Chapter 3
General Relativity and Astrophysics

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3.1 Introduction

In 1915 Einstein published the final version of his basic general theory of relativity. Although he was to further modify it in 1917 by adding the so-called 'lambda term' the framework of the theory for local applications remained the same. For our reference we will take the form of the 1915 field equations as

$$R_{ik} - \frac{1}{2}g_{ik}R = -\frac{8\pi G}{c^4}\,T_{ik}, \tag{3.1}$$

while those with the lambda term are:

$$R_{ik} - \frac{1}{2}g_{ik}R + \lambda\, g_{ik} = -\frac{8\pi G}{c^4}\,T_{ik}, \tag{3.2}$$

Here the spacetime coordinates are x^k, $k = 0, 1, 2, 3$, with 0 usually standing for the timelike dimension. The spacetime metric is g_{ik}. The signature of the metric is [+,–,–,–]. T_{ik} denotes the energy momentum tensor of the physical entities present in spacetime.

As is well-known, the general theory of relativity was considered esoteric by most physicists and for a considerable period of time very few people worked on it. The 1919 eclipse results on the bending of light, announced by Eddington went a long way towards gaining crdibility for the theory. The eclipse success also excited the popular mind while the media had a field day broadcasting news items related to the event.

In this connection it is worth recalling that in 1919, shortly after the announcement of the results by Eddington a popular account of the whole experiment was published by the *Statesman*, Calcutta on November 13, 1919 and the account was written by Meghnad Saha, then a young physics lecturer entering into the newly emerging field of astrophysics (AP). Indeed it would not be an exaggeration to say that Saha's ionization equation provided the basic foundations on which the theory of stellar atmospheres is based and which in turn leads to the theory of stellar structure, considered as central to AP.

3.2 The Calcutta School

However, general relativity (GR) came to India through the works of two applied mathematicians, Nikhil Ranjan Sen at Calcutta University and Vishnu Vasudeva Narlikar at the Banaras Hindu University. Sen was a contemporary of two other distinguished scientists of the future: Meghnad Saha and Satyendra Nath Bose at the Presidency College, Kolkata. In 1913 all three passed their B.Sc. examination with honours in mathematics, Bose standing first, Saha second and Sen third in order of merit.

For his academic career and research Sen opted for applied mathematics and theoretical physics. In 1921 he was awarded his first doctorate (D.Sc.) by Calcutta University, while holding a teaching position at the University's applied mathematics department. Subsequently he took study leave to visit and work at some of the well-known centres in Europe. His researches under von Laue on GR and cosmogony brought him a

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second doctorate (Ph.D.), from Berlin University. In Europe Sen had the benefit of coming in contact with the leading physicists of the day including Max Planck, Albert Einstein, Arnold Sommerfeld and Louis de Broglie.

Returning to India in 1924, Sen was appointed to the Rashbehari Ghosh Chair of Calcutta University and set up a flourishing school in applied mathematics and GR. Several young researchers joined his school and emerged well-qualified applied mathematicians and relativists. Sen's own contributions may be highlighted as follows:

His solution of Einstein's equations for the gravitational field of a spherical shell is well known (Sen 1924). It is shown that by a coordinate transformation the metric of the interior of the shell is the same as that of the de Sitter universe (Sen and von Laue 1924).

Sen analysed the equilibrium of a charged particle using the field equations that Einstein had modified in 1919. Assuming a charge distribution inside a finite-sized particle, Sen showed that under spherical symmetry, three-fourth of the total energy of the particle is electrical and one-fourth gravitational. Thus the mass of the electron was seen to consist of two components: the Lorentz mass and the other based on the charge distribution not uniquely determined by the theory (Sen 1927).

Sen worked on cosmology also and had a model of the universe which was static but not perfectly spherical (Sen 1934, 1935a, 1935b). He showed that the total mass of the model (which had a finite extent) was higher than that of the spherically symmetric Einstein universe. A stability analysis using this property then showed that the static Einstein model would lead to the expanding universe.

Studies of equilibrium configurations of constant density spheres led Sen (1933, 1937) to the conclusion that in contrast to classical Newtonian spheres, the relativistic ones have a maximum radius and a minimum mass for a given constant density. For a radius less than the maximum radius there are two possible configurations. The central pressures in these configurations sandwich the pressure present in the configuration of maximum radius. For a density corresponding to the maximum density of matter, the largest equilibrium configuration has a radius of 133 km, whereas the sphere of maximum mass is some 24 times the mass of the sun (Sen 1941). These conclusions are of interest in the modern context of limits on stellar masses at the end of nuclear energy generation.

Sen (1954) with T.C. Roy also studied the gravitational field of a stationary star cluster in which, following Einstein, each star follows a circular trajectory. They found a singularity-free solution which could be fitted to an extenal expanding spacetime. The gravitational mass of the cluster decreases as the cluster contracts, it being equal to the rest mass at infinite separation. The minimum gravitational mass is utmost 5% less than the rest mass. The velocities of the outermost stars approach the speed of light as the minimum is approached. If one puts the constraints that astronomically observed velocities only are found, then the difference between the two masses is insignificant and one may use Newtonian dynamics and gravitation. This type of work would have been found very interesting had it received suitable exposure in the early 1960s when equilibrium of supermassive objects was under consideration.

