Ludwig Burmester (1840–1927)

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Abstract Ludwig Burmester was a nineteenth century German mathematician. His *Lehrbuch der Kinematik, Erster Band, Die ebene Bewegung* (Textbook of Kinematics, First Volume, Planar Motion), of 1888, contains the first far-reaching attempt at a synthesis of theoretical kinematics and kinematics of mechanisms. His most influential technical contribution is the Burmester theory which deals with four or five discrete positions of a moving plane. The goal of the Burmester theory is the synthesis of mechanisms. In the present paper, Burmester's work is discussed against the background of the development of the theory of machines and mechanisms in the nineteenth century.

Biographical Notes

Ludwig Ernst Hans Burmester, Fig. 1, was born on May 5, 1840, as son of a gardener in the village of Othmarschen near Hamburg in Germany. He died on April 20, 1927, as a respected professor emeritus of Descriptive Geometry and Kinematics at the University of Munich and a member of the Bavarian Academy of Sciences. Between these dates lies a rich and varied professional life dominated by Geometry and Kinematics.

At the age of 14, Burmester became an apprentice in the workshop of a Hamburg precision mechanic. He was allowed to go to the Polytechnical Preparatory School in Hamburg run by Otto Jensen. Jensen was an excellent man who was not only a great teacher of mathematics but good at convincing rich citizens of the city of Hamburg to support talented children like Ludwig Burmester. Because Burmester thought that there was a good future in telegraph machines, he left Hamburg and went to Berlin where Siemens & Halske were building such machines. Yet, in the end his desire to study prevailed.

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Fig. 1 Ludwig Burmester (1840–1927) when he was a professor in Dresden (Courtesy of Dr. Klaus Mauersberger, T. H. Dresden)

In 1862 he enrolled at the Polytechnical School in Dresden in the department for future teachers of mathematics, science and technology. He attended the classes of Oskar Schlömilch on several areas of mathematics. It is remarkable that he seems to have acquired a considerable knowledge of geometry that later dominated his activities, all by himself. In 1864, Burmester received his diploma in Dresden with honours.

In the next years, Burmester studied in Göttingen where he obtained his doctor's degree in 1865.

In 1868, Burmester married Gabriele Schallowetz (August 4, 1848–November 15, 1919). Out of the marriage a daughter, who died very early and three sons were born. After 1865, Burmester worked for 4 years as a secondary school teacher in Lodz in Russian Poland.

In 1870, Burmester had to leave Lodz after the Russians had closed down the secondary school he was teaching in. In 1872, Burmester's luck changed, he became professor of descriptive and synthetic geometry in Dresden, where he had become Privatdozent after his Habilitation in 1871. In 1875, he started teaching projective geometry and in 1879 he started to teach kinematics. Much of his work in this period is related to kinematics. One of his earliest publications after his appointment in Dresden in 1873 combines a growing interest in kinematics with geometrical questions concerning projection and shadows. On 1 October 1887, Burmester was appointed to the chair of descriptive geometry and kinematics in Munich. For the events leading to his appointment, see Hashagen (2003, pp. 500–506). In Munich, where he lived in the Kaulbachstraβe 83, Burmester was highly respected. In 1905, he became a member of the Bavarian Academy of Sciences. In the last years of his life he developed a great interest in the mechanical engineering side of cinematography. He lectured on this subject and he studied different methods for guiding film in a projector.

Burmester died in 1927 of a heart attack. He was buried next to his wife in the cemetery of Kreuth near Lake Tegernsee. The grave is still there. Two necrologies appeared, written by Finsterwalder (1927) and Müller (1930). They show that one of the giants of nineteenth century German kinematics had died.

Burmester's Work

Burmester's Göttingen thesis is titled *Ueber die Elemente einer Theorie der Isophoten* (About the elements of a theory of isophotes). It deals with the representation of lighted surfaces in three-dimensional space. Isophotes on a surface are lines of constant light intensity: they are the sets of points for which the cosine of the angle between the light direction and the surface normal has a constant value. It is remarkable that Burmester's treatment of the subject is entirely analytical. The contrast with Burmester's later works, in which his method is more and more synthetic, is considerable.

In 1871, his first book appeared in Leipzig: the *Theorie und Darstellung gesetzmäßig gestalteter Flächen* (Theory and representation of well-defined surfaces). It was reprinted in 1875. The book is about the subject of his dissertation, the theory of isophotes.

Interesting is Burmester's distinction between lines of "true light intensity", the isophotes, and lines of "observed light intensity", which he called isophengs. They are the sets of points for which the product of, on the one hand, the cosine of the angle between the light direction and the surface normal, and, on the other hand, the cosine of the angle between the direction of the eye and the surface normal, has a constant value. Figure 2 is an example taken from (Burmester 1871).

Burmester was interested in all aspects of the theory of perspective as well. In 1883, in Leipzig he published his book *Grundzüge der Reliefperspective* (The Main Features of Relief Perspective), dealing with the principles of relief perspective. In ordinary perspective, a half space is mapped on one plane of projection; in relief perspective, a half space is mapped on the space between two parallel planes. Burmester built several models in order to demonstrate the effect of his theory (See Fig. 3).

In 1884, Burmester published *Grundlehren der Theaterperspective* (Foundations of Theater Perspective). It is said that he was involved in the design of the decor for some of Wagner's opera's.[1]

It is remarkable that both the subject of isophotes and the subject of relief perspective have recently attracted interest in the area of computer aided geometric design (Lordick, 2005).

[1]Cf. http://www.mathematik.tu-muenchen.de/~kaplan/fakul/node16.html. I have not been able to check this. Wagner died in 1883, before Burmester came to München. Yet the Germans took theatrical design very seriously and it is quite possible that during Wagner's last years or after his death, Burmester was involved in the scenography in Bayreuth.

