An Unpublished Astronomical Papyrus Contemporary with Ptolemy

Anne Tihon

Several years ago, Jean-Luc Fournet drew my attention to an unpublished astronomical papyrus which is preserved in the Library of the Institut Français d'Archéologie Orientale (IFAO) in Cairo (*P. Fouad* 267 A). This text appeared to me at once to be an interesting document, and we decided to make a joint publication. But I was far from guessing how difficult and important it would be. At first glance I believed that we had a fragment of an ordinary commentary on Ptolemy's *Handy Tables*, or a fragment of an astrological treatise explaining with an example the use of the tables. But quickly I realized that we had here something rather different, a sophisticated document which has, to our knowledge, no equivalent in the astronomical material coming from late Antiquity.

The papyrus is a folio from a codex, and we thus have two parts: Part *a* (recto) and Part *b* (verso). The papyrus is remarkable for a number of reasons: it is a long text (22 lines plus tables on the recto, 32 lines on the verso), and moreover it is an extract from an astronomical or astrological treatise. It mentions an observation of Hipparchus not known elsewhere, and uses tables different from those of Ptolemy. Finally it gives an example set in the year A.D. 130, making it contemporary with Ptolemy.

The analysis of the text proved to be especially difficult, and there remain some unsolved problems. However it seemed to us useful to make the document known and to present the results of our study, as well as the questions raised by the papyrus. This paper is a preliminary presentation of the document. I will give here a provisional translation of Part *a* with a short summary and a quick analysis of its content. The reading of Part *b* is much more difficult, and I am not able to give a provisional translation at this stage of my research; but I will summarize briefly the matters treated in this part. The text itself will be edited critically, with photos and full commentary, in a book by J. L. Fournet and myself to appear in the *Publications de l'Institut Orientaliste de Louvain* (Louvain-la-Neuve), with the collaboration of Raymond Mercier. The analysis of such a document is a difficult and perilous exercise: the text cannot be read properly without having a precise idea of its content while the content cannot be understood without a perfect reading of the text! What I present here is thus a first approach based on the securest possible readings of the text and confirmed by calculation. But many questions and details need improvement and correction.

A. Tihon (✉)
Université Catholique de Louvain, Louvain-la-Neuve, Belgium
e-mail: anne.tihon@uclouvain.be

A. Jones (ed.), *Ptolemy in Perspective*, Archimedes 23,
DOI 10.1007/978-90-481-2788-7_1, © Springer Science+Business Media B.V. 2010

Part *a*: Provisional Translation[1]

1. …
2. …
3. of the Sun…
4. at 9 h of the night[2] we have established <first of all> starting with the point
5. calculating the nativity… by using 365 days $^{1}/4$ less $^{1}/.9$ ($=^{1}/309?$) ; <secondly>, without fraction[3]
6. since only a quarter day elapsed beyond the year, and that is the average (value);
7. and thirdly starting with the solstices, based on 365 days $^{1}/4$ $^{1}/102$ which, conforming to the observations of Hipparchus
8. he realised; showing thus for the present years how many degrees…
9. the tropical points are displaced in the retrograde sense (εἰς τὰ προηγούμενα) since the time of Hipparchus
10. with the degrees from the beginning of the table of the Syntaxis up to
11. the observation of Hipparchus which occurred in the year 166 after the death of Alexander (= 26 June 158 B.C.)
12. 28 Pachon at …[4] hours of the day. (Thus) in the retrograde sense
13. the tropical points were displaced in longitude by …[5] degrees which one must subtract
14. from the sum of the (numbers) of the tropical longitude of the Sun as we will show in the example.
15. Therefore making (the sum) of the mean numbers of the three (motions) of the Sun
16. one obtains for each the mean motion of the concentricity of the Sun from the apogee and with the magnitude obtained
17. we enter into the second column (or table?) of the eccentricity,[6] to be subtracted up to 180°.
18. Take as example the <15th> year of Hadrian
19. Athyr 11, according to the Egyptians, Choiak 20, the night (of 20) to 21 at 9 h
20. There are for the Sun up to the death of Alexander $\overset{\Gamma}{M}$ Ƶ 334 years
21. and from Alexander up to the <15th> year of Hadrian 454 years. In all one has $\overset{\Gamma}{M}$ …
22. I have given the result (?) of the degrees for each year thus:

