3
Singular reference and pragmatically incomplete meaning

This chapter details how TIL analyses terms like ‘Charles’, ‘π’, ‘the tallest mountain’, ‘the largest prime’, and ‘it’. These terms are self-contained semantic units and must therefore have a construction assigned to them as their meaning; only which one? We finish by outlining how updating works within a dynamic discourse involving singular terms.

3.1 Definite descriptions

Consider the two sentences

(A) ‘Bill Gates is married.’

(B) ‘The richest man is married.’

The truth-conditions of (A) and (B) are distinct. That they are so should not be influenced by the fact that Bill Gates happens to be the richest man (as of 2009). The point is that ‘Bill Gates’ is a proper name[1] and so we cannot suppose that in distinct possible worlds this name would identify distinct individuals. Independently of any particular theory of proper names, it should be granted that a *proper* proper name (as opposed to a definite description grammatically masquerading as a proper name) is a rigid designator of a numerically particular individual. On the other hand, ‘the richest man’ as a (definite) *description* does offer an *empirical* criterion that both enables and forces us to establish which individual, if any, plays the role of the richest man at a particular world/time pair. If a pair $\langle W,T\rangle$ is such that Bill Gates is married at $\langle W,T\rangle$ and the man who is the richest man at $\langle W,T\rangle$ is not married at $\langle W,T\rangle$, then (A) is, and (B) is not, true at $\langle W,T\rangle$.

We can demonstrate this claim by associating (A) and (B) with two non-equivalent constructions. Let the types be: $BillGates/\iota$; $Married$, $Man/(o\iota)_{\tau\omega}$; $Richest/(\iota(o\iota))_{\tau\omega}$: the empirical function that, dependently on states-of-affairs, associates a class of individuals with at most one individual, namely the richest one.

(A′) $\lambda w \lambda t\ [{}^0Married_{wt}\ {}^0BillGates]$

(B′) $\lambda w \lambda t\ [{}^0Married_{wt}\ [{}^0Richest_{wt}\ {}^0Man_{wt}]]$,

[1] For a discussion of the semantics of proper names, see Section 3.2.

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or equivalently,

(B″) $\lambda w \lambda t\ [{}^0Married_{wt}\ [\lambda w \lambda t\ [{}^0Richest_{wt}\ {}^0Man_{wt}]]_{wt}]$.

The distinction between names and descriptions is of crucial importance due to their vastly different logical behaviour. This distinction is explicitly respected in TIL (a) type-theoretically (names lacking intensional character), and (b) by equipping analyses of empirical definite descriptions with the empirical indices w, t.

The contemporary discussion of the distinction between names and descriptions was triggered by Russell (1905). The relevant place is where Russell proposes eliminating the descriptive operator 'ι'. Where 'P' and 'A' are one-place predicates, Russell's standard formulation is

(Rus) $$P(\iota x\, Ax) \equiv \exists x\, (Ax\ \&\ \forall y\, (Ay \supset y = x)\ \&\ Px).$$

Russell's idea is that 'ι' possesses neither a self-contained meaning nor a denotation and that every context containing 'ι' can be replaced by an equivalent context lacking 'ι'. Below follows a set of comments outlining how we position ourselves in the contemporary debate on names and descriptions.

(a) *Frege vs. Russell*. Frege's conception of definite descriptions is referential; Russell's, quantificational. Frege assigns a dual, context-sensitive semantics to definite descriptions, while Russell argues that this sort of expressions must be done away with in the final logical analysis. We agree with Frege that definite descriptions are vehicles of reference. We find that Russell goes too far when arguing that they are syncategorematic expressions devoid of a semantics of their own. But we agree with Russell that definite descriptions do not denote the objects (if any) that they uniquely describe (even if we do not at all sympathise with his reasons for claiming so). Frege holds the same view, though only with regard to definite descriptions occurring in what he calls oblique (*ungerade*) contexts. In such contexts they denote what is in straight (*gerade*) contexts their sense, while in straight contexts they denote the unique objects, if any, they uniquely describe. Contextualism forces itself upon Frege because of his extensionalist semantics for straight contexts, which he himself acknowledges to fail to apply to oblique contexts. Despite their differences, a noteworthy feature shared by Frege's and Russell's conceptions is this. In oblique contexts Frege's definite descriptions denote a sense, which may be pre-theoretically construed as something like a *condition* satisfiable by the sort of objects that the descriptions denote in straight contexts. The gist of Russell's quantificational analysis of '$P(\iota x Ax)$' is that there is exactly one thing possessing the two properties A, P. (While there may be more than one P-object, there is to be exactly one A-object.) This analysis may likewise be construed pre-theoretically as forming the *condition* of being the unique thing that is both an A and a P. This is a very inspiring feature, because it suggests what kind of thing a definite description denotes, as soon as this is not to be whatever (if anything) it uniquely describes. In TIL this feature translates into the tenet that what is semantically salient about a definite description is its uniqueness clause, which is a condition, rather than what (if anything) satisfies it. To be more specific, the tenet is that empirical definite descriptions denote intensions (namely, offices or roles), which are the theoretical counterparts of pre-theoretically understood empirical conditions with a built-in uniqueness clause. In the case of mathematical definite descriptions, constructions figure as conditions while their denotations are the entities (if any) which are so constructed. Whether intensions or constructions figure as conditions, the principle that the semantic relation of denotation is a priori is heeded.

However, despite this common feature, Frege's and Russell's theories are inherently heterogeneous. Writing down the construction underlying the schema (Rus) in the empirical case brings out the fundamental distinction between Frege's and Russell's views. Let $P, A \rightarrow (o\iota)_{\tau\omega}$; $\iota/(\iota(o\iota))$; $x, y \rightarrow \iota$. Then:

(Rus')

Left-hand side:

$$\lambda w \lambda t\ [P_{wt}\ [\iota x\ [A_{wt} x]]].$$

Right-hand side:

$$\lambda w \lambda t\ [\exists x\ [[A_{wt} x] \wedge [\forall y\ [[A_{wt} y] \supset [y = x]]] \wedge [P_{wt} x]]].$$

Both sides construct *propositions* and not truth-values. Russell's insight (as opposed to Frege's) is that no individual makes any semantic or logical contribution to the analysis of definite descriptions.

(b) *Referential vs. attributive use*. Holding, as TIL does, that pragmatic problems are altogether different from semantic problems, referential use in Donnellan's sense is irrelevant to semantics. Our analyses terminate in constructions, so semantics affords no means to obtain the values of the particular intensions (as constructed by the constructions cited as meanings) in the actual world at the present time. So TIL is not able to accommodate Donnellan's bifurcation; nor ought any theory in the business of *logical* analysis of natural language to be able to do so, since the bifurcation can be upheld only in the sphere of pragmatics. In particular, Donnellan's famous example in 1966 of the man over there drinking martini is to be explained in terms of the pragmatics of communicative situations. The meaning of the phrase 'the man over there drinking martini' is an open construction which only v-constructs the individual office occupiable by whatever unique man is drinking martini. In order to be able to execute the construction, the parameter of valuation v must be added by a situation of utterance. Thus the phrase in and by itself does not denote an office prior to evaluation, it only denotes one in a given situation. If there is no such individual who at the given $\langle w, t\rangle$ and in the given situation occupies the office, then the phrase does not *refer to* anything (in our stipulative sense of *reference*), whereas the *speaker intends* to identify an individual.

'The man over there drinking martini' is a vehicle of reference that has recourse to such pragmatic factors as background knowledge shared by speaker and audience, gestures (like a nod in a particular direction), and perhaps a bit of charitable guesswork on the hearer's behalf. Absent such factors, the expressive power of 'the man over there drinking martini' is too feeble to enable the hearer to fix the speaker's reference.[2]

(c) *Eliminability of* 'ι'. (Rus') fails to apply, as soon as functions are allowed to be properly partial. We show this for two cases.

(c_1) *The Strawsonian case*. In TIL the functions corresponding to descriptive operators are of the polymorphous type $(\alpha(o\alpha))$ and not total. If the set that is the argument of ι is a singleton then ι returns the α-object that is the unique member of the set. Otherwise ι is *undefined*.[3] Thus the well-known proposition that the King of France is bald lacks a truth-value at such world-time pairs where there is no King of France: the set of Kings of France is empty at such worlds-times and so ι comes out undefined when applied to it. This result is in harmony with Strawson's criticism and, we might be so bold as to suppose, with people's untutored linguistic intuition as well. If that proposition were false (it cannot be true at such worlds/times) then its negation would have to be true. The King of France would not be bald, entailing that the King of France exists, thus colliding with the fact that there is no King of France.

Schiffer argues that Russell's theory cannot accommodate referential uses of definite descriptions, as it leaves it indeterminate what the entity intended by the speaker is (2005, p. 1179). But nor can Russell's theory, according to Schiffer, accommodate attributive uses, as it admits two interpretations of 'the *A*' in 'The *A* is a *P*', either as a quantifier phrase or a singular term. Instead Schiffer agrees with Frege that 'the *A*' is always a singular term, but adds that truth-gaps are acceptable.

(c_2) *Existential commitment and expressivity*. Suppose Charles is thinking about the Golden Mountain. He can do so if he thinks about the individual office[4] constructed by

$$\lambda w \lambda t\ \iota x\ [x = [[{}^0Golden_{wt}\, x] \wedge [{}^0Mountain_{wt}\, x]]].$$

Types: *Golden*, *Mountain*/$(o\iota)_{\tau\omega}$; $x \rightarrow_v \iota$.

The predicate corresponding to the left-hand side of (Rus') will denote the property that is had by a *y* being thought about by Charles; i.e.,

[2] See Section 3.4.

[3] Recall that 'ι' is abbreviated notation for '*Sing*', which denotes the function *singularizer*; see Definition 1.6. in Section 1.4.3.

[4] The following consideration holds even for the well-conceivable case that Charles is thinking about the *property* of being a golden mountain, perhaps wondering if there is any such property (as maybe the properties of being golden and being a mountain could not be co-instantiated).

$$\lambda w \lambda t\ \lambda y\ [{}^0Think_{wt}\ {}^0Charles\ y]$$

where $Think/(o\iota\iota_{\tau\omega})_{\tau\omega}$; $y \rightarrow_v \iota_{\tau\omega}$.[5] The left-hand side of (Rus') would be

$$\lambda w \lambda t\ [\lambda y\ [{}^0Think_{wt}\ {}^0Charles\ y]\ \lambda w \lambda t\ \iota x\ [x = [[{}^0Golden_{wt}\,x] \wedge [{}^0Mountain_{wt}\,x]]]].$$

What could Russell do with his right-hand side? He cannot distinguish between what we would call supposition *de dicto* and *de re*, for want of an equivalent mechanism, so he would use the existential quantifier to bind individual variables:

$$\lambda w \lambda t\ [\exists x\ [[{}^0Golden_{wt}\,x] \wedge [{}^0Mountain_{wt}\,x]] \wedge \\ \forall y\ [[[{}^0Golden_{wt}\,y] \wedge [{}^0Mountain_{wt}\,y]] \supset [y = x]] \wedge [{}^0Think_{wt}\ {}^0Charles\ x]]]].$$

For this right-hand side to come out true there must be a golden mountain (!). Thus in the worlds/times where there are no golden mountains the right-hand side of (Rus') would be false, whereas Charles can think about the office even in such world/time pairs.

A similar objection applies in all cases involving a construction of an office occurring *de dicto*, as in 'Charles wants to become the President of the USA'. Russell's solution involves the existence of the President of the USA, whereas Charles may well want to become President even if there is none.

(d) *Incomplete descriptions*. Most phrases of the form 'The *A* is a *P*', *P* some empirical property, can be conceived of as incomplete descriptions. The phrase 'The dog is dangerous' is obviously pragmatically incomplete in that it needs some contextual amendment: otherwise it would possess a truth-value only in a $\langle w, t\rangle$ having exactly one dog.[6] (We do not intend 'The dog is dangerous' to be synonymous with 'All dogs are dangerous'.)

(e) *Mathematical descriptions*. So far we have handled only empirical descriptions, as most problems with descriptions concern just those. In the case of mathematical descriptions, the question of whether referential, as opposed to attributive, use is possible does not arise. As an example consider the sentence

'The least prime number is even.'

Let ν be the type of natural numbers, and the other types as follows: (the)$Least/(\nu(o\nu))$; $Prime/(o\nu)$; $Even/(o\nu)$. We get

(C) $[{}^0Even\ [{}^0Least\ {}^0Prime]]$.

[5] This means that the construction of the office occurs *de dicto*; Charles is not thinking about whatever individual might be the occupant of the office, but about this office itself. For details on the *de dicto/de re* distinction, see Section 1.5.2.

[6] See Section 3.4.

This Composition obviously constructs the truth-value **T**.[7] What about the following sentence?

‘2 is even.’

An analysis of this sentence would be

(D) $[^0Even\ ^02]$.

(C) and (D) are simply two equivalent constructions. No problems analogous to those from the empirical sphere arise.

Another example:

‘The greatest prime is even.’

This time our sentence lacks a truth-value. *Greatest* is obviously of the same type as *Least* in the previous example, so we get

(E) $[^0Even\ [^0Greatest\ ^0Prime]]$.

The important difference between the sentence having and lacking a truth-value is not visible, the logical form being the same. The Composition (E) reflects, however, the fact that even when a sentence lacks a truth-value we *understand* the sentence: we know which procedure is to be executed. The fact that in this case the procedure would lead nowhere is given by the nature of the respective mathematical concepts.

3.2 Proper names

The formal semantics of TIL requires that every expression belonging to natural language that does not play an exclusively syntactic role must express a construction as its meaning and denote whatever is so constructed. The requirement presents us with an awkward problem in the case of *proper* proper names; that is, those so-called ordinary proper names whose semantics cannot be reduced to the semantics of any other sort of expression (typically definite descriptions) and which serve to pick out one numerically specific individual. Absent this requirement, however, a sentence containing an occurrence of a proper name would, due to the compositionality constraint, fail to express a sense and so would also fail to denote a proposition. Which cannot be right. In Section 3.3.1 it is shown how

[7] Throughout this book we mostly analyze simple *expressions* as expressing *simple concepts*. Thus the above analyses are *literal meanings* of the analyzed expressions. See Definition 1.10, Section 1.5.1. Though irrelevant here, a definition of *Prime* would help us to *refined* constructions of, e.g., the functions *Even* and *Least*.

'Hesperus' and 'Phosphorus' may fruitfully be construed as denoting individual offices; and this approach may no doubt be extended to several other ordinary proper names. But we may still have good reasons to preserve an irreducible category of proper names, for there are occasions when we wish to talk about some one particular individual (whatever may be true of this individual) rather than about whatever individual (if any) something is true of, relative to a given universe of discourse.

This prompts the question of which sort of construction to assign to *proper* proper names as their meaning. As we just suggested, the problem cannot be circumvented by simply declaring that names have no meaning at all, being mere 'labels', 'tags', or whatnot. For the meaning of a compound in which such a name occurs is a function of the meanings of its atoms, including names. Without assigning a construction to 'Charles', the sentence

'Charles is happy'

will elude semantic analysis. All we would have would be

$$\lambda w \lambda t\ [{}^0H_{wt} \ldots].$$

For lack of an argument for ${}^0H_{wt}$ the analysis would be nonsensical.

A second reason for assigning senses to proper names is that understanding the sense of a name is what enables a language-user to intellectually identify or select the bearer of the name. In keeping with our procedural semantics, identification or selection must take the form of executing a procedure whose product is the bearer.

The only two candidate constructions are Trivializations of individuals and variables ranging over individuals. This gives us either

$$\lambda w \lambda t\ [{}^0H_{wt}\ {}^0Charles]$$

or

$$\lambda w \lambda t\ [{}^0H_{wt}\ x].$$

The former pairs 'Charles' off with a construction directly of Charles the individual. The latter renders 'Charles' analogous to the occurrence of 'he' as in 'He is happy'. In Section 3.4 such an occurrence is paired off with a free occurrence of x.

There is a link between these two possible interpretations of 'Charles':

$$\frac{\begin{array}{c}\lambda w \lambda t\ [{}^0H_{wt}\ x] \\ \lambda w \lambda t\ [{}^0{=}\ x\ {}^0Charles]\end{array}}{\lambda w \lambda t\ [{}^0H_{wt}\ {}^0Charles].}$$

This sort of argument is needed in order to turn an open construction into a closed construction, which can then be evaluated for its truth-value at a $\langle w, t\rangle$.

What recommends the free-variable analysis is that often the mere name will not carry enough information to identify any particular individual. In a conversational context it will have to be settled, one way or the other, which particular individual is the denotation of a particular use of a particular name, whenever a name is just a string of characters formed from a vocabulary and not an ordered pair of such a string and a construction. The Achilles' heel of the free-variable analysis is that it cannot stand alone. 'Charles', as it occurs in 'Charles is happy' when this sentence is embedded in a particular conversational context, picks out one particular individual, and the only way to present this individual directly is by means of a Trivialization of him. Hence the second premise in the argument above. This suggests that the sense of a *proper* proper name is a Trivialization of an individual. To understand a name will then amount to knowing the numerical identity of the Trivialized individual.[8]

This construal of proper names offers a solution to the 'Cicero'/'Tully' puzzle. Let the meaning of 'Cicero' be ^{0}Cicero and the meaning of 'Tully', ^{0}Tully. Then if ^{0}Cicero and ^{0}Tully Trivialize the same individual, ^{0}Cicero and ^{0}Tully will be one and the same Trivialization, though encoded linguistically in two different manners, as '^{0}Cicero' and '^{0}Tully'. ^{0}Cicero and ^{0}Tully will be intersubstitutable in any sort of context, since anything may always be substituted for itself. The Closures

$$\lambda w\lambda t\ [^0B^*_{wt}\ ^0a\ ^0[\lambda w\lambda t\ [^0H_{wt}\ ^0Cicero]]]$$
$$\lambda w\lambda t\ [^0B^*_{wt}\ ^0a\ ^0[\lambda w\lambda t\ [^0H_{wt}\ ^0Tully]]]$$

are identical, $B^*/(o\iota^*_1)_{\tau\omega}$ ('to believe* constructionally' or explicitly, i.e., being related to the literal meaning of the embedded clause; see Section 1.5.1, Definition 1.10). It is irrelevant whether *a*, the attributee, knows either of the *words* 'Cicero' and 'Tully', since the belief ascription concerns (among other) what is constructed by ^{0}Cicero/^{0}Tully (to wit, Cicero the man) and not either of the names 'Cicero' or 'Tully' (see Section 5.1.1 for Mates' puzzle). Thus, to know that Cicero is Tully is only to know that Cicero is self-identical. Knowing that Tully is called both 'Cicero' and 'Tully' is something entirely different, concerning as it does linguistic competence with two English words. It is non-trivial to know that 'Cicero' and 'Tully' are synonymous (and therefore co-denoting). Only this is an a priori fact about English and not an a posteriori fact about empirical reality. If you wish to turn it into a discovery about empirical reality that Cicero was Tully, it is necessary to construe at least one of 'Cicero', 'Tully' as a name of an individual office, along the lines of the analysis of 'Hesperus is Phosphorus' in Section 3.3.1.

Ordinary proper names also occur in fictional literature. The sentence

'Sherlock Holmes is happy'

[8] For further discussion, see Jespersen (2000).

is not to be paired off with the closed construction

$$\lambda w \lambda t\ [{}^0H_{wt}\ {}^0a]$$

where a/ι. This would namely make it either true or false ('truth-apt') whether Holmes is happy. But there is no fact of the matter as to whether Holmes is happy. Nor should there be, as soon as we wish to uphold a demarcation between fact and fiction. The meaning of so-called fictional names such as 'Sherlock Holmes' is instead construed as a free variable ranging over individuals. The meaning of the sentence is the open construction

$$\lambda w \lambda t\ [{}^0H_{wt}\ x].$$

This analysis is analogous to the analysis of 'He is happy' up to this point. But the difference is that the step from open to closed construction is never made. It is, as it were, left hanging in the air *which* individual is Sherlock Holmes. For all the analysis says, *any* individual is a possible value of x. This allows both the author's and the reader's imagination free rein to identify Sherlock Holmes with any particular individual (e.g., the author or reader himself or herself) or no-one in particular. This analysis also removes the need for a parallel pseudo-universe of fictional entities as denotations of fictional names. The semantics of 'Sherlock Holmes' is that it expresses a free variable ranging over individuals as its sense, but fails to denote, since a free variable does not construct anything (until a valuation assigns a value to it). So 'Sherlock Holmes is happy' has a meaning, but lacks a denotation (a truth-condition) and, therefore, a reference (a truth-value). The attractive outcome is that fictional discourse may be meaningful without thereby lending itself to making assertions about factual matters.

A seemingly more tricky case is provided by occurrences in fiction of names familiar from extra-fictional discourse, such as 'London' as it occurs in, 'Sherlock Holmes lives in London'. But Conan Doyle's novels are not drama-documentaries about London. We speak sloppily, and misleadingly, when we say that the novels are set in London, if by this we mean that they literally take place in London. Rather we are to *imagine* the plots unfolding in London (and Yorkshire and Switzerland and wherever else). Also 'London' as it occurs in the novels expresses as its sense a free variable ranging over individuals (on the simplifying, but probably innocuous, assumption that cities are mere individuals). The analysis of 'Sherlock Holmes lives in London' is, therefore, this open construction ($Live\_in/(o\iota\iota)_{\tau\omega}$):

$$\lambda w \lambda t\ [{}^0Live\_in_{wt}\ x\ y].$$

But the respective values of x, y cannot just have any properties the reader cares to imagine. If x lives in y then x must be a person and y a house/village/town/city/country. It means that a sentence attributing the relation *Live_in* to a pair $\langle x, y\rangle$ of individuals comes with a presupposition, namely that x should be a person and y a venue[9].

$$\forall x \forall y\, [{}^0Presupposition\ \lambda w \lambda t\, [[{}^0Person_{wt}\, x] \wedge [{}^0Venue_{wt}\, y]]$$
$$\lambda w \lambda t\, [{}^0Live\_in_{wt}\, x\, y]]]]].$$

Types: $Presupposition/(o\ o_{\tau\omega}\ o_{\tau\omega})$; $Person$, $Venue/(o\iota)_{\tau\omega}$; $x \rightarrow \iota$; $y \rightarrow (o\iota)_{\tau\omega}$.

Gloss: 'For all *x, y*, in order that the proposition that x lives in y have a truth-value, the proposition that x is a person and y a venue has to be true.'

This means that when reading that Sherlock Holmes lives in London, the reader must *imagine* a person living in a city. Furthermore, to *read* the novels the reader must adapt his or her images to the predicates that the author uses to describe Sherlock Holmes and London. Within these two constraints, the reader is free to build up his or her own images of Sherlock Holmes and London.[10]

The use of free variables is what underpins poetic licence, enabling artists to separate a string like 'Amerika' from the pair $\langle$'Amerika', ^{0}America$\rangle$ and assign instead a free occurrence of z to it as its meaning to form the pair $\langle$'Amerika', $z\rangle$. This is what both enabled and entitled Franz Kafka to use the string 'Amerika' in his novel *Amerika* when conjuring up scenes from an imaginary country that at the best of times bears only superficial resemblance to the country that German-speaking Kafka knew as 'Amerika' without thereby making any claims about America. Only on an overly naïve interpretation of Kafka's 'Amerika', disregarding the fictional status of *Amerika*, would its sense be taken to be ^{0}America.[11]

3.2.1 Mathematical constants

Consider numerical constants like '1' and 'π'. What is their semantics? Since our general procedural semantics correlates sense and denotation as procedure and product, the resulting theory bears similarities to Moschovakis' as based on algorithm and value. At the same time we are in stark opposition to Kripke's unrealistic realist contention that the semantics of 'π' consists in nothing other than 'π' rigidly denoting π. For sure, 'π' does denote π—indeed, 'π' qualifies as a strongly rigid

[9] For the definition of presupposition, see Section 1.5.2, Definition 1.14.

[10] To settle just how free the reader is—for instance, as concerns inconsistent images of Sherlock Holmes and London—will include a discussion of *conceivability*, which we are not going to broach here.

[11] We do not pretend to have put forward anything like a semantics for fictional terms and expressions other than ordinary proper names. For instance, we have at this point nothing to say about the semantics of predicates or definite descriptions as they occur in fiction, or about the sense in which it seems somehow true (-in/about-fiction) to say that Sherlock Holmes' sidekick is Dr Watson and false that Sherlock Holmes plays the tuba. See, however, Tichý (1988, §49).

designator of π (cf. Kripke 1980, p. 48)—but there is substantially more to the semantics of 'π' than merely the denotation relation. In this section we focus on 'π', since our general top-down strategy is to develop a semantics for the hardest (or a very hard) case and then generalise downwards to increasingly less hard cases from there.

In outline, our procedural semantics says that 'π' expresses as its sense a procedure whose product is π. The procedure is, as a matter of mathematical convention, a *definition* of π and the product is, as a matter of mathematical fact, the (transcendental) *number* so defined. For comparison, '1' expresses as its sense the procedure consisting in applying the successor function to 0 once and denotes whatever (natural) number emerges as the product of this procedure.

The upside of a procedural semantics for 'π' is that to *understand*, as a reader or hearer, and to exercise *linguistic competence*, as a writer or speaker, one must merely understand a particular numerical definition and need not know which number it defines. Procedural semantics, whether realist or idealist, construes sense as an *itinerario mentis* abstracting from the itinerary's destination. Making the denotation of a numerical constant irrelevant to understanding and linguistic competence is not pressing in the case of '1', but it is so in the case of 'π'. The downside, however, is that at least two equivalent, but obviously distinct, definitions of π are vying for the role as *the* sense of 'π'. One is *the ratio of a circle's area and its radius squared*; the other is *the ratio of a circle's circumference to its diameter*. They are equivalent, because the same number is harpooned by both definitions. But the procedures are conceptually different, so they should not both be assigned to 'π' as its sense on pain of installing homonymy. This kind of predicament has become historically famous. Frege says, in analytic philosophy's single most notorious footnote:

> Solange nur die Bedeutung dieselbe bleibt, lassen sich diese Schwankungen des Sinnes ertragen, wiewohl auch sie in dem Lehrgebäude einer beweisenden Wissenschaft zu vermeiden sind und in einer vollkommenen Sprache nicht vorkommen dürften.
> (1986b, n. 2, p. 42.)

We shall suggest a solution to this predicament. The crust of the solution is to relegate each definition of π to individual *conceptual systems*.[12] Since an interpreted sign such as 'π' is a pair whose elements are a character (in this case the Greek letter 'π') and a sense, there will be as many such pairs as there are conceptual systems defining π. Disambiguation of 'π'-involving discourse will consist in making explicit which particular π-defining system should supply the sense of a token of the character 'π'.

