Application of Compressed Sensing in Cognitive Radio

Naveen Kumar and Neetu Sood

Abstract In the last few years, compressed sensing (CS) has been well used in the area of signal processing and image compression. Recently, CS has been earning a great interest in the area of wireless communication systems. CS exploits the sparsity of the signal processed for digital acquisition to reduce the number of measurement, which leads to reductions in the size, power consumption, processing time, and processing cost. This paper presents application of CS in cognitive radio (CR) networks for spectrum sensing and channel estimation. The effectiveness of the proposed CS-based scheme is demonstrated through comparisons with the existing conventional spectrum sensing and channel estimation methods.

Keywords Compressed sampling · Cognitive radio · Spectrum sensing · Channel estimation · Noncontiguous orthogonal frequency division multiplexing

1 Introduction

According to Nyquist's sampling theorem, a continuous-time band-limited signal $x(t)$ with bandwidth $B > 0$ can be exactly recovered from twice as many samples per second as the highest frequency present in the signal, i.e., $2B$ also known as the Nyquist rate [1]. However, around 2004, Donoho [2] proved that at given knowledge about sparsity of a signal, the signal may be reconstructed back with even fewer samples than required by Nyquist's sampling theorem, also known as compressed sensing (CS).

N. Kumar (✉) · N. Sood
Department of Electronics & Communication, Dr. B. R. Ambedkar NIT Jalandhar, Jalandhar, India
e-mail: naaveen@live.com

N. Sood
e-mail: soodn@nitj.ac.in

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To implement wideband spectrum sensing, cognitive radio (CR) needs fast analog-to-digital converter (ADC) but the achievable sampling rate of ADC is only 3.6 Gsps. Capitalizing on the wideband signal spectrum sparseness, CS technique can be employed in spectrum sensing in CR network [3]. Tian and Giannakis [4] firstly applied CS theory to wideband CR networks for acquiring spectrum at sub-Nyquist sampling rates. In most scenarios in CR networks, the number of used channels is comparatively much lesser than total channels; those are vacant at a particular time and space. Therefore, when dealing with channel estimation problem in CR system where the channel band is really wide and dynamic and occupation information of the channels is compressible, CS can be exploited, since CS does not require any knowledge of the underlying multipath channel, based on the fact that a sparse structure is exhibited by the physical multipath channels in angle delay Doppler spreading, especially at large signal space dimensions, it is advantageous to utilize sparse channel estimation method based on convex/linear programming, which can be proved to outperform the existing least square-based methods [5]. Jia et al. [6] presented channel estimation algorithm for OFDM-CR, based on OMP and applied sparsity adaptive matching pursuit (SAMP) algorithm for the first time for channel estimation in NC-OFDM systems. Moreover, for the reconstruction time-consuming of SAMP algorithm was too large, modified adaptive matching pursuit (MAMP) algorithm was introduced as an improved SAMP algorithm. Qi et al. [7] introduced sparse channel estimation (SCE) scheme in OFDM-CR, where pilot design was formulated as an optimal column selection problem and constrained cross entropy optimization-based scheme was proposed to obtain an optimized pilot pattern.

The remainder of this paper is structured as follows Sect. 2 presents CR system model for spectrum sensing and channel estimation. In Sect. 3, CS-based spectrum sensing and channel estimation scheme is proposed. Section 4 demonstrates and summarizes the performance advantages of proposed CS-based scheme over traditional energy detection spectrum sensing and maximum likelihood ratio-based channel estimation techniques. Section 5 concludes the paper.

2 System Model and Problem Statement

Suppose that CR system aims to find spectral holes (SHs) in the frequency range of 0 to W Hertz, as shown in Fig. 1. During the spectrum sensing interval, all CR nodes keep quiet. Thus, the continuous signal received at the receiver of CR network, i.e., $x(t)$, is composed of primary users' (PUs) signals and additive white Gaussian noise (AWGN).

