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Young Ketonen and his Supreme Logical Discovery

Introduction

Oiva Toivo Ketonen was born in Teuva January 21, 1913, into a family that all together raised 13 children. Oiva was child number eight. Based on his unfinished autobiography, there seems to have been much going on in the small village, Perälä, where he grew up.[1] The village saw some action during Ketonen's childhood despite its small size: during 1919–1932, the government passed a prohibition law on alcohol. This naturally led to a lot of illegal smuggling.[2] Perälä connected two important roads in the region, so it became something of a strategic hub for these local bandits. Ketonen still later remembered the village's law-enforcer roaming the roads on a sidecar-equipped Harley-Davidson motorcycle.

During his youth, Ketonen reveals in the autobiography, everyday experiences taught him the reality of life, in many respects. The law-governedness of nature etched itself deeply into his consciousness. "There were strange things, but also they are part of the natural order."[3] He recalls that these experiences proved to be extremely valuable: He noticed, for example, how "narrow-minded and strange conceptions some other students had" regarding theological questions and the individuals relationship with the church.

Ketonen graduated from Kristiinakaupungin Lukio (upper secondary school) in 1932, and enrolled in the Division of History and Philology (where philosophy in Helsinki was taught at that time). The current professor in philosophy in Helsinki was Eino Kaila, who was closely connected with the Vienna Circle. We can read in the autobiography, however, that Ketonen was not quite content with his studies. At that time, philosophy and psychology were not separate subjects, and Ketonen switched his main subject to mathematics. This should not be seen to suggest that he thought less of psychology – indeed, he reveals that Kaila's lectures on the psychology of personality made a deep impression on him. In his memoirs Ketonen also writes that, he suspected that mathematics and the natural sciences would be too "thin", that they would not contain the type of richness that would give life emotional and perspectival content. Ketonen began studying under the mathematician Rolf Nevanlinna, famous for his work on the theory of complex

1 The manuscript for the autobiography was kindly made available to the present writer by Oiva's son Timo Ketonen.

2 Products containing more than two percent alcohol were available only for medical, technical or scientific purposes.

3 Unfinished autobiography.

functions, and we can tell from preserved correspondence that Nevanlinna was extremely impressed by Ketonen's mathematical abilities.[4]

Regarding the teaching of logic at the University, Ketonen notes that the only textbook on logic available in 1932 was Thiodolf Rein's *Muodollinen Logiikka* (*Formal Logic*, my translation), which devotedly followed Aristotelian logic. There was, however a change in the university curriculum, and so new literature was introduced, including Bertrand Russell's *The Problems of Philosophy* and Kaila's *Nykyinen Maailmankäsitys (World Concepts of Today* – my translation*)*. Teaching in logic, Ketonen notes, was confined to the basics, and could not in such a form offer a subject of interest. Ketonen's study book reveals that he did not take a single course in logic.

According to discussions with Timo Ketonen, one of his sons, Oiva found himself interested in algebra and number theory. Ketonen's fellow student, Max Söderman, made Ketonen aware of Gödel's incompleteness theorem, and Nevanlinna later mentioned the theorem.[5] Gödel's fantastic result could be what sparked Ketonen's interest in formal logic. Ketonen began planning his Master's thesis fairly early in his studies, in 1935 at the latest (based on his study diary); this was after only two years of university studies. There is no doubt that the interest in logic was already serious: He writes in his study diary – cleverly entitled *Lahjomaton Tilintekijä* (*The Unbribable Accountant* – my translation) – that "Real mathematics begins with axioms and proceeds to prove from them, in the most direct way possible, the more complex propositions."[6] Ketonen writes in the autobiography that he frequently went to evening meetings of what he called "The philosophical club". These meetings seem to have been quite unofficial, usually the group gathered at the home of one of the professors, e.g. Kaila or prof. Yrjö Reenpää. They also gathered at least once at Söderman's home. In the study diary, we read that he later spent some evenings attending what he calls "mathematical-logical conferences". Where the members of these meetings the same? Logic was of course thoroughly discussed during these meetings, and Ketonen remembers a particular time (possibly May 5, 1936, based on an observation in the study diary) when he presented and defended one of his original ideas which will be expanded below:

In classical Hilbert-style propositional axiomatic logic, one has as the first axiom

$$A \supset (\neg A \supset B)$$

By this axiom, if A, and then from the negation $\neg A$, one can derive an arbitrary proposition B. The last instance in this derivation of B is thus intuitively modus ponens. Adapted to natural deduction it would, after suitable modification and

