

Advanced Parallel Structure Kalman Filter for Radar Applications

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Abstract Normally in tracking applications, the target motion is usually modeled in Cartesian coordinates but, most sensors measure target parameters in polar coordinates. In this paper two contributions are considered in target tracking. One depends on position measurements and another one is on Doppler measurements. The position measurements are measured by taking the range and bearing (angle) of the target depending on the sensor location. Tracking the target Cartesian coordinates by using this range and bearing measurements is a nonlinear state estimation problem. To calculate the position measurements (range and angle), it is preferred to convert them to Cartesian coordinates by considering the linear form values. This is done, to avoid using nonlinear filters. This method is called as converted position measurement Kalman filter (CPMKF). In this paper another contribution is Doppler (range rate) measurement in target tracking systems. In this contribution the nonlinear pseudo states are calculated. This method is called as Converted Doppler measurement Kalman filter (CDMKF). By considering these two methods a parallel filtering structure, called statically fused converted measurement Kalman filter (SF-CMKF) is proposed. The two methods are operated along with each other to construct the new state estimator SF-CMKF by a static estimator to obtain final state estimates.

Keywords Tracking · Kalman filter · Parallel filtering · Doppler measurement

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1 Introduction

In much Doppler type of radars, the measurements are considered in the form of polar values, which gives the measurements like range, range rate (Doppler), and one or two angles of its position during moving. Then the Cartesian components errors in the converted measurements are correlated with each other is explored in [4, 5, 9, 11, 13, 14]. The other one is done by using extended Kalman filter (EKF) presented in [3, 6, 8, 10, 12, 13]. In this approach we have to consider the measurements of the target state estimation in a nonlinear fashion, which results the mixed coordinate filter [7, 8]. These measured terms results are considered to compare with the first two moment approximations which are presented here. The new converted measurement Kalman filter (CMKF) [14], is having estimation errors, which are compatible with the calculated covariance of the measured terms. The EKF is different from this method, because it is consistent only for small errors. So that the CMKF is having the correct covariance, it processes all the target measurements with a gain, which is nearly optimal and gives smaller errors compared with the EKF [3]. In the moderately accurate sensors, the EKF performs very poorly in tracking the target at long range for RMS azimuth error of 1.5° or more [10]. But the CMKF [12] is consistent for 10° RMS azimuth error also.

In this paper to rectify these shortcomings, a new method is proposed. In the proposed method, the use of the nonlinear recursive filtering methods is avoided during the processing of Doppler measurements [6]. In the first one, a pseudo state vector is considered, in which the existing converted Doppler measurements of the target are linear functions and they are constructed. These pseudo state vectors consist of the converted Doppler measurements and its derivatives [7, 8]. The pseudo state equations are derived from the measurements and proven to be linear in two commonly used target motion models. One model is the constant velocity (CV) and the other one is constant acceleration (CA) models. By using these converted Doppler measurement Kalman filter (CDMKF), is proposed to estimate the pseudo states [7]. This is also used to filter the noise in the converted Doppler measurements Kalman filter. Finally, the CDMKF is combined with the CPMKF [13, 14] to construct a new filter which gives a new state estimator called as statically fused converted measurement Kalman filters (SF-CMKF).

2 Problem Description

2.1 System Formulation

In Cartesian coordinates target's parameters are considered by depending on the conversion measurements of the target from polar coordinates to Cartesian. It is modeled as

$$X(k+1) = \Phi(k)X(k) + \Gamma(k)V(k) \quad (1)$$

where $X(k) = [x(k), y(k), \dot{x}(k), \dot{y}(k)]^T$, here $X(k) \in R^n$ is the state vector consisting of target's position components and corresponding target's velocity components along x and y directions, respectively, at every time step k . If a moving target is considered, the state vector can be taken by other components such as acceleration. Here, $\Phi(k) \in R^{n \times n}$ is the target's state transition matrix, $v(k)$ is zero-mean Gaussian random process noise with covariance $Q(k)$, and $\Gamma(k)$ is noise gain matrix [2].

