CHAPTER I

MATHEMATICAL REALISM AND TRANSCENDENTAL PHENOMENOLOGICAL IDEALISM

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Abstract. In this paper I investigate the question whether mathematical realism is compatible with Husserl's transcendental phenomenological idealism. The investigation leads to the conclusion that a unique kind of mathematical realism that I call "constituted realism" is compatible with and indeed entailed by transcendental phenomenological idealism. Constituted realism in mathematics is the view that the transcendental ego constitutes the meaning of being of mathematical objects in mathematical practice in a rationally motivated and non-arbitrary manner as abstract or ideal, non-causal, unchanging, non-spatial, and so on. The task is then to investigate which kinds of mathematical objects, e.g., natural numbers, real numbers, particular kinds of functions, transfinite sets, can be constituted in this manner. Various types of founded acts of consciousness are conditions for the possibility of this meaning constitution.

The main question I would like to address in this paper is this: is mathematical realism compatible with transcendental phenomenological idealism or not? In the discussion that follows I will use the expressions "mathematical realism" and "mathematical platonism" interchangeably. In a moment I will be much more specific in speaking about both mathematical realism and transcendental phenomenological idealism but, for now, let me just say that I am mainly interested in forms of mathematical realism that have appeared in the recent literature in the philosophy of mathematics, and that in speaking of transcendental phenomenological idealism I am thinking of the philosophical view that Husserl began to develop around 1907 or so, and that appears especially in works such as *The Idea of Phenomenology*, *Ideas I*, *Cartesian Meditations*, Part II of *Formal and Transcendental Logic*, and portions of the lectures on *The Phenomenology of the Consciousness of Internal Time* (Husserl 1991).

The division between realism and idealism in philosophy has of course a long history. For my purposes in this paper, it is in Kant's transcendental philosophy that we find the most important approach to the realism/idealism debate prior to Husserl. In his *Critique of Pure Reason*

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Kant thought he could reconcile empirical realism with his transcendental idealism. Kant was not what we nowadays think of as a mathematical realist (platonist), and he was in fact critical of platonism in general (see Kant 1973 A4/B8-A6/B10). For Kant there would not have been a question of trying to reconcile mathematical platonism and transcendental idealism. By 1907 or so, however, it is quite possible to read Husserl as attempting to do such a thing. I would say that it is thanks to Kant that we can consider the possibility of reconciling realism with transcendental idealism at all, and it is thanks to Husserl that we can consider the possibility of reconciling mathematical realism and transcendental phenomenological idealism.

In Husserlian phenomenology there is an old division between supporters of realism and supporters of transcendental idealism, and this division has its roots in the changes in Husserl's thinking that, as indicated above, began to take place around 1907. There has been a line of thinking according to which realism in phenomenology is incompatible with transcendental idealism in phenomenology. You must choose one or the other. In my view, however, this issue of the compatibility or incompatibility deserves further study. In particular, the issue has not been explored deeply enough in connection with the Kantian background. It also has not been explored fully enough in the case of mathematics, and especially in connection with developments that occurred in the foundations of mathematics after Husserl began to lose touch with this area of research. Thus, what I would like to do in this paper is to (i) characterize some recent forms of mathematical realism, (ii) present some of the core claims of transcendental phenomenological idealism from *Ideas I* and other writings, and then (iii) examine in more detail some of the issues about the compatibility of mathematical realism and transcendental phenomenological idealism.

The starting positions are these: either you can be a mathematical realist and not a transcendental phenomenological idealist, or you can be a transcendental phenomenological idealist and not a mathematical realist, or you can in some sense be both. You can certainly be neither. There are many philosophers who would embrace neither view. Husserl's own early work (prior to roughly 1900) arguably falls into this latter category (see, e.g., Hua 12; Hua 21; Husserl 1994; also Tieszen 2004). I will not discuss this option here, however, since it puts us outside the circle of ideas in Husserl's later work that I want to discuss.

Let me now give a brief opening characterization of mathematical realism and, on the other hand, of a standard idealism (or anti-realism) about mathematics.

I. Standard Simple Formulations of Realism and Idealism (Anti-Realism) About Mathematics

Mathematical Realism: There are mind-independent abstract (or "ideal") mathematical objects or truths. Notice that I am formulating this specifically for *mathematical* objects or truths. By "mathematical" I just mean the kinds of objects or truths that practicing mathematicians typically take themselves to be thinking about. This includes geometric objects, natural numbers, real numbers, complex or imaginary numbers, functions, groups, sets, or categories, and truths about these objects. I do not necessarily want to exclude other kinds of objects that platonists might take to exist, such as meanings, propositions, properties, concepts, or essences, but I do want to note that mathematicians themselves (unlike some logicians) do not typically take themselves to be talking directly about such things in their theories. Logicians who are platonists are more likely to talk about such things. I do not want to make too much of the difference at the moment but I will note that at least part of what is involved here is that propositions, properties, concepts, essences, and the like are usually thought of as overtly *intensional* objects, whereas this is not typical in the case of standard mathematical objects in classical mathematics. We should note, in any case, that one can be a platonist about extensional objects, intensional objects, or both. Some platonists who recognize both kinds of objects might also prioritize the relationship between the two, holding that one kind of object is derivable from or dependent on the other.

