Attention and Intention in Learning About Teaching Through Teaching

John Mason

Introduction

Teaching is a marvelous action in which to engage, because it is possible to be learning on many levels simultaneously: About the content being taught, about the students who are learning, about teaching itself, about learning itself, and perhaps most importantly, about yourself. But learning from experience is highly problematic. The case is put forward that most human interaction is based on reacting to others in specific situations with habitual behaviors, and that if significant learning is to take place from and through experience, it is necessary to shift from reacting to responding thoughtfully, even mindfully (Claxton, 1984; Langer, 1997). This is the role of attention directed by intention. Furthermore, when faced with a novel situation, response rather than reaction is vital. The distinction between *reacting* and *responding* applies equally to students and to teachers, but is mainly developed here in relation to teachers learning about their practices while teaching.

In this chapter, I endeavor to show how it is possible to work on sensitizing oneself to notice what previously passed by unnoticed, and how that can be used to inform actions in the future, that is, to learn about teaching through teaching. The process calls upon the Discipline of Noticing (Mason, 2002) which can be used to learn intentionally from experience, by becoming aware of how attention shifts. For example, each technical term in mathematics signals the fact that those who developed the term needed a label to refer to particular features on which to focus attention or to a particular way of attending. Thus *angle* signals attention to turning between two limiting arms, and away from the space between them. It is a reasonable conjecture that learners need to experience a similar shift in their attention if they are to appreciate and understand the term and use it productively and effectively for themselves. Exactly the same applies to pedagogical strategies and didactic tactics. Labels signal shifts in attention, and similar shifts in the form and focus of

J. Mason (✉)
The Open University (emeritus), Milton Keynes, Buckinghamshire; University of Oxford, Oxford, UK

R. Leikin, R. Zazkis (eds.), *Learning Through Teaching Mathematics*, Mathematics Teacher Education 5, DOI 10.1007/978-90-481-3990-3_2,

attention are necessary in order to learn about teaching and to work at developing practices so as to make teaching more effective and pleasurable.

Teachers are subject to numerous forces: Legislation, inspection, practices in their school, expectations of parents and children, and, often submerged beneath these, their own ideals and intentions. Despite conclusions to the contrary, none of these forces preclude working on and developing one's teaching through enquiring into that teaching. What is required is a vision of what might be possible, a source of energy and support, and personal discipline. Discipline is amplified whenever, and to the extent that, appropriate support is present. The vision itself is most effective when dynamic, being renewed and modified periodically, rather than fixed. John Donne's famous meditation (1624 XVII) that "no man is an island, entire of itself" is an excellent reminder that even to work on one's own teaching by oneself, outside connections are necessary in order to avoid descent into solipsism.

Theoretical Underpinnings

I take learning to be the transformation of attention. That is, learning has taken place when people discern details, recognize relationships, and perceive properties not previously discerned, through attending in fresh or distinct ways, and when they have fresh possibilities for action from which to choose. Learning necessarily involves shifts in the form as well as the focus of attention.

I take as given that there is a complex interaction between student(s), content, and teacher within an environment of social, institutional, and political forces and practices. This triad of impulses within its environment is a dynamic system. Each component is constantly adjusting to maintain its Piagetian equilibrium. Each impulse acts at different times and in different ways upon one or other of the other two and is mediated in that action by the third (Bhagavad Gita 18:18 see Mascaró, 1962; see also Bennett, 1956–1966; Mason, 1979, 2008). In relation to this chapter, the environment within which I am working includes a conjecturing atmosphere, in which everything said (by me, and by readers in response) is said in order to consider it more carefully with the intention of altering or modifying it as necessary (Mason, Burton, & Stacey, 1982; Mason & Johnston-Wilder, 2006). Because of the one-way nature of printed text, this means that everything must be tested in your experience. To this end, my brief summary of theoretical underpinnings is followed by succinct descriptions of some phenomena in order to try to ground the subsequent analysis and proposals in experience.

Biologically, a dynamic equilibrium is sustained until there is a significant but not overwhelming disturbance. In an educational context, this means that no learning, no development takes place until there is some disruption of the working out of internalized actions (habits, automaticities), until some dissonance is experienced, whether cognitive (Festinger, 1957), affective (e.g. surprise: See Movshovitz-Hadar, 1988), enactive (Skinner, 1954), or some combination of all three. Following Piaget (1971, 1985), change (assimilation and accommodation) is a reaction or a response

to experienced variation, including spontaneous "mutation" in the organism or the environment. For the discipline of teaching (Mason, 2009) this means that it is only when some significant but not overwhelming challenge occurs (change of curriculum or other institutional demands, change in student behavior and attention, change in teacher behavior and attention) that significant development can occur. This applies just as much to student learning as to teacher learning. A student who is "going through the motions" of completing tasks as set by the teacher is not actually learning in any deep sense (such as described by the onion model of understanding see Pirie & Kieren, 1994). A teacher who is going through the motions of curricular or pedagogic reform is not actually disturbing current practices significantly and so is not learning about teaching.

There are dangers, however. When change is sought explicitly and overtly, reaction sets in (think of the last time someone else wanted you to change your behavior). What results is often the opposite of what was intended due to the multitude of possible unintended consequences. Rather, *change* is a partial description of a lived experience of dynamic and constant disequilibrium and equilibration. It comes about through the desire to make sense (note the body-based metaphor of sensation).

The key question in teaching is whether an action is generative or merely repetitive, that is, whether something fresh arises from the action, or whether it is a reworking of habit and internalized automaticities. To make sense of this, it is necessary to consider the commonplace (St. Maurice, 1991) of "learning from experience" and the role of disturbance, which will lead into the structure of the human psyche for inter-relating mindfulness and habit. This then leads me to how it is possible to learn from experience of disturbance. Throughout the chapter, I try to maintain a parallel between student learning and teacher development, because both are instances of learning. Sometimes it is easier to make observations of others learning, and sometimes easier to observe in oneself.

I take an unabashedly phenomenological or experiential stance. I am interested in lived experience and how that can be enriched. My theoretical frame draws eclectically on a range of pedagogical, psychological, and social constructs. The pedagogic constructs of *didactic contract, didactic tension, didactic transposition*, the psychosocial construct *zone of proximal development*, and the psychological constructs of *attention* and *intention*, combined with the Discipline of Noticing and *variation theory*, form the background theory for how teachers can endeavor to learn from their experience of teaching and learning, and can endeavor to promote a parallel action in their students so that they too have the best opportunities to learn. Each construct draws and labels one or more distinctions which remind me to discern details that might otherwise go unnoticed, thus enabling me to recognize relationships in situations, and even to perceive these as instantiations of properties. These, in turn, permit reasoning on the basis of agreed properties which characterizes a fully fledged discipline. Mathematics education still has some way to go to reach this agreement.

The notion of *didactic transposition* introduced by Yves Chevallard (1985) alerts me to the almost inevitable transformation that occurs when someone tries to give or reproduce their experience for someone else. In mathematics education, it

usually happens that an expert experiences an insight or surprise, or recognizes the potential pedagogic value of some mathematical exploration or insight, and then tries to reproduce that experience for students by constructing a worksheet or other form of task. Points at which the expert made use of their own powers to investigate and explore are naturally replaced by instructions telling students what to do. Similarly, when teachers try to re-enact a pedagogic strategy or a pre-planned teaching sequence, they tend to miss out on the opportunities to choose to respond sensitively and mathematically to particular students. I summarize the transposition as expert awareness is transposed into instruction in behavior and it applies just as much to policy makers specifying curricula and to teachers trying out a pedagogic strategy, didactic tactic, or rearranged curriculum, as it does to teachers setting tasks for students.

