Chapter 58
On the Numerical Prediction of Stability in Thin Wall Machining

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Abstract In this chapter, the numerical prediction of stability margin in thin wall machining is introduced. The Nyquist criterion is applied to the stability model presented by Adetoro, while a newly discovered damping prediction approach is presented, which when applied to the FEM and Fourier approach presented by Adetoro, would allow the prediction of stability margins without the need for experimentally extracted damping parameters.

Keywords Nyquist criterion · FEA · frequency response function · transfer function · damping ratio · damping matrix

58.1 Introduction

In aerospace, the manufacturing process is progressively limiting the use of joints through the manufacturing of structures as one monolithic piece. Machining is a very common operation in manufacturing, due to its versatility and its high material removal rate in producing parts of desired dimensions. Aircraft wing sections, fuselage sections, turbine blades and jet engine compressors, are all typical parts with sections produced from machined aluminium or titanium blocks. With environmental concerns and the general demand for higher efficiency, weight requirements compel the design of much thinner sections. In order to maintain the quality of the machined parts there is usually a dimensional tolerance, which the machined parts

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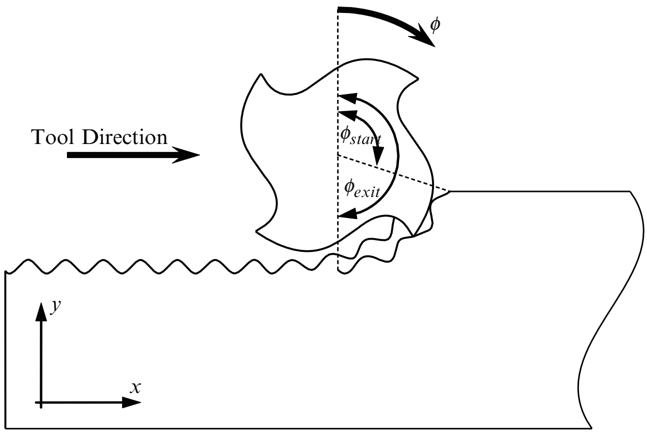

Fig. 58.1 Dynamic milling model

have to satisfy. To enforce this, it is a general practice for machined parts to undergo inspection before they are certified for use. While parts that fail this inspection are either scrapped or subjected to many hours of manual labour to remove the bad surface finish.

In milling the workpiece is fed past a rotating tool with one or more teeth (Fig. 58.1), which makes it possible to attain very high 'Material Removal Rate' (MRR). The tooth/teeth remove the material from the workpiece in the form of small individual 'chips'. The study into the numerical simulations of the machining of thin-walled sections [1] is the focus of this chapter. In thin wall machining, the cutting conditions used are very important and must be chosen with care as they directly influence the cutting forces. The cutting forces cause structural vibrations in the workpiece, tool and spindle. These vibrations can be classified as free vibrations (occur after an external energy source is removed), forced vibrations (occur during the presence of an external energy source) and self-excited vibrations [2]. The self-excited vibration has its source from the inherent structural dynamics of the machine tool-workpiece and feedback responses through the undulations left on the machined surface. The optimum cutting case is when the undulations left on the machined surface are in phase with the undulations from previous tooth pass. The worst case however is when the phase angle between the two undulations is out of phase by 90°. This leads to the phenomena known as "regenerative chatter" or simply "chatter". Chatter is usually characterised by a very bad surface finish and a drastic increase in both cutting forces and vibrations.

The prediction of stable conditions in the form of charts started when, Tobias [3] and Tlusty [4] simultaneously made the remarkable discovery that the main source of self-excited regenerative vibration/chatter was not related to the presence of negative process damping as was previously assumed. However, it is related to the structural dynamics of the machine tool-workpiece system and the feedback response between subsequent cuts. Their model is only applicable to orthogonal metal cutting where the directional dynamic milling coefficients are constant and not periodic. Other studies on the stability of orthogonal metal cutting were reported by Merritt [5].

