Issues of Reggeization in qq' Backward Scattering

M.V. Bondarenco

Abstract The Kirschner–Lipatov result for the DLLA of high-energy qq' backward scattering is re-derived without the use of integral equations. It is shown that part of the inequalities between the variables in the logarithmically divergent integrals is inconsequential. The light-cone wave-function interpretation under the conditions of backward scattering is discussed. It is argued that for hadron–hadron scattering in the valence-quark model the reggeization should manifest itself at full strength starting from $s_{hh} = 50\,\text{GeV}^2$.

Keywords: Quark–quark scattering, double-log asymptotics, Regge behavior

1 Introduction

Backscattering of high and intermediate energy, weakly radiating[1] particles (protons, X-ray) is known as a clean tool for atomic material structure analysis [1]. The clarity of the analysis owes exactly to the low scattered particle fraction. With the initial macroscopic luminosity, that poses no problem for detectability, but, more importantly, the relative background (actually, all the diverse kinds thereof) is suppressed.

In case when wave nature of the scattered particles is relevant, the backward scattering can sometimes get enhanced as compared with that at other large angles – due to some or the other kinematical symmetry (coherent backscattering).

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[1] Weakness of the radiation is, rather, a wish than a necessary condition. Backscattering, or large-angle scattering of electrons, of course, is widely applied, too, but requires proper calculation of radiative corrections.

For hadrons, which are objects composed of quarks, one can detect the events of single quark backscattering by separating (single-) flavor exchange reactions to high energy. Thereat, most probably, only one pair of quarks (of different flavor) scatters backwards and then recombines with the forward-moving hadron remnants. There is no external gluonic radiation in the fully exclusive reaction, because of color confinement. Besides, there is no necessity to rise the energy to extremely high values, where some internal radiative effects should eventually become important.

Owing to the hardness present in the process, a plausible approximation for it is one-gluon exchange. The latter is impact parameter conserving, which is convenient for the overlap representation of the scattering matrix element in terms of quark wave functions of hadrons, as was partially discussed elsewhere [2].

But yet, at energies high enough, the energy dependence of flavor-exchange reactions departs notably from the one-gluon-exchange prediction $\sigma \sim s^{-2}$, which is referred to as reggeization phenomenon. It is desirable to get it incorporated in the theory, within the impulse approximation treatment.

In 1967, Gorshkov, Gribov, Lipatov, and Frolov [3] (see also textbook [4]) had evaluated double-leading-logarithmic asymptotics (DLLA) of Feynman integrals corresponding to $e^-\mu^-$ backward scattering in QED, and resumed to all orders. They had found a power falloff slowdown (basically, t-independent).

Later, Kirschner [5], being generally interested in DLLA of QCD elementary scattering processes, examined quark-quark backward scattering, and quark-antiquark forward/backward annihilation, paralleling the framework of [3]. The negative signature amplitude was thereafter computed by Kirschner and Lipatov [6].

The amplitude of qq' backward scattering, which is the kernel of the hadron binary reaction overlap matrix element, has the asymptotics

$$M_{qq'\to q'q}\left(s_{qq'}\right) = \frac{1}{N_c}\delta_{m'l}\delta_{l'm}\sqrt{\frac{2\pi\alpha_s}{C_F}}\frac{8\pi}{\ln s_{qq'}}I_1\left(\sqrt{\frac{2\alpha_s C_F}{\pi}}\ln s_{qq'}\right) \quad \text{(DLLA)} \ (1)$$

$$\sim \frac{1}{\ln s_{qq'}}s_{qq'}^{\sqrt{\frac{2\alpha_s C_F}{\pi}}} \qquad\qquad\qquad (s\to\infty) \quad (2)$$

with I_1 the modified Bessel function, and

$$C_F = \frac{N_c^2 - 1}{2N_c}. \tag{3}$$

(Account of single logarithms can somewhat change the index in (1), but the effect of that correction is rather uncertain in view of our poor knowledge of the coupling constant α_s, anyway.)

Letting numerically $N_c = 3$, $\alpha_s \simeq 0.1 \div 0.2$, and assuming that for reactions such as $np \to pn$ small Feynman-x contribution is moderate (given that constituent quark models work rather well for nucleon), one obtains an estimate

$$\frac{d\sigma_{np\to pn}}{dt} \propto \frac{1}{s^2}\left|M_{du\to ud}\right|^2 \sim s^{-1.4\div-1.2}. \tag{4}$$

Fig. 1 Regge trajectory for
$np \to pn$ reaction. Data taken
from [7]. The straight line
shown for comparison is the
conventional ρ, A_2-trajectory
$0.5 + 0.8t$
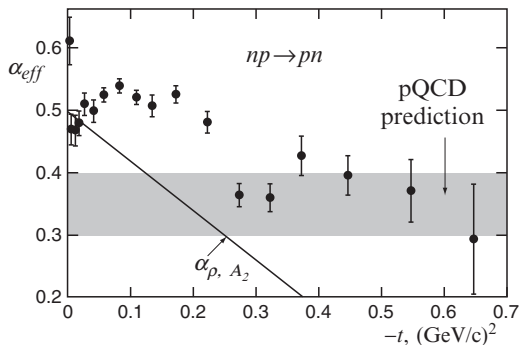
This asymptotics is expected to hold when $\sqrt{\frac{2\alpha_s C_F}{\pi}} \ln \left(s_{qq'} = \frac{s_{hh}}{N_1 N_2} \right) \gg 1$, where N_1, N_2 are valence quark numbers in the colliding hadrons. That numerically implies

$$s_{hh} \gg 50 \div 100 \text{ GeV}^2. \tag{5}$$

The correspondence of (4) with the experimental behavior is *not too bad*.