Since this chapter is about GR and its astrophysical applications we will not describe Sen's other works in quantum mechanics, fluid dynamics, etc. His reputation as a good teacher attracted students who worked with him sharing all his research interests. Amongst these a little known relativist was B. Datt.

Datt's work on gravitational collapse is described in his paper written in German and published in a German journal (Datt 1938). The work anticipated by an year, the much cited work of Oppenheimer and Snyder (1939) on the same topic. Unfortunately very little is known about Datt and his subsequent work. This paper was submitted from the Presidency College, Calcutta and the author thanks Dr J. Ghosh for consultation. We briefly describe this work as it is important in the context of relativistic AP.

For a spherically symmetric line element to decribe a spherically symmetric mass expanding or contracting with time, Datt writes

$$\mathrm{d}s^2 = \mathrm{e}^{\nu}\mathrm{d}t^2 - \mathrm{e}^{\lambda}\mathrm{d}r^2 - \mathrm{e}^{\mu}r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2). \tag{3.3}$$

Here the coordinates (r, θ, ϕ) are comoving coordinates of a typical pointlike component of the spherical object. The time coordinate t likewise denotes the proper time of this point. Datt then sets up the field equations and solves them in as general a form as possible. He then deduces special cases like the collapse of a dust ball (having pressure-free matter) which is similar to the Friedmann-Lemaitre models. Landau and Lifshitz in their book the *Classical Theory of Fields* (Third Edition) describe Datt's solutions in some detail.

We will return to the problem of gravitational collapse and other related issues later in this chapter.

3.3 The Banaras School

There was a parallel development, albeit a few years later, in the campus of Banaras Hindu University, where, in 1932 Vishnu Vasudeva Narlikar took charge as Professor and the Head of Department of Mathematics. Narlikar had just returned from Cambridge University, where, as the Isaac Newton Scholar he was working under Eddington. Narlikar was in fact due to go back to Cambridge and later spend a year in Caltech working at the Mount Wilson Observatory and the Department of Astronomy. However, the founder of Banaras Hindu University, the venerable Pandit Madan Mohan Malaviya, always on the look out for talented Indian academics peruaded him to join Banaras Hindu University and continue his researches there while attracting good students to mathematics. Narlikar accepted the challenge and did not go back to Cambridge.

While in a reminiscent mood, Narlikar recalled to this author the episode when, in 1930–31, as a fresh research student, he solved the Einstein field equations for a homogeneous and isotropic universe, *dropping* Einstein's condition of a static model. He got a family of solutions of the expanding universe models which would be of interest to the small family of cosmologists and extragalactic astronomers. His guide Eddington was impressed by the work and offered to communicate it to the *Monthly Notices of the Royal Astronomical Society*. Just then he received in post a paper by Abb*é* Lemaitre in which the very same problem had been solved. The paper in fact published in 1927, was in a little known Belgian journal (Lemaitre 1927) and Eddington had been unaware of it. So he regretfully told Narlikar that his work could not be published. However, to bring to attention of the larger English speaking astronomy community, Eddington communicated Lemaitre's paper after translating it in English, to the same journal MNRAS. As luck would have it, even Lemaitre turned out to have been anticipated by Alexander Friedmann (1922, 1924)! Relativistic cosmology was, indeed, a 'hot' subject in the 1920s–1930s.

After joining Banaras Hindu University Narlikar concentrated on the different aspects of GR. He recalled later that he had asked S. Chandrasekhar (who, in the 1930s had worked on stellar models) if there was any scope for applying GR to stars. Chandrasekhar replied in the negative, thus discouraging Narlikar from pursuing that line of research. The applications potential of GR for massive dense objects thus remained untested for three decades.

Some of the work from the Banaras school relevant to relativistic astrophysics may be described as below.

In 1922 the noted mathematician, T.Y. Thomas had proved that in Riemannian manifold of 4 dimensions only 14 independent curvature invariants can be constructed. But the explicit construction of these 14 invariants using the curvature tensor and the Weyl tensor was first given by Narlikar and Karmarkar (1949). However, this work was published in the *Proceedings of the Indian Academy of Sciences*, a journal which did not have much circulation outside India. Unaware of this work therefore, several years later Geheniau and Debever in 1956 did the same work for which they have been given credit. This was noticed by A.R. Prasanna, a student of Narlikar at Pune who pointed out to Professor Geheniau this fact when they met in 1972, at the Dirac Symposium at Trieste. Geheniau readily agreed that these invariants should be called 'Narlikar-Karmarkar invariants'.