A standard formulation of idealism or anti-realism about mathematics is now very easy to come by. Simply negate the formulation of mathematical realism: It is not the case that there are mind-independent abstract (or "ideal") mathematical objects or truths. Putting this as a positive statement, *Mathematical Idealism (Anti-Realism)*: Mathematical objects (which may be "abstract" in some sense but not eternal or atemporal) are mind-dependent.

On these formulations, mathematical realism and standard mathematical idealism (which is distinct from transcendental phenomenological idealism) are incompatible. They are incompatible at a level of generality that spares us the need to consider any further details.

Now if we had captured the essential features of mathematical realism and transcendental phenomenological idealism in these formulations then we would have an answer to our question and I could conclude this paper. Needless to say, I think we have hardly scratched the surface. Therefore, let us consider mathematical realism in somewhat more detail.

II. Mathematical Realism

The mind-independent abstract (or ideal) mathematical objects that are thought to exist by mathematical realists are usually taken to have the following properties. As the formulation obviously indicates, they are *mind-independent*. This means several things. First, they are not themselves mental entities. They are not the subjective ideas or thoughts or images of human beings. They are not immanent to human consciousness but they are supposed to *transcend* human consciousness. They are not internal to human consciousness but are in some sense external to it. They are supposed to exist whether there are minds in the universe or not. They would exist even if there were no minds or had never been any minds. The properties of "being expressed" or "being thought of" are not essential to mathematical objects. Mathematical objects are external to human consciousness but not in the sense of sensory, physical or material objects. This is what it means to say they are *abstract*. (Note that I'm using the term "abstract" as it is often used in the recent literature in the philosophy of mathematics, not in the sense of Husserl's theory of parts and wholes in which non-independent parts ("moments") of a whole are said to be "abstract.") To say they are abstract is to say that they are not spatial in nature, not involved in causal relations, as material objects presumably are, and not the kinds of objects that can be sensed with one or more of our five senses. "Concrete" objects, however, would have all of these latter properties. Not only are mathematical objects not in (physical) space but they are also not in time. Unlike objects in physical space or even the objects of "inner sense" (i.e., mental processes, thoughts, images, etc.), they do not have a temporal extension. They are not, as Plato would have said, subject to generation and decay. They are, on the contrary, unchanging. Some platonists say they are "eternal" or "timeless." As I will note below, Husserl has interesting things to say about the relationship of abstract objects to time.

Starting in 1900, in the *Logical Investigations*, Husserl draws a sharp distinction between *real* and *ideal* objects. Although this distinction is not widely used in the recent literature on mathematical realism it will be useful to note its relationship to some of the current terminology. The first thing to note is that the "ideal" in this distinction does not refer to "ideas" in a subjective sense. It does not refer to mental entities. It is rather a platonic use of "idealism" that is operative in this case and not, in spite of the potentially confusing language, the use involved in the realism/idealism division. The real/ideal distinction can be drawn in terms of the temporality of objects. Real objects are objects in time. They have temporal duration. This applies to the objects of "inner sense," i.e., thoughts, mental processes, and the like, but also to objects of "outer sense," i.e., objects in space and in external time. Ideal objects are not in time in the same sense. They do not come into being and pass away. Much of what I have said about abstract objects applies directly to the ideal objects that Husserl introduces in the *Logical Investigations*. Mathematical objects, as ideal in Husserl's sense, are not, as I indicated, abstract parts (moments) of real objects. Non-independent parts of real objects are just real parts even though we can speak and think of them in isolation from the wholes of which they are parts. This does not mean, however, that they can *exist* in isolation from the wholes of which they are parts. If Husserl is to be a mathematical realist (platonist) in the sense described above then mathematical objects, as ideal, could not depend for their existence on underlying real wholes. They must exist independently of real objects. Otherwise, Husserl's view would be closer to an Aristotelian realism. There are many remarks in the *Logical Investigations*, especially Investigations II and VI, and elsewhere in Husserl's later writings that indicate that he is not in this sense an Aristotelian realist about mathematical objects. § 52 of Investigation VI, for example, is entitled "Universal objects and their self-constitution in universal intuitions." In this section he contrasts the kind of abstraction involved in setting into relief a non-independent moment of a sensible object with *ideational abstraction* in which an *idea* or universal, not a non-independent moment, is brought to consciousness.