The best experimental representative of flavor exchange reactions is $np \to pn$, given the detailed data available for $d\sigma/dt$ and even some data for polarization for this reaction, and in addition – nucleon form-factors as an independent constraint for the wave function.

The Regge trajectory slope for $np \to pn$ is small (see Fig. 1). In contrast, for meson flavor exchange, particularly for $\pi^- p \to \pi^0 n$ (usually quoted as having an examplary linear Regge trajectory) the slope seems to be close to Chew–Frautchi substantial value 0.8 GeV^{-2}. But it is to be minded that in the pion charge exchange case there are cancelations between $ud \to du$ scattering and $u\bar{u} \to d\bar{d}$ annihilation, and in itself, pion is a more relativistic system then nucleon, probably, with a larger contribution from small x. Altogether, this makes the dynamics more intricate, and we refrain from discussing it here.

In this contribution we shall focus only on reggeization of two-free-quark scattering. That was the subject of Kirschner and Lipatov, but it is desirable to give it more dynamical interpretation, which can in future prove useful for scattering treatment in the spectator quark surroundings.

2 The origin of enhancements

Consider an ultra-relativistic collision of a free d-quark carrying momentum p_d with a free u-quark of momentum p_u, resulting in a near-backward elastic scattering to momenta p'_d, p'_u:

$$d(p_d) + u(p_u) \to u(p'_u) + d(p'_d),$$
$$\Delta_\perp = p_d - p'_d = p'_u - p_u \sim 1 \text{ GeV}.$$

As long as no other particles are concerned in the initial or final state, we shall throughout designate inter-quark kinematic invariants without hats or subscripts:

$$s = (p_d + p_u)^2 \gg \Delta_\perp^2.$$

Quark bispinors will be denoted as u for initial and u' for the final u-quark, and d, d' – the same for d-quark.

2.1 Loop structure. Collinear vs. infra-red large logarithms

The tree-level amplitude of quark–quark backward scattering is the single-gluon exchange:

$$M^{(1)} \approx -\frac{4\pi\alpha}{s} \left(\bar{d}'\gamma^\mu d\right) \left(\bar{u}'\gamma^\mu u\right) t_{l'l}^A t_{m'm}^A. \tag{6}$$

It scales with the collision energy $\sqrt{s}$ as s^0, which corresponds to cross-section decreasing as s^{-2}. But in higher orders there can arise loop enhancements of logarithmic kind, which are conventionally classified by two categories – soft and collinear ones. Soft divergences originate when some mass ratio tends to be large, $\frac{m}{\lambda} \to \infty$. Collinear ones require the high-energy limit $\frac{s}{m^2} \to \infty$; they correspond to an effective phase space extension with the energy.[2]

In general, a collinear divergence is encountered when a soft virtual particle connects two high-energy lines, provided the latter are sufficiently close to the mass-shell. Then, the high-energy line propagators admit eikonal approximation,[3] $\sim \frac{1}{pk}$ (p being the momentum of the high-energy line, and k – the momentum of the soft one), whereas the soft particle propagator decreases as $\sim \frac{1}{k^2}$ if it is a boson, or $\sim \frac{\not{k}}{k^2}$ if it is a fermion. When covered with 4d integration, by k-power counting it is seen to produce logarithmic divergences – in a triangle loop with two eikonal (fermion) and one soft boson lines (not counting possible hard propagators, which may be regarded as momentum-independent, and graphically represented as contracted into a point), and in quadrangular loops with two eikonal (boson) lines and two soft fermion lines.

In the first case, of triangular loops, the collinear divergence is merging with the soft one (IR). Although those can be given independent meaning, physically they both are related to emission and reabsorption of bremsstrahlung photons, with the energy smaller then the mass of the radiating particle (in the IR soft case), or then the

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[2] An often quoted definition of collinear divergence type is that it is inherent to the massless case, when the singularity of the integrand is encountered not in a single point (that would characterize soft divergence), but along an entire line. But from the viewpoint of initially massive, physical case, one yet needs to specify, in which order the massless limit is achieved. The answer is that ratios of all the masses stay finite and non-zero, while their ratios to the energy tend to zero – in contrast to the soft case when some mass ratio turns small. Again, that is equivalent to the growth of the effective phase space, in mass units. As for the method of identifying soft and collinear divergences by $\frac{d\omega}{\omega}$ and $\frac{d\theta}{\theta}$ factors, it is not manifestly Lorentz-invariant.

