Investigation of Optimum Phase Sequence for Reduction of PAPR Using SLM in OFDM System

Srinu Pyla, K. Padma Raju and N. Balasubrahmanyam

Abstract Orthogonal Frequency Division Multiplexing (OFDM) is able to mitigate the detrimental effects of multipath fading but, OFDM signal suffers from high peak to average power ratio (PAPR). Increase in PAPR, decrease the power amplifier efficiency otherwise leads to in-band distortion and out of band radiation due to signal clipping and spectral broadening respectively. There are many techniques to reduce PAPR and Selective Level Mapping (SLM) is a particularly promising technique. In this technique, array multiplication of data sequence with phase sequence reduces PAPR. The combination with the minimum PAPR is considered for transmission. There are many phase sequences for reduction of PAPR such as Riemann, Rudin Shapiro, Chaotic, Chu, Pseudorandom, Hadamard and Novel phase sequences. The selection of phase sequence is very crucial and it depends on PAPR improvement and simplicity in the recovery of signal. In this paper PAPR improvement for various phase sequences and digital modulation schemes is compared and a new phase sequence is developed.

Keywords OFDM · PAPR · Selective level mapping · Chaotic sequence · Riemann sequence · Chu sequence · Pseudorandom sequence · Rudin–Shapiro sequence · Modified sequence

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a technique in which high rate serial data is converted into low rate parallel data and increases symbol duration. International standards using OFDM in wireless LAN, WiMAX, Mobile broadband

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wireless access (MBWA) and Broadcasting Radio Access Network (BRAN) committees [1, 2]. OFDM provides multipath fading and impulse noise free transmission and minimizes complexity of equalizers (single tap equalizer is sufficient). Hardware complexity of OFDM implementation is greatly reduced by using Fast Fourier Transform (FFT). OFDM suffers from Inter carrier interference (ICI) due to frequency offsets and increase in Peak to Average Power Ratio (PAPR) due to coherent addition of subcarriers [1, 3]. Increase in PAPR causes in band distortion and out of band radiation due to clipping and spectral broadening respectively [4].

There are many techniques to reduce PAPR such as amplitude clipping, recursive clipping and filtering (RCF), coding, tone reservation (TR), tone injection (TI), active constellation extension (ACE), Partial Transmit Sequence (PTS), Selective Level Mapping (SLM) and interleaving. SLM and PTS reduce PAPR significantly without loss of any information [1–3], but in PTS latency increases with number of sub blocks. In this paper, SLM is considered for reduction of PAPR using various phase sequences. The PAPR improvement changes with phase sequences. Hence the selection of phase sequence is very important for better improvement of PAPR. This paper investigated for optimum phase sequences for better PAPR performance improvement. Firstly, the improvement of PAPR using SLM technique and pseudorandom phase sequence is carried out. Later the phase sequences namely Riemann, Hadamard, Chaotic, modified Chu, Rudin Shapiro, Novel and Modified phase sequences are replaced and the results are compared. Riemann, chaotic, modified Chu, Hadamard and Novel phase sequences are well described in [5–10] respectively.

2 PAPR Problem in OFDM System

OFDM System

Let total information bits (M) are grouped into N symbols (X_k, k = 0, 1, ….N − 1) and each symbol is modulated on one of a set of orthogonal subcarriers The total bandwidth of system W is divided into N sub bands, then the individual subcarrier bandwidth is W/N and it is less than the coherence bandwidth B_c. Hence the frequency selective fading channel is converted into flat fading channel (W/N < B_c), and OFDM symbol duration is increased by N times. OFDM signal can be defined as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{K=0}^{N-1} X_k e^{j2\pi f_k t}, \quad 0 \le t \le NT \tag{1}$$

where $j = \sqrt{-1}$. As input data streams are orthogonal, the real and imaginary parts of x(t) are uncorrelated and according to the central limit theorem, the distribution of these parts approach Gaussian distribution for large N with zero mean and variance $\sigma^2 = E\left[|Re\{x(t)\}|^2 + |Im\{x(t)\}|^2\right]/2$, where E[x] is the average value of x. the Probability Density Function (PDF) of OFDM signal is

$$P_r\{x(t)\} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{[x(t)]^2}{2\sigma^2}} \quad (2)$$

Original OFDM signal has Rayleigh nature, uniform phase and its PDF is

$$P_r(r) = 2re^{-r^2} \quad (3)$$

where 'r' indicates OFDM signal level.