From a point		ΔΓ[7]	mean	tropical	
$\overset{\Gamma}{M}$	240; 0, 0	$\overset{\Gamma}{M}$	.; . 0	$\overset{\Gamma}{M}$	264 [8]
Ƶ	8; 0, 0	Ƶ	.4 . 0	Ƶ	97
775	161; 36, 0	775	<1>69; 2, 20	775	171
13	356; 40, 19	13	356; 47, 48	13	356
Choiak	88; 42, 14	Choiak	88;4<2>, 22	Choiak	88
20	18; 43; 34	20	18;<43, 37>	20	18
h 9th night	0; 51, 45	h 9th night	0;<51, 45>	h 9th night	0
= beyond the circles	<154; 33, 52>	= beyond the circles		= beyond the circles	278
…	…	…	…	…	…

Commentary on Part *a*

Summary

In broad outline one can understand the text as follows:

(ll. 1–7) It is a question of calculating a nativity, that is an astrological *thema*. Here only the position of the Sun is involved. The text gives the method, then an example. The position of the Sun is calculated in three ways. We may complete the data in the following way:

(1) απο σημιου	"starting with a point"	360° / 365 1/4 1/102 d	sidereal year
(2) ομαλος	"mean"	360° / 365 1/4 d	mean year
(3) απο τροπων	"starting with the solstices"	360° / 365 1/4 - 1/<30>9 d	tropical year

The expression απο σημιου (1) means "starting at a point." In some ancient astronomical texts the expression is used for the revolution from one point of the celestial sphere to the same point. This is in contrast to the rotation from one solstitial or equinoctial point to the same solstitial or equinoctial point. The meaning of απο σημιου is clearly the sidereal revolution. In the text however there is a transposition between (1) and (3) since the value given for (3) is that of the sidereal year. As a philologist I do not like to suppose that the text is corrupted or erroneous. But here one can see by the title of the third column that the scribe erred, and corrected the title απο σημιου to τροπικος. We will see later how the numbers have been calculated, but they are rather clear in the papyrus.

(ll. 7–14) The value given for the sidereal year, 365 1/4 + 1/102 days, is said to agree with the observations of Hipparchus. The author of the "Syntaxis" used here has shown that the tropical points were displaced retrograde since the beginning of this Syntaxis up to the observation made by Hipparchus on Pachon 28 of the year of Alexander 166 (158 B.C. June 26). The number of degrees cannot be read in the text as preserved. These values must be subtracted from the tropical longitude of the Sun (col. 3).

We therefore have here three important data: (1) the length of the sidereal year according to Hipparchus: 365 1/4 + 1/102 days; (2) the mention of a "Syntaxis" different from that of Ptolemy; (3) a new observation of Hipparchus of the summer solstice, 26 June 158 B.C.

(ll. 15–17) We have the εγκεντροτης of the Sun, i.e. the "concentricity" of the Sun. With this quantity one must enter into a second table, a table of eccentricity, but this is not entirely clear. One finds a correction that must be subtracted up to 180°.

The terminology differs from that of Ptolemy: the word εγκεντροτης, "concentric," does not appear in Ptolemy, who uses "homocentric," but is used by Theon of Smyrna who often refers to Hipparchus.[9] Thus, as it seems, the papyrus assumes a model with an eccentric, with a correction table like that of Ptolemy, but the text is not perfectly clear.[10]

(ll. 18–22) The text gives an example for 11 Athyr (Alexandrian) = 20 Choiak Egyptian at 9 h of the night, that is A.D. 130, November 8. The text of the papyrus is therefore contemporary with Ptolemy. There is a problem concerning the Alexandrian day which should be the 12th of Athyr.

The chronological sum is the following:

334	from <*x*> to the death of Alexander
+ 454	from the death of Alexander to the year <15> of Hadrian
= 788	Total

The starting point of the tables—after the sections $\overset{\Gamma}{\mathrm{M}}$ and Ƶ which will be discussed later—is 658 B.C. Thoth 1 (this date is without historical significance, but is 500 Egyptian years before the observation by Hipparchus mentioned in the text.) Then follow the tables with calculation which will be commented later. This the content of Part *a*.

Explanation

Let us consider the following figure, which I find useful for understanding and commenting on the text. We suppose that at the beginning of the era the Sun (*S*) is placed at an equinoctial or solstitial point.[11] For convenience, we take Γ_0 as the Vernal Equinox, while E_0 is the corresponding point of the sidereal sphere, and M_0 the corresponding point on the "mean motion" sphere.