[3] The eikonal condition ($pk \gg k^2, p^2 - m^2$) is exactly the criterion of the line proximity to the mass shell.

collision energy (in the IR collinear case). So, it is natural that they obey the same cancelation principles. For backward scattering of equal-charge particles, or with perfect charge (color) exchange, IR cancelations must be working at full strength.

The second case, of quadrangular loop, instead, has no soft counterpart. Moreover, virtual corrections of that kind upon resummation should lead to enhancement rather then suppression of the cross-section, as we shall discuss in detail below.

In higher orders of perturbation theory, in order to obtain the leading logarithmic contribution, there must be an eikonal condition for each gluon line. Denoting by q_i – d-quark momenta on its course from p_d to p'_d (see Fig. 2 below),

$$-(q_{i-1} - q_i)^2 \approx 2q_{i-1}q_i \gg q_{i-1}^2, q_i^2. \tag{7}$$

Fulfilment of these conditions is possible if the intermediate quark (and gluon) momenta approximately belong to the plane formed by initial and final momenta. For backward scattering this plane approximately coincides with that of collision, and it may unequivocally be called longitudinal. It is profitable to define in it light-cone coordinates, and expand any vector $a^\mu = a^\mu_\parallel + a^\mu_\perp$, $a^\mu_\parallel = (a^0, a^3)$, $a^\mu_\perp = (a^1, a^2)$

$$a^\pm = \frac{a^0 \pm a^3}{\sqrt{2}},$$

$$a \cdot b = a^+ b^- + a^- b^+ + a_\perp \cdot b_\perp.$$

Then, Eq. (7) requires[4]

$$p_d^+ \gg q_1^+ \gg q_2^+ \ldots \gg q_{n-1}^+ \gg p'^+_d, \tag{8}$$

$$p_d^- \ll q_1^- \ll q_2^- \ldots \ll q_{n-1}^- \ll p'^-_d, \tag{9}$$

$$q_{i\perp}^2 \ll q_{i-1}^+ q_i^-, q_i^+ q_{i+1}^-. \tag{10}$$

In fact, Eq. (10) will be satisfied automatically if

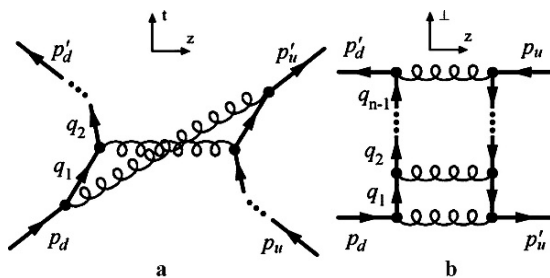$$q_{i\perp}^2 < 2q_i^+ q_i^- = q_{i\parallel}^2, \qquad q_i^2 > 0. \tag{11}$$

Fig. 2 The diagram giving leading logarithmic contribution in $2n$-th order: a – the temporal ordering representation; b – the spatial projection

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[4] Presuming that there are no fine cancelations between $\parallel$ and $\perp$ components in momentum squares, which would reduce the integration volume.

This is nothing but the usual multi-peripheral kinematics – the same as for the reggeization at forward scattering. That is quite natural, since the denominator structure for those cases is the same (for instance, in a scalar theory, with no propagator numerators, and all the particles identical, there would be no difference between forward and backward scattering).

2.2 The Feynman diagram topology

The ordering in rapidity guarantees uniqueness of the Feynman diagram, and the temporal order of boson emission from one fermion should be reverse to that of their absorption by the other fermion. Thereby, the concept of near-neighbor interaction in the phase space finds support.

The amplitude corresponding to the n-rung diagram (see Fig. 2) is

$$M^{(n)} = i(-4\pi i\alpha_s)^n C^{(n)} \int \frac{d^4q_1}{(2\pi)^4} \cdots \frac{d^4q_{n-1}}{(2\pi)^4} \frac{\mathcal{N}^{(n)}}{\mathcal{D}^{(n)}} \tag{12}$$

with

$$C^{(n)} = \left(t^{A_n} \ldots t^{A_1}\right)_{l'l} \left(t^{A_1} \ldots t^{A_n}\right)_{m'm}, \tag{13}$$

$$\mathcal{N}^{(n)} \approx \left[\bar{d}'\gamma^{\mu_n} \slashed{q}_{n-1} \ldots \gamma^{\mu_2} \slashed{q}_1 \gamma^{\mu_1} d\right] \left[\bar{u}'\gamma^{\mu_1} \slashed{q}_1 \gamma^{\mu_2} \ldots \slashed{q}_{n-1} \gamma^{\mu_n} u\right], \tag{14}$$

$$\mathcal{D}^{(n)} \approx (-2p_d q_1 + i0)(-2q_1 q_2 + i0)(-2q_{n-1}p'_d + i0)q_1^2(q_1 - \Delta)^2 \ldots q_n^2(q_n - \Delta)^2. \tag{15}$$

At this stage, certain insight can already be gained from the topology of Feynman diagrams. Drawing Feynman diagrams in accord with the process spatial projection, when the initial particle momentum directions are opposite, an arbitrary order diagram is depicted as a ladder, each cell of which is dissectible by two lines in the t-channel (for backward scattering, $|t| \ll s \approx |u|$). On the other hand, drawing Feynman diagrams according to the event temporal ordering, either the two fermion lines, or all the boson lines must cross. In any way, the diagram cannot be cut by two lines in the s-channel (which might be utilized for evaluation by unitarity). This is in contrast with the IR boson attachment order, where triangle loops (though not necessarily the entire diagram) can always be cut by two lines in the s-channel, and the concept of correspondence with the emitted real bosons through unitarity is useful.