PAPR Problem

In OFDM, the peak power increases with number of subcarriers and much larger compare to average power due to coherent addition of subcarriers. This high PAPR moves signal into non-linear region of amplifier and degrades its efficiency. The PAPR of OFDM signals x(t) is

$$PAPR[x(t)] = \frac{P_{peak}}{P_{average}} = 10\ \log_{10} \frac{\max[|X(n)|^2]}{E[|X_n|^2]} \quad (4)$$

where E[] denotes expected value. If this ratio is beyond threshold level, signal distortion due to clipping and radiation due to spectral broadening will occur. For large N, Rayleigh distributed OFDM signal peak will increase with non zero probability and its probability exceeds a threshold

$$P_0 = \frac{\sigma_0^2}{\sigma_n^2}. \quad (5)$$

$$P(PAPR \geq P_0) = 1 - (1 - e^{-P_0})^N \quad (6)$$

Relation Between PAPR and Subcarriers

Let there are N Gaussian independent and identically distributed random Variables $x_n; 0 \leq n \leq N-1$ with zero mean and unit power. Average power $E_n = (x[n])^2$ is

$$E\left[\{\frac{1}{\sqrt{N}}(|X_0 + X_1 + X_2 + \cdots + X_{N-1}|)\}^2\right] \quad (7)$$

$$\frac{1}{N} E(|X_0 + X_1 + X_2 + \cdots + X_{N-1}|)^2 = \frac{E|x_0|^2}{N} + \frac{E|x_1|^2}{N} + \cdots + \frac{E|x_{N-1}|^2}{N} = 1$$

Value increases with number of coherent subcarrier

$$\max\left[\frac{1}{\sqrt{N}}(|X_0 + X_1 + X_2 + \cdots + X_{N-1}|)\right]^2 = \left[\left|\frac{N}{\sqrt{N}}\right|\right]^2 = \left|\sqrt{N}\right|^2 = N$$

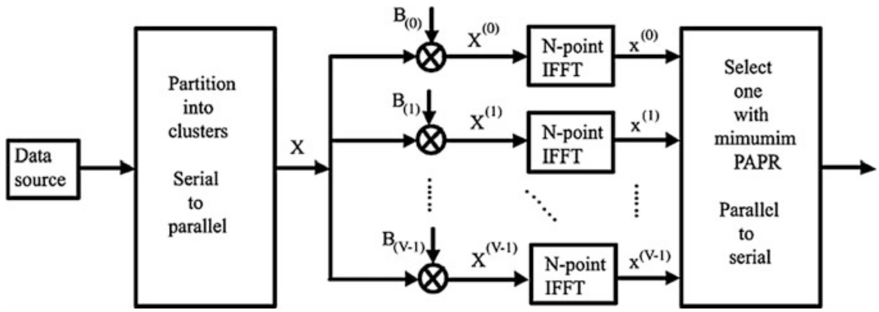

Fig. 1 Block diagram of OFDM transmitter with SLM

Therefore maximum PAPR of OFDM with N subcarriers is N.

Selective Level Mapping

In SLM, multiple data blocks of same information is generated and array multiplied with various phase vectors as shown in the Fig. 1. The sequence with minimum PAPR is considered for transmission. Every data block is array multiplied by V different phase sequences, each of length N, $V(m) = [v_{m,0}, v_{m,1}, \ldots, v_{m,N-1}]^T$, m = 1, 2, …, M, resulting in M new (modified) data blocks.

The new block for the mth sequence is

$$X(m) = [X_{0V_{m,1}}, X_{1V_{m,2}}, \ldots X_{N-1v_{m,N-I}}]^T \tag{8}$$

where m = 1, 2, …, M.

Among new data blocks X (m), m = 1, 2… M, the one with the minimum PAPR is considered for transmission. The amount of PAPR performance improvement depends on number of duplicate candidates (M) and type of phase sequence. The reduction of PAPR is proportional to M, but increase in M increases overhead information in terms of index length ($\log_2 M$) as side information. Hence M should not be large. Therefore selection of phase sequence play vital role in improving PAPR performance.