The construct of *didactic situation* introduced by Guy Brousseau (1997) acts as a reminder that the ancient triad of student–teacher–content takes place within a complex environment or *milieu* of social and institutional influences. The associated construct of *didactic tension* is a powerful reminder that the more precisely the teacher indicates the behavior being sought, the easier it is for students to display that behavior without generating it from themselves; when applied to educational reform it translates into the more precisely the policy indicates the specific behaviors (practices) desired, the easier it is for teachers to display that behavior, to go through the motions, without generating it from themselves and without making real contact with students.

It is possible to carry out actions mechanically, either because they have been internalized this way (the result of training of behavior) or because they are being reproduced through mimicry, through adopting the outer, visible form (which is the content of the didactic tension). Again this applies to teachers enacting prepared teaching as much as to students making their way through work sheets and exercise sets.

There are evident resonances with Vygotsky's *Zone of Proximal Development* as elaborated by (van der Veer & Valsiner, 1991, see also Mason, Drury & Bills, 2007). The intention of teaching is that students initiate actions or respond to situations from themselves for themselves rather than having to be triggered or cued by inputs from a worksheet or a more experienced 'other'. The "zone" refers to a state of potential for that transition from a dependent state based on assenting to an independent assertive stance (Mason, 2009). The whole point about learning is that what once had to be pointed out by a teacher becomes available to the learner as a choice, as an option that comes to mind. Attention is then restructured, giving rise to the possibility of a concomitant restructuring of intention. The difference between teachers teaching and students being taught is that whereas the students have a more experienced other (the teacher) on hand, teachers rarely have in-the-moment access to a mentor or a more experienced colleague. So in order to learn from teaching, it is necessary to develop an inner witness who observes without passing judgment or justifications. Put another way, progress in learning is an internalization of practices encountered in others (Vygotsky, 1978), but this internalization requires a transformatory action.

Following Bennett (1956–1966), I take actions to require three impulses: One that initiates or "acts upon," one that responds or is acted upon, and one that mediates between the two, bringing and perhaps holding them together, and so making the action possible. In educational settings these are usually instantiated by the elements of the traditional trio: Student, teacher, and content. By analyzing the combinatorial possibilities, six fundamental modes of interaction can be distinguished: Expounding, expressing, explaining, examining, exploring, and exercising. Here I shall make use of only the first three.

Metaphors

Finding a satisfactory way to articulate the role of the environment on actions is a real challenge. There are several metaphors which direct attention unhelpfully.

Cause and Effect

Cause and effect is a popular and deeply embedded metaphor in the discourse of education, but one which I find particularly misleading. It draws upon the natural sciences, particularly physics, and lies at the heart of why, despite experience, we try to cause learning by initiating teacherly actions, when it is patently obvious that the most we can do is provide opportunities, however sophisticated, for students to use their intention to engage their attention productively. No matter how precisely people try to specify the tasks, the conditions, and the ways of working in a classroom, it is impossible to predict the outcome, precisely because unlike physics which deals with inanimate objects, education involves human beings who can direct their attention by exercising intention, and who are profoundly influenced by the individual and collective energies of peers, and their own imagined assumptions about themselves and their situation.

Social Influence as Forces

I also find myself unconvinced by the physics-based metaphor of forces acting on people which is deeply embedded in sociological discourse. Physical forces are additive whereas the social milieu is a complex mix of influences whose combined effect is not merely the sum of all the parts. Various components interact with, interfere with, and serve to amplify or diminish each other. A slightly more appropriate metaphor would draw upon chemistry, because of the transformations that take place among the various components analogous to chemical reactions. However, in an ideal educational setting, even chemical metaphors are inadequate because, as Maturana and Varela (1972, 1988) suggested, student, teacher, and content form an auto-poetic system (self-constructing in both senses) within an environment which is much more analogous to a biological organism than even a chemical mixture, much less a collection of physical forces.

Sometimes people react automatically without affective or cognitive control; sometimes affect dominates reaction; sometimes cognition exercises control, enabling a response. Under certain conditions the student–teacher–content collective also takes on many of the qualities of a living organism (Davis & Sumara, 2007). The teacher can sometimes be seen as acting as the "consciousness of the collective" just as Bruner (1986, pp. 75–76) saw a tutor being able to act as "consciousness for two." This, of course, brings me back to Piaget (op cit.) and his biological metaphors of *equilibration* through *assimilation* and *accommodation* and to the necessity for some extra impulse in order to "learn from experience."

Learning as Change of State

A metaphor from physics that *is* useful in education is the way change of state comes about: Energy is added to a system and change is observed (for example, temperature rises), then further addition of energy produces no further change (the temperature stops rising while the material changes state), and then, subsequently, the change continues (temperature continues to rise). Often it seems as though learners act in a similar manner: Further instruction or advice produces no visible effect, and then suddenly students appear to have internalized something. They become more informed, more knowledgeable, more flexible, and more dextrous. Classic examples include accommodation of the conjugation of irregular verbs (Brown, 1973), and the gaining of independence with respect to the use of some mathematical technique. The role a teacher can play in this internalization process was elaborated as *scaffolding* (introduced by Wood, Bruner & Ross, 1976 inspired by Vygotsky 1978) then extended to include *fading* (Brown, Collins & Duguid, 1989) and independently expressed in Open University materials as a spectrum of teacher intervention moving through *directed–prompted–spontaneous* use of some action or probe (James in Floyd, Burton, James, & Mason, 1981; Love & Mason, 1992; Mason & Johnston-Wilder, 2004).

Human Psyche as Chariot

One way to see why training behavior alone is ineffective when trying to promote the development of students' powers of thinking mathematically is to make use of a framework developed in the Upanishads (Rhadakrishnan, 1953, p. 263), made use of in various places in Eastern and Western psychology, and underpinning the traditional terms *enaction*, *affect*, and *cognition* which pervade Western psychology.

The human psyche can be seen through the metaphor of a chariot. The chariot is connected by shafts to the harnesses of the horses, and the driver of the chariot uses reins connected to the harnesses in order to direct the horses. The driver attempts to carry out the instructions of the owner. The chariot can be seen as representing the body, or in more modern terms, enaction. It needs to be maintained in order to function properly. The horses are traditionally seen as the senses, or more loosely perhaps, as affect. They are the motive (cf. emotive) power that pulls the chariot, and if uncontrolled will pull the chariot in unintended directions. Thus emotions have to

be harnessed, as well as fed and cared for. The shafts can be taken to represent habits and internalized automaticities, the direct connection between chariot and action. The reins are imagination, the means by which we direct our energies through our intentions. The driver is consciousness, cognition, or as I prefer, awareness. The owner is will.

Caleb Gattegno's adage that

> Only awareness is educable

is entirely consonant with and resonant with this chariot metaphor, which also invites extensions:

> Only behaviour is trainable
> Only emotion is harnessable.

To discuss the significance and force of the "only" would take me too far astray, but it has to do with the disturbance these assertions might initiate if they act as *protases* (initial statements of syllogisms) when juxtaposed with specific incidents from your own experience, leading to some sort of conclusion or insight (the syllogistic action) (see Mason, 1998; Mason & Johnston-Wilder, 2006). This is an example of how an action can be initiated in a teacher or among a group of teachers, and which may lead to a transformation of perspective and/or a change of practice or way of articulating both ideals and practices (Mason, 1998).

