STRUCTURAL DAMAGE AND RISK ASSESSMENT AND UNCERTAINTY QUANTIFICATION

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Abstract. This article concentrates on the interplay between structural damage and risk assessment on one hand and numerical techniques, especially for uncertainty quantification, on the other hand. It shows the connection between damage assessment and risk quantification, touching on the methods of probabilistic risk assessment (PRA). It then details on how to initially asses the damage, which by necessity will involve some uncertainty, and how to update that initial assessment through additional testing. This is essential a statistical system identification process. The decision making process of finding whether the structure should be repaired or demolished is also mentioned shortly. It should involve a cost/benefit appraisal in the light of the information gained on the extent of the damage. Especially if the damage was caused by environmental forces, e.g. such as seismic action, it may be advantageous to determine the characteristic of this external action which caused the damage. This is a similar problem to the system identification of the structure, only that the testing is purely computational. Having identified the cause and the extent of the damage, one may want to draw lessons as to mitigate the hazard due future extreme environmental effect, in the form of robust design, minimizing vulnerability of life-lines and the fragility of structures.

Keywords: Risk assessment, uncertainty quantification, structural damage

1. Introduction

Structural Damage Assessment means to conduct damage and safety assessments of civil engineering structures and of infrastructure, and to perform structural inspections, and mitigation activities. The capability includes being able to provide contractor management, construction management, cost estimating, technical assistance, and other engineering services to support and manage response and recovery operations.

Here we will look at the role that uncertainty quantification can play in this process. The desired outcome of such an activity is the efficient

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implementation, management, and coordination of resources, aids emergency response, and recovery operations which restore the affected area to pre-event conditions.

For this to occur future hazards from the sustained damage has to be identified, the exposure to further threats has to be estimated, and the vulnerability of structures, infrastructure, and life-lines has to be considered. This is very similar to the usual risk assessment procedure, only that now the state of the system to be considered is also very uncertain, inasmuch as the amount of damage is not known yet.

For any kind of activity its benefit has to be weighed versus its risk (Starr, 1969). With structures and infrastructure that has been damaged, it is similar. These notes will concentrate on these more fundamental issues as well as on how a general procedure may actually be designed in structural damage and risk assessment, rather than give any specific prescriptions or guidelines on how to actually do the damage assessment, which may be found e.g. in FEMA (2004).

2. Damage, hazard, risk, and uncertainty

First of course it is necessary to dwell a bit on ontology, as to what is really meant by those terms *damage, hazard, risk,* and *uncertainty.*

2.1. ONTOLOGY

Let us start with the simplest of these, *damage*: Legally, as we are concerned with structures and infrastructure, it is a *property damage* we are talking about. It means that the object has changed in some way so that it cannot be used any more in the way originally designed or intended. A moment's thought shows that it is not only the *monetary* damage to objects (structures, infrastructure) which counts; as these may be life-lines for the population living there, a damage can threaten health and maybe even lives. Beyond that the damage maybe hampering economic possibilities, this one could term *opportunity costs* (King et al., 1997). All this of course enters into the decision which structures or infrastructure to repair first in case many have been damaged. Certainly on top of the priority list should be to avoid loss of life or hazards to health. When purely monetary terms count, then of course the opportunity costs have to enter the decision as well.

The term *hazard* is usually understood as the potential to cause harm in some way, i.e. the possibility of something happening which would cause further damage, or even loss of life. The larger the consequences or the potential of a damage or loss, the higher the hazard. To identify hazard, it is necessary to follow a proper way of reasoning (Pearl, 2000) in order to arrive at valid conclusions.

Hazard then has to be in some way combined with its frequency – or rather *probability* – of occurrence, and a combination of these two is usually taken to define *risk* (Kaplan and Garrick, 1981; Wilson and Crouch, 2001). Often it is the product of these two quantities which are taken as the definition of risk. In quite a number of cases, especially for very high or very low hazards, or very low or high probabilities, this simple combination seems not to be reasonable. It implicitly assumes that the decision-maker taking a certain risk is *risk-neutral*, which most people are not. In fact most people try to *avoid* risk, hence the institution of *insurance*.

The hardest to define of those terms seems *uncertainty* (Lindley, 2006), especially when contrasted with *risk*. While we have already defined – even though not completely specified – risk, it seems now the only thing left is to define uncertainty. But here we have the opinion of Frank Knight (1921), who in his view established a distinction between risk and uncertainty in this way:

> Uncertainty must be taken in a sense radically distinct from the familiar notion of Risk, from which it has never been properly separated.... The essential fact is that 'risk' means in some cases a quantity susceptible of measurement, while at other times it is something distinctly not of this character; and there are far-reaching and crucial differences in the bearings of the phenomena depending on which of the two is really present and operating.... It will appear that a measurable uncertainty, or 'risk' proper, as we shall use the term, is so far different from an unmeasurable one that it is not in effect an uncertainty at all.

