Chapter 56
A Multi-Parametric Analysis of Drift Flux Models to Pipeline Applications

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Abstract Several interactions occur between the constituents of effluents within a pipeline (fluid, particles, and the pipeline interface). These interactions are birthed from their constant motion in one point in a pipeline relative to another point within the same pipeline. These constant motions expressed through various Drift Flux models are amenable to multi-parametric analysis. This particular exercise successfully elucidates the working parameters used in obtaining the drift flux equations. It utilizes a step by step self explanatory method for calculating the terminal velocity of effluents, being the volumetric flux or relative velocity of fluid/fluid or fluid/particle or fluid-particle/wall interfacial flow contact. Thus, forces encountered as a result of these relative motions are then specifically examined within the parameters of drift flux models. This study, in further applying a multi-parametric analysis of these drift flux models therefore acts as a template which could be used for solving pipeline problems involving these relative motions, once the necessary data has been collated and subsequently computed.

Keywords Continuum Phase Flow · Drift Flux Models · Erosion Wear Rate · Pipeline · Terminal Velocity · Volumetric Flux · Volume Fraction

56.1 Introduction

A multi-parametric analysis involves the collective study of various investigations on drift flux models as applied to both vertical and horizontal pipelines. Drift flux models were first developed by Zuber in 1965. The Drift flux models fall into the category of Computational Fluid Dynamics CFD used for particle transport prediction equations. A drift flux model is employed to represent slip between fluid

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phases [4]. Brethour and Hirt [7] were of the opinion that the concept behind the Drift Flux models is that the relative motion between these components can be described as a continuum rather than by discrete elements. A continuum considers the entire process as a whole with no distinct exclusive or conclusive attention given to the elements or parts separately. It is uniquely used for the study of sedimentation, fluidised beds and other flow processes that involve relative motion (interaction) between phases that are controlled by buoyancy and fluid drag forces. The relative flow moves in a slip pattern propagated by Kinematic Shocks or Expansion Waves mostly caused by turbulent fluid motion aided by external and internal forces such as the inward radial pressure generated by interfacial surface tension of a fluid in a stable high thermal environment as in the case of a viscoplastic slurry or paste transport situation.

Applications range from scenarios and processes that see the occurrence of elements as simple as bubbles and slug particles, to more complex and subtly devastating affects such as pipeline erosion and the attendant wear rate. Drift Flux models consider the different densities and sizes of the volume fraction of particles assumed to be continuously slipping; in other words it considers the relative motion between and within the fluids or fluid/particle or fluid/particle/pipe wall at constant velocity due to gravitational and/or centrifugal forces [9].

The aforementioned Zuber considered a one dimensional flow of a mixture of two components, A and B. The volumetric fluxes of the two components, j_A and j_B, were related to the total volumetric flux, j, the drift flux, j_{AB} and the volume fraction, $\alpha_A + \alpha_B = 1$. To determine the relative motion (Drift Flux) by applying the theory of dynamics to the forces on the individual phases, the momentum and energy equations would have to be understood although not exclusively or conclusively.

Drift flux models are not without limitations in that some multiphase flows can not be approximated especially when the relative motion is intimately connected with the pressure and velocity gradients in the two or more phases. However, since Zuber, many researchers have applied the model to a two phase flow with success [2, 3, 8].

A fluid carrying pipeline is rife with opportunities for applying drift flux models. In this research work, the application of the various drift flux models to pipeline engineering is examined.

56.2 Application to a Vertical Pipe Considering the Buoyancy Effect

Since their introduction in the 1960s the Drift Flux models have proven very adaptable to various engineering challenges encountered. This versatility is of immense value in the prediction of expected and anticipated engineering failures, it is also important for examining post failure root cause analysis. The need for a full fledged parametric analysis of various drift flux models cannot be overemphasised.

The governing equations describing one dimensional two phase drift flux transport equations in vertical pipes are considered by stating the mass conservation, momentum and internal energy conservation equations; this is a basic start:

For Mass Conservation

$$\frac{\partial p_j}{\partial t} + \frac{\partial}{\partial z}\left(P_j U_j\right) = 0 \tag{56.1}$$

For Drift Flux Momentum Conservation

$$\rho_j \frac{\partial v_j}{\partial t} + \rho_j V_j \frac{\partial V_j}{\partial z} + \frac{\partial}{\partial z}\left(\frac{\rho_v \rho_1 \alpha V_{vj}^2}{\rho_j\,(1-\alpha)}\right) = -\sum_{i=1}^{N} F_{gl} - \frac{\partial P_j}{\partial z} - \rho_j g \cos\theta \tag{56.2}$$

where θ is the angle of contact between the surface of the liquid and the surface of the pipe which is assumed to be 0°.