4 The correspondence between Ketonen and Nevanlinna was, once again very kindly, made available by Timo Ketonen.

5 How well Nevanlinna was acquainted with logic, and what he thought of the new discipline, remains debated.

6 My translation from Finnish.

with the addition of the rule *Ex falso quod libet* in the form of the axiom $\bot\supset B$ and the definition of $\neg A$ as $A\supset\bot$, look like:

$$\dfrac{\bot\supset B \qquad \dfrac{\begin{matrix}\vdots\\ A\supset\bot\end{matrix} \quad \begin{matrix}\vdots\\ A\end{matrix}}{\bot}\supset E}{B}\supset E$$

Ketonen argued that there is something not quite right with this principle, because, although the derivation is formally correct, in order to correctly use rule $\supset E$ for concluding B, both premisses must be true. But $\bot$ is *never* true. This caused a heated debate during one of the evening gatherings, and Ketonen won over some participants to his side, but that is all that ever came of it, although Ketonen thinks it would have been worth developing.[7]

That Nevanlinna was impressed by Ketonen's mathematical abilities is demonstrated yet again in the study diary. The notes show that Ketonen and Nevanlinna discussed the topic of the Master's thesis repeatedly during the latter's office hours, and that he wanted Ketonen to take up function theory. One can presume that Nevanlinna would not recommend his own field of expertise to a student he did not consider up for the task.

The original plan for the thesis was to write something on pure axiomatics and prove, for example, the fundamental theorem of algebra. This is noted on December 18, 1935. Later, on March 21, 1936, he writes: "The thesis is changing like protoplasm". Roughly a month later – April 18, 1936 – we learn that "I will probably write the thesis on the theory of functions after all". Ten days later, the subject is changed again, this time back towards axiomatics, specifically towards the foundations of mathematics. Nevanlinna commented thus: "Quite a rare subject, since these questions are very scientific, not really intended for a work by a student." Ketonen's first note in the study diary that he has been studying Gödel's famous proof is from the May 4, 1936, 8 p.m. to 9.30 p.m. Two days earlier, he had had discussions with both Kaila and Nevanlinna. Five days later, he has discussed again his master's thesis with Nevanlinna, and his decision to write on axiomatic logic is re-affirmed and final. In the autobiography, he remembers having viewed the work ahead as "extremely interesting". One could speculate that after studying Gödel's proof personally, it made such an impression that it was no longer possible for him to even consider working in another field of mathematics. Formal logic and Gödel's first incompleteness theorem thus became the subject of Ketonen's master thesis. Ultimately then, the choice to take up formal logic seems to have been independent,[8] there is nothing in the study diary along the lines of, say, "After discussions with Mr. X I will take up logic" which one could assume to have been

7 He notes in the autobiography that he suspects that they lacked the necessary logical-philosophical tools at the time, but that later others have written about the subject.

8 Independent in the sense that no one actually suggested the topic to him. One can assume that every professor leaves some mark on his students.

the case had such discussions taken place. The actual writing on the master's thesis *Tutkimuksia Formaalisen Todistamisen Ristiriidattomuudesta* (*Investigations in the Consistency of Formal Proving*, my translation) began in May 1936. To the present writer's (and many others') dismay, the last line in the study diary reads "May 23, 9.00 – 10.00. Work on Master's thesis. See second notebook". No such notebook has been found.

The master's thesis concentrates on two main topics, namely Hilbert-style axiomatic propositional and predicate logic, and arithmetic and Gödel's theorem. It is not known exactly what the thesis looked like, because only part of it has survived in original form. When work on the thesis was finished and graded (Ketonen received the highest possible grade for it, Laudatur), approximately half of the pages were (probably) torn out. The reason for this mutilation was that this first half was going to be published in the *Ajatus* series (yearbook of the Philosophical Society of Finland) but Ketonen apparently wanted to change some passages, and had to alter the order of others, because the observations on Gödel incompleteness that were at the end of the original thesis were included in this published version. From the published version, he omitted the sections on arithmetic. Hence, the published version contained axiomatic propositional and predicate logic, and discussions on Gödel's incompleteness theorem. The original handwritten thesis (the cover and the pages that are left) has survived. When comparing this with the table of contents for the article published in *Ajatus* one quickly spots the differences and gets a picture of what has been changed. The article published in *Ajatus* is titled "Todistusteorian Perusaatteet" – "The Main Ideas of Proof Theory".[9]

Ketonen had received the impression from Nevanlinna that some mathematicians suspected that there was some fault in Gödel's proof, and that this fault might be worth uncovering. Ketonen believes that as a result of this investigative work, he somewhat succeeded in streamlining Gödel's proof. In the autobiography, he laments that he was given the highest grade for the thesis. This might seem odd, but the explanation is sound: Since he was given the highest grade, he thought the work to be 'complete', and so just put it in the bookcase and never gave it a second thought. Had he been given any other grade, he would have reworked the problems, trying to find out what went 'wrong'. He realised later, he writes, that this way of thinking had not been rational. Thus, he may have continued to pursue the task of clarifying Gödel's proof, and develop the ideas that he came to think of during the writing of the thesis.