A related predicament, which we shall also address, is whether 'π' is best construed as a *name* for π or as a shorthand for a *definite description*. If a name, the sense of 'π' will, in our semantics, be the Trivialisation $^0\pi$, i.e., the primitive procedure consisting in the instruction to obtain, or access, π in one step. The procedure

[12] See Definition 2.14, Section 2.2.3.

will not tell us *how* to obtain π, but only *that* π is to be obtained. This does not sit well with π being something as complicated as a transcendental number. But it does sit well with 'π' being itself a primitive, or simple, character not disclosing any information about its denotation. So at least on a naïve *literal analysis* (see Section 1.5.1, Definition 1.10), 'π' should be paired off with a non-complex sense. If 'π' is a definite description (in disguise), the sense of 'π' will, in our semantics, be a complex procedure consisting in the instruction to manipulate various mathematical operations and concepts in order to define a number. Only the problem, as we just pointed out, is, *which* procedure? Is it the instruction to calculate the ratio of a circle's area and its radius squared, or is it the instruction to calculate the ratio of a circle's circumference and its diameter, or is it some yet other instruction? Whichever it may be, though, the grammatical constant 'π' will be *synonymous* with the definite description 'the ratio…' chosen. The problem of homonymy does not rear its head in case the sense of 'π' is $^0\pi$, for then 'π' is only *equivalent* (co-denoting) with a particular definition. In fact, since all the variants of definitions co-denote the same number, 'π' will be equivalent with all such descriptions.

Whether 'π' be a name or a disguised definite description, it holds that its denotation needs to be *defined* and that an *algorithm* is required to bridge between definition and number. By showing how to calculate π, the algorithm shows, *ipso facto*, what the denotation of 'π' is. Our underlying semantic schema comes in two variants, one pure, the other impure. The pure one is

$$\begin{tikzcd} & \textit{constant} \arrow[dl, "\text{expresses}"'] \\ \textit{construction}\ (\textit{sense}) & \end{tikzcd}$$

The relation a priori of expressing as obtaining between constant and sense exhausts the pure semantics of the constant. Only its sense is semantically salient, so a semantic analysis of 'π' must make its sense explicit. However, as soon as a procedure is explicitly given, its product (if any) is implicitly given, for the relation from procedure to product is an internal one: a procedure can have at most one product, and that product is invariant. The pure schema depicts a constant expressing its sense and not also what the constant denotes. An impure schema includes not only constant and sense, but also denotation:

$$\begin{tikzcd} & \textit{constant} \arrow[dl, "\text{expresses}"'] \arrow[dr, dotted, "\text{denotes (names)}"] & \\ \textit{construction}\ (\textit{sense}) \arrow[rr, dotted, "\text{produces}"'] & & \textit{denoted entity}\ (\textit{if any}) \end{tikzcd}$$

The construction will produce its product independently of any algorithm; this is why the relation between construction and product is an internal one. But for epistemological reasons we will need some way or other of calculating its product to learn what it is, so we need a π-calculating algorithm to show us what number satisfies whatever π-defining condition. Such an algorithm will, *ipso facto*, reveal to us what the denotation of 'π' is. The number 3.14159… which is π is itself no player in the pure semantics of 'π'. The value of π is just whatever number rolls out as the value of the given procedure. The number 3.14159…is itself of little mathematical interest and of no semantic import. The *properties* of π, by contrast, are of great interest; e.g., whether π is normal in some base; and establishing that π is transcendental (and not merely rational) was a major mathematical achievement.

An algorithm may appear in one of two capacities. Either it is an intermediary between the definition and the number so defined: then the algorithm (whichever it is) is no player in the pure semantics of 'π'. Or an algorithm is the very sense of 'π': then the algorithm is a player in the pure semantics of 'π'. Our procedural semantics allows that a π-calculating algorithm may itself be elevated to playing the role of sense of 'π'. In such a case 'π' will have as its sense one particular way of calculating π. An algorithm is a particular kind of procedure and can as such figure as a linguistic sense relative to a procedural semantics.

In the former case, if the definition is a *condition* then the algorithm will calculate the *satisfier* of the condition. Full competence with respect to the definition *the ratio…* will yield knowledge of a condition to be satisfied by a real number, but will not yield knowledge of which number satisfies it. So the definition is, strictly speaking, a definition of something for a number to be; namely, the ratio of two geometric proportions. Hence three players need to be kept separate in the impure semantics of 'π': constant, sense, and number. If an algorithm is a sense then the sense is an effective mathematical procedure calculating π. Otherwise the sense is a logical procedure defining π in a non-effective way. Hence, if the sense defines π as the ratio between the area of a circle and its radius squared, a matching algorithm must calculate this ratio. Full linguistic competence with respect to 'π' neither presupposes, nor need involve, knowledge of how to calculate π. What competence consists in depends on whether the sense of 'π' is an atomic or a compound construction. If atomic, competence requires knowing which transcendental real 3.14159… is π. If compound, competence requires understanding the concept *the ratio of*, as well as either the concepts *the area of*, *the radius of*, *the square of*, or the concepts *the circumference of* and *the diameter of*, together with knowledge of how to mathematically manipulate them. A school child will understand such a complex procedure; it takes a professional mathematician to develop and comprehend a π-calculating algorithm. The task facing the mathematician is to come up with an algorithm equivalent with the sense of the definition defining the given ratio.

In the latter case, where an algorithm is the sense of 'π', full linguistic competence with respect to 'π' is to understand a definition of π and, again, not of the number so defined. But since the algorithm is now not an intermediary between definition and number, linguistic competence will be harder to come by, since the sense of 'π' is now likely to involve much more complicated mathematical notions than just, say, those of ratio, area, and circumference, such as the limit of an infinite series.

Assume now that the truth-condition of '…π…' requires π to exist as an independent, abstract entity. Assume, further, that we can have no epistemic access to entities that we can have no causal interaction with. Then next stop is Benacerraf's dilemma as formulated for π: we do not know what number is π; yet we want to dub π 'π' in order to talk about π in '…π…'.[13] So how is 'π' to be introduced into mathematese? Moreover, now that 'π' has actually been introduced into standard mathematical vocabulary and been in use for 300 years, what would a realist (as opposed to constructivist or otherwise idealist) construal of its semantics look like?

As language-users we can *baptise* an abstract entity E 'E', as well as *use* 'E' competently, provided the following two conditions are met.

First condition. In order to *introduce* 'E' into mathematese, we must have a complex *procedure P* at our disposal, such that the unique output of P is the entity E, making the procedure P an ontological definition of E. An ontological definition of E is a closed construction of E different from ${}^0E$.[14] Two examples of ontological definition of the real number π would be the right-hand sides of the equivalences

$$
\begin{aligned}
{}^0\pi &= \iota x\, [\forall y\, [x = [{}^0\mathit{Ratio}\, [{}^0\mathit{Area}\ y]\ [{}^0\mathit{Square}\, [{}^0\mathit{Radius}\ y]]]]];\\
{}^0\pi &= \iota x\, [\forall y\, [x = [{}^0\mathit{Ratio}\, [{}^0\mathit{Circumference}\ y]\ [{}^0\mathit{Diameter}\ y]]]].
\end{aligned}
$$

Types: π/τ; $x/*_1 \rightarrow \tau$; $y/*_1 \rightarrow \gamma$; *Ratio*/$(\tau\tau\tau)$; *Area*, *Radius*, *Circumference*, *Diameter*/$(\tau\gamma)$; *Square*/$(\tau\tau)$; $=/(o\tau\tau)$; γ is here the type of geometrical figures, whatever it may be.

The sense of any *program* computing π is going to be an algorithm *equivalent* to, but *not synonymous* with, ontological definitions such as *the ratio of the circumference of a circle to its diameter* or *the ratio between a circle's area to its radius squared*. To competently use 'π' is to know at least one of these definitions.

Second condition. In order to be able to *use* 'E', we must not kick off the definition(s) of E; for we need to know that the sense of 'E' is *equivalent* to, though *not synonymous* with, the respective definition(s). For instance, we can use 'is a prime', provided we know at least one of the possible definitions of the set of primes. That is, pretending that the three equivalencies below exhaust the possible

[13] See Benacerraf (1973).

[14] See Definition 2.13, Section 2.2.2.

definitions of the set of prime numbers, we must know at least one of them to qualify as competent with respect to 'is a prime'.

$^{0}Prime = \lambda x\ [[^{0}Cardinality\ \lambda y\ [^{0}Divide\ y\ x]] = {}^{0}2];$

$^{0}Prime = \lambda x\ [[x \neq {}^{0}1] \wedge \forall y\ [[^{0}Divide\ y\ x] \supset [[y{=}x] \vee [y = {}^{0}1]]]];$

$^{0}Prime = \lambda x\ [[x \neq {}^{0}1] \wedge \neg\exists y\ [y > {}^{0}1] \wedge [x \neq y] \wedge [^{0}Divide\ y\ x]]].$

In other words, we can baptise the set of primes 'is a prime', 'is a prôtos', 'is an euthymetric', 'is a rectilinear', or whatever other predicate may have been used, but without a complex procedure yielding the set as output, these concatenations of letters are semantically void and futile.

Similarly for the introduction of 'π' via an ontological definition of π. Any algorithm computing π is going to be equivalent to, but not synonymous with, ontological definitions such as *the ratio of the circumference of a circle to its diameter* or *the ratio between a circle's area to its radius squared*. To master 'π' is to know at least one of these definitions (and not the number).

It may be illustrative to compare our realist procedural semantics to Kripke's realist denotational semantics. Central to the latter is the distinction between *fixing the reference* and *giving the meaning/a synonym*. One of Kripke's illustrations is this:

> ['π'] is not being used as *short* for the phrase 'the ratio of the circumference of a circle to its diameter'… It is used as a *name* for a real number, which in this case is necessarily the ratio of the circumference of a circle to its diameter (1980, p. 60).

Kripke's semantics for 'π' is simple (simplistic, as it turns out):

$$\text{`}\pi\text{'} \xrightarrow[\text{rigidly designates}]{\qquad\qquad\qquad} \pi$$

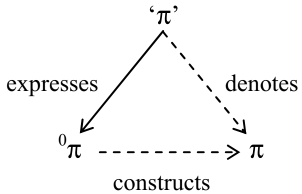
The description 'the ratio…' serves to single out the unique ratio shared by all circles, after which that number is baptised 'π'. The description is subsequently kicked off and so does not form part of the semantics proper of Kripke's 'π'. This is problematic. Nobody knows of some one particular real that it is π. So nobody knows of some one particular real that it is the reference of 'π'. So it is obscure what linguistic competence with respect to 'π' would consist in. Note that it is not an option to say that 'π' designates whatever real is the ratio of a circle's circumference to its diameter, for this uniqueness condition forms no part of Kripke's semantics for 'π'.[15] Kripke's introduction of 'π' is impeccable, and his 'π' does

[15] The Kripkean can have recourse to some causal theory of reference in the case of words for empirical entities like tigers, lemons and gold. But Benacerraf's second horn (the one that concerns knowledge and reference) blocks this avenue in the case of abstract entities like numbers. We hypothesise that Kripkean rigid designation cannot possibly be extended to numerical constants and other terms denoting abstract entities.

denote π. But we cannot use his ‘π’ to denote π, nor can we understand anyone else’s use of ‘π’, since we cannot know which particular transcendental number is π. In sum, Kripke’s ‘π’ has been severed radically from any humanly possible linguistic practice, so it is inoperative.[16]

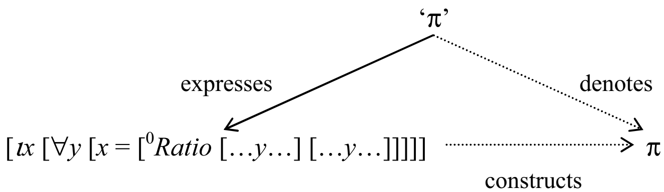
In the idiom of procedural semantics, Kripke focuses entirely on the *product* at the expense of the *procedure*. As a matter of mathematical fact, 3.14159… is π, but why introduce a non-descriptive name when that name severs the link between condition/procedure and satisfier/product? It seems that on Kripke’s semantics it will be a discovery, and not a convention, that π is the ratio of a circle’s circumference to its diameter (the template of the discovery being that *a* is the *F*). If so, it also seems that Kripke’s ‘π’ misconstrues mathematical practice.

Some π-producing procedure must figure in the semantics of ‘π’; but how? TIL faces a dilemma of its own, as we saw above. On the one hand, a literal analysis of ‘π’ would dictate that the sense of ‘π’ be $^0\pi$, yielding the schema



The advantage of this construal is that what looks like a constant *is* a constant (and not a definite description masquerading as one). However, this is too close to Kripke’s ‘π’ for comfort. We would be reinstating the problem that the semantics of ‘π’ pairs no mathematical condition off with ‘π’. To master ‘π’, $^0\pi$ would suffice. But, of course, this Trivialization merely instructs us to construct π and is silent on how to construct it.

On the other hand, not least epistemic concerns dictate that the sense of ‘π’ ought to be an *ontological definition* of π, yielding the schema



This makes ‘π’ a shorthand term synonymous with ‘the ratio…’, and its sense is an ontological definition of π. The advantage of this construal is that it pairs a mathematical condition off with ‘π’; but again, which one? There is no criterion to help decide which of the possible ontological definitions should be *the* sense of

[16] See Kripke (2008).

'π'. It would be arbitrary to select one and assign it as sense; but assigning them all would introduce homonymy.

It would seem evident that a language-user needs to know at least one definition of π in order to use and understand 'π'. If we go with the Trivialization-based analysis of 'π', the first step toward enhancing it is to make the logico-semantic fact that $^0\pi$ is *equivalent* with $[\iota x\ [\forall y\ [x = [^0Ratio\ [\ldots y\ldots]\ [\ldots y\ldots]]]]]$ part of the semantics of 'π'. $^0\pi$ is indifferent to how π is constructed by this or that compound construction, so as far as equivalence goes, any compound π-construction is as good as any.

'π' may be *introduced* as equivalent with

$$[\iota x\ [\forall y\ [x = [^0Ratio\ [\ldots y\ldots]\ [\ldots y\ldots]]]]],$$

or

$$[\iota x\ [\forall y\ [x = [^0Ratio^*\ [\ldots y\ldots]\ [\ldots y\ldots]]]]],$$

or any other compound π-constructing construction. *Understanding* is another matter. One thing is to understand $[\iota x\ [\forall y\ [x = [^0Ratio\ [\ldots y\ldots]\ [\ldots y\ldots]]]]]$; another thing is to understand $[\iota x\ [\forall y\ [x = [^0Ratio^*\ [\ldots y\ldots]\ [\ldots y\ldots]]]]]$. One may well know that 'π' is equivalent to this Composition without knowing, *ipso facto*, that it is equivalent to that Composition.

It is hopefully clear by now that both causal theory of reference and denotational semantics are neither here nor there as a theory of terms for abstract entities such as numbers. So we are putting forward a procedural semantics as a rival theory in order not to get gored by Benacerraf's horns or turning linguistic competence with mathematical constants into an enigma. We suggest, in the final analysis, that the semantics of 'π' ought to be that it is shorthand for, and therefore synonymous with, a definite description expressing a definition of π and denoting the number so defined. But for each definition$_n$ of π there is going to be a pair $\langle$'π', definition$_n$ (π)$\rangle$. So how do we handle the resulting homonymy? *Schwankungen des Sinnes* are neither here nor there in a regimented language such as mathematese. Our solution consists in relegating different definitions of π to different π-defining conceptual systems.

Relative to a particular conceptual system, a pair $\langle$'π', definition$_n$ (π)$\rangle$ is an unambiguous assignment of exactly one definition of π to 'π', provided the conceptual system is *independent* (as described in Section 2.2.3). Consequently, 'π' is not ambiguous, for this character must always be given together with a particular definition of π drawn from a particular conceptual system. The appearance of ambiguity arises only when two or more conceptual systems are invoked in the course of a discourse in which tokens of 'π' occur.

The upshot of our solution is that there are several π-denoting constants sharing the same first element, 'π'. So when two mathematicians are both deploying tokens of 'π', there is a risk of them talking at cross purposes, until and unless they compare notes and, in case of invoking different conceptual systems, come to agree on the same definition of π in the interest of synonymy. Yet the mathematical results they may have individually obtained with respect to π are bound to be equivalent, for any two definitions of π are bound to converge in the same number. The problem, after all, was always to do with *Schwankungen des Sinnes* and never *Schwankungen der Bedeutung*.

The more general morale we extract is that abstract entities cannot be dealt with without ontologically defining them first. Therefore, complex procedures are indispensable in the semantics of names for abstract entities.

3.3 Identities involving descriptions and names

TIL is a typed logic, so the identity relation = is of the polymorphous type (οαα). There is no such thing as being identical to something *simpliciter*; there is only being identical to an α-entity, α an arbitrary type. So, unlike type-free logics such as Bealer's, we cannot express that *everything* is self-identical. What we can express is that every α-entity is self-identical, that every β-entity is self-identical, and so on for each particular type. Two random instances of the type (οαα) would be (οιι), the self-identity of an individual, and $(\omicron(\omicron\iota*_1)_{\tau\omega}(\omicron\iota*_1)_{\tau\omega})$, the self-identity of a relation-in-intension between an individual and a first-order construction.

Here we take a closer look at various identity sentences culled from natural language. It would seem that the open-ended, seven-membered list below shall be able to cover a wide range of such sentences. TIL makes it possible to express that a particular individual bearing two names is self-identical; that a particular ι-Trivialization is self-identical (equivalently, that two different names are synonymous); that a particular individual is identical to the occupant of an office; that the occupant of one office is identical to the occupant of another office; that some particular individual is identical to the value of an attribute/$(\iota\iota)_{\tau\omega}$[17]; that the value of one attribute is identical to the value of another attribute; and that, necessarily, the occupant of one office is the occupant of another office.

Here is the list.

(1) $[{}^0a = {}^0b]$
(2) $[{}^{00}a =' {}^{00}b]$
(3) $\lambda w\lambda t\, [{}^0a = {}^0A_{wt}]$
(4) $\lambda w\lambda t\, [{}^0A_{wt} = {}^0B_{wt}]$
(5) $\lambda w\lambda t\, [{}^0a = [{}^0C_{wt}\, {}^0b]]$
(6) $\lambda w\lambda t\, [[{}^0C_{wt}\, {}^0a] = [{}^0D_{wt}\, {}^0b]]$
(7) $[{}^0Req_2\, {}^0A\, {}^0B]$.

[17] Attributes are here construed as empirical functions of type $(\alpha\beta)_{\tau\omega}$; α, β arbitrary types. *Father_of*, *Mother_of*/$(\iota\iota)_{\tau\omega}$, *Colour_of*/$((\omicron\iota)_{\tau\omega}\iota)_{\tau\omega}$ are examples of attributes.

Types: $=/o\iota\iota$; $='/(o*_1*_1)$; $a, b/\iota$; A, B ($A \neq B$)$/\iota_{\tau\omega}$; C, D ($C \neq D$)$/(\iota\iota)_{\tau\omega}$; $Req_2/(o\iota_{\tau\omega}\iota_{\tau\omega})$ (Section 4.1 explaining the subscript in 'Req_2').

Remark. It is also an option that C, D share the same argument: $[[^0C_{wt}\ ^0a] = [^0D_{wt}\ ^0a]]$. Alternatively, the argument of an attribute may be the value of an ι-office: $[^0C_{wt}\ ^0A_{wt}]$. Or a pair of attributes may be arranged in a requisite relation. This opens up the possibility of further (obvious) combinations.

Example 3.1 '*Leningrad is St Petersburg*', '*Praha is Prague*', '*Den Bosch is 's Hertogenbosch*'.

There are two options: either (1) or (2). (1) attributes self-identity to an individual bearing two different names and is, therefore, trivially true or trivially false. Since semantics, as TIL understands it, is a priori, (1) is knowable a priori, as it requires only linguistic competence to establish whether it is true. (2) says that ^0a is the same Trivialization as ^0b, attributing self-identity to the meanings of 'a', 'b'; i.e., that 'a', 'b' are synonymous. Whether (1) or (2), it makes no difference if the two names belong to two different languages, as with 'Prague' and 'Praha'. It constitutes a linguistic, and not empirical, discovery that 'Prague' and 'Praha' are synonymous expressions. (1), (2) are logically related, (1) trivially following from (2) and (2) from (1). We do not accept an interpretation to the effect that, say, 'Den Bosch is 's Hertogenbosch' would mean that Den Bosch is (also) called 's Hertogenbosch'.[18] This interpretation would require amending the intensional base {ο, ι, τ, ω} so as to include *linguistic types*. This is formally feasible, of course; but there is a philosophical-methodological reason not to make the amendment. Such a 'meta-linguistic' solution, as it is commonly dubbed in the literature, runs counter to the TIL tenet that semantics is a priori. It is not consonant with the tenet to include expressions *qua* expressions, or linguistic items, into a logico-semantic analysis. Expressions exhaust their role by expressing constructions. Expressions are gateways to constructions, which are the objects of logico-semantic study. The study of expressions (their grammar, etymology, etc.) belongs to linguistics. For instance, TIL is geared to logico-semantic analysis and cannot, without thoroughgoing alteration, analyse a linguistic sentence like, 'The English word 'word' is a monosyllabic word of Germanic origin'. The tenet is based on the assumptions that a logico-semantic analysis is a synchronic snapshot of the ⟨expression, meaning⟩ pairs of a given language (English, Dutch, and Czech, as it happens) at a given

[18] A real-life case would be registered trademarks involving not only a logo but also a name encodable in plain lettering as well, like 'Budweiser'. As the on-going legal battle over the string 'Budweiser' has shown, uniqueness does matter. Allegedly the US market is not big enough for two syntactically indistinguishable brand names, since name recognition is part of brand recognition. So the battle turns on over whether American Budweiser or Czech Budweiser will occupy the office *the unique beer named 'Budweiser' on the American market*. A TIL construction of that office would involve linguistic types, which is fine, since the string 'Budweiser' is obviously part of the analysandum. What we are opposed to is the trick of shifting the denotation of a name from an entity beyond the name to the name itself, as in the 'Den Bosch'—''s Hertogenbosch' case.

point in time, and that full linguistic competence with a language is tantamount to knowing the finite set of such pairs and the grammatical rules for combining atoms into molecules.

Since we are assuming that speakers are fully competent language-users, we disagree with Kripke when he claims that

> You certainly *can*, in the case of ordinary [non-Russellian] proper names, make quite empirical discoveries that… Hesperus is Phosphorus, though we thought otherwise. We can be in doubt as to whether Gaurisanker is Everest or Cicero is in fact Tully.
> (1971, p. 143.)

The only empirical discoveries our speakers make concern facts about the world and not facts about language. This stance is, of course, in stark opposition to the naturalistic conception of language made popular by Quine. So we also disagree that Kripke does right in agreeing with Quine when the latter says that

> When…we discover that we have tagged the same planet twice, our discovery is empirical… (ibid., p. 141.)

If 'Hesperus' and the rest are names of individuals, then we would be discovering empirically that Hesperus is self-identical and doubting whether Cicero is self-identical. It is not clear to us how self-identity might be established empirically or what it would mean to doubt anything's or anyone's self-identity. On the other hand, if 'Hesperus' and the rest are names of individual offices, then it must be established empirically whether Hesperus and Phosphorus happen to share the same occupant (see Section 3.3.1 for a worked-out analysis) and it may be rationally doubted whether the Cicero and Tully offices happen to share the same occupant.[19]

It might be tempting to analyse cases involving *pseudonyms* along the same lines. However, we suggest that they are best analysed as instances of (3).

Example 3.2 '*Samuel Langhorne Clemens is Mark Twain.*'

(3) says that *a* occupies the office denoted by *N*, where the office named by *N* is defined in terms of certain achievements, like authoring certain books. This analysis makes it analytically necessary that Mark Twain should pen *Huckleberry Finn*, or whatever else may define the office of Mark Twain. Similarly, it will be analytically necessary that Shakespeare should write *Richard III* (and all the rest traditionally attributed to Shakespeare). What constitutes a historical discovery will then instead be who occupied the Shakespeare office. It may even be a historical discovery that the Shakespeare office was occupied by two or more different

---

[19] On a charitable reading, Quine (and Kripke) perhaps had in mind another kind of empirical discovery, not concerning the state of the world but of the language in question. This would make a *linguistic* discovery of Quine's empirical discovery that two different names have been tagged to Venus. But this empirical linguistic discovery belongs to linguistics rather than semantics. Schematically, the formal semanticist lays down the rule that *if* two names NAME 1, NAME 2 co-denote *then* a sentence in which NAME 1, NAME 2 flank the identity sign expresses that the shared denotatum is self-identical. The field linguist instead establishes whether the antecedent is true of two actual names.

individuals at different points in time. In this respect there is something close to what Kripke calls a 'logical fate' hanging over Mark Twain and William Shakespeare (cf. 1980, p. 77.)

Example 3.3 '*Angela Merkel is the Bundeskanzlerin of Germany.*'

(3) is the right choice if we wish to express that some particular individual (denoted by a *proper* proper name) is, contingently, the occupant of some particular office (*in casu*, the *Bundeskanzleramt*). A deeper analysis would be (5), however, in order to also mention Germany in the analysis. The analysis is then

$$\lambda w \lambda t\ [{}^0a = [{}^0Bundeskanzlerin\_of_{wt}\ {}^0Germany]]$$

Types: $Bundeskanzlerin\_of/(\iota\iota)_{\tau\omega}$; $Germany/\iota$.

Example 3.4 '*The President of Turkmenistan is the Prime Minister of Turkmenistan*'.

This can be analysed in two different ways, one *de dicto* in terms of requisites, and the other *de re* in terms of contingent coincidence of two offices (see Section 1.5.2 for the corresponding dual *de dicto*/*re* analysis of, 'The King of France is a king'). The analysis *de dicto,* in accordance with (7):

$$[{}^0Req_2\ \lambda w \lambda t\ [{}^0Pres\_of_{wt}\ {}^0Turk]\ \lambda w \lambda t\ [{}^0PM_{wt}\ {}^0Turk]]$$

Types: $Req_2/(o\ \iota_{\tau\omega}\ \iota_{\tau\omega})$; $Pres\_of$, $PM\_of/(\iota\iota)_{\tau\omega}$: president of something, prime minister of something; $Turk/\iota$: Turkmenistan.

The analysis *de re*, in accordance with (6):

$$\lambda w \lambda t\ [[{}^0Pres\_of_{wt}\ {}^0Turk] = [{}^0PM_{wt}\ {}^0Turk]].$$

The analysis *de re* is a case of *co-reference*; i.e., of contingent co-occupation of two offices at a $\langle w, t\rangle$.

For a slightly more complicated case, consider

Example 3.5 '*Napoleon is the first Emperor of France.*'

$$\lambda w\ [{}^0a = [{}^0First_w\ [\lambda x\ \exists t\ [{}^0Emperor_{wt}\ {}^0France] = x]]].$$

Types: $First/(\iota(o\iota))_\omega$; $Emperor/(\iota\iota)_{\tau\omega}$; $France/\iota$; $x \rightarrow \iota$.