Drift flux internal Energy conservation

$$\begin{aligned}
&\frac{\partial}{\partial t}\left(\rho_j u_j\right) + \frac{\partial}{\partial z}\left(\rho_j u_j v_j\right) + \frac{\partial}{\partial z}\left[\frac{\alpha \rho_l \rho_v \left(u_v - u_l\right) V_{vj}}{\rho_j}\right] \\
&\quad + P\frac{\partial V_j}{\partial z} + P_m \frac{\partial}{\partial z}\left[\frac{\alpha\left(\rho_l - \rho_v\right) V_{vj}}{\rho_j}\right] \\
&= \sum_{i=1}^{N} q_l^n \frac{P_1}{A_l} + V_j\left(\sum_{i=1}^{N} F_{wi}\right)
\end{aligned} \tag{56.3}$$

where A is the surface area of the pipe; u_v and u_l are the vapor and liquid velocities respectively, V is the volumetric flow rate, F_g is the gravitational force, P is the operating pipe pressure, Z is the height of the pipe, ρ_l and ρ_v are the liquid and vapor density respectively, q is the heat flux whereas, F_{wi} is the wall shear force.

This study was done by examining the works of Holman [13] and DF Models [9]. In nucleate boiling, Holman [13] observed that bubbles are created by the expansion of entrapped gas or vapor at small cavities in the surface. The bubbles increase in size depending on the surface tension at the liquid vapor interface and the temperature and pressure.

In this scenario, a superheated liquid at its boiling point would have bubbles of vapor form on the heating element surface. These bubbles collapse as the heat increases, and the entrapped gases escape through the liquid to the surface of the vertical pipe being investigated. The volumetric drift flux of bubbles as they move through the liquid is represented by Eq. (56.4)

$$J_{VL} = \alpha\,(1-\alpha)\,U_{VL} \tag{56.4}$$

where J_{VL} is the drift flux, U_{VL} is the relative velocity and α is the volume fraction.

The relative velocity can also be represented in Eq. (56.5)

$$U_{VL} = U_{VLO}\,(1-\alpha) \tag{56.5}$$

In terms of the terminal velocity of single bubble in the dispersed vapor phase, U_{VLO}, as represented in Eq. (56.6) and the corresponding drift flux written in Eq. (56.7)

$$U_{VL} = U_{VLO}\,(1-\alpha)^{b-1} \tag{56.6}$$

$$J_{VL} = U_{VLO}\,(1-\alpha)^{b} \tag{56.7}$$

The term b is some constant of the order of 2 or 3. b takes on values from 3 for very minute bubbles to 2 for somewhat larger bubbles.

To determine the terminal velocity of individual bubbles rising, U_{VLO}, the first step here is to determine the radius of the bubble, R. The buoyancy force, F_b which propels the gas through the liquid is considered and expressed in Eq. (56.8)

$$F_b = \frac{4}{3}\pi R^3 g\,(\rho_L - \rho_V) \tag{56.8}$$

where R is the radius of the bubble, g is the acceleration due to gravity and ρ_L, ρ_V is the density of the liquid and vapor, respectively. The surface tension force F_σ, is also considered

$$F_\sigma = 2\pi R\sigma \tag{56.9}$$

where σ is the surface tension of the liquid and vapor interface

$$\sigma = \frac{1}{2}\left[(\rho_L - \rho_V)\,gR\left(H + \frac{R}{3}\right)\right]$$

where R is the radius of the bubble and H is the rise of the bubble [11].

Equating the two forces of Eqs. (56.8) and (56.9) gives a formula for R, to be

$$R = \left[\frac{3\sigma}{2g\,(\rho_L - \rho_V)}\right]^{\frac{1}{2}} \tag{56.10}$$

The second step here is to determine U_{VLO}. This is achieved by equating the drag force. F_D to the buoyancy force, F_b in Eq. (56.8)

$$F_D = \frac{C_D \pi R^2 \rho_L U_{VLO}^2}{2} \tag{56.11}$$

where C_D is the drag coefficient.