Ketonen kept himself occupied with Gödel's theorem also after he finished his master's thesis and the subsequent article for *Ajatus*. In 1941, Ketonen made a small improvement to Gödel's completeness theorem for the predicate calculus.[10] Gödel showed that that either a proposition A is provable, or it is impossible that there does not exist a counterexample. Ketonen improved this result so

9 Oiva Ketonen, "Todistusteorian perusaatteet", in: *Ajatus* 9, 1938, pp.28–108.
10 Oiva Ketonen, " Predikaattilogiikan täydellisyydestä", in: *Ajatus* 10, 1941, pp.77–92.

that this counter example can be found directly. Reportedly,[11] Söderman explained Ketonen's result to Gödel in Vienna, who admitted that it was indeed an improvement.

The Dissertation – Untersuchungen zum Prädikatenkalkül Studies in Göttingen

According to his autobiography, Ketonen had decided already in the spring of 1938 to go for a dissertation immediately. He went to the university of Göttingen to study under Gerhard Gentzen, most probably with the aid of Nevanlinna's contacts, who had worked at the University as a visiting professor in 1936–1937. Kaila had met Gentzen in 1936 in Münster. Some letters from Nevanlinna to Ketonen have survived[12] and they show conclusively in how high esteem the former held the latter (this respect of course also held in the other direction). Göttingen's mathematical 'omnipotence' had already somewhat diminished, in particular since several Jewish professors had already been expelled. The atmosphere was very 'mathematic-formalistic'. Morbidly, the very same night that Ketonen arrived in Göttingen, the night between December 9 and 10 in 1938, later became known as the infamous 'Kristallnacht' – 'crystal night', named after the shards of broken glass littering the streets of Germany the next morning after a horrific night of antisemitist violence. The following remark is found in the 1989-presentation in connection with the subject of the Master's thesis: "[…] I did not for a moment think that I would try to proceed along that road". This is an extremely puzzling remark, since, he did indeed proceed along that road immediately; the *voyage* included visits to Göttingen and Münster, a meeting with Heinrich Scholz, and then studying under Gerhard Gentzen's supervision resulting in the dissertation *Untersuchungen zum Prädikatenkalkül* published in 1944.[13] Why did Ketonen make this remark in 1989 of not having planned to proceed with research on mathematical logic, but then in his autobiography state the complete opposite?

There were of course recognised mathematicians still present in Göttingen, for example C.L. Siegel. Surprisingly however, according to Ketonen, no lectures on mathematical logic were given.[14] Ketonen recalls in the autobiography (my translation):

11 Jan von Plato, "Ein Leben, ein Werk – Gedanken über das wissenschaftliche Schaffen des finnischen Logikers Oiva Ketonen", in: Rudolf Seising (Ed.), *Form, Zahl, Ordnung: Studien zur Wissenschafts- und Technikgeschichte.* Stuttgart: Franz Steiner Verlag 2004, pp. 427-435.

12 I once again thank Timo Ketonen for providing copies of these letters.

13 Oiva Ketonen, *Untersuchungen zum Prädikatenkalkül*, Annales Acad. Sci. Fenn, Ser. A.I. 23 1944.

14 Note that Hilbert was retired, hence Ketonen's comment that Gentzen was the only logician *at the university*.

There were no lectures in mathematical logic. The field's only representative in the university was Gerhard Gentzen, a sympathetic relatively taciturn young man, who was Hilbert's personal assistant. He told me that his duties consisted mainly of the reading of popular scientific publications to him [Hilbert]. I saw Hilbert once when he was going, walking alone, to the city theatre to watch Cinderella and the Golden Slipper, where I was going myself.

The dissertation contains three parts. The first part presents and improves Gerhard Gentzen's sequent calculus, part two discusses a certain Skolem normal form, and the third part applies the results from parts one and two in order to produce a proof of the underivability of Euclid's parallel postulate from the Skolem-axioms for Euclidean geometry. Ketonen was the first to continue Skolem's work on geometry. The first part will be discussed in detail below.

Propositional Logic

Next, we will briefly discuss the notation for propositional logic and sequent calculus, so that the discussion on Ketonen's result is accessible also to the non-specialist.

We use the *capital Latin letters* A, B, C … to indicate *formulas* (either compound or atomic). We use the *connectives* &, $\vee$, $\supset$, $\neg$ for conjunction, disjunction, implication, and negation, respectively. Use these and parentheses to form *propositions*, for example.