The three "onlys" can be used on several levels: As constructs which promote discernment of details contributing to the psychologizing of subject matter (as Dewey so cogently described it in 1933) prior to teaching (Mason & Johnston-Wilder, 2006); as reminder when interacting and responding to learners to inform pedagogic and content-based choices; as stimuli to initiate disturbance as part of professional development; and as a way of thinking about the role of attention and intention in learning about teaching through teaching.

Phenomena

Because of my phenomenological stance, the data I have to offer consists of your response to stimuli. These are phenomena abstracted from specific situations that I have experienced directly myself, or that I have recognized in other people's descriptions. Specific situations are turned into phenomena by recognizing something similar in some respects to other situations. Phenomena are then interrogated through a combination of relationship to theoretical constructs and probing of my own experience in similar situations. Often these similar situations are contrived so as to be similar. At the same time I develop and refine tasks, which are challenges to participants in workshops with the aim of seeing whether or to what extent they recognize the phenomena and the associated proposals or actions which might be useful in the future. Participants are then responsible for actualizing the imagined

actions and for developing and modifying them to suit their own conditions, ways of working, perspective, etc.

Phenomena A (Own World)

> In the last ten minutes of a problem solving class I suddenly become sharply aware of a 'fog of non-understanding' rolling forward towards me from the audience. In the moment, I choose to keep going rather than stop and regroup.

For me, this is an instance of being so caught up in expressing current insights or mathematical material clearly as to be unaware of what it is like to be in the audience. Similarly, a teacher can be so caught up in orchestrating lesson activities as to be unaware of what students might be experiencing, or so caught up in some agenda such as "making the work interesting" (i.e. relevant, engaging, sufficiently easy) that the mathematical content is submerged. Keeping students on task and occupied may keep them busy and give the impression of engagement, but it may not significantly contribute to any learning. I chose to keep going because of the timing of the session, but then began the next session in a retrospectively reflective mode.

Phenomenon B (Expressing to Others)

> I had a sense of how a proof could be constructed, but each time I tried to write it down, I encountered a difficulty. I found it difficult to make it fully convincing and at the same time clear enough to follow.

For me this is an instance of a general phenomenon where I think I understand something, but it is only when I try to explain it to someone else that I really get to grips with it; it is only when I plan to expound or to guide exploration that I need to be sure about details, variations, ramifications. When struggling with expressing justification for a conjecture, or even with clarifying a conjecture, it can be so helpful to find a colleague who listens politely but need say nothing. This familiar phenomenon may be the source of resonance which makes group discussion and collaboration so popular in socio-cultural and social-constructivist stances, even though it depends for effectiveness on students experiencing a desire to express. It is closely related to the notion of *self-talk* identified by Goffman (1959). Put another way, in a sense, I really learn mathematics through "teaching," that is, expounding or explaining to others. This makes me wonder if one of the ways I learn about teaching is by expressing my thoughts about teaching to others.

Phenomenon C (Training Behavior)

Training learners in the behaviors required to pass specific examinations appears to be helping them achieve their goals, but may be setting up deep trouble for the future. As Herbert Spencer put it,

> . . . what with the mental confusion produced by teaching subjects before they can be understood, and in each of them giving generalizations before the facts of which they are generalizations – what with making the pupil a mere passive recipient of others' ideas, and

> not in the least leading him to be an active inquirer or self-instructor . . . . Examinations being once passed, books are laid aside; the greater part of what has been acquired, being unorganized, soon drops out of recollection; what remains is mostly inert – the art of applying knowledge not having been cultivated; and there is little power either of accurate observation or independent thinking (Spencer, 1878).

The phenomenon of students having learned less than expected has probably been with us since people congregated in groups. I have drawn on Spencer because he was a major influence on Edward Thorndike and John Dewey in the USA and on Alfred Whitehead in the UK, particularly in paying attention to the experience of the learner. Whitehead (1932) used the notion of *inert* knowledge in his own critique of education, in order to account for observed "deficiencies" in student performance. It may be that attempts to teach have been ineffective by failing to appeal to students' logical reasoning, or more generally, by failing to engage students' interests through using and developing their natural powers of specializing and generalizing, conjecturing and convincing, stressing and ignoring, characterizing and classifying, etc. (Mason, 2008a). It may also be that, as John Dewey put it, the effective teacher *psychologizes the subject matter* (Dewey, 1933) by taking into account the ebb and flow of student energies (Dewey, 1913).

Phenomenon D (Hidden Assumptions)

I offered some people the following task:

> I am thinking of two numbers, whose sum is one. I square the larger and add the smaller, then I square the smaller and add the larger. Which of my two answers is likely to be the largest?

After some discussion I took a vote, and almost everyone conjectured that the results would be the same. I assumed that they had done the algebra or perhaps quickly tried a few special cases. A short while later a perplexed voice said "It works for 0 and 1 and those are the only numbers that sum to one!" They had interpreted *number* to mean non-negative integer!

Note: I am deliberately not using a diagram, even though it is part of my way of working on this task in workshops, because my concern here is with the hidden assumptions, not the resolution!

Interpretation or meaning-making is an automatic action. In particular, students interpret tasks so as to make sense of them and so as to be able to do something, and this may mean making implicit assumptions based on a perceived range of permissible change (Watson & Mason, 2005), as here. Teachers also interpret subconsciously when they consider a task for possible use in a lesson, or when they read curricular requirements and reform statements. They too "construe so as to make it possible to act."

The impact of the incident had emotional, cognitive, and behavioral ramifications for me. I noticed, marked, and here am remarking upon that incident (Mason, 2002). I quickly formed a mental image of myself working on this task in the future and guarding against this whole-number assumption. The next time I used the task I

found myself sufficiently informed by the past to work with participants on interpretation of *number* before presenting the task itself. I have intentionally learned from experience.

General Comment

Phenomena *A* (*Own World*) refers to examples of momentary awarenesses that I have had, and I expect others have had, usually with a fleeting sense of "I won't do that again" which then fades, leaving the way open to doing it again and again, itself a phenomenon that could be labeled *Never Again*. Phenomenon *B* (*Expressing to Others*) is a common experience amongst mathematicians and can be observed in lessons in which working practices are based on a conjecturing atmosphere. Phenomenon *C* (*Training Behavior*) accounts for what is commonly observed in classrooms, justified by the pressures teachers experience in the struggle to hold onto ideals while retaining student trust and respect. Many theories and frameworks in mathematics education attempt to explain and provide ways to counteract the tendency to emphasize short-term goals over long-term aims; the procedural over the conceptual (Hiebert, 1986); rote learning over understanding. Most curricular reform movements begin with long-term goals based on *understanding*, but over time become rigid through the development of habits and routines. Phenomenon *D* (*Hidden Assumptions*) is a simple example of actually learning from experience.

Never Again is the heart of the matter of learning to teach through the act of teaching. It is not easy to "learn from experience." For example, despite massive evidence from experience, it is hard to learn that acts of teaching neither cause nor guarantee learning. Yet somehow we keep on trying.

Experience

It is commonly said that *we learn by (or from) experience*. My own experience suggests that *one thing we rarely learn from experience is that we do not usually learn from experience alone*. Put another way,

> a succession of experiences does not add up to an experience of that succession

which resonates well with the proposal that

> a succession of feelings is not, in itself, a feeling of succession (James, 1890, p. 628 paraphrasing Immanuel Kant).