2.3 Classical interpretation

In classical terms, the mechanism of enhancement may be thought of, roughly, as follows. In a high-energy collision, charged particles can shed their proper fields with the impart to them of the bulk of their energy, and slow down. In the slow state, they are turned around on a larger mutual distance, which results in the increase of

the scattering differential cross-section. Upon the reflection, the charged particles can again pick up each the comoving proper field from the other particle, and thus restore the high relative energy up to the initial value.

3 Numerators

Let us, in the first place, analyze matrix numerators, determining all the speciality of quark–quark scattering.

3.1 Spin factors

As long as fermion masses are neglected, their helicity must be conserved. But, in addition, we shall acquire strict correlation of helicities of colliding particles.

In addition to the light-cone decomposition, it is convenient to introduce chiral vector basis in the transverse plane:

$$a^R = -\frac{a^1 + ia^2}{\sqrt{2}}, \qquad a^L = \frac{a^1 - ia^2}{\sqrt{2}}, \qquad a_\perp \cdot b_\perp = a^R b^L + a^L b^R. \qquad (16)$$

Using in capacity of basic γ-matrices

$$\gamma^\pm = \frac{\gamma^0 \pm \gamma^3}{\sqrt{2}}, \qquad \gamma^R = -\frac{\gamma^1 + i\gamma^2}{\sqrt{2}}, \qquad \gamma^L = \frac{\gamma^1 - i\gamma^2}{\sqrt{2}} \qquad (17)$$

makes the covariant anticommutation relation $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ look like

$$\{\gamma^+, \gamma^-\} = 2, \qquad \{\gamma^R, \gamma^L\} = 2, \qquad (18)$$

with all other anticommutators zero:

$$\left(\gamma^+\right)^2 = \left(\gamma^-\right)^2 = \left(\gamma^R\right)^2 = \left(\gamma^L\right)^2 = 0, \qquad (19)$$

$$\{\gamma^\pm, \gamma^{R,L}\} = 0. \qquad (20)$$

Important for the future practice are cubic relations

$$\gamma^R \gamma^L \gamma^R = 2\gamma^R, \qquad \gamma^L \gamma^R \gamma^L = 2\gamma^L, \qquad (21)$$

and Dirac conjugation properties

$$\bar{\gamma}^\pm = \gamma^\pm, \qquad \bar{\gamma}^R = -\gamma^L, \qquad \bar{\gamma}^L = -\gamma^R. \qquad (22)$$

For in- and out-quark bispinors, which satisfy (massless) Dirac equations

$$\gamma^- d = 0, \qquad \bar{u}' \gamma^- = 0, \qquad \bar{d}' \gamma^+ = 0, \qquad \gamma^+ u = 0, \tag{23}$$

further, define polarization states as those of definite helicity (left and right):

$$\gamma^L d_R = \sqrt{2} d_L, \qquad \gamma^R d_L = \sqrt{2} d_R, \tag{24}$$

$$\bar{u}'_R \gamma^R = -\overline{\gamma^L u'_R} = -\sqrt{2} \bar{u}'_L, \qquad \bar{u}'_L \gamma^L = -\sqrt{2} \bar{u}'_R, \tag{25}$$

$$\gamma^R u_R = -\sqrt{2} u_L, \qquad \gamma^L u_L = -\sqrt{2} u_R, \tag{26}$$

$$\bar{u}'_R \gamma^L = \sqrt{2} \bar{d}'_L, \qquad \bar{d}'_L \gamma^R = \sqrt{2} \bar{d}'_R, \tag{27}$$

and the normalization should be

$$\bar{d}'_L d_R = \bar{d}'_R d_L = \bar{u}'_L u_R = \bar{u}'_R u_L = \sqrt{s}. \tag{28}$$

The important consequence of Eqs. (25–28) and (19) is

$$\gamma^R d_R = 0, \qquad \gamma^L d_L = 0. \tag{29}$$

$$\bar{u}'_R \gamma^L = 0, \qquad \bar{u}' \gamma^R = 0. \tag{30}$$

(The factor $\sqrt{2}$ in (25–28) comes from the relation $\{\gamma^R, \gamma^L\} = 2$, and the sign at it is the matter of bispinor normalization convention.)

We shall nowhere need the use of matrix γ_5, for which the chirality bispinors are eigenvectors. Thanks that all the momenta are contained in one hyper-plane, one can manage with matrices γ^R, γ^L alone, playing the role of (nilpotent) angular momentum raising and lowering operators.