Douglas Hubbard (2007) on the other hand gives the following distinctions, which by now have found widespread use in quantitative fields such as decision theory and statistics:

Uncertainty is a state where it is impossible to exactly or accurately describe the present state (of some system), or future outcome.

Quantification of uncertainty is the assignment of probabilities to each possible state or outcome.

Risk is a state of uncertainty where some possible outcomes have an undesirable effect, e.g. loss of life or property, damage to property.

Measurement of risk is the combination of quantified uncertainties which represent losses with their respective magnitudes of risk.

Often the term risk is used interchangeably with its measure; and the term state – unless qualified to mean the state of a system – is the state (of information) of the decision maker, or the one who might suffer the loss.

Of course the term *uncertainty* appears from philosophy, psychology, decision theory, probability, information theory to medicine and economics. It is hardly surprising that there are many different views (Tannert et al., 2007). This does not have to involve any philosophical statement on *determinism*, but may simply mean that certain things are not predictable, hence uncertainty is concerned with *unpredictability* and not a fundamental philosophical statement about the nature of the universe.

We will stick to the latter of the two views above (Kaplan and Garrick, 1981), and further only look at what is called *objective* uncertainty (Tannert et al., 2007), although *subjective* uncertainty in the form of *moral* uncertainty certainly enters many decisions concerned with structures and infrastructure and involving the hazards of loss of life and monetary costs. Objective uncertainty we will often think of as further subdivided on one hand into *aleatoric* uncertainty, where *it is in the nature of things* that we cannot give an exact description of future outcomes, and where also this uncertainty cannot be reduced arbitrarily through e.g. further measurements. The other aspect of objective uncertainty we will call *epistemic* uncertainty, i.e. it arises from our lack of information, and could possibly be reduced e.g. by further measurements. In any case one may take the uncertainty as a measure of information, our ability to predict a present state or future outcome.

While most scientists agree that aleatoric uncertainty can be measured via probability theory, and while there are convincing arguments that this is also the proper view for epistemic uncertainty (Jaynes, 2003), there are methods which are claimed to be suitable to model this kind of uncertainty, like propagation of convex sets (containing the uncertain information) (Natke and Ben-Haim, 1997; Ben-Haim and Elishakoff, 1990), or fuzzy methods (Zimmermann, 1992; Uncertainty in Engineering, 2000).

In relation to hazards, *vulnerability* is the extent to which changes can harm a system, i.e. in a certain sense its sensitivity to outside influences, or the susceptibility to damage. A very similar term is *fragility*, i.e. when small damages may yield a structure or infrastructure more or less completely useless.

2.2. INTERDEPENDENCE

To complete the picture, one should also point out the many interdependencies which exist between the various concepts.

A damage may incur further hazards, insomuch as a damage changes the state of the structure. This in turn may change the risk, as it changes maybe also the probabilities of other events stochastically dependent on the damage. It may also change the risk by changing the amount of subsequent loss. for many interdependent systems, such as an urban area, this may lead to a cascade of risk and uncertainty.

One may now see that structural damage assessment is very similar to any other model building (Pearl, 2000) exercise, and estimation of a state as treated in many control theory texts, and system identification (Ljung, 1999).

Also obvious are similarities with risk assessment (Kumamoto and Henley, 1996), integrity or structural health monitoring (Yao and Kawamura, 2001), and performance prediction and diagnosis (Natke and Cempel, 1997). And certainly risk assessment is an integral part of any kind of risk management (Flyvbjerg, 2006).

Damage and structural reliability (Ditlevsen and Madsen, 1996; Yao and Kawamura, 2001) are similarly linked, first because structural reliability is concerned with avoiding damage, or minimizing the probability of damage occurring, and secondly because after a damage the methods of structural reliability may be used to define subsequent and dependent risks.

3. Methods

A good general overview – although somewhat restricted – for a specific but fairly exemplary application area is contained in (Net, 2000).

