An Alternative Interpretation of the Weinberg–Salam Model

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Abstract A new interpretation of the Higgs field on the basis of a non-orthodox approach to the Weinberg–Salam (WS) model is suggested. It is argued that the masses of vector mesons can be generated without the use of the Higgs potential.

Keywords: Yang–Mills, Weinberg–Salam, Goldstone, vector meson, scalar field, Abelian transformation, ferromagnetism

Si nous ne trouvons pas des choses agréables,
nous trouverons du moins des choses nouvelles.
Cacambo to Candide before finding Eldorado

My wife and me found this phrase while reading on the Crimean beach during free time Voltaire's ironic description of the Candide's adventures. (What can you do on the beach but collecting pebbles and reading.) We could not help noting, that this sentence reflects feelings of the large part of physical community in wake of the results on LHC. In my talk, based on the joint paper with A. Niemi and M. Chernodub [1], I shall present a nonorthodox approach to WS model, proposing a new interpretation for the Higgs field. In particular I shall argue, that masses of vector bosons could be suplemented without use of the Higgs potential.

The paper [1] was produced by email correspondence and the version of A.N. and M.Ch. was published. Here I shall use my approach, which is fully equivalent to [1] and add some personal remarks.

The new interpretation concerns only bosonic part of WS model, so I shall consider only lagrangian for complex scalar dublet $\Phi = (\phi_1, \phi_2)$, abelian vector field Y_μ and SU(2) Yang–Mills triplet $B^a_\mu, a = 1,2,3$

$$\mathscr{L} = \left(\nabla_\mu \Phi, \nabla_\mu \Phi\right) + \frac{1}{4g^2} B^a_{\mu\nu} B^a_{\mu\nu} + \frac{1}{4g'^2} Y^2_{\mu\nu},$$

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where

$$\nabla_\mu \Phi = \partial_\mu \Phi + \frac{i}{2} Y_\mu \Phi + B^a_\mu t^a \Phi$$

$$B^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B_\mu + \varepsilon_{abc} B^b_\mu B^c_\nu$$

$$Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu$$

with $t^a = \frac{i}{2}\tau^a$, τ^a — Pauli matrices, $(\cdot,\cdot)$ — hermitian scalar product in $\mathbb{C}^2$, g and g' — coupling constants.

I do not introduce selfinteraction for the scalar field Φ. The interpretation below is exactly based on this omission. In fact all my friends among theoretical physicists hate ϕ^4 interaction, it is not asymptotically free and quite possible dissapears under proper renormalization.

The lagrangian $\mathscr{L}$ has U(2) gauge invariance with parameters: matrix Ω from SU(2) and real function ω:

$$\Phi^\Omega = \Omega\Phi, \quad \Phi^\omega = e^{i\omega}\Phi,$$
$$B^\Omega_\mu = \Omega B_\mu \Omega^{-1} - \partial_\mu \Omega \Omega^{-1}, \quad B^\omega_\mu = B_\mu,$$
$$Y^\Omega_\mu = Y_\mu, \quad Y^\omega_\mu = Y_\mu - 2\partial_\mu \omega.$$

Here $B_\mu = B^a_\mu t^a$. The idea of [1] is to make the change of variables, which leads to the gauge invariant degrees of freedom. Before presenting the explicite formulas I shall give a geometric reason for them. The "target" for the field Φ is $\mathbb{C}^2$ or $\mathbb{R}^4$ if we count real components. In radial coordinates $\mathbb{R}^4$ can be presented as $\mathbb{R}_+ \times \mathbb{S}^3$ and furthermore $\mathbb{S}^3$ is (almost) the same as SU(2). The SU(2) degrees of freedom, realized as matrix g, allow to introduce gauge transformation of the Yang–Mills field, leaving the gauge invariant vector field. In [1] one of realization of this idea was proposed. I am sure, that this comment is not original, a similar considerations were discussed for example long ago in [2]. However, I believe that the interpretation given below is new.