If we considered a 2D Doppler radar, which is assumed to report measurements of moving targets in polar coordinates, including range, range rate (Doppler) and angle presented in [3, 13, 14]. The measurement equation can be expressed as

$$\begin{aligned} z(k) &= [r_m(k), \theta_m(k), \dot{r}(k)]^T \\ &= h[X(k)] + w(k) = [r(k), \theta(k), \dot{r}(k)]^T + w(k) \end{aligned} \quad (2)$$

where

$$r(k) = \sqrt{x^2(k) + y^2(k)} \quad (3)$$

$$\theta(k) = \tan^{-1}[y(k)/x(k)] \quad (4)$$

$$\dot{r}(k) = [x(k)\dot{x}(k) + y(k)\dot{y}(k)] / \sqrt{x^2(k) + y^2(k)} \quad (5)$$

$$w(k) = [\tilde{r}(k), \tilde{\theta}(k), \tilde{\dot{r}}(k)] \quad (6)$$

Normally the measured range and bearing of the target [14] are considered by taking the true range r and bearing θ as

$$r_m = r + \tilde{r}, \quad \theta_m = \theta + \tilde{\theta} \quad (7)$$

2.2 Measurement Conversion Equations

The errors like range \tilde{r} and bearing $\tilde{\theta}$ are taken to get independent with zero mean and standard deviations presented in [1]. These polar measurements are converted into Cartesian coordinate measurements by using the following conversion techniques [14]

$$x_m = r_m \cos \theta_m; \quad y_m = r_m \sin \theta_m; \quad (8)$$

The errors can be found by expanding these terms

$$\begin{aligned}x_m &= x + \tilde{x} = (r + \tilde{r}) \cos(\theta + \tilde{\theta}); \\y_m &= y + \tilde{y} = (r + \tilde{r}) \sin(\theta + \tilde{\theta})\end{aligned}\quad (9)$$

The mean error of Cartesian positions becomes

$$\mu_t(r, \theta) = \begin{bmatrix} E[\tilde{x}|r, \theta] \\ E[\tilde{y}|r, \theta] \end{bmatrix} = \begin{bmatrix} r \cos \theta (e^{-\frac{\sigma_\theta^2}{2}} - 1) \\ r \sin \theta (e^{-\frac{\sigma_\theta^2}{2}} - 1) \end{bmatrix}\quad (10)$$

After some algebraic manipulation of the measurements, the elements of the targets converted measurement covariance [14] are given by

$$\begin{aligned}R_t^{11} &= \text{var}(\tilde{x}|r, \theta) = r^2 e^{-\sigma_\theta^2} [\cos^2 \theta (\cosh(\sigma_\theta^2) - 1) + \sin^2 \theta \sinh(\sigma_\theta^2)] \\ &\quad + \sigma_r^2 e^{-\sigma_\theta^2} [\cos^2 \theta \cosh \sigma_\theta^2 + \sin^2 \theta \sinh(\sigma_\theta^2)]\end{aligned}\quad (11a)$$

$$\begin{aligned}R_t^{22} &= \text{var}(\tilde{y}|r, \theta) = r^2 e^{-\sigma_\theta^2} [\sin^2 \theta (\cosh(\sigma_\theta^2) - 1) + \cos^2 \theta \sinh(\sigma_\theta^2)] \\ &\quad + \sigma_r^2 e^{-\sigma_\theta^2} [\sin^2 \theta \cosh \sigma_\theta^2 + \cos^2 \theta \sinh(\sigma_\theta^2)]\end{aligned}\quad (11b)$$

$$R_t^{12} = \text{var}(\tilde{x}, \tilde{y}|r, \theta) = \sin \theta \cos \theta e^{-2\sigma_\theta^2} [\sigma_r^2 + r^2 (1 - e^{\sigma_\theta^2})]\quad (11c)$$

Equations (10) and (11a, 11b, 11c) are the expressions of the bias and covariance of the targets converted measurements. The converted measurements of the target have a significant bias for long range and large bearing error. The true bias and covariance of the measured parameter values depend on the true range and bearing. They are denoted as with elements (10) and with elements (11a, 11b, 11c), respectively.