I briefly mention one other feature of the real/ideal distinction that is not always salient in the distinction between the concrete and the abstract. The real/ideal distinction embodies the difference between the inexact and the exact, or the imperfect and the perfect. This feature of Husserl's distinction has a distinctively platonic pedigree that is omitted from some modern versions of mathematical realism. Plato's forms were supposed to be perfect in relation to their imperfect or inexact instantiations in the material world. In relation to logic and mathematics, the idea is that logical or mathematical objects are exact and "perfect" in a way that instantiations of, expressions for, or thoughts about such objects cannot be. In Euclidean geometry, for example, the lines, triangles, circles, and so on, are supposed to be perfect or exact in a way in which drawings of circles and the like, which we can perceive visually, could never be. A globe, which we hold in our hands, could never be exact and perfect in the way that a sphere in Euclidean space is conceived to be perfect or exact. The instantiations can only approximate the ideal.

This will be enough for now about the general properties that mathematical objects are supposed to possess for the mathematical realist. Further specifications along different lines are possible, and I would now like to mention one such set of specifications that is, I think, quite important. Mathematical realists could agree with everything that has been said thus far about their realism and yet disagree about *which* mind-independent abstract or ideal mathematical objects exist. Among the types of mathematical objects about which one might be a realist are geometric objects of different kinds, natural numbers, real numbers, complex or imaginary numbers, sets of different kinds, functions of different kinds, groups, or categories. One might be a reductionist or eliminativist about some of the items on this list. For example, one might adopt a realist view about natural numbers but not about real numbers or imaginary numbers. One popular strategy has been to recognize the existence only of sets and then to define some of the other objects on the list in terms of sets.

Modern set theory is of special interest in connection with mathematical realism for a number of reasons. One of the principal reasons is that it compels philosophers to confront a distinctive and relatively new set of epistemological and ontological issues about mathematical realism. These are issues, by the way, which either emerged after Husserl's time or to which Husserl himself devoted very little if any attention. Modern set theory forces the mind-independence issue in a striking way. Human minds are finite and have finite capacities. Objects such as natural numbers are finite objects. Even if the human mind cannot actually grasp or form very large natural numbers we can *idealize* the notion of *finite capacity* to cover the grasp or formation of each natural number, thus imagining that there could be a *complete* grasp of each natural number. In modern set theory, however, we are faced with existence statements about huge transfinite sets. Suppose, for example, that we consider some of the existence axioms in Zermelo-Fraenkel set theory with the axiom of choice; in particular, the axioms of infinity, power set, and replacement. These latter three axioms allow us to show rather quickly that very large transfinite sets exist. Not only will denumerably infinite sets exist but also non-denumerably infinite sets will exist, and then power sets of non-denumerably infinite sets, and so on. There is a significant disanalogy with the case of natural numbers: we cannot idealize the *finite* mind or *finite* capacities in such a way as to cover the grasp or formation of such transfinite objects. Transfinite sets transcend the possibility of being known on the basis of acquaintance with all of their members. A much more substantial idealization has to be involved. If we return to our simple formulation of mathematical realism then, in connection with set theory, we should ask whether there are mind-independent abstract *infinite* objects. In particular, are there *actual, complete infinite sets*?

Many additional details come into focus once the question of realism about set theory emerges. There are of course the traditional worries about the axiom of choice. Furthermore, with the replacement axiom we also have impredicative specification of sets. Should we therefore hold as part of our mathematical realism that impredicatively specified transfinite sets exist or not? Should we recognize only the existence of predicatively specified sets and hold to only a predicative set theory?

Some philosophers and mathematicians, such as Gödel in his later work, are prepared to be realists about full impredicative set theory with the axiom of choice. Indeed, they might be prepared to adopt a realism that goes beyond the existential commitments of a theory such as ZFC, arguing for the need for new axioms to express more of what already exists in the universe of abstract, mind-independent transfinite sets. Gödel suggests that the search for new axioms depends on sharpening or clarifying our intuition of the concepts concerning this existing realm of objects or truths (see Gödel [1964] 1990; and Wang 1974, 189).

Now let me make some comments about transcendental phenomenological idealism.

III. Transcendental Phenomenological Idealism

What has been called transcendental phenomenological idealism emerges in the writings of Husserl in which he introduces the phenomenological reduction or *epoché*, starting around 1907. *The Idea of Phenomenology* (1907) (Husserl 1964) is a particularly interesting text because Husserl says in it that the way to solve the old, vexing philosophical problem of how we can be related to transcendent objects is through the phenomenological reduction. The only way to solve this problem is from within the reduction. This is perhaps the reason why Gödel refers to *The Idea of Phenomenology* as a "momentous lecture" (see Item 050120.1 in the Gödel *Nachlass*, Firestone Library, Princeton University).