At a world w, $\lambda x\ \exists t\ [[{}^0Emperor_{wt}\ {}^0France] = x]$ v-constructs the set of the individuals who were, are or will be Emperor of France in w. The function *First* then, dependently on worlds, picks out the first one from this set. If we wanted to make a still finer analysis, we would have to take into account the semantics of '*First*'; it picks out the individual who plays the role of Emperor of France at a time t' such that for all other times t when the role of Emperor is occupied it holds that $t' \leq t$. But the literal analysis would be the one above. On this analysis the proposition denoted by the sentence is, at a given world w, eternally true, or eternally false or eternally without a truth-value. Here we do not take into account the obvious semantic distinction between 'Napoleon *is* the first Emperor of France', 'Napoleon *will be* the first Emperor of France' and 'Napoleon *was* the first Emperor of France'. In other words, we are not analysing here the semantics of verbs with different grammatical tenses.[20]

For an instance of (6), consider

Example 3.6 'a*'s wife is* b*'s mother.*'

Assuming monogamy, the current state of bio-technology, and an atemporal copula so that it does not matter whether b's mother has passed away, a will have at most one wife and b exactly one mother. If a has no wife, then it is neither true nor false that a's wife is b's mother, since the identity relation will lack an argument. If a does have a wife, then it is true or else false that that individual happens to be the same as b's mother.

$$\lambda w \lambda t\ [[{}^0 Wife\_of_{wt}\ {}^0 a] = [{}^0 Mother\_of_{wt}\ {}^0 b]].$$

Type: (the)*Mother_of*; (the) *Wife_of* /$(\iota\iota)_{\tau\omega}$.

Remark. Example 3.6 should not be confused with the *predication*

'b's mother is a's sister.'

Individual a may have more than one sister, so the construction is

$$\lambda w \lambda t\ [[{}^0 Sister\_of_{wt}\ {}^0 a]\ [{}^0 Mother\_of_{wt}\ {}^0 b]].$$

Sister_of/$((o\iota)\iota)_{\tau\omega}$: given a $\langle w, t\rangle$ and an individual, we are given the set of that individual's sisters.[21] However, we can obtain uniqueness and so a case of *identity* by using a singularizer *Sing* of type $(\iota(o\iota))$:

$$\lambda w \lambda t\ [{}^0 Sing\ [{}^0 Sister\_of_{wt}\ {}^0 a] = [{}^0 Mother\_of_{wt}\ {}^0 b]].$$

This construction corresponds to the sentence, 'b's mother is the sister of a'.

[20] For details on tenses, see Section 2.5.2, or Tichý (1980a, b).

[21] Russell noted already in 1905, 'Now *the,* when it is strictly used, involves uniqueness; we do, it is true, speak of "*the* son of So-and-so" even when So-and-so has several sons, but it would be more correct to say "*a* son of So-and-so".' (1953, p. 44). So the English sentence 'Bertha's mother is the sister of Alfie' need not imply that Alfie would have one sister only.

3.3.1 Hesperus is Phosphorus: co-occupation of individual offices

In this section we discuss how to make it non-trivial that Hesperus is Phosphorus. Our solution is that 'Hesperus is Phosphorus' means that two individual offices—one named 'Hesperus', the other 'Phosphorus'—are co-occupied by the same individual at a given $\langle w, t\rangle$ of evaluation. The solution also makes it plain why it is vital to distinguish *denotation* from *reference*. Since *reference*, as we understand it, is extra-semantic and factual, while *denotation* is semantic and a priori, it is not an empirical fact that 'Hesperus' and 'Phosphorus' co-denote (for they never do), whereas it is an empirical fact that they co-refer (namely, when being co-occupied).[22]

Bealer (2004) discusses what it is we learn (i.e., get to know) when learning that Hesperus is Phosphorus. The conclusion of his discussion is that the best answer direct reference theory can offer is inconsistent. Bealer summarizes direct reference theory in these two tenets (tenet I, ibid., p. 575, tenet II, ibid., p. 576):[23]

Tenet 1. If 'a' and 'b' co-refer then '$a = a$' and '$a = b$' are synonymous.

Tenet 2. If $a = b$ then $(a = a) = (a = b)$.

He then raises the question:

> *How*, if $a = b$, can the proposition that $a = a$ and the proposition that $a = b$ be different? (Ibid., p. 575)

Well, they just cannot, if $a = a$ and $a = b$ are the proposition that a is self-identical. Yet what you learnt in astronomy class was certainly not that Venus, under whatever name, is self-identical. But this outcome is inescapable if one subscribes to Kripkean names and Russellian propositions. As Russell observed long ago,

> [I]f…'c' is a name for Scott, then the proposition [expressed by "Scott is the author of *Waverley*"] will become simply a tautology. It is at once obvious that if 'c' were 'Scott' itself, 'Scott is Scott' is just a tautology. But if you take any other name which is just a

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[22] Tichý points out that, according to Kripke, 'Hesperus is Phosphorus' comes out *necessarily* true, because it expresses that Phosphorus (Hesperus) is self-identical, and a posteriori because it is an empirical fact that 'Hesperus' and 'Phosphorus' co-refer. (1983, pp. 232–33; 2004, pp. 514–15). Kripke's observation concerning a posteriority is correct all right, but in our view also irrelevant to semantics. The semantic analysis must terminate at the *denotations* of 'Hesperus', 'Phosphorus', and not make the extra step from denotation to reference.

[23] The formulation of tenet 1 seems problematic. This is so because 'names do not contribute anything to the *meaning* of a sentence over and above their *reference*' (ibid., p. 575) (italics inserted). But we are far from sure that all direct reference theorists do, or must, hold that sentences have meanings at all. What is more, how can a *reference* contribute anything to a *meaning*? It would seem to fly in the face of any principle of semantic compositionality that the meaning of a sentence is a function of the meaning of some of its constituents together with the reference of its remaining constituents. For a critique of Bealer's critique of direct reference theory, see Jespersen and Zouhar (ms.).

> name for Scott, then if the name is being used *as* a name and not as a description, the proposition will still be a tautology. (1953, p. 245.)

This is sufficient to convince us that direct reference theory is a non-starter and that a more sophisticated alternative is called for. The alternative we are proposing takes its lead from the observation that

> When we are told that tan 45° = cot 45° …we learn something about the tangent and cotangent functions, not about the number one, which *is* the common value of those functions at 45°. (Tichý, 1986a, p. 254; 2004, p. 652.)

Let us agree, to begin with, that one constraint must guide our analysis, namely that it must be a discovery entirely due to astronomy that Hesperus is Phosphorus.[24] It ought to be neither logically nor analytically true that Hesperus is Phosphorus.[25] In particular, any viable theory must avoid that linguistic competence would suffice to establish whether Hesperus is Phosphorus. Otherwise the question whether Hesperus is Phosphorus would be prejudged. For all one learnt when learning that Hesperus is Phosphorus, the relevant individual might be Mercury, the moon, or any other celestial body visible from earth. It must require astronomical investigation to establish which of the candidate celestial bodies is the right one. Once the astronomers have done so, they will be sharing an additional, and logically independent, piece of information with you by adding that the co-extension that Hesperus and Phosphorus share in the actual world at the present moment is Venus. (Here we are assuming that 'Venus' does not name an individual office but an individual, which happens to be a planet.)

If the analysis of 'Hesperus is Phosphorus' is hedged in by the constraint that only astronomy will determine whether Hesperus is Phosphorus, then according to our proposal at least one of 'Hesperus' and 'Phosphorus' must denote an individual office on pain of perpetuating the self-identity analysis, according to which '*a* is *b*' can only mean that the co-referent of '*a*', '*b*' is self-identical. However, since 'Hesperus' and 'Phosphorus' ought to belong to the same semantic category to avoid arbitrarily making one a name of an individual office and the other a name of an individual, they must both denote an individual office, though emphatically not the same one on pain of reinstalling the self-identity analysis.

The following three tenets summarise our proposal.

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[24] This constraint is in keeping with Bealer's claim that '[n]early everyone agrees that the following at least seems intuitively obvious: It is not possible to know a priori that Hesperus = Phosphorus.' (*Ibid.*, p. 576.) But the analysis we offer is obviously incompatible with the widespread construal of the proposition that Hesperus is Phosphorus as being both ('metaphysically') necessary and not knowable a priori.

[25] So an analysis in terms of requisites – $[^0Req\ ^0Hesperus\ ^0Phosphorus]$, $Req/(o(\iota_{\tau\omega}\iota_{\tau\omega}))$ – is out of the question. (For *requisite*, see Section 4.1.) This would be philosophically and astronomically unreasonable, anyway. It ought to be conceptually and nomologically possible that a celestial body should satisfy the condition of being the brightest body in the evening sky without thereby satisfying the condition of being the brightest body in the morning sky, or the other way around. That is, these two conditions must be independent.

Tenet 1 (*individual offices*). 'Hesperus', 'Phosphorus' rigidly denote intensions (individual offices).

Tenet 2 (*co-extensionality*). 'Hesperus = Phosphorus' expresses the contingent co-extensionality of two named intensions coinciding in one (anonymous) individual, not the necessary self-identity of an individual bearing two names.

Tenet 3 (*contingency*). The contingency of the proposition that Hesperus is Phosphorus must be made explicit in the logical analysis of the sentence 'Hesperus is Phosphorus'. This is achieved by means of explicit intensionalization and temporalization (see Section 2.4).

The sought-after modality of contingency is acquired by relativizing the co-extensionality between the intensions Hesperus and Phosphorus to worlds and times. At some, but not all, worlds and times Hesperus and Phosphorus share the same extension. At some, but not all, worlds and times within this set of world-time pairs the shared extension is Venus. At other world-time pairs it is Mercury, or Titan, or UB313, or whatever else the universe may have in supply. In general, what makes a context modal is not exclusively the explicit presence of modal operators or modal expressions like 'necessary' and 'possible'. It is enough that the context concerns contingent truths and falsehoods.

The problem with neglecting the difference between empirical and non-empirical is that the specifically *empirical modality of contingency* is swept under the carpet. Yet the informational non-triviality of 'a is b', if true, can, at least in our opinion, be accounted for only in terms of its being contingently true.[26] When we know that $a = b$ then we know something that might have failed to be the case but in fact is the case. Therefore, if we fail to incorporate reference to contingency into our logical analysis of empirical sentences, our analysis is bound to be botched. This is evidenced by the direct reference analysis of 'Hesperus is Phosphorus', which can be nothing other than the self-identity analysis.

The fact that contingency is obviously pivoted on intensionality is at loggerheads with the prevalent tendency to treat 'a is b' ('a', 'b' empirical terms) as being on a par with non-empirical identity sentences. Our analysis is also at variance with the pre-theoretic conception of 'ordinary' proper names as (non-descriptive) names of individuals that not least direct reference theory has sought to underpin theoretically. In this book we do not engage in a large-scale confrontation with the various arguments that have been advanced in favour of this conception of 'ordinary' proper names. But the specific morale of our discussion of how 'Hesperus is Phosphorus' can be rescued from triviality is that the direct reference construal of 'Hesperus', 'Phosphorus' yields the wrong result. The failure of direct reference theory in this department is not a cogent argument for our approach, of course;

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[26] The account of the informational value of the mathematical proposition expressed by 'a^* is b^*', 'a^*', 'b^*' mathematical terms, cannot be cast in terms of contingency, and must be altogether different. As for the informativeness of mathematical sentences, see Section 5.4.

non-triviality can be achieved along alternative routes. But the case of ‘Hesperus is Phosphorus’ is a suitable platform for broaching one such route that has as yet not received its fair share of attention. The idea, again, is that ‘Hesperus’, ‘Phosphorus’ denote two distinct individual roles also when flanking the ‘is’ of identity in atomic sentences.

We claimed above that these two pieces of information are logically independent: (a) that Hesperus is Phosphorus and (b) that Venus is the shared extension of these two intensions. For comparison, consider how we do *not* intuitively construe a non-empirical claim such as ‘$7 + 5 = 4 \times 3$’, namely as claiming the self-identity of the number 12. It seems an intuitively appealing idea that what it claims is the coincidence in an anonymous number of the outcomes of two named operations, namely the operations of adding 7 to 5 and multiplying 4 by 3. That $7 + 5 = 12$ and that $4 \times 3 = 12$ constitute two further pieces of (analytic) information (see Section 5.4).

If we are agreed that the number 12 should play no role in the semantic analysis of ‘$7 + 5 = 4 \times 3$’, then, by analogy, Venus, or any other concrete celestial body, should likewise drop out of the picture. A second argument for leaving the actual extension out of the semantic analysis is Tichý’s modal argument to the effect that ‘the Morning Star’ does not denote Venus, or in general, that a definite description does not denote its actual *descriptum*.

> Those who hold that [‘The Morning Star is a planet’] does treat of Venus, the celestial body, will probably agree with one another that what [this sentence] *says* about the celestial body is … that it is a planet. It is easily seen, however, that [the sentence] might be true *without* that body’s being a planet. For consider a world in which Mars instead of Venus is the brightest celestial body one can see in the morning sky and in which Venus fails to be a planet. Clearly there are possible worlds of this sort. But in any such world [‘The Morning Star is a planet’] comes out true. Surely a sentence cannot come out true in a state of affairs where what it says is not the case. Hence what [the sentence] says cannot be to the effect that Venus is a planet (1975, p. 87; 2004, p. 214).

That is, if it is true that Venus is a planet, then it might have been false. If it is false that Venus is a planet, then it might have been true. Hence, it is a contingent truth or falsehood that Venus is a planet. The semantics in terms of which we analyse ‘Venus is a planet’ cannot confine itself to set membership, for the following is trivially true and trivially false, respectively: $a \in \{a, \ldots\}$, $a \notin \{b, c, d\}$, and $a \in \{b, c, d\}$, $a \notin \{a\ldots\}$. Let C be the set of all and only those individuals that are actually (and presently) a planet. Then consider a world (and a time) at which Venus is not a planet:

> In such a world (as in any world) it is true that Venus is a member of C (i.e. of the class consisting of Mercury, Venus, ..., and Pluto), yet (3) [“Venus is a planet”] is false. Now surely a sentence cannot be false in a state of affairs where what it says is the case. Consequently, what (3) [“Venus is a planet”] says cannot be to the effect that Venus is a member of C (or any other class) (1975, p. 83; 2004, p. 210).

Thus, according to Tichý, what *is* relevant to the truth-condition denoted by, ‘The Morning Star is a planet’ is the individual office *the Morning Star*; i.e., the condition of being the brightest body in the morning sky. The truth-condition of the sentence is that whatever celestial body is the brightest in the morning sky should be a planet. The analogy between the definite description 'the Morning Star' and the 'ordinary' proper name 'Phosphorus' is that neither denotes an individual, both denoting an individual office instead. So Tichý's modal argument above applies equally to 'Phosphorus' (and 'Hesperus'). If 'the Morning Star' and 'Phosphorus' are introduced as two names, in the same language, of the same individual office then one of them is redundant. That a definite description and an 'ordinary' proper name co-denote (even rigidly) the same entity is, of course, at odds with what Kripke and various proponents of direct reference have claimed; but it is a quite natural possibility within a semantic theory allowing both definite descriptions and 'ordinary' proper names to denote intensional entities.

However, one way of attempting to reinstate individuals as the denotations (as opposed to references) of definite descriptions would be to turn to the notion of *flexible designation*. Roughly, what a term denotes is then a function not only of linguistic *fiat* but also of the index at which the term is used (various subtleties of multi-dimensional semantics aside). Thus, at w 'the Morning Star' denotes Venus, while at w' 'the Morning Star' denotes Mars, say, since Mars is the brightest heavenly body in the morning sky at w'. (Similarly, if 'Phosphorus' is declared a flexible designator then it denotes Venus at w but Mars at w'.) The problem with flexible designation, though, is that it turns the designation relation into a part factual one. World-relative facts will in part determine a semantic property of flexible designators; namely, what their denotation is at a given world. Consequently, it takes not only knowledge of a linguistic convention but also knowledge of a world-relative fact to know what individual is predicated to be a planet in, 'The Morning Star is a planet'. Furthermore, a semantic theory boasting flexible designation ends up offering the self-identity analysis of sentences in the vein of, 'The F is the G'. If 'the F' and 'the G' both denote Mars at w' then the sentence just expresses that Mars is self-identical. This gets the modal profile of 'The F is the G' wrong. On the other hand, a pure and a priori semantics is available both for definite descriptions and 'ordinary' proper names by having them denote individual offices, since this relation between term and *denotatum* is independent of empirical indices.[27]

If, as we suggest, one goes for a pure semantics, what would the relevant portions of such a semantic theory look like? Let H, P be individual offices. Two scenarios involving some specific individual i would be

- individual i = individual i
- i-under-H = i-under-P.

However, an alternative, rival, scenario would be

- $[{}^0H_{WT} = {}^0P_{WT}]$.

[27] See Tichý (1986a, p. 255; 2004, p. 653) and Section 3.1 for the claim that also definite descriptions are rigid designators.

Where the first two scenarios include ('bare') individuals and individuals-as-occupants-of-offices, respectively, the third scenario includes offices-plus-extensionalization.[28] This scenario does not name the occupant of H_{wt}, P_{wt}, which is this or that particular individual. The only entities are H, P, T, W, Composition, and the identity relation.

If we allow that the offices may not be occupied at all worlds and times, we end up with the following five possibilities.

- At $\langle W, T\rangle$, $H_{WT} = P_{WT}$
- At $\langle W, T\rangle$, $H_{WT} \neq P_{WT}$
- At $\langle W, T\rangle$, there is an H_{WT} but no P_{WT}
- At $\langle W, T\rangle$, there is no H_{WT} but a P_{WT}
- At $\langle W, T\rangle$, there is neither an H_{WT} nor a P_{WT}.

Which of these five actually and presently obtains is a contingent matter, and one that must be settled a posteriori by astronomical research.

Let us compare the semantic analyses of 'Hesperus is Phosphorus' offered by direct reference theory and TIL. First, if 'Hesperus', 'Phosphorus' are Kripkean proper names denoting individuals, the analysis in TIL guise becomes

$$[{}^0H' = {}^0P'].$$

The analysis is cast in terms of functional application of the identity relation to H' and P' to obtain a truth-*value* as the product of this Composition. The truth-value is **T**, since this is the self-identity analysis.

Second, if 'Hesperus', 'Phosphorus' denote individual offices, the analysis becomes:

$$\lambda w \lambda t\ [{}^0H_{wt} = {}^0P_{wt}].$$

The so constructed truth-condition is that H_{wt} and P_{wt} should be one and the same celestial body for a given choice of values for $\langle w, t\rangle$ as points of evaluation.

It may be illuminating to compare Bealer's approach to ours. Bealer's question was this: if $a = b$, how can the proposition that $a = a$ be different from the proposition that $a = b$? Restricting ourselves to the empirical case, our answer was because our a and b are not individuals but two distinct individual offices alias conditions to be satisfied by individuals. Trivializations of these two individual

[28] The idea of operating directly with specified individual offices and only indirectly with unspecified individuals (in TIL, via extensionalisation) is one of the three approaches that Aloni considers in her (2005). She both rejects operating with 'bare individuals' and 'ways of specifying [bare] individuals' (i.e., individual offices), opting for 'individuals specified in one determinate way' (see, for instance, p. 27). Her stance appears to square with the second scenario adumbrated above. However, despite the length of her paper, we are still not sure we fully understand the idea of identifying an individual-under-a-description with an individual-under-a-different-description. In particular, it is not clear how the self-identity analysis is to be avoided.

offices are constituents of the Closure $\lambda w\lambda t\ [{}^0H_{wt} = {}^0P_{wt}]$, as are variables ranging over worlds and times, together with a Trivialization of the identity relation defined over individuals. The other construction would be $\lambda w\lambda t\ [{}^0H_{wt} = {}^0H_{wt}]$. The so constructed proposition is true at all worlds and times at which H is occupied, lacks a truth-value at those at which it is vacant, and is never false. These two constructions contain the same number of occurrences of subconstructions, but not entirely the same constituents. $\lambda w\lambda t\ [{}^0H_{wt} = {}^0H_{wt}]$ contains two occurrences of Trivialization of the same individual office, whereas $\lambda w\lambda t\ [{}^0H_{wt} = {}^0P_{wt}]$ contains two occurrences of a Trivialization of two different individual offices. A Trivialization of Venus (or of any other particular celestial body) is a component of neither construction. Thus, in direct reference parlance, these two propositional constructions are no 'singular propositions', in contrast to the status direct reference theory would bestow upon its ⟨Hesperus, =, Phosphorus⟩.[29]

Bealer does not attempt to answer his own question in 2004. However, we may get enough of an impression of the tack of his answer from his (1993, 1998). We will briefly sketch it to compare Bealer's intensional logic with Tichý's, as far as the analysis of 'Hesperus is Phosphorus' is concerned.

Part of Bealer's grand-scale project of establishing a logic that is both first-order and hyperintensional is to devise a semantics, according to which

> [P]roper names do not have Fregean senses, and predicates do not have Fregean references or Millian denotations. Nevertheless, a sentence like 'Cicero is a person' does have a meaning not shared with 'Tully is a person' and 'Tully is a person' has a meaning not shared with 'Cicero is a person'. (1993, p. 43.)

Bealer wishes to accommodate both the alleged intuition that 'Cicero is a person' means something different from 'Tully is a person' as well as Kripke's claim that 'Cicero = Tully' expresses a proposition that is simultaneously ('metaphysically') necessary and knowable a posteriori only. To this end Bealer introduces what he dubs 'non-Platonic modes of presentation', which encompass what he calls 'intentional naming trees', 'causal naming chains' (1993, pp. 35ff), 'living names' (1998, pp. 16ff) and 'conventional naming practices' (1993, p. 36). For instance, two such practices P, P' may present the same individual, Cicero/Tully, but do so in two different ways: $P_{\text{'Cicero'}} \neq P'_{\text{'Tully'}}$.

Bealer discusses very briefly the Hesperus/Phosphorus case in (1993, p. 45, 1998, pp. 28–29). We are to imagine that the inception of the non-Platonic mode of presentation $P_{\text{'Phosphorus'}}$ comes after the inception of $P'_{\text{'Hesperus'}}$. The solution is that

> [T]he relevant non-Platonic modes of presentation are different: the … newly instituted practice is different from [the] standing practice $P_{\text{'Hesperus'}}$ … Accordingly, descriptive predications involving the new non-Platonic mode of presentation … result in propositions that are different from those which result from descriptive predications

[29] We are here following the direct reference practice of encoding what this theory considers 'structured propositions' as ordered n-tuples.

involving instead [the] standing non-Platonic mode of presentation [$P_{\text{'Hesperus'}}$]. (1993, p. 45.)

Bealer and Tichý both make use of the resources of intensional logic by descending from modes of presentation to extensions such as individuals rather than 'giving' individuals straightaway. Thus, the fine-grained content of 'Hesperus is Phosphorus' is both in Bealer and Tichý to do with two different intensions converging in the same entity.

However, we have two objections. The first is this. We feel uneasy about the sort of modes of presentation Bealer invokes. To get off the ground, Bealer's proposal requires that the logical operation of *descriptive predication*, pred_d, may operate in part on historical chains of linguistic practice and the like to form (hyper-) propositions.[30] Bealer in effect 'semanticizes' his non-Platonic modes of presentation by making them denizens of the subdomain D_1 of the domain D of one of his intensional algebras. Otherwise he would not be in a position to claim that the solutions he offers to various puzzles are 'purely semantical' (1993, p. 43).[31] Although it is trivial that Cicero = Tully, it is (supposedly) not trivial that $P_{\text{'Cicero'}}$, $P'_{\text{'Tully'}}$ present the same individual. However, the sort of non-Platonic modes of presentation that Bealer invokes, such as naming chains and naming practices, undeniably belong to the pragmatics department of semiotics. This is to say that Bealer somewhat strains the notion of semantics so as to include also certain pragmatic entities.

The second objection is this. All the elements of D are to be thought of as *primitive*, irreducible items (cf. 1993, p. 25). Indeed, they must be, for otherwise the very project of erecting a (hyper-) intensional *first*-order logic would be a non-starter. But the philosophical price exacted is that we must possess a firm pre-theoretic grasp of those non-Platonic modes of presentation, among other. We are not sure our grasp is firm enough for us to understand in sufficient theoretical detail what, e.g., $P_{\text{'Hesperus'}}$ might be. We would have much preferred an intra-theoretic explanation.[32]

We find that these two objections provide enough reason not to include non-Platonic modes of presentation into the domain D of a Bealer-style intensional algebra. If one is reluctant to include them, then Bealer's solution grinds to a halt, since pred_d needs them as arguments.

Bealer shares the conviction with direct reference theory that 'Cicero' and 'Tully', 'Hesperus' and 'Phosphorus' are mere labels of individuals. Hence, any differences between 'Hesperus' and 'Phosphorus', or 'Cicero' and 'Tully', must be located somewhere other than in the semantics of the reference relation to steer

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[30] If $x, y \in D_1$ then pred_d predicates x of y. See Bealer (1998, p. 14).

[31] Bealer's account of why his solutions are not metalinguistic 'in any of the standard senses' needs in our view to be made clearer in order to become part of his theory. At this point we need to content ourselves with 'a type of proposition that is 'metalinguistic without being metalinguistic'' (1993, fn. 62).

[32] See also Section 2.4.2 for a similar objection to Bealer's theory of predication.

clear of triviality. This is the same tack as followed by direct reference theory, which also has two options. The difference between 'Hesperus' and 'Phosphorus' is either to do with these two words having different orthographic shapes or with their being guided by two different sets of pragmatic rules for their correct use. Bealer explores both avenues in (1993, pp. 35ff, 1998, pp. 16ff). A third option, which is neither based on syntax nor pragmatics but on semantics, is not available, namely that 'Hesperus' and 'Phosphorus' would denote two different entities to begin with. Bealer is, at the end of the day, closer to direct reference theory than to TIL, as far as the analysis of 'Hesperus is Phosphorus' goes. This is little wonder, after all, since Bealer explicitly has his mind set on a Russellian semantics, just as direct reference theory.[33] But then, while it is clear that Tichý's intensional logic is in a position to offer a purely semantic explanation of the non-triviality of 'Hesperus is Phosphorus', it is far from clear this holds for Bealer's.[34]

Our solution is neo-Fregean, insofar as it has recourse to individual offices and accounts for the non-triviality of '$a = b$' in terms of the identity of the respective extensions of a and b rather than in terms of identity between a and b construed as extensions. But our solution is only broadly neo-Fregean, because we eschew reference shift. Nonetheless, it would seem that Bealer is anticipating a position similar to ours when saying,

> The two propositions differ because those two senses differ (the concept of being $A \neq$ the concept of being B). So goes the Fregean solution to … Frege's puzzle (2004, p. 573).

Though not quite. We balk at labelling our solution 'Fregean', or even 'neo-Fregean', in any narrow sense for the simple reason that Frege's own solution to his famous 1892 puzzle is a half-solution at most, although this point tends to be overlooked. Here is why. If you know that Hesperus is a planet then you definitely do not know that the usual *sense* of 'Hesperus' is a planet. Yet this is exactly the upshot of Frege's shifting the reference from an individual to a sense without accompanying the shift with some means of bending the sense toward a celestial body within sentences whose reference is a thought (*Gedanke*) and not a truth-value. Frege's semantics provides the wrong sort of subject of predication in the subclauses that Frege considers.[35]

If making empirical expressions denote intensions instead of extensions is the first half of the solution, the other half is making the so denoted intensions descend to extensions. If H is the individual office of Hesperus, then H_{wt} is an individual,

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[33] See Bealer (1993, pp. 40ff).

[34] Interestingly, Bealer says that, 'I myself defended a form of direct reference theory…but have since abandoned it in favour of a *semantical* account.' A reflection of this change in orientation is that the proposition that Hesperus is Phosphorus counts as knowable a priori in 1993 and as knowable only a posteriori in 2004. (See Bealer 1982, pp. 161–66, for his pro-Russellian, anti-Fregean stance on 'ordinary proper names' at the time.) Yet the paper he cites as where he pursues his new, 'purely semantical' orientation is none other than his 1998.