Just because I engage in mathematical activity, it does not follow that I am aware of the activity itself as a whole. As many teacher educators have found, some people are disposed to reflect on their experience, to pick out moments and ponder them, and others seem not to be so disposed. That does not mean that they do not process their experience, but if they do, they do it in some subtle manner which is hard to detect.

The sentiment is backed up by a plethora of wise utterances through the ages.

> What experience and history teach is this – that nations and governments have never learned anything from history, or acted upon any lessons they might have drawn from it. (Hegel, 1830/1975)
>
> If men could learn from history, what lessons it might teach us! But passion and party blind our eyes, and the light which experience gives is a lantern on the stern, which shines only on the waves behind us. (Coleridge, 1835, 20 Jan 1834)

An almost lone voice can be found in Roger Ascham, a medieval schoolmaster:

> Learning teacheth more in one year than experience in twenty: Ascham (1570).

We might rephrase this as *real learning* integrates experience and includes making sense of it, which is more informative than a lifetime of experience alone. Most generally, Oscar Wilde beautifully captures the whole point:

> Experience is the name [everyone] gives to their mistakes: Wilde (1893 act 3).

Phenomena *A* (*Own World*) summarizes examples of situations in which it is erroneously assumed that engagement or immersion in activity will produce a transformatory action which, in turn, will result in what is commonly referred to as learning, evidenced through changed behavior in the future. Something more is needed. In order for experience in the past to inform actions in the future, some transformation of attention, intention, and disposition is required.

Reflection

In mathematics education, one transformatory action has been variously referred to as *reflective abstraction* (Piaget, 1970, 2001), *reification* (Sfard, 1991, 1994), *compression* (Thurston, 1994), *retrospective learning* (Freudenthal, 1991, p. 118), *looking back* (Pólya, 1945), *reflection-on-action* (Schön, 1983), and so on and traced back to Locke (1710, Chapter VI) by von Glasersfeld (1991). But before all this retrospective work can be done, there has to be some data. Something has to be discerned and noticed in order to be recalled "in tranquillity." Tranquillity, or at least a change of state from local goal-oriented activity to a more global perspective is at least advantageous and often necessary for there to be any lasting impression. In order to provide energy for this transformation, a disposition to reflect or otherwise process past experience also seems necessary. Attention has to draw back from immersion in immediate action.

Distanciation

Distanciation appears to be a term coined by Bertolt Brecht (1948), which has crept into biblical exegesis (Carson, 1996) and which is currently infiltrating educational discourse. The essence, as the word suggests, is a pulling back from immersion in action in the moment, what Schön (1983) referred to as *reflection-in-action* as distinct from retrospective *reflection-on-action*. In mathematics education it seems to

capture the state of awareness-in-the-moment when something triggers the awakening of attention and you participate in a mathematical or pedagogical choice. The best way to prepare for distanciation in the midst of a teaching situation is through retrospective distanciation. An articulation of a systematic approach to this can be found in Mason (2002).

Construal Through Story Telling

One form of transformatory action which supports learning from experience is re-expressing things for yourself, as in *phenomenon B* (*Expressing to Others*). There is a commonplace adage that you only really learn when you try to teach, which I take as an articulation of the insight that it is when trying to express things for and from yourself that you really make contact with the content and clarify your own thinking. This fits with two of the six modes of interaction: *Expressing* and *expounding*. In expressing, there is something about the content (and your affective state) which simply has to be told to someone. Here the content can be seen as taking the initiative, the teacher as respected and experienced other mediates through being the reason as respected audience, and the student comes to appreciate more clearly. In expounding, you try to draw your audience into your own world of perceptions and conceptions. The fact of the audience (students or colleagues) amplifies the need to get things sorted out for yourself as teacher, so their presence (virtual, imminent, or actual) helps to bring you in contact with the subject matter, hopefully in a fresh way. When students take on an expository role, they are taking the role of teacher. Note that the usual meaning of *explaining* (to make plain) overlooks an important feature of this form of interaction, because it involves the teacher endeavoring to enter the world of the student by means of common concern about the content (Leinhardt, 2001). In effective explanation, the content mediates an interaction between student and teacher in which the teacher is drawn into the world of the student.

Two sets of distinctions (frameworks) which can inform pedagogical choices concerning learning from experience through story telling are *Do–Talk–Record* and *Manipulating–Getting-a-sense-of–Articulating* (Floyd et al., 1981; Mason & Johnston-Wilder, 2004). The first can act as a reminder that doing is amplified by talking about what you are doing (whether to yourself or to others) and talking clarifies so that recording becomes easier. Rushing to written records can actually obstruct meaning-making. The second can act as a reminder that the purpose of manipulating material objects, diagrams symbols, or virtual objects is to get a sense of relationships which may be instantiations of general properties. The goal is not to achieve the manipulation but to make contact with underlying generalities in the form of mathematical structure. Another way of approaching this is through the slogans *going with the grain* and *going across the grain* (Watson, 2000), which similarly act as a reminder that recognizing a pattern (going with the grain) is only a precursor to sense-making by examining what structure is revealed (going across the grain).

It is one thing to see students' manipulation of various objects as part of activity which may act as fodder for sense-making through constructing personal stories to account for what happens. It is perhaps another to see acts of teaching as fodder for teacher sense-making through trying to bring to expression, to articulate what is observed about student behavior, mathematical topics, mathematics more generally, and the management of effective interactions with students. Careful observation reveals that there can be moments of collective consciousness (Davis op cit.) as well as individual insights. The latter are of course notoriously unstable over time.

Construal through story telling is an age-old phenomenon. Egan (1986) used it explicitly to propose an alternative stance to teaching in primary schools; Bruner (1990, 1991) put forward the case that human beings are *narrative animals* and Norretranders (op cit.) exploits it in his critique of our illusions about the role of consciousness. We tell stories to ourselves and to other people. Most human narratives involve *accounting-for* actions (Mason, 2002) and are intended to reinforce the coherence of personal identity through the justification of acts. As Norretranders suggests, consciousness as the director of actions and the maker of choices is (often) an illusion. Consciousness lags behind enaction and affect in reacting to events. It takes explicit and significant work to bring consciousness or awareness into play in the moment when a choice is possible. The term *accounts-of* is intended to act as a reminder that what is most valuable is description which is sufficiently brief-but-vivid that others can recognize similar situations in their own experience. Explanations and theory-based *accounting-for* can be indulged in later once colleagues recognize the data they are being offered (accounts-of). As Husserl intimated "describe don't explain" is a basis for phenomenological enquiry. Perhaps the struggle to develop a scientific method within mathematics education could be seen as an attempt to disentangle *accounts-of* observations and *accounting-for* them.

Attention

At the heart of the matter of learning from experience is the person's attention. What is attended to, in what ways, and with what intention and disposition? (Mason 1982) If I am attending to the specifics of some sequence of actions, I may not be in a position to be aware of the fact of those actions or of that sequence, especially when I am being directed by a worksheet or following a prepared lesson plan. Here the *didactic transposition* beautifully captures part of the gap between teaching and learning, and the *didactic tension* describes how attention is directed towards accomplishing tasks as set, rather than using tasks to engage in activity which may promote experience from which it is possible to learn, whether as student or as teacher. Phenomena *A* (*Own World*) and *D* (*Hidden Assumptions*) provide access to specific examples. Teachers whose attention is fully occupied by organizing resources and managing classroom behavior are in a similar position. In order to learn about teaching through their teaching, they need to extract some of their attention to form an inner witness.