Equating Eqs. (56.8) to (56.11) generated Eq. (56.12)

$$U_{VLO} = \left[\frac{8Rg\,(\rho_L - \rho_v)}{3\rho_L C_D}\right]^{\frac{1}{2}} \tag{56.12}$$

When Eq. (56.12) is substituted into Eq. (56.6) and Eq. (56.7), the values of the volumetric drift flux and the relative velocity of the bubble and liquid interface would be obtained.

However, Holman [13] was of the opinion that when a liquid is heated above the saturation temperature, boiling occurs and the heat flux will depend on the difference in temperature between the surface and the saturation temperature. Zuber and Findlay [21] proposed an equation to determine the peak heat flux in nucleate boiling as expressed in Eq. (56.13)

$$\left(\frac{q}{A}\right)_{\max} = \frac{\pi}{24} h_{fg} \rho_v \left[\frac{\sigma g \left(\rho_L - \rho_V\right)}{\rho_v^2}\right]^{\frac{1}{4}} \left(1 + \frac{\rho_V}{\rho_L}\right)^{\frac{1}{2}} \tag{56.13}$$

where q is the heat flux, A is the surface area of the pipe, represented in Eq. (56.14)

$$A = \pi d L \tag{56.14}$$

d and L is the diameter and length of the pipe respectively. The heat transfer coefficient, h_{fg} is expressed in Eq. (56.15) as

$$h_{fg} = 2.54 \left(T_V - T_l\right)^3 e^{\frac{p}{1.551}} \left(W/m^2 {}^{\circ}C\right) \tag{56.15}$$

$(5 < P < 170 \text{ atm})$
where P is the pressure in meganewtons per square meter. T_v, T_L are the vapor and liquid temperature, respectively.

Holman [13] stated that in saturated boiling, when the bubbles break away from the surface because of the buoyancy action, the bubbles move back into the body of the liquid. This results when the temperature of the surrounding liquid is lower than the saturated temperature in the bubble. This can be explained by deriving an expression for the pressure gradient that exists between the interface of the vapor and liquid phase.

The pressure force F_p and the surface tension force, F_σ are considered at equilibrium

$$F_\rho = \pi R^2 \left(P_V - P_l\right) \tag{56.16}$$

where, P_v is the vapor pressure inside the bubble and P_L is the liquid pressure.

Equating Eqs. (56.9) to (56.16) generated Eq. (56.17)

$$P_v - P_L = \frac{2\sigma}{R} \tag{56.17}$$

Holman [13] was of the opinion that Eq. (56.17) indicates that when the pressure inside the bubble is reduced, the corresponding vapor temperature will also reduce. This implies that the bubble will rise and move further away from the heat source to where the liquid temperature is lower. This means that heat is conducted out of the bubble and the vapor inside the bubble condenses and collapses back to the liquid especially in a forced convective condition.

In this condensed state, to determine the drift flux and relative velocity of the vapor–liquid interface. The terminal velocity, U_{VLO} should be obtained.

Here, the net gravitational force, F_g is equated to the drag force, F_D

$$F_g = \frac{4}{3}\pi R^3 g\left(\rho_L - \rho_V\right) \tag{56.18}$$

Eq. (56.18) is same as Eq. (56.8)

$$F_D = \frac{C_D \pi R^2 \rho_v U_{VLO}^2}{2} \tag{56.19}$$

Equating Eqs. (56.18) to (56.19) generated Eq. (56.20)

$$U_{VLO} = \left[\frac{8Rg\left(\rho_L - \rho_v\right)}{3\rho_v C_D}\right]^{\frac{1}{2}} \tag{56.20}$$

The value for R, as determined by Zuber et al. [20] is expressed in Eq. (56.21)

$$R \approx \lambda\alpha\left[\frac{\sigma}{g\left(\rho_L - \rho_V\right)}\right]^{\frac{1}{2}} \tag{56.21}$$

where λ is the wavelength in its unstable state related to Rayleigh–Taylor unstable surface and it is assumed to be equal to the size of water droplets at the vapor/liquid interface [9].