A conversion of the Doppler measurements is also made in this paper to yield the converted Doppler measurements as [7]:

$$\eta_c(k) = r_m(k) \dot{r}_m(k) = \eta(k) + \tilde{\eta}(k)\quad (12)$$

where $\eta(k)$ is the converted Doppler (i.e., the product of range and range rate), given by

$$\eta(k) = x(k) \dot{x}(k) + y(k) \dot{y}(k)\quad (13)$$

and $\tilde{\eta}(k)$ is the error in the converted Doppler Measurement $\eta_c(k)$, [10].

The use of the zero-mean expressions of measured one cannot be taken for the bias of the target at long ranges with the bearing error and the covariance approximation (13) is poor. The expressions in (10) and (11a, 11b, 11c) cannot be used because; the fact is that they are conditioned on the target's true values of

range and bearing [14]. But these are not available in practice. The results become useful, when the expected values of target true moments are evaluated with the measured position. The expected bias and covariance are examined as [5]:

$$E[\mu_t(r, \theta) | r_m, \theta_m] = \mu_a \quad (14)$$

$$E[R_t(r, \theta) | r_m, \theta_m] = R_a \quad (15)$$

Then the expected value of the target's true bias is considered with elements of its classical approximations and target's true covariance is considered with elements of the same, which are conditioned on the measured position. These are called as the target's average true bias and target's average true covariance. Expanding the expected bias and covariance using (1) and applying the trigonometric identities gives the mean (14) as [5]

$$\mu_a = \begin{bmatrix} r_m \cos \theta_m (e^{-\sigma_\theta^2} - e^{-\sigma_\theta^2/2}) \\ r_m \sin \theta_m (e^{-\sigma_\theta^2} - e^{-\sigma_\theta^2/2}) \end{bmatrix} \quad (16)$$

and the covariance as [5]:

$$\begin{aligned} R_a^{11} = & r_m^2 e^{-2\sigma_\theta^2} [\cos^2 \theta_m (\cosh 2\sigma_\theta^2 - \cosh \sigma_\theta^2) + \sin^2 \theta_m (\sinh 2\sigma_\theta^2 - \sinh \sigma_\theta^2)] \\ & + \sigma_r^2 e^{-2\sigma_\theta^2} [\cos^2 \theta_m (2 \cosh 2\sigma_\theta^2 - \cosh \sigma_\theta^2) \\ & + \sin^2 \theta_m (2 \sinh 2\sigma_\theta^2 - \sinh \sigma_\theta^2)] \end{aligned} \quad (17a)$$

$$\begin{aligned} R_a^{22} = & r_m^2 e^{-2\sigma_\theta^2} [\sin^2 \theta_m (\cosh 2\sigma_\theta^2 - \cosh \sigma_\theta^2) + \cos^2 \theta_m (\sinh 2\sigma_\theta^2 - \sinh \sigma_\theta^2)] \\ & + \sigma_r^2 e^{-2\sigma_\theta^2} [\sin^2 \theta_m (2 \cosh 2\sigma_\theta^2 - \cosh \sigma_\theta^2) \\ & + \cos^2 \theta_m (2 \sinh 2\sigma_\theta^2 - \sinh \sigma_\theta^2)] \end{aligned} \quad (17b)$$

$$R_a^{12} = \sin \theta_m \cos \theta_m e^{-4\sigma_\theta^2} [\sigma_r^2 + (r_m^2 + \sigma_r^2)(1 - e^{\sigma_\theta^2})] \quad (17c)$$

Note that the average covariance (17a, 17b, 17c) is larger compared to the covariance (11a, 11b, 11c), which is conditioned on the exact position; it gives the additional errors by evaluating it by the measured position [7]. This is difficult in showing the consistency later. The bias and increase in the covariance, is always significant for long ranges and also for large bearing errors. So the new polar-to-Cartesian conversion [8], is an unbiased consistent conversion [8], with the correction of the average bias which is taken, instead of (7), given by

$$z^c = \begin{bmatrix} x_m^c \\ y_m^c \end{bmatrix} = \begin{bmatrix} r_m \cos \theta_m \\ r_m \sin \theta_m \end{bmatrix} - \mu_a \quad (18)$$

where the elements of μ_a are taken from (16) and the average covariance of the converted measurements is R_a with elements (17a, 17b, 17c).