Altintas and Budak [6–8] later proposed an analytic approach to predict stability margin. Perhaps, the first analytical approach in which the zeroth order term in the Fourier series expansion of the time varying coefficients was adopted. The analytical model was later extended to include three directions by Altintas [9], where the axial immersion angle was assumed to be constant. Except for flat end mills however, the axial immersion angle, is a function of the axial depth of cut. Campa et al. [10] later proposed an averaging approach to calculating the axial immersion angle in order to solve the stability model analytically. However, the axial immersion angle was still assumed to be a constant. This is the main analytical approach generally used in predicting stable cutting conditions in machining [11, 12]. The model has recently been improved by Adetoro et al. [13, 14] to include the nonlinearity of the cutting force coefficients, axial immersion angle and system dynamics.

The accuracy of the predicted stable region relies on the dynamic parameters identified at the cutter-workpiece contact zone. The cutter and workpiece dynamics consist of its damping, stiffness and mass parameters. Damping is the dissipative factor present in every real-life system/structure. Unlike the well developed mass/inertia and stiffness forces, the damping forces are at present extracted through experiments known as modal testing/analysis. This is because the physics behind the damping forces are not fully understood especially for a wide range of systems. It is however always desirable for an analyst to be able to predict the damping ratio (either analytically or numerically) for any given geometry without having to rely solely on experimental results.

A significant contribution at the early development of modal analysis was proportional damping model. It was first proposed by Lord Rayleigh in 1878, where he indicated that if the viscous damping matrix is proportional to mass and stiffness matrices (the damping forces are proportional to the kinetic and potential energies of the system) then it can be expressed [15] as,

$$\mathbf{C} = \alpha_0 \mathbf{M} + \alpha_1 \mathbf{K}, \tag{58.1}$$

where α_0 and α_1 are real positive constants. The model is termed 'Rayleigh damping' or 'classical damping'. The significance of this model is that the damped system would have the same mode shapes compared to its undamped counterpart, thus the system is said to possess 'classical normal modes'.

The equation of motion for a multi degree of freedom (MDoF) system can be expressed as,

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}, \tag{58.2}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the viscous damping matrix, $\mathbf{K}$ is the stiffness matrix, $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$, $\mathbf{x}$ and $\mathbf{F}$ are the acceleration, velocity, displacement and excitation force vectors respectively.

In 1960, Caughey and O'Kelly [16] provided a generalization of Rayleigh's condition for discrete systems in form of the series,

$$\mathbf{C} = \mathbf{M} \sum_{u=0}^{L-1} \alpha_u \left(\mathbf{M}^{-1}\mathbf{K}\right)^u, \tag{58.3}$$

where L is the number of identified modes used in the curve fitting, α_u are real positive constants obtained through using experimentally identified damping parameters. The Rayleigh damping model is the first two series of the expansion.

Thus this chapter is structured as follows; the Nyquist Criterion is applied to the dynamic milling model; a newly discovered approach to predicting the structural damping parameters is presented.

58.2 Application of the Nyquist Criterion

The dynamic milling model is explained in the paper by Budak and Altintas [6] and summarised by Adetoro et al. [1, 17]; while a new 3-D mode was developed in the paper by Altintas [9], with improvements in the paper by Adetoro et al. [13]. For purposes of applying the Nyquist stability criterion the dynamic milling model may be re-stated in terms of the dynamic forces acting the machine tool in the form,

$$\mathbf{F}(t) = (1/2)aK_t\mathbf{A}\mathbf{G}(D)\left(\mathbf{F}(t) - \mathbf{F}(t-T)\right) + \mathbf{F}_c(t), \tag{58.4}$$

or as,

$$\mathbf{F}(t) = (1/2)\,aK_t\mathbf{A}\mathbf{G}(D)\left(\mathbf{I} - \exp\left(-DT\right)\right)\mathbf{F}(t) + \mathbf{F}_c(t), \tag{58.5}$$

where $\mathbf{F}(t)$ is the dynamic force vector acting on the cutting tool, a is the axial depth of cut, K_t is the tangential cutting force coefficient, $\mathbf{A}(t)$, is the immersion dependent directional cutting force coefficient matrix, which could in general be a periodic function of time satisfying the condition, $\mathbf{A}(t+T) = \mathbf{A}(t)$, $\mathbf{G}(D)$ is the direction dependent and frequency dependent transfer function relating the static and dynamic cutting force vector, T is the inter-tooth time of passage, $\mathbf{F}_c(t)$ is the control force vector acting on the cutting tool. Equation (58.5) is expressed in terms of the force vector. Further information on the formulation of the force equations for a generic case, including the application of Floquet theory to machine tool chatter may be found in the paper by Minis and Yanushevsky [18].