3 PAPR Mitigating Phase Sequences

Riemann Sequence

Riemann matrix (**R**) can be defined as

$$R(p,q) = \begin{matrix} \mathrm{p} -1 & \text{if p divides q} \\ -1 & \text{otherwise} \end{matrix} \tag{9}$$

Here first row and column of the Riemann matrix have to remove to obtain required matrix.

Chaotic Sequence

In this approach concentric circle constellation (CCC) based mapping is considered to obtain the data symbols. This sequence exhibits random behaviour. The M-ary chaotic sequence C_n €{0, 1, 2,…M − 1}, $0 \le n \le N - 1$ and C_n is given by

$$\begin{aligned} C_n &= \left[\frac{M_{y_{n+1}}}{2}\right] + \frac{M}{2}, \quad y_{n+1} = f(y_n) = 1 - \alpha y_n^2, \\ \alpha &\in [1.4015, 1.99], \quad y_n \in (-1, 1) \end{aligned} \tag{10}$$

C_r, the *r*th element of first phase sequence becomes $P_r^1 = \exp(j2\pi C_r/M)$.

Rudin Shapiro Sequence

Every element of this sequence is either +1 or −1. The *n*th term of the sequence, b_n, is

$$a_n = \sum \varepsilon_i \varepsilon_{i+1} \quad \text{and} \quad b_n = (-1)^{a_n} \tag{11}$$

where ε_i indicates the digits of *n* (binary form). Therefore a_n gives the number of occurrences of the sub-string 11 in the binary expansion of *n*.

$b_n = +1$ for a_n is even and $b_n = -1$ for a_n is odd. This sequence can be generated by

$$b_{2n} = b_n \quad \text{and} \quad b_{2n+1} = (-1)^n b_n \tag{12}$$

Chu Sequence

In this scheme, the phase sequence is defined as $B^{(u)} = \left[B_0^{(u)}, B_1^{(u)}, \ldots\ldots\ldots B_{N-1}^{(u)}\right]$. The $b_k^{(u)}$ is the kth element of a phase sequence. It is defined as

$$b_k^{(u)} = \begin{cases} \exp\left(i\frac{2\pi}{N}\left[\frac{uk^2}{2}\right]\right), & N \text{ even} \\ \exp\left(i\frac{2\pi}{N}\left[\frac{uk(k+1)}{N}\right]\right), & N \text{ odd} \end{cases} \tag{13}$$

Pseudo Random Sequence

This sequence can be generated randomly in terms of exponential sequence. The phase sequence is defined as

$$p(u) = \exp\left(\frac{\pi i}{n}\right) \tag{14}$$

where n is a random angle value to generate the exponential phase sequence.

Modified Sequence

In the proposed approach, the matrix elements obtained by the multiplication of the normalised matrices is considered as phase rotation sequence. The first matrix is defined as

$$\begin{aligned} \mathrm{A(p,\ q)} &= 0 \ \text{ if } \ \mathrm{k} = 0 \\ &= -1 \ \text{ if } \ \mathrm{k} < 0 \\ &= 1 \ \text{ if } \ \mathrm{k} > 0 \text{ where } \mathrm{k} = \mathrm{p} - \mathrm{q}. \end{aligned}$$

The second matrix is defined as

$$\begin{aligned} \mathrm{B(p,\ q)} &= 1 \ \text{ if } \ \mathrm{p} \text{ divides } \mathrm{q} \\ &= -1 \text{ else} \end{aligned}$$

The final matrix (phase elements) is obtained by multiplying the above matrices.

$$C(i,j) = A(i,j) * B(i,j) \tag{15}$$

4 Digital Modulation Schemes

Digital modulation schemes are of binary and M-ary. If the bandwidth of the channel is not sufficient for desired data transmission, M-ary digital modulation schemes are preferred. In general OFDM systems are designed for high data rate applications, so M-ary signaling schemes especially M-ary PSK and QAM are used. Detailed description about M-ary PSK and QAM performance are given in [11].