Now, the smallest block in the matrix element

$$\gamma^\mu d_R \bar{u}'_R \gamma^\mu = -2 d_L \bar{u}'_L, \tag{31}$$

$$\gamma^\mu d_R \bar{u}'_L \gamma^\mu = 0. \tag{32}$$

Eq. (31) implies that fermion angular momentum projection onto the collision axis must flip after the vector boson exchange, and the spins of the opposing fermions must exactly correlate. Physically, that is natural, since a vector boson emitted by a $M_z = +\frac{1}{2}$ fermion has $M_z = +1$, so after the vector boson emission the fermion acquires $M_z = -\frac{1}{2}$, and the opposite fermion must initially have $M_z = -\frac{1}{2}$ to be able to absorb the $M_z = +1$ boson.

Hence,

$$\mathcal{N}^{(1)}_{RR,RR} = \mathcal{N}^{(1)}_{LL,LL} = -2s, \tag{33}$$

whereas all the other helicity amplitudes equal zero.

The next larger block

$$\gamma^\nu \, \slashed{q}_1 \gamma^\mu d_R \bar{u}'_R \gamma^\mu \, \slashed{q}_1 \gamma^\nu = -2 \gamma^\nu \, \slashed{q}_1 d_L \bar{u}'_L \, \slashed{q}_1 \gamma^\nu = 4 \, \slashed{q}_1 d_R \bar{u}'_R \, \slashed{q}_1. \tag{34}$$

The non-zero part of r. h. s. of (34)

$$\slashed{q}_1 d_R \bar{u}'_R \, \slashed{q}_1 = \left(q_1^- \gamma^+ + q_1^R \gamma^L \right) d_R \bar{u}'_R \left(q_1^- \gamma^+ + q_1^L \gamma^R \right). \tag{35}$$

Now, matrix-vectors $\slashed{q}_i$ sandwiching this expression have components q_i^+ at γ^-, which are negligible as compared to then $q_{i\perp}$. Then, it is possible to (anti-)commute the matrices γ^+ in (36) outwards to a position next to on-mass-shell bispinors $\bar{d}'$ and u, action on which, by virtue of (23), gives zero. So, block (34) equals

$$\slashed{q}_1 d_R \bar{u}'_R \, \slashed{q}_1 = q_1^R q_1^L \gamma^L d_R \bar{u}'_R \gamma^R = \mathbf{q}_\perp^2 d_L \bar{u}'_L, \tag{36}$$

which is proportional to (31).

Ultimately, it is understood that in the arbitrary order

$$\mathcal{N}^{(n)}_{RR,RR} = \mathcal{N}^{(n)}_{LL,LL} = (-2)^n s\, \mathbf{q}_{1\perp}^2 \cdots \mathbf{q}_{n-1\perp}^2. \tag{37}$$

Note that the $2\mathbf{q}_\perp^2$ factors emerge here without the appeal to the azimuthal averaging, or reasoning that $q_\parallel$ components cancel the logarithmic singularities in the integral (cf. [3]). As is known, vector interaction at hard momentum transfers (compared to the mass) is predominantly magnetic – similarly to the conventional separation of electric and magnetic form-factors:

$$J^\mu_{fi} = \bar{u}_f \left[F_e\left(Q^2\right) \gamma^\mu_\parallel + F_m\left(Q^2\right) \gamma^\mu_\perp \right] u_i. \tag{38}$$

Since in our case polarizations of all the virtual particles are completely fixed by that of initial ones, the problem is equivalent to some scalar field theory. The vector character of the bosons does not entail any momentum-dependent numerators, and merely secures helicity conservation.

3.2 Color matrix factor

As had been discussed in [3], [5, 6], in the perfect charge (color) exchange situation the infra-red vector boson exchange contributions mutually cancel. Here, let us neglect them altogether, and consider only the hard ladder.

Embarking on the Fierz-type identity for color generators

$$t^A_{l'l} t^A_{m'm} = \frac{1}{2} \delta_{l'm} \delta_{m'l} - \frac{1}{2N_c} \delta_{l'l} \delta_{m'm}, \tag{39}$$

by induction one proves[5]

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[5] Here, C_F and $-\frac{1}{2N_c}$ are just the values of Kirschner's matrix τ_2 (defined in a basis of convenience for him [5]), and $\delta_{m'l} \delta_{l'm}$ together with $t^A_{m'l} t^A_{l'm} - \frac{1}{2N_c} \delta_{m'l} \delta_{l'm}$ are its eigenvectors.

$$C^{(n)} = \left(t^{A_n} \dots t^{A_1}\right)_{l'l} \left(t^{A_1} \dots t^{A_n}\right)_{m'm}$$
$$= C_F^n \frac{1}{N_c} \delta_{m'l} \delta_{l'm} + 2 \left(-\frac{1}{2N_c}\right)^n \left[t_{m'l}^A t_{l'm}^A - \frac{1}{2N_c} \delta_{m'l} \delta_{l'm}\right], \qquad (40)$$

with C_F given by Eq. (3). Obviously, $C_F > -\frac{1}{2N_c}$, both by sign, and in magnitude. At $n \geq 2$, i.e., in any loop, it suffices to keep only the first term in the r.h.s. of (40). As the Kronecker symbols indicate, the leading term requires exchange of color.[6]

The underlying reason for the law that the color exchange is assured at the given ordering of gluon emission and re-absorption, and when $N_c \to \infty$, is also transparent. For each quark, the first ladder gluon emitted by it carries away its color, and in addition has arbitrary (except at the tree level) anticolor. The final quark moving in the same direction will absorb this gluon last of all, and must annihilate its anticolor whatever it is (by color conservation), and accept its color. Thereby, the color of the final quark will coincide with that of the comoving initial one.

