Phenomenology of the Heavy Flavored Spin 3/2 Baryons in Light Cone QCD

T.M. Aliev, K. Azizi, and A. Ozpineci

Abstract Motivated by the results of the recent experimental discoveries for charm and bottom baryons, the masses and magnetic moments of the heavy baryons with $J^P = 3/2^+$ containing a single heavy quark are studied within light cone QCD sum rules method. Our results on the masses of heavy baryons are in good agreement with predictions of other approaches, as well as with the existing experimental data.

Keywords: Magnetic moments, Light cone QCD sum rules, heavy bottom baryons, heavy charm baryons

1 Introduction

In the recent years, considerable experimental progress has been made in the spectroscopy of baryons containing a single heavy quark. The CDF Collaboration has observed four bottom baryons $\Sigma_b^{\pm}$ and $\Sigma_b^{*\pm}$ [1]. The DO [2] and CDF [3] Collaborations have seen the Ξ_b. The BaBar Collaboration discovered the Ω_c^* state [4]. The CDF sensitivity appears adequate to observe new heavy baryons. Study of the electromagnetic properties of baryons can give noteworthy information on their internal structure. One of the main static electromagnetic parameters of the baryons is their magnetic moments. Magnetic moments of the heavy baryons in the framework of different approaches are widely discussed in the literature. In the present work, we study the magnetic moments and masses of the ground state baryons with total angular momentum 3/2 and containing one heavy quark within light cone QCD sum rules. The paper is organized as follows. In section 2, the light cone QCD sum rules for mass and magnetic moments of heavy baryons are calculated. Section 3 is devoted to the numerical analysis of the mass and magnetic moment sum rules

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and discussion. Detailed analysis of the mass and magnetic moments of the baryons containing single heavy quark is presented in the original work in [5].

2 Light cone QCD sum rules for the mass and magnetic moments of the heavy flavored baryons

To calculate the magnetic moments of the heavy flavored hadrons, we start considering the correlation function which is the basic object in LCSR method. In this correlator, the hadrons are represented by their interpolating quark currents.

$$T_{\mu\nu} = i \int d^4x e^{ipx} \langle 0 \mid T\{\eta_\mu(x)\bar{\eta}_\nu(0)\} \mid 0 \rangle_\gamma, \tag{1}$$

where η_μ is the interpolating current of the heavy baryon and γ means the external electromagnetic field. In QCD sum rules method, this correlation function is calculated in two different ways: (1) In terms of quark-gluon language (QCD side), (2) In terms of hadrons, where the correlator is saturated by a tower of hadrons with the same quantum numbers as their interpolating currents (phenomenological side). The magnetic moments are determined by matching two different representations of the correlation function, i.e., theoretical and phenomenological forms, using the dispersion relations.

From Eq. (1), it follows that to calculate the correlation function from QCD side, we need the explicit expressions of the interpolating currents of heavy baryons with the angular momentum $J^P = 3/2^+$. The main condition for constructing the interpolating currents from quark field is that they should have the same quantum numbers of the baryons under consideration. For the heavy baryons with $J^P = 3/2^+$, the interpolating current is chosen in the following general form

$$\eta_\mu = A\varepsilon_{abc} \left\{ (q_1^{aT}C\gamma_\mu q_2^b)Q^c + (q_2^{aT}C\gamma_\mu Q^b)q_1^c + (Q^{aT}C\gamma_\mu q_1^b)q_2^c \right\}, \tag{2}$$

where C is the charge conjugation operator and a, b and c are color indices. The value of A and quark fields q_1 and q_2 for each heavy baryon is given in Table 1.

Table 1 The value of A and quark fields q_1 and q_2 for the corresponding baryons

	A	q_1	q_2
$\Sigma_{b(c)}^{*+(++)}$	$1/\sqrt{3}$	u	u
$\Sigma_{b(c)}^{*0(+)}$	$\sqrt{2/3}$	u	d
$\Sigma_{b(c)}^{*-(0)}$	$1/\sqrt{3}$	d	d
$\Xi_{b(c)}^{*0(+)}$	$\sqrt{2/3}$	s	u
$\Xi_{b(c)}^{*-(0)}$	$\sqrt{2/3}$	s	d
$\Omega_{b(c)}^{*-(0)}$	$1/\sqrt{3}$	s	s

The phenomenological part of the correlation function can be obtained by inserting the complete set of states between the interpolating currents in (1) with quantum numbers of heavy baryons.