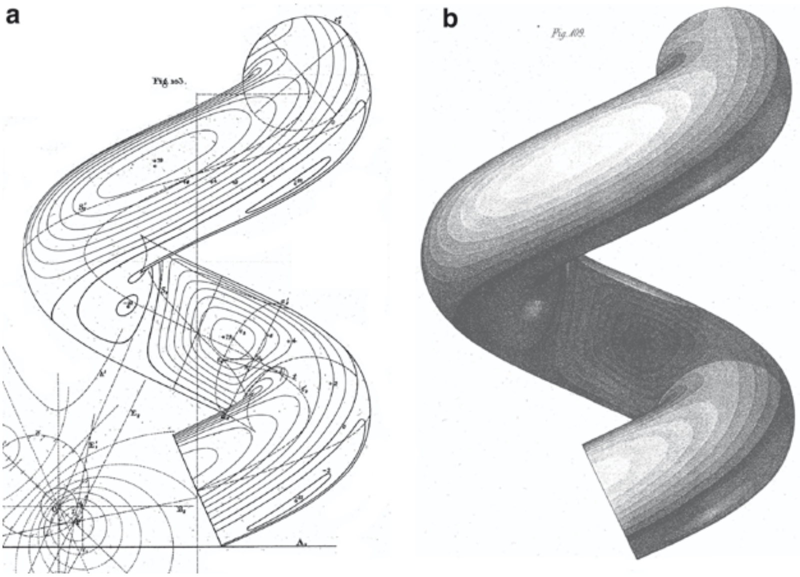


Fig. 2 Figures from Burmester's book on isophotes: (**a**) lines of equal light intensity; (**b**) a shadow representation

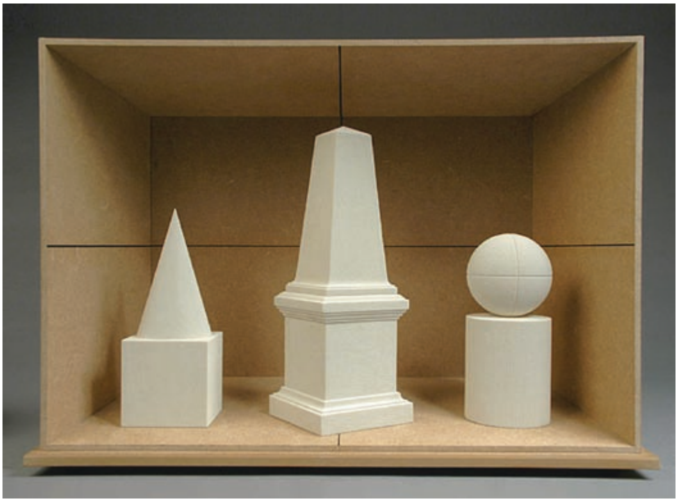
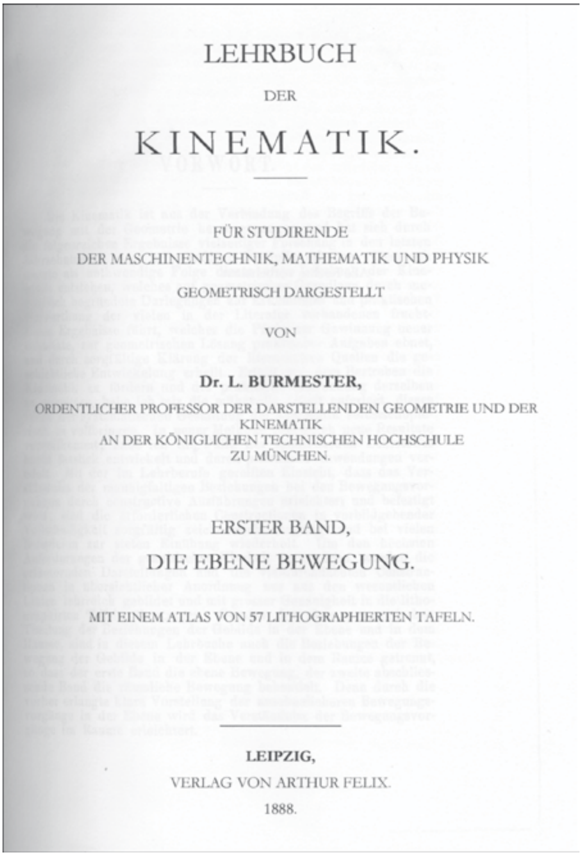


Fig. 3 A reconstruction by Dr. Daniel Lordick (TU Dresden) of one of Burmester's relief perspective models (Lordick 2005) (Courtesy of Dr. Lordick)

The turn to kinematics was crucial in Burmester's life. Before the 1960s of the nineteenth century, kinematics of mechanisms was hardly a coherent discipline and results from theoretical kinematics were not applied systematically to the kinematics of mechanisms. In the second half of the nineteenth century, things changed.

Kinematics enjoyed great popularity among both mechanical engineers and mathematicians. The work of the German engineer Franz Reuleaux (1829–1905) was in particular very influential. Reuleaux's *Theoretische Kinematik* (a theoretical treatise on kinematics of mechanisms, in spite of its title. See below), published in book form in 1875, paved the way for further mathematization of kinematics of mechanisms. On the other hand mathematicians also played an essential role. Ludwig Burmester was a prominent example.

Burmester's most important contribution to kinematics is undoubtedly his *Lehrbuch der Kinematik, Erster Band, Die ebene Bewegung* (Textbook of Kinematics, First Volume, Planar Motion). It was originally published in three installments; the first two installments appeared in 1886 in one volume of 560 pages and the last installment appeared at the end of 1887. In 1888 the three installments appeared in one volume. In 941 pages accompanied by an atlas with 863 figures presented Burmester gave a survey of everything he knew about planar kinematics, which included many of his own results (Fig. 4).



LEHRBUCH

DER

KINEMATIK.

FÜR STUDIRENDE
DER MASCHINENTECHNIK, MATHEMATIK UND PHYSIK
GEOMETRISCH DARGESTELLT

VON

Dr. L. BURMESTER,

ORDENTLICHER PROFESSOR DER DARSTELLENDEN GEOMETRIE UND DER KINEMATIK
AN DER KÖNIGLICHEN TECHNISCHEN HOCHSCHULE
ZU MÜNCHEN.

ERSTER BAND,
DIE EBENE BEWEGUNG.

MIT EINEM ATLAS VON 57 LITHOGRAPHIERTEN TAFELN.