Summarizing this section, re-absorption of gauge bosons in the inverse order stipulates transfer of all the quantum numbers between the scattered quarks. The large-N_c limit here is sufficiently robust, and within it the picture is equivalent to that of QED, the coupling constant correspondence being $\alpha_{QED} \to \alpha_s C_F$.

4 Loop integrals in DLLA

Using the numerator kinematical factors, we are in a position to treat the loop integrals.

4.1 One-loop integral reduction. Wave-function interpretation

By far the simplest approach for of high-energy asymptotics derivation and understanding is infinite momentum frame quantum field theory. One might anticipate its applicability for the backward scattering, as well, inasmuch as the denominator structure in Feynman integrals is the same as for forward scattering. But, because of the occurrence of factors $\mathbf{q}_\perp^2$ in the numerator (see section 3.1), application of LCPT is obstructed by the divergence of the eikonal integral, over $d^2 q_\perp$. To keep the treatment consistent, one may, first, straightforwardly carry out the q^- integration in Feynman integrals. In one loop,

$$M^{(2)}/C^{(2)} \approx -2is(4\pi\alpha)^2 \int \frac{d^4 q}{(2\pi)^4} \frac{2q_\perp^2}{(2p_d' q - i0)(2p_d q - i0)q^2(q-\Delta)^2}$$
$$\approx (4\pi\alpha)^2 \int \frac{dq^-}{2\pi i(q^- - i0)} \frac{dq^+}{2\pi q^+} \frac{d^2 q_\perp}{(2\pi)^2} \frac{2q_\perp^2}{q_\perp^2 (\Delta_\perp - q_\perp)^2}.$$

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[6] The second term of (40), in fact, is not yet related to a self-consistent scattering amplitude since it is devoid of infra-red DL corrections.

Upon the integration (in the exact expression) over q^-, reducing, essentially, to taking residue in a single pole $q^- \approx 0$, we derive a restriction on q^+: $p_d^- \le q^+ \le p_d^+$. Then, one can pass to the eikonal approximation. At that, the condition

$$-(p_d' - q)^2 \approx 2p_d'q \approx 2p_d'^- q^+ \gg q^2, q_\perp^2$$

yields ordering of $q_\perp^2$ and q^+, which secures convergence of the integral over $q_\perp^2$ at large $q_\perp^2$. Within the (double-) logarithmic accuracy,

$$M^{(2)}/C^{(2)} \approx \frac{(4\pi\alpha)^2}{(2\pi)^2} \int_{p_d'^+}^{p_d^+} \frac{dq^+}{q^+} \int_1^{p_d'^- q^+} \frac{dq_\perp^2}{q_\perp^2} \tag{41}$$

$$= \frac{(4\pi\alpha)^2}{8\pi^2} \ln^2 s = \frac{\alpha}{4\pi} \ln^2 s \cdot M^{(1)}/C^{(1)}.$$

In the final representation (41) valuable is the separation of hard and soft physics, which does not in fact depend on our choice of prior integration over q^-, or q^+. The longitudinal hard gluons pertain to hard physics, whereas the braking fermions – to the soft. Soft physics is most conveniently interpreted in terms of wave functions and their overlaps. If one invokes the analogy with the non-relativistic (or old-fashioned) perturbation theory, Eq. (41) may be compared with the expression for the second-order transition matrix element

$$\langle 2| V |1\rangle = \sum_n \frac{\langle 2| V |n\rangle \langle n| V |1\rangle}{E_0 - E_n}. \tag{42}$$

The role of the perturbation operator V in our case is played by the coupling constant $4\pi\alpha$. The energy denominator finds an analog in the factor $\frac{1}{q^+}$, which, however, is positive, not negative, i.e., the intermediate states reside *under* the mass-shell. As for the intermediate state wave functions $|n\rangle$, their counterparts are the factors $\frac{\sqrt 2}{q^1 \pm iq^2}$. Finally, the phase space volume element is $\frac{dq^+ d^2 q_\perp}{(2\pi)^3}$. It should be noted that the phase space available for $q_\perp^2$ is restricted by the value of the "energy" q^+. That reflects the circumstance that soft and hard physics are not separated absolutely, but only within the logarithmic accuracy. A similar situation (not encountered at forward scattering) is often met at description of exclusive hadronic processes with a large momentum transfer (see, e.g., [9]).