Mathematically, using sub-Nyquist sampling rate f_S ($f_S < 2W$), the compressed samples y ($y \in C^{M \times 1}$, $M = \tau f_S \ll N$) can be written as

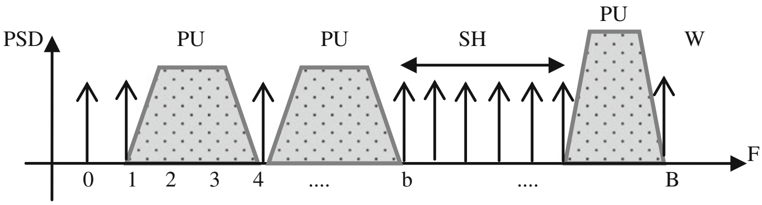

Fig. 1 Frequency frame of wideband cognitive radio

$$y = \varphi x = \varphi \psi_S \quad (1)$$

where φ is an $m \times N$ sensing matrix, ψ_S is sparsifying matrix and y is the measurement vector of m measurements. Let $\mu(\varphi, \psi)$ be the coherence between φ and ψ, $S = k/N$ be the sparsity measure, then we can choose minimum number of measurements required for reconstruction of x from y

$$m \geq C_0 \cdot \mu^2(\varphi, \psi) \cdot S \cdot \log N \quad (2)$$

for a Gaussian measurement matrix m, where C_0 is a constant. An estimate can be obtained by solving the CS reconstruction problem [8]:

$$\hat{x} = \arg\min||x||_1 \quad \text{s.t. } y = \varphi\, \psi\, x \quad (3)$$

After spectrum reconstruction secondary users (SUs) sense the recovered channel spectrum in order to identify frequency holes. If K symbol periods are allocated for channel sensing, the problem can be described as the hypothesis testing problem, mathematically:

$$\mathcal{H}_0 : z_i = n_i \quad i = 1, 2, \ldots, K \quad (4)$$

$$\mathcal{H}_1 : z_i = x_i + n_i, \quad i = 1, 2, \ldots, K \quad (5)$$

After spectrum sensing, CR adopts noncontiguous orthogonal frequency division multiplexing (NC-OFDM) technique that decomposes wideband into orthogonal sub-channels. The sub-channels are activated when the spectrum is idle and when it is not available corresponding sub-channels are deactivated.

For S number of nonzero elements, the vector is S-sparse, discrete Fourier transform (DFT) size is N, active sub-carriers are M and pilot sub-carriers (c_p) are K ($K \leq M$). The CP length is greater than the maximum possible path delay (Fig. 2).

OFDM symbol data $X(n)$ contains mapping signals and pilot signals. After removing CP, discrete Fourier transform (DFT) is applied to the received time-domain signal y_n for $n \in [0, N-1]$ to obtain $k \in [0, N-1]$. The discrete-time channel model is:

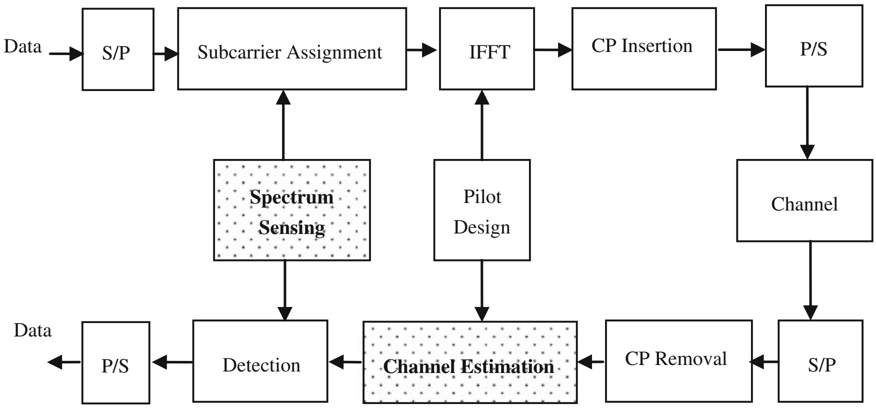

Fig. 2 NC-OFDM-based CR system

$$h(n) = \sum_{l=0}^{l-1} h_l \delta(n-1) \tag{6}$$

where the impulse response vector of the channel $h = [h_0, h_1, \ldots, h_{l-1}]^{\mathrm{T}}$ remains unchanged in multiple OFDM symbol period of time reflects the slow time variation of the channel. The relation between the transmitted pilots and received pilots can be written as:

$$\begin{bmatrix} y(c_{p_1}) \\ y(c_{p_2}) \\ \vdots \\ y(C_{pk}) \end{bmatrix} = \begin{bmatrix} x(c_{p_1}) & 0 & \ldots & 0 \\ 0 & \ddots & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \ldots & x(c_{pk}) \end{bmatrix} \cdot F_{K\times L} \cdot \begin{bmatrix} h(1) \\ h(2) \\ \vdots \\ h(L) \end{bmatrix} + \begin{bmatrix} Z(1) \\ Z(2) \\ \vdots \\ Z(K) \end{bmatrix} \tag{7}$$