Most teachers will have experienced frustration that students don't seem to learn from experience, or that any such learning is short lived, as Spencer (*op cit*) observed in *phenomenon C* (*Training Behavior*). One way to account for this failure to learn from experience is to blame institutional factors (assessment, organization, curriculum pressures, etc.) or even to blame others (students, parents, policy makers). A more productive approach is to attempt to change what can be changed. What can be changed is yourself, because all other components in the auto-poetic system are even more problematic.

One issue is what to do about frustration. In parallel with students, it is most natural for teachers to be caught up in the details of getting through the day, setting tasks, sustaining activity, evaluating and assessing and keeping discipline, as in phenomena *A* (*Own World*). The result is that they too end the day with only a succession of experiences rather than an experience of that succession. Alerted to issues in student learning, and informed by distinctions such as those mentioned here, augmented by many others available in the literature it is always possible to work on changing the conditions in which students are learning. But it takes more than further instances of *Never Again*!

An intelligently designed educational system would recognize that the community as a whole, and the people involved in various roles, all need both time and stimulus to reflect, to reconstruct, and re-enter a succession of experiences in order to locate where their attention could most effectively be concentrated so as to break out of the many stable cycles which constitute what we currently call teaching.

This is where the human being as narrative animal is so important. The negative aspect is that because our psyche is fragmented, and because awareness is the slow mover compared to enaction and affect, we spin stories to account for the functioning of our multiple selves so as to make "our-selves" feel better, feel coherent, and unified. One explanation for the *Never Again* phenomenon is that the self who formulates the intention is not the self in charge later when the change is needed. A positive aspect is that it is possible to learn from experience by engaging all aspects of the psyche: Enaction, affect, awareness, attention, and intention. This requires work on self-integration, including the development of an inner witness or monitor who observes without acting, yet is able to divert attention out of immersion in activity. Again the roots go back to ancient times, such as this stanza from the Rg Veda:

> Two birds, close-yoked companions, both clasp the self-same tree.
> One eats of the sweet fruit; the other looks on without eating (Bennett, 1964, p. 108).

While some interpretations see the tree as the tree of immortality with people ignoring the potential by failing to eat of the sweet fruit, the tree can also be seen as the human psyche with immersion in action (eating of the sweet fruit) accompanied by an inner witness or monitor who merely comments on the action (the bird looking on). As with most Eastern teaching stories, multiple and conflicting interpretations are the source of significance and richness of meaning.

Story telling is one of the best ways to coordinate experiences, to draw back from immersion in activity, to reflect upon rather than simply engage in. By trying to articulate what has been noticed, by trying to re-enter salient moments, by trying to express connections, similarities, and differences between otherwise apparently disparate objects (mathematical concepts, objects, etc.; classroom incidents, mathematical obstacles, pedagogical and didactic choices, etc.), in short by assembling narratives, we can indeed withdraw from the action sufficiently in order to learn from experience. However, it is vital to avoid the negative side of narratives: Self justification, self-calming, and self-promotion. Explaining why some action was impossible or inappropriate so easily degenerates into draining away the energy that comes from noticing, from having attention suddenly sharpened through making distinctions, recognizing relationships and perceiving properties which are instantiated as relationships among particulars.

Reacting and Responding; Habit and Choice

Since learning through teaching involves experiencing some sort of disturbance to the flow of internalized and habitual actions, it is useful, as indicated in the introduction, to distinguish between *reacting* and *responding*. To respond is to make an intentional, conscious, considered choice of action. It is very rare. Usually we react. Something in a situation may trigger a metonymic association, and there may be resonance with some past experiences. Together these produce action. The body acts before consciousness is even aware. Evidence for this can be found in the most ancient of psychological studies such as the Upanishads, through personal investigation (see, for example, Mason, 2002, Chapter 12), or through neurological studies (Norretranders, 1998). Response requires awareness in addition to action.

Habits are the repetition of choices made previously, often a long time previously. The more deeply ingrained or internalized, the harder they are to counteract. But they only need to be counteracted if they are working against desire and intention. Indeed, sometimes they take place because desire and intention are in conflict. For example, the automaticities of arithmetic facts (e.g. single digit additions, multiplication tables, use of associativity, commutativity and distributivity, factoring) and algebraic manipulation are essential for making progress in mathematics. So too is the flexibility to move with facility between process and object (as in $^3/_4$ as a division and as the answer to a division). The same applies to acts of teaching such as using placement in the classroom to calm or quieten potentially noisy students, or waiting after asking a question. Habits and automaticities are essential, which is why Skinnerian stimulus-response (I prefer stimulus-reaction) works so well for training behavior.

Teaching which focuses on internalizing actions, so as to create automaticities is essentially training behavior. The trouble with training behavior alone is that it tends not to be flexible when conditions change. For example, training children to react to specific language patterns in word problems is more likely to blinker them

than to enrich their problem solving flexibility and creativity. Training behavior in concert with educating awareness, including "integration through subordination" (in Gattegno's memorable phrase, 1970), is much more powerful than either dimension alone. The procedural-conceptual dichotomy (Hiebert op cit.) arises from imbalanced evocation of the human psyche.

Various constructs have arisen as a result of failure to train behavior successfully. For example, the rise of constructs such as *transfer* (Detterman & Sternberg, 1993), *situated cognition* (Lave & Wenger, 1991; Watson & Winbourne, 2008), and *situated abstraction* (Noss & Hoyles, 1996) are attempts to account for why what seems to be learned in one context is not called upon or made use of in another (Marton, 2006).

Complication

Any impression that habits are necessarily bad and that flexibility is necessarily good need to be challenged. We cannot function without habits because the effort required to process every stimulus consciously and freshly would overwhelm even our complex brains. It is absolutely vital to internalize responses in order to be able to react quickly and effectively to common situations. If you have to pause and divert attention in order to carry out a simple computation or to simplify an algebraic expression, you are unlikely to plumb the depths of subtle structural relationships. However it is equally important to be able to question and modify some of these internalized automaticities, especially when they begin to obstruct rather than facilitate. One of the weaknesses of our current educational system is that we do not support adolescents in developing techniques for and dispositions to interrogate habits and reactions.

In mathematics, for example, it is vital when manipulating arithmetic or algebra to have a monitor awake to slips and ready to ask "Why are we doing this?" when the going gets tough and perhaps something has gone wrong. The same applies to trying out some new pedagogic strategy: Just because everything does not go swimmingly (notice the metaphor of immersion), there is no reason to abandon ship and conclude that the strategy cannot work, only that it did not work fully in that instance. This, in turn, parallels the way students label themselves and their efforts: "Did not" is much more helpful that "could not," which all too readily turns into "will not," creating mathematics-refusal behavior and affect (Dweck, 2000). Teachers working on their own teaching can also fall into this cycle of decline, by allowing "could not" to displace "did not," leading to "will not" and disaffection with trying to push boundaries and to respond sensitively to students.

Intention and Will

Attention is often described as the manifestation of will. Whatever we are attending to is "where we are" in that moment (James, 1890). It is what "we will it to be," though the experience is very often of being at the mercy of some outside influences

which attract our attention and tempt our horses. Intention is a major component of the psyche and a major factor in the effectiveness of teaching and learning, yet curiously difficult to get a handle on (Anscome, 1957). Seen as the means through which the will directs attention, intention is the guide behind the making of choices and the changing of habits, propensities, and dispositions to act, which are components of affect. In the metaphor of the human psyche as a chariot, will is the owner who sets goals for the driver (cognition) who manifests intention through the use of the reins to direct the horses, the sources of motive power.