In many applications to machine tool chatter the immersion dependent directional cutting force coefficient matrix $\mathbf{A}$ may be approximated by a constant coefficient matrix. The problem can be reformulated as a closed loop control problem and the control law defining the control force vector acting on the cutting tool, $\mathbf{F}_c\,(t)$ may be synthesized using standard techniques of control law synthesis. Thus if the control law takes the form,

$$\mathbf{F}_c(t) = -\mathbf{K}_{cl}(D)\mathbf{F}(t), \tag{58.6}$$

and it follows that,

$$\mathbf{F}(t) = (1/2)aK_t(\mathbf{I} + \mathbf{K}_{cl}(D))^{-1}\mathbf{AG}(D)\,(1 - \exp(-DT))\,\mathbf{F}(t) \tag{58.7}$$

Equation (58.7) may be expressed as,

$$\begin{aligned}&\left(\mathbf{I} - (1/2)aK_t(\mathbf{I} + \mathbf{K}_{cl}(D))^{-1}\mathbf{AG}(D)\,(1 - \exp(-DT))\right)\mathbf{F}(t)\\&\quad\equiv (\mathbf{I} + \mathbf{\Phi}(D))\mathbf{F}(\mathbf{t}) = 0\end{aligned} \tag{58.8}$$

The conditions for stability may now be stated in terms of the Nyquist stability criterion. The return difference equation is given by,

$$[\mathbf{I} + \mathbf{GA}] = (\mathbf{I} + \mathbf{\Phi}(D)) \tag{58.9}$$

Thus the multi-input multi-output Nyquist plot may be obtained by computing the eigenvalues, λ, defined by,

$$[\mathbf{\Phi}(D) + \lambda\mathbf{I}]\,\mathbf{x} = \mathbf{0} \tag{58.10}$$

As $\mathbf{\Phi}(D)$ is a complex matrix, the above is a complex eigenvalue problem. To obtain the Nyquist plot corresponding to an eigenvalue it is plotted on the complex plane as ω traverses the Nyquist contour. In the classical Nyquist plot the gain crossover point (i.e. when the gain exceeds unity in magnitude) and phase crossover point (i.e. when phase increases from less than 180° to greater than 180°) are critical in the assessment of relative stability. The gain margin of stability is measured at the phase crossover point and the phase margin at the gain crossover point. The gain margin is the additional gain factor in dB that force the plot to pass through crossover point while the phase margin is the additional phase which when added to the phase would force the phase plot to pass through the phase crossover point. Moreover when the gain crossover and the phase crossover points are relatively close to each other, it signifies that a point of neutral stability is in the vicinity of the gain crossover point.

58.3 System's Transfer Function

The system's transfer function matrix, $\mathbf{G}(D)$ in Eq. (58.4) is required in order to predict the system's stability margin. It is therefore defined for the cutter and the workpiece as,

$$\mathbf{G}(D) = \mathbf{G}_c(D) + \mathbf{G}_w(D), \tag{58.11}$$

Where for a 2-D system,

$$\mathbf{G}_\beta(D) = \begin{bmatrix} G_{\beta xx}(D) & G_{\beta xy}(D) \\ G_{\beta yx}(D) & G_{\beta yy}(D) \end{bmatrix}, \quad (\beta = c, w), \quad (D = i\omega_c) \tag{58.12}$$

When considering only the tool's dynamics, the cutter's transfer function $\mathbf{G}_c(D)$ can be assumed to be constant and generally extracted experimentally, while the workpiece's transfer function matrix $\mathbf{G}_w(i\omega_c)$ is simply ignored. In the case of thin wall machining however, the workpiece dynamics cannot be ignored and the current adopted experimental methods would be inefficient as the dynamics are not constant along the thin wall as shown by Adetoro et al. [14]. An FEM and Fourier approach to extracting the system's transfer function was presented by Adetoro et al. [1, 17]. The main drawback of this approach is that it requires the damping parameters of the structure, therefore a new approach to predicting the damping parameters is presented in the next section.