The three ways of calculating the longitude of the Sun are shown in Fig. 1:

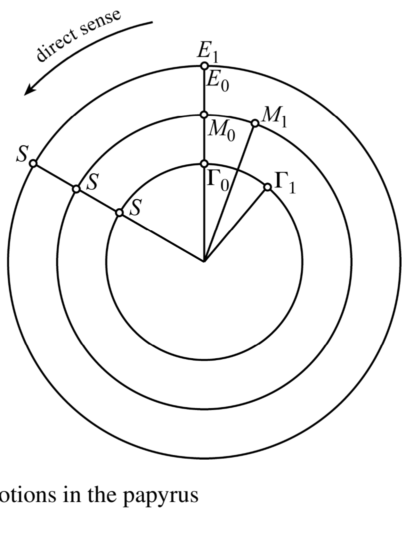

Fig. 1 The three mean motions in the papyrus

The points M_0 and Γ_0 are subject to a retrograde motion, that of Γ_0 being the precession. So at a given date the motion in longitude (over complete circles) during the periods of time given in the tables is:

$E_1 - S$ for the sidereal longitude of the Sun given in col. 1
$M_1 - S$ for the mean longitude given in col. 2
$\Gamma_1 - S$ for the tropical longitude given in col. 3

and $\Gamma_0 - \Gamma_1$ is the precession.

Dating and Chronological Divisions

As we have seen, the text gives an example, according to the Alexandrian and Egyptian calendar, using the regnal year of Hadrian; it also refers to the death of Alexander. The date is thus November 8, A.D. 130—exactly the time of Ptolemy. The chronological division is partly identical to that of the *Handy Tables*: groups of 25 years, single years, Egyptian month, day, and hour, as one can see in the tables: 775 years, 13 single years, Egyptian month (Choiak), day (20) and hours (9th of the night). The years here are elapsed years, contrary to the usage of the *Handy Tables* (where one would find for example 776 instead of 775).

The Length of the Years Used in the Tables

As we have already seen, the length of the different years used for those calculations is given in the text itself, but it can be reconstructed also from the values given in the tables. The concordance between text and calculation is a guarantee of the validity of our reconstruction. Since we have the motion of the sun during periods of 775 Egyptian years, 13 years and so on, it is possible to derive the following values:

Tab. 1	Tab. 2	Tab. 3
365;15,35d	365;15d	365;14,46 $>$ *n* $<$365;14,50d
= 365 $^{1}/_{4}$ + $^{1}/_{102}$ d	= 365 $^{1}/_{4}$ d	(probably 365 $^{1}/_{4}$ - $^{1}/_{309}$ d)
sidereal year	mean year	tropical year

The value of the tropical year remains somewhat uncertain, because the right edge of the papyrus is cut off, and the sexagesimal fractions are missing. But the value must be very close to the Ptolemaic value and the last figure: 9 (Θ) is clearly readible in the papyrus. I thus suggest reading $^{1}/_{309}$ (<T>Θ). With help of these numbers, we may reconstruct the daily motion of the Sun and the yearly motion (in 365 days) as follows:

	Tab. 1	Tab. 2	Tab. 3
daily motion	0.98560004867°	0.98562628…°	0.98563501645…°
	= 0;59,8,9,36,37°	= 0;59,8,15,16,…°	= 0;59,8,17,9,48,…°
yearly motion (365d)	359;44,38,27,45,5°	359;45,12,52,20°	359;45,24,24,37,0°
	sidereal year	mean year	tropical year

The Value of Precession

The text mentions explicitly the precession of the tropical points: between the start of this *Syntaxis* and the Summer Solstice of B.C. 158 June 26 observed by Hipparchus, the solstitial points are displaced in the retrograde direction; unfortunately the value of the displacement is not identifiable in the papyrus.

However the value of the movement of precession can be inferred with a good approximation from the data in the text. If one calculates the difference between the sidereal year of the papyrus and the tropical year one obtains an approximate value of

$$0;0,45,56,51,55^\circ \cong 0\,;0,46^\circ$$

which is $1^\circ/78y$. At this stage of our research, other values (such as $1^\circ/79y$) are still acceptable, but the next step of our analysis will decide in favour of $1^\circ/78y$.

The Meaning of the Abbreviations $\overset{\Gamma}{\mathrm{M}}$ *and* Ƶ

In the two first lines of the tables, one finds two signs, the meaning of which is the most mysterious question posed by this document. The Greek letter M (*mu*) with a small Γ (*gamma*) written above is a common way of writing 3 *myriades* (30,000), but *prima facie* such a huge number made no sense in this kind of astronomical calculation, which seemed rather similar to any calculation performed with Ptolemy’s tables. The second line marked by a Ƶ (*zêta* crossed with a bar) raised the same problem.

I had the idea that $\overset{\Gamma}{\mathrm{M}}$ could be an abbreviation for μέγας, and Ƶ for ζύγος (or ζεῦγος), both being well attested in the papyri. One would have had here an equivalent of the Sanskrit *mahayuga* (a great period), and *yuga* (a smaller period). In the end, I realized that these are indeed large periods, the magnitudes of which are given by the two symbols $\overset{\Gamma}{\mathrm{M}}$ = 30,000 and Ƶ = 7,000. So the comparison with the Sanskrit *yuga* had to be abandoned.