Similarly, one can get the bias and variance of the converted Doppler measurements as [8]

$$\mu_n(k) = \rho\sigma_r\sigma_{\dot{r}} \quad (19)$$

$$R^{\eta\eta}(k) = r_m^2(k)\sigma_{\dot{r}}^2 + \sigma_r^2\dot{r}_m^2(k) + 3(1 + \rho^2)\sigma_r^2\sigma_{\dot{r}}^2 + 2r_m(k)\dot{r}_m(k)\rho\sigma_r\sigma_{\dot{r}} \quad (20)$$

The debiased converted position measurements are given as

$$z_c^{\eta}(k) = \eta_c(k) - \mu^{\eta}(k) \quad (21)$$

The covariance between the converted position measurements and the converted Doppler measurements can be given as [8]

$$R^{p\eta}(k) = \begin{bmatrix} R^{x\eta}(k) \\ R^{y\eta}(k) \end{bmatrix} = \begin{bmatrix} [\sigma_r^2\dot{r}_m(k) + r_m(k)\rho\sigma_r\sigma_{\dot{r}}] \cos \theta_m(k)e^{-\sigma_0^2} \\ [\sigma_r^2\dot{r}_m(k) + r_m(k)\rho\sigma_r\sigma_{\dot{r}}] \sin \theta_m(k)e^{-\sigma_0^2} \end{bmatrix} \quad (22)$$

2.3 *Converted Doppler Kalman Filter*

Normally, nonlinear filtering methods like the EKF and UKF are taken to deal with Doppler measurements. Here, the nonlinear problem is solved by building a pseudo state vector, which is having a linear relationship with the already existing converted measurements of the target and by deriving the particular linear filtering equations. This shows the results of the CDMKF. In this particular section, the two commonly used target motion models are, one is nearly constant velocity (NCV) and the other one is nearly constant acceleration (NCA) models [2], are evaluated. Now, the pseudo state equations of derivatives are considered by taking the second order and third order derivatives are as zero [2].

$$\begin{bmatrix} \ddot{x}(k) \\ \ddot{y}(k) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (23)$$

for CV model and

$$\begin{bmatrix} \ddot{x}(k) \\ \ddot{y}(k) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (24)$$

for CA model.

The pseudo state vector of the dynamic system for a CV model and CA model is considered as [9, 10]

$$\eta(k) = \begin{bmatrix} \eta(k) \\ \dot{\eta}(k) \end{bmatrix} = c[X(k)] = \begin{bmatrix} x(k)\dot{x}(k) + y(k)\dot{y}(k) \\ \dot{x}^2(k) + \dot{y}^2(k) \end{bmatrix} \quad (25)$$

$$\eta(k) = \begin{bmatrix} \eta(k) \\ \dot{\eta}(k) \\ \ddot{\eta}(k) \\ \dddot{\eta}(k) \end{bmatrix} = c[X(k)] = \begin{bmatrix} x(k)\dot{x}(k) + y(k)\dot{y}(k) \\ \dot{x}^2(k) + \dot{y}^2(k) + x(k)\ddot{x}(k) + y(k)\ddot{y}(k) \\ 3\dot{x}(k)\dot{x}(k) + 3\dot{y}(k)\dot{y}(k) \\ 3\ddot{x}^2(k) + 3\ddot{y}^2(k) \end{bmatrix} \quad (26)$$

The derivatives of the NCV and the NCA in Cartesian coordinates are considered by using zero-mean white noise [2]. Then the NCV and NCA are expressed as follows