These invariants are important in deciding if a spacetime manifold has singularities. The question of singularities became relevant to reality by the discovery of collapsed massive objects in the form of quasars in 1963. Will the spacetime in the neighbourhood of such massive objects develop a singularity? If so how to spot it in a coordinate-invariant fashion? This is where the curvature invariants become important.

Work of a more mathematical nature came out of the studies of Narlikar and his students Ramji Tiwari and Kamala Prasad Singh. Tiwari was concerned with the unified field theory proposed by Einstein in the late 1940s and examined in detail the interaction between gravitation and electromagnetism (Narlikar and Tiwari 1949). Singh, on the other hand, worked on metric invariants. His work of the Christoffel symbols is of interest in bringing out the role of coordinate transformation that lead to indeterminateness (Narlikar and Singh 1951).

General relativity has the unique feature that it contains the equations of motion of the sources and the method of deriving them was indicated by Einstein, Infeld and Hoffmann (1938). Narlikar's student, B.R. Rao worked on the details of this problem and pointed out some corrections to the Einstein, Infeld and Hoffmann (EIH) work. This was recognized by Infeld and Hoffmann. The Narlikar-Rao paper (1955) appeared in print in the year that Einstein died.

But perhaps the most important paper to come out of the Banaras school was by P.C. Vaidya, one of the earliest students of V.V. Narlikar. We will describe it next.

3.4 The Vaidya Solution

P.C. Vaidya started his research career as a student of V.V. Narlikar. Himself a postgraduate of Bombay University, Vaidya enroled himself as an external research student of Narlikar in the Banaras Hindu University in the year 1942–43. Essentially living on his savings he made them stretch out to this period during which he also had to support a family of wife and child. Yet during those 2 years Vaidya was able to produce work that was to prove to be of very special interest to relativistic AP about 25 years later.

Basically the ‘Vaidya solution’ is a generalization of the Schwarzschild solution, the main difference between the two being that while the exterior of the gravitating sphere in the Schwarzschild solution is empty, the sphere in the Vaidya solution is radiating. Evidently, the situation described in the Vaidya solution is time-dependent; not static. We summarise this work below: for details of this solution see Vaidya (1943, 1950)

The 1950 paper quotes V.V. Narlikar (1939): ‘If the principle of energy is to hold good, that is, combined energy of the matter and the field is to be conserved, the system must be an isolated system surrounded by flat spacetime. A spherical radiating mass would probably be surrounded by a finite and non-static envelope of radiation with radial symmetry. This would be surrounded by a radial field of gravitational energy becoming weaker and weaker as it runs away from the central body until at last the field is flat at infinity. It has to be seen whether and how this view of the distribution of energy is substantiated by the field equations of relativity.’ This conjecture was borne out by the Vaidya solution.

To start with take the four spacetime coordinates to be $x^0 = t, x^1 = r, x^2 = \theta, x^3 = \phi$. A star of mass M and radius r_0 is supposed to start radiating at time t_0 and as time goes on, the zone of radiation increases in thickness, its outer surface at time $t = t_1 > t_0$ being given by $r = r_1 > r_0$. For $r_0 \le r \le r_1$ and $t_0 \le t \le t_1$, the line element is given by

$$ds^2 = e^\nu\, dt^2 - e^\lambda\, dr^2 - r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{3.4}$$

where both λ and ν are functions of r, t only. The outflowing radiation is described by the energy momentum tensor

$$T^{ik} = \rho v^i\, v^k, \tag{3.5}$$

where ρ is the density of radiation and v^i is the null vector representing its flow direction. For radial flow, we have $v^2 = v^3 = 0$. The field equations then give (with $G = 1, c = 1$),

$$e^{-\lambda} = 1 - \frac{2m}{r},\ m = m(r,t), \tag{3.6}$$

and

$$e^{\nu/2} = -\frac{\dot m}{m'}\left(1 - \frac{2m}{r}\right)^{-1/2}, \tag{3.7}$$

where m satisfies the relation

$$m'\left(1 - \frac{2m}{r}\right) = f(m). \tag{3.8}$$

The dot and dash denote differentiations with respect to t and r, respectively.

The function $f(m)$ is so far arbitrary but needs to be specified by the physical conditions that lead to the radiation from the star, whose mass m decreases at a rate determined by the amount of energy radiated by it. The radiation envelope of the star is described by the line element

$$ds^2 = \frac{\dot m^2}{f^2}\left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{3.9}$$

The energy conservation relation is described by the condition

$$\frac{dm}{d\tau} \equiv v^0\frac{\partial m}{\partial t} + v^1\frac{\partial m}{\partial r} = 0. \tag{3.10}$$

Of course, this equation is automatically satisfied if all field equations are satisfied. This formalism has been useful in the context of bright radiating objects in AP, such as quasars, active galactic nuclei, gamma ray bursts, etc.