Methods to identify damage range from the purely educated visual (FEMA, 2004) which will always be needed as a first guess (Revadigar and Mau, 1999), to further testing (Pearl, 2000), maybe involving the theory of design of experiments (Box and Hunter, 2005), and sequential optimum design (of experiments) (Chernoff, 1972). These techniques, as well as those already mentioned in Section 2.2 in (Ljung, 1999), as well as those used to estimate the state of a system, rely heavily on *Bayesian* statistics (Lindley, 1972), enforcing the view to treat epistemic uncertainties on an equal footing via probability theory, see Section 2.1.

Eventually we want to arrive at a probabilistic risk assessment (*PRA*) (Kumamoto and Henley, 1996). To perform a PRA, an analysts may go through the following steps:

1. Specify the *hazard*, the outcome(s) to be prevented or reduced.
2. Identify initiating events, those that could possibly lead to the specified consequence.
3. Estimate the *frequency/probability* of each initiating event.
4. Assuming that the initiating event has occurred, identify the *combinations* of failures that lead to a specific outcome.

5. Compute the *likelihood* of each combination. The *probabilities* of all those sequences that lead to the same outcome are added. To determine how often this outcome might occur, these probabilities are multiplied by the frequency of the initiating event(s).

As already mentioned, probability theory (Loève, 1977; Krée and Soize, 1986; Jaynes, 2003) is used in most cases to address the uncertainty (Augusti et al., 1984; Ditlevsen and Madsen, 1996), usually on a Bayesian setting (Lindley, 1972; Jaynes, 2003). But combinations with supposedly simpler theories (usually yielding considerably less information) are also proposed. These are e.g. bounds – interval analysis, convex sets – (Natke and Ben-Haim, 1997; Ben-Haim and Elishakoff, 1990), fuzzy analysis (Zimmermann, 1992), and combinations such as *fuzzy probability*, (Uncertainty in Engineering, 2000; Möller and Beer, 1998, 2004; Möller and Reuter, 2007), see also (Elishakoff, 1999). A modification of the Bayesian point of view, which tries to accommodate conflicting information and is based on *belief* and *plausibility* is the *Dempster-Shafer* theory (Dempster, 1968; Shafer, 1976).

The uncertainty then has to be computationally propagated through the system(s). Ideally, given the probability distributions of the input, the numerical models should produce the probability distributions of the outputs, or any other stochastic information desired. This may in the simplest case be sensitivity analysis (Kleiber and Hien, 1992). It may pertain only to (asymptotically) very small probabilities (Ditlevsen and Madsen, 1996). Conceptually the simplest is the Monte Carlo method or its variants (Caflisch, 1998). Overviews are given in (Matthies and Soares, 1997; Schueller, 1997), and more recently in (Matthies, 2007a, b). Modern developments relate to the view of random variables (stochastic processes, random fields) as elements of a vector space, to be approximated through finite dimensional subspaces (Ghanem and Spanos, 1991; Matthies and Bucher, 1999; Matthies and Keese, 2005; Xiu and Karniadakis, 2002) in the sense of weighted residual Galerkin methods, which then also includes collocation methods. In some ways this may be seen as a systematic way of producing *response surfaces* (Khuri and Cornell, 1987).

4. Identifying the damage

As already mentioned, identifying the damage is very similar to system identification. In case damage occurs to a structure, there has to be a quick initial assessment of the kind

– Unsafe, dangerous to enter
– Safe, but unfit for human habitation – occupancy should be restricted

– Lightly damaged, but habitable – occupancy permitted
– Not damaged, not requiring any or only cosmetic repair

This may also involve posting the structures as “unsafe to enter” or just “inspected”, meaning safe to enter.

For life-lines and infrastructure similar criteria may be drawn up, and especially for life-lines it is critical to quickly establish whether and/or to what extent are they damaged, are they completely unusable, partly usable, what is the most urgent repair to get them partly on line again? Damage to critical infrastructure includes

– Water and sewage (leaks, damaged pipes de-pressurized?)
– Electric (e.g. exposed power lines de-energized?)
– Oil and gas pipelines (e.g. line breaks sealed?, leaks contained?)
– Gas and propane storage (e.g. tanks inspected, secured by qualified experts?)

This may involve identifying qualified contractors offering damage assessment services, developing damage assessment procedures, developing mitigation plans and procedures, conducting debris assessment, and assessing the requirement for decontamination or safe demolition, removal, and disposition of contaminated debris.