Intention can usefully be seen as related to affect in the same way that awareness is related to enaction: Awareness guides enaction and enables actions to come to mind in response to problems when they arise; intention guides or flavors affect as energy arising from emotions so as to be directed productively. On the other hand, intention provides a cognitive component to accompany or to challenge desire, which is how energy is accessed and harnessed to make things happen. In that sense, intention is a cognitive dimension of will. Intention is often weaker than the flow of energies of desire, however. The best of intentions may be submerged or diverted by perceived exigencies; ideals may be compromised by pragmatic response to perceived conditions.

For example, everyone recognizes how resolutions (such as those made at New Year) are so quickly forgotten or over-ridden. This can be accounted for by seeing the self that made the resolution as being superseded by other selves with no such commitment. This is probably the most controversial dimension because it challenges the very notion of identity. Deeply embedded in ancient and modern psychology is the notion that the psyche of human beings is not single but composite. Versions can be found in Minsky (1975) and Hudson (1968), as well as in more esoteric sources such as Bennett (1964), Shah (1978), and many others. The idea is that in response to different local environments, we each develop distinct persona or selves. These selves then compete for control of the whole organism. For example, the self who is in charge when chairing a meeting at work is not the self who cooks dinner for the family: There are different dispositions and propensities, different sensitivities as to what is noticed and attended to, different actions skilfully executed, even different epistemological stances. Each self can be thought of as a network of energy flows, directing energy through different channels, thereby activating different collections of habits and actions (see also De Geest, 2006). In Plato's version of the chariot, the servants of a mansion compete to play the role of butler, or even try to usurp the role of the owner (will) who is away; the owner cannot return until the organization of the house is in order and functioning properly.

In order to strengthen the guidance of intention, people have long realized that it is necessary to provide some discipline, whether externally (and not usually effective in the long run) or internally.

The Rg Veda metaphor of the two birds suggests that one of the ways that the will–intention link can be strengthened is through the growth of an inner witness or monitor who, as it were, sits on your shoulder and asks "Why are we doing this?" or "What are we doing?" This, in turn, is fostered by intentional preparation by *prospective* planning based on *retrospective* reflection. The Discipline of

Noticing (Mason, 2002) is an articulation of techniques for enhancing the possibility of noticing opportunities in the moment and participating in a choice to act freshly.

Disciplined Development

The core issue in learning from any experience, and particularly in learning about teaching through teaching, is how to participate in actual choices rather than being dragged by the horses in whatever direction attracts them. Put more sharply, how can one be present in the moment when a choice is being made, an action initiated? Applied to teachers, an answer would shed light on why it is that despite espousing some stance towards teaching and learning, what is observed being enacted is often in considerable contrast. Applied to learners, it would shed light on why it is that despite having been shown or taught some technique or concept, students often do not make use of it when appropriate. This is the age-old problem of transfer: Just because I see the possibility of using technique *T* or concept *C* in some situation, what is it that could enable students to be similarly aware (Detterman & Sternberg, 1993, Marton, 2006)?

The chariot metaphor suggests that mental imagery (the reins) is important, together with maintenance of the chariot and the harness and contact between will and awareness. Put another way, attention and intention are crucial elements, the whole of which is encompassed in the Discipline of Noticing. As George Bernard Shaw put it:

> We are made wise not by the recollection of our past, but by the responsibility for our future. (Shaw, 1921, p. 250).

It is the desire to act differently "next time" that drives personal and professional development, but that desire (part of affect) has to be amplified in order to influence intention beyond the particular "self" experiencing the desire in the moment.

In brief, what is required is sufficient discipline to engage in three actions in order to facilitate the important action:

> Collecting pedagogic strategies, didactic tactics and other awarenesses that could inform practice if only they came to mind when needed.
> Engaging in retro-spective distanciation/reflection in order to amplify the energy released by noticing a missed opportunity, including intentionally re-entering moments as fully as possible without judgement or explaining away what happened, in order to locate actions that you wish in retrospect you had tried, or to locate specific relationships that seem problematic (Tripp, 1993);
> Engaging in pro-spective preparation by intensely imaging yourself in some future situation acting in some desired or intended manner.

The latter two actions call upon that fundamental human power of mental imagery: The ability to place ourselves in a remembered past and in an imagined future. The third is how will power is developed, slowly building on fragments of energy

released through noticing. Together all three actions contribute to enriching the possibility of having something "come to mind" through a combination of metonymic triggering and metaphoric resonance.

Externalizing and labeling salient moments in retrospect is part of bringing to expression, of story telling. Labels act as axes or foci around which experiences can gather (Mason, 1999). They can come to be associated with distinctions that might be informative, actions that might be relevant, and stances that might be fruitful. The value-laden terms being used here are of course relative to the value system of the individual within the encompassing institutional and hence socio-political milieu.

The combination of these three actions to produce the fourth is the essence of the Discipline of Noticing (Mason, 2002) which provides a detailed structure, method, and philosophical justification.

Maintenance

Following the chariot metaphor, the chariot or carriage, harness, shafts, and reins all require maintenance in order to function efficiently, and, of course, the horses themselves need looking after. Much could be made of this aspect of the metaphor, but suffice it to say here that work on dispositions and propensities (the horses) arises from work on developing the inner witness associated with awareness in conjunction with the strengthening of mental imagery (the reins). The components of the psyche are complexly inter-related beyond what is revealed by the carriage metaphor.

Skills need to be rehearsed because they may atrophy through lack of use. Habits need to be inspected and renewed or replaced as conditions change. This applies to dominantly cognitive skills such as specialized techniques in mathematics and topic specific didactic tactics, to dominantly affective skills such as holding still or poker-facing when a student makes a good conjecture and asks for validation, and to dominantly enactive skills such as classroom behavior management.

When pedagogic strategies and didactic tactics are tried, modified, and found to have potential, they need to be integrated into functioning. In the words of Gattegno (1970) "to integrate through subordination" is achieved through drawing attention out of and away from the carrying out of the skill. In this way, it truly becomes habit. If awareness is not extended and enriched (educated) then training provides a habit on which to call, but may not be sufficiently flexible to cope with changed conditions such as non-routine problems.

One message from this is that a regular practice of working on mathematics at your own level for yourself and with colleagues is an excellent way to rehearse skills, refresh awarenesses, and maintain the chariot! In a sense this is the companion to learning about teaching from and through teaching, because it provides a library of recent personal experience on which to draw and through which to be reminded about what students may be experiencing at their own level.

What Can Teachers Do?

Teachers are expected to do many things, but what is the core of what they can hope to do? I consider this question in relation to students and then in relation to themselves and their teaching apropos of learning about teaching from and through teaching.

What Can Teachers Do for Students?

Teachers can edit and amplify what students do and say (Hewitt, 1996). They do this by how they respond to students through reformulating and rephrasing, through reflecting specific elements, phrases, and objects back to a group of learners. They can pick out some expressions while ignoring others, and by using the same expressions over a period of time in order to enculturate students into using the discourse to express their own awareness. A conjecturing atmosphere, an ethos of enquiry and collegiality is one way to support and amplify this editorial role of a teacher.