3.5 The Raychaudhuri Equation

A major advance in the field of gravitational collapse came from the work of A.K. Raychaudhuri (1955). Before describing it we may mention that Raychaudhuri was essentially a loner when he did this work. Having studied in Calcutta, Raychaudhuri naturally started research in N.R. Sen's group. However, he soon discovered that he worked better in isolation. So, while at the Indian Association for Cultivation of Science, he turned his attention to the singularity problem. Raychaudhuri's work was originally concerned with relativistic cosmology but was later found to be applicable to finite massive distributions of matter as well. The aim was to find out if the presence of shear and rotation would prevent the gravitational collapse of such distributions. We will now briefly describe this work and its significance for relativistic AP.

The Raychaudhuri equation arises in relativistic cosmology when we study the bundle of timelike geodesics defined by the Weyl postulate. If u^i is the unit tangent to the geodesic, we define the spin-vorticity 3-tensor for the cosmic fluid by $\omega_{\mu\nu} = \frac{1}{2}[u_{\mu;\nu} - u_{\nu;\mu}]$. Writing the line element in the form

$$ds^2 = dt^2 + 2g_{0\mu}dtdx^\mu + g_{\mu\nu}dx^\mu dx^\nu, \tag{3.11}$$

where the geodesics are specified by x^μ = constant and t is the cosmic time, the (0,0) component of field equations in the case of dust of density ρ then becomes

$$\frac{\ddot{Q}}{Q} = \frac{1}{3}(2\omega^2 - 4\pi G\rho - \phi^2) \tag{3.12}$$

where $Q^6 = -g$ and

$$2\omega^2 = -g^{\lambda\mu}g^{\sigma\tau}\omega_{\lambda\sigma}\omega_{\mu\tau},$$

$$\phi^2 = \frac{1}{4}g^{\mu\nu}\dot{g}_{\nu\sigma}g^{\sigma\lambda}\dot{g}_{\lambda\mu} - \frac{1}{3}\left(\frac{\partial}{\partial t}\ln\sqrt{-g}\right)^2. \tag{3.13}$$

The ϕ term is identified with shear and it goes the opposite way (to the spin term) through promoting singularity by helping the scale of the cosmic volume, Q approach zero. It vanishes when the expansion is isotropic.

The Raychaudhuri equation can be stated in a slightly different form as a *focussing theorem*. In this form it describes the effect of gravity on a bundle of null geodesics spanning a finite cross-section. Denoting the cross-section by A, we write the equation of the surface spanning the geodesics as $f =$ constant. With the normal to the cross-sectional surface being $k_i = \partial f/\partial x^i$. By the analogue of the hydrodynamic conservation law, we deduce

$$k^l A_{,l} = [k^l{}_{;l}]A. \tag{3.14}$$

Additionally we also have from the null geodesic condition

$$k^l k_{i;l} = 0. \tag{3.15}$$

Using a calculation similar to that which leads to the geodetic deviation equation, we now get the focussing equation as

$$\frac{1}{\sqrt{A}}\frac{d^2\sqrt{A}}{d\lambda^2} = \frac{1}{2}R_{im}k^i k^m - |\sigma|^2, \tag{3.16}$$

where

$$|\sigma|^2 = \frac{1}{2}k_{i;m}\,k^{i;m} - \frac{1}{4}[k^n_{;n}]^2. \tag{3.17}$$

The jvneq16 is similar to the Raychaudhuri equation with $|\sigma|^2$ being the square of the magnitude of shear. With Einstein's equations, we can rewrite jvneq16 as

$$\frac{1}{\sqrt{A}} \frac{d^2\sqrt{A}}{d\lambda^2} = -4\pi G(T_{im} - \frac{1}{2}g_{im}T)k^i k^m - |\sigma|^2. \tag{3.18}$$

For focussing of the bundle of rays we need $A \rightarrow 0$, so that the right hand side should be negative. This is helped by the shear term in the above equation just as Raychaudhuri had found. The first term on the right hand side of the focussing equation also has this property if

$$(T_{im} - \frac{1}{2}g_{im}T)k^i k^m \geq 0. \tag{3.19}$$

For dust we have $T_{im} = \rho u_i u_m$ and this condition is satisfied with the left hand side equalling $\rho(u_i k^i)^2$. (Remember that k_i is a null vector, so that $g_{im}k^i k^m = 0$.) Thus the normal tendency of matter is to focus light rays by gravity.

The *singularity* theorems of Penrose and Hawking (see Hawking and Ellis 1973) use this basic feature to state conditions that inevitably lead to spacetime singularity. The condition of the positivity of the T_{ik} term in equation (3.19) above plays a crucial role in general. We will not go into these details except to highlight it as a field deserving further research. In particular, the positive energy condition suggests that there may be non-singular spacetimes if it is violated and there are negative energy fields. We will now describe a line of thinking where such fields are used to avoid the initial (or *any*) singularity.

The Raychaudhuri equations for geodesic congruences has been around for more than half a century. In 1995, it was shown by Capovilla and Guven, that a generalization of these equations for families of extremal timelike membranes (D dimensional surfaces in a N dimensional background) is possible. Several illustrative examples and generalizations were worked out by Sayan Kar (1995, 1996a, 1996b). Later he also proved a focusing theorem for timelike surface congruences (Kar 1997).