58.3.1 Damping Ratio Prediction

An approach to predicting the damping ratio was discovered by Adetoro et al. [19, 20], which can be used to predict the damping parameters used in the FEM simulations. In many practical situations the use of the Nyquist stability criterion is unnecessary if one can extract the damping ratio of all the vibration modes relatively quickly. The approach proposed by Adetoro et al. [19] is a quick, simple and yet significantly accurate approach to predicting the damping ratio in terms of the frequency for a given wall; based on the use of the known damping ratios of a wall with same height (provided only the wall thickness is changed). From a range of extracted structural dynamics, it was discovered that there was a certain trend between the different damping ratios for different wall thicknesses. It was found that a new set of parameters, $\overline{\zeta}_p$ and $\overline{\omega}_p$, can be defined as follows,

$$\overline{\zeta}_p = \frac{\zeta_p^a}{t_a}, \tag{58.13}$$

for damping ratio and

$$\overline{\omega}_p = \frac{\omega_{np}^a}{t_a}, \tag{58.14}$$

for natural frequency, where t_a is the reference current wall thickness, ζ_p^a is the modal damping ratio and ω_p^a is the natural frequency for the reference wall respectively. These parameters ($\overline{\zeta}_p$ and $\overline{\omega}_p$) are then used to predict the damping ratio, ζ_p^b in terms of frequency, ω_p^b for any new geometry (provided only the wall thickness is changed) by simply multiplying $\overline{\zeta}_p$ and $\overline{\omega}_p$ by the new wall thickness t_b as follows,

$$\zeta_p^b = \overline{\zeta}_p \cdot t_b, \tag{58.15}$$

$$\omega_p^b = \overline{\omega}_p \cdot t_b \tag{58.16}$$

It should be noted that ζ_p^b and ω_p^b are not necessarily the precise modal damping and natural frequencies of the new wall. Studying the series in Eq. (58.3) propose by Caughey [16], the zeroth order approximation gives,

$$\mathbf{C}_0 = \alpha_0 \mathbf{M}, \tag{58.17}$$

which is not realistic as there the stiffness term has to always exist in whatever level of approximation, hence in an attempt to preserve the stiffness and mass terms a new series is proposed, which is defined as,

$$\zeta_p = \frac{1}{2} \sum_{u=1}^{L/2} \left(\alpha_{2u-1} \cdot \omega_n^{-u} + \alpha_{2u} \cdot \omega_n^{u} \right), \tag{58.18}$$

where the first term expands out to,

$$\zeta = \frac{1}{2} \left(\frac{\alpha_1}{\omega_n} + \alpha_2 \omega_n \right) \tag{58.19}$$

Therefore, Eq. (58.19) can be written for the first series as,

$$C_p = \alpha_1 M_p + \alpha_2 K_p, \tag{58.20}$$

which, shows that both mass and stiffness retained in the first series. Expanding the proposed series gives,

$$C_p = \alpha_1 M_p + \alpha_2 K_p + \alpha_3 M_p^{1.5} + \alpha_4 K_p^{1.5} + \alpha_5 M_p^2 + \alpha_6 K_p^2 + \cdots \tag{58.21}$$

Therefore by dividing Eq. (58.19) through by the wall thickness t_a, as done in Eqs. (58.13) and (58.14) we obtain the following series expansion,

$$\overline{\zeta} = \frac{1}{2} \sum_{u=1}^{L/2} (\alpha_{2u-1} \cdot \overline{\omega}^{-u} + \alpha_{2u} \cdot \overline{\omega}^{u}), \tag{58.22}$$

where constants α_{2u-1} and α_{2u} are real constants obtained using least squares method. This series expands out in the form,

$$\overline{\zeta} = \frac{1}{2} \left(\frac{\alpha_1}{\overline{\omega}} + \alpha_2 \overline{\omega} + \frac{\alpha_3}{\overline{\omega}^2} + \alpha_4 \overline{\omega}^2 + \frac{\alpha_5}{\overline{\omega}^3} + \alpha_6 \overline{\omega}^3 + \cdots \right) \tag{58.23}$$

Therefore, by dividing the different numerically extracted natural frequencies (refer to the FEM approach in the paper by Adetoro et al. [1, 17]) for each identified mode for the new wall thickness by t_b to obtain $\overline{\omega}$ in Eq. (58.22) and then multiplying the calculated $\overline{\zeta}$ by t_b, the corresponding damping for that mode is obtained. This significance of this new damping modelling approach is its application in thin wall machining, as the workpiece thickness reduces and its damping parameters change. Several case studies are presented in the paper by Adetoro et al. [19, 20].