5 Results and Conclusions

The probability of the OFDM Signal PAPR exceeds a threshold value γ is

$$P_r\{PAPR[x(n)] > \gamma\} = 1 - (1 - e^{-\gamma})^N$$

Here the PAPR improvement is verified by considering 4096 bits for various phase sequences. From the results it is observed that PAPR reduction improvement is better for Riemann sequence and also it is observed that improvement increases with order of modulation. Increase in order improves the spectral efficiency. Modified phase sequence performance is better over other sequences at higher order modulation schemes (Figs. 2, 3, 4, 5, 6, 7 and 8).

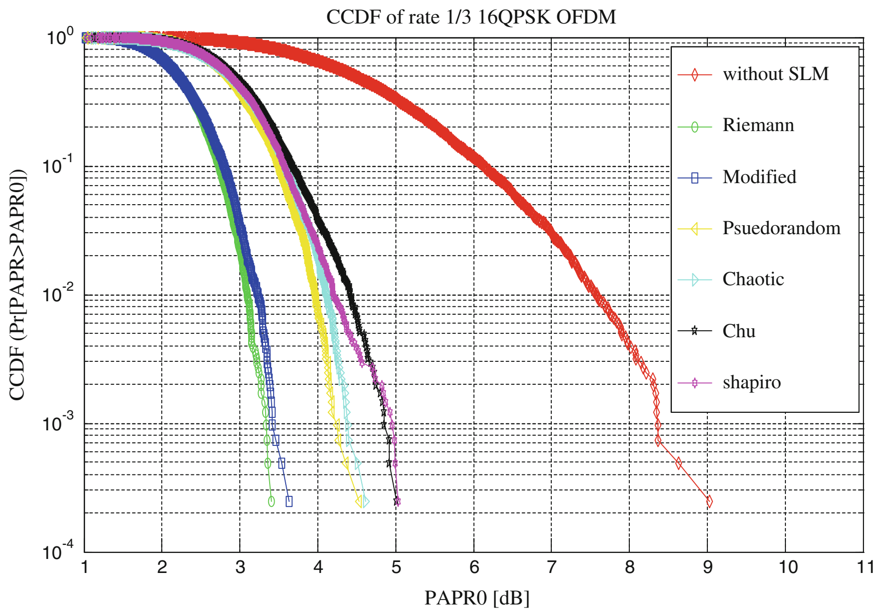

Fig. 2 Comparision of PAPR reduction for various phase sequences with 16PSK