$$T_{\mu\nu} = \frac{\langle 0 \mid \eta_\mu \mid B(p_2) \rangle}{p_2^2 - m_B^2} \langle B(p_2) \mid B(p_1) \rangle_\gamma \frac{\langle B(p_1) \mid \bar{\eta}_\nu \mid 0 \rangle}{p_1^2 - m_B^2}, \tag{3}$$

where $p_1 = p + q$, $p_2 = p$ and q is the photon momentum. The vacuum to baryon matrix element of the interpolating current is defined as

$$\langle 0 \mid \eta_\mu(0) \mid B(p,s) \rangle = \lambda_B u_\mu(p,s), \tag{4}$$

where λ_B is the residue and $u_\mu(p,s)$ is the Rarita–Schwinger spinor. The matrix element $\langle B(p_2) \mid B(p_1) \rangle_\gamma$ entering Eq. (3) can be parameterized in terms of the form factors f_i and G_i as follows

$$\langle B(p_2) \mid B(p_1) \rangle_\gamma = \varepsilon_\rho \bar{u}_\mu(p_2) \left\{ -g_{\mu\nu} \left[\gamma_\rho(f_1 + f_2) + \frac{(p_1 + p_2)_\rho}{2m_B} f_2 + q_\rho f_3 \right] \right.$$
$$\left. - \frac{q_\mu q_\nu}{(2m_B)^2} \left[\gamma_\rho(G_1 + G_2) + \frac{(p_1 + p_2)_\rho}{2m_B} G_2 + q_\rho G_3 \right] \right\} \bar{u}_\nu(p_1), \tag{5}$$

where ε_ρ is the photon polarization vector and $q^2 = (p_1 - p_2)^2$. To obtain the explicit expressions of the correlation function, summation over spins of the spin 3/2 particles is performed. Using the above equations in principle one can write down the phenomenological part of the correlator. But, the following two drawbacks appear: (a) all Lorentz structures are not independent, (b) not only spin 3/2, but spin 1/2 states also contribute to the correlation function. Indeed the matrix element of the current η_μ between vacuum and spin 1/2 states is nonzero and is determined as

$$\langle 0 \mid \eta_\mu(0) \mid B(p,s=1/2) \rangle = \alpha(4p_\mu - m\gamma_\mu)u(p,s=1/2), \tag{6}$$

where the condition $\gamma_\mu \eta^\mu = 0$ is imposed.

There are two different ways to remove the unwanted spin 1/2 contribution and retain only independent structures in the correlation function: (1) Introduce projection operators for the spin 3/2 states, which kill the spin 1/2 contribution, (2) Ordering Dirac matrices in a specific order and eliminate the structures that receive contributions from spin 1/2 states. In this work, we will follow the second method and choose the ordering for Dirac matrices as $\gamma_\mu \not{p} \not{\varepsilon} \not{q} \gamma_\nu$. With this ordering for the correlator, we get

$$T_{\mu\nu} = \lambda_B^2 \frac{1}{(p_1^2 - m_B^2)(p_2^2 - m_B^2)} \left[g_{\mu\nu} \not{p} \not{\varepsilon} \not{q} \frac{g_M}{3} \right.$$
$$+ \text{other structures with } \gamma_\mu \text{ at the beginning and } \gamma_\nu \text{ at the end}$$
$$\left. \text{or which are proportional to } p_{2\mu} \text{ or } p_{1\nu} \right], \tag{7}$$

where $g_M/3 = f_1 + f_2$ and at $q^2 = 0$, g_M is the magnetic moment of the baryon in units of its natural magneton. The factor 3 is due the fact that in the non-relativistic limit the interaction Hamiltonian with magnetic field is equal to $g_M B = 3(f_1 + f_2)B$.