Let us proceed. We should extract the SU(2) degrees of freedom from Φ in most convenient way. Here is my proposal. First write Φ as

$$\Phi = \rho\chi,$$

where ρ is a positive function and χ — normalised as follows

$$(\chi,\chi) = \bar{\chi}_1\chi_1 + \bar{\chi}_2\chi_2 = 1.$$

The matrix

$$g = \begin{pmatrix} \chi_1 & -\bar{\chi}_2 \\ \chi_2 & \bar{\chi}_1 \end{pmatrix} = |\chi, \sigma\bar{\chi}|$$

is unimodular and unitary. Here

$$\sigma = \frac{1}{i}\tau_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The doublet $\sigma\bar{\chi}$ transforms under the nonabelian gauge transformation exactly as χ, indeed

$$(\sigma\bar{\chi})^{\Omega} = \sigma\bar{\Omega}\bar{\chi} = -\sigma\bar{\Omega}\sigma\sigma\bar{\chi} = \Omega\sigma\bar{\chi},$$

where I use the properties $\sigma^2 = -I$ and $\sigma\bar{\tau}\sigma = \tau$ for all Pauli matrices. Thus the whole matrix g transforms as

$$g^{\Omega} = \Omega g.$$

The abelian transformation is different for χ and $\bar{\chi}$, so that

$$g^{\omega} = g \begin{pmatrix} e^{i\omega} & 0 \\ 0 & e^{-i\omega} \end{pmatrix} = g e^{i\omega\tau_3}.$$

We see, that covariant derivative of g assumes the form

$$\nabla_\mu g = \partial_\mu g + \frac{i}{2} Y_\mu g \tau_3 + B_\mu g$$

and $|\nabla_\mu \Phi|^2$ can be rewritten as

$$|\nabla_\mu \Phi|^2 = \frac{\rho^2}{2} \mathrm{tr}\big((\nabla_\mu g)^*(\nabla_\mu g)\big) + \partial_\mu\rho\partial_\mu\rho.$$

Introduce the new vector field

$$W_\mu = g^* B_\mu g + g^* \partial_\mu g.$$

It is easy to check, that

$$\begin{aligned} W_\mu^{\Omega} &= W_\mu, \\ W_\mu^{\omega} &= e^{-i\omega\tau_3} W_\mu e^{i\omega\tau_3} + i\partial_\mu\omega\tau_3 \end{aligned}$$

and direct calculation shows, that

$$\frac{1}{2}\mathrm{tr}\big((\nabla_\mu g)^*(\nabla_\mu g)\big) = \frac{\rho^2}{4}\mathrm{tr}\big(Z_\mu^2 + [W_\mu, \tau_3]^2\big) = \frac{\rho^2}{4}\big(Z_\mu^2 + W_\mu^+ W_\mu^-\big),$$

where

$$Z_\mu = Y_\mu + W_\mu^3$$

and we introduce the components of field W_μ

$$\begin{aligned} W_\mu &= \frac{i}{2}(W_\mu^1\tau_1 + W_\mu^2\tau_2 + W_\mu^3\tau_3) \\ W_\mu^{\pm} &= W_\mu^1 \pm W_\mu^2. \end{aligned}$$

In these components

$$(W_\mu^{\pm})^{\omega} = e^{\pm 2i\omega} W_\mu^{\pm}, \quad (W_\mu^3)^{\omega} = W_\mu^3 + 2\partial_\mu\omega,$$

so that abelian vector field Z_μ is completely gauge invariant and $W_\mu^\pm$ and W_μ^3 behave under abelian transformation as charged and abelian gauge vector field, correspondingly.

The vector part of the lagrangian can be rewritten as

$$\frac{1}{4g'^2}Y_{\mu\nu}^2 + \frac{1}{4g^2}(W_{\mu\nu}^3 + H_{\mu\nu})^2 + \frac{1}{4g^2}(\nabla_\mu W_\nu^+ - \nabla_\nu W_\mu^+)(\nabla_\mu W_\nu^- - \nabla_\nu W_\mu^-),$$

where

$$W_{\mu\nu}^3 = \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3$$
$$H_{\mu\nu} = \frac{1}{2i}(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^-)$$

and

$$\nabla_\mu W_\nu^\pm = \partial_\mu W_\nu^\pm \pm i W_\mu^3 W_\nu^\pm.$$

To express the sum of quadratic forms of $Y_{\mu\nu}$ and $W_{\mu\nu}^3$ via Z_μ it is convenient to introduce the combination

$$A_\mu = \frac{1}{g^2 + g'^2}(g'^2 W_\mu - g^2 Y_\mu)$$

such that

$$\frac{1}{4g'^2}Y_{\mu\nu}^2 + \frac{1}{4g^2}(W_{\mu\nu}^3)^2 = \frac{1}{4(g^2+g'^2)}Z_{\mu\nu}^2 + \frac{g^2+g'^2}{4g^2g'^2}A_{\mu\nu}^2$$

with as usual

$$Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu, \quad A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

The abelian vector field A_μ transforms in the same way as W_μ^3

$$A_\mu^\omega = A_\mu - 2\partial_\mu \omega$$

and enters into lagrangian via $\frac{1}{4e^2}A_{\mu\nu}^2$ with $e^2 = \frac{g^2g'^2}{g^2+g'^2}$. It is clear, that A_μ can be interpreted as electromagnetic field with electric charge given by e the ω-action having meaning of electromagnetic gauge transformation.