where Z is additive white Gaussian noise (AWGN) and is $F_{K\times L}$ is a DFT sub matrix given by:

$$F_{K\times L} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & w^{c_{p1}} & \ldots & w^{c_{p1}\cdot(L-1)} \\ 1 & \ddots & & w^{c_{p2}\cdot(L-1)} \\ \vdots & & \ddots & \vdots \\ 1 & w^{c_{pK}} & \ldots & w^{c_{pK}(L-1)} \end{bmatrix} \tag{8}$$

where $w^{nl} = \mathrm{e}^{-j\frac{2\pi nl}{N}}$. Let $A = XF_{K\times L}$, then (7) can be written as:

$$y = Ah + Z \tag{9}$$

Since the channel delay spread is much larger than sampling period [9], particularly for OFDM systems with over sampling, most components of h are either zero or nearly zero, which implies that h is sparse. With this a priori condition, CS theory can be applied to estimate h.

3 Compressed Spectrum Sensing Algorithm

Consider a discrete-time signal $x \in \mathcal{P}^N$, which can be expressed as $X(n)$ where $n = 1, 2, \ldots, N$. The claim of compressive sensing is that from m ($m \ll n$) measurements, we can reconstruct the original signal x with nonadaptive linear measurements. This does not violate the Shannon–Nyquist sampling theorem as reconstruction of only sparse signals is possible. According to Eq. (1):

$$y = \varphi \hat{x} = \varphi \psi_s = \varphi \psi_s F^{-1} \hat{X} \tag{10}$$

where $F^{-1}\hat{X}$ is the inverse Fourier transform of $\hat{x}$. Using Eq. (10) problem reconstruction of $\hat{x}$ can be converted into the problem of reconstruction of $\hat{X}$:

$$\hat{x} = \arg_x \min ||\hat{X}||_0 \quad \text{s.t. } y = \left(\varphi F^{-1}\right)\hat{X} \tag{11}$$

Basis pursuit (BP) [10] can be used for signal reconstruction, which transforms the sparseness constraint on into a convex optimization problem solvable by linear programming:

$$\hat{x} = \arg_x \min ||\hat{X}||_1 \quad \text{s.t. } y = \left(\varphi F^{-1}\right)\hat{X} \tag{12}$$

To deal with the signals with noise components, some variants of LASSO algorithm can be developed by minimizing the usual sum of squared errors:

$$\hat{x} = \arg \min ||\hat{X}||_0 \quad \text{s.t. } ||\, \varphi F^{-1}\hat{X} - y||_2 \leq \varepsilon \tag{13}$$

where ε is recovery error threshold. The problem can be solved with a two-step scheme: first, use compressed measurements y to estimate the sparse sequence and second, reconstruct signal $\hat{x}$ according to ψ_s.

Orthogonal matching pursuit (OMP) algorithm [11–13] suggests the reconstruction under the conditions of a given iteration number, as the iterative process is forced to stop, OMP algorithm needs a lot of linear measurement to ensure accurate reconstruction. The basic idea of the OMP algorithm is to select the columns of measurement matrix with greedy iterative algorithm, make sure the correlative value between the columns selected in each iteration and the current redundant vector is maximum, and then subtract the correlative value from the sampling vector and repeat iteration until the number of iterations achieves the sparse degree s. OMP algorithm selects an atom in each iteration to update the atom collection, which will certainly pay a large time for reconstruction. The number of iterations is closely related to sparse degree S and the number of samples m, with their increase, time consumption will also increase significantly.

Problem with OMP algorithm is that it is not adaptive, pre-estimate of the sparse degree of the sparse signal is needed, and the reconstruction accuracy is not satisfactory. In reality, the sparse degree of the sparse channel is usually unknown. Sahoo et al. [14] proposed extended OMP-CS algorithm in order to improve the accuracy of reconstruction and make the algorithm adaptive. In the ExtOMP-CS algorithm, one key issue is how to choose the step size. Unlike the OMP-CS algorithm, the iteration times of ExtOMP-CS algorithm is not certain and is related to step size, and computational complexity and computational time are higher in the ExtOMP-CS algorithm than OMP-CS algorithm.

An extension to OMP algorithms is the compressed sampling matching pursuit (CoSaMP) algorithm [15]. The basis of the algorithm is OMP but CoSaMP can be shown to have tighter bounds on its convergence and performance [16].