On QCD side, the correlation function (1) can be evaluated using operator product expansion. After simple calculations, we get the following expression for the correlation function in terms of quark propagators

$$
\begin{aligned}
\Pi_{\mu\nu} = -iA^2 \varepsilon_{abc}\varepsilon_{a'b'c'} \int d^4x e^{ipx} \langle 0[\gamma(q)] \, | \, \{ S_Q^{ca'} \gamma_\nu S'^{bb'}_{q_2} \gamma_\mu S_{q_1}^{ac'} \\
+ S_Q^{cb'} \gamma_\nu S'^{aa'}_{q_1} \gamma_\mu S_{q_2}^{bc'} + S_{q_2}^{ca'} \gamma_\nu S'^{bb'}_{q_1} \gamma_\mu S_Q^{ac'} + S_{q_2}^{cb'} \gamma_\nu S'^{aa'}_{Q} \gamma_\mu S_{q_1}^{bc'} \\
+ S_{q_1}^{cb'} \gamma_\nu S'^{aa'}_{q_2} \gamma_\mu S_Q^{bc'} + S_{q_1}^{ca'} \gamma_\nu S'^{bb'}_{Q} \gamma_\mu S_{q_2}^{ac'} + Tr(\gamma_\mu S_Q^{ab'} \gamma_\nu S'^{ba'}_{q_2}) S_Q^{cc'} \\
+ Tr(\gamma_\mu S_Q^{ab'} \gamma_\nu S'^{ba'}_{q_1}) S_{q_2}^{cc'} + Tr(\gamma_\mu S_{q_2}^{ab'} \gamma_\nu S'^{ba'}_{Q}) S_{q_1}^{cc'} \} \, | \, 0 \rangle,
\end{aligned}
\tag{8}
$$

where $S' = CS^T C$ and $S_Q(S_q)$ is the full heavy (light) quark propagator. In calculation of the correlation function from QCD side, we take into account terms linear in m_q and neglect quadratic terms. The correlator contains three different contributions: (1) Perturbative contributions, (2) Mixed contributions, i.e., the photon is radiated from freely propagating quarks at short distance and at least one of quark pairs interact with QCD vacuum non-perturbatively. The last interaction is parameterized in terms of quark condensates. (3) Non-perturbative contributions, i.e., when photon is radiated at long distances. In order to calculate the contributions of the photon emission from large distances, the matrix elements of nonlocal operators $\bar{q}\Gamma_i q$ between the photon and vacuum states are needed, $\langle \gamma(q) \, | \, \bar{q}\Gamma_i q \, | \, 0 \rangle$. These matrix elements are determined in terms of the photon distribution amplitudes (DA's). For these matrix elements and also the photon DA's see [6].

Using the expressions of the light and heavy full propagators and the photon DA's and separating the coefficient of the structure $g_{\mu\nu} \not{p} \not{\varepsilon} \not{q}$, the expression of the correlation function from QCD side is obtained. Separating the coefficient of the same structure from phenomenological part and equating these representations of the correlator, sum rules for the magnetic moments of the $J^P = 3/2^+$ heavy baryons is obtained. In order to suppress the contribution of higher states and continuum, Borel transformation with respect to the variables $p_2^2 = p^2$ and $p_1^2 = (p+q)^2$ is applied. The sum rules for the magnetic moments is obtained as

$$
-\lambda_B^2 \frac{\mu_{B_Q}}{3} e^{\frac{-m_{B_Q}^2}{M^2}} = A^2 \Pi^{B_Q}.
\tag{9}
$$

The functions $\Pi_i(q_1, q_2, Q)$ can be written as

$$
\Pi_i = \int_{m_Q^2}^{s_0} e^{\frac{-s}{M^2}} \rho_i(s) ds + e^{\frac{-m_Q^2}{M^2}} \Gamma_i,
\tag{10}
$$

where the explicit expressions for the ρ_i and Γ_i functions are given in [5]. In the above relations M^2 and s_0 are the Borel mass square and continuum threshold, respectively.

For calculation of the magnetic moments of the considered baryons, their residues λ_B as well as their masses are needed (see Eq. (9)). Note that many of the considered baryons are not discovered yet in the experiments. The residue is determined from analysis of the two point sum rules. For the interpolating current given in Eq. (2), we obtain the following result for λ_B^2:

$$\lambda_B^2 e^{\frac{-m_{B_Q}^2}{M^2}} = A^2 \left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2) \right],$$ (11)

where the explicit expression for Π' is presented in [5]. The masses of the considered baryons can be determined from the sum rules. For this aim, one can get the derivative from both side of Eq. (11) with respect to $-1/M^2$ and divide the obtained result to the Eq. (11), i.e.,

$$m_{B_Q}^2 = \frac{-\frac{d}{d(1/M^2)} \left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2) \right]}{\left[\Pi' + \Pi'(q_1 \longleftrightarrow q_2) \right]}.$$ (12)