We know that the difference between col. 3 and col. 1 gives the precession during the given period. So, if we combine such periods with the value of the precession ($^{1°}/_{78\text{y}}$), we get the following results which are quite convincing:

$$30{,}000\ (\overset{\Gamma}{\mathrm{M}})\ \text{years} / 78 = 384;36,55,\ldots° - 360° = 24;36,55,\ldots°$$

This corresponds to the difference between col. 3 and col. 1:

$$264° - 240° = 24°$$

which gives the value of the precession for this period. In the same way,

$$7{,}000\ (\text{Ƶ})\ \text{years}/78 = 89;44,36\ldots°$$

corresponding to the difference between col. 3 and col. 1:

$$97° - 8° = 89°$$

which is again the value of the precession for this period. If we apply the same procedure using another rate for the precession, for example, $^{1°}/_{79\text{y}}$, the results are very far from the papyrus.

These periods are consistent with the results found if we recalculate the longitude for each tables, for these periods:

	daily	$\overset{\Gamma}{\mathrm{M}}$ (30,000 × 365 days)	papyrus	Ƶ (7,000 × 365 days)	papyrus
Tab. 1	0.9856°	(29,978 × 360°)	240°	(6,995 × 360°)	8°
	= 0;59,8,9,36°	+ 240°		+ 8°	
Tab. 2	0.98562628…	(29,979 × 360°)	… ?	(6,995 × 360°)	<7>4;2(?),0°
	= 0;59,8,15,16,…°	+ 167;48,10,…°		+ 75;9,14,…°	
Tab. 3	0.98563501645	(29,979 × 360°)	264;<…>°	(6,995 × 360°)	97;<…>°
	= 0;59,8,17,9,48…°	+ 263;25,25,8,20,…°		+ 97;27,51,56,40,…°	

The Tropical Longitude of the Sun at the Date of the Example and at the Beginning of the Era

Now we have elucidated all the lines of the calculations written at the bottom of the Part *a*. We are able more or less to complete the numbers—but some problems are caused by the fact that we do not know how many sexagesimal terms must be considered. For large periods like 30,000 or 7,000 years, it makes a real difference if one neglects the fourth or the fifth sexagesimal place.

Another problem arises from the fact that one expects to find as a final result the position of the Sun at the given date, namely A.D. 130 November 8, expressed in either sidereal, mean, or tropical longitude. This is not the case. According to a modern calculation, the tropical longitude of the Sun must be around 15° of Scorpio; according to Ptolemy's tables, at 225;49,35° (mean position) and 224;58,35° (true position).[12] None of the tables directly gives such a position. But two positions of the Sun are given in Part *b*, both in Scorpio (approximately 14° and 18°) but the reading of them is not well established.

Between the end of the tables in Part *a* and the legible text of Part *b* there is quite a large a gap, and one may suppose that the last part of the calculation was given there. One may think that, as in the *Almagest*, the epoch position at the beginning of the era is not built into the tables. At the end of a calculation made with the tables of the *Almagest*, one has to add this epoch position which is written at the top of the tables. Here, we know the time of the origin: the 1st of Thoth 37,788 Egyptian years (30,000 + 7,000 + 788) before the date of the example (A.D. 130), but we do not know the epoch position of the Sun. However it may be reconstructed from the data of the text. We may take the beginning of the third row of the tables which correspond to the 1st of Thoth 658 B.C. (4th of February 658 B.C.). The tropical longitude of the Sun at that moment, calculated with the *Almagest*, is 309;6,42° (mean position) and 311;16,42° (true position). Since the sum of the two first rows for the tropical longitude is around 361°, the approximate mean tropical longitude of the Sun was around 308° at the beginning of the era—though one would like to have these figures recalculated with more precision. If one adds this position, the result is:

$$\sim 308° + \sim 278° = 226° \text{ or Scorpio } 16° \text{ (mean position)}$$

a result which is roughly approaching the expected tropical longitude of the Sun at the date of the example.[13]

Sidereal Longitude

A sidereal longitude implies a reference star, but no name of a star is given in the text. In our explanation, we have taken an arbitrary starting point, E_0, which was the corresponding point of Aries 0° Aries (Γ_0) on the sidereal sphere at the beginning of the era. But we do not know if that point corresponds to a special star, and we do not know which was the shift between the tropical zodiac and the "sidereal zodiac" at the date of the example.