Varun Sahni has also been interested in the singularity problem in higher dimensions. Recently Sahni and his colleagues have demonstrated that the braneworld cosmology, in which our 3+1 dimensional universe is a 'brane' embedded in a five dimensional 'bulk', has many important consequences (Sahni and Shtanov 2002). For instance, an expanding universe can encounter a new class of *quiescent* singularities at which the Hubble parameter and the density and pressure of matter remain finite, while the deceleration parameter and the Kretchmann invariant $R_{iklm}R^{iklm}$ diverge (Shtanov and Sahni 2002).

Another approach to singularities is through a generalization of the Hoyle-Narlikar concept of the C-field which has negative energy and stresses (see Hoyle and Narlikar 1962 for the first paper on this topic). In today's terminology such a scalar field is called a *phantom field*. M. Sami (Sami and Toporensky 2004) has worked in this area in relation to cosmology.

3.6 Naked Singularities

While the Penrose-Hawking theorems suggest the ubiquitousness of spacetime singularities, the question arises as to whether the singularity can be seen by an external observer in the case of a collapsing massive object. While the 'cosmic censorship hypothesis' suggests that the singularity will always be hidden by a black hole type horizon, some recent work by P.S. Joshi and his collaborators has turned up significant counter-examples.

A key result that emerges from these investigations is that visible ultra-dense regions or naked singularities arise naturally and generically as the final outcome of gravitational collapse in Einstein's gravity, in a wide variety of physically realistic situations. While it predicts their existence, the GR may no longer hold in these very late stages of collapse, and quantum gravity should take over to resolve the classical spacetime singularity. The quantum effects from a visible extreme gravity region could then propagate to outside observers to provide a possible laboratory to understand Planck scale physics and quantum gravity.

Joshi and Dwivedi (1993) carried out detailed analysis of naked singularities in dust collapse models to gain insights into the issue. Later this work was extended to models with pressure and also to anisotropic collapse. The question is of basic significance to black hole physics since the deductions like the second law of black hole physics are based on the validity of the cosmic censorship hypothesis. Details of such approaches may be found in Joshi's recent book (Joshi 2007). The implication of this type of work for the generally accepted behaviour of collapsing bodies is immense.

A. Banerji and collaborators (2003) studied the Tolman-Bondi type gravitational collapse in the spacetime with more than five dimensions and found that close to the centre in a marginally bound case naked singularity never appears. In another case Banerji et al. (2002) discussed a fluid sphere with outgoing heat flux and showed that trapping surface can be avoided till the singularity is attained at the end of the collapse. The non-occurrence of the horizon may be due to the mass-energy loss being balanced by the fall of the boundary radius.

3.7 Black Hole Astrophysics

The popularity of black holes amongst astrophysicists since the 1970s is reflected in the work done by Indian astrophysicists. The early work by Vishveshwara in 1970 on the quasi-normal modes (QNM) of oscillations of black holes is quite well cited. They offer a method of observing black holes directly. An important question that arises is in regard to the sensitivity of QNM to perturbing influences. This has been examined by Aguirregabiria and Vishveshwara (1996) using different equivalent potentials expected to be generated by the perturbations.

A phenomenal amount of research has been carried out on isolated black holes. Gravitational fields of such black holes are asymptotically flat and time independent. However, one cannot rule out black holes that are surrounded by matter distributions or imbedded in the cosmological background. Under these circumstances, the conditions of time symmetry and asymptotic flatness would have to be relaxed. A systematic study of such black holes shows how the properties of isolated black holes are significantly modified (Nayak et al. 2000; Ramachandra and Vishveshwara 2002, 2003; Ramachandra et al. 2003).

In the context of accretion dynamics A.R. Prasanna studied the stability of the accretion disc around compact objects with or without associated electromagnetic fields. In this context Abramowicz and Prasanna (1990) investigated the inertial forces very close to a black hole and found that there is the possibility of reversal of the centrifugal force in this region. This could affect the overall viscous drag and inwards advection during accretion. This in turn could influence the fluid flow towards the last stable circular orbit around the compact object.

N.K. Dadhich and collaborators revisited the Penrose Process (PP) of extracting energy from a black hole. The original PP had the unrealistic demand that the fragment of the original piece of matter falling into a negative energy orbit had to be accelerated to half the speed of light. In the absence of any mechanism for doing so, the PP became rather esoteric. Dadhich showed that if there is (electro)magnetic field around inside the black hole ergosphere, that could do the job of sending the piece into the required negative energy orbit, without having to accelerate it unrealistically. This modified PP is known as the 'Magnetic Penrose Process' and it has been described in an article by Dadhich (1989).

Jishnu and Mira Dey and colleagues (see Bagchi et al. 2008) have provided a rather exotic explanation of the afterglow seen after a gamma ray burst (GRB). They invoke a strange star which collapses into a black hole depending on its mass and temperature. A black hole sucks in matter and in the process creates an outward jet causing GRB. The ejected material in the GRB gives rise to afterglows that bear the signature of the GRB.