3 Numerical analysis

In this section, we perform numerical analysis for the mass and magnetic moments of the heavy flavored baryons. Firstly, we present the input parameters used in the analysis of the sum rules: $\langle \bar{u}u \rangle(1\ GeV) = \langle \bar{d}d \rangle(1\ GeV) = -(0.243)^3\ GeV^3$, $\langle \bar{s}s \rangle(1\ GeV) = 0.8 \langle \bar{u}u \rangle(1\ GeV)$, $m_0^2(1\ GeV) = (0.8 \pm 0.2)\ GeV^2$ [7], $\Lambda = 1\ GeV$ and $f_{3\gamma} = -0.0039\ GeV^2$ [6]. The value of the magnetic susceptibility was obtained in various papers as $\chi(1\ GeV) = -3.15 \pm 0.3\ GeV^{-2}$ [6], $\chi(1\ GeV) = -(2.85 \pm 0.5)\ GeV^{-2}$ [8] and $\chi(1\ GeV) = -4.4\ GeV^{-2}$[9].

Before proceeding to the results for the magnetic moments, we calculate the masses of heavy flavored baryons predicted from mass sum rule. Obviously, the masses should not depend on the Borel mass parameter M^2 in a complete theory. However, in sum rules method the operator product expansion (OPE) is truncated and as a result the dependency of the predictions of physical quantities on the auxiliary parameter M^2 appears. For this reason one should look for a region of M^2 such that the predictions for the physical quantities do not vary with respect to the Borel mass parameter. This region is the so called the "working region" and within this region the truncation is reasonable and meaningful. The upper limit of M^2 is determined from condition that the continuum and higher states contributions should be small than the total dispersion integral. The lower limit is determined by demanding that in the truncated OPE the condensate term with highest dimension remains small than sum of all terms, i.e., convergence of OPE should be under control.

These both conditions conditions for bottom (charmed) baryons are satisfied when M^2 varies in the interval $15\ GeV^2 < M^2 < 30\ GeV^2$ ($4\ GeV^2 < M^2 < 12\ GeV^2$). In Figs. 1–5, we presented the dependence of the mass of the heavy flavored baryons on M^2. From these figures, we see very good stability with respect to M^2.

Fig. 1 The dependence of mass of the Ω_b^* on the Borel parameter M^2 for two fixed values of continuum threshold s_0
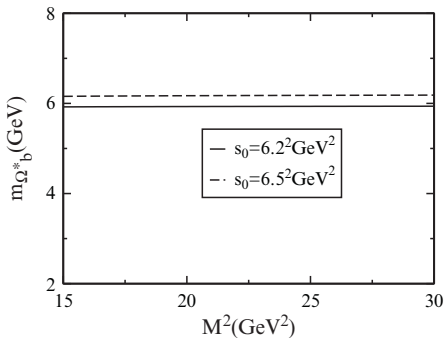

Fig. 2 The dependence of mass of the Ω_c^* on the Borel parameter M^2 for two fixed values of continuum threshold s_0
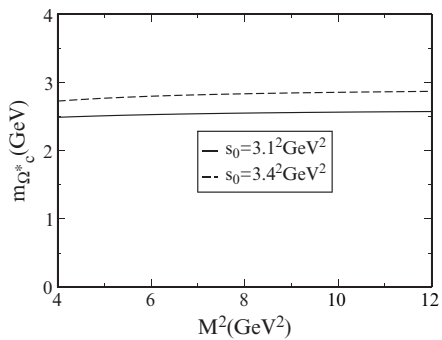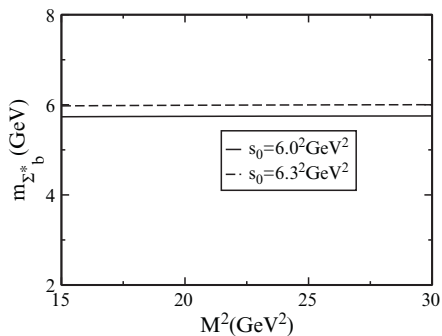

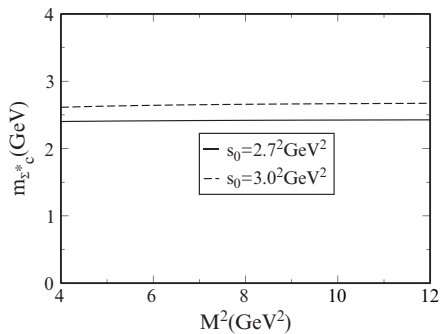**Fig. 3** The same as Fig. 1, but for Σ_b^*