A&B	$C \vee (D \supset E)$	$\neg (A \vee B)$
'A and B'	'C or D implies E'	'not A or B'

Notice how the parentheses remove the ambiguity of natural language. Consider next the proposition $(A \supset B) \supset (\neg B \supset \neg A)$. It reads 'If A implies B, then B implies not-A'. This proposition is always valid, and we call such propositions *tautologies*. Before we can perform any derivations, however, we require formal *rules of inference*. For this, we introduce a Gentzen-style[15] *sequent calculus*. A *sequent* is of the form

$$A_1, A_2, \ldots, A_n \rightarrow B_1, B_2, \ldots, B_m$$

The formulas to the left of $\rightarrow$ make up the *antecedent*, those to the right the *succedent.* The formulas in the antecedent can be viewed as *assumptions*, those in the succedent as *possible cases*. Thus, 'A, B $\rightarrow$ C' reads 'from A and B together, C follows'. The sequent arrow can also be read as 'gives'. Greek capital letters Γ, Δ, Θ,

15 Gerhard Gentzen, "Investigations into Logical Deduction", in: Manfred Szabo (Ed.), *The Collected Papers of Gerhard Gentzen*. Amsterdam, London: North Holland Publishing Company 1969, pp. 68-131. The article was originally published in *Mathematische Zeitschrift* in 1934–1935 and accepted as Dissertation by the university of Göttingen.

… are lists of formulas, and can be interpreted as a *context* for the derivation. The only axiom is the *initial sequent* A → A, which states that from the assumption A, the case A follows. We can think of the sequent as a generalization of the concept of derivability. If we put n=1 in the sequent above, we get the standard case of a single conclusion as in natural deduction. Below are two examples of inference rules, along with an intuitive explanation of how they are applied.

$$\frac{\Gamma \rightarrow \Theta, A \quad \Delta \rightarrow \Lambda, B}{\Gamma, \Delta \rightarrow \Theta, \Lambda, A\&B}\ R\&$$

If something, call it Γ, gives Θ and A as possible cases, and something else, call it Δ, gives Λ and B as possible cases, then Γ and Δ together give Θ, Λ, and A&B as possible cases.

Another example:

$$\frac{A, \Gamma \rightarrow \Theta \quad B, \Delta \rightarrow \Lambda}{A \vee B, \Gamma, \Delta \rightarrow \Theta, \Lambda}\ L\vee$$

When the assumptions A and Γ give Θ and the assumptions B and Δ give Λ, then A∨B together with Γ∨ and Δ will give Θ and Λ as possible cases. The symbols R& and L indicate which rules has been applied.

The inference rules are divided into two groups, *logical rules* and *structural rules*. Roughly, the logical rules are applied on connectives, while the structural rules are applied on the formulas.

Logical Rules for Gentzen's Calculus LK

$$\frac{\Gamma \rightarrow \Theta, A \quad \Gamma \rightarrow \Theta, B}{\Gamma \rightarrow \Theta, A\&B}\ R\&$$

Right conjunction

$$\frac{A, \Gamma \rightarrow \Theta \quad B, \Gamma \rightarrow \Theta}{A \vee B, \Gamma \rightarrow \Theta}\ L\vee$$

Left disjunction

$$\frac{A, \Gamma \rightarrow \Theta}{A\&B, \Gamma \rightarrow \Theta}\ L\&_1$$

Left conjunction 1

$$\frac{B, \Gamma \rightarrow \Theta}{A\&B, \Gamma \rightarrow \Theta}\ L\&_2$$

Left conjunction 2

$$\frac{\Gamma \rightarrow \Theta, A}{\Gamma \rightarrow \Theta, A \vee B}\ R\vee_1$$

Right disjunction 1

$$\frac{\Gamma \rightarrow \Theta, B}{\Gamma \rightarrow \Theta, A \vee B}\ R\vee_2$$

Right disjunction 2

$$\frac{A, \Gamma \rightarrow \Theta}{\Gamma \rightarrow \Theta, \neg A}\ R\neg$$

Right negation

$$\frac{\Gamma \rightarrow \Theta, A}{\neg A, \Gamma \rightarrow \Theta}\ L\neg$$

Left negation

$$\frac{A, \Gamma \rightarrow \Theta, B}{\Gamma \rightarrow \Theta, A \supset B}\ R\supset$$

Right implication

$$\frac{\Gamma \rightarrow \Theta, A \quad B, \Delta \rightarrow \Lambda}{A \supset B, \Gamma, \Delta \rightarrow \Theta, \Lambda}\ L\supset$$

Left implication

Structural Rules for Gentzen's Calculus LK

$$\frac{\Gamma \to \Theta}{A, \Gamma \to \Theta}\ LW \qquad \frac{\Gamma \to \Theta}{\Gamma \to \Theta, A}\ RW$$