58.3.2 Damping Matrix

The damping ratio, ζ_p^b in terms of frequency can be readily used directly by most commercial Finite Element (FE) packages, however the damping matrix **C** in Eq. (58.2) is sometimes required. To obtain the damping matrix, the numerically extracted natural frequency of the new structure for each mode is divided by the wall's thickness t_b, to obtain $\overline{\omega}$ and used in Eq. (58.22), to calculate $\overline{\zeta}$, which is then multiplied back by the wall's thickness t_b, to obtain the new modal damping ratio for the corresponding mode.

The modal damping C_p is simply calculated in a similar fashion as in SDoF. The damping matrix **C** is finally obtained by pre-multiplying by the modal matrix and then post-multiplying by the transpose of the modal matrix or eigenvectors obtained in FEM simulations. This orthogonal property only applies to systems that possess classical normal modes or proportional damping.

58.3.3 Examples

To demonstrate the new damping prediction approach, the FEM approach presented by Adetoro [1] was used. The workpiece material used in the FEM model is "Aluminium Alloy 7010 T7651" and the properties are: Density, $\rho = 2.823 \times 10^3$ (kg m^{-3}), Young's Modulus, $E = 69.809$ (GPa) and Poisson Ratio, $\nu = 0.337$. Two different examples taken from the paper by Adetoro [20] are shown here. The dimensions are given in the paper by Adetoro [20] and the corresponding experimentally identified damping parameters. The experimentally extracted damping parameters for the reference wall are used to predict the damping parameters, ζ_p^b for other structures using the approach presented in previous section. The damping parameters predicted, ζ_p^b and the force data, $f(t)$ measured by the instrumented hammer (in time domain) during experimental impact tests were used in each corresponding FE analysis.

During the experimental impact test, the workpiece was bolted at the back surface to a milling machine, hence in the FEM simulations it was assumed to be perfectly clamped (characterised by stiffness values of 1×10^{36} for the corresponding degrees of freedom) and that the resonant frequency of the machine is much higher than the excited frequencies during impact tests.

Two FEM simulations were carried out for each structure; one using the experimentally extracted damping parameters and the second using the predicted damping parameters. The comparison between the two (Figs. 58.2 and 58.3) shows the accuracy of the new approach to predicting damping parameters. The FEM simulations are also compared with experimentally measured accelerations to depict the accuracy of the FEM simulations.