In a significant work, A. Banerji and S.B. Duttachoudhury (1989) studied the boundary conditions at the surface of a charged viscous fluid sphere with outgoing heat and radiation flux. Thus the exterior solution is given by the Reissner-Nordstrom-Vaidya metric and the Israel-Darmois boundary conditions are used. In Romesh Kaul's work the Chern-Simons theory finds application in the study of black holes in quantum geometric formulation of gravity. In fact, $SU(2)$ Chern-Simons theory represents the boundary degrees of freedom of a non-rotating black hole in four dimensions. An exact formula for the dimensionality of the Hilbert space of the boundary states of $SU(2)$ Chern-Simons theory has been derived. This provides a way of calculating entropy of a four dimensional Schwarzschild black hole. While the formula obtained gives an *exact* quantum entropy for any size of the area, in the limit of large areas, the result does indeed agree with the semi-classical Bekenstein-Hawking entropy proportional to the horizon area A. The next order term for large areas is lnA with a definite coefficient, $-3/2$ (Kaul and Majumdar 2000).

After this pioneering result was obtained, some other black holes have been shown to posses this log (horizon area) correction to their entropy with the same coefficient $-3/2$. This includes those in string theories

(Kaul 2003). Like the Bekenstein-Hawking formula, this correction to the black hole entropy appears to be universal as argued by Carlip (2000).

This work is typical of the interest amongst particle physicists in the notion of a black hole viewed in the framework of quantum theory. Although talking about black holes, the ideas developed following Stephen Hawking's pioneering work, have little common ground with the black holes discussed by astrophysicists. As our interest in this chapter is limited to relativistic AP, we will have to skip the very productive work in quantum fields in curved spacetime, quantum gravity, *per se*, string theory, etc.

3.8 White Holes

The simplest way of describing a white hole is as a time reversed version of a black hole. Thus instead of gravitational collapse of a massive object ending in spacetime singularity, one is talking here of an object emerging from a spacetime singularity with high outward velocities. In a sense the Big Bang universe is the supreme example of a white hole. How will a white hole appear to an external observer?

In the mid-1970s J.V. Narlikar and his collaborators (see Narlikar et al. (1974); Narlikar and Apparao (1975)) showed that initially there will be high blueshift of light emitted by the white hole and that as the expansion proceeds the magnitude of the blueshift will decline and ultimately the shift will be towards the red end. The solution of the white hole emission problem is exact if the white hole is made of dust (i.e. it is pressure-free).

Somewhat similar work was done by A. Banerji (1966) showing that the frequency shift observed by an external observer is a mixture of Doppler shift and gravitational redshift.

D.M. Eardley (1974) had argued in a short paper that a white hole is unstable and gets converted into a black hole because of strong accretion near the horizon as it tries to emerge from it. T.K. Dey and S. Banerji (1991, 1993) have shown, however, that a white hole may or may not convert to a black hole as it encounters a collapsing spherical dust shell, depending on the initial conditions.

3.9 Gravitational Lenses

Although 1979 saw considerable publicity being given to gravitational lensing in quasar astronomy, there had been earlier workers in the field also. Thus one may start with Fritz Zwicky (1937), Refsdal (1964) and Barnothy (1965). A direct application of gravitational lensing to quasar observations was suggested first by J.V. Narlikar and S.M. Chitre (1978) who argued that the apparent superluminal separation of VLBI sources in a quasar may have been magnified. Thus a subluminal separation may be seen as superluminal because of lensing by an intermediate galaxy.

After 1979 and the realization that the so-called twin quasars may be the two lensed images of a single source, there were several attempts at modelling the suspected lensed cases. In this field Indians have played a lead role and the main players in this field have been S.M. Chitre, K. Subrahmanian, D. Narasimha and S. Nair. General relativistic bending of light is invoked in constructing these models.

In Einstein's general theory of relativity, light rays can get bent by the gravitational influence of mass distributions. This effect called gravitational lensing can lead to several interesting phenomena. One of the most interesting consequences of gravitational lensing is the multiple imaging of a distant quasar by an intervening galaxy. The first case of such multiple imaging was discovered in 1979 by Walsh, Carswell and Weymann. Optical observations of two quasars 0957+561 A and B, separated by about 6 arc seconds, showed almost identical spectra. This was surmised to be one quasar whose light has been bent by an intervening galaxy such that it arrives along two different paths. This discovery of multiply imaged quasars led to an explosion of interest in gravitational lensing. Subsequent to the discovery of the twin quasar, detailed lens models for a large number of observed cases of multiple imaging were constructed by Indian astrophysicists (Narasimha, Subramanian and Chitre 1982, 1984a, 1984b; Subrahmanyan et al. 1990; Nair, Narasimha and Rao 1993; Nair and Garrett 1997; Nair 1998). The gravitational lens models probed the general properties of the intervening galaxies and clusters. In the process the Indian group developed one of the first numerical codes to extensively model cases of such strong lensing.