[35] See Frege (1986b, pp. 51–4). Bealer touches upon the problem of mismatch between property and subject of predication in passing (1993, p. 34).

namely the celestial body that is the extension of H at $\langle w, t\rangle$. Notice, though, that while the first half concerns semantics—assigning a denotation to 'Hesperus'—the other half is a logical matter: identifying a logical operation that will take intensions to their extensions.[36] While 'Hesperus' invariably refers to the individual office H and exhausts its purpose by picking out its denotation, H may, or may not, be extensionalized. This was the difference between H_{wt} and H, respectively. Whether it does is a matter of whether the abstract entity H has, or has not, been subjected to extra-semantic, logical manipulation in the form of extensionalization. One of the essential purposes of a logically perspicuous notation is, therefore, to flag whether H occurs extensionalized or not. This way we steer clear of an ambiguous notation in which 'H' refers to an individual in one sort of context and to an individual office in another sort of context.

By way of summary, our analysis of 'Hesperus is Phosphorus' is:

$$\lambda w \lambda t\ [{}^0H_{wt} = {}^0P_{wt}],$$

where H, P are two different individual offices, which are extensionalized in order to pick up the individual (if any) that occupies the respective offices at a given $\langle w, t\rangle$ of evaluation. The Trivializations of the offices, namely ${}^0H$, ${}^0P$, are the respective senses of 'Hesperus' and 'Phosphorus'. Venus (or any other specific celestial body) is no part of the semantic analysis.[37]

On a polemic note, if you go along with the general drift of our analysis of 'Hesperus is Phosphorus', the answer to the direct reference theorist Jonathan Berg's rhetorically intended question, 'But does anybody ever explicitly mention notions?' (1999, p. 463) is straightforward: 'Everybody does it all the time!'

[36] A logical operation taking intensions to extensions should not be conflated with an empirical operation whereby an agent executes such a logical operation. The former operation qualifies as a *procedure*; the latter, as a *process*, which is the actual execution of a procedure by an agent relative to a world and a time. What is intended above is a *logical* operation.

[37] Tichý himself offers an alternative analysis of 'Hesperus is Phosphorus' in 1983. His analysis is in effect a two-dimensionalist one: the same sentence may *express* (or *denote*, in mature TIL parlance) one proposition and be *associated* with another. The expressed proposition is that Venus is self-identical; hence, a necessary and a priori one. The associated proposition is that 'Hesperus is Phosphorus' is a true sentence of English; hence, a contingent and a posteriori one. The analysis allows 'Hesperus', 'Phosphorus' to be two names of the same individual, which some may consider an asset of the analysis. However, the notion of associated proposition remains intuitive in 1983 and does not re-appear in later works, so it smacks of adhockery. Besides, the analysis is superfluous, since the one we present above is more in the spirit of TIL. All that is needed is the proposition *denoted* by 'Hesperus is Phosphorus', because 'Hesperus', 'Phosphorus' denote two different individual offices and not the same individual. (For comments on Tichý as a very early, 'very strong two-dimensionalist', see Soames, 2005 pp. 171ff.)

3.4 Pragmatically incomplete meanings

TIL is thoroughly anti-contextualistic, which may seem to be unrealistic at least when dealing with anaphoric terms or sentences containing indexicals. In this section we show that the analysis of sentences containing indexicals is, indeed, compatible with the anti-contextualism of TIL. Sentences containing anaphoric reference will be analysed in the same spirit in Section 3.5.

As far as indexicals are concerned, would it not be true to say that the *meaning* of sentences like 'The man over there is drinking beer', 'I am hungry', etc., depends on the context that is the situation of utterance in which the truth-conditions of such sentences are to be evaluated? No, it would not. It is an old truth, for sure, that the empirical *evaluation* of sentences in a given situation of utterance belongs to the realm of pragmatics rather than (logical) semantics. However, at the same time, as argued in this book, the meaning of a sentence should make it possible to evaluate the proposition denoted by the sentence in any state of affairs. This holds, *provided*, of course, that there is a proposition to evaluate in the first place. However, sentences containing indexicals like 'over there', 'I', 'he', etc., do not express a *closed* construction constructing a proposition susceptible to being evaluated in any state of affairs. This is just because indexicals are what might be called 'pragmatic gaps'. Sentences containing such 'gaps' are not pragmatically complete, in the sense that a value to be supplied to fill the gap by the state of affairs serving as point of evaluation has not been supplied. Thus from the logical point of view, these pragmatic gaps are to be paired off with free variables that do not construct an entity, but only v-construct one. Only after valuation has been supplied by a situation of utterance assigning values to these free variables is a proposition susceptible to evaluation obtained. For this reason we will assign an *open* construction with one or more free variables to sentences containing indexicals as their meaning, and we will say that such sentences have a *pragmatically incomplete meaning.*[38]

All semantic analyses undertaken so far in this book have been couched within *pure semantics*. We did not need to study events like *utterances* of expressions. A brief recapitulation of our pure semantics might be helpful now to make clear how pragmatically incomplete meanings fit into the bigger picture.

The meaning of an unambiguous expression E is a *construction* C expressed by E. If E is an empirical expression, then C v-constructs an α-intension, i.e., a function of type $\alpha_{\tau\omega}$ denoted by E.[39] In particular, if E is an empirical sentence S then the meaning of S is a construction C_P of a *proposition* P of type $o_{\tau\omega}$. The construction

[38] Since 'incomplete' denotes a privative modifier (see Section 4.4), an incomplete meaning is not a meaning. However, by 'pragmatically incomplete meaning' we do mean a meaning, though one that is pragmatically incomplete.

[39] Recall that we reserve the term 'denote' for the a priori relation between E and an α-intension and the term 'refer' for the a posteriori relation between E and the α-value (if any) of the intension in the actual world at the present moment.

C_P is an instruction of how to evaluate the truth-conditions denoted by the sentence for any state of affairs $\langle w, t \rangle$. So C_P makes it in principle possible to determine the value of P (if any) at any $\langle w, t \rangle$ pair. Which truth-value, if any, the denoted proposition has in particular circumstances is not a matter of a priori logical investigation; rather it is a matter of a posteriori empirical investigation. Disclosing the expressed construction is a matter of logic, hence a priori. The process of *executing* the procedure at $\langle w, t \rangle$ is in turn a posteriori.

Frege's semantic schema was essentially modified in Section 1.1.1 by

(a) letting *constructions* play the role of Fregean *Sinn*;
(b) distinguishing between *denotation* and *reference* (in the case of empirical expressions);
(c) letting *intensions* (in the case of empirical expressions) play the role of *Bedeutung* (so that intensions are *denoted*).

Further, the Parmenides principle is a vital step towards finding for every meaningful expression its best literal analysis (see Section 2.1).

Yet natural language contains an important class of expressions where what is denoted is dependent on contexts of utterance. The members of this class are expressions that contain *indexicals*. These are mostly pronouns; for example, personal pronouns ('I', 'you', 'they', etc.), demonstratives ('this', 'those', etc.), possessive pronouns ('my', 'their', etc.), as well as some adverbs (e.g., 'here', 'there', 'now'[40]). Clearly, the construction expressed by an expression that contains indexicals cannot be evaluated if such an expression is simply written on the blackboard, say, without being wrapped within a context of use to provide determinate references. This goes to show that empirical expressions containing indexicals have a *pragmatically incomplete* meaning. We propose construing the meaning of an empirical expression containing indexicals as an *open construction*. An intension is, then, constructed only after a valuation of the free variable(s) has been provided by the situation of utterance. The valuation fills in the pragmatic gaps and thus closes the open construction so that it constructs an intension. (Or, properly speaking, the open construction is *replaced* by a closed construction.) Thus the *denotation* of an expression E_I containing indexicals *is* context-dependent, which, however, does not make the *meaning* of E_I context-dependent. The open construction is context-*in*variably assigned to E_I as its meaning, which complies with our anti-contextualistic stance.

For instance, the sentence

'He is a logician'

[40] In systems equipped with explicit temporalisation (such as TIL), 'now' does not have an indexical character. Instead 'now' denotes the identity function of type $(\tau\tau)$ taking every instant of time to itself, since the time that is present at t is t itself.

has a pragmatically incomplete meaning and expresses, thus, the following open construction with a free variable $he \rightarrow_v \iota$; $Logician/(o\iota)_{\tau\omega}$:

$$\lambda w \lambda t\ [{}^0Logician_{wt}\ he].$$

This Closure only *v-constructs* a proposition. It is not possible to evaluate the truth-condition of the sentence unless and until a value of the parameter *he* has been provided by a context.

The context can be one of two kinds: *a pragmatic context* (a situation of utterance[41]) or a *linguistic-discourse context*, which is the case of anaphora (see Section 3.5). If the sentence is uttered in a situation where a hearer succeeds, in whatever manner, in identifying the particular individual Charles, then the *pragmatic meaning* of the sentence *in that situation* is the closed construction

$$\lambda w \lambda t\ [{}^0Logician_{wt}\ {}^0Charles].$$

On the other hand, the sentence

'Charles is a logician'

has a complete meaning, and expresses the closed construction $\lambda w \lambda t\ [{}^0Logician_{wt}\ {}^0Charles]$ independently of contextual embedding.[42]

The two sentences are not equivalent; the meaning of the former is an open construction whereas the meaning of the latter is a closed one. In another situation of utterance or in another linguistic context, the variable *he* may *v*-construct another individual.[43] Thus the sentences are only $v(Charles/he)$-congruent.

The following Fig. 3.1 sums up our conception of the semantics and pragmatics of empirical sentences. By '$C(x)$' we denote an open construction with the free variable x. For the sake of simplicity, we consider only one free variable here. Remember that by '$C \rightarrow_v o_{\tau\omega}$' we mean that the construction C *v*-constructs a proposition, whereas by '$C \rightarrow o_{\tau\omega}$' we mean that C constructs a proposition independently of valuation.

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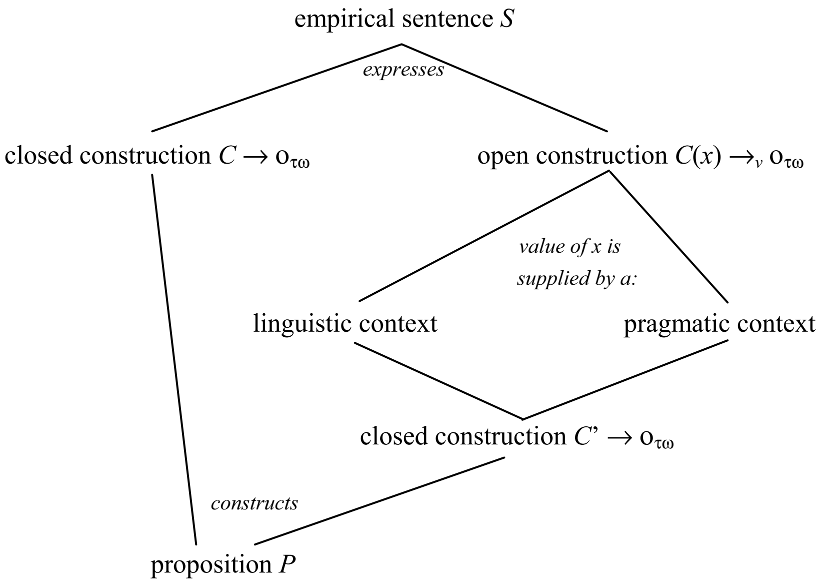
[41] By 'pragmatic context' we mean only a situation of utterance. Hence we do not take into account, e.g., the interrogative, imperative, emotional and other intentions of a speaker, as well as other pragmatic aspects to do with the speaker's reasons for making a particular utterance. See Materna et al. (1976).

[42] 'Charles' is paired off with ${}^0Charles$; see Section 3.2.

[43] For details, see Materna (1998, pp. 115–21).



Outside the scope of logic: empirical (a posteriori) evaluation of the proposition P at $\langle w, t\rangle$, resulting in True, False, or no value at all.

Fig. 3.1 The semantics and pragmatics of empirical sentences

3.4.1 Indexicals

To get the ball rolling, consider the sentence

(1) ‘This hat is blue’.

Following our method of analysis informed by the Parmenides principle, we first assign types to the entities that the sentence talks about. The first attempt might be this one:

Types: *Blue*, *Hat*/$(o\iota)_{\tau\omega}$; *This_Hat* $\rightarrow_v \iota_{\tau\omega}$.

Such a type assignment comes down to the following coarse-grained schematic analysis:

$$\lambda w \lambda t\ [{}^0Blue_{wt}\ This\_Hat_{wt}].$$

What speaks against this attempt is that we cannot write down ‘^{0}*This_Hat*’, because the indexical term ‘this hat’ does not denote an individual office; it has a pragmatically incomplete meaning and so does not denote anything. (Remember that it is internal to a construction what it constructs.) There is no definite individual office to be Trivialized, and the construction *This_Hat*, whatever it may be, only *v*-constructs an individual office.

Above we explained that the meaning of (1) is an open construction containing the free variable *this*. Only what type of entity is *v*-constructed by this variable? The answer is that what is constructed is a property of individuals that should pragmatically complete the description of the hat in question. To make the situation clearer, let us rephrase the sentence as

'The only individual with this property and the property of being a hat is blue'.

This yields the additional types $Hat/(o\iota)_{\tau\omega}$; $this \rightarrow_v (o\iota)_{\tau\omega}$; $\iota/(\iota(o\iota))$: singularizer; $x \rightarrow \iota$. The individual office in question is *v*-constructed by

$$\lambda w\lambda t\ \iota x\ [[this_{wt}\ x] \wedge [{}^0Hat_{wt}\ x]].$$

Gloss: 'In any w at any t, pick up the only individual x that has at $\langle w, t\rangle$ *this* property and the property of being a hat.'

Thus the analysis of (1) is:

(1′) $\lambda w\lambda t\ [{}^0Blue_{wt}\ [\lambda w\lambda t\ \iota x\ [[this_{wt}\ x] \wedge [{}^0Hat_{wt}\ x]]]_{wt}]$,
or its β-reduced form,

(1″) $\lambda w\lambda t\ [{}^0Blue_{wt}\ \iota x\ [[this_{wt}\ x] \wedge [{}^0Hat_{wt}\ x]]]$.

Now imagine that (1) is uttered by our friend Charles in a situation where there is just one hat lying on the table in front of him, and Charles points at that hat. In such a situation one can agree or disagree with Charles, because the situation of utterance makes it possible to assign the property of lying on the table in front of Charles to the variable *this*. This assignment pragmatically completes the meaning of (1), and yields another sentence:

(1^P) 'The hat lying on the table in front of Charles is blue'.

Observe that (1) and (1^P) are not co-denoting, because they are not equivalent. In fact, they are not even co-referring. Since due to its pragmatically incomplete meaning (1) does not denote a proposition (unlike (1^P)), it cannot be said to refer to a truth-value at a given $\langle w, t\rangle$ of evaluation.

If $Lying\_on\_table/(o\iota)_{\tau\omega}$ is the property of lying on the table in front of Charles, then (1^P) expresses the closed construction

($1^{P}{}'$) $\lambda w\lambda t\ [{}^0Blue_{wt}\ \iota x\ [[{}^0Lying\_on\_table_{wt}\ x] \wedge [{}^0Hat_{wt}\ x]]]$.

Thus the situation S described above makes (1′) and ($1^{P}{}'$) *v*(*Lying_on_table*/*this*)-congruent.[44]

[44] See Section 1.5.1, Definition 1.5.

Materna (1998, pp. 118–19) talks about the ‘pragmatic meaning’ and the ‘pragmatic denotation’ of an expression E_I containing indexicals *in a situation S.* Thus he would have said at the time that ($1^{P\prime}$) is the pragmatic meaning of (1) in the situation *S.* Accordingly, he would have said that (1′) and ($1^{P\prime}$) are co-referring in the situation *S.*

In this book, though, we are not going to adopt this terminology, nor are we including situations of utterance into our semantic theory. The latter would most probably amount to enriching the base of the TIL type hierarchy with an additional atomic type σ of situations. For instance, Montague (1974a, pp. 95–118) has among his *indices* not only possible worlds and times but also context-dependent indices for a speaker in a situation of utterance. The reason why we do not want to include the pragmatic meaning of a sentence *S* is that, strictly speaking, a pragmatic meaning is not a or the meaning of *S* at all. Rather, it is the meaning of another sentence. Thus we will not say that, for instance, ($1^{P\prime}$) is the pragmatic meaning of (1) in the situation *S*, because ($1^{P\prime}$) is *not* the meaning of (1), but of (1^{P}), and the two sentences are neither synonymous nor equivalent, for the reasons explained above. Instead we will say that ($1^{P\prime}$) is the pragmatic meaning *associated with* (1) *in the situation S.*

For comparison, one of the most elaborate theories of indexicals is David Kaplan’s, as set out in his 1978 and 1989. Kaplan’s conception shares some common ground with the functional approach of TIL. For example, what he calls *character* is a function from tuples of contextual parameters to contents, while what he calls *content* is a function from empirical parameters (‘circumstances of evaluation’) to individuals, truth-values or whatever the case may be. The general idea is that, e.g., a sentence containing indexicals will, relative to a context, express a particular proposition and that proposition may then be evaluated to obtain a truth-value. But a major difference between Kaplan and us is that the sort of proposition that an indexical-involving sentence picks out relative to a context is a so-called singular proposition, which counts among its constituents the individual referred to by the indexical. This makes one wonder whether the content of an indexical is an individual or an individual-in-intension (what TIL calls as an ‘individual office’ or ‘individual role’).[45] On the other hand, contents are supposed to be functions: contrast (1989, p. 523) with (ibid., p. 546). Where Γ is either a term or a formula, Kaplan writes ‘$\{\Gamma\}^{\mathsf{A}}{}_{cf}$’ for the content of Γ in the context *c* (under assignment *f* and in the structure A). The structure is an ordered *n*-tuple involving sets of contexts, $\mathcal{C}$, worlds, $\mathcal{W}$, individuals, $\mathcal{U}$, positions, $\mathcal{P}$, times, $\mathcal{T}$, as well as a function, $\mathcal{I}$, assigning intensions to predicates and functors. Hence, “If α is a term, $\{\alpha\}^{\mathsf{A}}{}_{cf}$ = that function which assigns to each $t \in \mathcal{T}$ and $w \in \mathcal{W}$, $|\alpha|_{cftw}$.” (Ibid., p. 546). More specifically,

[45] We are indebted to Marian Zouhar for alerting us to Kaplan’s apparent oscillation between function and functional value and for providing exact references.

> Where Γ is either a term or a formula, *the Content of* Γ *in the context c (in the structure* A*) is Stable* iff for every assignment f, $\{\Gamma\}^{\mathsf{A}}_{cf}$ is a constant function (i.e., $\{\Gamma\}^{\mathsf{A}}_{cf}(t, w) = \{\Gamma\}^{\mathsf{A}}_{cf}(t', w')$, for all t, t', w, w' in A). (Ibid., p. 547.)

Here it is plain: contents are always functions, either constant or not. It would seem as though Kaplan is simply taking the liberty of identifying a constant function with its value, in order to uphold his theses that indexicals refer directly to individuals and that individuals are constituents of singular propositions. This oscillation, if that is what it is, between function and functional value would be symptomatic of the awkwardness of the combination of directly referring terms, singular propositions, and contents as functions.

It may also be illustrative to briefly compare our theory of indexicals to Castañeda's distinction between the speaker's *execution*, or *production*, of indexical reference and the hearer's *interpretation*, or *consumption*, of it.[46] In TIL, thanks to the pragmatic assignment of the proper value to the free occurrence of the pragmatic variable x in an open construction, the hearer is able to close the construction and obtain the propositional construction which is the meaning of the sentence as viewed from the speaker's perspective. The speaker intends to assert a proposition to be true when assertorically uttering, 'He is a logician', though leaving it to the hearer to assign to 'he' the referent that the speaker intended. The speaker must be able to spell out whom he or she intended by 'he', and there are various ways of doing so. Two would be to cite either a proper name of the individual or a definite description denoting an individual office that the individual occupies. Two other ways would be to either use a demonstrative (whether simply 'This one!' or 'That guy in the corner') or point at the individual. The reason for the requirement is that the speaker must, upon request, be able to display the closed construction that constructs the proposition which the speaker asserted to be true. In principle, any way of identifying an individual (whether a numerically specific individual or whatever individual satisfies some condition) goes, provided 'he' is matched by a construction (rather than a non-construction like a nod or other non-verbal, pragmatic vehicles of communication). Of course, if communication is to succeed, the speaker and the hearer need to be talking about the same individual. There are various routes leading up to the same individual, and speaker and hearer may well use different routes. However, nothing in the propositional construction that the hearer completes reflects the speaker's perspective, if x is replaced by a Trivialization of an individual (Trivialization being a non-perspectival mode of presentation; see Section 1.3). Perspectives are reinstalled, if a construction of an individual office is used. In case individual a co-occupies two different individual offices at $\langle w, t\rangle$ then speaker and hearer may identify a from these two different perspectives. Only this notion of perspective does not correspond to the notion of perspective that Castañeda operates with as regards indexicals. In TIL, since indexicals do not verbally or literally reflect perspectives, perspectives are not reflected semantically, either. This is a departure from Castañeda's perspectival,

[46] See Castañeda (1989) and Kapitan (2001, 2004).

dual-sense theory of indexicals. According to his theory, an indexical comes with a two-pronged sense, one prong being executive, the other interpretative. For instance, 'I' has an executive sense, which the speaker uses, and an interpretative one, which the hearer uses. Whatever the details of these two senses, the hearer uses the interpretative sense to track the individual who is the author of 'I' as (tokens of) 'I' occurs (occur) in '…I…', while the speaker uses the executive sense for indexical self-identification.[47] Note, however, that unlike the hearer, the speaker has already fixed the references of 'I', 'they' 'this', 'that', etc., when '…I…', '….they…', etc., are uttered and does not interpret his or her own utterances. Interpreting one's own utterances would be pretty much like putting the cart before the horse; for pieces of language are produced (by a speaker) before they are consumed (by a hearer), and producers are not consumers of their own products. At the same time, it may be helpful for speaker and hearer to 'compare notes' by realising how, on some particular occasion, the speaker identifies whatever he or she refers to by means of 'that', as in 'That's country-and-western music at its best!' and how the hearer identifies it from his or her particular vantage point. As Tomis Kapitan writes,

> [F]ully successful communication with an indexical token requires both parties to utilize both meanings of the associated type, and it is this coordinated duality, in addition to the peculiar sorts of context-dependence, that distinguishes indexicals semantically (2001, p. 297).

This brief comparison is not intended to imply that TIL is eventually going to veer off into the general direction of Castañeda's position. But an intensionalist theory such as Castañeda's probably fits the edifice of TIL better than an extensionalist one like Kaplan's. Going intensionalist in this manner would, in the parlance of TIL, amount to expressions containing indexicals assuming a dual meaning; one for the speaker, the other for the hearer. But such a reform would not be a straightforward undertaking. In particular, the notion of pragmatic meaning would have to be altered. For instance, the pragmatic meaning associated with 'I am hungry' when uttered by Albert Einstein is $\lambda w \lambda t\ [{}^0 Hungry_{wt}\ {}^0 Albert\_Einstein]$.[48] But ${}^0 Albert\_Einstein$ obliterates the differences that are bound to exist between

[47] Tichý analyses Castañeda's (1968) example, 'The Editor of *Soul* knows that he* is a millionaire' in 1971, p. 290, 2004, p. 130. Tichý puts the difference between '…he*…' and '…he…' down to 'The Editor of *Soul*' occurring *de re* and *de dicto*, respectively. This analysis obviates the need for a first-person sense of 'he*'. But it might be objected that the Editor of *Soul* does not identify himself as the Editor of *Soul* (cf. Castañeda, ibid., p. 441). There is a shift in perspective involved. On Tichý's analysis the Editor must identify himself from a third-person perspective (along the lines of, 'I am identical to whoever individual is the Editor of *Soul*') to have any thoughts about himself. In Castañeda the Editor identifies himself via a first-person perspective. Tichý's analysis gets the truth-condition, though arguably not the sense, of '…he*…' right. Cf. Kapitan (1992, esp. p. 127).

[48] See Materna (1998, p. 120). Thus the pragmatic meaning associated in a given situation with an expression containing indexicals is a (TIL) concept arising from replacing free occurrences of variables.

how the hearer interprets this token of 'I' and *how* the speaker identifies himself. On the other hand, the substitution of $^0Albert\_Einstein$ for x in $\lambda w \lambda t\ [^0Hungry_{wt}\ x]$ fixes the denotation shared by speaker and hearer in successful communication.[49]

3.4.2 Indefinite descriptions

The problem of indefinite descriptions has been the subject of much dispute among philosophers and logicians just in connection with anaphoric reference. Neale characterizes indefinite descriptions as follows:

> The label 'incomplete description' is misleading. But we need to begin somewhere, so let us have some preliminary definitions. Let us say for the moment that a description is *proper* if, and only if, its nominal—or its superficial matrix in some standard system of representation—is true of exactly one thing, and *improper* otherwise. And let us say that an *improper* description is *empty* if it is true of nothing, and *incomplete* if it is true of more than one thing (2004, p. 32).

However, the condition of a description being 'proper', namely 'its nominal—or its superficial matrix in some standard system of representation—[being] true of exactly one thing' is not clear. From the point of view of TIL, there are two options. Either a description expresses *analytical uniqueness*, which means that in every state of affairs $\langle w, t\rangle$ there is *at most one* entity of which the description is true. Or a description can be *contingently true* of one entity at some $\langle w, t\rangle$, while at another $\langle w, t\rangle$ it is true of more entities (cf. Neale's *incomplete description*) or even none (cf. Neale's *empty description*). The former are *definite descriptions* that denote α-offices (for a type $\alpha \neq (o\beta)$ for any β); these were dealt with in Section 3.1. The latter are *indefinite descriptions* that denote $(o\alpha)$-properties; we are going to discuss them in this section.

Neale goes on to characterize in which sense a description can be *incomplete*:

> So what sorts of things have we really been attributing *incompleteness* to for the past sixty years? [R]emarks by Quine and Sellars … suggest we have been talking all along about incomplete *uses or utterances* of descriptions. Recall that they brought the suggestive word 'elliptical' into the debate in the course of sketching their own answers to the question the Russellian must answer. They talk of elliptical 'uses' (Quine) or elliptical 'utterances' (Sellars) of descriptions, and not of descriptions *per se* being elliptical. According to Sellars, an utterance of 'the table' will typically be elliptical for an utterance the speaker could have made of a richer description such as 'the table over here' or 'the table beside me'. The connection between ellipsis and incompleteness in Sellars's thinking manifests itself when he says (i) that 'in *ellipsis* the context completes the utterance and enables it to say something which it otherwise would not, different contexts enabling it to say different things,' (ii) that some 'utterances … are *not* complete and are only made complete by the context in which they are uttered,' and (iii) that 'statements which are non-elliptical … do not depend on their contexts for their *completion*'. Drawing

[49] Bjørn is indebted to Tomis Kapitan for discussion of Castañeda's theory (March 2007).

upon these early discussions, we might talk of incomplete 'utterances' of descriptions (Ibid., p. 36).