What the reduction shows us, to a first approximation, is how to restrict ourselves in a non-naturalistic manner to the sphere of appearances, to what is immanent and absolute. How does it do this? In *Ideas I* the epoché is motivated by way of some comparisons with Descartes' method of doubt. This Cartesian approach to explicating the phenomenological reduction can be contrasted with other paths to the reduction in later writings, such as the path indicated in the *Crisis of the European Sciences and Transcendental Phenomenology (Crisis)*. In *Ideas I* Husserl notes that the reduction is not the same thing as the Cartesian method of doubt but the Cartesian method, even though it was intended for different purposes, can get us into the neighborhood of what he wishes to obtain. The epoché, for example, plays no role in establishing substance dualism but is used instead to make us aware in a non-naturalistic way of mental phenomena as phenomena. As Husserl says in the *Cartesian Meditations*, Descartes did not make the transcendental turn (§ 10). As he says in *Formal and Transcendental Logic* (§ 100), Kant did make the transcendental turn but he neglected to carry it out with respect to the ideal objects of logic and mathematics. In the Introduction to *Formal and Transcendental Logic* Husserl says that mathematics and logic are positive sciences that require a foundation in transcendental phenomenology. What the modern sciences lack is a true logic, i.e., a transcendental logic that investigates the cognition behind science and thereby makes science understandable in all its activities. This logic does not intend to be a mere pure and formal logic, a *mathesis universalis*, for while *mathesis* may be a science of logical idealities it is still only a "positive" science. Transcendental phenomenology should bring to light the system of transcendental principles that gives to the sciences the possible sense of genuine science. The positive sciences are completely in the dark about the true sense of their fundamental principles. Transcendental phenomenology is supposed to make it understandable how the positive sciences can bring about only a relative, one-sided rationality.

What can be accomplished with the phenomenological reduction, which is fundamental for transcendental phenomenology, is this: as we attempt to doubt everything we notice that in fact not everything is doubtful (see *The Idea of Phenomenology*, Hua 2, 23; *Ideas I*, § 31). If I think that everything is doubtful then while I am thinking that everything is doubtful it is indubitable that I am so thinking. In every case of a definite doubt it is indubitable that I am having this doubt. The same is in fact true of every instance of cognition. If I am perceiving or judging, for example, then whether these activities are veridical or not, whether they have objects that exist or not, it is nonetheless clear that I am *perceiving* this or that, or *judging* this or that. The awareness that I am perceiving or judging implies that I have the capacity to *reflect* on my cognitive activities. In this reflection something is given to me that I cannot doubt. It is given, Husserl says, "absolutely" and with certainty. In this manner we are able to find a way to focus on what appears to us, just as it appears. If we are conscious we cannot doubt that something or other appears to us in our cognitive activities but of course we can very well doubt that what appears in the appearing is actually the case. In this manner, we can affect a "suspension" or "bracketing" of the (natural) world and everything in it. This means that we also bracket the natural, psychophysical ego or self, the self that is the object of natural science. The ego that is directed toward objects after the reduction is the "transcendental ego."

The method is thus to restrict ourselves to what is "immanent," to disengage from the natural attitude in which we naively and without reflection take ourselves to be experiencing transcendent objects. In the phenomenological attitude, obtained by the reduction, we experience, on the basis of reflection, the immanent. Husserl then goes on to say that the immanent is absolute, while the transcendent is not. What is transcendent is always relative to consciousness.

Many passages in *Ideas I* express the new transcendental idealism that results from taking the epoché seriously. In § 46, for example, Husserl argues that any physical thing that is given "in person" can be non-existent but that no mental process which is given "in person" can be non-existent. The non-existence of the world is conceivable but the existence of what is immanent—the absolute being of mental processes—would in no respect be altered thereby. In fact, there is a distinct manner, in which mental processes would always remain presupposed in any effort to doubt the existence of various phenomena. Consider the case, which is certainly possible, in which a perception is corrected by a subsequent perception. Now imagine that this process of correction continues to occur. In § 49 of *Ideas I* Husserl says that it is conceivable, due to such conflicts, that experience might dissolve into illusion not only in detail but globally. In this case no natural world would be constituted in our experience. There would be no experience of a natural world but in all of this there would still be consciousness. Consciousness would indeed be necessarily modified by the "annihilation" of the world of physical things but its own existence would not be touched. The absolute being of the mental processes would in no way be altered thereby. Thus, in § 47 Husserl says that "no limits check us in the process of conceiving of the destruction of the Objectivity of something physical—as the correlate of experimental consciousness." Whatever things are, they are as experienceable things. It is experience alone that prescribes their sense. We must not let ourselves be deceived by speaking of the thing naively as something that transcends consciousness and exists in itself, apart from any possible relation to consciousness. The genuine concept of transcendence can only be derived from the contents of our experience itself. "*An object existing in itself is never one with which consciousness or the ego pertaining to consciousness has* *nothing to do*" (§ 47). In § 49 Husserl says that the whole spatiotemporal world and each of its constituents is thus, according to its sense, a merely intentional being. It is a being posited by consciousness in its experiences. Each constituent of the world, of essential necessity, can be determined and intuited only as something identical through motivated multiplicities of appearances. It is something invariant for consciousness through a manifold of appearances. Beyond that it is nothing.