LEIPZIG,
VERLAG VON ARTHUR FELIX.
1888.

Fig. 4 Title page of Burmester's *Lehrbuch der Kinematik* (1888)

In 1890, he and the other mathematicians from München were involved in the foundation of the *Deutsche Mathematiker-Vereinigung* (Association of German Mathematicians).

Review of Main Work Related to Mechanism Design

The Background

With the completely new classification of mechanisms by means of kinematical chains in the 1860s, Reuleaux had introduced a very abstract point of view in kinematics of mechanisms. For a mathematician, this boiled down to viewing a planar mechanism as a collection of coinciding Euclidean planes all moving (with one degree of freedom) with respect to each other. This was a major and not trivial step forward which helped to turn kinematics of mechanisms into a more coherent discipline. It involves two related elements: considering the frame of a mechanism as a link and, moreover, abstraction from the particular shape of the links in a mechanism and concentration on the way in which the links are connected. Without mentioning Reuleaux, Sylvester wrote in 1875:

> The true view of the theory of linkages is to consider every link as carrying with it an indefinitely extended plane and to look upon the question as one of relative motion [...] Fix any of these planes and the linkage becomes a link-work [...]. (Sylvester, 1875)

In the investigation of bar-mechanisms, Sylvester attributed the extension of the consideration to the planes connected with bars to Samuel Roberts. Yet the priority belongs to Reuleaux who developed his ideas at least 5 years before Roberts turned to kinematics at the end of the 1860s.

Burmester's turn to kinematics in Dresden is related in different ways to Reuleaux' influence. Undoubtedly, the abstract point of view introduced by Reuleaux in the theory of mechanisms combined with Burmester's great interest in the recently developed geometrical theories enabled Burmester to do his kinematical work. As for geometry, Theodor Reye's *Geometrie der Lage* (Projective geometry) of 1866, revised and reprinted several times, is representative of the geometrical background of Burmester's work in kinematics). It seems to have been Rittershaus who stimulated Burmester to do kinematics. Trajan Rittershaus (1843–1899), professor of kinematics and electro-technical machines in Dresden, had been Reuleaux' student in Zürich and his assistant in Berlin.

The Lehrbuch der Kinematik: A Characterization

Nowadays it is not uncommon to distinguish between *theoretical kinematics* dealing primarily with the general properties of motion and *kinematics of mechanisms* which concentrates on the kinematical properties of (classes of) specific mechanisms. As we will see below, Reuleaux used the term kinematics in a different way. We will use the modern terminology. Burmester's *Lehrbuch der Kinematik* contains the first far-reaching attempt at a synthesis of theoretical kinematics and kinematics of mechanisms. The planned second volume on spatial kinematics was never written. The content of the kinematical papers that Burmester wrote before the publication of his book, are all included in the book. In 1874 and 1875, he published on affine and equiform kinematics: the moving plane moves and is subjected to affine or equiform transformations as well, while the fixed plane remains Euclidean. The results return in the twelfth chapter of the *Lehrbuch der Kinematik*. In 1876 and 1877, he wrote two very original papers on what would later be called the *Burmester theory*. The Burmester theory returns in the ninth chapter of the book. In 1880 he published a systematic investigation of the instantaneous velocities of n rigid systems moving with respect to each other in the plane. Aronhold had discovered the three centers theorem when he prepared a course on theoretical kinematics in Berlin, which took place in 1866–67. Twenty years later, Kennedy would independently discover the same theorem. In Burmester's 1880, paper the full potential of the theorem in velocity analysis became clear. The results return in the seventh chapter of the book. It is remarkable that this paper exerted some influence in civil engineering. The results were used for the calculation of forces in a framework by means of the principle of virtual displacements (Kurrer 2002, pp. 521–525). After 1888, Burmester continued to publish on kinematics. In 1893, he thoroughly investigated a class of overconstrained mechanisms introduced by Kempe in 1878. He returned to affine kinematics, this was in 1902, a subject that he had studied in 1874 and 1875. In 1911 and in 1925, he published on acceleration in planar motion. The *Lehrbuch der Kinematik* consists of 12 chapters. It is a compilation of almost everything that was known on planar kinematics at that time. On the one hand, Burmester gave a survey of the results from theoretical planar kinematics. On the other hand, he systematically studied their application to practically all planar mechanisms that had been identified at the time. Examples of figures from the atlas are given in Figs. 5 and 6.

The Lehrbuch der Kinematik: Its Contents

Let us consider the structure of the *Lehrbuch der Kinematik* in more detail. The first chapter is devoted to the general properties of planar motion: velocity, the instantaneous center of rotation (the pole), the polhodes, the motion of three coinciding planes with respect to each other, construction of tangents and centers of curvature, with many examples of special motions.

Chapter two concerns the different cycloids and their properties.

Chapter three deals with cylindrical gears, cycloid gearing, involute gearing.

Chapter four deals with the gear in internal gear pumps (*Kapselraeder*). In this chapter, Burmester discusses all existing types of such pumps.