Fig. 4 The same as Fig. 2, but for Σ_c^*

Fig. 5 The same as Fig. 1, but for Ξ_b^*
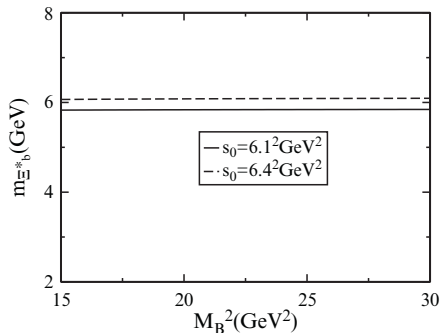
Table 2 Comparison of mass of the heavy flavored baryons in *GeV* from present work and other approaches and with experiment

	$m_{\Omega_b^*}$	$m_{\Omega_c^*}$	$m_{\Sigma_b^*}$	$m_{\Sigma_c^*}$	$m_{\Xi_b^*}$	$m_{\Xi_c^*}$
This work	6.08 ± 0.40	2.72 ± 0.20	5.85 ± 0.35	2.51 ± 0.15	5.97 ± 0.40	2.66 ± 0.18
[10]	$6.063^{+0.083}_{-0.082}$	$2.790^{+0.109}_{-0.105}$	$5.835^{+0.082}_{-0.077}$	$2.534^{+0.096}_{-0.081}$	$5.929^{+0.088}_{-0.079}$	$2.634^{+0.102}_{-0.094}$
[11]	6.088	2.768	5.834	2.518	5.963	2.654
[12]	-	-	5.805	2.495	-	-
[13]	6.090	2.770	5.850	2.520	5.980	2.650
[14]	-	2.768	-	2.518	-	-
[15]	6.083	2.760	5.840	-	5.966	-
[16]	6.060	2.752	5.871	2.5388	5.959	2.680
Exp[17]	-	2.770	5.836	2.520	-	2.645

The sum rule predictions of the mass of the heavy flavored baryons are presented in Table 2 in comparison with some theoretical predictions and experimental results. Note that the masses of the heavy flavored baryons are calculated in the framework of heavy quark effective theory (HQET) using the QCD sum rules method in [10].

After determination of the mass as well as residue of the heavy flavored baryons our next task is the calculation of the numerical values of their magnetic moments. For this aim, from sum rules for the magnetic moments it follows that the photon DAs are needed [6]. The sum rules for magnetic moments also contain the auxiliary parameters: Borel parameter M^2 and continuum threshold s_0. Similar to mass sum rules, the magnetic moments should also be independent of these parameters. In the general case, the working region of M^2 and s_0 for the mass and magnetic moments should be different. To find the working region for M^2, we proceed as follows. The upper bound is obtained requiring that the contribution of higher states and continuum should be less than the ground state contribution. The lower bound of M^2 is determined from condition that the highest power of $1/M^2$ be less than say 30% of the highest power of M^2. These two conditions are both satisfied in the region $15 \ GeV^2 \leq M^2 \leq 30 \ GeV^2$ and $4 \ GeV^2 \leq M^2 \leq 12 \ GeV^2$ for baryons containing b and c-quark, respectively. The working region for the Borel parameter for mass and magnetic moments practically coincide, but again we should stress that, this circumstance is accidental.