By stressing and consequently ignoring teachers can direct student attention in ways which are indicative or representative of the way that mathematicians use their attention. It is a reasonable conjecture that each technical term, each definition, each technique, or method arose originally through a shift in what people attended to and how they attended to it (otherwise, there would neither have been nor be any struggle, any problematicity). Students usually have to experience some similar shift in attention in order to internalize and exploit the new concept or approach. So what a teacher offers is a more sophisticated awareness, more discerning distinctions, wider recognition of relationships, and more insightful perception of properties which are instantiated as relationships. They also display the kind of attention and the kind of reasoning that marks out mathematical thinking from other types of thinking.

Put another way, teachers display higher psychological processes (sometimes called "modeling behavior" but this is problematic because of what people attend to) which over time may be internalized by students, and there are various pedagogic devices for fostering and promoting internalization, including scaffolding and fading and the use of a range of pedagogic constructs (see Mason & Johnston-Wilder, 2006). One popular device for drawing student attention out of immersion in activity, and useful as a contribution to the fading aspect of scaffolding, is meta-questioning such as asking "What question am I going to ask you?" or "What am I going to suggest you do?" If, after a period of using the same or similar prompt, one of these meta-questions is used, student attention can be diverted to experience of the prompt rather than immediately reacting to it.

To learn about teaching through teaching is in large part to work on becoming ever more aware of what as a teacher it is possible to do for students, and to withdraw from trying to do things for students that actually block their learning. Examples include trying to do for students what they can already do for themselves, and usurping their powers in the name of efficiency by specializing and generalizing,

conjecturing and reasoning for them. This is not to say that the teacher never does these things, but rather, that when the teacher does these things, not to assume that the students can consequently now repeat them for themselves. Learning about teaching from teaching is a lifetime process of refining sensitivities to students and to the conditions in which learning is fostered and sustained.

What Can Teachers Do for Themselves?

There are parallel actions that teachers can do for themselves. First and foremost, they can maintain their interest and pleasure in mathematics by engaging in mathematics for themselves at their own level. The purpose is not to "learn more mathematics" but to sensitize themselves to the struggles that students experience. Working on mathematics for themselves is an instance of a more general program of engaging in retro-spection and pro-spection in order to support *spection*. This is both a process of alerting oneself to issues that may need probing and actions to take in order to promote responding freshly and more sensitively to situations that emerge when teaching, whether when planning or when engaging with students.

What Can Teachers Do for Each Other?

By collaborating in their enquiries, teachers can display higher psychological processes to and with each other so that as a collective they grow in and into community. They can, for example, edit and amplify descriptions of incidents so as to enrich labels that can serve as triggers to awaken their inner monitor and enable participation in choices in the moment. They can reflect specific elements back to each other or to themselves, drawing them out of the specific action and into states in which mental imagery can be used to prepare for future action. They can direct each other's attention to salient features so that finer distinctions can be made. The power and value of distinctions needs to be tested again and again, especially those that have become so ingrained that they activate even when not appropriate. It is easy to make distinctions, but distinctions which inform future effective action are not so readily located.

Conclusion

For me, Alfred Tennyson (1842) beautifully sums up the role and importance of experience:

> I am a part of all that I have met;
> Yet all experience is an arch wherethro'
> Gleams that untravell'd world whose margin fades
> Forever and forever when I move. [*Ulysses* lines 18–21]

To learn from or attend to experience in this way is to strive to move beyond that arch, an intention which re-inspires:

> And this gray spirit yearning in desire
> To follow knowledge like a sinking star,
> Beyond the utmost bound of human thought. [Lines 30–33]

By working together on mathematics and on pedagogical and didactic choices in a conjecturing and collegial atmosphere, teachers can indeed learn about teaching from and through teaching. But no one says that it is easy!

References

Anscome, G. (1957). *Intention*. London, UK: Harvard University Press.

Ascham R. (1570). *The scholemaster. Or plaine and perfite way of teachyng children, to vnderstand, write and speake, the Latin tong*. Printed for W. Innis at the west end of St. Paul's street and S. Birt in Ave-Maria Lane.

Bennett, J. (1956–1966). *The dramatic universe* (four volumes). London, UK: Hodder & Stoughton.

Bennett, J. (1964). *Energies: Material, vital, cosmic*. London, UK: Coombe Springs Press.

Brecht, B. (1948). *Petit Organon pour le theatre*. Paris, France: Éd. De L'Arche.

Brousseau, G. (1997). *Theory of didactical situations in mathematics: didactiques des mathematiques, 1970–1990*. In N. Balacheff, M. Cooper, R. Sutherland, & V. Warfield, (Trans.), Dordrecht, The Netherlands: Kluwer.

Brown, R. (1973). *A first language*. Cambridge, MA: MIT Press.

Brown S., Collins A., & Duguid P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, *18*(1), 32–41.

Bruner, J. (1986). *Actual minds, possible worlds*. Cambridge, MA: Harvard University Press.

Bruner, J. (1990). *Acts of meaning*. Cambridge, MA: Harvard University Press.

Bruner, J. (1991). The narrative construction of reality. In M. Ammanti & D. Stern (Eds.), *Rappresentazioni e narrazioni* (pp. 17–42). Roma-Bari: Laterze.

Carson, D. (1996). *Exegetical fallacies*. Carlisle, UK: Paternoster.

Chevallard, Y. (1985). *La transposition didactique*. Grenoble, France: La Pensée Sauvage.

Claxton, G. (1984). *Live and learn: An introduction to the psychology of growth and change in everyday life*. London, UK: Harper and Row.

Coleridge, S. (1835). *Specimens of the table talk of the late Samuel Taylor Coleridge*. London: John Murray.

Davis, B., & Sumara, D. (2007). Complexity science and education: Reconceptualizing the teacher's role in learning. *Interchange*, *38*, 53–67.

De Geest, E. (2006). *Energy transactions in the learning of mathematics*. Unpublished PhD Thesis, Milton Keynes, UK: Open University.

Detterman, D., & Sternberg, R. (Eds.) (1993). *Transfer on trial: Intelligence, cognition, and instruction*. Norwood, NJ: Ablex.

Dewey, J. (1933). *How we think*. London: D.C. Heath & Co.

Dewey, J. (1913). *Interest and effort in education*. Boston, MA: Houghton Mifflin. Reprinted by Kessinger Publishing – Rare Reprints of Hard to Find Books.

Donne, J. (1624). *Deuotions vpon emergent occasions, and seuerall steps in my sicknes*. London: Thomas Iones.

Dweck, C. (2000). *Self-theories: Their role in motivation, personality and development*. Philadelphia, PA: Psychology Press.

Egan, K. (1986). *Teaching as story telling: An alternative approach to teaching and curriculum in the elementary school*. Chicago, IL: University of Chicago press.