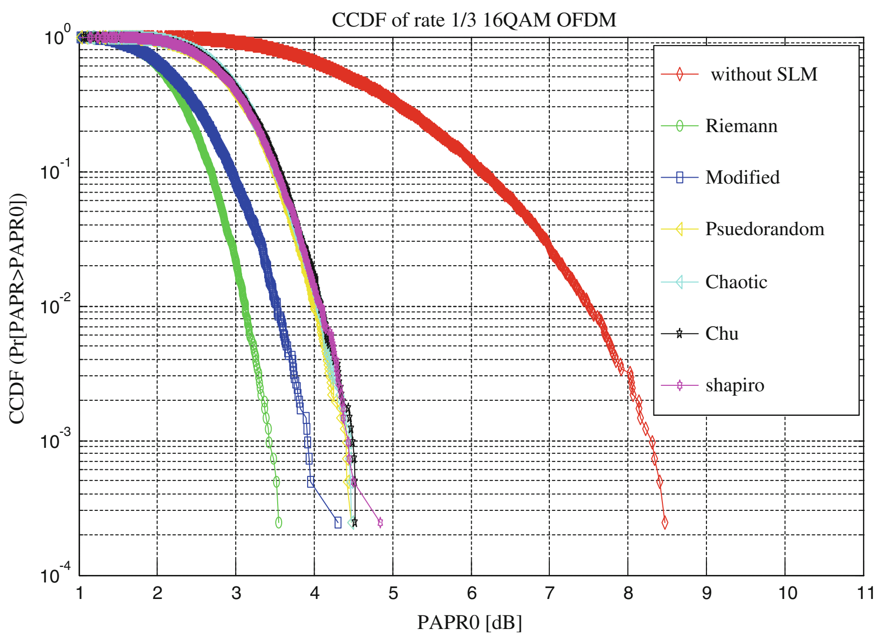

Fig. 3 Comparision of PAPR reduction for various phase sequences with 16QAM

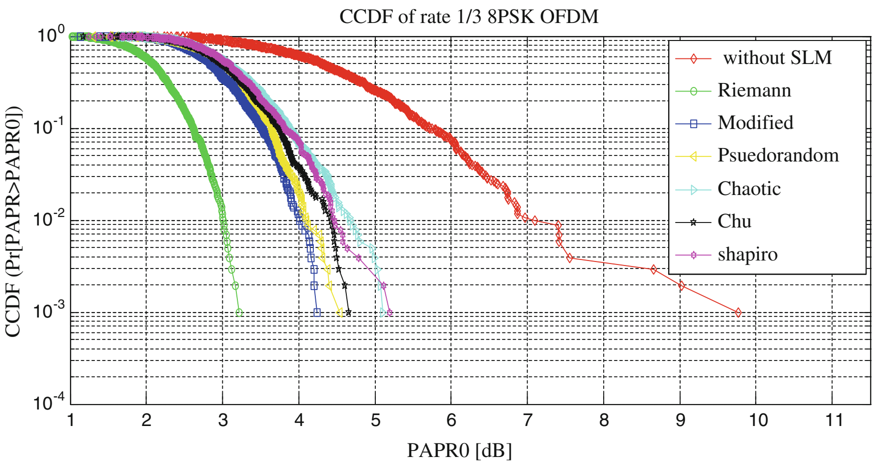

Fig. 4 Comparision of PAPR reduction for various phase sequences with 8PSK

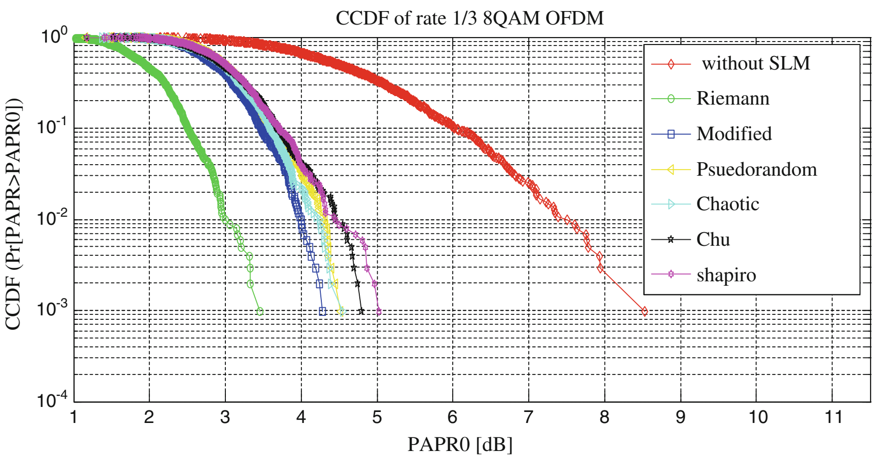

Fig. 5 Comparision of PAPR reduction for various phase sequences with 8QAM

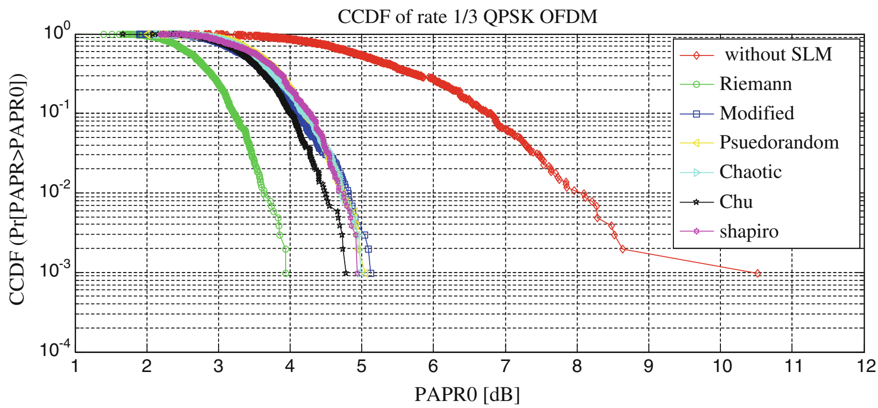

Fig. 6 Comparision of PAPR reduction for various phase sequences with QPSK

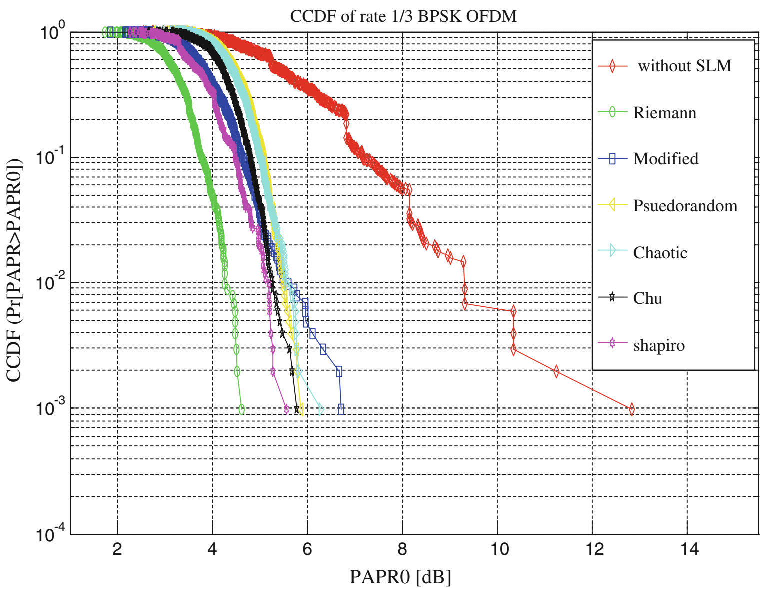

Fig. 7 Comparision of PAPR reduction for various phase sequences with BPSK

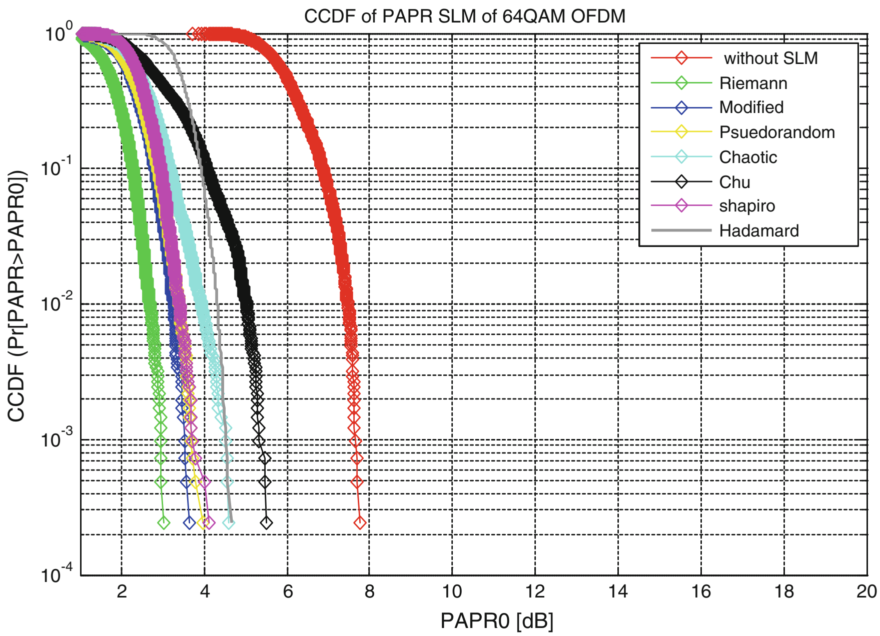

Fig. 8 Comparision of PAPR reduction for various phase sequences with 64QAM

6 Conclusions

In this paper, the main problem of OFDM system, high PAPR and its significance is discussed. SLM scheme is implemented for reduction of PAPR and the reduction performance for various PAPR minimization phase sequences is compared. For higher order modulation schemes, the PAPR performance improvement is better. The PAPR reduction improvement using Modified phase sequence is far better for higher order modulation schemes compare to other phase sequences and competent with Riemann phase sequence. This sequence is less complex and easier for implementation and transmission of side information along with desired data is not required. Hence it is more bandwidth efficient.

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