Sun and Lienhard [16] proposed an equation for determining q_L, to be

$$q_L = \frac{0.061}{K} \tag{56.22}$$

where

$$K = \frac{d}{\left[\dfrac{\sigma}{g\left(\rho_l - \rho_v\right)}\right]^{\frac{1}{2}}} \tag{56.23}$$

where d is the diameter of the tube. Equation (56.23) should be used when $K < 2.3$, however where $K < 0.24$, there is no nucleate boiling.

56.3 Drift Flux Models as Applied to Wear Rate in Horizontal Pipelines

In this case, the wear rate effect on the interface between the volume fraction of particles immersed in a transport fluid and the internal walls of a pipeline have been studied [1]. However, the Eulerian continuum flow model, the particle equation of motion and the erosion prediction equation are explained here in detail.

56.3.1 The Continuous Model

This model describes the behaviour of fluid flow patterns in a continuous phase. In this phase the conservation equations for mass and momentum in combination with transport equations for a turbulence model are applied. Tian [17] was of the opinion that in CFD model equations, governing equations are fundamentally based on fluid dynamics, which represents the mathematical statements of the conservation law of physics. These laws have been derived from the fact that certain measures must be conserved in a particular volume, known as a control volume. The governing equations for axisymmetric turbulent flow were expressed as follows [4, 18].

$$\frac{\partial}{\partial x_j}\left(\rho u_j\right) = 0 \qquad (56.24)$$

where U_j is the average or mean velocity component and ρ is the fluid density.

Equation (56.24) is expanded as expressed in Eq. (56.25)

$$\frac{\partial}{\partial x_j}\left(\rho_f u_{if} u_{jf}\right) = \frac{-\partial P_t}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j}\right) - \frac{\partial}{\partial x_j}\left(\rho u_i u_j\right) \qquad (56.25)$$

where P is the static pressure and the stress tensor was further expanded as written in Eq. 56.26 as proposed by Hinze [12]

$$-\rho u_i u_j = \left[\mu_{f,t}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] - \frac{2}{3}\rho k \delta_{ij} \qquad (56.26)$$

where δ_{ij} is the Kronecker delta and μ_{gt} is the eddy viscosity or turbulent viscosity. The turbulent quantity, K which is the Kinetic energy of turbulence expressed in Cartesian tensor notation as

$$K = \frac{1}{2}\overline{u_i u_i} \qquad (56.27)$$

Can be simply expressed as

$$K = 0.01 u_f^2 \qquad (56.28)$$

The rate of production of turbulent kinetic energy, P_k is given by

$$P_k = -\rho_f \overline{u_i^1 u_j^1} \frac{\partial u_{if}}{\partial x_j} \text{ and}$$

the rate of dissipation of turbulent energy, ε is expressed as

$$\varepsilon = \frac{\mu_{ft}}{\rho_t}\overline{\left(\frac{\partial u_i^1}{\partial x_j}\right)\left(\frac{\partial u_i^1}{\partial x_j}\right)} \qquad (56.29)$$

ε can simply be calculated from Eq. (56.30)

$$\mu_{ft} = \rho_f C_\rho \frac{K^2}{\varepsilon} \tag{56.30}$$

where C_ρ is the specific heat capacity of the fluid and was given by Reynolds [14] as 0.0845.

The turbulent viscosity, if not given can be determined from Eq. (56.31)

$$\mu_{f,t} = \upsilon \rho_t \tag{56.31}$$

where υ is the Kinematic viscosity of the fluid. Considering Eqs. (56.28)–(56.31), the term K can be obtained and substituting the value of K into Eq. (56.28), U_m would be obtained.

The mass rate of flow, $\dot{m}$ is calculated from Eq. (56.32)

$$\dot{m} = \rho_t u_f A \tag{56.32}$$

where A is the cross sectional area of the pipe given as πdl, where d and l have been previously defined.

The mass flow velocity, G is given in Eq. (56.33) as

$$G = \frac{\dot{m}}{A} = \rho_f u_m \tag{56.33}$$

The term U_f is the fluid terminal velocity.

56.3.2 Particle Equation of Motion

In deriving this equation, two assumptions were employed

1. The solid particles do not interact with each other.
2. The influence of particle motion on the fluid flow field is very small and could be neglected.

These assumptions were also adopted by Edwards et al. [10] and Wallace et al. [19] in their various research works.