Festinger, L. (1957). *A theory of cognitive dissonance*. Stanford, CA: Stanford University Press.
Floyd, A., Burton, L., James, N., & Mason, J. (1981). *EM235: Developing mathematical thinking*. Milton Keynes, UK: Open University.
Freudenthal, H. (1991). *Revisiting mathematics education*: *China lectures*, Dordrecht, The Netherlands: Kluwer.
Gattegno, C. (1970). *What we owe children: The subordination of teaching to learning*. London, UK: Routledge & Kegan Paul.
Goffman, E. (1959). T*he presentation of self in everyday life*. Garden City, NY: Anchor Books.
Hegel, G. (1830/1975) H. Nisbet (Trans.) *Lectures on philosophy of world history: Introduction*. Cambridge, UK: University of Cambridge Press.
Hewitt, D. (1996). Mathematical fluency: The nature of practice and the role of subordination, *For the Learning of Mathematics*, *16*, 28–35.
Hiebert, J. (1986). *Conceptual and procedural knowledge: The case of mathematics*. Hillsdale, NJ: Erlbaum.
Hudson, L. (1968). *Frames of mind*. London, UK: Methuen.
James, W. (1890 reprinted 1950). *Principles of psychology* (Vol. 1). New York, NY: Dover.
Langer, E. (1997). *The power of mindful learning*. Reading: Addison Wesley.
Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.
Leinhardt, G. (2001). Instructional explanations: A commonplace for teaching and location for contrast. In V. Richardson (Ed.). *Handbook of research on teaching* (4th edn., pp. 333–357). Washington, DC: American Educational Research Association.
Locke, J. (1710). In P. Nidditch (Ed.), *Essay on human understanding* (1975). Oxford, UK: Clarendon Press,
Love, E., & Mason, J. (1992). *Teaching mathematics: Action and awareness*. Milton Keynes, UK: Open University.
Marton, F. (2006). Sameness and difference in transfer. *Journal of the Learning Sciences*, *15*, 499–535.
Mascaró, J. (Trans.) (1962). *The Bhagavad Gita*. Harmondsworth, UK: Penguin.
Mason, J. (1979). Which medium, which message. *Visual Education*, *2*, 29–33.
Mason, J. (1982). Attention. *For the Learning of Mathematics*, *2*(3), 21–23.
Mason, J. (1998). Protasis: a technique for promoting professional development, in C. Kanes, M. Goos, & E. Warren (Eds.) *Teaching mathematics in new times: Proceedings of MERGA 21* (Vol. 1, pp. 334–341). Australasia: Mathematics Education Research Group of Australasia.
Mason, J. (1999). The role of labels for experience in promoting learning from experience among teachers and students. In L. Burton (Ed.), *Learning mathematics: From hierarchies to networks* (pp. 187–208). London: Falmer.
Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London, UK: Routledge Falmer.
Mason, J. (2008). Making use of children's powers to produce algebraic thinking. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 57–94). New York, NY: Lawrence Erlbaum.
Mason, J. (2008a). From concept images to pedagogic structure for a mathematical topic. In C. Rasmussen & M. Carlson (Eds.), *Making the connection: research to practice in undergraduate mathematics education*. MAA Notes (pp. 253–272). Washington, DC: Mathematical Association of America.
Mason, J. (2009). From assenting to asserting. In O. Skvovemose, P. Valero & O. Christensen (Eds.), *University science and mathematics education in transition* (pp. 17–40). Berlin, Germany: Springer.
Mason, J. (2009). Teaching as disciplined enquiry. *Teachers and Teaching: Theory and Practice*, *15*(2–3), 205–223.
Mason, J., Burton, L., & Stacey, K. (1982). *Thinking mathematically*. London, UK: Addison Wesley.

Mason, J. Drury, H., & Bills, E. (2007). Explorations in the zone of proximal awareness. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice. Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia*. (vol. 1, pp. 42–58). Adelaide SA, Australia: MERGA.

Mason, J., & Johnston-Wilder, S. (2004). *Fundamental constructs in mathematics education*. London, UK: Routledge Falmer.

Mason, J., & Johnston-Wilder, S. (2006). *Designing and using mathematical tasks* (2nd edn.). St. Albans, UK: Tarquin.

Maturana, H., & Varela, F. (1972). *Autopoesis and cognition: The realization of the living*. Dordrecht, The Netherlands: Reidel.

Maturana, H., & Varela, F. (1988). *The tree of knowledge: The biological roots of human understanding*. Boston, MA: Shambala.

Minsky, M. (1975). A framework for representing knowledge. In P. Winston, (Ed.) *The psychology of computer vision* (pp. 211–280). New York, NY: McGraw Hill.

Movshovitz-Hadar, N. (1988). School mathematics theorems – an endless source of surprise. *For The Learning of Mathematics*, *8*(3), 34–40.

Norretranders, T. (1998). (J. Sydenham Trans.). *The user illusion: Cutting consciousness down to size:* London, UK: Allen Lane.

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings: Learning cultures and computers*. Mathematics Education Library, Dordrecht, The Netherlands: Kluwer.

Piaget, J. (1970). Genetic Epistemology. New York: Norton.

Piaget, J. (1971). *Biology and knowledge*. Chicago, IL: University of Chicago Press.

Piaget, J. (1985). *The equilibration of cognitive structures*. Chicago, IL: University of Chicago Press.

Piaget, J. (2001). *Studies in reflective abstraction*. Sussex, UK: Psychology Press.

Pirie, S., and Kieren, T. (1994). Growth in mathematical understanding: How can we characterize it and how can we represent it? *Educational Studies in Mathematics*, *26*(2–3), 165–190.

Pólya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton: Princeton University Press.

Rhadakrishnan S. (1953). *The principal upanishads*. London, UK: George Allen & Unwin.

Schön, D. (1983). *The reflective practitioner: How professionals think in action*. London, UK: Temple Smith.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics. 22*, 1–36.

Sfard, A. (1994). Reification as the Birth of Metaphor, *For the Learning of Mathematics*, *14*(1), 44–55.

Shah, I. (1978). *Learning how to learn*. London, UK: Octagon.

Shaw, B. (1921). *Back to Methuselah*. London, UK: Constable.

Skinner, B. F. (1954). The science of learning and the art of teaching. *Harvard Educational Review*, *24*(2), 86–97.

St. Maurice H. (1991). A Guide to Commonplaces: On the use of loci in educator's discourse. *Journal of Curriculum Studies*, *2–3*(1), 41–54.

Tennyson, A. (1842). *Poems, In two volumes*. London, UK: Moxon.

Thurston, W. (1994). Proof and progress in mathematics. *Bulletin of the American Mathematical Society*, *30*(7), 161–177. Reprinted in 1995 by *For the learning of mathematics*, *15*(1), 29–37.

Tripp, D. (1993). *Critical Incidents in Teaching: Developing professional judgement*. London, UK: Routledge.

van der Veer, R., & Valsiner, J. (1991). *Understanding Vygotsky*. London, UK: Blackwell.

Vygotsky L. (1978). *Mind in Society: The development of the higher psychological processes*. London, UK: Harvard University Press.

Von Glasersfeld, E. (1991). Abstraction, re-presentation, and reflection. In L. Steffe (Ed.), *Epistemological foundations of mathematical experience* (pp. 45–67). New York, NY: Springer.

Watson, A., & Mason, J. (2005). *Mathematics as a constructive activity: Learners generating examples*. Mahwah, NJ: Erlbaum.
Watson, A., & Winbourne, P. (2008). *New directions for situated cognition in mathematics education*. New York, NY: Springer.
Watson, A. (2000). Going across the grain: Mathematical generalization in a group of low attainers, *Nordisk Matematikk Didaktikk*, *8*, 7–22.
Whitehead, A. (1932). *The aims of education and other essays*, London: Williams and Norgate.
Wilde, O. (1893). *Lady Windermere's fan: A play about a good woman*. London, UK: Elkin Mathews and John Lane.
Wood, D., Bruner, J., & Ross, G. (1976). The role of tutoring in problem solving, *Child Psychology*, *17*, 89–100.