Left weakening *Right weakening*

$$\frac{A, A, \Gamma \to \Theta}{A, \Gamma \to \Theta}\ LC \qquad \frac{\Gamma \to \Theta, A, A}{\Gamma \to \Theta, A}\ RC$$

Left contraction *Right contraction*

$$\frac{\Delta, B, A, \Gamma \to \Theta}{\Delta, A, B, \Gamma \to \Theta}\ LE \qquad \frac{\Gamma \to \Theta, B, A, \Lambda}{\Gamma \to \Theta, A, B, \Lambda}\ RE$$

Left exchange *Right exchange*

$$\frac{\Gamma \to \Theta, B \quad B, \Delta \to \Lambda}{\Gamma, \Delta \to \Theta, \Lambda}\ Cut$$

Cut

In 1943, Ketonen discovered that all rules can be made **invertible**, i.e. such that, if a sequent matches the conclusion of a rule, and if it is derivable, then the corresponding premisses are derivable. Not all of Gentzen's rules are invertible, consider this counterexample. The sequent $A \to A \lor B$ is clearly derivable from the initial sequent $A \to A$. However, if the Gentzen's rule $R\lor_2$ were invertible, it would mean that also the sequent $A \to B$ is derivable. This cannot be: $A \to B$ is not at all an initial sequent if A and B are non-identical atomic formulas.

Gentzen's LK rules for left conjunction and right disjunction are not invertible, and Ketonen chose to simplify the rule for left implication so that it has shared contexts in the premiss. The modified rules receive the following form:

$$\frac{A, B, \Gamma \to \Delta}{A \& B, \Gamma \to \Delta}\ L\& \qquad \frac{\Gamma \to \Delta, A, B}{\Gamma \to \Delta, A \lor B}\ R\lor$$

$$\frac{\Gamma \to \Delta, A \quad B, \Gamma \to \Delta}{A \supset B, \Gamma \to \Delta}\ L\supset$$

Below two proofs of $\to (A \supset B) \supset (\neg B \supset \neg A)$ with Ketonen's invertible rules are given.

Example 1: Proof of $\to (A \supset B) \supset (\neg B \supset \neg A)$

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{A \to A}{\neg B, A \to A}\ LW}{\neg B \to \neg A, A}\ R\neg}{\to \neg B \supset \neg A, A}\ R\supset \qquad \cfrac{\cfrac{\cfrac{B \to B}{B \to B, \neg A}\ RW}{B, \neg B \to \neg A}\ L\neg}{B \to \neg B \supset \neg A}\ R\supset}{A \supset B \to \neg B \supset \neg A}\ L\supset}{\to (A \supset B) \supset (\neg B \supset \neg A)}\ R\supset$$

Example 2: Another proof of $\to (A \supset B) \supset (\neg B \supset \neg A)$

$$
\dfrac{\dfrac{\dfrac{\dfrac{A \to A}{\neg B, A \to A}\,LW}{\neg B \to \neg A, A}\,R\neg \qquad \dfrac{\dfrac{B \to B}{B \to B, \neg A}\,RW}{B, \neg B \to \neg A}\,L\neg}{A \supset B, \neg B \supset \to A}\,L\supset}{\dfrac{A \supset B \to \neg B \supset \neg A}{\to (A \supset B) \supset (\neg B \supset \neg A)}\,R\supset}\,R\supset
$$

With the invertible system established, one can now construct the proof of a proposition root-first, beginning from the conclusion (thought of as the root of the proof tree) and applying the rules ‘upwards’ until one reaches initial sequents. This proof search terminates trivially, because each rule, when applied in reverse direction, reduces the number of connectives in the proposition. Ketonen calls this kind of proof search decomposition.[16] We can now construct proofs mechanically; we don’t need to think (that much) about what we are doing when constructing a proof. Another feature is, naturally, that the system permits us to investigate whether a proposition is provable or not. Since this is a terminating process, it can be done by a computer. The computer would have difficulties with Gentzen LK since it would have to ‘guess’ what is missing in the premiss *qua* the conclusion. The examples above show that the order in which the rules are applied in the decomposition does not matter. The calculus is neither deductive nor reductive, but *deduktionsgleich*.[17] In the thesis, as an example of the application of his invertible sequent calculus, Ketonen applies root-first proof search to axiomatic geometry (based on work by Skolem from the 1920s) in order to show the independence of the parallel postulate. The earliest reference to Ketonen’s work internationally is probably in Karl Popper’s “New Foundations for Logic” from 1947. Beth, in his work on the tableau method, cites Kleene, but not Ketonen, despite Ketonen’s work being relevant (and cited by Kleene).