This sphere of absolute consciousness that remains as a residuum after the conceivable annihilation of the world is what provides the subject matter for pure phenomenology. From this point of view, Husserl says, we think of all reality as existent by virtue of a *sense-bestowing* consciousness which, for its part, exists absolutely and not by virtue of another sense-bestowal. Consciousness *constitutes* the sense of objectivity. Although this is a form of idealism it is not, Husserl says, a Berkeleyan subjective idealism. Rather, it is transcendental-phenomenological idealism. It recognizes that not everything is constituted as a mental phenomenon and it also recognizes the role of the overlapping horizons of different egos in the constitution of a common, objective world.

In Part II of *Formal and Transcendental Logic*, in the context of his investigations of logic, Husserl says similar things. Transcendental phenomenological idealism is represented in *FTL* as the view that it is only in our own experience that things are "there" for us, given as what they are, with the whole content and mode of being that experience attributes to these things. In § 94 of this work Husserl says that "nothing exists for me otherwise than by virtue of the actual and potential performance of my own consciousness." Whatever I encounter as an existing object is something that has received its whole sense of being from my intentionality. Illusion also receives its sense from me. Experience teaches me that the "object" could be an illusion. Objects can be thought of as intentional *poles of identity* through the manifold activities of consciousness. There is no conceivable place where the life of consciousness could be broken through so that we might come upon a transcendent object that had any other sense than that of an intentional unity making its appearance in the subjectivity of consciousness. Thus, if what is experienced has the sense of *transcendent being* then it is experience itself that constitutes this sense. If an experience is "imperfect" in the sense that an object is given only partially, then it is only experience that teaches me this.

One of the most interesting features of this transcendental-phenomenological idealism is that it does not deny that there is objectivity or objective truth but rather it makes of objectivity a problem that is to be grasped from what is absolutely given. It enjoins us to investigate how consciousness constitutes the sense of objectivity. We must now engage in constitutional analysis. We must do this, furthermore, for any kind of objectivity. It is not the case that some objects are supposed to escape the phenomenological reduction. Thus, not only are we supposed to analyze the constitution of the sense (meaning) of the being of objects of ordinary founding acts of sensory perception but we are also supposed to analyze the constitution of the sense of the being of objects of founded forms of consciousness which are based on acts of abstraction of different types, acts of generalization, reflection, and idealization. In particular, we are supposed to analyze the constitution of the sense of being of categorial objects, of ideal objects, and of mathematical objects in particular.

If we start with the ordinary physical objects given to us in founding acts of sensory perception then we see that they are given to us only partially and as transcendent, as objects that are in space and external (world) time. They are not given to us as subjective or as mental entities. We do not need to hold, as we noted, that everything is a mental phenomenon or a subjective idea. We can recognize that physical objects transcend mental phenomena (as do mathematical objects) only now we say they are constituted by consciousness in this manner. That is, the meaning of the being of physical objects is constituted by consciousness in such a manner that physical objects are not mental entities. They are not meant as mental entities. They are constituted as external objects, as objects that are in space and in external time. We are led, in this sense, to a kind of realism about physical objects. This is different, however, from a naive realism. It is, rather, a phenomenological or "constituted" realism that has its origins in transcendental subjectivity itself. Thus, starting with physical objects we can say that it is only *naïve* forms of realism about the natural world that take physical objects to somehow exist in themselves, totally independently of consciousness. If we are operating from the position of transcendental-phenomenological idealism then, for the reasons discussed above, we cannot be naïve realists. We also cannot be crude empiricists, naïve naturalists, or positivists.

Taking our lead from Husserl's comments in *The Idea of Phenomenology*, *Ideas I*, *Formal and Transcendental Logic*, and elsewhere, we can say that ideal objects are also constituted as such by consciousness. Let us apply Husserl's words from these texts to mathematical objects in particular: Whatever things are, mathematical objects included, they are as experienceable things. It is experience alone that prescribes their sense. The genuine concept of the transcendence of mathematical objects can only be derived from the contents of mathematical experience itself. Nothing exists for me otherwise than by the actual and potential performance of my own consciousness. Whatever is given as an existing object in mathematics is something that has received its whole sense of being from my intentionality. There is no conceivable place where the life of consciousness could be broken through so that we might come upon a transcendent mathematical object that had any other sense than that of an intentional unity making its appearance in the subjectivity of consciousness. We need to explicitly note the new twist here: If what is experienced has the sense of "transcendent being" then it is experience itself that constitutes this sense. If what is experienced has the sense of being "ideal," "non-mental," "acausal," "unchanging," "non-spatial," (possibly "partially given") and "non-material" then it must be experience itself that, in a non-arbitrary manner, constitutes this sense. If mathematical objects are considered to be objects that existed before we became aware of them and that would exist even if there were no human subjects then it must be the case that this sense of mathematical objects is constituted in a motivated and non-arbitrary manner.