Stars in the lensing galaxy can also further amplify the light from the background quasar. Since the stars move in the galactic potential, this amplification can vary on timescale of a few years. This phenomena called microlensing was also extensively studied (Narasimha, Subramanian and Chitre 1984a; Nityananda and Ostriker 1984; Subramanian, Narasimha and Chitre 1985), and even used to probe the central black holes in lens galaxies (Narasimha, Subramanian and Chitre 1986). If the source which is being microlensed is moving relativistically then fast variations can result even in radio wavelengths (Gopal-Krishna and Subramanian 1991).

By the end of the 1980s a new phenomenon was discovered, the lensing of extended sources like galaxies, by intervening massive galaxy clusters, to produce giant arcs (Lynds and Petrosian 1986, 1989). These were also modeled by Narasimha and Chitre (1988, 1993) and used to probe the matter distribution in clusters. Gravitational lensing has proved to be one of the most important ways in which to probe the dark matter distribution of galaxy clusters.

A very interesting case of multiple imaging was also discovered by Indian astronomers (Subrahmanyan and Rao 1988), first through radio observations. This lens (PKS 1830-211) called by some as the "Ooty lens" was modeled extensively by Subrahmanyan et al. (1990) and Nair, Narasimha and Rao (1993), was later confirmed to be an example of an Einstein ring.

Another important advance, made by Nityananda (1990a, 1990b) and Blandford and Narayan (1986), was to formulate the gravitational lensing equations as an application of Fermat's principle. General theorems on the conditions for multiple imaging by a smooth, bounded gravitational lens were also proved by Subramanian and Cowling (1986). Padmanabhan and Subramanian (1988), extended these considerations to cosmologically extended thick gravitational lenses.

3.10 Gravitational Waves

In the last decade or so Indians have begun making their presence felt amongst the small international community of workers in the field of gravitational waves (GWs). On the instrumentation side, the detectors need to meet very demanding criteria for dealing with low signal to noise ratio. On the software side frontier level work is going on in the area of data analysis. S.V. Dhurandhar has played a lead role in this field.

The detectors are optimally looking at cosmic sources in the form of inspiralling compact binaries; for inspiralling compact (neutron stars, black holes) binaries are considered to be the most promising sources for ground-based detectors. In two papers, the foundation of a computationally optimal scheme in searching for inspiralling waveforms was presented. Further, efficient hierarchical methods in searching for inspirals were developed which reduces the computational cost several times has become the standard reference for designing numerical codes. Another important contribution is coherently extracting inspiralling binary signals from a network of detectors. This work enabled Inter-University Center for Astronomy and Astrophysics (IUCAA) to join the LIGO Science Collaboration (LSC) which is the most important worldwide collaboration and will run the US-based LIGO detectors. The coherent search takes into account the phase information into building up the network statistic and therefore is superior in performance to the coincidence search where each detector is treated in isolation. A quantitative comparison of the two strategies is being performed currently with the Japanese.

Recent work comprises the radiometric search for stochastic GWs and also for periodic sources such as pulsars or rotating neutron stars. This work is within the LSC collaboration with Caltech and Albert Einstein Institute, Potsdam, Germany. The radiometric search has the advantage that the search is possible in the sensitive frequency band of GW detectors. This is a directed search involving a network of detectors.

In LISA (Laser Interferometric Space Antenna) data analysis, an important problem is cancellation of laser frequency noise which arises because of the impossibility of LISA to maintain equal armlengths. The cancellation is achieved with a scheme known as time-delay interferometry (TDI). A rigorous mathematical foundation of this scheme was laid out with the use of Gröbner basis methods and modules over polynomial rings. The TDI combinations form the first module of syzygies well known in commutative algebra and algebraic geometry. In the case of stationary LISA, its generators were obtained so that all TDI combinations could be available in principle. Work is in progress when the armlengths change with time. This leads to non-commutative operators. One way is to optimise the spacecraft orbits so that the rate of change of armlengths is acceptably small and the other approach is to generalise the previous results to the non-commutative case. This work is in collaboration with the French.