It may include the provision of geo-coded status reports of community, homes and facilities identified as safe or unsafe to re-enter and re-occupy. Situation assessments are conducted using one of following methods (FEMA, 2004):

1. Aerial reconnaissance
2. Remote sensing
3. Computer modelling (e.g., HAZUS [FEMA, 2004])
4. Rapid field assessments/windshield surveys

4.1. INITIAL ASSESSMENT OF DAMAGE

Hereby we mean a more formal assessment than the *first* assessment just sketched, trying to identify the extent of damage and the state of the structure. In a Bayesian setting it may be described as guessing à priori information. Methods used beyond visual inspection are those used in any other system identification process (Revadigar and Mau, 1999), such as vibration test, (ultra-)sound testing, thermal image analysis, etc. (Natke and Cempel, 1997).

The approach yielding finally the most information is to apply a probabilistic modelling, usually on a Bayesian setting (Lindley, 1972; Jaynes, 2003). This then means assigning so-called à priori probabilities, reflecting the state of information. A modification of this is using two measures, belief and plausibility instead of just probability (Dempster, 1968; Shafer, 1976).

Also sometimes used are supposedly simpler theories – yielding less information. These include simple bounds in convex sets (Natke and Ben-Haim, 1997; Ben-Haim and Elishakoff, 1990) fuzzy analysis (Zimmermann, 1992) using possibility instead of probability, and combinations like fuzzy probability (Möller and Beer, 1998, 2004; Möller and Reuter, 2007), see (Elishakoff, 1999) for some examples.

4.2. FURTHER TESTING AND UPDATING

If after the initial assessment described in Section 4.1 the uncertainty on how to class a structure is still too large, especially when deciding whether it is safe or not, and whether it should be demolished or repaired, further testing may be required. This is intended to demonstrate the structural static/dynamic properties of the structure. It is a method to obtain à posteriori information (Lindley, 1972; Jaynes, 2003), through carefully selected experiments. This may use the techniques known as *design of experiments*, and *optimal sequential design* (Chernoff, 1972; Pearl, 2000; Box and Hunter, 2005). The à posteriori information in this approach will come through Bayesian updating of the à priori distributions as new information is included.

Computational models of the structure become very important in this whole phase, as they serve to refine the estimate of the state of the structure. They have to be able to propagate probability/uncertainty information, see the description in Section 3 and the overviews in (Matthies and Soares, 1997; Schueller, 1997). These computational models can be of differing complexity, but they may help in refining the assessment made as described in the beginning of this Section 4.

5. Contribution of decision theory

Decision theory may be used to decide on possible courses of action (Pearl, 2000; Flyvbjerg, 2006), taking care to properly estimate costs, be they due to repair, or demolition and rebuilding. There is a long history of cost-overruns in civil engineering work, and there are methods to obtain an *outside view* (Flyvbjerg, 2006). In any of these decisions, there will be a cost/benefit appraisal (Starr, 1969; Wilson and Crouch, 2001). It involves the allocation of resources for rehabilitation, rebuilding and strengthening.

Again computational models of the structures/infrastructure are very important here, as they help in evaluating different "what if?" scenarios.

If the only purpose of damage assessment was to decide what to do with damaged structures or infrastructure, the task is finished here. But if one wants to learn for the future, especially important if the damage was caused by extreme environmental forces, such as floods, storms, or earthquakes, more analysis is necessary.

6. Reconstructing/identifying the cause

In case of environmental loads leading to damage, it is often desirable to identify the magnitude of those forces, in order to learn for future events. The damaged structure or infrastructure may be used as a "measurement device". Otherwise the whole process is very similar to the one described in Section 4. Only instead of the structure, here the objective is to identify the cause for the damage, or the magnitude and nature of the environmental force. In this process, a computational model capable of propagating uncertainty/stochastic information is definitely needed.

6.1. INITIAL ASSESSMENT OF CAUSE

The initial assessment is similar to the state of the structure an "educated guess" as to the nature and magnitude of the causative action which resulted in the damage, see Section 4.1. This should ideally involve a modelling of the uncertainty of this educated guess, as to express the variation deemed possible for the environmental forces.

Concerning environmental forces, there is common agreement that probabilistic modelling is adequate, usually on a Bayesian setting (Lindley, 1972; Jaynes, 2003). This means assigning so-called à priori probabilities to actions, based on the usually extensive knowledge on how they are generated – e.g. storms, floods, earthquakes.

6.2. COMPUTATIONAL TESTING AND ACTION IDENTIFICATION

This step parallels the one in Section 4.2, only that the testing and updating is performed numerically. The numerical results then may be compared to the estimated state of the damaged structure. The whole process is very akin to system identification, this time not of the structure, but of the environmental action.

The numerical models have to propagate the probability through the system(s), as described in Section 3. As already mentioned the simplest is sensitivity analysis (Kleiber and Hien, 1992). For very small probabilities, asymptotical methods as in structural reliability may be used (Ditlevsen and Madsen, 1996).