Here Neale talks about the difference that was introduced at the beginning of the previous section, namely the difference between a *pragmatically incomplete/complete meaning* (dependent/independent of a situation of utterance) and a *pragmatic meaning in a given situation of utterance*.

It ought to be obvious that the sentence 'The mountain is high' has a pragmatically incomplete meaning. It expresses an open construction with a free variable, as it does not express a complete instruction for evaluating truth-conditions in *any* empirical context $\langle w, t\rangle$. If the sentence is used out of context, one cannot evaluate its truth-condition, unless additional information is provided on *which mountain*, *among several other mountains*, is predicated to be high. If somebody asserts, out of context, that the mountain is high, the audience is entitled to an answer to the question '*Which* mountain is high?' Even if there happened to be just one mountain in the entire universe, the question would be legitimate, because the noun 'mountain' does not semantically reveal such a contingent uniqueness. This goes to show that in terms like 'the *F*', where '*F*' denotes a property of individuals rather than an individual office, the definite article 'the' functions as a demonstrative like 'this' in 'this *F*'. Roughly speaking, there are in principle (at least) two ways of using the definite article in English:

(a) The expression '*F*' of 'the *F*' denotes an office *F* of type $\alpha_{\tau\omega}$ (where $\alpha \neq (o\beta)$ for any type β). The description is analytically (hence, necessarily) *unique.* The value of the office *F* is *necessarily*, at *every* $\langle w, t\rangle$, *at most one object* of type α. Expressions like 'the Pope', 'the President of the USA', 'the highest mountain on earth' may serve as examples. In Slavic languages (such as Czech), which for the most part lack articles, this way of using 'the' does not correspond to any expression; instead the necessary uniqueness is determined by the *meaning* of '*F*'. Thus 'President České republiky' expresses a meaning determining uniqueness such that the Czech Republic can have at most one president at a time. Hence, the definite article is redundant. This is the case of *definite descriptions*.

(b) The expression '*F*' denotes a property *F* of type $(o\alpha)_{\tau\omega}$, which can *contingently at some* $\langle w, t\rangle$ pairs have a singleton as its value, while at other $\langle w, t\rangle$ pairs the value of *F* is of more than one element or the empty set. If there may be more than one *F* or none, the expression 'the *F*' has a pragmatically incomplete meaning. A sentence in which 'the *F*' is used does not denote a proposition; it has as yet no truth-condition to evaluate at *any* $\langle w, t\rangle$ pair unless an additional piece of information is provided that uniquely selects an α-object (an element of a many-valued population). In Slavic languages this way of using 'the' corresponds to using a definite pronoun (like 'ten', 'ta', 'to', in Czech). Hence the definite article is not redundant, as it signals the need for additional specification. This is the case of *indefinite descriptions*.

Sentences containing definite descriptions were analysed in Section 3.1. In this section we discuss indefinite descriptions. When analysing, for instance, the sentence

(2) 'The mountain is high'

we have got a case *ad* (b); i.e., the expression 'the mountain' serves as an indefinite description. Its meaning is thus an open construction with a free variable *the*, and the analysis of 'the mountain' obtains in the same way as the analysis of 'this hat' provided in Section 3.4.1. The sentence expresses the construction

(2′) $\lambda w \lambda t\ [{}^0 High_{wt}\ \iota x\ [[the_{wt}\ x] \wedge [{}^0 Mountain_{wt}\ x]]]$.

Types: $High$, $Mountain/(o\iota)_{\tau\omega}$; $x \rightarrow \iota$; $the \rightarrow (o\iota)_{\tau\omega}$; $\iota/(\iota(o\iota))$.

A valuation of the subsidiary parameter *the* must provide an additional property so that the set *v*-constructed by the construction $\lambda x\ [[the_{wt}\ x] \wedge [{}^0 Mountain_{wt}\ x]]$ becomes a singleton.

When there is just one mountain on the skyline, (2′) is *v*(*Skyline*/*the*)*-congruent* with the construction ($Skyline/(o\iota)_{\tau\omega}$):

(2^P) $\lambda w \lambda t\ [{}^0 High_{wt}\ \iota x\ [[{}^0 Skyline_{wt}\ x] \wedge [{}^0 Mountain_{wt}\ x]]]$

which is the *pragmatic meaning* associated with the sentence *in the described situation.* If the sentence occurs in a linguistic context, the article 'the' has an anaphoric character; it refers to the meaning of an antecedent expression that denotes a property.[50] Here we just outline the substitution method that serves to complete the meaning of an expression with anaphoric reference. The method was first encountered in Section 1.4.3. The meaning of the sentence

(3) 'There is just one mountain on the skyline and *the* mountain is high'

becomes

(3′) $\lambda w \lambda t\ [\exists y\ [[{}^0 Mountain_{wt}\ y] \wedge [{}^0 Skyline_{wt}\ y]] \wedge {}^2[{}^0 Sub\ {}^{00}Skyline\ {}^0 the$
$\quad {}^0[\lambda w \lambda t\ [{}^0 High_{wt}\ \iota x\ [[{}^0 Mountain_{wt}\ x] \wedge [the_{wt}\ x]]]]]_{wt}]$.

The *Sub* function, here of type $(*_1 *_1 *_1 *_1)$, associates constructions C_1, C_2 and C_3 with the construction C which is the result of substituting C_1 for C_2 into C_3. Here the construction ${}^0 Skyline$ is substituted for variable *the* into the Composition $[\lambda w \lambda t\ [{}^0 High_{wt}\ \iota y\ [[{}^0 Mountain_{wt}\ y] \wedge [the_{wt}\ y]]]]$. As a result, the construction $\lambda w \lambda t$ $[{}^0 High_{wt}\ \iota x\ [[{}^0 Mountain_{wt}\ x] \wedge [{}^0 Skyline_{wt}\ x]]]$ is returned, which must be executed in order to obtain a proposition; this explains the use of Double Execution. Finally, the so constructed proposition has to undergo intensional descent in order to yield a truth-value, which is the second argument of the conjunction.

[50] For anaphoric reference, see Section 3.5.

Note that (3′) is equivalent to (2^P), because the product of the Double Execution of the substitution is the proposition constructed by

$$\lambda w \lambda t\ [{}^0 High_{wt}\ \iota x\ [[{}^0 Mountain_{wt}\ x] \wedge [{}^0 Skyline_{wt}\ x]]].$$

If this proposition has at a particular $\langle W, T\rangle$ pair a truth-value (**T** or **F**), then the class $v(W/w,T/t)$-constructed by $\lambda x\ [[{}^0 Mountain_{wt}\ x] \wedge [{}^0 Skyline_{wt}\ x]]$ is a singleton, which is a non-empty set. Thus the first conjunct of (3′) $v(W/w,T/t)$-constructs **T**. At those $\langle w, t\rangle$ pairs where the set of mountains on the skyline is not a singleton, the Composition $[{}^0 Sing\ \lambda x\ [[{}^0 Mountain_{wt}\ x] \wedge [{}^0 Skyline_{wt}\ x]]]$ is v-improper and so is the entire conjunction of (3′). Thus the propositions constructed by (3′) and (2^P) are identical.

By contrast, the sentence

(4) ‘There is a mountain on the skyline *which* is high’

is not equivalent to (2) and (3). It is simply a case of anaphoric reference to a quantified variable:

(4′) $\lambda w \lambda t\ \exists x\ [[{}^0 Mountain_{wt} x] \wedge [{}^0 Skyline_{wt} x] \wedge {}^2\mathbf{[}{}^0 Sub\ {}^{\mathbf{0}}x\ {}^0 which$
$\ {}^{\mathbf{0}}\mathbf{[}\lambda w \lambda t\ [{}^0 High_{wt}\ which]]\mathbf{]}_{wt}]$

which is equivalent to

$$\lambda w \lambda t\ \exists x\ [[{}^0 Mountain_{wt} x] \wedge [{}^0 Skyline_{wt} x] \wedge [{}^0 \mathrm{High}_{wt}\ x]].$$

Additional type: $which \rightarrow_v \iota$.

The proposition constructed by (4′) is false at those $\langle w, t\rangle$ pairs where there are no mountains on the skyline, and at those $\langle w, t\rangle$ pairs where there are some mountains on the skyline it is true or false, according as some of them are high.

An indefinite description can be combined with a pragmatic (indexical) variable, as is, for instance, the case in the sentence

(5) ‘*The boy* believes that *he* is immortal’.

The sentence has a pragmatically incomplete meaning due to the indefinite description ‘the boy’ that is assigned an open construction with the free variable *the*:

$$\lambda w \lambda t\ [{}^0 Sing\ \lambda x\ [[the_{wt} x] \wedge [{}^0 Boy_{wt} x]]],$$

or in abbreviated form:

$$\lambda w \lambda t\ \iota x\ [[the_{wt} x] \wedge [{}^0 Boy_{wt} x]].$$

This construction must be substituted for the variable *he* into the meaning of ‘He is immortal’. Assuming that *Believe* is an intensional attitude, i.e. an attitude to a possible-world proposition, the analysis of the sentence comes down to this:

(5′) $\lambda w \lambda t\, [{}^0 Believe_{wt}\ \iota x\, [[the_{wt} x] \wedge [{}^0 Boy_{wt} x]]$
${}^2[{}^0 Sub\, [{}^0 Tr\ \iota x\, [[the_{wt} x] \wedge [{}^0 Boy_{wt} x]]]\ {}^0 he\ {}^0[\lambda w \lambda t\, [{}^0 Immortal_{wt}\, he]]]]$.

Types: $Boy/(o\iota)_{\tau\omega}$; $Believe/(o\iota o_{\tau\omega})_{\tau\omega}$; $Immortal/(o\iota)_{\tau\omega}$; x, $he \to \iota$; $the \to (o\iota)_{\tau\omega}$.

Now if the construction $\iota x\, [[the_{wt} x] \wedge [{}^0 Boy_{wt} x]]$ is *v*-improper, then the whole Composition $[{}^0 Believe\ldots]$ is *v*-improper and the so *v*-constructed proposition is undefined. In another situation, if the construction $\iota x\, [[the_{wt} x] \wedge [{}^0 Boy_{wt} x]]$ is *v*-proper, it *v*-constructs an individual. Let Charles be this individual. Then the function $Tr/(*_1\iota)$ takes Charles to his Trivialization, ${}^0 Charles$. Finally, the *Sub* function applied to ${}^0 Charles$, *he* and $[\lambda w \lambda t\, [{}^0 Immortal_{wt}\, he]]$ returns $\lambda w \lambda t\, [{}^0 Immortal_{wt}\ {}^0 Charles]$, which is the *pragmatic* meaning associated with the embedded clause. This construction has to be executed in order to obtain the proposition to which Charles is related; hence Double Execution is called for. The *pragmatic* meaning associated with the whole sentence in this situation is then

$$\lambda w \lambda t\, [{}^0 Believe_{wt}\, [{}^0 Charles\ \lambda w \lambda t\, [{}^0 Immortal_{wt}\ {}^0 Charles].$$

If we analyzed the sentence as a hyperintensional attitude $Believe^*/(o\iota *_1)_{\tau\omega}$ to a propositional construction, we would simply omit the second step (after the substitution). Thus, we would not use Double Execution:

$$\lambda w \lambda t\, [{}^0 Believe^*_{wt}\ \iota x\, [[the_{wt} x] \wedge [{}^0 Boy_{wt} x]]$$
$$[{}^0 Sub\, [{}^0 Tr\ \iota x\, [[the_{wt} x] \wedge [{}^0 Boy_{wt} x]]]\ {}^0 he\ {}^0[\lambda w \lambda t\, [{}^0 Immortal_{wt}\, he]]]].$$

Attitude sentences will be analysed in detail in Chapter 5, and sentences with anaphoric references in the next Section 3.5.

3.5 Anaphora and meaning

Here we take on sentences containing anaphoric reference to the meaning of an expression previously used in linguistic discourse (the antecedent of the anaphoric reference). In principle, there are two problems connected with the analysis of anaphoric sentences.

The first problem is how to combine the meaning of an antecedent with the meaning of the clause where the anaphorically referring pronoun is used. We encountered this problem in the previous Section 3.4 when analysing the sentences 'There is a mountain on the skyline *which* is high' and 'The boy believes that *he* is immortal'. Our solution consisted in applying the substitution method. Thus in the analysis of the first sentence we substituted the existentially bound variable *x* for the free variable *which*, in order to predicate of some mountain *x* that it is high. Similarly, in the analysis of the second sentence we substituted the Composition

$\iota x\ [[the_{wt}\, x] \wedge [^0Boy_{wt}\, x]]$, v-constructing a particular boy, for the variable *he* which is the meaning of the anaphoric pronoun 'he'.

The second problem is how to determine the antecedent of an anaphoric reference. The problem is a well-known hard nut of *linguistic* analysis, because the antecedent is often not unambiguously determinable. For instance, the sentence

'The boy and his daddy saw a dragon, and the boy thought that *he* was immortal'

is ambiguous. If the second clause stood alone, the anaphoric pronoun 'he' would unambiguously refer to the boy, but in this compound sentence it might refer to a dragon rather than the boy.

Thus it is often said that anaphora constitute a pragmatic problem rather than a problem of (logical) semantics. We agree that *logical* analysis cannot disambiguate the above sentence. Actually, logical analysis does not, and cannot, disambiguate any sentence in the sense of privileging one particular meaning. What a logical analysis does is enumerate all the unambiguous individual readings of an ambiguous sentence, or any other kind of expression. Our method of logical analysis can contribute to disambiguation in this manner: type-theoretical analysis of the entities that receive mention in the sentence and/or a specification of some of the requisites of these entities serve to unambiguously determine which of the possible meanings of a homonymous expression is used in a sentence.[51] Thus when analysing a sentence which is ambiguous by having n different meanings, we simply propose n different constructions as expressed by the sentence. As shown in Sections 1.5.2 and 2.6.2., one kind of logically interesting ambiguity feeds on the distinction between *de dicto* and *de re* readings.

Here it will be shown that the same kinds of disambiguation apply to sentences involving anaphoric reference. If the sentence is unambiguous, a type-theoretical analysis determines unambiguously the antecedent of the anaphoric reference, and we propose a method of *logically* analysing such a sentence. As outlined above, the method consists in substituting an appropriate construction for the anaphoric variable. Which construction is to be substituted is determined by the meaning of the antecedent and the type of the object which is the subject of predication in the embedded anaphoric clause. In other words, we perform a *semantic pre-processing* of the embedded anaphoric clause based on the *meaning* of the respective antecedent. In this sense anaphora *are* a *semantic* problem. For the sake of simplicity, we will presuppose that the antecedent is the first expression to the left of the anaphoric reference which denotes a type-theoretically appropriate object whose construction is to be substituted. Hence we will not address the pragmatic problem of disambiguation when the anaphoric reference is ambiguous. However, at the end of this section we outline how to *implement* our method in a way that takes into account the need to make other possible readings explicit as well.

[51] The notion of requisite has been introduced in Section 1.5.2. We will deal in details with requisites and the logic of intensions in Chapter 4, where requisites are defined in Section 4.1.

3.5.1 Semantic pre-processing of anaphora

As explained in Section 3.4, the sentence 'He is a logician' has a pragmatically incomplete meaning and so expresses the *open* construction $\lambda w\lambda t\ [{}^0Logician_{wt}\ he]$, where $Logician/(o\iota)_{\tau\omega}$; $he/*_1\to\iota$. If the sentence is uttered in a situation where the speaker succeeds, in whatever manner, in identifying Charles, then the *pragmatic meaning* associated with the sentence *in this situation* is the construction $\lambda w\lambda t\ [{}^0Logician_{wt}\ {}^0Charles]$, which, though *v(Charles/he)*-congruent with the construction $\lambda w\lambda t\ [{}^0Logician_{wt}\ he]$, is not equivalent to the open construction. In another situation we may well obtain a different construction, because the variable *he* will *v*-construct another individual. Hence the *pragmatic meaning* associated with the sentence *in the given situation of utterance* is a closed construction, whereas *the meaning* of the sentence is the open construction.

If the sentence 'He is a logician' occurs in a *linguistic context*, does it also have an incomplete meaning? Since we advocate an anti-contextualist approach, the answer is Yes. The sentence has the same meaning in every context, which is to say that it expresses, always and in every context, one and the same *open* construction. However, when the sentence occurs in a linguistic context then we, as readers or hearers, are able to get to know only from the linguistic context what the anaphoric pronoun 'he' refers to. This is possible only if the whole sentence has a *complete* meaning. For instance, in the following sentence (1) the pronoun '*he*' refers to Charles:

(1) 'If Charles is rational, then *he* is a logician',

and to understand the sentence completely we do not need any situation of utterance. The sentence encodes a *complete procedure* for evaluating the truth-condition for any $\langle w, t\rangle$. Hence its meaning has to be a *closed construction* constructing a proposition without the mediation of pragmatic or empirical factors.

Note that the meaning of (1) is *not* construction (2′):

(2′) $\lambda w\lambda t\ [\lambda w\lambda t\ [{}^0Rational_{wt}\ {}^0Charles]_{wt} \supset \lambda w\lambda t\ [{}^0Logician_{wt}\ {}^0Charles]_{wt}]$

(or, β-reduced: $\lambda w\lambda t\ [[{}^0Rational_{wt}\ {}^0Charles] \supset [{}^0Logician_{wt}\ {}^0Charles]]$), because then (1) would be synonymous with

(2) 'If Charles is rational then *Charles* is a logician',

which it obviously is not.
Types: (Being) *Rational*, $Logician/(o\iota)_{\tau\omega}$; $Charles/\iota$.

The common objection to such a solution that the first occurrence of the name 'Charles' can denote a different individual than the second one can readily be set aside. The construction ${}^0Charles$ is a simple concept of the particular individual Charles, regardless of how, or whether, the individual is named. It constructs—in every context, without exception—one and the same individual.

But, there is a more serious objection. If (2′) were the meaning of (1), then the meaning of the embedded clause ‘he is a logician’ would in *this context* have to be the Closure $\lambda w \lambda t\ [{}^0Logician_{wt}\ {}^0Charles]$, rather than $\lambda w \lambda t\ [{}^0Logician_{wt}\ he]$. Otherwise we would have to give up the compositionality principle. In keeping with this principle, we hold that the meaning of (1) has to be derived in part from the meaning of ‘he is a logician’; and we have seen that this meaning is not the meaning of ‘Charles is a logician’.

It seems that we either have to give up the compositionality constraint or else the anti-contextualist transparency constraint. Yet our goal is to propose a solution that is in full accordance both with compositionality and transparency. Much is at stake. If no such solution is forthcoming, then TIL will turn out to be inapplicable to a key fragment of natural language.

A moment’s reflection on the way we *understand* sentence (1) indicates where to look for a solution. Since the whole sentence has a complete meaning, a complete procedure for evaluating its truth-condition in *any* $\langle w, t\rangle$ is encoded. This means that as soon as we understand (1), we know that a *semantic pre-processing* of the anaphoric reference has been specified. The pre-processing must be specified *neither by a pragmatic factor nor* be performed at *the empirical level* of evaluation of reference a posteriori. The procedure of pre-processing the anaphoric reference must be specified at the *semantic level*, since the (sub-) procedure is a *constituent of the meaning* of the whole sentence. So as language-users we understand how an open Closure, $\lambda w \lambda t\ [\ldots he \ldots]$, is to be converted into a closed Closure, $\lambda w \lambda t\ [\ldots {}^0X \ldots]$, X the specific individual cited by the anaphoric pronoun ‘he’. The fact that we understand the sentence is evidence that also an open construction is a procedure. This fact, in turn, is further evidence that the concept of procedural semantics has much going for it.

In the present case the meaning of the antecedent ‘Charles’, i.e., the Trivialization ${}^0Charles$, is to be substituted for the variable *he*. This suggests to us that an anaphoric pronoun is a *semantic* abbreviation. Accordingly, the sentence encodes a two-phase procedure:

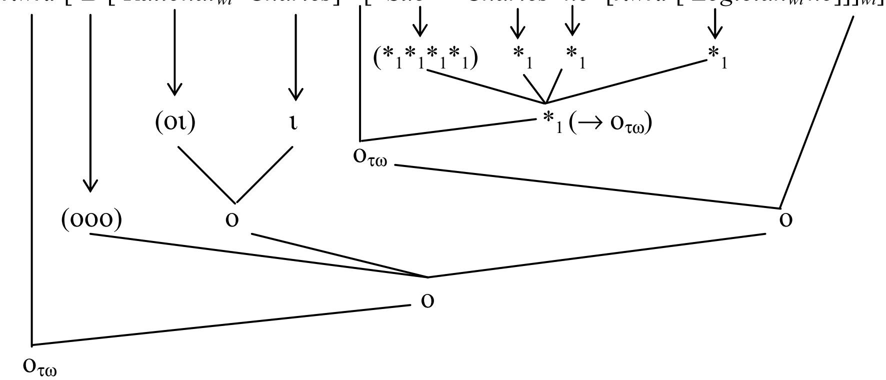
(i) pre-process the anaphoric reference by means of the meaning of the antecedent expression;
(ii) execute the adjusted meaning, which is the pre-processed construction.

To specify phase (i) we use the substitution function Sub_n introduced in Section 1.4.3. In the case of sentence (1) we have $n = 1$, hence $Sub_1/(*_1 *_1 *_1 *_1)$. The meaning of (1) is the Closure (1′):

(1′) $$\lambda w \lambda t\ [[{}^0Rational_{wt}\ {}^0Charles] \supset {}^2[{}^0Sub\ {}^{00}Charles\ {}^0he\ {}^0[\lambda w \lambda t\ [{}^0Logician_{wt}\ he]]]_{wt}].$$

Since (1′) may appear rather complicated at first sight, we first run a type check (using prefix notation) and then show that (1′) is an adequate analysis meeting our three requirements of heeding compositionality, anti-contextualism and being a purely semantic solution.

$\lambda w \lambda t\, [{}^0\!\supset [{}^0 Rational_{wt}\ {}^0 Charles]\ {}^2[\,{}^0 Sub\ \ {}^{00}Charles\ {}^0 he\ {}^0[\lambda w \lambda t\, [{}^0 Logician_{wt}\ he]]]_{wt}]$



The constituent (S) of (1′)[52]

(S) $[{}^0 Sub\ {}^{00}Charles\ {}^0 he\ {}^0[\lambda w' \lambda t'\, [{}^0 Logician_{w't'}\ he]]] \to *_1$

constructs a *construction* of order 1, namely the one obtained by the substitution of ${}^0 Charles$ for the variable *he* into the Closure $\lambda w' \lambda t'\, [{}^0 Logician_{w't'}\ he]$. The result is the construction

(S') $\lambda w' \lambda t'\, [{}^0 Logician_{w't'}\ {}^0 Charles]$,

which constructs a proposition *P*. But an argument of the truth-function of implication ($\supset$) can be neither a propositional construction, nor a proposition, but must be a truth-value. Since (S) constructs the construction (S′), and (S′) constructs *P*, the execution steps have to be:

(a) execute (S) to obtain the propositional construction (S′),
(b) execute the result (S′) to obtain *P* (hence we need Double Execution of (S) to construct *P*),
(c) extensionalize *P* with respect to the external *w*, *t* in order to *v*-construct a truth-value:

$$[[{}^2[{}^0 Sub\ {}^{00}Charles\ {}^0 he\ {}^0[\lambda w' \lambda t'\, [{}^0 Logician_{w't'}\ he]]]w]\ t] \ \to_v \ o.$$

This construction *v*-constructs the truth-value **T** at those $\langle w, t\rangle$, at which *Charles* is a logician, just as it should in accordance with the three requirements.

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[52] To make things clearer by displaying which of the occurring Closures λ-bind which variables, we α-renamed the *w*, *t* variables.

The meaning of a sentence containing a clause with an anaphoric reference is the procedure which is, in this case, a two-phase procedure, as specified by Double Execution.[53] The procedure comes down to this:

- first, execute the substitution based on the meaning of the antecedent for the anaphoric variable;
- second, execute the result (a propositional construction) again to obtain a proposition.

If $=_o/(ooo)$ is the identity of truth-values, then for any valuation v of variables w, t it holds that

$$^2[^0Sub\ ^{00}Charles\ ^0he\ ^0[\lambda w'\lambda t'\ [^0Logician_{w't'}\ he]]]_{wt} =_o$$
$$\lambda w'\lambda t'\ [^0Logician_{w't'}\ ^0Charles]_{wt} =_o [^0Logician_{wt}\ ^0Charles].$$

Hence constructions (1′) and (2′) are equivalent, yet the *meaning* of the sentence (1) is *not* the construction (2′), but (1′). In Section 3.5.2 we will show that it is not always possible to equivalently transform the meaning of an anaphoric sentence into the construction obtained after executing the substitution, because its execution may depend on a particular valuation v. This is another good reason for assigning a construction with an explicit specification of substitution to an anaphoric sentence as its meaning. Thus we have a unique method for analyzing sentences containing occurrences of anaphoric reference.

However, at this point the analysis might be objected to. We said that the way how we analyse expressions is in accordance with the Parmenides principle.[54] An adequate analysis of an expression E contains only constructions of those objects that receive mention in E. One may wonder, though, which subexpression of (1) expresses the instruction to perform the substitution (S).[55] Our answer is this. The sentence (1) is a semantic abbreviation, and its full, unpacked meaning expresses the semantic substitution. When unpacking the abbreviation, the sentence can be read as follows: ‘If Charles is rational then he (‘he’ referring to Charles) is a logician’.

At the beginning of this section we said that the type-theoretical analysis facilitates an assignment of a proper antecedent to the anaphorically referring term. This term expresses a variable for which the construction of a respective entity is substituted *via* the meaning of the antecedent expression. If the anaphoric variable v-constructs an α-entity, then the construction of an entity of the type α must be substituted. The type α can be any of the type hierarchy, even the type of a construction. However, until now we analysed only examples of substituting

[53] In what follows we show that the second step (Double Execution) can be absent in an adequate analysis of an anaphoric sentence. This holds for cases where the meaning of the antecedent is *mentioned* in a hyperintensional context.

[54] For details, see Section 2.1.1.

[55] We are grateful to Jaroslav Peregrin for this remark.

individuals into an extensional context. Let us now analyze, by the method described above, some more examples of anaphoric reference as they occur in (A) a hyper-intensional context, (B) an intensional context, and (C) an extensional context. First up is (A):

(A) ‘5 + 7 = 12, and Charles knows *it*.’

The embedded clause ‘Charles knows it’ does not express Charles’ relation (-in-intension) to the truth-value **T**, but to the *procedure* of calculating the result of 5 + 7 = 12. Hence the pronoun ‘*it*’ refers anaphorically to the meaning of ‘5 + 7 = 12’, and *knowing* is here a relation-in-intension between an individual and a construction, in this case the Composition $[[^0+\ ^05\ ^07] = {}^012]$. The meaning of (A) is thus the closed construction

(A′) $$\lambda w \lambda t\, [[[^0+\ ^05\ ^07] = {}^012] \wedge {}^2[^0Sub\ ^{00}[[^0+\ ^05\ ^07] = {}^012]\ ^0it\ ^0[\lambda w \lambda t\, [^0Know^*_{wt}\ ^0Charles\ it]]]_{wt}]$$

Types: $Know^*/(o\iota *_1)_{\tau\omega}$; $Sub/(*_2 *_2 *_2 *_2)$; $it/*_2 \rightarrow *_1$.