In Figs. 6–16, we present the dependence of the magnetic moment of heavy flavored baryons on M^2 at two fixed values of continuum threshold s_0. From these figures, we see that the magnetic moments weakly depend on s_0. The maximal change of results is about 10% with variation of s_0. The magnetic moments also are practically insensitive to the variation of Borel mass parameter when it varies in the working region. We should also stress that our results practically don't change considering three values of χ which we presented at the beginning of this section. Our final results on the magnetic moments of heavy flavored baryons are presented in Table 3. For comparison, the predictions of hyper central model [18] are also presented. The quoted errors in Table 3 are due to the uncertainties in m_0^2, variation of s_0 and M^2 as well as errors in the determination of the input parameters.
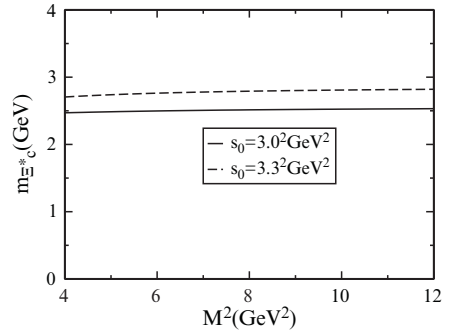
Fig. 6 The same as Fig. 2, but for Ξ_c^*
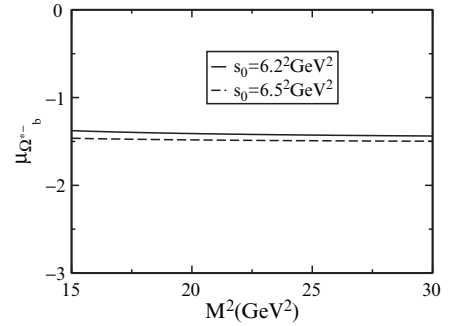
Fig. 7 The dependence of the magnetic moment of Ω_b^{*-} on Borel parameter M^2 (in units of nucleon magneton) at two fixed values of s_0
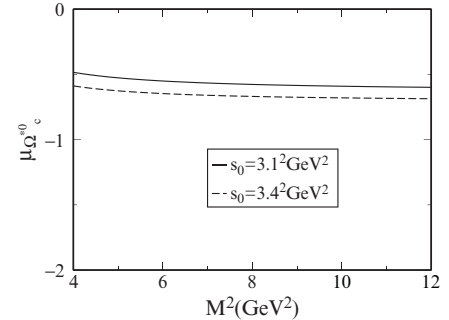

Fig. 8 The dependence of the magnetic moment of Ω_c^{*0} on Borel parameter M^2 (in units of nucleon magneton) at two fixed values of s_0

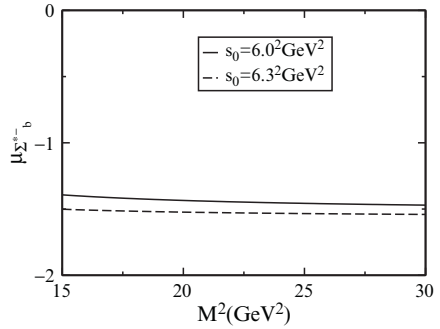**Fig. 9** The same as Fig. 7, but for Σ_b^{*-}

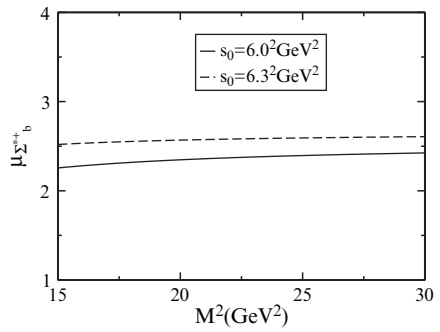**Fig. 10** The same as Fig. 7, but for Σ_b^{*+}

Fig. 11 The same as Fig. 8, but for Σ_c^{*0}
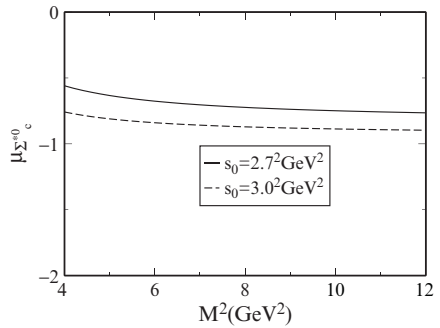
Fig. 12 The same as Fig. 8, but for Σ_c^{*++}
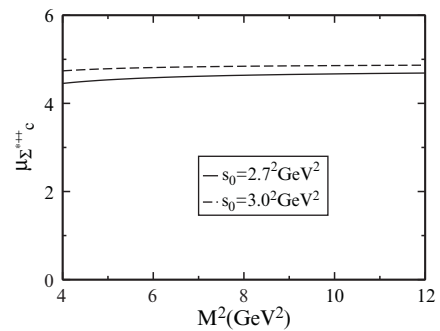
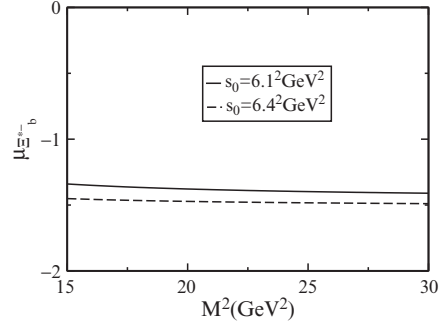**Fig. 13** The same as Fig. 7, but for Ξ_b^{*-}