Thus in new variables the list of fields consists of real positive field ρ, neutral abelian fields Z_μ and A_μ and charged vector field $W_\mu^\pm$. The lagrangian assumes the form

$$\mathscr{L} = \partial_\mu\rho\partial_\mu\rho + \frac{\rho^2}{4}(Z_\mu^2 + W_\mu^+ W_\mu^-) + \frac{1}{4g^2}(\nabla_\mu W_\nu^+ - \nabla_\nu W_\mu^+)(\nabla_\mu W_\nu^- - \nabla_\nu W_\mu^-)$$
$$+ \frac{1}{4(g^2+g'^2)}Z_{\mu\nu}^2 + \frac{1}{4e^2}A_{\mu\nu}^2 + \frac{2}{4g^2}(H_{\mu\nu}, W_{\mu\nu}^3) + \frac{1}{4g^2}H_{\mu\nu}^2,$$

where W^3_μ should be changed to its expression via Z_μ and A_μ. Now we can begin discussion.

The change of variables, which I undertook, eliminates 3 degrees of freedom, leaving positive field ρ instead of four real components of scalar Φ. The functional measure

$$d\mu = \prod_x d\phi_1\, d\bar{\phi}_1\, d\phi_2\, d\bar{\phi}_2\, dY_\mu\, dB^a_\mu$$

used before gauge fixing, looks in new variables as

$$d\mu = \prod_x \rho^2 d\rho^2\, dZ_\mu dW^+_\mu\, dW^-_\mu\, dA\, dg,$$

where $\prod_x dg$ is volume of the gauge group, which is completely separated from measure without any gauge fixing. We are just to drop it to write nonsingular functional integral.

We see, that ρ^2 is not an ordinary scalar field. Besides being positive, it enters the functional integral with local factor. This requires some interpretation. In particular the reason for nontrivial expectation value

$$\langle \rho^2 \rangle = \Lambda^2,$$

supplying mass to vector fields Z_μ and $W^\pm_\mu$, must be elucidated.

In [1] we proposed to interpret ρ^2 as conformal factor of the metric in space-time

$$g_{\mu\nu} = \rho^2 \delta_{\mu\nu}.$$

Indeed, in 4-dimensional space-time we have $\sqrt{g} = \rho^4$, and contravariant vector and tensors have factors

$$\chi^\mu = \rho^{-2}\chi_\mu, \quad \chi^{\mu\nu} = \rho^{-4}\chi_{\mu\nu},$$

so that $\rho^2 Z^2_\mu = Z_\mu Z^\mu \sqrt{g}$, $H^2_{\mu\nu} = H_{\mu\nu}H^{\mu\nu}\sqrt{g}$, etc. Moreover the scalar curvature is given by

$$R = \frac{1}{6}\frac{\partial_\mu \rho \partial_\mu \rho}{\rho^4} + \text{divergence}.$$

Finally, the Christoffel's symbols, entering the definition of the field strengths $A_{\mu\nu}$, $Z_{\mu\nu}$, $W^\pm_{\mu\nu}$ via covariant derivatives, cancel due to antisymmetry. Thus the lagrangian can be rewritten in manifestly covariant form.

In this interpretation it is natural to require, that

$$\rho^2|_{r\to\infty} \to \Lambda^2$$

at spacial infinity to maintain the asymptotical flatness. Parameter Λ^2 enters as a new parameter of the model.

An alternative argument for nontrivial expectation value Λ^2 looks as follows. The field ρ enters the lagrangian either via derivatives or being multiplied by another field. So the classical vacuum configuration, given by

$$\rho^2 = \Lambda^2, \quad Z_\mu = 0, \quad W^\pm_\mu = 0, \quad A_\mu = 0$$

is degenerate. The choice of particular value of Λ^2 corresponds to the concrete choice of the vacuum. All this looks as noncompact analogy of ferromagnetism.

Thus, one way or another we see, that the nonzero expectation value for the ρ^2 can be envoked without the Higgs potential. The fundamental question which remains, is the origin of the excitations for the field ρ. In both interpretations the most natural answer is massless scalar–analogy of dilaton in the first interpretation or kind of Goldstone mode in the second.

I hope, that more experienced phenomenologist can consider seriously this hypothesis.

References

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