As mentioned above, inspiralling compact binaries comprising neutron stars and black holes are the most promising sources for ground-based laser interferometric GW detectors like LIGO and Virgo and space-based detectors like LISA. Data analysis issues must therefore be supplemented by theoretical analysis of these sources. Their detection and parameter estimation employs matched filtering and this requires the construction of effectual, faithful and efficient template banks. The theoretical input for building template banks are high accuracy phasing formulas for the evolution of the GW phase under gravitational radiation reaction far beyond the leading radiation reaction at 2.5 post-Newtonian (PN) order used in the timing analysis of the binary pulsar. Starting with the 2PN generation of GW (providing the 4.5PN terms in the equations of motion and forming the starting point for all GW data analysis in LIGO and Virgo), over a decade the multipolar post-Minkowskian formalism matched to a PN source has been systematically extended to 3.5PN order. For many years the 3PN results (Blanchet et al. 2002, Blanchet and Iyer 2005) remained partial due to the presense of undetermined parameters arising from the incompleteness of the Hadamard regularization used to deal with the divergent self-field effects. Recently, by the use of a more powerful dimensional regularization these 3PN parameters have been determined (Blanchet et al. 2004) completing the GW phasing to 3.5PN. Further, the full gravitational waveform generated by inspiralling compact binaries moving in quasi-circular orbits is computed at the 2.5PN and 3PN approximation (Blanchet et al. 2008). This 3PN amplitude accurate and 3.5PN phase accurate phase waveform will form the basis for searching and deciphering the GW signals in the current and future network of GW detectors and also for calibrating and interpreting the recent exciting results of numerically generated waveforms for the merger and ringdown of binary black holes (Blanchet et al. 2004). More generally, for quasi-elliptic orbits the formalism can describe both their secular evolution under 3PN gravitational radiation reaction (Arun et al. 2008) and the smaller but fast orbital scale periodic terms (Damour et al. 2004). Resummation techniques like Pade approximants and effective one body methods have been used to extend the validity of standard PN approximants (Damour et al. 1998) and template families more critically classified for comparison (Damour et al. 2001).

The implications of the full waveform that brings into play harmonics beyond the dominant have been explored for LISA. By inclusion of higher harmonics in matched filters more massive systems that were previously thought to be *not* visible in LISA are detectable with reasonable SNRs. Moreover, the angular resolution of LISA increases by more than a factor of 10 thereby making it possible for LISA to identify the host galaxy/galaxy cluster. Thus, LISA's observation of certain binary supermassive black hole (SMBH) coalescence events could constrain the dark energy equation of state to within a few percent, comparable to the level expected from other dark energy missions (Arun et al. 2007). Finally, LISA provides a unique opportunity to probe the non-linear structure of PN theory both in the context of GR and its alternatives (Arun et al. 2006).

3.11 The Present GR-AP Infrastructure in India

I have briefly described the range of subjects covered in India in the field of GR and AP. Additionally work is being done in such frontier areas like quantum gravity via loops (G. Date) as well as via strings, alternative models of the universe (J.V. Narlikar), quantum fields in curved spacetime (Padmanabhan, Sriramkumar), multi-dimentional cosmologies (M. Sami, V. Sahni), and the mathematical/geometrical aspects of GR and other gravity theories (Pankaj Joshi). I have mentioned the names of a few leading workers in the parentheses after each topic. It will take me well beyond the scope of this chapter to touch those areas.

It is worth recording here that Indians have been regularly participating in the international essay competition conducted by Gravity Research Foundation of USA. Several honourable mentions have been won as well as prizes. T. Padmanabhan has received prizes four times in this contest during the last 7 years including the first prize in 2008. This is the first time work done in India (Padmanabhan 2008) has received the top award from Gravity Research Foundation.

I end this chapter brief review of where in India important centres of GR-AP exist. The Tata Institute of Fundamental Research, Mumbai has scientists in this field working largely in Astronomy Department. Another group exists in the Raman Research Institute, Bengaluru and the Indian Institute of Astrophysics, Bengaluru. The former of the two is more theoretical while the latter is inclined towards AP. Then we have the Physical Research Laboratory, Ahmedabad and the Institute of Physics, Bhuvaneswar with a few workers, again inclined towards AP in the former and GR in the latter. More mathematically oriented work is being done in the Institute of Mathematical Sciences, Chennai and the Harishchandra Research Institute in Allahabad.

These places, as will be noticed, are all *outside* the University Sector, in contrast to the early days when work was done mainly in the universities like Calcutta and Banaras. The flagship for the University Sector in this repect is the IUCAA in Pune which has strong research base in both GR and AP. Smaller groups do exist in the universities, for example, in Kolkata there are the Calcutta University and Jadavpur University both of which have research in GR-AP going on. A new Centre for Theoretical Physics has been opened at the Jamia Millia Islamia in Delhi with GR as the thrust area. Thanks to IUCAA the pedagogical activities in GR-AP have increased considerably, ranging from introductory summer schools to advanced specialist workshops.

Two national organizations look after the interests of GR and AP. The Indian Association of General Relativity and Gravitation (IAGRG) was founded in 1969, whereas the Astronomical Society of India started in 1973. Both have many common members and each society meets once in about 18 months. Besides, several international meets have been successfully held in India, including the General Assembly of the IAU in Delhi in 1985, the International Conference on Gravitation and Cosmology in Goa in 1987, the Asia Pacific Regional Meeting of the IAU in 1993, the GR-15, of the International Society of General Relativity and Gravitation in 1997. The last two were hosted by IUCAA.

Given these data, the situation looks brighter for the future, except for the caveat: Can we continue to attract young talent to GR-AP? India's participation in major international facilities like GW detectors, ultra large telescopes, large particle accelerators, etc. will help towards this goal.

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