An Erdős–Ko–Rado theorem for matchings in the complete graph

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We consider the following higher-order analog of the Erdős–Ko–Rado theorem [1]. For positive integers r and n with $r \leq n$, let $\mathcal{M}_n^r$ be the family of all matchings of size r in the complete graph K_{2n}. For any edge $e \in E(K_{2n})$, the family $\mathcal{M}_n^r(e)$, which consists of all sets in $\mathcal{M}_n^r$ containing e is called the star centered at e. We prove the following result:

Theorem 1. *For $r < n$, if $\mathcal{A} \subseteq \mathcal{M}_n^r$ is an intersecting family of r-matchings, then $|\mathcal{A}| \leq \phi(n, r)$ with equality holding if and only if $\mathcal{A} = \mathcal{M}_n^r(e)$ for some $e \in E$.*

We note that the case $r = n$ is settled (as part of a stronger theorem for uniform set partitions) by Meagher and Moura [3]. To prove Theorem 1, we use an analog of Katona's cycle method [2]. As in Katona's original proof of the Erdős–Ko–Rado theorem, the main challenge is to come up with a class of objects over which to carry out the double counting argument. We use the notion of Baranyai partitions to construct these objects.

References

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