In conclusion, let us remark that in [3] the extraction of DLLA contributions is conducted by prior integration over $d^2 q_\perp$, in analogy with the Sudakov's vertex asymptotic treatment [10]. That renders the framework more symmetric appearance, but the wave-function interpretation gets obscured.

4.2 All-order treatment

Integrals for higher orders of perturbation theory may also be calculated via first q^--integration, but it requires more detailed considerations (cf. [8]). Instead of the

variables q_i^-, $q_{i\perp}^2$, in the present case it is convenient to introduce

$$q_i^-, \quad \kappa_i = \frac{\mathbf{q}_{i\perp}^2}{2q_i^-}, \tag{43}$$

and only then carry out the q_i^- integration. Then, a strong ordering condition ensues

$$\kappa_i \gg \kappa_{i+1} \tag{44}$$

(corresponding to the multiperipheral condition $q_i^- \ll q_{i+1}^-$ for q_i^-, which have been integrated over), and

$$\kappa_i < q_i^+ \tag{45}$$

(corresponding to the eikonal condition $\mathbf{q}_{i\perp}^2 < 2q_i^+ q_i^-$). The integral of the $2n$-th order of perturbation theory, in DLLA assumes the form

$$M^{(n)}/C^{(n)} = \left(M^{(1)}/C^{(1)}\right) \left(\frac{\alpha}{2\pi}\right)^{n-1} \int_{p_d'^+}^{p_d^+} \frac{dq_1^+}{q_1^+} \int_{p_d'^+}^{q_1^+} \frac{d\kappa_1}{\kappa_1} \cdots$$
$$\cdots \int_{p_d'^+}^{q_{n-3}^+} \frac{dq_{n-2}^+}{q_{n-2}^+} \int_{p_d'^+}^{\min(q_{n-2}^+,\kappa_{n-3})} \frac{d\kappa_{n-2}}{\kappa_{n-2}} \int_{p_d'^+}^{q_{n-2}^+} \frac{dq_{n-1}^+}{q_{n-1}^+} \int_{p_d'^+}^{\min(q_{n-1}^+,\kappa_{n-2})} \frac{d\kappa_{n-1}}{\kappa_{n-1}}. \tag{46}$$

For evaluation of this integral, it is convenient to recast the i-th pair of integrations

$$\int_{p_d'^+}^{q_{i+1}^+} \frac{dq_i^+}{q_i^+} \int_{p_d'^+}^{\min(q_i^+,\kappa_{i+1})} \frac{d\kappa_i}{\kappa_i} \cdots = \int_{p_d'^+}^{q_{i+1}^+} \frac{dq_i^+}{q_i^+} \int_{p_d'^+}^{\kappa_{i+1}} \frac{d\kappa_i}{\kappa_i} \cdots - \int_{p_d'^+}^{\kappa_{i+1}} \frac{dq_i^+}{q_i^+} \int_{q_i^+}^{\kappa_{i+1}} \frac{d\kappa_i}{\kappa_i} \cdots$$
$$\equiv \int_{p_d'^+}^{q_{i+1}^+} \frac{dq_i^+}{q_i^+} \int_{p_d'^+}^{\kappa_{i+1}} \frac{d\kappa_i}{\kappa_i} \cdots - \int_{p_d'^+}^{\kappa_{i+1}} \frac{d\kappa_i}{\kappa_i} \int_{p_d'^+}^{\kappa_i} \frac{dq_i^+}{q_i^+} \cdots \tag{47}$$

(the integral over a trapezium represented as an integral over the rectangle minus the integral over the triangle). But, as is easy to demonstrate by changing the order of variables,

$$\int_{p_d'^+}^{\kappa_{i+1}} \frac{d\kappa_i}{\kappa_i} \int_{p_d'^+}^{\kappa_i} \frac{dq_i^+}{q_i^+} \left\{ \int_{p_d'^+}^{q_i^+} \frac{dq_{i-1}^+}{q_{i-1}^+} \int_{p_d'^+}^{\kappa_i} \frac{d\kappa_i}{\kappa_{i-1}} - \int_{p_d'^+}^{\kappa_i} \frac{d\kappa_{i-1}}{\kappa_{i-1}} \int_{p_d'^+}^{\kappa_{i-1}} \frac{dq_{i-1}^+}{q_{i-1}^+} \right\} \cdots \equiv 0, \tag{48}$$

so, we can drop the terms $-\int_{p_d'^+}^{\kappa_{i+1}} \frac{d\kappa_i}{\kappa_i} \int_{p_d'^+}^{\kappa_i} \frac{dq_i^+}{q_i^+}$ at all the $dq_i^+ d\kappa_i$ integrations but the $(n-1)$-th.[7] The $(n-1)$-th double integration gives

$$\int_{p_d'^+}^{q_{n-2}^+} \frac{dq_{n-1}^+}{q_{n-1}^+} \int_{p_d'^+}^{\kappa_{n-2}} \frac{d\kappa_{n-1}}{\kappa_{n-1}} - \int_{p_d'^+}^{\kappa_{n-2}} \frac{d\kappa_{n-1}}{\kappa_{n-1}} \int_{p_d'^+}^{\kappa_{n-1}} \frac{dq_{n-1}^+}{q_{n-1}^+} =$$