Algorithm 1 Compressed Sampling Matching Pursuit Algorithm

1. Input: S, y, ϕ
2. $x(0) \leftarrow 0$
3. $v \leftarrow y$
4. $k \leftarrow 0$
5. while Halting condition false do
6. $k \leftarrow k + 1$
7. $z \leftarrow \phi^T v$:signal proxy
8. $\Omega \leftarrow z^{2S}$:find the largest 2S components of the signal proxy (*Identification*)
9. $\Gamma \leftarrow \Omega$ supp $(x^{(k-1)})$:merge the support of the signal proxy with the support of the solution from the previous iteration (*Support Merge*)
10. $\bar{x} \leftarrow \arg\min ||(\phi x - y)||_2$:estimate a solution via least squares with the constraint that the solution lies on a particular support (*Estimation*)
11. $x^k \leftarrow x^s$:takes the solution estimate and compresses it to the required support (*Pruning*)
12. $v \leftarrow y - \phi x$: update the sample, namely the residual in F-space (*Sample Update*)
13. end while
14. $\hat{x} \leftarrow x(k)$
15. Output $\hat{x}$:such that it is S-sparse and $y = \phi x$

For any $2S$-sparse channel vector x, CoSaMP algorithm produces the channel estimator $\hat{x}$ that satisfies

$$||x-\hat{x}||_2 \leq C \;\; \max\{\varepsilon, 1/\sqrt{s}||x-\hat{x}_{2S}||_1 + ||z||_2\} \tag{14}$$

for a given parameter ε, and x_{2S} is a best $2S$-sparse approximation to x.

Having estimated $\hat{x}$, SU finds the presence of PUs in a certain sub-band using energy detector. Let the energy $E_\mathrm{p} = \sum_{i\,\varepsilon\,\mathrm{sub-channel}\,m} \hat{x}_i$ received in sub-channel M. The spectrum availability is decided by:

$$\mathcal{H}(n) = \begin{cases} \mathcal{H}_0 & Z_i \geq \lambda \\ \mathcal{H}_1 & Z_i \leq \lambda \end{cases} \tag{15}$$

The threshold λ is a decision threshold and is a design parameter for the CR receiver system. P_d probability of detection and P_f probability of false alarm are two probabilities used for performance evaluation of the scheme.

$$P_\mathrm{f} = P_\mathrm{r}\{Z \geq \lambda|H_0\} \tag{16}$$

$$P_\mathrm{d} = P_\mathrm{r}\{Z < \lambda|H_1\} \tag{17}$$

After deactivating, the active sub carriers random pilots are assigned. As wireless channels are rapidly decaying, the channel response h is highly sparse because of the small number of significant multipath components. A sparse high-resolution signal h can be recovered with high probability with a constraint from the measurements y. The corresponding model can be written as

$$\min_{h\,\in\,c^N} ||h||_0 \quad \text{s.t. } y = Ah \tag{18}$$

Problem (18) is NP-hard problem and even for moderate N, it is not possible to solve. ℓ_1 relaxation model with the same constraints can be used as an alternative.

$$\min_{h\,\in\,c^N} ||h||_1 \quad \text{s.t. } y = Ah \tag{19}$$

The reconstruction can never be exact due to the noise present in the measurements. Using a final de-noising step based on least square problem, noise can be eliminated.

4 Performance Analysis and Simulation Result

In NC-OFDM, the power spectral density (PSD) of P_{th} sub-carrier signal is characterized of the form

$$\Gamma_k(f) = K \cdot \mathrm{sinc}^2((f - f_{\mathrm{P}})T_{\mathrm{S}}) \tag{20}$$

where K is the signal level, fk is the sub-carrier center frequency, T_{S} is the OFDM symbol duration and TG is guard interval. Assuming independent symbols in different sub-carriers, the PSD of an NC-OFDM signal is obtained as

$$\Gamma(f) = \sum_{P} \Gamma_P(f) \tag{21}$$

where index P is the number of active subcarriers. In this paper, a wideband spectrum of 0–100 MHz is considered with total six sub-bands (B_1–B_6). Among these sub-bands, B_1, B_3, and B_5 located at 1–10 MHz, 20–35 MHz, 70–75 MHz, have relatively high PSD in the range of 0.0277–0.1126, as shown in Fig. 3 by level 16–25. For CS, the compression ratio is set to 75 % and the noise level is 8 dB.