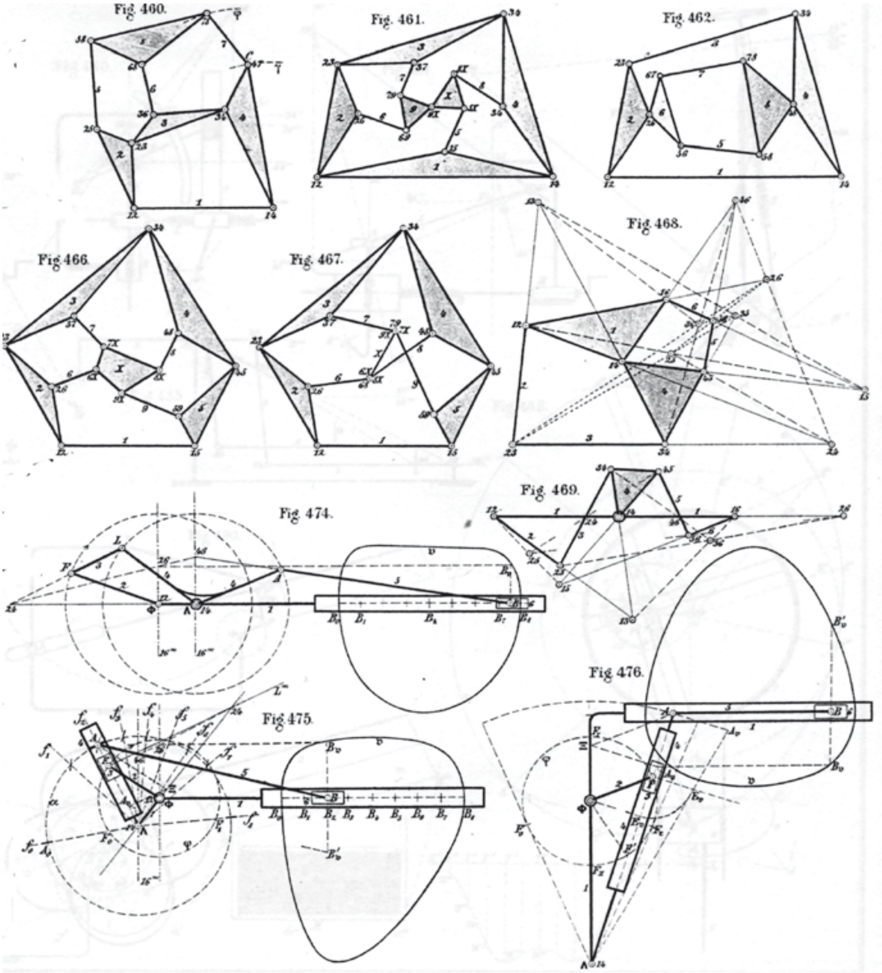


Fig. 5 Figures accompanying Burmester's seventh chapter of the *Lehrbuch der Kinematik* on compound planar mechanisms

He always moves from the very general to the specific. The theories given in the first chapter are applied in chapters two to four. In the fifth chapter, Burmester again starts from very general considerations, which are applied in chapters six and seven.

In the fifth chapter, he deals with the general theory of constraints (*Zwanglauf*): restrictions on the motion of a system. The final goal of this theory, which was defined by Reuleaux, is to determine the most favorable ways to realize a particular one-degree-of-freedom constrained motion. Reuleaux' notions of kinematic pair (consisting of two elements that constrain each other's motion) and kinematic chain

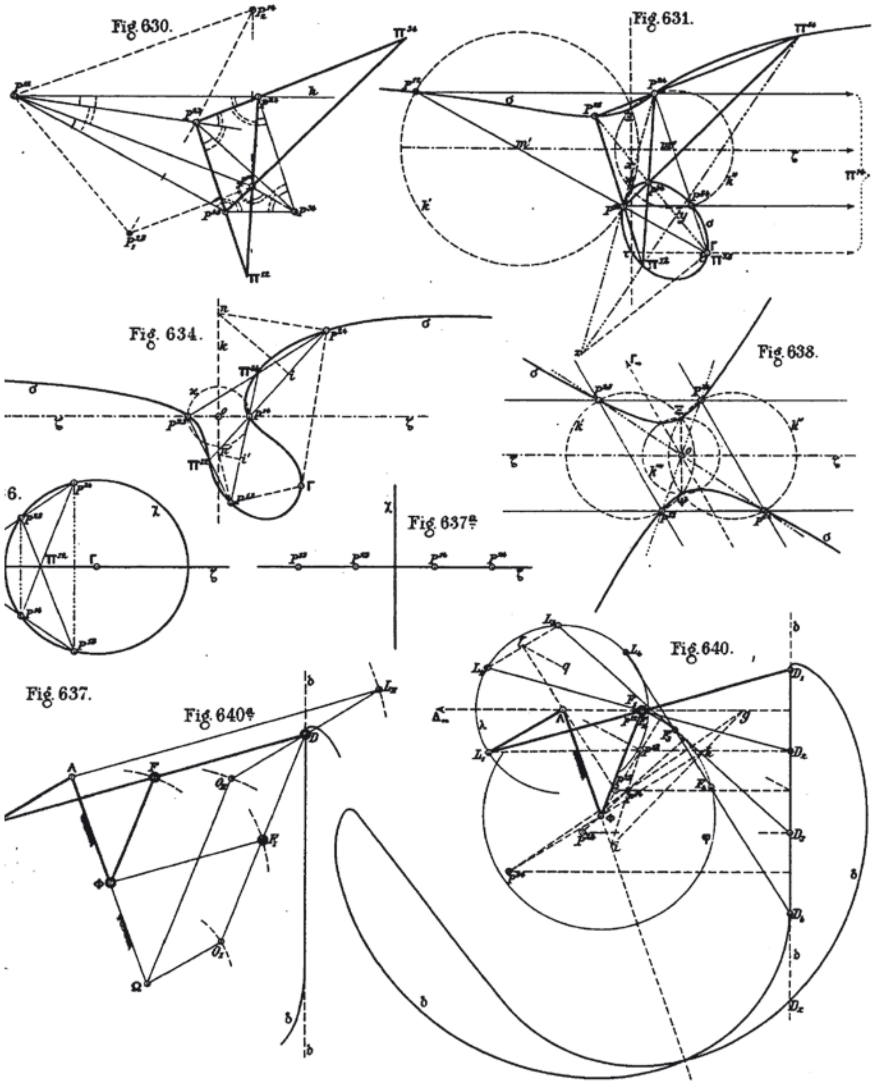


Fig. 6 Figures accompanying the treatment of the Burmester theory in the ninth chapter of the *Lehrbuch der Kinematik*. The curve σ in Fig. 634 is the centre point curve. In Fig. 638, σ consists of the line at infinity and a hyperbola

are the basic notions. There are, however, some terminological differences between Burmester and Reuleaux. Instead of Reuleaux' word 'kinematic chain', for example, Burmester uses the word 'mechanism', something Reuleaux would later interpret as an insult that revealed that Burmester had not understood the significance of Reuleaux' kinematics at all. For more details see the section on the reception of Burmester's work below.