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \dot{x}(k+1) \\ \dot{y}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \\ \dot{x}(k) \\ \dot{y}(k) \end{bmatrix} + \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} v_x(k) \\ v_y(k) \end{bmatrix} \quad (27)$$

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \dot{x}(k+1) \\ \dot{y}(k+1) \\ \ddot{x}(k+1) \\ \ddot{y}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & T & 0 & T^2/2 & 0 \\ 0 & 1 & 0 & T & 0 & T^2/20 \\ 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 1 & 0 & T \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(k) \\ y(k) \\ \dot{x}(k) \\ \dot{y}(k) \\ \ddot{x}(k) \\ \ddot{y}(k) \end{bmatrix} + \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_x(k) \\ v_y(k) \end{bmatrix} \quad (28)$$

Now by considering the mean of the squared noise as a known input, the state equation can be written as

$$\eta(k+1) = \Phi_\eta \eta(k) + Gu(k) + \Gamma_x v_x(k) + \Gamma_s v_s(k) \quad (29)$$

where for NCV model

$$\begin{aligned} \Phi_\eta &= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad G = \Gamma_s = \begin{bmatrix} T^3/2 & 3T/2 \\ T^2 & T^2 \end{bmatrix}, \quad u(k) = E \left(\begin{bmatrix} v_x^2(k) \\ v_y^2(k) \end{bmatrix} \right) = \begin{bmatrix} q \\ q \end{bmatrix} \\ \Gamma_x(k) &= \begin{bmatrix} T & 3T^2/2 \\ 0 & 2T \end{bmatrix}, \quad v_x(k) = X_\Gamma v(k) = \begin{bmatrix} x(k) & y(k) \\ \dot{x}(k) & \dot{y}(k) \end{bmatrix} \begin{bmatrix} v_x(k) \\ v_y(k) \end{bmatrix} \\ v_s(k) &= \begin{bmatrix} v_x^2(k) - q \\ v_y^2(k) - q \end{bmatrix} \end{aligned}$$

for NCA model

$$\phi_\eta = \begin{bmatrix} 1 & T & T^2/2 & T^3/6 \\ 0 & 1 & T & T^2/2 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 3 \end{bmatrix}, \quad G = \Gamma_s = \begin{bmatrix} T^3/2 & T^3/2 \\ 3T^2/2 & 3T^2/2 \\ 3T & 3T \\ 3 & 3 \end{bmatrix}, \quad u(k) = E\left(\begin{bmatrix} v_x^2(k) \\ v_y^2(k) \end{bmatrix}\right) = \begin{bmatrix} q \\ q \end{bmatrix}$$

$$\Gamma_x = \begin{bmatrix} T & 3T^2/2 & T^3 \\ 1 & 3T & 3T^2 \\ 0 & 3 & 6T \\ 0 & 0 & 6 \end{bmatrix}, \quad v_x(k) = X_\Gamma v(k) = \begin{bmatrix} x(k) & y(k) \\ \dot{x}(k) & \dot{y}(k) \end{bmatrix} \begin{bmatrix} v_x(k) \\ v_y(k) \end{bmatrix},$$

$$v_s(k) = \begin{bmatrix} v_x^2(k) - q \\ v_y^2(k) - q \end{bmatrix}$$

3 Statically Fused Converted Measurement Kalman Filters

3.1 Filtering Structure

The CDMKF provides a new method to exploit Doppler measurements. But the resulting pseudo states from the CDMKF are quadratic [7, 8], not linear, in Cartesian states. Additional processing is needed to extract the final target states from the pseudo states. The Cartesian states can be provided by the CPMKF, which is used along with the CDMKF, leading to a new tracking filtering approach, the SF-CMKF [13].

Figure 1 illustrates the structure of the SF-CMKF. The original sensor measurements (i.e., range, Doppler, and angle) are divided into two parts to be processed separately by two linear filters first.