Nevertheless we note that the distance between Γ_1 and Γ_0 (the precession), which is here about 124°, is not far from the longitude of Regulus as given in the *Handy Tables* for the year A.D. 130:

25 y = 451	122;	24		
+ 3 y	+ 0;	1,	48	
454 y	122;	25,	48	

On the other hand, one may also remark that during the 37,500 Egyptian years counted from the beginning of the era to the observation of Hipparchus in 158 B.C., the precession at the rate of ${}^{1^\circ}/_{78y}$ is

$$37{,}500/78 = 360^\circ + 120;46,9\ldots^\circ$$

This could explain the bizarre choice of these unusual periods of 30,000 and 7,000 years. At the starting point of the system, the unknown author of this *Syntaxis* would have assumed that Regulus and the Spring equinox are both placed at the point E_0. But this hypothesis is rather uncertain. What we can say is that in the *Handy Tables* Ptolemy uses two systems for counting the longitude of the planets: they are first calculated from Regulus (α Leonis) and then adjusted to the usual tropical longitude measured from Aries 0°.

There are other questions which need more investigation, for example, why calculate also a "mean" longitude with a year of 365 ¼ days?[14] What was the underlying construction with an "enkentros" and an "ekkentros"? There is nothing in the text which can be interpreted as a reference to an apogee, but a construction with an eccentric certainly implies fixing an apogee and a perigee.[15]

Part *b*

The second part of the text (Part *b*) is difficult to read, but the problems treated in the text are rather clear: it is a matter of correcting the time, meaning seasonal and equinoctial time, and the author uses a table of oblique ascensions for the *clima* of Alexandria. The data are close to the *Handy tables*, but not exactly the same. One must note that the ascensions are expressed in terms of zodiacal signs—in the papyrus, the sidereal circle, the tropical ecliptic, and the equator are all divided in "zodiacal signs."

After converting the seasonal time in equinoctial hours, the author corrects the position of the Sun. There are two different positions given here, Scorpio 14° <…> and 18° <…>, but the sexagesimal figures are not clearly established.[16] Moreover, it is hard to decide which one is the sidereal longitude, and which one is the tropical longitude. After adjustment of the Sun's position due to the correction of the given time, the author continues with the calculation of the declination. Obviously, his table is very close to the *Almagest* table, but not exactly the same. The text is coming to an end, as it seems, with an explanation concerning the "direction" of the Sun, how to know if the Sun is in the "ascending" or the "descending" part of the ecliptic, and in the northern or the southern part, a problem which has no great interest for a modern scientist, but which is commonly discussed in the ancient astronomical commentaries.

As a conclusion, I would like to underline the exceptional interest of this papyrus. We find here a striking evidence of the fact that many different tables and astronomical *Syntaxeis* did exist at the time of Ptolemy.[17]

Notes

1. Underlined words are suggestions for a better understanding of the text.
2. Uncertain.
3. Uncertain.
4. The number is missing.
5. Illegible.
6. In the text "concentricity"?
7 Uncertain.
8 The border of the papyrus is damaged and the sexagesimal rows have disappeared.
9. Hiller 1878, 181 lines 7–9.
10. Since this paper was written, I have succeeded in identifying in the papyrus the symbol for the apogee of the Sun; so it is clear that the text implies a model with an eccentric. But the terminology still has to be clarified.
11. This starting point has been taken arbitrarily, and the real position of the Sun at the beginning of the era will be estimated later.
12. Calculation made to 2 sexagesimal places.
13. The figures given here are rough approximations.
14. Nallino 1903, vol. 1, 40: Hipparchus autem longitudinem anni ex 365 diebus et ¼ diei tantum constare fecit.
15. As I remarked above, I am now able to identify in the papyrus a symbol which represents the apogee; doubt about an eccentric model can be eliminated (see note 10).
16. The two positions of the Sun have been identified in Cairo as Scorpio 18;30° (sidereal longitude) and Scorpio 14;20,18° (tropical longitude), but they still have to be confirmed by a new examination of the original document.
17. Postscript: Since the meeting at Caltech in June 2007, I had the opportunity to examine the papyrus in Cairo with Jean-Luc Fournet in February 2008. Thanks to his talent, almost magical, and his expertise in the reading of papyri, significant progress was made in the edition of the text, especially in Part b. As a result, changes will have to be introduced in the translation and in many points of my analysis even if the general interpretation presented here can be maintained. Points which need to be improved or modified are indicated in the footnotes. I would like to express my warm thanks to Mme L. Pantalacci, Directrice of the IFAO in Cairo, who allowed me to work on the papyrus in the Institut Français d'Archéologie Orientale.