4.1 Recovered PSD

Using algorithm 1 based the CS scheme, wideband spectrum can be successfully reconstructed back at sub-Nyquist rate, as shown in Fig. 4.

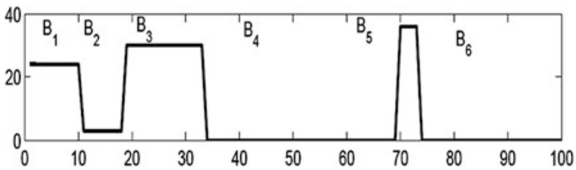

Fig. 3 Wideband spectrum $\hat{X}$

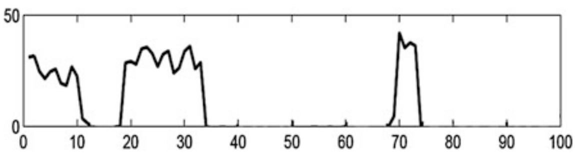

Fig. 4 Reconstructed spectrum

4.2 Probability of Detection Performance

The performance of the conventional generalized likelihood ratio test scheme and CS-based spectrum sensing scheme is evaluated and compared via the probability of detection P_d for a constant false alarm rate of $P_f = 0.08$ (Fig. 5).

4.3 BER Performance

An OFDM-based CR system is considered with $M = 1024$ subcarriers, after spectrum sensing without any false alarm or missing detection and deactivating

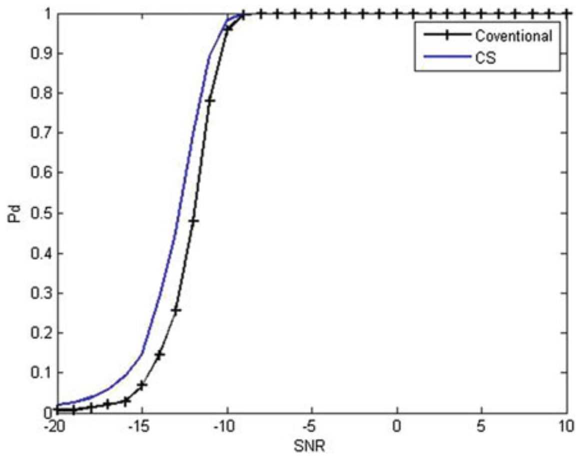

Fig. 5 Comparison of probability of detection performances

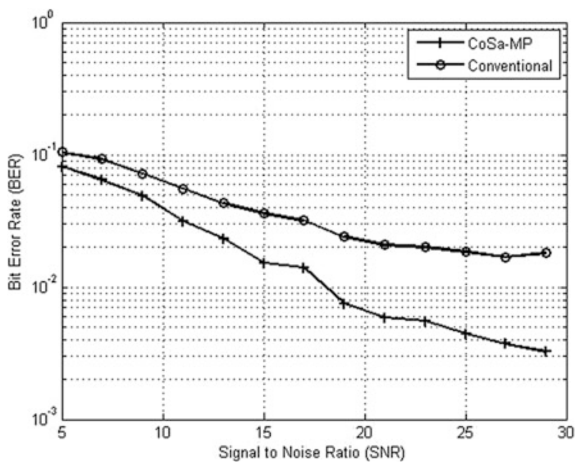

Fig. 6 Comparison of BER performances

those subcarriers occupied by PUs, there are 512 remaining OFDM subcarriers for SUs, including three noncontiguous subcarrier blocks, i.e., {1, 2, …, 256}, {513, 514, …, 640} and {897, 898, …, 1024}, with the number of subcarriers in each block being 256, 128, and 128, respectively. A sparse multipath channel h is considered with $L = 60$ taps where 5 nonzero taps are placed randomly. The channel estimation performance is evaluated now using the designed pilot patterns.

Figure 6 shows BER performance of two schemes—proposed CS channel estimation and channel estimation scheme based on LS. Improved BER performance of the proposed CS-based scheme can be seen from the above figure, over the conventional LS-based scheme.

5 Conclusion

CS is a very promising technique in wireless communication networks. However, the studies on the applications of CS are just in fewer areas. Even in these areas, a lot of problems are still not been fully settled, limiting the performance of CS. In CR network systems, if the number of channels, is not large enough, the requirement of sparsity cannot be guaranteed, which limits the advantages of CS. In this paper, the application of CS is demonstrated in CR networks and based on the advantages of the proposed scheme, the problem of designing a high-performance CR receiver indicates that the approach should work both for spectrum sensing and channel estimation.

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