The sixth chapter is very long, 122 pages. It deals with *simple planar mechanisms*. When the links (or elements including the one that is fixed, the frame) of a mechanism form a closed sequence and all elements execute a one-degree-of-freedom constrained motion with respect to each other, the mechanism is by definition a simple mechanism. First Burmester discusses the different planar four-bar linkages, Robert's theorem and the graphical velocity analysis of a planar four-bar linkage. Then he discusses special cases like the slider–crank mechanism and the Scotch-yoke mechanism with the graphical velocity analysis. He treats simple mechanisms with cams, with non-circular gear wheels and simple belt-driven mechanisms. There is also a section on locking and switching devices.

Burmester then proceeds in the seventh long chapter of 143 pages to compound planar mechanisms. In such mechanisms there will be at least one element that is connected through closure to more than two others. At the beginning of the chapter, Burmester discusses Grübler's results concerning constraints and results concerning the configuration of poles at a particular instant in the case of several coinciding moving planes. Then he turns to the discussion of all the compound mechanisms that he was familiar with. Wherever possible, Burmester applied the pertinent theoretical results to the many special mechanisms that he discusses. Watt's mechanism, Stephenson's mechanism and many others are treated. Figure 5 gives an idea of the variety of mechanisms that Burmester studied.

In the eighth chapter, Burmester deals with what he calls "guided mechanisms", like the pantograph. A one-degree-of-freedom kinematic chain is attached to the fixed plane by means of only one hinge. Then if we guide a point of one of the other links along a curve, we have a guided mechanism. In the same chapter, he discusses overconstrained mechanisms.

In the ninth chapter he deals with straight-line mechanisms. Also, here he develops a very general theory before it is applied. This time it is quite new: it is the theory nowadays called Burmester's Theory. Burmester's *Lehrbuch der Kinematik* is the culmination of a long development and is frequently based on papers that Burmester had written earlier. The treatment of the Burmester theory is based on the content of two papers from 1876 and 1877. Below, we will look at these papers in some detail.

In the twentieth chapter, of the *Lehrbuch der Kinematik*, he discusses slider controls for locomotives like Stephenson's control or Heusinger von Waldegg's control. In the eleventh chapter of 122 pages, Burmester gives an extensive theory of acceleration with applications to many simple and compound mechanisms. Also, here Burmester starts from the general theory and only then considers its applications. Finally in the twelfth chapter, Burmester deals with equiform and affine kinematics.

The Burmester Theory and the Burmester Points

The 1876 paper, which clearly represents Burmester's growing interest in applications of kinematics in mechanical engineering, consists of three parts. The paper is on the design of four-bar straight-line mechanisms taking into consideration the discrete positions of a moving plane. In 1876, Burmester first considered three discrete positions S_1, S_2 and S_3 of a moving plane and determined the points in the moving plane that are in those positions on a straight line. Right from the start, Burmester attacked the problem by means of projective geometry. By considering projective pencils of points and lines, he proved that the set of all points in S_2 that are in the three positions on a straight line, is a conic section. Because the circle points I and J are on it, it is a circle C_2 in S_2. This result was already known; it had been published without proof by Grouard in 1870. In his 1876 paper, Burmester went on to four positions, although he restricted himself to a proof of the theorem which says that, in general, precisely one point is in the four positions on a straight line.

In the 1877 paper, Burmester attacked the more general problem of the loci of points that are in a number of discrete positions on a circle. First he considered three positions and proved that the locus of the centers of the circles that are determined by triples of homologous points on three homologous lines is a conic section through the three poles. This theorem immediately yields: There exist either one or three circles that go through four homologous points on four homologous lines. The circles correspond to real points of the intersection of conic sections. By applying this theorem to the homologous lines of a pencil, the locus of centers of quadruples of four homologous points turns out to consist of the points of intersection of corresponding elements of two pencils of conic sections. Burmester analytically proved (the methods in the rest of the paper are synthetic) that this *Mittelpunktskurve* (centre point curve) is a circular curve of third degree. He also shows that it is a so-called focal curve, which is the locus of the foci of all conic sections that touch four given straight lines. Burmester then showed that the locus of points that are in four positions on a circle is also a focal curve, which he calls the *Angelpunktkurve* (pivotal point curve). In his book, he called this curve the "circle point curve". From the text of the 1877 paper, it is clear that Burmester's ideas were still being developed. For example, Burmester identifies the fixed plane with position 1, which means that he did not seem to realize that by considering the situation from the point of view of the moving plane, we are also dealing with four positions of the fixed plane with respect to the moving one and it is because of that the *Angelpunktkurve* must be of the same nature as the "centre point curve".

Finally, Burmester considered five positions and he found the points that are all five positions on a circle by intersecting two focal curves. Because from the nine points of intersection the circle points and three poles must be subtracted, he easily proved the existence of the four points that were later called the *Burmester points* by R. Müller.

One of the admirable characteristics of Burmester's work is the fact that he combined a great interest in theoretical results with an interest in applications. His 1877 paper on discrete position theory is no exception. The Burmester theory is immediately applied to Stephenson's link mechanism for controlling the steam valve of a locomotive.

Kempe's Focal Mechanism

In 1878, Alfred Bray Kempe published a paper which contains a remarkable over-constrained linkage (see Fig. 7). ABCD is a four-bar mechanism. If we choose arbitrary points S, R, V and U on the bars and connect them with bars of a fixed length to a joint P, we create, in general, a rigid structure. Kempe showed that P can be chosen in such a way that quadrilateral ASPV is similar to quadrilateral PRCU and quadrilateral BSPU is similar to quadrilateral PRDV. He, moreover, showed that for such points P, the mechanism has one degree of freedom. Obviously P moves in a circle with respect to AB. Kempe also showed that if the centers of this circle moves to infinity (on the extension of AB) and we remove the bars PR and PS (one has to remove them in fact, because their lengths become infinite), the mechanism becomes the straight-line linkage that Hart had found a year earlier.

It is easy to show that the similarity of the quadrilaterals implies that for the point P inside the quadrilateral we have:

$$\sphericalangle APB = \pi - \sphericalangle CPD.$$

In other words, P is a point from which opposite sides of the quadrilateral AB and CD are seen under angles that are each other's complement. Moreover, the converse holds as well: for all such points we can determine points S, U, R and V such that we get an over-constrained linkage consisting of a four-bar linkage divided into two pairs of similar quadrilaterals.

