]]]].$$

[65] See Section 3.5.

Double Execution is indispensable here, because the result of the substitution is a propositional *construction*, whereas the second argument of $\angle$ is a proposition. The analysis of (Cond1) is thus as follows:

(Cond1″) $\lambda w \lambda t\ [\lambda w^* \lambda t^*\ [\neg \exists x\ [{}^0Own_{wt}\ {}^0Charles\ x] \wedge \exists x\ [{}^0Own_{w^*t^*}\ {}^0Charles\ x]] \angle\ {}^2[{}^0Sub_1\ {}^{00}Charles\ {}^0he\ {}^0[\lambda w^* \lambda t^*\ \forall y\ [[{}^0Own_{w^*t^*}\ he\ y] \supset [{}^0Care_{w^*t^*}\ he\ y]]]]]$.

Counterfactuals do not always have the form of conditionals. Thus, another common kind of counterfactual modality is expressed by sentences of the form

‘The *F* might not have been an *F’*

e.g., ‘Smith’s murderer might not have murdered Smith’.

We construe ‘the *F*’ as expressing the following Closure and denoting the individual office so constructed ($F/(o\iota)_{\tau\omega}$):

$$\lambda w \lambda t\ [\iota x\ [{}^0F_{wt}\ x]].$$

The sentence ‘The *F* might not have been an *F’* is standardly considered ambiguous between these two readings:

Possibly, the *F* is not an *F*.

The *F* is such that it possibly is not an *F*.

The first construal is *de dicto*, the sentence expressing the Closure

$$\lambda w \lambda t\ [\exists w'\ \exists t'\ \neg [{}^0F_{w't'}\ [\iota x\ [{}^0F_{w't'}\ x]]]].$$

Gloss: ‘The *F* at <*w′*, *t′*> is a member of the set of those *x* that are not an *F* at <*w′*, *t′*>.’

What is constructed? Very simple: a proposition that yields **T** for *no* world/time pair; i.e., an impossible proposition.[66] No *F* can be a non-*F* at the same world/time pair.

The second construal is *de re*. On this reading the sentence comes close to being necessarily true: it denotes a proposition that takes **T** at all the worlds/times at which the *F* exists. The modality is ascribed to a *res*, *in casu* the individual that is picked out as the *F* at the first of two world/time pairs. The point is that the predication of possibly not being an *F* demands, on the construal *de re*, that ‘is not an *F*’ be evaluated at a world/time different from the one at which the unique *F* was identified. So in the first world/time the set of *F*-objects is singled out and the

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[66] There are many impossible propositions, which differ only by being false or undefined at different $\langle w, t\rangle$ pairs.

unique F is identified. Next, in the second world/time the individual so identified is predicated to be a member of the set of those individuals who, at the second world-time, are not F's (i.e., the set that is the complement of the F-set at the second world/time pair).

Our analysis of the sentence, 'The F might not have been an F' when understood *de re* is

$$\lambda w \lambda t\ [\lambda x\ [\exists w'\ \exists t'\ \neg[{}^0F_{w't'}\,x]]\ [\iota y\ [{}^0F_{wt}\,y]]].$$

Gloss: 'The F at $\langle w, t\rangle$ is a member of the set of those x that are not an F at $\langle w', t'\rangle$.'

Juggling (at least) two world/time pairs simultaneously is what underpins counterfactual modality, and if restricted to just worlds is what has become known as 'two-dimensional', or 'multi-dimensional', modal logic.[67] The semantic actualist argues that without reference, implicitly or explicitly, to the actual world, locutions in the vein of 'The F might not have been an F' read *de re* possess an expressive power that can be accommodated only by an actualist semantics. We wish to show that the amount of expressive power required can be obtained otherwise, without recourse to the actual world as a semantic component.[68]

For illustration, consider the following two examples. First, H.T. Hodes's 'There could be something which doesn't actually exist' (1984, p. 28). His formula is this ('@' being explained ibid., p. 27):

$$\Diamond(\exists x)@\neg\mathrm{E}x.$$

We interpret the sentence as, 'Of all the things that do not exist, some might have' or 'Something which does not exist might have existed', and stating insofar a necessary truth about the contingency of existence. For example, although no unicorns exist, unicorns might well have existed, and although zebras do exist, zebras might not have existed. Following Frege, and differing from Hodes, TIL treats existence as a second-degree property of intensions of type $(o\alpha_{\tau\omega})_{\tau\omega}$: unicorns exist at $\langle w, t\rangle$ iff the intension *Unicorn* returns a non-empty set at $\langle w, t\rangle$; the Queen of Belgium exists at $\langle w, t\rangle$ iff the intension *The Queen of Belgium* returns exactly one individual at $\langle w, t\rangle$; etc.[69] The formalization in TIL, x ranging over a given intension (i.e., $x \rightarrow \alpha_{\tau\omega}$, $Exist/(o\ \alpha_{\tau\omega})_{\tau\omega}$), is:

$$\lambda w \lambda t\ [\exists x\ [\exists w^*\ \exists t^*\ [[{}^0Exist_{w^*t^*}\,x] \wedge \neg[{}^0Exist_{wt}\,x]]]].$$

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[67] Cf. Davies and Humberstone (1980), as well as Segerberg's seminal (1973): '[I]n "two-dimensional" modal logic one wants to evaluate formulas at two points: at a point x, with respect to a point y.' (Ibid., p. 79.)

[68] See Section 2.4.1 for objections to semantic actualism.

[69] see Section 2.3.2.

Gloss: 'There is an intension x and a world/time couple $\langle w^*, t^*\rangle$ such that x exists at $\langle w^*, t^*\rangle$ and x does not exist at $\langle w, t\rangle$.'

Next up is M. Davies's 'It is possible that everything which is actually red should have been shiny' (1981, pp. 220–1). His formula is ('*A*' being explained ibid., p. 221)

$$\Diamond(\forall x)\ (A(x \text{ is red}) \rightarrow x \text{ is shiny}).$$

We read the sentence as, 'It is possible that everything which is red should have been shiny'. It is easy to make sense of the idea that, for instance, all the individuals that are red at one world/time pair are shiny at another. We use a set to single out some individuals that we then insert into another set. This idea forms the foundation of this formalization:

$$\lambda w \lambda t\ [\exists w^*\ \exists t^*\ [\forall x\ [[{}^0Red_{wt}\ x] \supset [{}^0Shiny_{w^*t^*}\ x]]]].$$

Gloss: 'Possibly, every x that is a member of the set of red things at $\langle w, t\rangle$ is a member of the set of shiny things at $\langle w^*, t^*\rangle$.'