If we consider all of the general features of mathematical realism that we outlined at the beginning of § 2 then we can now say that mathematical objects possess these features except that we must add the crucial qualification that they are constituted non-arbitrarily in this manner in the consciousness of the transcendental subject. One feature that we must now modify, however, concerns the temporality of mathematical objects. Since we are within the sphere of possible experience for transcendental subjects we are within the sphere of temporality. This means that mathematical objects are also objects that must be in time, only now we will say that they exist at all times. Thus, instead of saying that mathematical objects are atemporal or eternal or timeless—somehow outside of time (and all possible experience) altogether—we will now say that they are omnitemporal (see *EU*, § 64). As transcendental phenomenological idealists we cannot speak about the existence of objects that are somehow outside of all possible appearance or outside of all possible consciousness, and hence outside of all possible time.

We thus appear to arrive at a wholly unique kind of "platonism" about mathematics, which I will call "*constituted platonism*." This is, as it were, a platonism embedded within transcendental idealism. In a remarkable new twist in the age-old debate about platonism, we look to the transcendental ego as the source (origin) of platonism about logic and mathematics, where logic and mathematics are built up non-arbitrarily through acts of abstraction, idealization, reflection, and so on. Just as the "realism" about physical objects is not a naïve realism, so this unique kind of platonism about mathematical objects is not a naïve platonism.

IV. Mind-Independence and Mind-Dependence in Formulations of Mathematical Realism

Since mathematical realism and mathematical idealism are viewpoints expressed in terms of mind-independence or mind-dependence, I would now like to single out these characteristics in order to arrive at an explicit formulation of how a form of mathematical realism might be compatible with transcendental phenomenological idealism. As I mentioned above, Husserl says in *The Idea of Phenomenology* that the only way to solve the problem of how we can be related to transcendent (or mind-independent) objects is from within the phenomenological reduction. Once we restrict ourselves to the sphere of appearances, to what is immanent, on the basis of the epoché, we see that consciousness exhibits intentionality. We find that (transcendental) subjects are directed by the contents (or noemata) of their acts toward objects that *transcend* these very subjects. In the language of *Ideas I*, we find the noetic-noematic-hyletic structure at work in our experience of ordinary sensory objects. In the case of the founded pure categorial or ideal objects this same structure will be present, without the constraints of sensory hylé but not without grammatical, formal, meaning-theoretic, and other structural constraints. In other words, within the sphere of the immanent and absolute that we obtain after the reduction we can draw a new distinction between the immanent and transcendent, i.e., we can distinguish *what appears as immanent* from *what appears as transcendent*. We can distinguish *what appears as mind-dependent* from *what appears as mind-independent*. Of course the terms "immanent" and "transcendent," or "mind-dependent" and "mind-independent" in this context will have a sense different from their sense prior to the reduction. Similarly, if we view the reduction as depending on a distinction between appearance and reality (in the naïve sense of the natural attitude) and then restricting us to the sphere of appearances, then we find that within the sphere of appearances we can still distinguish appearance from "reality." This will be true in ordinary sensory experience but also in the case of our experience in mathematics and logic.

How does this work? We can start with an example in sensory experience. Suppose that at a certain stage of your experience you perceive a snake lying in a garden. At a later stage, however, you perceive that it is not really a snake lying in a garden but a coiled garden hose. Now what usually happens in situations such as this is that our experience settles down so that we do not have a continuous series of misperceptions of this sort. Instead, there is typically a more or less harmonious course of experience involving transcendent objects. This opens up the possibility of making an appearance/reality distinction after the epoché. Looking back on the experience, we can say that there was merely an appearance of a snake at the earlier stage in the perception and that what we have "in reality" is a coiled garden hose. We cannot simply say that "to be is to be perceived" because in a case such as this subsequent experience shows that there was no snake. It is not the case that in fact I was perceiving a snake at the earlier stage. It only appeared that I was. What seems to be mind-independent, given the evidence thus far, is the coiled garden hose. The perception of the coiled garden hose, however, could itself be overturned in future experience. That is, there might be evidence (experience) in the future that would show us that it is also not a coiled garden hose. Its being a coiled garden hose is *not absolute* even if the coiled garden hose is given as what is "real" and mind-independent in accordance with all of our evidence thus far. Our evidence that the coiled garden hose is mind-independent is in this sense presumptive.

What this shows is that from within the epoché everything is indeed understood as appearance or phenomenon and that appearances are corrected or verified only by further appearances. Within the sphere of appearance, however, we can still distinguish the "real," the transcendent, or the mind-independent from the "merely apparent," the immanent, or the mind-dependent, on the basis of what stabilizes or becomes invariant in our experience. This is a key idea of transcendental phenomenology, and it holds for both sensory experience and mathematical experience. There are illusions and corrections and refinements in mathematical experience just as there are in sensory experience. We cannot somehow get outside of appearances to an appearance-independent thing-in-itself. The "real" will be that for which we have evidence across places, times, and persons. This will not hold for the "merely apparent." Rational justification depends on evidence. Imagine a form of experience in which nothing ever stabilizes or becomes invariant. This would be a form of experience that is without reason. It would be experience in which there is no order and no rational connection among the contents of consciousness. We are nonetheless not entitled to say that what is stable or invariant is the final, absolute reality. We cannot have a realism that recognizes an appearance-independent absolute reality. At best, the notion of "absolute reality" might be preserved as an infinite ideal. Thus, transcendental phenomenology recognizes an appearance/reality distinction after the reduction that allows for a kind of realism, only it is not naive or absolute realism. It is also not a naive idealism for the same reason: it makes an appearance/reality distinction after the epoché.