Ketonen received his Ph.D. in March 1944, in the middle of the bombing of Helsinki. Only the day before, the old part of the main building of the university had been hit, the main hall and the rooms nearby had been badly burnt and so the dissertation was moved to an auditorium on the ‘new side’ of the university where one could still sense the smoke. There is a peculiar statement in the autobiography concerning the dissertation: “I did not expect much from it, but it appears that someone actually read it”. These “someones” included Bernays, Curry, Feys, Kleene etc. In any case, Ketonen’s opinion of his dissertation was consistent with that of his master’s thesis – he promptly put it in the bookcase. He notes that there were indeed some ideas that could have been developed further (we can read in the introduction to the dissertation that he at least at the time of publication intended to extend his results), but he says that they did no longer interest him.

16 Translated from the German word *Zerlegung*.

17 The conclusion of a rule is derivable if and only if the premiss is derivable

Ketonen's thesis originally became known through Bernays' favourable review from 1945.[18] Arend Heyting also wrote a review of Ketonen's thesis in 1947 for *Mathematical Reviews*.[19] One must wonder, however, whether Heyting studied the thesis thoroughly. The review is three sentences long (or short), and does not at all point out the fact that Ketonen's results amount to significant progress in proof theory, it more or less resembles a table of contents.

There is no evidence that Gentzen would have had any other students except Ketonen. Thus, he was one of the first to work with and extend Gentzen's calculus. Ingebrigt Johansson published a work related to Gentzen in 1937.[20] Kleene notes explicitly in 1952 that he knows of Ketonen's work only through Bernays' review.[21] Curry began using Ketonen's calculus by 1950, and the present writer has seen a letter[22] by Curry to Ketonen dated September 29, 1947, where the former asks for any material Ketonen might have written on logic in any language – "even in Finnish!". Curry reportedly[23] held Ketonen's work to be the best thing in proof theory since Gentzen.[24]

No More Logic? Lost Works

When one reads Ketonen's works published after the dissertation, one notes that no more original logical work is to be found. As stated earlier, Ketonen intended to continue along the logical path, but the plans changed. We will probably never know exactly why. As the story goes, whenever someone later asked him why he shifted his interests away from logic, the reply was "Logic gives me such headache". However, combining bits and pieces from survived correspondence, and notes in the autobiography, we can make some observations regarding what might have been the cause of this headache. First, Ketonen did not at all completely cease with research on logic and mathematics. Apart from giving lectures on mathematical logic in the 1950's and 1960's (attended by several professors), correspondence with his son Jussi Ketonen from the period 1969–1971 reveals that he

18 The review appeared in *The Journal of Symbolic Logic*, Vol. 10, No.4, Dec. 1945, pp.127-130.

19 Heyting's review is known to the writer via the American Mathematical Society's electronic database MathSciNet.

20 Ingebrigt Johansson, "Der Minimalkalkül, ein reduzierter intuitionistischer Formalismus", in: *Compositio Mathematic*, tome 4, 1937, pp.119-136.

21 See Stephen Kleene, "Permutability of inferences in Gentzen's calculi LK and LJ", in: *Memoirs of the American Mathematical Society*, vol. 10, 1952, pp. 1-26. See also (by the same author) *Introduction to Metamathematics*, Noordhoff, Groningen: North-Holland Publishing Co., Amsterdam, 1952

22 The letter is kept in Finland's National Archive in Helsinki.

23 See note 11 above.

24 Ibid.

has been working on the theory of numbers, on Kaila's work on the paradoxes of relativity theory, and on the logical concept of forcing. Especially forcing seems to have caught his interest: He writes that he has come up with some interesting distinctions and concepts, but suspects that they have probably been published elsewhere. We will never know, since none of this work has survived. A brief description of some post-thesis work has survived, however, in letters by Ketonen to Georg Henrik von Wright during the former's stay in the US 1949–1950 on a Rockefeller grant. Ketonen mentioned that he sent two works to Kaila for evaluation, and based on von Wright's expert opinion statement in connection with Ketonen's application for the professorship Kaila must have passed these works on to von Wright: One can compare the description of the works with each other and find that they converge.

What is then treated in these lost works? The works sent to Kaila were titled "On Analytic and *a Priori* Knowledge" and "On the possibility of a three-valued logic". The former comprises, according to von Wright, about 50 pages of material. The first two chapters discuss, in connection with C.I. Lewis' and Rudolf Carnap's work, basic concepts in the theory of meaning such as extension, intension, comprehension and significance. These are then in the third chapter employed to define analytic knowledge. The first three chapters serve as an introduction, the following two are more complex. In these the suggestion is made and argued for, that analytic knowledge is knowledge *a priori* and vice versa. Von Wright applauds the exposition for its comprehensiveness: Although it is a tad rough on the edges as a piece of research, it is most clear and readable due to Ketonen's ability to produce clear and concise formulations. Thus, Ketonen is able to link the work's main subject to related interesting questions such as the new nominalistic approach to knowledge analysis, the subjectivity of the concept of meaning etc.