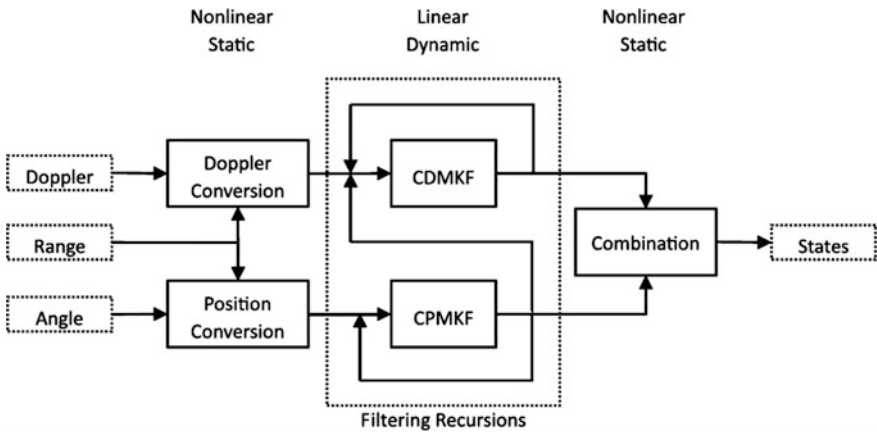


Fig. 1 Filtering of SF-CMKF

The prior mean of the state to be estimated is

$$\bar{x}(k+1) = E[x(k+1)/\hat{x}_p(k+1, k+1)] \quad (30)$$

Debiased converted measurement is

$$z(k+1) = \eta(k+1) - \tilde{\eta}(k+1, k+1) \quad (31)$$

The covariance between the states to be estimated and the measurement is

$$P_{XZ} = E[(x - \bar{x})(z - \bar{z})^T] \quad (32)$$

The covariance of the measurement is

$$P_{ZZ} = E[(z - \bar{z})(z - \bar{z})^T] \quad (33)$$

The static nonlinear estimation equation is obtained as

$$\hat{X} = \hat{x}_p + P_{xz}(P_{zz})^{-1}(\hat{\eta} - \bar{z}) \quad (34)$$

4 Simulation Results

Considering the target starting and moving with two trajectories which gives the effectiveness of the CDMKF and CPMKF methods in the forms nearly constant velocity trajectory and also nearly constant acceleration trajectory which are starting at (10, 10 km) and the target moves with a speed of 10 m/s heading to 60°. In the second scenario the acceleration is of 0.2 m²/2. The process noise of the target is assumed to be zero-mean white Gaussian noise with standard deviation 0.001 m/s². The sensor is located at origin (0, 0 km) and the sampling interval is T = 1 s. The standard deviations of target's range, azimuth, and Doppler measurements are taken as, $\sigma_r = 50$ m, $\sigma_\theta = 2.5^\circ$, $\sigma_{\dot{r}} = 0.1$ m/s. The correlation coefficient of the target between range and bearing is taken as $\rho = 0$. Simulations are performed here, over 200 time steps with the 50 Monte Carlo experiments.

The motion of the targets and the root mean squared (RMS) errors of the following methods are considered and are shown in Figs. 2, 3, 4, and 5. The results are shown with Root Mean Square (RMS) error for the NCV and NCA trajectories, respectively. The effectiveness of the SF-CMKF as tracking filter is illustrated by comparing the performance of this method with that of the sequential nonlinear filtering method based on the sequential extended Kalman filter (SEKF) and the sequential filtering approach with the sequential unscented Kalman filter (UKF). The three tracking filters are having approximately the same RMSEs.