The meaning of the sentence ‘Charles knows it’ is the open construction

$$\lambda w \lambda t\, [^0Know^*_{wt}\ ^0Charles\ it].$$

The variable *it* is free here either for a pragmatic valuation (by the situation of utterance) or for a substitution of the meaning of the antecedent that is referred to in the linguistic context. The object—*what* is known by Charles—can be completed by a situation of utterance or by a linguistic context. If the sentence occurs within another linguistic context, then *Sub* substitutes a different construction for the variable *it*, namely the construction to which ‘*it*’ anaphorically refers. Next up is (B):

(B) ‘Charles sought the Mayor of Dunedin but (*he*) did not find *him*.’

Consider the *de dicto* reading of (B), which is that Charles’ search concerned the office of Mayor of Dunedin and not the location of its holder. Charles wanted to find out who the Mayor of Dunedin is, that is, who is the occupant of the individual office of Mayor. Thus *seeking* and *finding* are here relations-in-intension of an individual to an *individual office*, of type $(o\iota\iota_{\tau\omega})_{\tau\omega}$, and the context under scrutiny is an intensional one.[56]

The function *Sub* creates a new construction from constructions and so can easily be iterated. The *de dicto* analysis of (B) is:

[56] The attitudes of seeking and finding will be analyzed in details in Section 5.2.2, where other types of these attitudes will be examined as well. For the sake of simplicity, in this section we again disregard tenses. See, however, Section 2.5.2.

(B^d) $\lambda w \lambda t\, [[{}^0Seek_{wt}\, {}^0Ch\, \lambda w \lambda t\, [{}^0Mayor\_of_{wt}\, {}^0D]] \wedge {}^2[{}^0Sub\, {}^{00}Ch\, {}^0he$
$[{}^0Sub\, {}^0[\lambda w \lambda t\, [{}^0Mayor\_of_{wt}\, {}^0D]]\, {}^0him\, {}^0[\lambda w \lambda t\, \neg[{}^0Find_{wt}\, he\, him]]]]_{wt}]$.

Types: *Seek*, *Find*/$(o\iota\iota_{\tau\omega})_{\tau\omega}$; *Ch*(*arles*)/$\iota$; *Mayor_of* (something)/$(\iota\iota)_{\tau\omega}$; *D*(unedin)/$\iota$; *he*/$*_1 \to \iota$; *him*/$*_1 \to \iota_{\tau\omega}$.

Again, the meaning of (B) is the closed construction (B^d), and the meaning of the embedded clause '*he* did not find *him*' is the open construction $\lambda w \lambda t\, \neg[{}^0Find_{wt}\, he\, him]$ with the two free variables *he* and *him*.

Of course, another refinement is thinkable. The variables *he* and *him*, ranging over individuals and individual offices, respectively, reduce the ambiguity of 'to find' by determining that here we are concerned with finding the occupant of an individual office. But the expressions 'he', 'him', or 'she', 'her', also indicate that the finder as well as the occupant of the sought office are male and female, respectively. Thus, e.g., a refined meaning of 'He found her' might be

$$\lambda w \lambda t\, [[{}^0Find_{wt}\, he\, her] \wedge [{}^0Male_{wt}\, he] \wedge [{}^0Female_{wt}\, her_{wt}]].$$

Additional types: *Male*, *Female*/$(o\iota)_{\tau\omega}$; *her*/$*_1 \to \iota_{\tau\omega}$.

The meaning of the *de dicto* reading of the sentence

'Charles sought the Mayor of Dunedin and he found her'

refined in the way just described is then

$$\lambda w \lambda t\, [[{}^0Seek_{wt}\, {}^0Ch\, \lambda w \lambda t\, [{}^0Mayor\_of_{wt}\, {}^0D]] \wedge$$
$${}^2[{}^0Sub\, {}^{00}Ch\, {}^0he\, [{}^0Sub\, {}^0[\lambda w \lambda t\, [{}^0Mayor\_of_{wt}\, {}^0D]]\, {}^0her\, {}^0[\lambda w \lambda t\, [[{}^0Find_{wt}\, he\, her] \wedge$$
$$[{}^0Male_{wt}\, he] \wedge [{}^0Female_{wt}\, her_{wt}]]]]]]_{wt}]$$

which is equivalent to

$$\lambda w \lambda t\, [[{}^0Seek_{wt}\, {}^0Ch\, \lambda w \lambda t\, [{}^0Mayor\_of_{wt}\, {}^0D]] \wedge [{}^0Find_{wt}\, {}^0Ch\, \lambda w \lambda t\, [{}^0Mayor\_of_{wt}\, {}^0D]]$$
$$\wedge [{}^0Male_{wt}\, {}^0Ch] \wedge [{}^0Female_{wt}\, [{}^0Mayor\_of_{wt}\, {}^0D]]].$$

Since such a refinement is obvious, we shall not make these additional specifications in the following analyses.

Now perhaps a more natural *de re* reading of the 'seeking clause' of (B) can be reformulated as

(B^r) 'Charles is looking for the Mayor of Dunedin (namely, his location)'.

This sentence is understood as uttered in a situation where Charles knows who the Mayor is, and is striving to locate this individual. Unlike the *de dicto* case, the sentence when understood *de re* comes with an *existential presupposition*: in order for (B^r) to have a truth-value, the Mayor must exist.[57] The object of Charles's search is now a μ-office, μ being the type of location/position.[58] The μ-office is *v*-constructed by $[\lambda w\lambda t\ [^0Loc\_of_{wt}\ him]]$. This time we must not substitute the *de re* occurrence of the construction $[^0Mayor\_of_{wt}\ ^0D]$. We would be drawing an extensional occurrence of $[^0Mayor\_of_{wt}\ ^0D]$ into the intensional context of $[\lambda w\lambda t\ [^0Loc\_of_{wt}\ him]]$, which is not a valid substitution.[59] Instead we must use the function *Tr* in order to substitute the Trivialization of the individual (if any) *v*-constructed by $[^0Mayor\_of_{wt}\ ^0D]$. The Composition $[^0Tr\ [^0Mayor\_of_{wt}\ ^0D]]$ fails to *v*-construct anything if $[^0Mayor\_of_{wt}\ ^0D]$ is *v*-improper (the Mayor failing to exist); otherwise it *v*-constructs the Trivialisation of the occupant of the office. By using the substitution technique, we can obtain the adequate analysis of (B^r):

$$\lambda w\lambda t\ [^0Look\_for_{wt}\ ^0Ch\ ^2[^0Sub\ [^0Tr\ [^0Mayor\_of_{wt}\ ^0D]]\ ^0him\ ^0[\lambda w\lambda t\ [^0Loc\_of_{wt}\ him]]]].$$

Additional types: $Look\_for/(o\iota\mu_{\tau\omega})_{\tau\omega}$; $Tr/(*_1\iota)$; $him\ /*_1\rightarrow\iota$; $Loc\_of/(\mu\iota)_{\tau\omega}$.

When the clause '*He* did not find *him*' occurs in a different linguistic context, its meaning is the same. For instance, the *de dicto* reading of the sentence

(B_1) 'Whomever Charles is seeking, *he* is not finding *him*',

where *Seek* is again a relation to a ι-office, is analysed as

(B_1^d) $\lambda w\lambda t\ \forall z\ [[^0Seek_{wt}\ ^0Ch\ z] \supset\ ^2[^0Sub\ ^{00}Ch\ ^0he\ [^0Sub\ ^0z\ ^0him\ ^0[\lambda w\lambda t\ \neg[^0Find_{wt}\ he\ him]]]]_{wt}]$.

Types: $z \rightarrow \iota_{\tau\omega}$; otherwise as above.

The construction (B_1^d) is again equivalent to the construction resulting from the substitution

$$\lambda w\lambda t\ \forall z\ [[^0Seek_{wt}\ ^0Ch\ z] \supset [\lambda w\lambda t\ \neg[^0Find_{wt}\ ^0Ch\ z]]_{wt}],$$

which is β-equivalent to

$$\lambda w\lambda t\ \forall z\ [[^0Seek_{wt}\ ^0Ch\ z] \supset \neg[^0Find_{wt}\ ^0Ch\ z]].$$

The meaning of (B_1) is, however, the construction (B_1^d), in which the semantic pre-processing of the anaphora is specified.

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[57] See Section 1.5.2, Definition 1.14.

[58] We do not specify this type. It can be, for instance, the GPS coordinates of an individual.

[59] The rules of valid substitution are found in Section 2.7.

An example of a relation-in-intension to an extensional entity is easily found and easily analyzed:

(C) 'Charles met the Mayor of Dunedin and *he* talked to *him*.'

Types: $Meet$, $Talk\_to/(o\iota\iota)_{\tau\omega}$); $he, him \rightarrow \iota$.

The meaning of the embedded clause is again an open construction:

$$[\lambda w \lambda t\ [{}^0 Talk\_to_{wt}\ he\ him]].$$

Now suppose that the sentence has been disambiguated into 'Charles met the Mayor of Dunedin and *he* (namely, Charles) talked to *him* (namely, the Mayor of Dunedin).' The substitution of the meaning of the first antecedent (${}^0Ch$) for the anaphoric variable *he* is not a problem. But, for the variable *him* we are to substitute the construction of that (unspecified) individual (if any) who is referred to by 'the Mayor of Dunedin' at $\langle w, t\rangle$. In other words, we need to substitute a construction of the individual (if any) v-constructed by the Composition $[{}^0Mayor\_of_{wt}\ {}^0D]$ into the construction $\lambda w \lambda t\ [{}^0Talk\_to_{wt}\ he\ him]$. There are two equivalent alternatives. The first alternative uses the function Tr to substitute the Trivialization of the individual v-constructed by the Composition $[{}^0Mayor\_of_{wt}\ {}^0D]$. The resulting analysis is

$$(C_1)\quad \lambda w \lambda t\ [[{}^0Meet_{wt}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]] \wedge$$
$${}^2[{}^0Sub\ {}^{00}Ch\ {}^0he\ [{}^0Sub\ [{}^0Tr\ [{}^0Mayor\_of_{wt}\ {}^0D]]\ {}^0him$$
$${}^0[\lambda w \lambda t\ [{}^0Talk\_to_{wt}\ he\ him]]]]_{wt}].$$

If there is no Mayor of Dunedin at a given $\langle w, t\rangle$, then $[{}^0Mayor\_of_{wt}\ {}^0D]$ is v-improper, and due to compositionality the whole Composition

$$(S_1)\quad [{}^0Sub\ {}^{00}Ch\ {}^0he\ [{}^0Sub\ [{}^0Tr\ [{}^0Mayor\_of_{wt}\ {}^0D]]\ {}^0him$$
$${}^0[\lambda w \lambda t\ [{}^0Talk\_to_{wt}\ he\ him]]]]$$

is v-improper. The so constructed proposition has a truth-value gap. Of course, if there is no Mayor of Dunedin then there is no Mayor of Dunedin to talk to, nor is there any Mayor of Dunedin not to talk to. On the other hand, if the Mayor is, e.g., Mr Taylor, then $[{}^0Tr\ [{}^0Mayor\_of_{wt}\ {}^0D]]$ v-constructs ${}^0Taylor$ and the result of the substitution is the construction $[\lambda w \lambda t\ [{}^0Talk\_to_{wt}\ {}^0Charles\ {}^0Taylor]]$. Yet, note that (C) does not mention Mr Taylor. The Closure $\lambda w \lambda t\ [{}^0Talk\_to_{wt}\ {}^0Charles\ {}^0Taylor]$ is not equivalent to (S_1), but only v-congruent.

The analysis (C_1) is thus equivalent to

$$(C')\quad \lambda w \lambda t\ [[{}^0Meet_{wt}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]] \wedge$$
$$[{}^0Talk\_to_{wt}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]]].$$

The second alternative analysis makes use of two things: (a) both ${}^0Meet_{wt}$ and ${}^0Talk_{wt}$ are constituents occurring extensionally in the Compositions $[{}^0Meet_{wt}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]]$, $[{}^0Talk\_to_{wt}\ he\ him]$, respectively; (b) the constructions $[{}^0Mayor\_of_{wt}\ {}^0D]$, *him* must be *v*-congruent in the analysis of (C). Thus we can substitute the former for the latter. In order to prevent collision of variables, we must rename the variables *w*, *t*.[60] The upshot is the analysis

$$(C_2)\quad \lambda w\lambda t\ [[{}^0Meet_{wt}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]] \wedge {}^2[{}^0Sub\ {}^{00}Ch\ {}^0he\ [{}^0Sub\ {}^0[{}^0Mayor\_of_{wt}\ {}^0D]\ {}^0him\ {}^0[\lambda w'\lambda t'\ [{}^0Talk\_to_{w't'}\ he\ him]]]]_{wt}],$$

which is again equivalent to (C′).

Note that (C_1) differs from (C_2) only in using the function *Tr*, which is applied to the individual (if any) *v*-constructed by $[{}^0Mayor\_of_{wt}\ {}^0D]$. In (C_2) we substituted directly the Composition $[{}^0Mayor\_of_{wt}\ {}^0D]$ for *him* into the intensional context of the Closure $[\lambda w'\lambda t'\ [{}^0Talk\_to_{w't'}\ he\ him]]$. One may wonder whether such a substitution is correct. To be sure, while the Composition $[{}^0Mayor\_of_{wt}\ {}^0D]$ may be *v*-improper for some valuations *v* due to ${}^0Mayor\_of$ occurring *de re* in it, the above Closure is never *v*-improper. In Section 2.7 we warned against dragging a construction occurring *de re* into an intensional context on pain of ending up with a non-equivalent construction. Yet the two constructions just considered are equivalent. Here is why. The result of applying the *Sub* function twice is here the Closure

$$\lambda w'\lambda t'\ [{}^0Talk\_to_{w't'}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]].$$

Due to Double Execution, this Closure is *used* in (C_2) to *v*-construct a proposition. In the next step this proposition is subjected to intensional descent with respect to *w*, *t*, and the result is that the Closure occurs extensionally in (C_2). The Composition $\lambda w'\lambda t'\ [{}^0Talk\_to_{w't'}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]]_{wt}$ is equivalent to $[{}^0Talk\_to_{wt}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]]$. Thus the second conjunct of (C_2), namely

$${}^2[{}^0Sub\ {}^{00}Ch\ {}^0he\ [{}^0Sub\ {}^0[{}^0Mayor\_of_{wt}\ {}^0D]\ {}^0him\ {}^0[\lambda w'\lambda t'\ [{}^0Talk\_to_{w't'}\ he\ him]]]]_{wt}$$

is equivalent to

$$[{}^0Talk\_to_{wt}\ {}^0Ch\ [{}^0Mayor\_of_{wt}\ {}^0D]].$$

Hence both analyses are equivalent.

Anaphoric reference can occur not only in an embedded clause, but also be part of a sentence. Some further examples:

(D_1) ‘John loves *his* mother’

[60] See again the rules of valid substitution in Section 2.7.

(D_2) 'Everybody loves *their* mother'.

The subexpression 'loves his/their mother' denotes a property $LM/(o\iota)_{\tau\omega}$, and a coarse-grained analysis of the above sentences is:

(D_1') $\lambda w \lambda t\ [{}^0LM_{wt}\ {}^0John]$

(D_2') $\lambda w \lambda t\ [\forall x\ [{}^0LM_{wt}\ x]]$.

Additional types: $John/\iota$; $x \to \iota$.

${}^0LM$ must be refined by Composing Trivializations of $Love/(o\iota\iota)_{\tau\omega}$ and $Mother\_of/(\iota\iota)_{\tau\omega}$; $y \to \iota$:

(LM) $\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\, y\ [{}^0Mother\_of_{wt}\, y]]$.

Replacing ${}^0LM$ by the construction (LM) we obtain these finer analyses of (D_1) and (D_2):

(D_1'') $\lambda w \lambda t\ [\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\, y\ [{}^0Mother\_of_{wt}\, y]]_{wt}\ {}^0John]$

(D_2'') $\lambda w \lambda t\ [\forall x\ [\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\, y\ [{}^0Mother\_of_{wt}\, y]]_{wt}\ x]]$.

And after β-reduction:

(D_{1red}'') $\lambda w \lambda t\ [\lambda y\ [{}^0Love_{wt}\, y\ [{}^0Mother\_of_{wt}\, y]]\ {}^0John]$

(D_{2red}'') $\lambda w \lambda t\ [\forall x\ [\lambda y\ [{}^0Love_{wt}\, y\ [{}^0Mother\_of_{wt}\, y]]\ x]]$.

Further β-reducing is valid, because ${}^0John$ and x are not v-improper for any v. However, in case of (D_{1red}'') the result

$$\lambda w \lambda t\ [{}^0Love_{wt}\ {}^0John\ [{}^0Mother\_of_{wt}\ {}^0John]]$$

will be an analysis of another sentence, in which the anaphoric reference is lost:

'John loves John's mother'.

Moreover, consider the following valid argument:

(D_3) 'John loves his mother and *so does* Peter';

hence

(D_4) 'John and Peter share a common property'.[61]

By means of this example we are going to show that the adequate analyses of the above sentences are the non-reduced constructions.

The antecedent of 'so does' in (D_3) is 'loves his mother', which denotes the property of individuals constructed by (LM). Thus the analysis of the embedded clause 'so does Peter' is an open construction with a free variable $so\_does/*_1 \rightarrow (o\iota)_{\tau\omega}$:

$$\lambda w \lambda t\ [so\_does_{wt}\ {}^0Peter].$$

To analyze (D_3), we must substitute the construction (LM) for the variable *so_does*:

(D_3') $\lambda w \lambda t\ [[[\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\ y\ [{}^0Mother\_of_{wt} y]]]_{wt}\ {}^0John] \wedge$ ${}^2[{}^0Sub\ {}^0[\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\ y\ [{}^0Mother\_of_{wt} y]]]\ {}^0so\_does$ ${}^0[\lambda w \lambda t\ [so\_does_{wt}\ {}^0Peter]]]_{wt}]$.

The construction (D_3') is equivalent to the construction that emerges *after* semantic pre-processing:

(D_3'') $\lambda w \lambda t\ [[[\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\ y\ [{}^0Mother\_of_{wt} y]]]_{wt}\ {}^0John] \wedge$ $[\lambda w \lambda t\ [[\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\ y\ [{}^0Mother\_of_{wt} y]]]_{wt}\ {}^0Peter]]_{wt}] =_\beta$

(D_3''') $\lambda w \lambda t\ [[[\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\ y\ [{}^0Mother\_of_{wt} y]]]_{wt}\ {}^0John] \wedge$ $[[\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\ y\ [{}^0Mother\_of_{wt} y]]]_{wt}\ {}^0Peter]]$.

The consequence that Peter and John share a common property,

$$\lambda w \lambda t\ [\exists p\ [[p_{wt}\ {}^0John] \wedge [p_{wt}\ {}^0Peter]]],$$

is now trivially derivable by existential generalisation. Obviously, the property *LM* constructed by $[\lambda w \lambda t\ \lambda y\ [{}^0Love_{wt}\ y\ [{}^0Mother\_of_{wt} y]]]$ is here the common property shared by John and Peter.

Note that if the reduced construction $\lambda w \lambda t\ [{}^0Love_{wt}\ {}^0John\ [{}^0Mother\_of_{wt}\ {}^0John]]$ were assigned to the first clause of (D_3) as its meaning, then there would be no construction of *LM* to be substituted for *so_does* into $\lambda w \lambda t\ [so\_does_{wt}\ {}^0Peter]$, and the consequence would not be directly derivable.

With the exception of (B^r), the above analyses containing the constituent ${}^0Sub$ were equivalent to the construction obtained *after* the execution of the substitution (and, as the case may be, after the execution of intensional descent). The meaning

[61] We include here only the 'strict reading' of (D_3), on which Peter loves his own mother, and exclude the 'sloppy reading', on which Peter loves John's mother. See Neale (2004, p. 63).

of the antecedent to which the anaphoric term refers has been (a) mentioned in a hyperintensional context, (b) used with *de dicto* supposition in an intensional context, or (c) used with *de re* supposition in an extensional context. The equivalence mentioned above is due to the fact that the respective substitutions are homogeneous. We inserted (a) a construction into a hyperintensional context, (b) an intension into an intensional context, or (c) an extension into an extensional context. According to the (constructional, intensional and extensional) rules introduced in Section 2.7.1, these substitutions are valid and thus result in equivalent constructions.

However, not all sentences containing anaphoric reference are this simple, as (B^r) illustrates. Problems may crop up when there is a need to substitute a construction of a lower-order entity into a higher-order context, namely of an extension into an intensional or hyperintensional context, or of an intension into a constructional context, because the higher-order context is dominant. This problem comes to the fore not least when analysing (propositional and notional) attitudes *de re* (see Chapter 5).

3.5.2 Donkey sentences

The following example is a variant of the well-known problem of Peter Geach's *donkey sentence*s:

(D) 'If somebody has got a new car then *he* often washes *it*.'

The analysis of the embedded clause '*he* often washes *it*', containing the anaphoric pronouns 'he' and 'it' is again an open construction with the two free variables *he* (who washes), *it* (what is washed), *he*, *it* $\rightarrow \iota$; $Wash/(o\iota\iota)_{\tau\omega}$:

$$\lambda w \lambda t\, [{}^0 Wash_{wt}\; he\; it].$$

If we also want to analyze the frequency of washing, i.e., the meaning of 'often', then we use the function $Often/((o(o\tau))\tau)$.[62] The function *Often* associates each time t with a set of those time intervals (of type $(o(o\tau))$) that are frequent at t (for instance, once a week). The analysis of '*he* often washes *it*', is then

$$\lambda w \lambda t\, [{}^0 Often_t\, \lambda t' [{}^0 Wash_{wt'}\; he\; it]].$$

However, since rendering the frequency of washing does not influence how the problem of anaphora in donkey sentences is solved, we will use, for the sake of simplicity, the first construction.

[62] See Section 2.5, or Tichý (1986a, pp. 261–63) and Duží (2004).

The problem of donkey sentences consists first and foremost in discovering their *logical form*,[63] because it is far from clear how to understand them. Geach, (1962, p. 126), proposed a structure that can be rendered in 1st-order predicate logic as follows (*NC*, new car):

$$\forall x \forall y \, ((NC(y) \wedge Has(x, y)) \rightarrow Wash(x, y)).$$

However, Russell objected to this analysis that the expression 'a new car' is an *indefinite description*, something which does not come across in Geach's analysis. Hence Russell proposed an analysis that corresponds to this formula of 1st-order predicate logic:

$$\forall x \, (\exists y \, (NC(y) \wedge Has(x, y)) \rightarrow Wash(x, \boldsymbol{y})).$$

But the last occurrence of the variable y (marked in bold) is *free* in this formula and so out of the scope of the existential quantifier supposed to bind it.

Neale (1990) proposes a solution that combines both of the above proposals. On the one hand, the existential character of an indefinite description is saved (Russell's demand), and on the other hand, the anaphoric variable is bound by a general quantifier (Geach's solution). Neale introduces so-called *restricted quantifiers*[64]:

[every x: man x and [a y: new-car y](x owns y)]
([whe z: car z and x owns z] (x often washes z)).

The sentence (D) does not entail that if a man owns more than one new car then some of these cars are not washed by him. Hence we can reformulate the sentence into (D_1):

(D_1) 'Everybody who owns some new cars often washes *all of them* [each of the new cars he owns].'

However, the following sentence (D_2) obviously means something else:

(D_2) 'Everybody who owns some new cars often washes *some of them* [some, though not all, of the new cars he owns].'

The analysis of (D_1), which in principle corresponds to Geach's proposal, is

[63] See Section 1.5.1, or Duží and Materna (2005).

[64] Neale (1990, p. 236). Neale takes it into account that the sentence is true even if a man owns *more than one* new car. To avoid singularity he thus claims that the description used in his analysis need not be singular (definite), but may be plural: his abbreviation 'whe *F*' stands for 'the *F* or the *F*s'.

$$(D_1') \quad \lambda w \lambda t\ \forall x \forall y\ [[[{}^0NC_{wt}\, y] \wedge [{}^0Own_{wt}\, x\, y]] \supset {}^2\mathbf{[}{}^0Sub\ {}^0x\ {}^0he\ \mathbf{[}{}^0Sub\ {}^0y\ {}^0it\ {}^0[\lambda w \lambda t\ [{}^0Wash_{wt}\ he\ it]]\mathbf{]]}_{wt}],$$

which is β-equivalent to this construction after executing the substitution:

$$(D_1'') \quad \lambda w \lambda t\ \forall x \forall y\ [[[{}^0NC_{wt}\, y] \wedge [{}^0Own_{wt}\, x\, y]] \supset [{}^0Wash_{wt}\, x\, y]].$$

Types: $Own/(o\iota\iota)_{\tau\omega}$; $Wash/(o\iota\iota)_{\tau\omega}$; NC (being a new car)$/(o\iota)_{\tau\omega}$; $x, y, he, it \to \iota$; $\forall/(o(o\iota))$.

But then an objection due to Neale can be levelled against these analyses, namely that in the original sentence (D) the anaphoric pronoun 'it' stands *outside* the scope of the quantifier occurring in the antecedent. Moreover, Russell's objection applies as well. To overcome these objections, we use a different type of quantifier. Apart from the common quantifiers $\forall, \exists/(o(o\iota))$ that operate on a set of individuals (returning **T** iff this set is the whole universe ($\forall$)/non-empty ($\exists$), respectively), we use quantifiers of another type, namely the restricted quantifiers $Some, All/((o(o\iota))(o\iota))$, which were introduced in Section 1.4.3.

To recapitulate, *Some* is the function that associates an argument (a set *S*) with the set of all those sets sharing a non-empty intersection with *S*. *All* is the function that associates an argument (a set *S*) with the set of all those sets containing *S* as a subset. For instance, the sentence 'Some students are happy' is analyzed as ($Student$, $Happy/(o\iota)_{\tau\omega}$)

$$\lambda w \lambda t\ [[{}^0Some\ {}^0Student_{wt}]\ {}^0Happy_{wt}].$$

Similarly, the sentence 'All students are happy' is analyzed as

$$\lambda w \lambda t\ [[{}^0All\ {}^0Student_{wt}]\ {}^0Happy_{wt}].$$

Back to the car washer. We first analyze the embedded clauses of (D_1), (D_2), namely:

(E_1): '*he* washes all of *them*'

(E_2): '*he* washes some of *them*'.

The anaphoric pronoun '*them*' refers here to a *set of individuals*, *viz*. the set of new cars that a man owns. Thus we use the variable $them \to (o\iota)$ as the meaning of 'them'. The analyses of (E_1), (E_2) are:

$$(E_1') \quad \lambda w \lambda t\ [[{}^0All\ them]\ \lambda it\ [\lambda w \lambda t\ [{}^0Wash_{wt}\ he\ it]]_{wt}],$$

$$(E_2') \quad \lambda w \lambda t\ [[{}^0Some\ them]\ \lambda it\ [\lambda w \lambda t\ [{}^0Wash_{wt}\ he\ it]]_{wt}]$$

or, if β_i-reduced,

(E_1'') $\lambda w\lambda t\ [[{}^0All\ them]\ \lambda it\ [{}^0Wash_{wt}\ he\ it]]$

(E_2'') $\lambda w\lambda t\ [[{}^0Some\ them]\ \lambda it\ [{}^0Wash_{wt}\ he\ it]]$.