Conceptually the simplest method which can give complete distributional information is the Monte Carlo method or its variants (Caflisch, 1998). Overviews on all such methods is given in (Matthies and Soares, 1997; Schueller, 1997), and more recently in (Matthies, 2007a, b).

Newer developments relate to directly approximating random variables (stochastic processes, random fields) as elements of a vector space through finite dimensional subspaces (Ghanem and Spanos, 1991; Matthies and Bucher, 1999; Matthies and Keese, 2005; Xiu and Karniadakis, 2002). As already mentioned such approximations of the random quantities may be seen as a systematic way of producing response surfaces (Khuri and Cornell, 1987).

In any case, the computational models should help answer the question what are the likely distributions and magnitudes of the environmental actions, in the form of an Bayesian updating to à posteriori distributions of the à priori distributions of Section 6.1, see (Lindley, 1972; Jaynes, 2003).

7. Damage tolerant design

Having identified the type of damage, as well as the environmental actions which caused it, together with their uncertainty quantification, it is now possible to do some kind of improvement/optimization.

7.1. MINIMISE EXPECTED DAMAGE

One way to reduce risks is to reduce the hazard, e.g. to make the structure or infrastructure less prone to certain kinds of exterior actions. Especially for infrastructure life-lines this can often achieved by choosing carefully where they are installed, using redundancy, etc.

The other aspect of risk reduction is the minimization of potential loss in case the hazard *does* occur.

All of these items require a cost/benefit analysis (Starr, 1969); a guideline here may be the so-called "ALARP" principle, an acronym meaning **A**s **L**ow **A**s **R**easonably **P**racticable. It is a very common sense approach regularly practised in the off-shore industry. After a probabilistic risk assessment, the risks – of course forming a continuum – are divided into three main categories. One are those which are considered negligible, here nothing has to be done, no design optimization is necessary. At the other end are risks which are considered unacceptable, and they have to be avoided categorically. In between is the ALARP region, where risks are not negligible, but the amount of reduction follows the ALARP principle. This means that a risk has to be reduced if this incurs no substantial costs, and all risks are reduced to the point where further efforts yield only marginal risk reductions. At this point it is usually meaningless to try to reduce the risk any further, at least not with the methods applied up to that point. Here a decision has to be taken, as to whether the residual risk is acceptable; if not the activity it is pertaining to has to be stopped, or a completely different design has to be chosen.

7.2. REDUCE VULNERABILITY AND FRAGILITY

The reduction of possible losses has at least two aspects (Douglas, 2007), one is the vulnerability, and the other the fragility.

Reduction of vulnerability may be achieved by reducing the possible impact of the hazard, in general terms moving things out of harms way. Infrastructure development considering vulnerability is especially important, to ensure that life-lines remain operable to as large an extent as possible. Structures may be valued in reference to vulnerability with impact elements, to arrive at some overall idea of vulnerability.

Fragility is a property of the structures or the infrastructure. It may be reduced through robust design, i.e. designs which do not suddenly increase the risk due to smaller damages. It can often represented through so-called fragility curves, showing the risk versus larger magnitude of the actions. In these circumstances a design which has only slowly varying risk with a gradual increase in magnitude is less fragile, there cannot be any unexpected failures. Often, for structures, this boils down to the existence of several load paths, i.e. statically highly indeterminate designs. This is often at variance with traditional design methods, and also often contradict “optimal designs” which are optimal with respect to other criteria, like minimum weight or minimal cost.

Methods used here are similar to those in structural reliability analysis (Augusti et al., 1984; Ditlevsen and Madsen, 1996), where it is also important to estimate the sensitivity of the risk to design parameters.

8. Conclusion

This article has tried to illuminate the issues involved in structural damage and risk assessment, and on the interplay between structural damage and risk assessment on one hand and numerical techniques, especially for uncertainty quantification, on the other hand.

Damage assessment and risk quantification are closely interrelated, once the basic terms have been defined this becomes very clear. Damage assessment and risk quantification go hand in hand from very simple to more and more sophisticated, such as time and possible expenses allow, and as is meaningful for the questions at hand. The similarities to statistical system identification have been highlighted. Any decision on possible actions involves a cost/benefit appraisal on the basis of present information.

If the damage was caused by environmental forces such as seismic action, one may use the damaged structures as a "measurement device" to determine the characteristic of this external action. This is similar to the system identification of the structure with testing purely computational. Reduction of risk is possible by considering the vulnerability and fragility of design of life-lines, infrastructure, and structures.

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