In 1893, Burmester wrote a long and very thorough paper about the mechanism. He showed that for a given quadrilateral, the locus of all points P that yield an over-constrained mechanism of this type is a circular curve of the third degree. Moreover, such points P are always the focus of a conic section that touches the sides of the quadrilateral. In other words, the third-degree curve is the set of all foci of the conic sections to which the four sides of the quadrilateral or their extension are tangent. Burmester introduced the term *focal mechanism* for this mechanism because, during the motion of the mechanism, the point P is always the focus of a

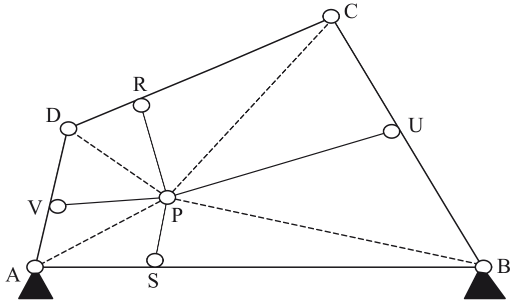


Fig. 7 Kempe's focal mechanism

conic section (changing during the motion) that touches the sides (or their extension) of the four-bar linkage.

Although beyond any doubt, Kempe discovered the mechanism, Wunderlich wrote a paper in 1968 in which he called it "Burmester's focal mechanism" without mentioning Kempe. Dijksman noticed Wunderlich's oversight and called the mechanism *Kempe's focal mechanism*. I prefer that name. The mechanism is also called the Kempe-Burmester focal mechanism (cf. Mallik et al. 1994, p. 121)

On the Reception of His Work

Background

Burmester's book is part of the development of scientific mechanical engineering in the nineteenth century. Before the nineteenth century machines had been studied traditionally in mechanics. When in the eighteenth century, rational mechanics was created, the investigation of machines had become an application of rational mechanics. Because of the difficulties involved, the results were limited. In the nineteenth century, it became clear that the geometrical aspect of machines and mechanisms could be investigated very successfully. Yet the investigation of machines remained applied mathematics or applied mechanics. There was no independent science of machines with a status comparable to the status of the established disciplines. Moreover, in the nineteenth century, the theory of machines was taught on the European continent at polytechnical schools, which in general, had a lower status than universities.

Scientific mechanical engineering was not born easily. For example, in 1877, J. Lüders wrote a booklet of 88 pages with the title *Wider Herrn Reuleaux* (Against Mr. Reuleaux) in which he described Reuleaux' *Theoretische Kinematik* as a book without any value, not written for the specialist but meant to impress the public at large. Others criticized Reuleaux as well because they failed to see the value of his work for mechanical engineering in practice.

In the second half of the nineteenth century, one can distinguish three different views of technology in Germany (Hensel et al. 1989, p. 167). The first view, represented by Franz Grashof (1826–1893), for a long time chairman of the Association of German Engineers (Verein Deutscher Ingenieure), defines technology in the tradition of French positivism: technology is applied natural science and applied mathematics. Burmester must have shared this view. In the second view, represented by Reuleaux, the machine is in the development of mankind, the essential element that determines man's relation with nature. Reuleaux can indeed be seen as one of the first philosophers of technology, who attempted to characterize the general development of mankind on the basis of the fundamental notion of machine. As a result of this position, he emphasized the need for an independent, unified theory of the machine. This theory would reserve a precise place for its application. The third view, represented by Alois Riedler (1850–1936), is opposed to the dominance of theory in the first two views. For Riedler, technology is more a socio-economic system of which theory is only one of the components. "Knowledge is a daughter of application, not the other way around", Riedler wrote (Hensel et al. 1989, p. 180). Words like 'reality', 'organisation' and 'labour' frequently occurred in his writings. He also opposed the specialization that resulted in curricula at technological universities that consisted of completely separate courses.

What Grashof, Burmester and also Reuleaux, however, had in common was the belief in the value of theoretical considerations and in deduction from the general to the particular. The difference was that Reuleaux defended a general theory of machines, independent of natural science and mathematics. Riedler was sceptical of all general theory. Another representative in Germany of the latter view was Th. Beck (1839–1917), factory owner and engineer from Darmstadt. Reuleaux had written (1875, p. 586): "What has 'advantage' to do with science?" Beck argued that the main task of mechanical engineering is to understand how to build a machine in such a way that it yields the greatest advantage (Hensel et al. 1989, p. 199).

When Reuleaux and Burmester wrote their books, the gap between the theory of machines and the practice of building, using and maintaining machines was still considerable. This created tensions and Reuleaux felt under attack from Riedler for not being practical enough, while, on the other hand, he felt that his work was not taken seriously enough by people like Grashof and Burmester. In this particular position, Reuleaux was unable to appreciate the value of Burmester's book.

Reuleaux' Severe Criticism of Burmester

In general, Burmester's book was well received. For example, in 1886, Schumann wrote a very positive review of the first two installments in the *Jahrbuch für die Fortschritte der Mathematik* and in 1887 he repeated his positive opinion with respect to the third installment. There was, however, one remarkable exception. In 1889 Reuleaux wrote:

> His [Burmester's – T. K.] book represents a phenomenon that probably has not yet occurred in our literature. As the main result of my investigation I must point out that *the book does not contain one new thought, not even a small one, in the area covered by the title of the book*. Yes, in order to honor the truth I must say that none of the laws of kinematics is treated completely correctly. Mr. Burmester turns out to be an amateur in kinematics (1894, p. LIII, italics are mine- T. K.).

This quotation is preceded by almost 20 (sic!) pages of severe criticism. Some of the criticism is undoubtedly justified. Wherever Burmester saw a possibility to apply his general theoretical considerations to a particular mechanism he did so, even if such an application seemed miles away from the practice of machine building. In this respect, Burmester's book was written clearly by a mathematician and not by an engineer. However, Reuleaux mainly criticized Burmester for using the word kinematics in an improper way. The above quotation in fact says: "the book does not contain any new contribution in the area of knowledge that I, Reuleaux, call kinematics; it may contain many new thoughts concerning other areas of knowledge".