These considerations show us that there are weak and strong senses of "appearance-independence" or "mind-independence." There could not be mind-independent objects in the strong or absolute sense of lying outside of all possible experience (or appearance). We simply cannot say anything about the possibility of such radically independent things-in-themselves. On the other hand, there are objects that are mind-independent in a weaker sense according to which objects are invariants in a manifold of appearances. We could be mistaken about objects in our experience, so that we could at some later stage come to see that we had been under an illusion, that we had mere appearances at an earlier stage. To say that there are weak and strong senses of "mind-independence" or "mind-dependence" will then affect the formulations of mathematical realism and mathematical idealism. In transcendental phenomenology we must set aside the strong (or naïve) sense of mind-independence. The weaker sense, however, will allow us to preserve important insights of realism.

V. Compatibility or Incompatibility?

To summarize the discussion thus far we can say that we need to index our conceptions of the mind-dependent and mind-independent, of the immanent and transcendent, and of appearance and reality.

With the phenomenological reduction we turn to phenomena, to appearances, to the immanent. So we first distinguish between appearances (or the immanent), and the naïve view of appearance-independent reality (or of the transcendent). On the one side of this distinction we have appearances, the immanent and mind-dependent, and on the other side of the distinction we have appearance-independent reality or the transcendent as mind-independent. Now, restricting ourselves to the sphere of phenomena, to the immanent and absolute, we find that consciousness exhibits intentionality. Transcendental subjects are directed toward objects that transcend subjects. We find the noesis-noema-object structure, minus sensory hylé in the case of mathematical objects. Intentionality in pure mathematics is not constrained by sensory hylé but there are still grammatical, formal, meaning-theoretic, and other structural constraints on it. One of the marks of objectivity in both sensory and mathematical experience is that we find our awareness to be constrained in certain ways. It is not possible to will objects or states of affairs in either sensory or mathematical experience to be just anything we want them to be. We find all of these moments of experience after the epoché. Within the sphere of appearances we can then draw a distinction between the immanent and the transcendent. Here we introduce a new distinction between the immanent and transcendent, the mind-dependent and mind-independent, between appearance and reality. Some things *appear to us* as immanent, some as transcendent.

We can depict the situation in the following diagram, which I will formulate for the mind-dependent/mind-independent distinction, since the issue of mathematical realism is typically described in these terms:

Mathematical objects are

mind-dependent$_1$ $\square$ mind-independent$_1$ $\Diamond$

/ \

mind-dependent$_2$ $\square$ mind-independent$_2$ $*$

It is inconsistent to say that abstract mathematical objects are mind-dependent$_1$ and mind-independent$_1$. Formulated in this way, mathematical realism and mathematical idealism are incompatible. Most of the debate about realism and idealism, including recent debate, seems to take place at this level. It is also inconsistent to say that abstract mathematical objects are mind-dependent$_2$ and mind-independent$_2$. Formulated in this way, mathematical realism and mathematical idealism are still incompatible. It is not inconsistent, however, to say that abstract or ideal mathematical objects are mind-dependent$_1$ and mind-independent$_2$. Indeed, mind-independence$_2$ falls under mind-dependence$_1$. What this means is that mathematical realism, in this sense, is compatible with transcendental phenomenological idealism. Mathematical realism in this sense, which we can call "*constituted mathematical realism*" or "*constituted platonism*," is concerned with non-arbitrarily or rationally motivated constituted mind-independence.

What we are now to investigate is the *constitution of the sense* of mind-independence from within the epoché. We need to investigate the rationally motivated *constitution of the sense of the existence of ideal mind-independent mathematical objects*.

As we look back from this viewpoint, we can say that the standard positions of mathematical realism and mathematical idealism that we set out in our initial formulations are too simple. They are ambiguous. If we make the distinctions just indicated then the assertion that mathematical objects are mind-independent$_1$ is naïve (or pre-critical) mathematical realism and is untenable. The assertion that mathematical objects are mind-dependent$_1$, with no further qualification, is naïve (or pre-critical) mathematical idealism and is untenable. The third position that we outlined combines a transcendental phenomenological idealism and a mathematical realism in which neither the realism nor idealism is any longer naïve. We have left naïve metaphysics behind. It also follows that transcendental phenomenological idealism is not compatible with naïve mathematical realism.

Once we make these distinctions then existence claims, whether in ordinary perception or in the case of mathematics, will have to be understood accordingly. There are of course important disanalogies between sensory objects and mathematical objects but in either case the existence of mind-independent objects will now have to be understood in the sense of mind-independence$_2$. If these phenomenological considerations are correct then what other sense could we legitimately give to existence claims?