Fig. 2 RMS position error in NCV scenario

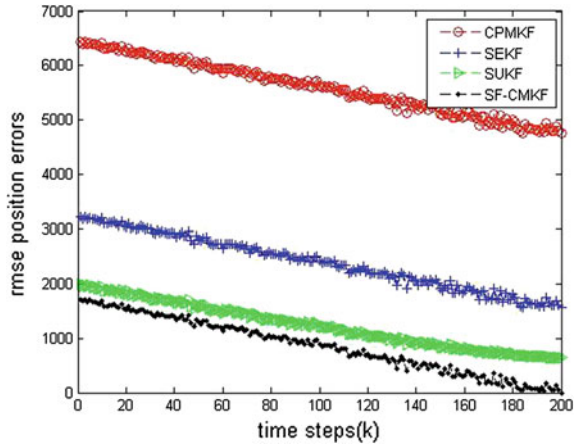


Fig. 3 RMS velocity error in NCV scenario

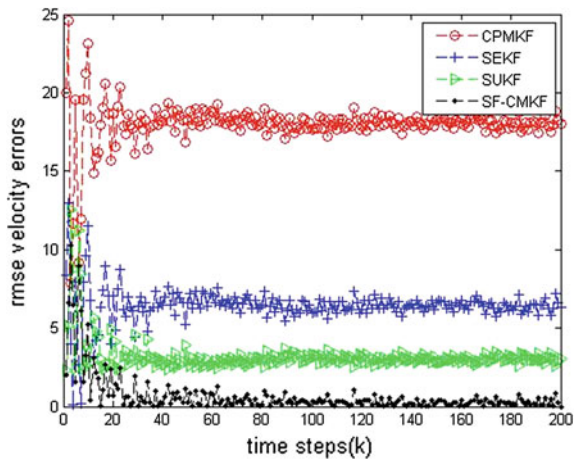


Fig. 4 RMS position error in NCA scenario

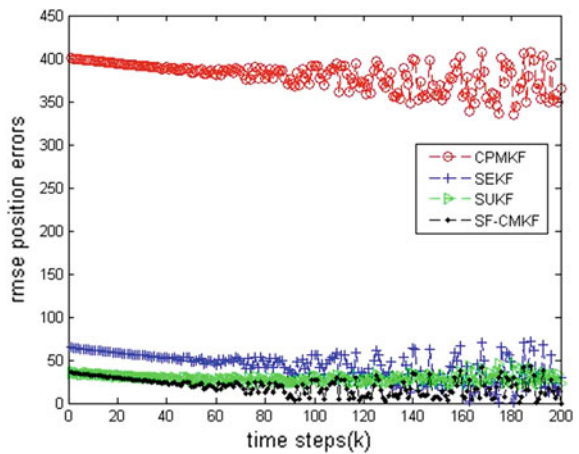
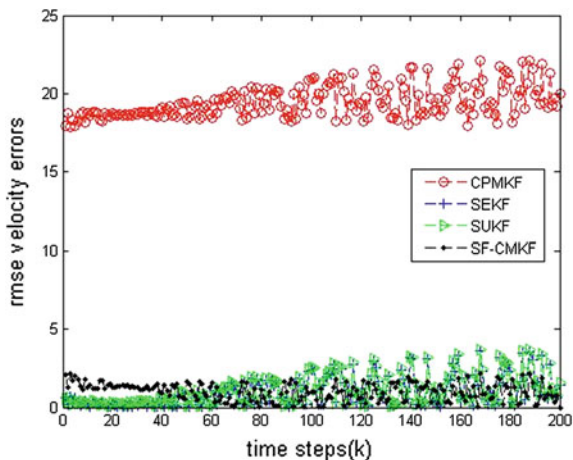


Fig. 5 RMS velocity error in NCA scenario



5 Conclusion

In this paper, the use of nonlinear recursive filtering approaches is avoided while processing the Doppler measurements. A linear filter, the converted Doppler measurement Kalman filter (CDMKF), is proposed to estimate the pseudo states and filter the noise in the converted Doppler measurements.

CDMKF can be used to operate along with the CPMKF to construct a new state estimator, statically fused converted measurement Kalman filter (SF-CMKF). Cartesian state and pseudo state estimates are produced by CPMKF and CDMKF, respectively, and are then combined by a static estimator to obtain final state estimates. The non-linearity of the pseudo states is quadratic and is handled by expanding the pseudo states up to the second term around the estimated states of the CPMKF.

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