Now we need to substitute a construction of the set of new cars owned by the man for the variable *them.* Moreover, we have to substitute the variable x ('anybody') for the variable *he* ('*who* washes'), and then the pre-processed construction must undergo Double Execution. Finally, the so v-constructed proposition must undergo intensional descent to a truth-value in order to obtain the second argument for the connective $\supset$. To prevent collision of variables, we rename the internal variables w, t.

The analysis of (D_1):

(D_1^A) $\lambda w\lambda t\ [{}^0\forall\lambda x\ [[[{}^0Man_{wt}\ x] \wedge [{}^0\exists\lambda y\ [[{}^0NC_{wt}\ y] \wedge [{}^0Own_{wt}\ x\ y]]]] \supset$
${}^2[{}^0Sub\ {}^0[\lambda y\ [[{}^0NC_{wt}\ y] \wedge [{}^0Own_{wt}\ x\ y]]]\ {}^0them\ [{}^0Sub\ {}^0x\ {}^0he$
${}^0[\lambda w'\lambda t'\ [[{}^0All\ them]\ \lambda it\ [{}^0Wash_{w't'}\ he\ it]]]]]_{wt}]]$.

Gloss: 'For every man x, if the man x owns some new cars then each of them [i.e., the new cars owned] is such that he [i.e., the man x] washes it.'

This construction can be viewed as the most adequate analysis of (D_1), because it meets Russell's requirement of an indefinite description in the antecedent, while the scope of $\exists$ does not exceed the antecedent. Now (D_1^A) is equivalent to the construction that would be obtained after pre-processing (i.e., execution of the respective substitutions):

($D_1^{A'}$) $\lambda w\lambda t\ [{}^0\forall\lambda x\ [[[{}^0Man_{wt}\ x] \wedge [{}^0\exists y\ [[{}^0NC_{wt}\ y] \wedge [{}^0Own_{wt}\ x\ y]]]] \supset$
$[[{}^0All\ [\lambda y\ [[{}^0NC_{wt}\ y] \wedge {}^0Own_{wt}\ x\ y]]]]\ \lambda it\ [{}^0Wash_{wt}\ x\ it]]]]$.

Gloss: 'For every man x, if the man x owns some new cars then each of these new cars is such that x washes it.'

The second possible reading of (D) is now analyzed in a similar way using *Some* instead of *All*:

(D_2^A) $\lambda w\lambda t\ [{}^0\forall\lambda x\ [[[{}^0Man_{wt}\ x] \wedge [{}^0\exists\lambda y\ [[{}^0NC_{wt}\ y] \wedge [{}^0Own_{wt}\ x\ y]]]] \supset$
${}^2[{}^0Sub\ {}^0[\lambda y\ [[{}^0NC_{wt}\ y] \wedge [{}^0Own_{wt}\ x\ y]]]\ {}^0them\ [{}^0Sub\ {}^0x\ {}^0he$
${}^0[\lambda w'\lambda t'\ [[{}^0Some\ them]\ \lambda it\ [{}^0Wash_{w't'}\ he\ it]]]]]_{wt}]]$.

Gloss: 'For every man x, if the man owns some new cars then some of them [i.e., the new cars owned] is such that he [i.e., the man x] washes it.'

(D_2^A) is also equivalent to the construction that would be obtained after pre-processing:

$$(D_2^{A'}) \quad \lambda w \lambda t\, [^0\forall \lambda x\, \big[[[^0Man_{wt}\, x] \wedge \big[^0\exists y\, [[^0NC_{wt}\, y] \wedge [^0Own_{wt}\, x\, y]]]] \supset [[^0Some\, [\lambda y\, [[^0NC_{wt}\, y] \wedge [^0Own_{wt}\, x\, y]]]\big]\, \lambda it\, [^0Wash_{wt}\, x\, it]]]].$$

As we pointed out above, it is not clear how to exactly understand (D). We thus offered several analyses to disambiguate it. Whether these readings are the only possible ones is not for us to decide. In our opinion the reading (D_1) is more plausible, and Neale considers only this one. However, our method makes it possible to easily analyse particular variants of donkey sentences like '… none of them …', '… most of them…', and suchlike.

It might be objected, however, that in the interest of disambiguation we actually analysed two variants of the original sentence (D). Therefore we are going now to supply a deeper analysis of (D). Gabriel Sandu (1997) formulates two principles that every compositional procedure for analysing natural language sentences should obey:

(a) there is a one-to-one mapping of the surface structure of a sentence of (a fragment of) English into its logical form which preserves the left-to-right ordering of the logical constants;
(b) the mapping preserves the nature of the lexical properties of the logical constants, in the sense that an indefinite is translated by an existential quantifier, etc.

Evidently, our analyses (D_1^A) and (D_2^A) obey these principles with respect to the glossed variants, but not with respect to the original sentence (D):

(D) 'If a man has got a new car then *he* often washes *it*.'

Regardless of the disambiguation concerning some/all new cars being washed, principle (b) is violated because 'a man' is analysed as 'every man'. In this respect the analyses (D_1^A), (D_2^A) deviate as much from the above principles as does an analysis couched in standard first-order logic:

$$\forall x \forall y\, ((Man(x) \wedge NC(y) \wedge Own(x, y)) \supset Wash(x, y))).$$

Whereas it is generally admitted that traditional first-order predicate logic is not a satisfactory tool for the analysis of natural-language sentences, dynamic predicate logic (DPL) is considered superior to other competing first-order theories of discourse semantics.

While referring for details to Kozen and Tiuryn (1990) and Sandu (1997), we briefly summarise how DPL analyses donkey sentences. From the syntactic point of view, DPL is a first-order predicate logic. The basic difference between the two concerns the semantics, in particular the scope of the existential quantifier and binding conventions. DPL is often characterised as a logic of programmes, for the interpretation of a DPL formula is a programme. Thus it might seem as though

DPL embedded a procedural semantics as found in TIL or Moschovakis. However, a DPL programme is understood as a set of pairs of assignments in a model *M*, where an assignment is a function from the set of variables to the universe of *M*. The model *M* is construed as the set of all the input/output pairs of the states of a computation. A formula is interpreted as a set of pairs of assignments; that is, as a programme. Therefore, the semantics of DPL is an enhanced version of the denotational semantics of modal logics, where the role of Kripke-like possible worlds is played by assignment functions. Roughly speaking, a pair $\langle i, j \rangle$ satisfies a formula φ if and only if the evaluation of φ with respect to the input state i results in the output state j.

Atomic formulae and formulae composed of negation, disjunction, implication or universal quantification are called 'tests'. When evaluated with the input assignment i, they only examine whether i satisfies the condition specified by the formula and, if so, do not change the assignment, and otherwise reject it. Existentially quantified formulas and conjunctions have a non-standard interpretation, since they pass on assignments of variables and their semantic bindings. The 'conjunction' of the programmes φ and ψ is not a commutative operation, but a sequence of programmes (i.e., something akin to progressive conjunction). Similarly, a formula '$(\varphi \wedge \psi)$' is interpreted as a sequence of programmes: φ, when evaluated on an initial assignment g, returns an output assignment h that serves as an input for ψ yielding an output assignment k. Similarly, a formula '$\exists x P(x)$' yields an output assignment $h(x)$ that may serve as an input assignment for a succeeding formula. Thus, as Sandu says (1997, p. 150), a formula '$\exists x P(x) \wedge Q(x)$' is interpreted, or rather 'computed', as follows:

$$||\exists x P(x) \wedge Q(x)|| = \{(g,h) \mid h[x]g \wedge h(x) \in F(P) \wedge h(x) \in F(Q)\},$$

where $h[x]g$ is an assignment which differs from the assignment g at most with respect to the value it assigns to x, while F is the interpretation function that assigns to the non-logical symbols of a formula the respective denotation in the model *M*.

The occurrence of x in the second conjunct '$Q(x)$' is thus 'syntactically free' and at the same time 'semantically bound'. The DPL approach to the problem of anaphora makes use of just this kind of non-standard binding. Thus the pair of sentences

'A man is walking. He whistles.'

receives in DPL the logical form

$$\exists x\ (Man(x) \wedge Walk(x)) \wedge Whistle(x).$$

The last occurrence of x, though syntactically free, is semantically bound by $\exists$. Similarly, the donkey sentence (D) has the DPL logical form

$$\exists x \exists y\ (Man(x) \wedge NC(y) \wedge Own(x, y)) \supset Wash(x, y).$$

Unfortunately, since non-standard binding applies only to DPL conjunction and existential quantification, this approach fails to generalize. In particular, it does not work for an anaphor whose antecedent contains functionally dependent quantifiers. Sandu (1997, p. 151) adduces the example

‘Every player chooses a pawn. *He* puts *it* on square one.’

The DPL logical form constructed as above would be

$$\forall x\,[Player(x) \supset \exists y\,[Pawn(y) \wedge Choose(x, y)]] \wedge Put(x, y, a).$$

But since the general quantifier does not pass on binding, the last occurrences of x and y are semantically free. Therefore, it is said that the DPL analysis has to be as the standard one in

$$\forall x\,[Player(x) \supset \exists y\,[Pawn(y) \wedge Choose(x, y) \wedge Put(x, y, a)]].$$

One pressing question is whether the anaphoric pronouns should be, in general, syntactically/semantically bound, and if so, another pressing question is whether this is to be in a standard or non-standard way. DPL does not provide an answer. But even if we put this fundamental question aside,

> The main question of anaphora is not, in our opinion, how to represent in the symbolism of some logic the anaphoric relation between a pronoun and its head, but to formulate general principles predicting when an anaphorical interpretation of a pronoun is possible and when it is not (Ibid., pp. 151–2).

Sandu further argues that

> One of the praised merits of DPL is that it preserves compositionality. In the game-theoretical tradition, compositionality is not a desired outcome. Hintikka (1991) has argued that to try to maintain compositionality is merely an attempt to enforce a paradigm which has already proved too narrow. The latest developments in GTS have led to a logic (i.e. the independence-friendly logic) which is non-compositional. The key idea on which this logic is based is the idea of informational independence, which *ipso facto* involves a violation of compositionality. For if a quantifier or a connective is independent of another, its interpretation depends on the latter one, which is located further out in the sentence in question, hence violating compositionality (Ibid., p. 152).

However, as we consider compositionality not only desirable but adamantly non-negotiable, we are not going to dispute the necessity of this principle. Suffice it to say that, of course, compositionality is ‘too narrow’ if we restrict ourselves to a first-order approach, or to an approach close to the first-order one, only slightly exceeding it, like GTS does. Our priorities are different, so we preserve compositionality by applying a higher-order logic.

In Section 3.5.1 we argued that anaphoric pronouns are bound by Trivialization and processed semantically by substitution based on the meaning of the antecedent. Thus our answer to Sandu’s questions is: if a pronoun is anaphoric then the substitution method can always be applied (as we illustrated by examples). To put our arguments on a still more solid ground, we now propose analyses of the sentences adduced by Sandu and Hintikka as examples where the compositional treatment allegedly fails.

First, as mentioned above, a literal compositional analysis of the sentence (D)

(D) 'If a man has got a new car then *he* (often) washes *it*' is called for. Here is how.

The analysis of the antecedent 'A man has a new car' is as follows:

(NC) $\lambda w \lambda t\ [{}^0\exists \lambda xy\ [[{}^0Man_{wt}\, x] \wedge [{}^0NC_{wt}\, y] \wedge [{}^0Own_{wt}\, x\ y]]]$.

Types: $\exists/(o(o\iota\iota))$; Man, $NC/(o\iota)_{\tau\omega}$; $Own/(o\iota\iota)_{\tau\omega}$.

Gloss: 'The set of couples <*man*(*x*), *new_car*(*y*)> such that *x owns y* is non-empty.'

The consequent '*he* washes *it*' expresses the open construction

$$\lambda w \lambda t\ [{}^0Wash_{wt}\ he\ it].$$

Types: $Wash/(o\iota\iota)_{\tau\omega}$; he, $it/*_1 \rightarrow \iota$.

The sentence (D) expresses that *if* the former is true, *then all* the pairs <*he*, *it*> which belong to the set of pairs mentioned by the former are such that *he washes it*. Using a variable $pairs/*_1 \rightarrow (o\iota\iota)$, and a quantifier $All^p/((o(o\iota\iota))(o\iota\iota))$, we have:

$$\lambda w \lambda t\ [[{}^0All^p\ pairs]\ \lambda he\ it\ [{}^0Wash_{wt}\ he\ it]].$$

The problem now consists in how to Compose the two constructions so as to construct the proposition denoted by (D). In order to obey the Parmenides principle, we must apply implication. To ensure that the pairs <*he*, *it*> belong to the respective set of *pairs* we need to apply the substitution method. Hence we substitute the construction of the set of pairs constructed by the Closure of (NC) for the variable *pairs*:

(D′) $\lambda w \lambda t\ [{}^0\exists \lambda xy\ [[{}^0Man_{wt}\, x] \wedge [{}^0NC_{wt}\, y] \wedge [{}^0Own_{wt}\, x\ y]] \supset$
${}^2[{}^0Sub\ {}^0[\lambda xy\ [[{}^0Man_{wt}\, x] \wedge [{}^0NC_{wt}\, y] \wedge [{}^0Own_{wt}\, x\ y]]]\ {}^0pairs$
${}^0[\lambda w \lambda t\ [{}^0All^p\ pairs]\ \lambda he\ it\ [{}^0Wash_{wt}\ he\ it]]_{wt}]]$.

The analysis can be simplified by removing the redundant η-expansion:

$$\lambda he\ it\ [{}^0Wash_{wt}\ he\ it] = Wash_{wt}$$

(D″) $\lambda w\lambda t\ [[{}^0\exists\lambda xy\ [[{}^0Man_{wt}\,x] \wedge [{}^0NC_{wt}\,y] \wedge [{}^0Own_{wt}\,x\ y]]] \supset$
$^2[{}^0Sub\ {}^0[\lambda xy\ [[{}^0Man_{wt}\,x] \wedge [{}^0NC_{wt}\,y] \wedge [{}^0Own_{wt}\,x\ y]]]\ {}^0pairs$
$^0[\lambda w\lambda t\ [{}^0All^p\ pairs]\ {}^0Wash_{wt}]_{wt}]]$.

To illustrate the adequacy of our analysis, imagine that at a given $\langle w, t\rangle$ there are five men, M_1, …, M_5, and six cars, C_1, …, C_6, related to each other as follows (h – has, w – washes):

	C_1	C_2	C_3	C_4	C_5	C_6
M_1		$h + w$		$h + w$		
M_2	$h + w$				h	
M_3			w			
M_4			$h + w$			
M_5						

For this $\langle w, t\rangle$ the Closure $\lambda xy\ [[{}^0Man_{wt}\,x] \wedge [{}^0NC_{wt}\,y] \wedge [{}^0Own_{wt}\,x\ y]]$ v-constructs the set H of pairs:

$$H = \{<M_1, C_2>,\ <M_1, C_4>,\ <M_2, C_1>, <M_2, C_5>, <M_4, C_3>\}.$$

The result of substitution, Double Execution and application of intensional descent in the consequent construction is equivalent to

$$[{}^0All\ \lambda xy\ [[{}^0Man_{wt}\,x] \wedge [{}^0NC_{wt}\,y] \wedge [{}^0Own_{wt}\,x\ y]]]\ \lambda he\ it\ [{}^0Wash_{wt}\ he\ it]]].$$

The constituent $[{}^0All\ \lambda xy\ [[{}^0Man_{wt}\,x] \wedge [{}^0NC_{wt}\,y] \wedge [{}^0Own_{wt}\,x\ y]]]$ v-constructs the set of sets of pairs containing H as a subset. Let it be H':

$$H' = \{\{<M_1, C_2>,\ <M_1, C_4>,\ <M_2, C_1>, <M_2, C_5>, <M_4, C_3>\},$$
$$\{<M_1, C_2>,\ <M_1, C_4>,\ <M_2, C_1>, <M_2, C_5>, <M_4, C_3>, <M_3, C_3>, <M_5, a>\},$$
$$\{<M_1, C_2>,\ <M_1, C_4>,\ <M_2, C_1>, <M_2, C_5>, <M_4, C_3>, <M_3, b>, <M_5, a>\}, \ldots\}.$$

The set v-constructed at $\langle w, t\rangle$ by $Wash_{wt}$ is the set of pairs $<he, it>$ such that *he washes it*:

$$Wash_{wt} = \{<M_1, C_2>,\ <M_1, C_4>,\ <M_2, C_1>, <M_3, C_3>, <M_4, C_3>\}.$$

Now the construction of the consequent v-constructs **T** if $Wash_{wt}$ is an element of H', which is not the case here. This is due to the fact that man M_2 has car C_5, but does not wash it. If he did, then the set $Wash_{wt}$ would be

$$\{<M_1, C_2>,\ <M_1, C_4>,\ <M_2, C_1>, <M_3, C_3>, <M_4, C_3>, <M_2, C_5>\}$$

and this set would be an element of H'.

As is seen, (D′) is fully compositional. Our constituents operate on constructions of sets of pairs of individuals, as well as on constructions of particular individuals, which is impossible within a first-order theory. In this respect Hintikka is right when claiming that the compositional treatment does not work; it does not work within a first-order framework. But as soon as we have a powerful higher-order system like TIL at our disposal, there is no need to give up compositionality.

Note that (D′) provides at the same time an explication of DPL's mechanism of passing on binding. As mentioned above, in DPL an existentially quantified formula yields an output assignment that may serve as an input assignment for a succeeding formula. Indeed, the antecedent of (D′), $^0\exists\lambda xy\ [[^0Man_{wt}\ x] \wedge [^0NC_{wt}\ y] \wedge [^0Own_{wt}\ x\ y]]$, yields an output assignment for the consequent: the set of pairs constructed by $\lambda xy\ [[^0Man_{wt}\ x] \wedge [^0NC_{wt}\ y] \wedge [^0Own_{wt}\ x\ y]]$ is substituted for the variable *pair* into $[^0All^p\ pairs]\ \lambda he\ it\ [^0Wash_{wt}\ he\ it]]$.

Thus the variable *pairs* is bound in (D′), but the binding is of another kind. It is not directly bound by the existential quantifier. Formally, the variable is bound by Trivialization; semantically, it is bound by the condition that the pairs of individuals it *v*-constructs must be those which belong to the set mentioned by the antecedent clause.

The other example in Sandu (1997) was

(P) 'Every player chooses a pawn. *He* puts *it* on square one.'

Obviously, 'he' and 'it' anaphorically refer to 'any player' and 'a pawn', respectively. However, in

(W) 'Every man walks. He whistles.'

the pronoun 'he' cannot be interpreted as anaphorically referring to 'every man'. Sandu's worries concern the lack of a universal method to determine when an anaphoric pronoun refers to an antecedent, and when not. The only answer we can give is that (P) is *understood* as being equivalent to

'Every player chooses a pawn and (*he*) puts *it* on square one',

unlike (W). The sentence

'Every man walks and whistles'

has obviously a different meaning than (W).

The respective analyses are:

(P′) $\lambda w\lambda t\ [[^0Every\ ^0Player_{wt}]$
$\lambda x\exists y\ [[^0Choose_{wt}\ x\ y] \wedge [^0Pawn_{wt}\ y] \wedge [^0Put_{wt}\ x\ y\ ^0Sq_1]]]$

(W′) first sentence: $\lambda w\lambda t\ [[{}^0Every\ {}^0Man_{wt}]\ {}^0Walk_{wt}]$;
second sentence: $\lambda w\lambda t\ [{}^0Whistle_{wt}\ he]$;
a pragmatically incomplete meaning.

Types: *Every*/((ο(οι))(οι)) is a restricted quantifier; *Player*, *Man*, *Pawn*, *Walk*, *Whistle*/$(οι)_{τω}$; *Choose*/$(οιι)_{τω}$; *Put*/(οιιι); Sq_1/ι; *he*, *x*, *y*/$*_1$→ι.

Note that in (P′) we do not need *Sub*. Yet an adequate analysis of (P) should heed the anaphoric status of the pronouns '*he*' and '*it*'. By applying the same method as above, we obtain an analysis involving *Sub*. First, the second sentence of (P) expresses the open construction (*it*/$*_1$→ι)

$$\lambda w\lambda t\ [{}^0Put_{wt}\ he\ it\ {}^0Sq_1].$$

The first sentence expresses

$$\lambda w\lambda t\ [[{}^0Every\ {}^0Player_{wt}]\ \lambda x\exists y\ [[{}^0Choose_{wt}\ x\ y] \wedge [{}^0Pawn_{wt}\ y]]].$$

The gloss is that the application of the restricted quantifier *Every* to the set of *Players* at a given ⟨*w*, *t*⟩ gives as a result the set *S*/(ο(οι)) of supersets of $Player_{wt}$. Further, the application of *S* to the set *v*-constructed by $\lambda x\ \exists y\ [[{}^0Choose_{wt}\ x\ y] \wedge [{}^0Pawn_{wt}\ y]]$ returns **T** or **F**, according as the set of those who choose a pawn belongs to *S*.

Now, in order to analyze (P), *x* must be substituted for *he* and *y* for *it*:

(P″) $\lambda w\lambda t\ [[{}^0Every\ {}^0Player_{wt}]\ \lambda x\exists y\ [[{}^0Choose_{wt}\ x\ y] \wedge [{}^0Pawn_{wt}\ y] \wedge$
${}^2[{}^0Sub\ {}^0x\ {}^0he\ [{}^0Sub\ {}^0y\ {}^0it\ {}^0[\lambda w\lambda t\ [{}^0Put_{wt}\ he\ it\ {}^0Sq_1]]]]_{wt}]]$

The result of the Double Execution of the application of *Sub* is obtained as follows (=/(οοο), the identity of truth-values):

$${}^2[{}^0Sub\ {}^0x\ {}^0he\ [{}^0Sub\ {}^0y\ {}^0it\ {}^0[\lambda w\lambda t\ [{}^0Put_{wt}\ he\ it\ {}^0Sq_1]]]]_{wt} =$$
$${}^2[{}^0[\lambda w\lambda t\ [{}^0Put_{wt}\ x\ y\ {}^0Sq_1]]]_{wt} = [\lambda w\lambda t\ [{}^0Put_{wt}\ x\ y\ {}^0Sq_1]]]_{wt} = [{}^0Put_{wt}\ x\ y\ {}^0Sq_1].$$

Thus the analysis (P′) is equivalent to (P″). The literal analysis of the disambiguated variant of the sentence (P) is (P″).

3.5.3 Dynamic discourse

In this section we outline a method for computing the complete meaning of anaphoric sentences. This is a method for implementing the substitution of an appropriate antecedent to accompany an anaphoric reference. Our method is similar to the one applied in general by Hans Kamp's Discourse Representation Theory

(DRT).[65] 'DRT' is an umbrella term for a collection of logical and computational linguistic methods developed for a dynamic interpretation of natural language, where each sentence is interpreted within a certain discourse, which is a sequence of sentences uttered by the same speaker. Interpretation conditions are given *via* instructions for updating the discourse representation. DPL, as described briefly above, is a logic belonging to this group of theories.[66] DRT as presented in Kamp (1981) addresses in particular the problem of anaphoric links crossing the sentence boundary. It is a first-order theory, and it is provable that the expressive power of the DRT language with negation is the same as that of first-order predicate logic.[67] Thus, actually only expressions denoting individuals (indefinite or definite noun phrases) can introduce so-called discourse referents, which are free variables that are updated when interpreting the discourse. Anaphoric pronouns are also represented by free variables linked to appropriate antecedent variables. There are various extensions of the basic theory which are now more or less assimilated to the existing formalism, in particular treatments of plurality and presupposition. For instance, the system of Brasoveanu (2007a, 2007b) deals with plural discourse reference to a quantificational dependency between sets of objects. The system is based on classical type logic that extends the compositional DRT of Muskens (1996). In principle, our approach to dynamic discourse representation is similar to that of Brasoveanu and Muskens.

Muskens proposes tackling explicit attitudes as attitudes to what he calls 'propositions', where a 'proposition' is a primitive entity individuated in a finer way than by co-entailment. Thus more 'propositions' can identify the same set of possible worlds. However, there is no hint of *what* kind of entity a 'proposition' is. Muskens draws upon Thomason's primitive type p, whose elements are hyperpropositions. Thomason (1980) defines the granularity of p-objects negatively (as being finer than logical equivalence), and he says nothing about the substance of p-objects.[68] Thus introducing hyperpropositions as primitives is to acknowledge the very need for entities with certain properties, but the theory is barred from saying much at all about them. TIL, unlike Thomason, has a substantial philosophical theory to tell in terms of hyperintensions as procedures, and this theory has, furthermore, been worked out in great technical detail in terms of TIL constructions. Moreover, it is obvious that co-entailment would be too crude a criterion for hyperpropositions, so we agree with Muskens on that point.

Since our semantics is procedural, hence hyperintensional and higher-order, not only individuals, but entities of any type, like properties of individuals, propositions, relations-in-intension, and even constructions (i.e., meanings of antecedent expressions), can be linked to anaphoric variables. Moreover, the thoroughgoing

[65] For details, see Kamp (1981) and Kamp and Reyle (1993).

[66] See also Grenendijk and Stockhof (1991).

[67] See Eijck (2006, p. 666).

[68] See Jespersen (2010) for comments on Thomason's p.

typing of the universe of TIL makes it possible to *determine the respective type-theoretically appropriate antecedent*.

The specification of the implementation algorithm proposed here is imperative.[69] Similarly as in DRT, we update the list of potential antecedents, or rather the constructions expressed by them, in order to substitute the type-theoretically appropriate entities for anaphoric variables whenever needed. For each type, *viz.* ι, μ, $(o\iota)_{\tau\omega}$, $o_{\tau\omega}$, $(o\iota(o\iota)_{\tau\omega})_{\tau\omega}$, $(o\iota\iota)_{\tau\omega}$, $*_n$, etc., a list of *discourse referents* is formed. These discourse referents are free variables which serve a dual purpose. First, similarly as the variables of an imperative programming language, discourse referents function as memory cells to which a program stores objects in order to temporarily remember them. Thus each closed constituent of the meaning of a message becomes a temporal value of a type-theoretically appropriate discourse-referent variable. The method substitutes these values for anaphoric variables to complete the meanings of anaphoric clauses. Here our substitution method is applied so that discourse-referent variables serve their second purpose, *viz.* as ordinary constituents of the Composition [^{0}*Sub* …]. The completed closed construction becomes in turn a new value of a discourse-referent variable of an appropriate type. In this way the discourse variables are gradually updated.