The governing particle equation of motion is given as

$$\frac{du_p}{\partial t} = F_D(u_f - u_p) + \frac{g\,(\rho_\rho - \rho_f)}{\rho_\rho} + \sum F_x + \Delta P + F_d \tag{56.34}$$

where $F_D\ (u_f - u_p)$ is the drag force per unit particle mass and F_D is given by

$$F_D = \frac{3C_D \mu \mathrm{Re}_p}{4\rho_p d_p^2} \tag{56.35}$$

where ρ_p is the density of particle material, d_p is the particle diameter, u_p is the particle velocity and Re_p is the relative Reynolds number written as expressed in Eq. (56.36)

$$\mathrm{Re}_p = \frac{\alpha_f \rho_f d_p (u_p - u_f)}{\mu_{f,t}} \tag{56.36}$$

where α_f is the volume fraction of the fluid, U_p is the particle velocity and C_d is the drag coefficient, this is defined in Eq. (56.37)

$$C_D = \begin{cases} 0.44 & Re_p > 1000 \\ \frac{24}{Re_p}\left(1.0 + \frac{1}{6}\mathrm{Re}_p^{0.66}\right) & \mathrm{Re}_p \leq 1000 \end{cases} \tag{56.37}$$

$\frac{g(\rho_p - \rho_f)}{\rho_p}$ represents the particle buoyancy force that keeps the particles in continuous flow suspension when it is at equilibrium with the pressure force F_d is the Saffman lift force proposed by Saffman [15].

$\sum F_x$ is the increase in momentum flux in the fluid around the particles. This could be represented as

$$\sum F_x = \frac{\partial(mu_T)x}{\partial\tau} \tag{56.38}$$

where U_T is the friction velocity and τ is the shear force due to flow.

Most of the energy loss takes place during the algebraic particle-wall collision at the interface. This causes the disintegration of particles and pipewall deformation. However, large energy loss due to molecular level forces, such as adhesion is not reversible and occurs primarily during rebound [6, 17].

The friction velocity or relative velocity at the interface can be calculated from Eq. (56.39)

$$U_T = \sqrt{\frac{\tau_w}{\rho_f}} \tag{56.39}$$

where τ_w is the wall shear stress

$$\text{The pressure gradient,}\Delta P = f\frac{L}{d}\rho_f\frac{U_T^2}{2g} \tag{56.40}$$

where f is the frictional force, L is the length of the pipe and d is the diameter of the pipe.

Here Eqs. (56.36)–(56.41) is used to determine the velocity of the particle, U_p.

The value of U_p is used to compute the Erosion wear rates of the pipeline depending on the angle of contact between the particle transported by the fluid and the internal pipewall, α.

56.3.3 The Erosion Prediction Equation

The erosion prediction equations suggested by Wallace et al. [19] were used because of their simplicity and high level of accuracy for the prediction of erosion rates. These equations are given in Eqs. (56.41) and (56.42).

$$E = \left\{ \frac{\frac{1}{2} U_p^2 \cos^2 \alpha \sin 2\alpha}{\Upsilon} + \frac{\frac{1}{2} U_p^2 \sin^2 \alpha}{\sigma} \right\} \tag{56.41}$$

For $\alpha \leq 45°$

And

$$E = \left\{ \frac{\frac{1}{2} U_p^2 \cos^2 \alpha}{\Upsilon} + \frac{\frac{1}{2} U_p^2 \sin^2 \alpha}{\sigma} \right\} \tag{56.42}$$

For $\alpha > 45°$.

Where Y and σ are the cutting wear and deformation wear coefficients having the values 33,316.9 and 77,419.7, respectively.

From the study made by Bitter [5] peak erosion rates have been measured to occur at impact angles of 25–30°, indicating that cutting wear dominates. The difference between U_p and U_f gives the drift flux velocity. The term of $\left| U_f - U_p \right|$ can be used to replace the term $\left| U_p \right|$ in Eqs. (56.41) and (56.42) to obtain erosion rates based on the relative motion of fluid/particle interface.