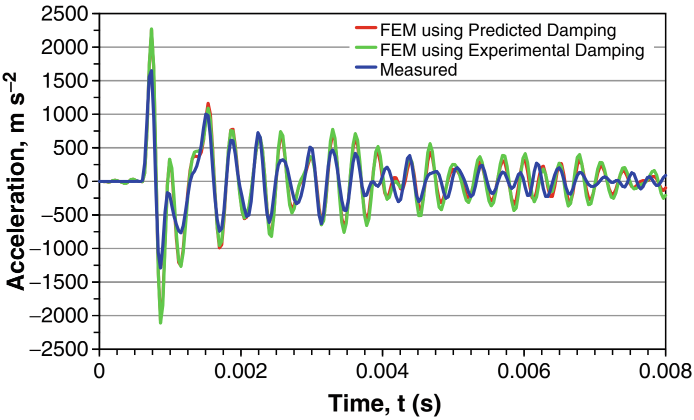

Fig. 58.2 Example 1: Wall 30 height, 3.0 thickness

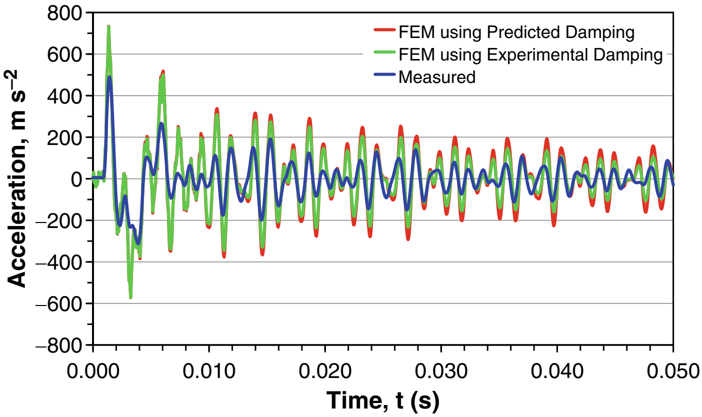

Fig. 58.3 Example 2: Wall 70 height, 3.5 thickness

References

1. Adetoro, O.B., Wen, P.H., Sim, W.M., Vepa, R.: Stability lobes prediction in thin wall machining. Proceedings of the International Multi-Conference of Engineers and Computer Scientist **1**, 520–525 (2009). www.iaeng.org/publication/WCE2009_pp20–525.pdf
2. Huo, D., Cheng, K.: Fundamentals, Applications and Practices, Springer Series in Advanced Manufacturing, pp. 7–20 (2009)
3. Tobias, S.A., Fishwick, W.: Theory of regenerative machine tool chatter. Eng. London **205**, 109–203 (1958)
4. Tlusty, J., Polacek, M.: The stability of machine tools against self excited vibrations in machining. Int. Res. Product. Eng. **41**, 465–474 (1963)
5. Merritt, H.E.: Theory of self-excited machine tool chatter. Trans. ASME J. Eng. Indus. **87**, 447–454 (1965)
6. Budak, E., Altintas, Y.: Analytical prediction of stability lobes in milling. CIRP Annal. Manufact. Technol. **44**(1), 357–362 (1995)
7. Budak, E., Altintas, Y.: Analytical prediction of chatter stability in milling – Part I: General formulation. Trans. ASME – J. Dynam. Syst. Measure. Contr. **120**, 22–30 (1998)
8. Budak, E., Altintas, Y.: Analytical prediction of chatter stability in milling – Part II: Application of the general formulation to common milling systems. Trans. ASME J. Dynam. Syst. Measure. Contr. **120**, 31–36 (1998)
9. Altintas, Y.: Analytical prediction of three dimensional chatter stability in milling. JSME Int. J. Ser. C: Mechan. Syst. Mach. Element. Manufact. **44**(3), 717–723 (2001)
10. Campa, F.J., Lopez de Lacalle, L.N., Lamikiz, A., Sanchez, J.A.: Selection of cutting conditions for a stable milling of flexible parts with bull-nose end mills. J. Mater. Process. Technol. **191**(1–3), 279–282 (2007)
11. Bravo, U., Altuzarra, O., Lopez de Lacalle, L.N., et al.: Stability limits of milling considering the flexibility of the workpiece and the machine. Int. J. Machine Tools Manufact. **45**(15), 1669–1680 (2005)
12. Lacerda, H.B., Lima, V.T.: Evaluation of cutting forces and prediction of chatter vibrations in milling. J. Brazil. Soc. Mechan. Sci. Eng. **26**(1), 74–81 (2004)
13. Adetoro, O.B., Sim, W.M., Wen, P.H.: Stability lobes prediction for corner radius end mill using nonlinear cutting force coefficients. Machin. Sci. Technol. (2009, submitted for publication)
14. Adetoro, O.B., Sim, W.M., Wen, P.H.: Accurate prediction of stability lobes using nonlinear thin wall dynamics. J. Mater. Process. Technol. (2009, submitted for publication)
15. Rayleigh, L.: Theory of Sound, vol. 2. Macmillan, New York (1878) (Reprinted 1945 by Dover Publications, New York)
16. Caughey, T.K., O'Kelly, M.E.J.: Classical normal modes in damped linear systems. Trans. ASME J. Appl. Mechan. **27**:269–271 (1960)
17. Adetoro, O.B., Wen, P.H., Sim, W.M., Vepa, R.: numerical and experimental investigation for stability lobes prediction in thin wall machining. Engineering Letters, **17**(4) (2009)
18. Minis, I., Yanushevsky, T.: A new theoretical approach for the prediction of machine tool chatter in milling. Trans. ASME J. Eng. Indus. **115**, 1–8 (1993)
19. Adetoro, O.B., Sim, W.M., Vepa, R., Wen, P.H.: Numerical and experimental investigation of damping parameters for thin wall structures. J. Advance. Manufact. Syst. (2009, submitted for publication)
20. Adetoro, O.B., Sim, W.M., Vepa, R., Wen, P.H.: A new damping modelling approach and its application in thin wall machining. Int. J. Advan. Manufact. Technol. (2009, submitted for publication)