We now illustrate the method by a simple dialogue between three agents, *Adam*, *Berta* and *Cecil*. The agents communicate by exchanging messages of various kinds. Basic kinds are 'inform', 'query', 'reply' and 'order'. The *content* of a message is a sentence that is analysed using TIL and pre-processed by the substitution method. We use the sign ':=' to indicate the type of entities the constructions of which are being assigned to a discourse referent variable by the algorithm. From the logical point of view, these variables are of type $*_n$ and *v*-construct constructions of entities of the indicated type. For instance, the discourse-variable *ind* serves to keep track of individuals that receive mention in the dialogue. Thus we should write '$ind/*_n \rightarrow *_{n\text{-}1}$; $^2ind \rightarrow \iota$'. Instead we write '$ind{:=}\iota$'. If the algorithm assigns to *ind* the Trivialization ^{0}*Berta*, then *ind* *v*-constructs ^{0}*Berta*, where *Berta*/ι. The list of discourse-referent variables used in the dialogue is this:

- $ind{:=}\iota$, to keep track of individuals;
- $loc{:=}\mu$, to keep track of locations of the type μ;
- $pred{:=}(o\iota)_{\tau\omega}$, $prof{:=}(o\iota)_{\tau\omega}$, to keep track of individual properties; the former keeps track of properties denoted by simple predicates, the latter of properties denoted by complex predicates;
- $rel_1{:=}(o\iota(o\iota)_{\tau\omega})_{\tau\omega}$, to keep track of relations-in-intension of an individual to a property of individuals;
- $rel_2{:=}(o\iota\mu)_{\tau\omega}$, to keep track of relations-in-intension of an individual to a location;
- $rel_3{:=}(o\iota o_{\tau\omega})_{\tau\omega}$, to keep track of relations-in-intension of an individual to a proposition;
- $prop{:=}o_{\tau\omega}$, to keep track of propositions;
- $constr{:=}*_n$, to keep track of constructions.

[69] The algorithm was first proposed in Křetínský (2007).

Adam to Cecil: 'Berta is coming. ***She*** is looking for a parking space'.
'Inform' message content (first sentence):

$$\lambda w \lambda t\ [{}^0Coming_{wt}\ {}^0Berta];$$

(Relevant) discourse variables updates:

$$ind{:=}{}^0Berta;\ pred{:=}{}^0Coming;\ prop{:=}\ \lambda w \lambda t\ [{}^0Coming_{wt}\ {}^0Berta];$$

'Inform' message content (second sentence):

$$\lambda w \lambda t\ {}^2[{}^0Sub\ ind\ {}^0she\ {}^0[{}^0Looking\_for_{wt}\ she\ {}^0Park\_Space]] \Rightarrow$$
(is transformed into)
$$\lambda w \lambda t\ [{}^0Looking\_for_{wt}\ {}^0Berta\ {}^0Park\_Space].$$

(Relevant) discourse variables updates:

$$rel_1{:=}\ {}^0Looking\_for;\ pred{:=}{}^0Park\_Space;$$
$$prop{:=}\ \lambda w \lambda t\ [{}^0Looking\_for_{wt}\ {}^0Berta\ {}^0Park\_Space];$$
$$prof{:=}\ \lambda w \lambda t\ \lambda x\ [{}^0Looking\_for_{wt}\ x\ {}^0Park\_Space];$$

Cecil to Adam: '***So*** am I.'
'Inform' message content:

$$\lambda w \lambda t\ {}^2[{}^0Sub\ prof\ \ {}^0so\ {}^0[so_{wt}\ {}^0Cecil]] \Rightarrow$$
$$\lambda w \lambda t\ [{}^0Looking\_for_{wt}\ {}^0Cecil\ {}^0Park\_Space]$$

(Relevant) discourse variables updates:

$$ind{:=}{}^0Cecil;$$

Adam to both: 'There is a car park with vacant slots at P_1'.
'Inform' message content:

$$\lambda w \lambda t \exists x\ \ [[[{}^0Vac\ {}^0Car\_Park]_{wt}\ x] \wedge [{}^0At_{wt}\ x\ {}^0P_1]]$$

(Relevant) discourse variables updates:

$$loc{:=}{}^0P_1;\ pred{:=}[{}^0Vac\ {}^0Car\_Park];$$
$$prop{:=}\ \lambda w \lambda t\ [\exists x\ [[{}^0Vac\ {}^0Car\_Park]_{wt}\ x] \wedge [{}^0At_{wt}\ x\ {}^0P_1]]$$

Cecil to Adam: 'I don't think ***so***. I have just been ***there***'.
'Inform' message content (first sentence):

$$\lambda w \lambda t\ [{}^2[{}^0Sub\ prop\ {}^0so\ {}^0[\neg[{}^0Think_{wt}\ {}^0Cecil\ so]]] \Rightarrow$$
$$\lambda w \lambda t\ \neg[{}^0Think_{wt}\ {}^0Cecil$$
$$[\lambda w \lambda t\ [\exists x\ [[{}^0Vac\ {}^0Car\_Park]_{wt}\ x] \wedge [{}^0At_{wt}\ x\ {}^0P_1]]]],$$

'Inform' message content (second sentence):

$$\lambda w \lambda t\ \exists t'[[t' \leq t] \wedge {}^2[{}^0Sub\ loc\ {}^0there\ {}^0[{}^0Been\_at_{wt'}\ {}^0Cecil\ there]]] \Rightarrow$$
$$\lambda w \lambda t\ \exists t'[[t' \leq t] \wedge [{}^0Been\_at_{wt'}\ {}^0Cecil\ {}^0P_1]].$$

Berta to Adam: 'What do you mean by 'car park with vacant slots'?'

'Query' message content:

$$\lambda w \lambda t\ [{}^0 Unrecognized_{wt}\ {}^0[{}^0 Vac\ {}^0 Car\_Park]]$$

(Relevant) discourse variables updates:

$$constr := {}^0[{}^0 Vac\ {}^0 Car\_Park]$$

Adam to Berta: 'A car park with vacant slots is a parking lot some of whose parking spaces are not occupied'.

'Reply' message content:

$$[{}^0 Refined_{wt}\ {}^0[{}^0 Vac\ {}^0 Car\_Park]] =$$
$${}^0[\lambda w \lambda t\ \lambda x\ [[{}^0 Car\_Park_{wt}\, x] \wedge \exists y\ [[{}^0 Part\_of_{wt}\ y\ x] \wedge \neg[{}^0 Occupied_{wt}\, y]]]]$$

And so on.

Note that our hyperintensional procedural semantics makes it possible to easily specify and implement *agents' learning* by experience. The sort of agents we are considering here learn not only empirical facts, but also new *concepts*. They come equipped with a minimal ontology of primitive concepts and in the course of their life cycle enrich their ontology with new compound concepts. This is done in particular by messaging and consulting fellow agents. If an agent *a* does not have a concept *C* in his or her ontology, then *a* sends a query message announcing that the concept *C* has not been recognized by *a*. The appropriate reply provides a concept *C*' serving as an explication of *C*. To arrive at explications we use two functions that have concepts as arguments, *viz.* $Unrecognized/(o*_n)_{\tau\omega}$ and $Refined/(*_n*_n)_{\tau\omega}$. The former is a property of concepts (of not being known by an agent), the latter is a function that dependently on worlds and times returns a concept *C*' which is an explication of the argument concept. Thus we need to mention concepts as arguments and values, which means that the content of these messages must be hyperintensional.

In our example, upon receiving Adam's reply, Berta learns the refined meaning of the predicate 'is a car park with vacant slots', i.e., she updates her ontology by the respective compound construction defining the property of being a car park some of whose parking spaces are still vacant.

Moreover, our method makes it possible to work with *multi-lingual ontologies*. The content of an agent's knowledge is not a piece of syntax, but its meaning. And since a construction is what synonymous expressions (even of different languages) have in common, agents behave in the same way independently of the language in which their knowledge and ontology is encoded. For instance, if we throw some Czech in, the underlying constructions are *identical*:

$${}^0[{}^0 Vac\ {}^0 Car\_Park] = {}^0[{}^0 Voln\acute{e}\ {}^0 Parkovi\check{s}t\check{e}].$$

Of course, improvements of the above method are possible. For instance, in the above dialogue, for each type we kept track only of the *last* type-theoretically appropriate entity that had been mentioned. If we wanted to take into account possible ambiguities of the anaphoric references, we might store into the discourse-representation file a list of variables for each type, so as to be able to spell out more meanings of an ambiguous sentence, and thus to contribute further to its disambiguation.

3.6 Questions and answers

In the previous section we adduced an example of a dialogue in order to illustrate our implementation method of dynamic discourse representation and pre-processing of anaphora. In the dialogue, there was a 'query message' the content of which was analysed in the same way as the content of a corresponding 'inform message'. A question arises here, though. Is it plausible to analyse interrogative sentences in the same way as declarative ones? In this section we provide an answer.

There are many *logics of questions* (*interrogative* or *erotetic logics*).[70] The question, however, is whether it is necessary to build up specific systems in which to semantically analyze interrogative sentences and call each of them a *logic*. TIL answers this question in the negative.[71] Our principal tenet is that

> Logic investigates logical objects and ways they can be constructed. Its findings apply regardless of what people *do* with those objects: whether they exploit them in asserting, desiring, commanding, or questioning. (Tichý, 1978b, p. 278, 2004, p. 298.)

To motivate this stance, consider the declarative sentence

(1) 'Bill walks.'

and the corresponding interrogative sentence

(2) 'Does Bill walk?'

Tichý argues that the syntactic difference between these sentences 'reflects no difference in the *logic* of the two sentences' (ibid., p. 275/p. 295).[72] Instead the

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[70] See the overview in Harrah (2002).

[71] Here we confine ourselves to setting out our general approach to the semantics of interrogative sentences. Some consequences of this approach can be found in Materna (1981) and Materna, Hajičová and Sgall (1987). It might well prove fruitful to compare ours to the approaches offered by, e.g., Belnap et al. (See Harrah, 2002).

[72] He quotes the general claim advanced by Fitch that '[W]e do not need a special 'logic of imperative statements', 'logic of performative statements', and so on, as logic over and beyond, or basically different from the standard logic of propositions.' See Fitch (1971, p. 40).

difference between (1) and (2) is to do with the *pragmatic* use made of them. Thus, (1) is used to *assert* that Bill walks, while (2) is used to *ask* whether Bill walks.

Borrowing the terms 'concern' and 'topic' from Leonard (the former used for the pragmatic, the latter for the semantic aspect),[73] Tichý claims that (1) and (2) have the same topic but not the same concern. Logic is interested exclusively in topics. In the above example the topic is the proposition constructed by

$$\lambda w \lambda t\ [{}^0Walk_{wt}\ {}^0Bill].$$

Types: $Walk/(o\iota)_{\tau\omega}$; $Bill/\iota$.

Groenendijk and Stokhof (1994) advocate the thesis that interrogatives have a semantics of their own and so do not share their semantics with, e.g., indicatives. The difference in semantics is the one between truth-conditional content and so-called answerhood (cf. ibid., pp. 30ff). Their stance is at odds with the one expounded in Tichý (1978b), which they categorize as being a 'reductionist view' (ibid., p. 19). They level an objection against what they argue to be Tichý's position, which runs as follows:

> Consider 'John knows that Bill walks' and 'John knows whether Bill walks'. If the embedded interrogative and the embedded indicative really have the same semantic value, then each of these sentences should have the same value, too. If Bill walks, and John knows this, we might say that that is indeed the case: both are true. But if Bill does not walk, and John knows this, then they differ in value: in that case the first sentence is false, whereas the second is true. Such a simple example suffices to show that there are semantic differences between interrogatives and indicatives, and that the semantic content of interrogatives needs to be accounted for. (Ibid., p. 19.)

Tichý's position, however, is that no custom-built *semantics* for interrogatives, and no special erotetic *logic*, is needed. Propositions wrapped inside interrogatives, as in 'Is *P* true?', will suffice. So whether what John knows is that Bill walks or whether Bill walks, the semantic value embedded in the indicative or the interrogative is of the same *kind*, namely a (hyper-) proposition. It cannot be the same (hyper-) proposition, obviously, and their respective truth-values may well differ, but while Tichý's view does qualify as 'reductionist', it is not true to say that it requires that the complements be 'the same semantic value'.

To amplify the point, 'knowing that' and 'knowing whether' denote *different* relations-in-intension, and are for this reason assigned different (hyper-) propositions. Whereas 'knowing that *P*' not only implies, but also *presupposes* that *P* should be true, it is not so with 'knowing whether'. *Contra* Groenendijk and Stokhof, if Bill does not walk then the proposition denoted by 'John knows that Bill walks' has *no* truth-value. If the sentence is false then it is true that John does not know that Bill walks, which entails that Bill does walk. Thus if John knows that

[73] See Leonard (1959).

Bill does *not* walk then John neither knows that Bill walks, nor does John not know that Bill walks.[74]

First, we deal with sentences containing empirical expressions. We define:

Definition 3.1 (*topic of an interrogative empirical sentence)* The *topic* of an interrogative empirical sentence *S* is the intension denoted by *S*. □

Remark. The topic is an intension whose value in the actual world and present time the questioner would like to know. Thus what is usually called a 'question' is, properly speaking, just the topic of an interrogative sentence.

In the case of Yes/No interrogative sentences, the topic is a proposition. Any other kind of (essentially '*wh-*') interrogative sentence is connected with another kind of topic. In general, the topic is indicated by the type of the subject of an admissible answer.[75] If the type of the subject is α, then the topic of the interrogative sentence is an intension of type $\alpha_{\tau\omega}$. Here is a survey of several kinds of '*wh-*' interrogative sentences.

A. *Who is ...* ?

'Who is the father of the Pope?'

The syntactic means, namely the phrase 'Who is' and the question mark, do not possess any semantic significance. They are pragmatic indicators, instructing us what to do with the topic. In this case the topic is not a proposition. Since the type of the expected answer is ι, the topic is an *individual office* of type $\iota_{\tau\omega}$. The questioner wants to know the value of the intension denoted by

'The father of the Pope',

which is the individual office constructed by

$$\lambda w\lambda t\ [{}^0Father\_of_{wt}\ {}^0Pope_{wt}].$$

Types: $Father\_of/(\iota\iota)_{\tau\omega}$; $Pope/\iota_{\tau\omega}$.

We can use the phrase 'the father of the Pope' indicatively, e.g., when answering the question, 'Who is your favourite person?', or interrogatively, i.e., when wishing to know who occupies the office. Grammatical means then indicate particular kinds of use (full stop in the former case, 'who is' and question mark in the latter case).

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74 See Section 5.1 for our analysis of 'knowing that' and 'knowing whether'.

75 Groenendijk and Stokhof (1994) correctly classify Tichý (1978b) as a 'categorial' theory of questions: 'Tichý prefers to identify the category of an interrogative with that of its characteristic answers.' (Ibid., p. 54.)

B. *Which ... are ...* ?

‘Which mountains are higher than Makalu?’

Since an admissible answer is a set of individuals, an $(o\iota)$-object, the topic is a *property of individuals*/$(o\iota)_{\tau\omega}$, namely *being a mountain higher than Makalu*, constructed by

$$\lambda w \lambda t\ \lambda x\ [[^0Mountain_{wt}\ x] \wedge [^0Higher\_than_{wt}\ x\ ^0Makalu]].$$

Types: $Mountain/(o\iota)_{\tau\omega}$; $Higher\_than/(o\iota\iota)_{\tau\omega}$; $Makalu/\iota$.

C. *... or...* ? (*Alternative questions*)

‘Is Charles a composer or a dancer?’

Such sentences are ambiguous. They can be construed either as Yes/No questions or alternative questions.[76] If the former, then ‘or’ denotes an inclusive disjunction; we say ‘Yes’ if at least one of the alternatives holds and ‘No’ otherwise.

Now we are interested in the case of alternative questions, *viz.* the questions involving exclusive disjunction. Here the topic is a little bit more complex. The questioner wants to know *which* of the alternative propositions is the case. Since admissible answers are ‘Charles is a composer’ or ‘Charles is a dancer’, both denoting $o_{\tau\omega}$-objects, the topic is now a *propositional office*/$(o_{\tau\omega})_{\tau\omega}$, constructed by

$$\lambda w \lambda t\ \iota' p\ [p_{wt} \wedge [[p = \lambda w \lambda t\ [^0Composer_{wt}\ ^0Ch]] \vee [p = \lambda w \lambda t\ [^0Dancer_{wt}\ ^0Ch]]]]].$$

Types: $p/*_1 \rightarrow o_{\tau\omega}$; $=/(oo_{\tau\omega}o_{\tau\omega})$; $Ch(arles)/\iota$; $Composer$, $Dancer/(o\iota)_{\tau\omega}$; $\iota'/(o_{\tau\omega}(oo_{\tau\omega}))$.

The topic cannot be a proposition; for this would mean that when answering the question in a correct way we would be saying either ‘Yes’ or ‘No’, because to answer the question correctly means to determine the value of the topic in the actual world at the present time. Instead a correct answer will be one of the sentences ‘Charles is a composer’, ‘Charles is a dancer’. ‘Charles is both a composer and a dancer’ is not an option, since the question is stipulated to be an *alternative* question requiring as an answer exactly one of the disjuncts and not their conjunction. Thus, the value of the topic in the actual world at the present time is a proposition. The sort of intension whose value at a world/time pair is a proposition is a propositional office/$(o_{\tau\omega})_{\tau\omega}$.

Other cases would be ‘Why…?’, ‘How…?’, ‘When…’, ‘How long…?’. Similar cases can be reformulated so that the character of the topic is made clear; for example, ‘What is the cause of ...?’, ‘What is the length of…?’.

Logical analysis of (empirical) interrogative sentences unveils constructions of the *topics* of the interrogative sentences. Since we are now dealing

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[76] The disambiguation can be realized on the phonetic level. The Yes/No case obtains if the pitch of the voice rises at the end and the alternative case if the pitch goes down.

with *empirical* questions, the interrogative sentences are empirical expressions and so it holds that

the topics of empirical interrogative sentences are non-constant intensions.[77]

Tichý introduced the terminological convention that the topic of an interrogative sentence is to be called a *question*, such that 'People's questions are ... propositions, individual offices, properties, and the like.' (Tichý 2004, p. 297.) Tichý concedes that this terminology may be objected to; for instance, we do not ask propositions but questions. But Tichý rebuts the objection by pointing out that one could then likewise insist that what one believes are *beliefs,* what one wishes are *wishes*, and what one conjectures are *conjectures,* not propositions. Yet what *a* believes to be the case, what *b* wishes to be the case, and what *c* conjectures to be the case may obviously be *one and the same* thing. So we propose the thesis that *questions are intensions* in the following particular sense: every intension may be used as the topic of an interrogative sentence. The notion of question is in this sense a *pragmatic* one.[78]

That the notion of interrogative sentence has to be distinguished from the notion of question is obvious. The interrogative sentences

'Who is the father of the Pope?'

and

'Wer ist der Vater des Papstes?'

are distinct, for sure, but the respective question is the same (viz. the topic constructed in our example above). Thus we can say that these two interrogative sentences share the same topic as well as the same meaning. To correctly translate an interrogative sentence *S* from one language *L* into another language *L′* means finding in *L′* an expression whose meaning constructs the same topic as does the meaning of *S* in *L* and to add, furthermore, the syntactic signals of the interrogative attitude in *L′*.[79]

A question can also be embedded in an indicative sentence. For instance, the above question can be embedded in

'Charles asked: Who is the father of the Pope?'

Then the topic of the question constructed by $\lambda w \lambda t\, [^0 Father\_of_{wt}\, ^0 Pope_{wt}]$ is an argument of $Asked/(o\iota\iota_{\tau\omega})_{\tau\omega}$, relating an individual to an individual office, and the analysis comes down to:

77 Recall that empirical expressions denote non-constant intensions.

78 See Materna et al. (1976, p. 177), '[T]his sphere of pragmatics, which deals with potential attitudes of potential language users and which directly manifests itself in the syntactic component of an ordered triple, is to be termed internal (pragmatic) indices. ... [I]n communicative situations, we have to introduce the notion of external pragmatics, characterizing the respective situation in which the given sentence has been uttered.'

79 For instance, if *L* is English and *L*' is Spanish, then the correct translation of '…?' is '¿…?'.

$$\lambda w \lambda t\ [{}^0Asked_{wt}\ {}^0Charles\ \lambda w \lambda t\ [{}^0Father\_of_{wt}\ {}^0Pope_{wt}]].$$

Interrogative sentences, just like other kinds of expressions of natural language, can have a pragmatically incomplete meaning. Then the analysis is an open construction *v*-constructing the topic. In case of indexicals their analysis contains—as is the case with other indexicals (see Section 3.4.1—free variables whose valuation is given by a situation of utterance. The answer then depends on the same situation. For example, the interrogative sentence 'Who is this man?' depends for its correct answer on some one particular man being singled out as the one about whom the speaker wishes to know who he is. The correct answer is going to be his name or a definite description identifying him as the occupant of an individual office. In the case of anaphoric reference to a discourse, the meaning is completed by substitution based on the meaning of the antecedent phrase, as described in Section 3.5.3. For instance, the two atomic sentences of the discourse,

'The richest man in the world came to Prague on Monday.'

'Where does *he* come from?'

express the following constructions:

$$\lambda w \lambda t\ [\lambda x\ [{}^0Past_t\ \lambda c\ \exists t'\ [[ct'] \wedge [{}^0Come\_to_{wt'}\ [x\ {}^0Prague]]]\ {}^0Monday]\ {}^0RichestMan_{wt}]$$

and

$$\lambda w \lambda t\ \iota x\ {}^2[{}^0Sub\ {}^{00}RichestMan_{wt}\ {}^0he\ {}^0[{}^0Come\_from_{wt}\ he\ x]].$$

Types: *Past*/$((o(o(o\tau))(o\tau))\tau)$; *c*/$*_1 \rightarrow (o\tau)$; *Come_to*/$(o\iota\iota)_{\tau\omega}$; *Prague*/$\iota$; *Monday*/$(o\tau)$; *RichestMan*/$\iota_{\tau\omega}$; *he*/$*_1 \rightarrow \iota$; *Come_from*/$(o\iota\iota)_{\tau\omega}$.[80]

As outlined above, questions and answers are type-theoretically interlocked, namely in the following fashion: interrogative sentences denote questions, while true answers cite the values of these questions at the given $\langle w, t\rangle$ of evaluation. If no particular such pair is mentioned, it is assumed that the intended pair is the actual world and the present moment.

Definition 3.2 (*complete answer*) Let *S* be an interrogative sentence whose topic—and so the respective question *Q*—is of type $\alpha_{\tau\omega}$. Then a *complete answer* to the question *Q* is an expression that cites an object of type α. □

Remark. We use the neutral verb 'to cite' on purpose. A complete answer has to guide us to the value of the respective intension in the actual world at the present moment. But in general we cannot say in which manner it will do so. With the

[80] For an analysis of the simple past tense, see Section 2.5.2.

exception of Yes/No questions, where we can *denote* the respective object ('Yes', 'No' being names of **T**, **F**, respectively), we have no means to do so. In general, the object that is the actual value of an intension is not accessible as the *denotation* of an expression (with the possible exception of proper names). Empirical expressions denote intensions, never their actual values, so we often use indexicals and rely upon pragmatic factors for identification. For example, a complete answer to the question

'Who is the head of al-Qaida?'

has to cite an individual; it can do so by saying

'Osama bin Laden'

or

'This one'.

In the latter case we rely upon the given situation to unequivocally fix an individual.

If the cited α-object is the value of the respective intension in the actual world at the present time, then we say that the answer is *right*, otherwise *wrong*. Some particular cases will justify this notion of complete answer.

The type of Yes/No questions is $o_{\tau\omega}$. Hence the type of the object cited by an answer is o. To cite such an object is tantamount to saying 'Yes' or 'No'.

If the type of the question is $\iota_{\tau\omega}$, then the type of the object cited by an answer is ι. Citing an individual qualifies as an answer to the question. If the type of the question is $(o\iota)_{\tau\omega}$, then the type of the object cited by an answer must be $(o\iota)$. Thus citing a class of individuals (in the above example *ad* (B), the class of mountains that are actually higher than Makalu) counts as an answer to the question. A different class of mountains would also be an answer, only not the right one. The type of alternative questions is $(o_{\tau\omega})_{\tau\omega}$. To answer such questions is to cite a proposition.

The notion of *incomplete answer* is easily derivable from the notion of complete answer. Let the type of the object cited by a complete answer be α. Then an incomplete answer will offer (in some way or other) a *class* of α-objects (different from a singleton). The answer is right only if this class contains the object cited by the right complete answer. Notice, however, that an incomplete answer to a Yes/No question is uninformative. Such a question must be answered by 'Yes' or 'No'.

Examples. First, let a complete (right or wrong) answer to the question 'Who is Charles's father?' be 'Abraham'. An incomplete answer will be, e.g., 'Balthazar or Abraham'. If the complete answer was right then the incomplete answer would be right as well, for the cited class contains Abraham. Second, if the question is a property of individuals, then offering more than one class of individuals amounts to offering an incomplete answer. Thus the following schematic answers must be distinguished. Let the class cited by the *right complete* answer to the question *Q* be {A, B, C, D}. Citing the class {A, B, C, D, E} is a *wrong complete* answer to *Q*. Offering the classes {A, B, C, D} *or* {A, B, C} *or* {A, B, C, D, E} is a *right* *incomplete* answer to *Q*. Offering {A, B, C} *or* {A, B, C, D, E} is a *wrong incomplete* answer to *Q*.[81]

The term ‘question’ is mostly used just in the sense of empirical question. Questions in logic and mathematics are oftentimes *examinatorial* questions and rather more like imperatives—‘Prove Fermat’s last theorem!’, ‘Define the De Morgan laws!’—and they are answered correctly if the ordered task is fulfilled.[82] The examiner is not trying to get to know what the De Morgan’s laws (etc.) are, for these he or she already knows. Instead the examiner wants to get to know whether the examined student knows them. So the examiner might ask the non-examinatorial question, ‘Do you know the De Morgan Laws?’

A non-examinatorial question concerning mathematical/logical objects is a *construction* (rather than an intension) which is an analysis of the respective interrogative sentence divested, as the case may be, of interrogative phrases like ‘which is’, ‘which are’, etc. The correct answer denotes the object (if any) constructed by this construction. So, for example, the question asked by means of the interrogative sentence ‘Which are the roots of the equation $8x^2 + 8x + 2 = 0$?’ is the construction

$$\lambda x\, [{}^0{=}\, [{}^0 Add\, [{}^0 Add\, [{}^0 Mult\, {}^0 8\, [{}^0 Power\_of\, x]]\, [{}^0 Mult\, {}^0 8\, x]]\, {}^0 2]\, {}^0 0].$$

Types: *Add*, *Mult*/$(\tau\tau\tau)$: the functions of adding and multiplying, respectively; *Power_of*/$(\tau\tau)$; 0,2,8/τ; =/$(o\tau\tau)$; $x \to \tau$.

The correct answer is the singleton {– ½}.

[81] To *react* to a question does not automatically mean to *answer* the question. Reactions which do not satisfy the definitions of complete/incomplete answers may be called *replies* or *responses*. For instance, punishing silence or a ‘I am not going to dignify that question with an answer’ would be replies and not answers. In everyday transaction we may well succeed in converting a reply into an answer, given a sufficient supply of background information and suchlike. This is still not to say that, e.g., silence on behalf of the one who was asked the question qualifies as an answer; the audience must still make explicit to themselves what the answer implied by the silence is, if indeed there is an answer to be teased out.

[82] See also Materna (1981).