Constituted platonism, unlike naive metaphysical platonism, does not cut off the possibility of knowledge of mathematical objects. Knowledge involves intentionality. Mathematical knowledge is to be spelled out in terms of intentional directedness toward ideal or abstract objects, where the objects are to be thought of as (founded) invariants in mathematical experience. What we are describing here is a position about mathematical *experience*. Note how different this is, for example, from a position that starts with neuroscience and then asks how the brain could be related to abstract objects. How could brains be causally related to abstract objects? From my point of view, this is the wrong question. There is a reason for wanting to suspend or bracket natural sciences of the mind such as neuroscience. The reason is not to avoid neuroscience in particular or natural sciences of cognition in general. We of course need such important sciences. The reason is rather to avoid a reductionistic, eliminativist, and one-sided philosophy of mind that leaves out consciousness and intentionality. Such sciences abstract away from *experience*. There is much more to say about these matters but further discussion will have to wait for another occasion (see, however, Tieszen 2005a, 2005b, 2006).

On the kind of realism described above objects can be mind-dependent$_1$ and mind-independent$_2$. Before concluding this section I would like to note that such a realism bears more than a passing resemblance to Hilary Putnam's "internal realism" (see, e.g., Putnam 1981, 1987). Although it is not possible to do so here, it would be worthwhile to compare the views in some detail. I am arguing, for example, that the notions of mind-dependence$_1$ and mind-independence$_2$ can be applied in the case of *mathematical* objects or states of affairs. Does Putnam apply his internal realism to the question of mathematical realism? Does he have anything like the idea of constituted mathematical platonism? There are certainly developments in the phenomenological analyses that would not be found in Putnam's internal realism but there might also be many points on which the views in fact reinforce one another.

VI. Brief Interlude: Where to Place Gödel, Brouwer, and Other Mathematical Realists and Idealists in our Schematization?

It is of some interest to consider where various philosophers, logicians, and mathematicians would fall within the set of distinctions I have drawn. In terms of the diagram above, I have suggested that the later Husserl is at the position marked by "∗," or at least that the principles I have discussed would lead him there, even if he did not explicitly analyze all of the consequences of the position. Brouwer and Gödel are frequently regarded as antipodes on the mathematical realism/mathematical idealism issue. It seems that Brouwer would be at the position marked by "□" because he does not have the distinction between weak and strong senses of mind-independence. The idea that mathematical objects could be non-mental and yet not be appearance-independent does not seem to be part of his view. There are very interesting connections between ideas of Brouwer and Husserl but if it is not part of Brouwer's view that mathematical objects could be non-arbitrarily constituted by subjects as non-mental then Brouwer would still be what we have called a naive idealist. But perhaps Brouwer's position could or should be modified. On the other hand, there are various philosophers would who place Gödel at the position marked by "◊." Perhaps that is where he belongs, in which case his position will be subject to all of the problems associated with naive metaphysical platonism. We know, however, that Gödel was interested in aspects of Kant's transcendental idealism and that from 1954 to 1959 he corresponded with Gotthard Günther at some length about transcendental philosophy (see Gödel [1954–59] 2003). Starting in 1959, he became an avid reader of Husserl's philosophy, and was especially interested in transcendental phenomenology (see, e.g., Gödel [∗1961/?] 1995). Thus, we should perhaps put him in the position marked by "∗," or at least regard him as groping toward such a position (see also Tieszen, Introduction and Part II of 2005b).

VII. A Conclusion and an Introduction

If it is therefore possible to see how a form of mathematical realism is compatible with transcendental phenomenological idealism we can still ask about which kind of mathematical realism can be supported from this point of view. About which kinds of mathematical objects and states of affairs, that is, can we be constituted realists? This is now a question of meaning constitution. What kinds of mathematical objects, e.g., geometric objects, natural numbers, real numbers, complex or imaginary numbers, functions of different types, sets of different types, and the like, can the mind constitute?

Here things are much more complicated. This is where the real work of constitutional analysis in the case of mathematics must begin. It could be argued that Husserl would hold that natural numbers and the geometric objects of Euclidean geometry could in principle be constituted as particular objects. Which other (alleged) mathematical objects on our list could be constituted? Husserl himself does not have much to say about the constitution of such objects. He does present some ideas on the origin and constitution of sets in *Experience and Judgment* and other places but, relative to modern set theory, they do not take us very far. They leave a lot undetermined. It is not clear that they would lead to the existence of any sets beyond those that constructivists would be prepared to recognize.

There are many questions about the constitution of mathematical objects and the constitution of generalities about mathematical objects that need to be considered. To mention just one question of this type, for example, it might be asked whether it is possible to constitute generalities about mathematical objects even if we cannot constitute such objects individually. Could there even be a kind of objectivity in mathematics without objects? I will not try to address the many questions that could be asked here. What I have tried to do in this paper is to show how, at a *general* level, a form of mathematical realism can be compatible with transcendental phenomenological idealism. One can then enter into issues about constituted realism in the case of different kinds of mathematical objects.[1]

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