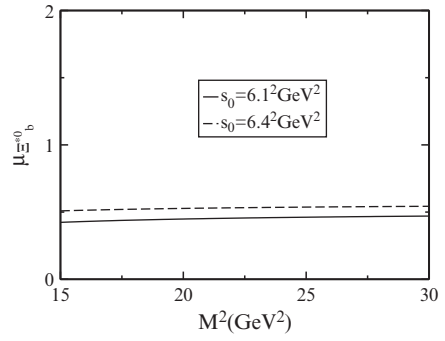**Fig. 14** The same as Fig. 7, but for Ξ_b^{*0}

Fig. 15 The same as Fig. 8, but for Ξ_c^{*0}
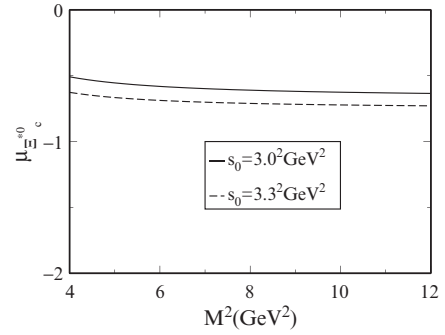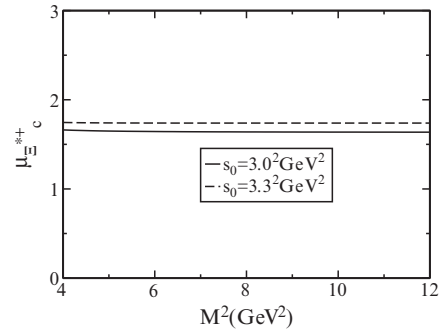
Fig. 16 The same as Fig. 8, but for Ξ_c^{*+}

Table 3 The magnetic moments of the heavy flavored baryons in units of nucleon magneton

	Our results	Hyper central model[18]
$\mu_{\Omega_b^{*-}}$	-1.40 ± 0.35	$-1.178\div-1.201$
$\mu_{\Omega_c^{*0}}$	-0.62 ± 0.18	$-0.827\div-0.867$
$\mu_{\Sigma_b^{*-}}$	-1.50 ± 0.36	$-1.628\div-1.657$
$\mu_{\Sigma_b^{*0}}$	0.50 ± 0.15	$0.778\div0.792$
$\mu_{\Sigma_b^{*+}}$	2.52 ± 0.50	$3.182\div3.239$
$\mu_{\Sigma_c^{*0}}$	-0.81 ± 0.20	$-0.826\div-0.850$
$\mu_{\Sigma_c^{*+}}$	2.00 ± 0.46	$1.200\div1.256$
$\mu_{\Sigma_c^{*++}}$	4.81 ± 1.22	$3.682\div3.844$
$\mu_{\Xi_b^{*-}}$	-1.42 ± 0.35	$-1.048\div-1.098$
$\mu_{\Xi_b^{*0}}$	0.50 ± 0.15	$1.024\div1.042$
$\mu_{\Xi_c^{*0}}$	-0.68 ± 0.18	$-0.671\div-0.690$
$\mu_{\Xi_c^{*+}}$	1.68 ± 0.42	$1.449\div1.517$

Although the $SU(3)_f$ breaking effects have been taken into account through a nonzero s-quark mass and different strange quark condensate, we predict that $SU(3)_f$ symmetry violation in the magnetic moments is very small, except the relations $\mu_{\Sigma_c^{*+}} = \mu_{\Xi_c^{*+}}$ and $\Pi^{\Sigma_c^{*++}} + \Pi^{\Omega_c^{*0}} = 2\Pi^{\Xi_c^{*+}}$, where the $SU(3)_f$ symmetry violation is large. For the values of the magnetic moments, our results are consistent with the results of [18] except for the $\mu_{\Omega_b^{*-}}$, $\mu_{\Xi_b^{*-}}$ and especially for the $\mu_{\Sigma_c^{*+}}$, $\mu_{\Xi_b^{*0}}$ which we see a big discrepancy between two predictions.

In summary, inspired by recent experimental discovery of the heavy and flavored baryons [1–3], the mass and magnetic moments of these baryons with $J^P = 3/2^+$ are calculated within the QCD sum rules. Our results on the masses are consistent with the experimental data as well as predictions of other approaches. Our results on the masses of the Ω_b^*, and Ξ_b^* can be tested in experiments which will be held in the near future. The predictions on the magnetic moments also can verified in the future experiments.

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