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[7] This means that inequalities (45) are inconsequential, except for the first and the last one.

$$= \ln \frac{q_{n-2}^+}{p_d'^+} \ln \frac{\kappa_{n-2}}{p_d'^+} - \frac{1}{2} \ln^2 \frac{\kappa_{n-2}}{p_d'^+}. \tag{49}$$

Passing to the self-suggestive variables

$$\eta_i = \ln \frac{q_i^+}{p_d'}, \quad \xi_i = \ln \frac{\kappa_i}{p_d'}, \tag{50}$$

the DLLA amplitude of $2n$-th order is calculated quite trivially:

$$M^{(n)}/C^{(n)} = \left(M^{(1)}/C^{(1)} \right) \left(\frac{\alpha}{2\pi} \right)^{n-1}$$
$$\times \int_0^{\ln s} d\eta_1 ... \int_0^{\eta_{n-3}} d\eta_{n-2} \int_0^{\ln s} d\xi_1 ... \int_0^{\xi_{n-3}} d\xi_{n-2} \left\{ \eta_{n-2}\xi_{n-2} - \frac{1}{2}\xi_{n-2}^2 \right\}$$
$$= \left(M^{(1)}/C^{(1)} \right) \left(\frac{\alpha}{2\pi} \ln^2 s \right)^{n-1} \left\{ \frac{1}{[(n-1)!]^2} - \frac{1}{(n-2)!n!} \right\}$$
$$= \left(M^{(1)}/C^{(1)} \right) \left(\frac{\alpha}{2\pi} \ln^2 s \right)^{n-1} \frac{1}{(n-1)!n!}. \tag{51}$$

Invoking the series expansion for the modified Bessel function of first order,

$$I_1(z) = \sum_{n=1}^{\infty} \frac{\left(\frac{z}{2} \right)^{2n-1}}{(n-1)!n!}, \tag{52}$$

we arrive at the result of [3], which we have thus re-derived by straightforward resummation, without the recourse to the formalism of integral equations.

Post factum, it is important to check the self-consistency of the adopted multi-peripheral approximation (8–10). When $z = \sqrt{\frac{2\alpha C_F}{\pi}} \ln s \gg 1$, the largest terms in sum (52) have numbers

$$\bar{n} \sim \frac{z}{2}. \tag{53}$$

Equally, and independently of the overall energy, one can say that each gluon typically shifts the quark rapidity by

$$\Delta y = \frac{Y = \ln s}{\bar{n}} \sim \sqrt{\frac{2\pi}{\alpha C_F}} \simeq 5 \div 7. \tag{54}$$

This implies that for the given problem the multi-peripheral approximation is very safe.

The transverse motion of quarks in the ladder rails is usually regarded as random walk. At that, the rung gluons propagate nearly forward (since, in the eikonal approximation, their propagators do not depend on transverse momenta), and so, impact parameters of the final u-quark must coincide with that of the initial d-quark, and impact parameter of the final d-quark – with that of the initial u-quark. Thereby,

the walk is not completely random. Yet, the walk step is small as compared to typical hadronic radius (recall that large $\mathbf{q}_\perp^2$ dominate), so the initial quark impact parameters must be close to one another, anyway.

5 Discussion and summary

The mechanistic picture of reggeization is observed to fall into certain contrast with the analyticity and duality expectations. In particular, transversal hardness excludes exact analogy with meson exchange in the t-channel, and yet suggests that the quark-exchange reggeization phenomenon, and the relevant intercept, may be universal, or there can be a few universal reggeons (much less numerous then the host of mesons). The similarity of the Regge ladder diagram with that of the Bethe-Salpeter equation must not be deluding, given the dominance in the present case of large $q_\perp$ (let alone the excessive hardness of the ladder u-channel gluons). In their own turn, mesons, being strongly bound relativistic states, for which the interaction radius is not small compared to the average inter-constituent distance must not necessarily obey a Bethe–Salpeter-like equation at all.

In what concerns the hadron wave function overlap representation, the hardness of the Regge ladder implies that one can rather safely exploit the notion of coincidence of colliding quark impact parameters – unless the energy becomes super-high, giving the short-step transverse random walk eventually a spread comparable to the hadron size. Another feature important at hadron wave function overlap computations is that the reggeized kernel (1) is not scale-invariant, and does not factorize in terms of Feynman-x of the active quarks:

$$M_{qq' \to q'q}\left(s_{qq'} = s_{hh} x_q x_{q'}\right) \neq f_1\left(s_{hh}\right) f_2\left(x_q\right) f_3\left(x_{q'}\right), \tag{55}$$

and it is only in far asymptotics (2), where some noninteger-power scaling law and factorization set in. Finally, note that amplitude (1) is neither even, nor odd function of s, in contrast to the kernel in Born approximation. The latter property matters at calculations of meson flavor exchange amplitudes.

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