In a letter dated April 22, 1950, (kept in the National Library in Helsinki) sent from the US, Ketonen makes his own summary of this manuscript: First, one establishes the transfiniteness of the definition of analytic knowledge, the equivalence analytic–*a priori*, that this equivalence is non-constructive and not suitable as a guideline for analysis and does not hold unless one considers meaning as intensional i.e. not valid on its own (context independent). Finally, if meaning is to be restricted to the extensional – the nominalists – then the whole concept of analytic knowledge changes so that the question disappears.

"On the Possibility of a Three-valued Logic" treats, according to von Wright, the works of Lukasiewicz and Post published in the 1920's. Ketonen has produced a commendable presentation of the formal aspects of the structure and interpretation of the calculi, and formulates a condition the calculi must fulfil in order to be applicable. This condition is constructive (exactly in which way 'constructive' is to be read is not made clear in the description), and Ketonen does not comment on the probability to actually realize it in a calculus. We find Ketonen's own description of this work in the same letter mentioned previously. He notes that three-valued logic should be reduced to the 'applicability' or 'non-applicability' of certain concepts, and that there for this reason should exist some translation of a proposition in a three-valued logic into a two-valued logic, in order for it to be held as true. If it is true, it is not inconsistent. Post's translation, Ketonen notes, does not fulfil this requirement. Furthermore, Ketonen points out that these questions are of such a kind as to be solved *a priori*, so that we can say what it means to apply non-classical to logic to experience.

In the said letter, Ketonen also mentions a 12 page presentation on the philosophical interpretations of scientific disciplines, and, perhaps more interestingly, on the interpretations of consistency proofs. The main point in the work is to show that if logic is understood analytically (non-formally), then consistency proofs say something, namely, the same as all other proofs. Logic is then treated only through the interpretation of expressions and symbols. Ketonen notes that this would perhaps have turned out as a better piece of work, had he only used more pages for it.

One Recovered Manuscript

As discussed above, hints of some later on work on logic are to be found in various places. The present writer was happy to discover a manuscript, comprising 16 pages, titled "Tietomme apriorisista aineksista" ("Our Knowledge of *a Priori* Elements"[25]) in the National Archives in Helsinki. The contents of this manuscript closely resemble von Wright's description of chapters 4 and 5 of the 50-page manuscript "On Analytic and *a Priori* Knowledge" included in the application for the professorship, and thus obviously also match Ketonen's description of the work sent to Kaila from the US. Ketonen writes in the previously discussed letter dated April 22, 1950, that "these things have been lying around for a while" (referring to the work sent to Kaila), so we can assume that they have been written before he travelled to the US. The manuscript is in an extremely unfinished form, written on typewriter but containing several corrections by hand, especially towards the end. The changes and additions are quite clearly indicated however, so quite a high readability is preserved.

Ketonen begins the manuscript with the following question: Is everything that we prove [in mathematical logic] based on our modes of speech, that reflect, cleverly hidden, but without deeper connections, in a sense only by chance the invariance of reality? Or, Ketonen continues, does our knowledge include other, more higher elements, that are necessarily true in all experience, notwithstanding that we cannot at all analyse the nature of this knowledge? [26] These questions are, he notes, as old as philosophy itself. Ketonen proceeds to clarify, with the aid of

25 My translation from Finnish.

26 One of the main proposals of Kaila's logical empiricism, according to Ketonen, is that all *a priori* knowledge is analytic.

proof theory and axiomatic geometry, how one could interpret the equality of the analytic and the *a priori*. He constructs a model of *a priori* knowledge – a simple and idealized world containing only points and lines – with some initial configuration of these. He then invites us to assume that this world is governed by the laws of elementary geometry that allow to add to (or construct from) some arrangement of points and lines new points and intersecting lines. Think of the logical proposition

$$A_1 \& A_2 \& \ldots \& A_k \supset B_1 \vee B_2 \ldots \vee B_l$$

as in the antecedent describing a multitude of possible different initial configurations of points and lines, and in the consequent describing other, possibly more complex, configurations. Assume now that some of the configurations in the consequent are known to us to have been realized in our world. Let us also assume that we select, from the proposition above, those elements from the antecedent that together describe our assumed initial condition, i.e., what is true in our idealized world. Ketonen now gives us a process by which we, applying *all* the axioms on the initial configuration, and subsequently again on the resulting configuration, and so on, exhaustively can examine whether or not some configuration can be constructed from some initial one. If now one of the disjuncts on the right is also realized in our world, the proposition is true. Assume also this to be the case. Now, if we, by Ketonen's construction procedure, can reach from the initial configuration the configuration defined by the true disjunct in the consequent, the knowledge that the disjunct on the right is true is analytic. If the construction process goes on *ad. inf.,* this particular piece of knowledge could be called synthetic *a priori*: At least, Ketonen notes, this model would at least come close to a logical model of such a situation: "It is sufficient for most classical cases". If Kant was correct about the about the parallel postulate, Ketonen continues, it would be impossible for human beings to even imagine non-Euclidean geometry (how could we then imagine, say, a sphere?).[27] The parallel postulate would assume the position of some mysterious "property of nature as a whole".