Burmester defined kinematics in 1886 when he announced his book as follows:

> Kinematics, which encompasses the geometrical theory of motion and its application to machines, was born from the connection of geometry with the notion of motion. (Quoted by Reuleaux 1894, p. XXXII)

Reuleaux had defined kinematics as *Zwanglauflehre*, which can be translated as "the theory of constrained motion". Kinematics was, from Reuleaux' point of view, identical with the theory of mechanisms. Reauleux wrote that kinematics is

> The science that deals with the question how a machine should be composed such that the movements, i.e. changes of position, of the parts with respect to each other are completely determined (Reuleaux 1894, p. XVI)

The basic notions in kinematics are "kinematical pair", "kinematical chain" and "mechanism". In a kinematical pair, two elements are connected in such a way that the motion of two elements with respect to each other is constrained but keeps at least one degree of freedom. In a closed kinematical chain kinematical pairs are connected in such a way that one degree of freedom is left in the chain. By fixing one of the elements of a closed kinematical chain we obtain a mechanism. The mechanism becomes a machine when we use it to force the forces of nature do work by means of precisely defined movements.

Reuleaux distinguished between *theoretical and applied kinematics*. Applied kinematics dealt with the design of machinery in practice on the basis of theoretical kinematics.

It is clear that Reuleaux had defined kinematics as the core discipline in the new emerging science of machines that he envisaged. In Reuleaux' view of kinematics, the geometry of motion was most useful but it played only a supporting role in kinematics. Kinematics encompassed in this view much more than geometrical considerations.

Reuleaux' goal in life was the creation of a science of machines with a status comparable to the other sciences. That is why he gave his *Theoretische Kinematik* a deductive structure; he had tried to make the science of machines with kinematics at its heart as rigid as Euclid's *Elements*. In Reuleaux' perspective, Burmester did not simply use the word kinematics in a different way. No, Reuleaux saw Burmester's book as an attempt to reduce the science of machines to merely applied mathematics. In 1890 he wrote about Burmester and others:

> At the same time geometry imagined, that is its self-appointed imperialist protagonists imagined, to easily unharness the numerous recently derived kinematical theorems with the right of the conqueror and drive them like patient lambs into their sheepcote. (Reuleaux 1890, p. 248)

Reuleaux felt he had to defend the science of machines against enemies like Burmester. It seems that Reuleaux sent his criticism to the Rector and the Senate of the Technische Hochschule in Munich and later he even sent a copy of his 20-page criticism of Burmester's book to the Royal Bavarian Ministery of Education in Munich (Königliche Bayrische Kultusministerium) so that the ministry would see how incompetent a professor they had on their payroll. Burmester, well-positioned in Munich, Fig. 8, reacted to this with a public defense, which obviously did not satisfy Reuleaux. Actually, Burmester was not the real enemy. When he criticized Burmester, Reuleaux was professor in Berlin at the Technische Hochschule and he was at the top of his power. In 1890–1891, he was even rector of the Hochschule. In 1888, however, Alois Riedler had become professor at the Hochschule. Riedler had completely different views for mechanical engineering. He defended a curriculum with much less theory and more practical exercises. Riedler was a formidable opponent who clashed vehemently with Reuleaux. In the end, Riedler won. He succeeded in drastically reducing the number of classes devoted to mathematics and other theoretical subjects. In 1896 Reuleaux retired and soon kinematics was no longer an obligatory subject in Berlin.

H.–J. Braun characterized the situation in mechanical engineering in late nineteenth century Germany as a conflict about the correct method. That is what it was. Yet, one gets the impression that Reuleaux' self-image included infallibility which, in combination with his overambitious theoretical program and his willingness to fight his opponents any time anywhere at great lengths, turned the opposition between points of view into real battles. In particular, in Berlin the clash was very serious. Elsewhere things developed differently, as we will see below.

The Development in Dresden

It is interesting to compare the dramatic developments in Berlin with the situation in Dresden in the same period (Mauersberger 2001). Kinematics classes started in Dresden in 1870. They were given by Ernst Hartig (1836–1900) who was a specialist and pioneer with respect to measurement and test engineering. As we have seen, Burmester became professor for descriptive geometry in Dresden in 1872. In 1874, Reuleaux' pupil Trajan Rittershaus became professor of kinematics and electrotechnical machines. In the same period, Otto Mohr (1835–1918), specialist in graphical methods in statics and strength of materials, was professor of technical mechanics for the civil engineers. Moreover, in 1873, Gustav Zeuner (1828–1907) had become director of the Polytechicum in Dresden. Instead of fighting each other, these men stimulated each other in different ways. Although, in the course of time, the subject played a lesser role in the curriculum, for example, because dynamical aspects of machines became more important, in Dresden kinematics continued to be an important discipline. In 1894, Mohr had succeeded Zeuner, who had returned to teaching, as professor of technical mechanics. In 1900, the successor of Otto Mohr was Martin Grübler (1851–1935), famous for his contributions to the theory of constraints. A younger representative of the kinematical tradition in Dresden is Hermann Alt (1889–1945) who worked on the Burmester theory, as we will see below.

The Long-Term Reception of Burmester's Work

As for the long term reception of Burmester's work and in particular the Burmester theory, I will restrict myself. In the twentieth century, his work and the elaborations by others became an established part of modern kinematics. I will restrict myself to remarks about Italy, Germany and the United States.

Lorenzo Alievi (1856–1941) wrote his treatise *Cinematica della Biella Piana* (Kinematics of the plane coupler) in Rome in 1892. He published the book in Naples in 1895. It deals with the higher-order properties of the curvature of the four-bar coupler curve. Allievi was familiar with Burmester's *Lehrbuch der Kinematik* and with Schoenflies' *Geometrie der Bewegung* of 1886. Schoenflies had pointed out that Burmester's discrete position theory implied analogous, elegant results in instantaneous kinematics, for example, points of stationary curvature are on a curve of the third degree, the instantaneous analogue of Burmester's circle point curves. Moreover, the analytical methods of the differential calculus can be applied in instantaneous kinematics. Allievi drew the conclusion that the higher-order properties of the curvature of the four-bar coupler curve ought to be accessible to an analytical investigation. He started at the second-order level with the Euler-Savary equation and by differentiating, he reached higher-order results. This was a good idea. Half a century later, A. E. Richard de Jonge rightly praised Allievi's book (De Jonge 1943, p. 667).