56.4 Conclusion

The parametric analysis of these select drift flux models has introduced a wide range of applications. The application of drift flux models of fluid/fluid flow in a stagnant position and fluid/particle flow in a continuum phase to pipeline have been clearly expressed. The effect of interfacial motion between the fluid-particle and pipe wall as related to the relative motion of the fluid velocity and the particle velocity was applied to the erosion wear equation to determine the resistance of the pipe wall material to wear. These models therefore act like templates for solving pipeline engineering problems whenever they arise or when anticipated.

References

1. Achebo, J.I.: Computational analysis of erosion wear rate in a pipeline using the drift flux models based on eulerian continuum equations. In: Proceedings of the World Congress on Engineering (I) 1–3 July pp. 719–721 London, UK (2009)
2. Ahmad, K.M.: Effect of solid catalyst on bubble rise velocity and gas drift flux in three phase columns. Eng. Tech. **25**(1), 97–102 (2007)

3. Ambroso, A., Chalons, C., Coquel, F., Galie, T., Godlewski, E., Raviart, P., Segun, N.: The Drift Flux asymptotic limit of barotropic two phase two pressure models. Commun. Math. Sci. **6**(2), 521–529 (2008)
4. Bahr, H.M., Habib, M.A., Ben-Mansour, R., Said, S.A.M.: Effect of flow velocity and particle size on erosion in a pipe with sudden contraction. In: The 6th Saudi Engineering Conference, KFUPM, Dhahran held in December (5), 79–88 (2002)
5. Bitter, J.G.A.: A study of erosion phenomena parts 1 and 2. Wear **6**, 5–21 & 169–190 (1963)
6. Brach, R.M., Dunn, P.F.: Models of rebound and capture for oblique microparticle impact. Aerosol. Sci. Tech. **29**, 379–388 (1998)
7. Brethour, J.M., Hirt, C.W.: Drift flux model for two component flow. http://www.flow3d.com/pdfs/tn/flosci-TN83.pdf) (2009)
8. Dai, W., Woodward, P.R.: A high order iterative implicit – explicit hybrid scheme for magneto hydrodynamics. SIAM J. Sci. Comput. **19**, 1827–1846 (1998)
9. DF Models: Chapter 14: drift flux models. http://www.caltechbook.library.caltech.edu/51/1/chap14.pdf (2009)
10. Edwards, J.K., Mclanry, B.S., Shirazi, S.A.: Evaluation of alternative pipe bend fittings in erosive service. In: Proceedings of ASME Fluids Engineering Summer Meeting. Paper No. FEDSM 2000–11245, Boston, MA, 11–15 June (2000)
11. Experiment 446.1.: Surface tension of liquids. http://www.udel.edu/pchem/C446/Experiments/exp1.pdf (2006)
12. Hinze, J.O.: Turbulence. McGraw-Hill, New York (1975)
13. Holman, J.P.: Heat Transfer, 4th edn, pp. 364–379. McGraw-Hill, Tokyo (1976)
14. Reynolds, W.C.: Fundamentals of turbulence for turbulence modeling and simulation. In: Lecture Notes for Von Karm. Inst, Agard. Report No. 755 (1987)
15. Saffman, P.G.: The lift on a small sphere in a slow shear flow. J. Fluid Mech. **22**(2), 385–400 (1965)
16. Sun, K. H., Lienhard, J.H.: The peak boiling heat flux on horizontal cylinders. Int. J. Heat Mass Trans. **13**, 1425 (1970)
17. Tian, Z.: Numerical modelling of turbulent gas – particle flow and its applications. PhD Thesis in the School of Aerospace Manufacturing andMechanical Engineering, RMIT University (2006)
18. Versteeg, H.K., Malalasekera, W.: An Introduction to Computational Fluid Dynamics: the Finite Volume Method. Longman, London (1995)
19. Wallace, M.S., Peters, J.S., Scanlon, T.J., Dempster, W.M., McCulloch, S., Ogilvie, J.B.: CFD-based erosion modelling of multi-orifice choke valves. In: Proceedings of ASME Fluids Eng Sum. Meeting, Paper No. FEDSM2000–11244, Boston, MA, 11–15 June (2000)
20. Zuber, N. et al.: The dynamics of vapor bubbles in non uniform temperature fields. Int. J. Heat Mass Trans. **2**, 83–98 (1961)
21. Zuber, N., Findlay, J.A.: Average volumetric concentration in two phase flow systems. J. Heat Trans. **68**, 453–468 (1965)