Ketonen also presents C.I. Lewis' argument for this position, from his 'newest' book at the time, *An Analysis of Knowledge and Valuation* from 1946. It runs, simplified: Assume the concept B not to be deducible from the concept A, but when one experiences A, then B follows. The first mistake here is, then, that the meaning of A is extended to objects in general, which means that the impossibility of – in experience – presenting A together with the opposite of B does not say anything of the connection between A and B. Furthermore, A should be limited to A as 'phenomenon' in order to be *a priori*, in which case it is no longer synthetic since the phenomenon of A includes all relevant conditions in order for A to be identified in experience.

27 The surface of a sphere is an example of a model of non-Euclidean geometry.

We note that this argument goes through in a very simple manner when compared to the somewhat complex formal-logical analysis above. Its emphasis is only on realization in experience, there is no mention of 'finite steps' or 'properties of nature as a whole'. If we reconstruct this example in the form of a logical model, however, this difference can be spotted and we are able to see what makes the proof so simple. The bottom line of the argument seems to be that since there is only one type of knowledge by experience, which includes all 'layers' of knowledge simultaneously, logical distinctions disappear. The "sense meaning" of a concept is based solely on what is true in experience and what is not. Based on the disappearance of these distinctions, one may say that all *a priori* knowledge is analytic.[28] There is, Ketonen notes, one problematic consequence of such reasoning: We will get as result a model which is philosophically impeccable, but presents paradoxes for the exact sciences: We would be *forced* to accept as universal such laws of nature that we have recognised *only in experience*, although it may be the case that they are completely incomprehensible, i.e., we are unable to construct any type of theory for them. It may further be the case that we could not even imagine such a model in which these laws were not valid, i.e., we would be unable to negate these laws. This possibility is not excluded by the previous proof that all *a priori* knowledge is analytic.

War, Disappointment, and Ethics

In the letters from the US to von Wright, Ketonen is quite clear about the fact that he is broadening his philosophical horizon, and reconsidering the most important elements of philosophy. This is due to the fact he was extremely disappointed with the lectures on the philosophy of science given at Columbia University in the fall of 1949. He found it "hard to digest" on the whole, and on February 12, 1950, he actually writes that he has "had enough of it", and he feels that such a thing as philosophy of science does not exist. He writes, in the same letter, that there either has to exist a positive natural science, or a philosophy of science existing as just another practice, investigating one aspect side by side with other more current topics of interest. He still believes in logical empiricism, but sees it as being perhaps too limitative. He writes that philosophy does not exist, unless it practices and involves ethics and the life of man in general. He writes: "I don't mean that philosophy should present rules of life, I mean that ethics is more important than the philosophy of science". In the next letter, dated March 15, 1950, he writes, however, that logic and the foundations of logic are what he really respects in philosophy. He admits that the words in the previous letter were quite strong, but he insists that he "cannot consider as philosophical anything which explicitly forbids

28 Ketonen is not entirely clear on this point. With 'layers' is probably meant something like 'level of logical complexity'.

the study of ethics […]" He remembers that he, when mentioning to Kaila a discussion on sociology with a graduate student, felt as if he had been "down a dark alley looking for forbidden company". It is clear from letters to the Rockefeller foundation that Ketonen planned not only to go to the US, but also to visit Gödel in Princeton. This, at least to the writer, constitutes proof that Ketonen was serious about continuing his research in logic up until the visit to the States.

Ketonen was not at the frontline during the Finnish winter war, but later (briefly), in the continuation war (1941–1944) he served in the artillery, at the Ladoga archipelago, and at the ballistics office (which at the time was part of the air force). Recall that bombs were raining down on Helsinki in regular daily intervals during the days of Ketonen's defence of his doctoral thesis, and that the work on it began with the arrival in Göttingen during the Kristallnacht. One cannot even imagine how these events must have affected the young logician! He writes in the autobiography how the war and everything it brought with it had a profound effect on him. One can speculate that the horrors of the war combined with his dissatisfaction with the philosophy of science prompted a need for a turn towards a broader philosophy incorporating ethical studies, and logic became a spare time activity instead of an object of full-time academic research.

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