In 1930, the German Rudolf Beyer published his *Technische Kinematik* (Technical Kinematics). Beyer starts his preface, next to a picture of Reuleaux, with the statement:

> In the spirit of Franz Reuleaux,
>
> with Burmester in mind,
>
> in the footsteps of Ferdinand Wittenbauer.

The first 147 pages are devoted to Reuleaux' theory of constraints in the spirit of the great man. The next 280 pages are written with Burmester in mind and the last 63 pages are devoted to dynamics in the footsteps of Wittenbauer. In the 1930s, Rudolph Beyer was the spokesman of the German kinematicians (Kerle 2007, p. 184) and the situation is perfectly clear. The quarrels of the second half of the nineteenth century were forgotten and Reuleaux and Burmester were both celebrated as great kinematicians. Reuleaux' lasting contribution turned out to have been his abstract approach of the machine: look at mechanisms and the problem of their classification in terms of kinematical pairs and kinematical chains. The bulk of the theory, however, at the time turned out to be, with Burmester in mind, the development of the general kinematical properties of motion in combination with their application to specific mechanisms.

The first half of the twentieth century is exactly the period in which kinematics was really applied in practice. In the 1990s, Riedler had been right: much of the theory that was taught at the Technische Hochschulen was not applied in practice. This had changed drastically in the 1930s. The nineteenth century graphical methods were really applied both in the analysis and the synthesis of mechanisms. Moreover, there were new developments. Worth mentioning is the fact that Herman Alt (1889–1954), professor in Dresden, further developed the Burmester theory (Mauersberger 2006).

In 1942 and 1943, A. E. Richard de Jonge wrote two papers in which he introduced the kinematics that had been developed in Europe to the American study of Mechanical Engineers in the United States. Writing about the US, De Jonge called “kinematic synthesis” the least known but most interesting branch of kinematics. The Burmester theory gets much attention in De Jonge’s description. The author’s intention was to wake up the Americans by pointing out that in Europe, in particular in Germany, but elsewhere as well, important work in kinematics had been done. In America hardly any published work existed. In the author’s closure, he expressed his disappointment at the lack of interest for his papers.

However, the man who most successfully introduced modern kinematics in America was Ferdinand Freudenstein (1926–2006). When de Jonge published his papers on kinematics, he was adjunct professor at the Polytechnic Institute of Brooklyn. In 1954, he worked for the Reeves Instrument Corporation in New York. Freudenstein was enthusiastic about De Jonge’s papers. He wrote:

> The greatest benefit of the use of the author’s method is obtained when computing machines can be utilized. (Roth 2007, p. 242)

Through Freudenstein and his pupils, modern kinematics, including many of Burmester’s ideas, conquered America. Roth (2007) contains Freudenstein’s academic family tree at the time of his death. It consisted of 500 members. Worth mentioning in this context is a 1959 paper written by Freudenstein and Sandor in which he started an important series of papers on the vector-loop methods in synthesis and the modern development of the classical Burmester theory (Roth 2007, p. 175).

In 1954, De Jonge saw what was coming. In the course of the years the graphical methods were replaced by analytical methods and the solutions were calculated by means of computers.

Spherical and spatial analogues of the planar Burmester theory have been developed. In a good modern textbook like McCarthy’s 2000 the theorem

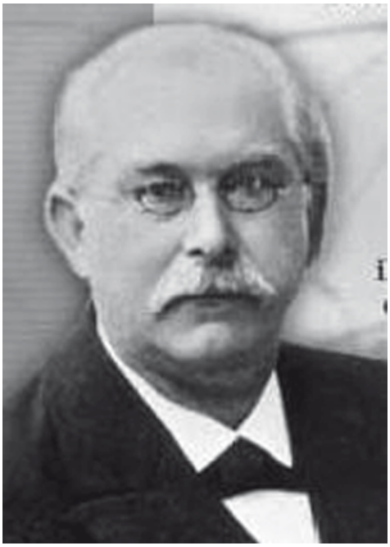


Fig. 8 Ludwig Burmester when he was professor in Munich

that makes it possible to easily find the center points of RR-chains that correspond to four given positions of a body in the plane, is called *Burmester's theorem* (see p. 60). In 1967, Bernard Roth derived the spherical analogue of this theorem. McCarthy calls it the *Burmester-Roth theorem* (see p. 176). As for space, given five positions of a moving body, there are six fixed axes that can serve as central axes of CC-chains that can guide the body through given positions. McCarthy calls these lines the Burmester lines (p. 244).

The Burmester Curves

In German, *French curves* are called *Burmester curves*. A French curve is a template made out of wood or plastic composed of different curves. It is used in manual drafting to get smooth curves. It is not known exactly how the German Burmester sets that consist of three templates acquired this name. Yet it seems very probable that it was indeed Ludwig Burmester whose name was given to the templates because, in his *Grundzüge der Reliefperspective* (The Main Features of Relief Perspective) of 1883, Burmester remarked in a footnote:

> For exact mathematical drawings the suitably formed curved ruler is a useful aid like the circle and can be much recommended; because in a drawing made with ruler and compass the curves drawn by hand form an ugly contrast which greatly influences the beauty and the homogeneity of the drawing. […] Experience teaches that three suitably formed curved rulers are sufficient for drawing curves in almost all cases. (1883, p. 17)

Burmester made many drawings. The experience he mentioned must have been his own experience. The T.U. Dresden possesses a considerable collection of "Burmester curves" (See Fig. 9).

Fig. 9 Burmester curves (Courtesy of Dr. Klaus Mauersberger)

Fig. 10 A commercial set of Burmester templates (Courtesy of Faber-Castell)

It is possible that someone in Dresden or Munich took the set that Burmester used and started to produce it and sell it under the name *Burmester curves* (Fig. 10).

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